Video Transmission for Quantized Variable Bandwidth Networks

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Transmission of variable bit-rate sources, such as video, over channels with quantized rates is discussed. An optimal scheme for transmission, based on the Viterbi algorithm, is described. A sliding window sub-optimal algorithm is introduced which significantly reduces algorithmic complexity while maintaining near-optimum performance. Simulation of the performances of the optimal and sub-optimal algorithms, as well as comparisons to a constant bit rate channel is completed.
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1 Introduction

Transmission of video information is a challenging task due to the overwhelming amount of data and real-time constraints. Compression algorithms are utilized to reduce the number of bits required to be transmitted. Compression techniques output varying amounts of data from frame to frame, depending on the scene content of the video. Typically, if there is high motion in the video, the encoder outputs a large amount of data, whereas if there is low motion, considerable compression can be achieved by exploiting the temporal redundancy and as a consequence the encoder outputs a relatively small amount of data. Transmitting a variable-bit-rate (VBR) source over a constant-bit-rate (CBR) channel has been a well studied problem [FM86], [SG88], [Zde91].

A common configuration for transmitting variable bit rate data over CBR channel is shown in Figure 1. A buffer is implemented to regulate the variations in the amount of data output by the video encoder. A feedback loop, monitoring the buffer fullness, controls the quantization step-size of the video encoder. As the buffer reaches capacity, the feedback loop increases the quantization step-size, and the video encoder outputs less data. This is done to prevent buffer overflow, which would cause a loss of bits. More sophisticated transform (e.g. Discrete-Cosine Transform, Wavelets) based schemes distribute bits among the frequency coefficients with the constraint of minimizing the mean squared error of the reconstructed video at the receiver end [DB89]. The buffer monitor feedback determines the bit allocation for the encoder. A smaller bit allocation tends to cause a degradation of the image quality of the encoded video. Fluctuations in image quality is the typical tradeoff for transmitting a VBR source over a CBR channel.
It is believed by some, that a variable bandwidth network is well suited for variable bit-rate sources, such as compressed video. Existing network systems are generally constant bit rate channels. Only recently, as technology for fast switching devices have become available, has the development of a variable bandwidth network, ATM (Asynchronous Transfer Mode), been so seriously pursued. A major objective for the development of ATM is that by matching the variable bit rate output of video encoders, constant image quality can be maintained.

A variable bandwidth network will provide greater flexibility. Extending this flexibility, however, may come at a cost in terms of network management complexity. An approach advocated by Lea [Lea92], is to consider a network that can transmit data at a discrete set of fixed rates. This will provide many of the advantages of a variable bandwidth network, but will simplify the system control mechanisms. In this paper, we address the issue of constant video image quality transmission over variable bandwidth network with quantized rates.

2 Buffer Dynamics

Following the development of Reibman and Haskell [RH92], we define system behavior for video transmission. Let $E(t)$ be the number of bits output by the video encoder at time $t$. Define $R(t)$ to be the transmission rate of bits at time $t$. Let $T$ be the time for one frame to be displayed. We can then define $E_i$ ($i = 1, 2, \ldots$) to be the number of encoded bits in the
Define $E_i$ to be the transmission capacity of bits of the channel during time interval $(i - 1)T$ to $iT$. We have:

$$E_i = \int_{(i-1)T}^{iT} E(t) \, dt$$  \hfill (1)

Define $R_i$ to be the transmission capacity of bits of the channel during time interval $(i - 1)T$ to $iT$. We have:

$$R_i = \int_{(i-1)T}^{iT} R(t) \, dt$$  \hfill (2)

In the case where the channel has more capacity than there are bits to be transmitted, an underflow condition has occurred. This does not pose a problem to the system as bits can be stuffed. We define $R'_i$ as the actual number of bits transmitted. Note, if underflow condition does not occur, we have:

$$R_i = R'_i$$  \hfill (3)

Let $B^e(t)$ be the encoder buffer fullness at time $t$. We have the following relationship:

$$B^e(t) = \int_0^t [E(s) - R'(s)] \, ds$$  \hfill (4)

$B^e_i$ is the buffer fullness after encoding of frame $i$, and is:

$$B^e(iT) = \int_0^{iT} [E(s) - R'(s)] \, ds$$  \hfill (5)

