The three-dimensional structure of the turbulent motion of fluid flowing close to a surface has been studied using the technique of linear mean square stochastic estimation to find estimates of the flow field when certain events occur in the flow. The structures given the velocity at one point have been documented, and their dynamic evolution is being studied. It is shown that they are long-lived. One-point events provide insight into the physical bases of closure approximations in one-point moment closures, and they have been used to provide a new closure approximation for the fast pressure-strain correlation. Stochastic estimation is used as a means of defining three-dimensional, dynamic wall boundary conditions for large eddy simulations.
STRUCTURE AND DYNAMICS OF TURBULENT WALL LAYERS

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STRUCTURE AND DYNAMICS OF TURBULENT WALL LAYERS

Goals

The goals of this investigation were to

a) use stochastic estimation to study the structure of turbulent motion close to a wall given velocity data at one and two points;

b) investigate the dynamics (i.e., the evolution) of the conditional eddies found from stochastic estimation;

c) develop a theory and procedure for using stochastic estimates of the near wall layer to provide boundary conditions for large eddy simulations, and evaluate the accuracy of this procedure.

In addition to these goals, the work was extended until May 31, 1994, and this extension was used to explore:

d) the scaling of the two-point correlation tensor with Reynolds number.

Dynamics of Conditional Eddies.

One-point conditional eddies are defined by conditional averages of the form
\[ \langle u(x') \mid u(x) \rangle, \]
and by means of stochastic estimation the conditional average can be approximated in terms of the two-point spatial correlation tensor, \( R_{ij} \). It was established in the first two years of the grant that the one-point eddies possess kinematic features similar to all of the various wall structures that have been observed previously: Close to the wall \( (y^+ < 20) \) they are long, high-speed and low-speed streaks with spanwise wavelengths of approximately 100 viscous units. Farther up \( (20 < y^+ < 100) \) they are hairpin vortices with long legs laying behind them on the wall. If a spanwise velocity is included in the event \( u(x) \), they are cane-like, one-legged structures. Still farther from the wall \( y^+ > 100 \) they become vortex rings.

To study the dynamics of eddies in channel flow, one-point stochastic estimates were used to provide an initial fluctuating velocity field which was solenoidal. It was added to a mean turbulent flow profile \( U(y) \) to create the complete initial field. The given velocity vector \( u(x,0) \) would be prescribed any value, and the evaluation of the field could be studied parametrically as a function of \( u(x,0) \) and its strength in relation to the mean flow.

In the work of Kendall (1992)*, the hairpin vortices that are associated with maximum Reynolds stress events were found to initially strengthen, lift slightly, and grow, followed by a period of decaying peak vorticity. The convection velocity of such a condition eddy was about 20% less than the local mean velocity at the center of the eddy for \( y^+ = 49 \). The lifetime of a hairpin was very long -- longer than the time it takes to fill

* References cited can be found in the bibliography of Appendix A
a 1900 $y^+$ long computational domain. There was no tendency for the eddies to become unstable and break apart, nor did we observe obvious spawning events. These are important properties because they imply that the conditional eddies live long enough to contribute substantially to time averages as single coherent entities. However, unless the number of eddies per unit area exceeds a critical value, the mean flow relaminarizes.

Further studies by J. Zhou focused on the dynamics of conditional eddies which were considerably stronger than those investigated by Kendall (1992). It was discovered that there exists a critical threshold level of $u(x,t)$, above which the conditional eddy ceases to be a passive, long-lived form, and becomes, instead, very active dynamically. Specifically, the eddy spawns new eddies of several kinds: hairpins fore and aft of the original hairpin, streamwise vortices on the outboard sides of the legs of the original hairpin, and shear layers above the hairpin. This behavior seems consistent with observations that have been reported separately by Brooke and Hanratty (1992), Smith and Lu, and others. The principal significance of this work is two-fold:

a) It offers a mechanism for auto-regeneration of hairpin vortices and maintenance of low speed streaks.

b) It offers a unified picture of wall vortex structures that encompasses several seemingly disparate types of structure observed in earlier investigations by others.

