Changes in Microstructural Parameters of Snow During Deformation

FINAL REPORT

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This study utilized quantitative and morphology theory to describe the microstructure of cohesive granular materials and to relate microstructure to mechanical and physical properties. Snow was used as the experimental material. A unique feature of the theory developed in this grant involved the description of the necked areas which are the intergranular restricted regions which tie the grains together to form a bonded granular material. Using a set of criteria to specify what would constitute a neck, the formulation was developed to automatically identify the necks and determine their geometry as they appeared in surface sections. For the three-dimensional properties, the necks were modeled as double truncated cones in order to provide an accurate description of the neck geometry and volume. Stereology theory was then used to calculate the three dimensional properties of the necks based on this model. In addition, other features such as pore size, grain size, coordination number and density were found. The final computer code consisted of approximately 150,000 lines of code and represents a very effective and automated means of describing cohesive granular materials. Finally an elastic-viscoplastic constitutive relation utilizing microstructure of the material was developed and validated against experimental data.
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1.0 PROBLEM STATEMENT

This project was undertaken to investigate the microstructure of snow, i.e., the small features such as grain size, intergranular bonding, grain shape, etc. Snow is a granular material consisting of an assemblage of ice grains bonded together to form a porous solid material. When initially deposited on the ground the snow particles make contact with each other through minute points of contact. These contacts between grains slowly grow into necks of a finite length due to a number of physical processes, including volume diffusion, surface diffusion and vapor diffusion to transport mass from the grain bodies to the bonds. An additional mechanism which often is of major importance is sintering, in which mechanically applied loads force the grains together and thereby induce plastic flow in the bonds. All of these processes result with the formation of necks, regions of finite length which connect the particles and hold them together. It is generally accepted that mechanical sintering produces intergranular bonding which is different than that produced by the other processes, although no comprehensive data is available to support that contention.

The characteristics of these necks are the dominant factors which determine material properties such as strength, elastic moduli and viscosity. Important microstructural parameters include grain diameter $d_g$, bond diameter $d_b$, neck length $l_n$, pore diameter $d_p$, free surface area per unit volume $S_v$, and the number of bonds per grain (coordination number, $N_3$). The necks are the weak links that hold the material together and where most of the material deformation takes place. Stresses in the neck may be an order of magnitude larger than in the grain bodies. Since strain rate in ice varies approximately with the square or cube of the stress, actual strain rates in the necks may be 100 to 1000 times as large as those in the grain bodies, depending on the neck dimensions. In addition to mechanical properties, many physical properties are also dependent upon the microstructure. This includes thermal conductivity and diffusivity.

The primary purpose of this grant was the formulation of a stereology theory for cohesive, granular materials and the development of a computer software package which utilizes advanced concepts in quantitative stereology theory and morphology theory to determine the parameters $d_g$, $d_b$, $l_n$, $S_v$, $N_3$ and $d_p$. In order to do this, a number of advancements in stereology theory and the implementation of them would have had to be developed. With such advancements, these theories would allow one to utilize two-dimensional information such as seen in a surface section (Figure 2(b)) to determine many of the three-dimensional properties of the material granular structure.

A second objective of this project was the development of a constitutive theory utilizing microstructural processes to determine how the microstructure ($d_g$, $d_b$, $l_n$, $S_v$, $N_3$ and $d_p$) of snow changes during deformation. As snow is compressed, the grains are forced together. At very low deformation rates, this can be accomplished without integrgranular bond fracture. The necks are plastically deformed in both shear and compression, thereby altering the size and geometry of the necks connecting the particles. Anisotropic behavior can result for most deformations if allowed to continue long enough. As this takes place, the actual properties continue to change. To date there has been no careful evaluation of the relationship between the microstructure and the mechanical and physical properties.
At higher deformation rates, intergranular fracture with resultant intergranular slip may occur. When this happens, substantial differences in mechanical properties may result, since new deformation mechanisms are now present and in fact may dominate. In addition, it is possible that the evolving microstructure may be substantially different than for the very low strain rates which due not cause significant bond fracture.

In summary, the primary purpose of this study was the development of an advanced stereology theory which could accurately measure those microstructural parameters which determine properties in cohesive granular materials. If this could be done, than the ability to answer question about the relationship between microstructure and properties could be addressed.

2.0 RELEVANCE TO ARMY MISSION

Snow plays an important part in military operations in alpine and polar regions. Quite often the military has to perform operations on snow, be it in deep snow, snow over wet soils, or on specially prepared snow roads and snow landing strips. Central to such operations is the material strength, its durability, and how factors such as temperature affect the ability of snow to withstand repeated vehicle loads. A better understanding of the relationship between the material microstructure and its mechanical properties may help provide answers to such questions. For snow roads and landing strips, snow must be processed (disaggregated, mixed with a binding material, and then laid down, vibrated and compacted) in order to make it suitable as a roadway or as a landing strip for wheeled aircraft. A better understanding of the relationship between microstructure and properties of processed snow would help determine effective means of maximizing strength.

Snow has also been found to rapidly attenuate shock waves from explosives and to very effectively impede rifle fire and other military projectiles. It has also been determined that the Army’s fuses for artillery rounds, while they work in soils, mud, and water, fail to detonate in snow or at least give delayed detonations, thereby diminishing their effectiveness. The findings of this study may help provide a more precise understanding of the properties of snow which reduce fuse effectiveness.

The Antarctic ice pack, which is largely responsible for controlling the earth’s climate, is insulated from direct interaction with the atmosphere by a 100-meter layer of snow, referred to as firn. How this firn layer responds to global warming is a question that is of concern to scientists around the world. The ability to accurately measure and monitor snow microstructure and how it changes with time will help provide answers about how firn will respond to future global warming.