Alternatively, this is:

$$B^e(iT) = \sum_{j=1}^{i} [E_j - R'_j]$$  \hfill (6)

At time $i$, encoded bits will be entering the buffer at rate $E_i$, and the channel will be emptying the buffer at rate $R'_i$. The buffer state at time $i$, $B^e_i$, will therefore be:

$$B^e_i = B^e_{i-1} + E_i - R'_i$$  \hfill (7)

In the case of underflow, the system will stuff bits and the buffer will be left empty. We thus have equivalently:

$$B^e_i = \max(B^e_{i-1} + E_i - R_i, 0)$$  \hfill (8)

We further define $B^e_{\text{max}}$ as the maximum size of the encoder buffer, which cannot overflow. Thus, we have the condition:

$$0 \leq B^e_i \leq B^e_{\text{max}} \ \forall i.$$  \hfill (9)
3 Optimal Trellis Algorithm

Consider the case where the statistics of a sequence of video data is completely known, and the buffer size is constrained. This scenario might be a situation where the processing delay is not of concern, but buffer size is limited due to implementation considerations (i.e. cost, size, etc). Let $E_i$ be the number of bits encoded for frame $i$ and $R_i$ be the transmission rate when the bits for frame $i$ enter the buffer. Suppose we have $M$ allowable transmission rates and denote these as $R(j)$ ($0 \leq j < M$). At each time instance for frame $i$, $R_i$ is assigned to be some $R(j)$. The entire sequence contains $N$ frames.

Problem Formulation:

\textit{minimize cumulative transmission rate without incurring buffer overflow or underflow}

Let,

\[ R_{Total} = \sum_{i=0}^{N} R_i \]  

The problem formulation can be stated as:

\[ \text{min}(R_{Total}) \text{ subject to the condition } 0 \leq B_i^e \leq B_{max}^e \forall i \]  

An optimal solution can be determined by exhaustively determining for each combination of $R(j)$'s if buffer overflow occurs, and among those solutions, select the combination that has the smallest cumulative transmission rate. Although this is optimal, the computation complexity is unreasonable for real-time applications. For $N$ frames and $M$ rates, we have $N^M$ combinations, and the computational load grows exponentially in $N$.

A method of reducing the number of combinations to calculate is to use a trellis-approach, similar to Ortega [Ort94]. As such, the problem can be solved by application of the Viterbi algorithm, a well-known dynamic programming method [LM94]. The computational load is reduced to order $N$, a linear relationship. The algorithm can be described as follows:

1. Begin at frame 1, and construct a path for each rate, to form $M$ branches. From each of those branches, we again have $M$ branches.
2. Anywhere along the path there occurs a buffer overflow, we can immediately prune the entire path, as this branch will not lead to an admissible solution.

3. Upon completion after \( N \) frames, we select among the surviving paths the one with the minimum cost, which by construction, did not incur buffer overflow at any time.

Figure 2 shows an example of a constructed trellis. At each frame iteration, the effective buffer maximum is decreased by the number of encoded bits entering the buffer at that time. The slope of each path is proportional to the transmission rate. Any path that terminates above the effective buffer maximum at any stage, corresponds to an overflow condition and the associated branch can be pruned. Referring to figure 2, a path during frame 2 is in an overflow state. Thus, branches from this node can be pruned and subsequent paths do not need to be computed.

Unlike the problem formulation of Ortega and Vetterli, the problem that is posed in this paper has more structure in the following sense: if no underflow is assumed to have occurred, it is recognized that there exist paths with the same cost \( R_{\text{Total}} \). For example, let \( N = 3 \): the sequence of transmission rates \( (R_4, R_5, R_3) \) and the sequence \( (R_2, R_4, R_5) \) have the same cumulative transmission rate. For a given path, define:

\[
\begin{align*}
n_1 &= \text{number of } R(1)'s \\
n_2 &= \text{number of } R(2)'s \\
\text{etc.}
\end{align*}
\]

with,

\[
n_1 + n_2 + \ldots + n_M = N
\]  

We have:

\[
R_{\text{Total}} = \sum_{i=1}^{n_1} R(1) + \sum_{j=1}^{n_2} R(2) + \ldots + \sum_{i=1}^{n_M} R(M) 
\]  

The number of permutations is:

\[
\binom{N}{n_1, n_2, \ldots, n_M}
\]
Figure 2: trellis
Once a path has been determined to satisfy the given conditions, all other paths that provide the same $R_{Total}$ do not have to be computed. A lower bound on the computation can be determined as:

$$\binom{N + M - 1}{M}$$

(17)

We also note, all paths with values of $R_{Total}$ greater than any $R_{Total}$ that is a possible solution, do not have to be traversed.