Much of the previous studies of the near-wall region of TBL have been primarily limited to, on one hand experimental or direct numerical simulations of fully turbulent flows which is comprised of many interacting coherent structures. To date, the kinematics of these structures has been fairly well understood but these turbulent flows are in general so complex that a complete dynamical picture of the interacting structures has been hard to extract. On the other hand, theoretical models and inviscid computations capture the dynamical details, but the effect of important assumptions inherent in these approaches needs to be evaluated. Much of our current knowledge of the dynamical details is from careful experiments where the evolution of one or few of these structures is followed in an otherwise nonturbulent background. Corresponding numerical simulations which follow the evolution of isolated structure(s) is in general lacking. Hence, we considered the dynamics of near-wall structures through direct numerical simulations. Initial conditions for these simulations corresponding to single and multiple vortical elements of different types were obtained through stochastic estimation technique. The initial size, strength, location and background environment of these vortical structures can therefore be systematically changed to assess their role in the overall dynamics. Significant advantages of this approach are as follows:

- The present approach is based on Navier-Stokes simulations and therefore inviscid assumption is not involved. Finite core size of the vortical elements, effect of vortical perturbations on the background flow and near-wall viscous effects on the vortical structures are naturally included in this approach.

- By providing initial conditions based on stochastically estimated average structure the present approach is rendered deterministic. Evolution of near-wall structures in a variety of background flows can be studied without the clutter of noise from background turbulence. These simulations can be considered as analog of "kernel" experiments by Smith and coworkers where dynamics of individual vortical structure is followed in an otherwise nonturbulent flow.
The key to understanding the dynamical details of the development of the near-wall structures lies in our ability to view all the relevant structures simultaneously as they evolve in time. A complete understanding of the relative spatial location of all the structures in three-dimensions as a function of time is important in assigning the cause-and-effect property. Towards this end Adrian and coworkers (Zhong, Adrian and Huang 1994) have developed a new vortex identification methodology which can be used to track the vortical structures unambiguously over time. This methodology avoids the pitfalls of investigating just two-dimensional slices and three-dimensional contours of vorticity magnitude.

The present methodology allows direct computation of the three-dimensional Eulerian velocity field from which the vorticity field can be extracted accurately. Thus the indirect inference of vortical structures based on Lagrangian data as in smoke wire and hydrogen bubble flow visualization is avoided.

By varying the event vector for the stochastic estimation the initial vortical structure, whose evolution is under investigation, can be carried from symmetric to asymmetric, from single to multiple and from weak to strong. Depending on the nature of the initial structure different dynamical scenarios are possible. Thus the present approach provides an opportunity to link the various observations made in different experimental approaches with the theoretical and computational results - thereby exposing all the near wall structures and their dynamics in a unified set of computational experiments.

A series of computations investigating the temporal evolution of single symmetric initial hair-pin vortex obtained from linear stochastic estimation has been performed. In these computations the initial location of the head of the hair-pin vortex and its strength are systematically varied to investigate their effect on the dynamics. Results obtained from these simulations can be summarized as:

- A threshold behaviour is observed for the regeneration process to initiate. Hair-pin vortices of strength below the threshold are observed to decay after an initial growth in their strength and do not participate in the generation of new vortical structures. Never-the-less even these weak vortices are very long lived (O(1000) t+ units) and decay on a slow time scale.

- Hair-pin vortices of initial strength greater than the threshold evolve rapidly in time with an initial lift-up of the head with subsequent generation of secondary and tertiary vortices. This vortex regeneration process is general agreement with the eruptive mechanism observed by Acalar & Smith. The newly generated vortices undergo rapid growth and spawn-off a newer generation of vortices thus forming a hierarchy of hair-pin vortices, providing support to the hierarchical lambda-vortex model proposed by Perry & Chong (1982).

- A detailed investigation of the secondary vortex generations reveals that as the legs of the primary vortex are stretched by the mean shear they develop kinks in the x-y plane. Subsequently, the upstream portion of the legs break-off resulting in a shorter parent vortex which by now has lifted up to the center of the channel. The parent vortex attains a near vertical orientation and with its characteristic W shape resembles a vortex ring. The upstream sections continue to lift up and undergo vortex reconnection to form the secondary hair-pin vortex. This scenario closely resembles the evolution of parabolic vortex filaments studied by Moin et al. (1986).

