This report addresses the functional behavior of Cellular Neural Networks (CNN). The impact of variable convergence times on the proper operation of the network is discussed. A test method is presented to determine the functionality of the network. The functional fault models assume that the cells are unable to switch between limiting states. The proposed method attains 100% stuck-at failure coverage without any extra hardware for its implementation. Moreover, the required number of test vectors is constant and independent of the array size which makes it suitable for practical implementations. The report discusses the new fault model, presents the algorithmic procedures and shows simulated testing results.
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Hardware Implementation of a Desktop Supercomputer for High Performance Image Processing

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Behavioral Testing of Cellular Neural Networks

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1. Introduction

The converged final state of a Cellular Neural Network relies on the interaction between cells as determined by the templates. Some applications depend on the output value of cells to determine the final state of the Network. This paper will look at the effect of variances in the convergence rate of cells and its impact on the final state of the Network.

The testing of CNNs has been scarcely addressed. The only existing approach has limited applications such as only orthogonal interaction with neighboring cells is tested and it requires additional hardware to implement [1]. The test described in this article overcomes the previous limitations and achieves 100% fault detection. Using the concept of C-Testability, it is possible to determine the functionality of a processing array by applying a constant number of predetermined vectors independent of the array size and then comparing the actual output values to the predicted output values [2]. Under C-Testability, the input is propagated through the network to arrive at a final output state. If the actual final state is the one predicted by the given input vector, then the network is determined to be operating properly. However, if a given cell is faulty, its faulty state value will also be propagated and the fault will appear at the output.

2. Background

A CNN is an analog cellular nonlinear dynamic processor array. The basic circuit unit is called the cell [3,4]. It contains linear and nonlinear circuit elements. Any cell, \( C(i,j) \), is connected only to its neighbor cells, i.e. adjacent cells interact directly with each other, see Fig. 1. This intuitive concept is called neighborhood and is denoted as \( N(i,j) \).

Cells not in the immediate neighborhood have an indirect effect because of the propagation effects of the dynamics of the network. Each cell has state \( x \), input \( u \), and output \( y \). The state of each cell is bounded for all time \( t > 0 \) and, after the transient has settled down, a cellular neural network always approaches one of its stable points. The dynamics of a cellular neural network have both output feedback (A) and input control (B) mechanisms. The first order nonlinear differential equation defining the dynamics of a cellular neural network is shown by (1).

\[
\frac{dx_{ij}(t)}{dt} = -\frac{1}{R}x_{ij}(t) + \sum_{C(k,l) \in N(i,j)} A(i,j;k,l) y_{kj}(t) + \sum_{C(k,l) \in N(i,j)} B(i,j;k,l) u_{kl} \tag{1}
\]

\[
y_{ij}(t) = \frac{1}{2}(|x_{ij}(t) + 1| - |x_{ij}(t) - 1|)
\]

![](image)

**Fig. 1** Cellular Neural Network 4 x 4 processing array with border cell inputs
A set of inputs is necessary to simulate interaction with imaginary cells outside the processing array to ensure that the cells on the perimeter of the processing array achieve proper convergence. These imaginary cells are called border cells and form a ring around the processing array. A border cell, such as border cell C(0,0) of Fig. 1, outputs a constant voltage to mimic the output voltage \( V \) that a properly converged cell would produce as well as a voltage \( V_u \) to represent the input image voltage that border cell C(0,0) would receive if it were a functioning cell. Therefore, a cell on the perimeter of the processing array uses the image voltage and dynamic output voltage of neighboring cells as well as the static output and image voltages of the border cells to arrive at the proper final state. The border cells are treated as members of the array for initialization purposes and template implementation, but are not considered in the final state analysis.

3. Convergence Variance

Cell convergence is achieved when \( \frac{dx(t)}{dt} = 0 \). The rate of convergence is determined by many factors. The amount of current that is flowing into the cell, as governed by the templates, determines if the cell converges at its maximum rate \( \tau \). \( \tau \) is defined as the amount of time it takes for the state of a cell to change from its most positive state to its most negative state. The maximum convergence rate of a cell C(i,j) is determined by \( C \) the capacitor, as defined in (1), and the equivalent resistance, \( R_{eq} \), seen by \( C \). If another cell C(m,n) in the CNN has a variance in either \( C \) or \( R_{eq} \) the result will be a change in \( \tau \), defined as \( \Delta \tau \).

The proper final state of a CNN can vary depending on \( \Delta \tau \). If the output of a cell is used to determine the final state of a CNN, as is the case when \( \Delta \tau \neq 0 \), then it is possible that the cells can converge to a wrong value. The dependence of a CNN on \( \Delta \tau \) is determined by the architecture of the array as well as the templates used to implement the given function.