4 Sub-Optimal Moving Window Algorithm

The above formulation is an optimal solution that requires knowledge of the statistics of the entire sequence. In many scenarios, this will not be the case. In real-time applications, the statistics of only a few frames will be available at any given moment. A proposed sub-optimal solution is to determine the optimal path for a window of frames that are known, and slide the window as each frame is encoded. The sub-optimal algorithm of window length $n$ is outlined as follows:

1. Determine all $K$ optimal paths for the sequence of $n$ frames and denote the $i$th path as $R_1^i, R_2^i, \ldots, R_n^i$. Choose $\min(R_j^i)$ for $1 \leq j \leq K$ as the rate. Slide window of $n$ frames and repeat step 1. If no optimal path exists, go to step 2.

2. Choose $R_{max}$ as the rate. Slide window of $n$ frames and go to step 1.

The performance of the sub-optimal algorithm depends on the length of the window, the variability of the data, and the buffer size. Although parameterizing video data is difficult due to its non-stationary characteristic, the mean rate ($E_\mu$) and then peak rate ($E_{max}$) are reasonable first-order parameters. The window length, "$n$", needs to be long enough to enable the system to empty the buffer in the event that the next frame is $E_{max}$. We have:

$$E_{max} - B_{max} + \sum_{i=1}^{n-1} E_i \leq n \cdot R_{max}$$

(18)

Taking expected value:

$$E_{max} - B_{max} + (n - 1) \cdot E_\mu \leq n \cdot R_{max}$$

(19)
We can determine a first-order lower bound for $n$:

$$n \geq \frac{E_{\text{max}} - B_{\text{max}} - E_{\mu}}{R_{\text{max}} - E_{\mu}}$$

(20)

We note, that the sub-optimal solution for window of length 1, degenerates to a Greedy algorithm, where the minimum rate to satisfy no buffer overflow for the next frame is selected.

5 Simulation

An important development in the effort in variable bandwidth networks, is the accurate modeling of variable bandwidth sources, such as video. Characterizing video data is crucial to the analysis of such systems. This task, however, is difficult due to the nature of video. Statistics of video data are a function of the coding scheme employed, as well as the scene content. Autoregressive (AR) models are a common approach to modeling video [Yeg93], [Jab93]. Correlation between successive frames is typically high, except during a scene change. These scene changes are typically modeled as a Markov chain. Each state of the Markov chain has associated parameters which in turn, modulates the AR model.

For the simulation in this report, the encoded bit rate of a video source is modeled simply as a first-order Gaussian AR process:

$$x(n) = a \cdot x(n-1) + \epsilon(n)$$

The parameters are governed by a Markov chain consisting of three states. A plot of the video source modeled as AR process is shown in Figure 3.

A buffer size of 50 kbits is chosen, and a channel with 3 discrete rates of $R_1 = 20$, $R_2 = 40$, and $R_3 = 80$ kbits per frame is considered. The rates approximate the mean of each state of the Markov chain. A plot of the buffer dynamics for a sequence of 100 frames is shown in Figure 6. The path determined by the optimal algorithm does not incur buffer overflow or underflow, as required. The aggregate transmission rate for the optimal path is $36.2 \times 100$ kbits.