- During the above described process of vortex break-up and reconnection, alongside
on the outboard side of the original streamwise vortex pair two new streamwise vortices are generated. Streamwise location of this secondary vortex generation very nearly coincides the location of break-up and reconnection, but closer to the wall than the lifted sections of the original vortex. This generation mechanism can be compared to the flow oriented vortex generation mechanism addressed by Brooke & Hanratty (1993), Benard et al. (1993) and Jimenez & Orlandi (1993).

- In addition to these mechanisms, which have been observed in earlier studies, the present simulations also exhibit other, relatively weaker mechanisms of structure generation, which might have gone unobserved in earlier studies. Each fully developed hair-pin vortex is observed to generate one or two additional ring vortices on the downstream side. These ring vortices lift up from the wall, agglomerate and eventually weaken along with the primary hair-pin vortex. In addition to the evolution of symmetric hair-pin structures, limited number of simulations were also performed with an initial asymmetric stochastically estimated hair-pin vortex obtained from a Q2 event with nonzero spanwise velocity. Evolution of multiple interacting hair-pin vortices was also considered. Results from these simulations in general qualitatively followed those of symmetric structures. But some important differences can be observed:

- The evolution of asymmetric structures closely resemble those of turbulent boundary layer and channel flow simulation results. As observed by Robinson, the initial vortex develops into a one-sided cane like vortical structure with only the stronger leg of the original vortex and the corresponding shoulder surviving. Further generated new vortical structures are also highly asymmetric, which none resembling a complete hair-pin vortex. The flow appears to be dominated by lifted streamwise vortices. This provides support for the conjecture that the lifted cane shaped streamwise vortices are nothing but strongly asymmetric hairpin vortices.

- For the same initial amplitude the asymmetric structures are observed to evolve more rapidly, spawning strong new secondary vortices in an efficient manner. This observation if generalized to the natural selection (or preference) of asymmetric structure by the TBL, could explain the more common observance of asymmetric structures in experiments and direct numerical simulations.

- An interesting phenomenon observed in the multiple structure simulation, where initially two hair-pin vortices are placed in a staggered arrangement, is the generation of new hair-pin vortices to the side original upstream vortex, well behind the original downstream vortex. This sideways generation of vortical structure in many ways resemble the subsidiary vortex generation observed in experiments (Acarlar & Smith) and inviscid simulations (Hon & Walker).

It cannot yet be stated conclusively that the structure we are observing is the mechanism of wall turbulence, but the results suggest that we are seeing an important part. Ongoing work is computing interactions of several hairpins, and we are very interested in extending this computation to higher Re and to include outer flow interactions with the earlier flow.

Stochastic Boundary Condition for Large Eddy Simulation.

The structure of the region of the flow between the wall and the lowest plane of grid points in an LES computation can be established by stochastic estimation in terms of the LES grid data and the two-point spatial correlation tensor. The latter must be provided as empirical input, but if it is fairly universal, this input can be used for many flow
conditions. The intent of this method is to represent algebraically the SGS wall motions with a scheme that takes full account of the coherent structures in the wall layer.

The first problem that has been looked at is that of estimating the wall layer of a DNS channel flow computation. This eliminates the SGS modeling in the core region and focuses on the accuracy of the estimate relative to the full DNS field, and the effects of parameters such as the location of the lowest grid (plane(s) and the spacing of the event data grid.

The conventional no-slip boundary conditions on \( u \) and \( w \) were replaced by stochastic estimates of the wall shear stresses in the \( x- \) and \( z- \) directions given velocity data on a plane \( y = y_e \). The normal velocity was kept at zero. This approach simulates boundary conditions for the resolved scale field in an LES computation in which the stresses produced by the sub-grid scale wall layer are estimated. Since there must be some error in the wall shear stress estimates, the fluid at the wall eventually begins to slip a small amount. The mean square error (MSE) in estimating the streamwise wall shear by means of stochastic estimation is less than the model by Piomelli et al. Stochastic estimation reduces the error substantially, especially if data from several planes are used. Comparing the \( \text{rms} \) velocities from the exact, no-slip DNS to the estimated shear stress DNS shows that close to the wall, the \( \text{rms} \) of \( u \) is non-zero, indicating the fluctuating slip allowed by the estimated stresses, but this error becomes small outside of the buffer layer.