If the CNN relies heavily on the B template, then the effect of \( \Delta \tau \) is minimum. This is because the network is being driven by the constant value of \( u_i \), representing the image which can offset any slowly converging cells.

The architecture used to implement a CNN can also increase the effects of \( \Delta \tau \). If the slope of the nonlinear output limiting function \( f_n(\cdot) \) is very steep, then the effects of \( \Delta \tau \) are greatly increased. A very steep slope implies that the output, \( y \), of a cell changes very rapidly when the value of the state, \( x \), is close to the origin. Therefore, at some point in time, say \( \tau \), the output of the cell changes very rapidly from -1 to +1 without the cell being fully converged. If a cell C(i,j) has a convergence time of \( \tau + \Delta \tau \), then using the same assumption, the output changes at time \( \tau + \frac{\Delta \tau}{2} \). If cell C(i,j) is surrounded by cells with the quicker convergence time of \( \tau \), then the outputs of the surrounding cells change states before cell C(i,j).

If the output of cell C(i,j) is dependent only on the output of the cells surrounding it, then the final state of cell C(i,j) is effected by the improper data that it receives in the time period between \( \frac{\tau}{2} \) and \( \tau + \frac{\Delta \tau}{2} \). The following simulation results show this effect.

A 5x5 CNN was simulated using HSPICE. The template multipliers were voltage controlled current sources. The output function \( f_n(\cdot) \) was implemented using a transistor level inverter. The slope of the output function is very steep as discussed above. The output, \( y(t) \), transition from the negative rail to the positive rail takes place when the state, \( x(t) \), is between -10mV and +10mV.

An edge detection template was used on the diamond image of Fig. 2a. The initial conditions placed on the cells are the values for the pixels of the image itself. A black pixel is represented by a normalized value of +1 volt and white pixel is represented by a normalized value of -1 volt. The simulation was ran with all cells having the same value of \( \tau \) with \( C=1.5\mu F \). The correct final state of the processed image is shown in Fig. 2b.
The τ of cell C(3,3) was then altered by changing the value of the capacitor in the cell to 1.05C. This had the effect of making Δτ equal to 5%. Fig. 2c shows that the slower convergence rate of cell C(3,3) caused the CNN to converge to an improper final state.

**Fig. 2** Edge detection using initial conditions (a) input image (b) final results with Δτ = 0 (c) final results with cell C(3,3) having Δτ = 5%

Fig. 3 shows a comparison between the state of cell C(3,3) and its surrounding cells. As the figure shows, at 1.7us cell C(3,3) has decayed to .4V but the surrounding cells have already reached the transition point. Once the surrounding cells reach the transition point, their output changes to -1. This change causes the surrounding cells to inject current into cell C(3,3). The injected current causes the state of cell C(3,3) to begin to rise. The graph shows the impact of this change on the cell.

**Fig. 3** State value comparison with cell C(3,3) having a slower convergence time. The state of cell C(3,3) is shown as the dashed line and the state of all surrounding cells is shown as the solid line.

**4. Convergence Test Method**

The test method presented next test a CNN for variances in convergence. The test is intended to be used in a production environment in conjunction with the functional test presented in the following sections. The test uses an edge detection type template to detect slowly converging cells. After the test, the slow converging cells appear black while the rest of the array turns white. The convergence test algorithm is presented next.

1. Let \( \mathbf{A} = \begin{bmatrix} 0 & 2 & 0 \\ 2 & 4 & 2 \\ 0 & 2 & 0 \end{bmatrix} \) \( \mathbf{B} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \) \( i = -4, 7 \)
2. Set the input image and border cells to black
   \( \forall_{y} u_{ij} = 1 \ i = 0, 1, ... n + 1 \ j = 0, 1, ... m + 1 \)
3. Set the initial conditions to black
   \( \forall_{y} x_{ij}(0) = -1 \ i = 1, 2, ... n \ j = 1, 2, ... m \)
4. Allow the CNN to converge
5. All cells, \( C_{ij} \) remaining at black are considered slow to converge
   \( \forall_{y} v_{ij} = -1 \rightarrow C_{ij} \ i = 1, 2, ... n \ j = 1, 2, ... m \)
The test works much in the same way as edge detection. The test image is a solid black box with the border cells forming the edge of the image. The initial conditions placed on the cells cause the dynamic convergence of the array to drive the cells to their final steady state. Cells which are slow to converge remain at black as discussed in the above example. The test can detect variances in $\Delta t$ as small as 5% as long as the cell does not have two or more slowly convergent orthogonal neighbors.

5. Simulated Convergence Test

A 5x5 CNN simulator implemented using HSPICE was used to confirm the above test procedure. All cells had a 1pF integrating capacitor except for cells C(1,1), C(2,3), C(2,4), and cell C(5,4) which had a capacitor value of 1.05pF. This resulted in a $\Delta t$ of 5%.