Finally, the capability that is being developed in this program has direct application to the use of powdered metal compacts. These are high-strength materials that are formed by compacting powdered metals at elevated temperatures to form a bonded granular material with a high strength-to-weight ratio. The capability developed in this grant has direct applicability to such areas of study.
3.0 SUMMARY OF FINDINGS

3.1 Development of Image Analysis System

Snow is considered here to be a three phase material. The phases consisted of an air phase, ice grain phase, and neck phase. The air phase is the pore space, and the ice grain phase is taken to be the individual ice grains, excluding the inter-grain bonding regions. The neck phase is therefore the inter-grain bonding regions. The inter-grain bonding region was taken to be the constricted region joining two ice grains and lying between the points at which the curvature changes from convex to concave with respect to the outward surface normal as depicted by Figure 1.

The location of a bond is determined by locating the disk of minimum diameter with center on the skeleton and lying between two connected grains, as indicated in Figure 1.

![Figure 1. Three dimensional view of two bonded grains. The bond between the grains is the cross hatched area and is shown projected onto an intersecting plane.](image)

One of the essential features of the stereology analysis involves the construction of a skeleton structure to represent the surface section. The skeleton consists of a series of line segments which contain the radii of all the maximal disks contained in G, the set of all points in a surface section representing the sum of the ice grains and necks. This sequence of line segments form a center line of the bonded grains. Each point x of the skeleton contains the value of a disk of radius r(x) representing the distance to the nearest grain surface or neck surface. The skeleton therefore contains the information needed to reconstruct the microstructure of the material as seen in the surface section. A surface section and its corresponding skeleton is shown in Figure 2.
Figure 2. Figure 2(a) is the skeleton of 2(b) prior to segmenting. In 2(b) bonds are in gray.

Development of the software to construct the skeleton structure required a very significant development and programming effort, since the software had to be very robust in order to form a representative skeleton structure when complicated and highly variable grain sizes and grain shapes were involved. However, the skeleton structure allows one to record the important features of the surface section while at the same time requiring a minimum of computer memory. The skeleton can be used to reconstruct the original surface section.

Once the skeleton structure is formed, the intergranular bonds can be found within the surface section with information obtained from the skeleton structure (Figure 2(a)). A set of criteria must be satisfied in order to qualify a point on the skeleton as a bond. Then, using the mathematical variation of the distance r(x) on either side of the bond, the necks can be defined as they exist in the surface section. While it is difficult to see with the degree of reduction necessary to include both parts (a) and (b) in Figure 2, the figure shows the necks as the gray areas.

Once the two-dimensional information has been extracted for the surface section by means of the skeleton structure, morphology theory and stereology theory are used to calculate the three-dimensional variables. In order to do this, a geometrical model is needed to define the necks. A model of relatively simple geometry but which would produce a good degree of resolution in defining the neck geometry and volume was chosen. The necks are represented as two truncated cones as shown in Figure 3. In this figure, half of the necked region is shown, and the truncated cone is shown overlaid over the actual neck. Criteria for constructing the neck to fit the actual neck dimensions for a wide variety of grain and neck geometries also required the development of a very robust software package. Such parameters as density, mean grain size, bond diameter, and mean pore size are
readily calculated, but determination of mean size of necks based on the double truncated cone geometry and coordination number also required considerable development.

\[ D(n) \]

Figure 3. Two- and three-dimensional neck configurations. The dotted lines forming a trapezoid are the cross section of a neck as seen on a surface section. It is generated by determining the corresponding corner points lying on the surface boundary.

In all, the computer software comprises approximately 150,000 lines of computer code. However, this system can quickly and accurately calculate the three-dimensional parameters which describe the material microstructure. The computer program is essentially completed and is now being finalized under an EPSCoR program through ARO funding.

### 3.2 Formulation of a Constitutive Theory

One of the objectives was the development of a constitutive relation that would utilize deformation processes which occur at the microstructural level and therefore make use of the material microstructure. A formulation was developed, and this work has demonstrated a significant improvement over earlier models. In this formulation, the microstructure of the material is described in terms of a typical ice grain. A distribution function, \( D(n) \), describes the probability of contact with other neighboring grains in terms of the orientation of unit vector, \( n \), on the grain's surface. Figure 4 shows a typical grain with an oriented neck that contacts another grain. By use of a variational principle, one can calculate the stresses applied to the necks distributed around an ice grain, given the globally applied stresses. The deformation of the necks can then be calculated with the appropriate use of a constitutive relation for ice. This formulation allows for bond fracture with ensuing intergranular glide if the neck stresses become sufficiently large. Therefore, neck sintering, bond fracture and intergranular slip are all included in the description of the material response.

It should be noted that the geometry of the necked region is that of a cylinder as opposed to a double truncated cone as was assumed in the development of the image analysis system. Future work is needed to continue to improve the constitutive model to more accurately approximate the actual material microstructure.
Figure 4. Schematic of a typical ice grain shown with a model if a neck with its centerline having the orientation given by the vector $\mathbf{n}$.

Figure 5. Strain vs. time for uniaxial tensile stress of 0.012 MPa at -10°C.

Figure 5 shows typical results for the comparison of theory with results. Generally, this equation gave very good comparisons with data. This theory was compared with data for compression, tension, shear, and hydrostatic loads with generally good results. Finally, it was used to model settlement of a building foundation into a snow cover. While the results of this analysis were successful, the constitutive relation was found to be quite complicated and computationally difficult to use.
4.0 PUBLICATIONS RESULTING FROM GRANT


"Non-equilibrium thermodynamics applied to metamorphism of snow", Proceedings of the International Conference on snow & its Ramifications, Manali, India, September, 1994 (with M. Barber and E. Adams).


5.0 LIST OF DEGREES GRANTED


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