The wall-layer in a large eddy simulation (LES) was also estimated through stochastic estimation in a \( Re_\tau = 640 \) channel flow. In the LES simulations, the wall-layer subgrid scales and the core region subgrid scales are modeled. The core region momentum retardation created by the viscous wall-layer is stochastically estimated through the wall shear stresses. These cases have been compared to the shifted model of Piomelli et al. (1989). Due to the limited amount of empirical data, the necessary correlations for the LSE had to be extrapolated from a lower Reynolds number (\( Re_\tau = 180 \)) to a higher Reynolds number (\( Re_\tau = 640 \)). As in the DNS case, the LES coefficients are localized in real space. Although this was necessary from a practical standpoint, it is also an important step in assessing the viability of the LSE model. If the method is to be successful, it must be possible to extrapolate empirical data from one Reynolds number to another. From dimensional analysis, it can be shown that the two point spatial correlation tensor in channel flow should be dependent upon five other dimensionless groups. The simplest possible extrapolation of the correlation data is to assume that a scaling transformation exists that will make the correlation tensor an invariant function of the Reynolds number:

\[
R_i(r_1, x_2, x_2', r_3, Re_\tau) = u^2 F_j(r_1/r_1^s, x_2/x_2^s, x_2'/x_2'^s, r_3/r_3^s),
\]

where \( u^2, r_1^s, x_2^s, \) and \( r_3^s \) are the appropriate scaling variables that successfully remove the Reynolds number dependence.

Three versions of the correlation tensor scaling are investigated: a full inner scale correlation tensor, a mixed scale correlation tensor and a two scale correlation tensor. The mixed scale and the two scale assumptions are motivated by the results of Naguib & Wark (1992), where the longer wavelengths of the turbulent spectrum tend to scale in terms of the inviscid scales (\( u^2 = u_\tau, r_1^s = d/2, x_2^s = d/2, r_3^s = d/2 \)) and the shorter wavelengths scale with viscous scales (\( u^2 = u_\tau, r_1^s = \nu u_\tau, x_2^s = \nu u_\tau, r_3^s = \nu u_\tau \)). The wall boundary conditions have a substantial impact on the mean flow, and hence the study of
the boundary conditions is warranted. The LSE results in a better approximation than the
shifted model, if the correlation tensor is approximately scaled in terms of the mixed set:
\( \tau = u, \ \tau_1 = d/2, \ x_2 = v \tau, \ \tau_3 = d/2. \) When the correlation tensor is assumed to scale in terms
of inner variables, \( \tau = u, \ \tau_1 = v \tau, \ x_2 = v \tau, \ \tau_3 = v \tau \), the LSE model was found to be
inferior to the shifted model. When the correlation tensor is assumed to scale in terms of
the two scale arguments (completely inviscid for long wavelengths and completely
viscous for short wavelengths), the results are also inferior to the shifted model. All of the
scaling assumptions have justification from experiments, but are crude assumptions since
the Reynolds number effect upon the correlation is not fully understood. The results show
that extrapolating the correlation data to different points in its parameter space is a
difficult process. From the boundary condition viewpoint, this implies that the model is
sensitive to its parameters. However this should be expected since the LSE boundary
conditions are a direct statement of the structural characteristics of the wall shear stress.

A full description of the work on LES boundary conditions is contained in
Appendix A which is a copy of T. Bagwell's Ph.D. thesis.

Publications

The following publications were written under the grant:


T. G. Bagwell, R. J. Adrian, R. D. Moser and J. Kim, "Stochastic Estimation of Near

Z. C. Liu, R. J. Adrian and T. J. Hanratty, "Reynolds Number Simulation of Proper
Orthogonal Decomposition of the Outer Layer of Turbulent Wall Flow," TAM Rept 747,


S. Balachandar and R. J. Adrian, "Structure Extraction by Stochastic Estimation with Adaptive

R. J. Adrian, "Stochastic estimation of conditional structure", in Eddy Structure Identification in
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APPENDIX A

T. G. Bagwell Ph.D. Thesis,
"Stochastic estimation of near wall closure in turbulence models"
University of Illinois (1994).
STOCHASTIC ESTIMATION OF NEAR WALL CLOSURE
IN TURBULENCE MODELS

BY
TED GLYN BAGWELL
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