![Figure 4](image)

Fig. 4 Slow converging cells (a) original test image to be processed (b) final result showing cells C(2,3), C(2,4) and cell C(5,4) are slow converging cells

As Fig. 4 shows, all slowly convergent cells remained black except cell C(1,1). This is due to the fact that this cell had the constant driving force of the border cells to eventually force the cell to a white value. This is similar to the effect of having the $B$ template present. Fig. 5 shows the state value of cell C(1,1). The convergent rate of the cell is slowed when its neighbors outputs changes, but its direction is not reversed. Due to this phenomenon the corner cells of the array will always converge to the proper value.

![Figure 5](image)

Fig. 5 State variable value of cell C(1,1) showing the effect of the border cells on its convergence. The state of cell C(1,1) is shown as $- - - - - - -$, and its output is shown as the solid line

6. Fault Models

A CNN processor has only two output states. Commonly, in image processing applications these states appear as white or black pixels. If a cell is unable to change from one state to the other, it is defined to be "stuck-at-white" or "stuck-at-black", depending on its current value. With the proposed test method it is possible to detect 100% of the stuck-at faults in the processor.

7. Functional Test Methods

The test procedure has two separate methods to detect faulty cells, a local method using the $B$ template and a propagation method using the $A$ template. The entire array can be tested using either of these methods regardless of its size. The advantage of the $A$ template method is that it verifies that each cell is responding correctly to its neighbors output. The propagation test should be used if the processing array does not appear to be disseminating information throughout the network properly.

6
The local method uses the input image and the $B$ template to predict the final output state, $y_{ij}$, of each cell in the array. The algorithm for the local test procedure is shown next.

1. Let $A = \begin{bmatrix} 0 & 0 & 0 \\ \cdot & \cdot & \cdot \\ 0 & 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} -1 & -1 \\ -1 & -1 \\ -1 & -1 \end{bmatrix}$ $I = 0$

2. Set the input image and border cells to white

   $\forall_{ij} u_{ij} = -1 \quad i = 0, 1, \ldots, n + 1; \quad j = 0, 1, \ldots, m + 1$

3. Set the initial conditions to white

   $\forall_{ij} x_{ij}(0) = -1 \quad i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, m$

4. Allow the CNN to converge

5. All cells, $C_{ij}$ remaining at white are considered faulty

   $\forall_{ij} y_{ij} = -1 \quad C_{ij}$ is faulty \quad $i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, m$

The normalized value corresponding to a white input is $-1$. The $B$ template values are also $-1$. The positive product of these two values results in current being injected into cell $C(i,j)$ from each of its eight neighbors. As the current is injected into the cell, the integrator voltage rises and the cell output reaches the normalized value of 1, which corresponds to black. If cell $C(i,j)$ fails to change under such overwhelming circumstances the conclusion that must be drawn is that the cell is "stuck-at-white". The result of this test does not depend on the output of any cell, therefore if cell $C(i,j)$ is "stuck-at-white" its output will not effect the output of its neighbors. The same algorithm can be applied to find "stuck-at-black" cells by changing all instances of white to black and $-1$ to $1$ in steps 2 thru 5 of the local test algorithm.

The test of the CNN array using the $A$ template uses the idea of propagation of information across the network. The propagation ability of CNNs has been described before [5]. Here we use the same concept although the templates are different since we only want propagation and not "full dragging" as described in [5]. When propagating information across the network, the effects of $\Delta t$ do not effect the final state of the CNN they only delay the final result due to the slower convergence time of any cells in the propagation path. In this case the input image does not matter and the border cells and initial conditions of the network are black. The $A$ template shown in the algorithm below causes each cell, $C(i,j)$, to look at the cell behind it, $C(i-1,j)$, and change to the color of that cell. The algorithm for the propagation test method is outlined next.

The process starts at the left edge of the array and propagates across the network to the right side. Since the border cells, $C(i,0)$, are black, the predicted result should be an all black image. If a "stuck-at-white" fault is detected, then all properly functioning cells to the right of the stuck cell should also remain white. The faulty cell in effect cast its "shadow" across the array. The situation where two or more faulty cells lie on the same row can be detected by rotating the $A$ template clockwise $45^\circ$ and repeating the test. The template should be rotated in this manner $360^\circ$ in order to assure complete coverage of the array. The "stuck-at-white" cells can be determined by performing the logical OR of the resulting eight output images. The same procedure can be used to detect "stuck-at-black" cells by changing all occurrences of white to black and $-1$ to $1$ in steps 2 thru 8 in the propagation test algorithm and performing a logical AND in step 7.

8. Simulated Functional Test Results

Using a CNN simulation program the above tests were applied to a 5x5 CNN network. In both the local and propagation tests, cells $C(3,2)$ and $C(2,3)$ were intentionally forced into the "stuck-at-white" state. Fig. 6a shows the input image used for both tests. Fig. 6b shows the resulting final image after the simulation of the local test. It is clearly seen that all properly working cells have successfully implemented the $B$ template and made the transition from white to black. Cell $C(2,2)$ was able to make the transition even though two of its...
1. Let \( A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \) and \( B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \) and \( k = 1 \)

2. Set the input image to white and the border cells to black
   \[
   \forall i \in (1, 2, \ldots, n) \quad i = 0, n + 1; \quad j = 1, 2, \ldots, m + 1 \\
   \forall j \in (1, 2, \ldots, m) \quad i = 0, 1, 2, \ldots, n + 1; \quad j = 0, m + 1
   \]

3. Set the initial conditions to black
   \[
   \forall i \in (1, 2, \ldots, n) \quad \forall j \in (1, 2, \ldots, m) \quad V_{ij}(0) = 1
   \]

4. Allow the CNN to converge and save the results of rotation \( k \)
   \[
   \forall i \in (1, 2, \ldots, n) \quad \forall j \in (1, 2, \ldots, m) \quad V_{ij}(t) = C^{(k)}_{ij}
   \]

5. Add 1 to \( k \) and rotate A template clockwise 45°

6. If \( k > 8 \), continue to step 7; otherwise, go to step 2

7. Perform the logical OR of the results
   \[
   \forall i \in (1, 2, \ldots, n) \quad \forall j \in (1, 2, \ldots, m) \quad V_{ij}(C) = V_{ij}(1) \Rightarrow C_{ij}
   \]

8. All cells, \( C_{ij} \) remaining at white are considered faulty
   \[
   \forall i \in (1, 2, \ldots, n) \quad \forall j \in (1, 2, \ldots, m) \quad V_{ij} = 0 \Rightarrow C_{ij}
   \]

neighbors where faulty. This is because the output of each cell is due only to the input image and is independent of any cells output.

Fig. 6 Local test method (a) input image (b) final result

Fig. 7 shows the results of the simulation after the propagation test. Fig. 7a shows the “shadow” effect discussed earlier. It is safe to assume by viewing Fig. 7a that cells \( C(3,2) \) and \( C(2,3) \) are faulty. However, it is unclear if any of the remaining cells on rows 2 or 3 are “stuck-at-white” due to the “shadow” of the faulty cells. Fig. 7b shows the result after rotating the template 45° and the new direction of propagation. It is still unclear as to whether cell \( C(3,4) \) is functioning properly. Fig. 7c confirms that cell \( C(3,4) \) is functioning properly. For this example, ORing the three images of Figs. 7a–c is enough to show all faulty cells. In more complicated fault location patterns however, the template must be rotated completely to detect all possible faults. The results of the test are shown in Fig. 7i and it is clear that cells \( C(3,2) \) and \( C(2,3) \) are faulty.

Fig. 7 Propagation test clockwise template rotation (a)0° (b) 45° (c) 90° (d) 135° (e) 180° (f) 225° (g) 270° (h) 315° (i) final result
9. Conclusion

The variance in convergence rates of the cells of a CNN has been shown to have an impact on the final state of the Network. If cell $C(i,j)$ is slower to converge to its final state than the cells around it and if the architecture is sensitive to variances in $\tau$, then cell $C(i,j)$ may fail to make the transition to its proper final value. A convergence test method to detect dependence on $\Delta \tau$ has been proposed. The test method can detect variances in convergence rates as small as 5%.

A testing method for CNNs has been presented which provides 100% fault detection with no additional hardware required. Only five input vectors are needed $x(0)$, $u$, $A$, $B$ and $I$. The image vector $u$ contains only two components, the color of the image and the color of the border cells. The initial conditions are always the same color for the entire array therefore the vector $x(0)$ needs to only represent the chosen color. The template vectors $A$ and $B$ always contain the nine values necessary to interact with the surrounding cells of $C(i,j)$. $I$, the independent current source vector is 0 in all cases. Since none of the input vectors have any dependence on the array size, any size array can be tested and the number and size of the input vectors will remain constant.

Both functional testing methods give 100% fault detection. The local testing method provides 100% fault isolation and the diagnosis is available by simply looking at the final image achieved after the CNN converges. There are some fault location configurations that could impede fault isolation using the propagation test, i.e. if the faults form a complete rectangle, the status of the cells inside the rectangle would be unknown due to the shadow effect. This fact lowers the fault isolation capabilities of the propagation method but, does not change the fault detection percentage. The diagnosis for the propagation test is available after the $A$ template has been rotated $360^\circ$ and the proper logic function has been performed on the results.

References