For the sub-optimal algorithm, we use a window of length three. The path for the sub-optimal algorithm also does not incur buffer overflow or underflow. The aggregate transmission rate for the sub-optimal path ($36.4 \times 100$ kbits) is slightly greater than the
Figure 3: video source

Figure 4: AR process modulated by Markov chain
<table>
<thead>
<tr>
<th>state</th>
<th>$\alpha$</th>
<th>$\mu$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.55</td>
<td>26000</td>
<td>2400</td>
</tr>
<tr>
<td>2</td>
<td>0.79</td>
<td>49000</td>
<td>2600</td>
</tr>
<tr>
<td>3</td>
<td>0.83</td>
<td>79000</td>
<td>11000</td>
</tr>
</tbody>
</table>

Figure 5: AR parameters

Figure 6: buffer state
<table>
<thead>
<tr>
<th></th>
<th>average rate per frame (100 bits)</th>
<th>total buffer overflow (100 bits)</th>
<th>total buffer underflow (100 bits)</th>
</tr>
</thead>
<tbody>
<tr>
<td>optimal</td>
<td>362</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>sub-optimal</td>
<td>364</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>constant-rate</td>
<td>400</td>
<td>52.8</td>
<td>86.4</td>
</tr>
<tr>
<td>constant-rate</td>
<td>800</td>
<td>0</td>
<td>433.2</td>
</tr>
</tbody>
</table>

Figure 7: simulation for 100 frames

optimal path. A constant rate of $R_3 = 800$ (aggregate transmission rate of $80.0 \times 100$ kbits) is required to prevent buffer overflow. When the path is traversing along the x-axis, this corresponds to an underflow condition. The constant-rate path is in this state frequently, reflecting the fact that to prevent encoder buffer overflow, it is required to transmit at an inefficient rate. Figure 7 compares the performance of the optimal, sub-optimal, and constant rate paths.

Another aspect to consider is a constant bit-rate channel where the aggregate transmission rate is equal to the optimal path. For the example above, we have $R_{Total}$ for the optimal path is 3620 kbits for 100 frames, so equivalently a CBR channel of 36.2 kbits per frame. Given this rate, we plot the buffer dynamics in Figure 8. In this plot, overflow condition occurs when the path is clipped at buffer maximum (50 kbits), and underflow condition happens when path is below the x-axis.

6 ATM Network

For variable-bit-rate sources, the ATM network achieves aggregate bandwidth utilization by statistical multiplexing of many source bit streams. The network is able to improve throughput at the risk of congestion and ATM cell loss. By the Law of Large Numbers the network is able to achieve improved aggregate throughput with small probability of cell loss if the sources are independent stationary processes. Unfortunately, sources are often correlated.
Figure 8: lost efficiency for CBR channel

<table>
<thead>
<tr>
<th></th>
<th>average rate per frame (100 bits)</th>
<th>total buffer overflow (100 bits)</th>
<th>total buffer underflow (100 bits)</th>
</tr>
</thead>
<tbody>
<tr>
<td>optimal</td>
<td>362</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>constant-rate</td>
<td>362</td>
<td>65.2</td>
<td>61.9</td>
</tr>
</tbody>
</table>

Figure 9: simulation for 100 frames
and are non-stationary, particularly in the case of video. This problem has created a need for studies in network management strategies for congestion control and prevention of cell loss [HS91], [Rob91].

A feature of ATM is the ability to integrate multimedia, by transporting all information into common units, or ATM cells. It is expected that a large proportion of the traffic on ATM networks will be video data [Kis89], [Zha91], [Nom91]. The ability to accurately characterize incoming traffic will facilitate network management [KV99], [Led94]. As discussed in the development of the sub-optimal algorithm, examples of first order statistics include the mean, or peak rate. These types of quantities are known as “traffic descriptors”. Higher order statistics are being developed to capture the non-stationary nature of various variable-bit-rate type sources [Luc94].

7 Conclusion

A motivation in considering a channel with a discrete set of transmission rates rather than allowing completely arbitrary rates, is to simplify the network management issue. With this basis, an optimal algorithm has been outlined for transmitting a variable-bit-rate source with a finite buffer. The approach is a version of the Viterbi algorithm, a dynamic programming scheme. The optimal path achieves the minimum aggregate transmission rate without buffer overflow or underflow.

A sub-optimal solution is described by invoking the optimal algorithm for a sliding window of \( n \) frames. A formulation for a reasonable \( n \) is discussed. In general, \( n \) should be large enough where sudden variations in the number of bits of the source can be absorbed by the buffer and by the appropriate choice of subsequent transmission rates. As can be observed, buffer size, the set of possible transmission rates, and the statistics of the source, all affect the performance of the sub-optimal algorithm.

Simulated data is generated to provide a variable-bit-rate source. For a sequence of 100 frames, the optimal path and sub-optimal path with window of length 3 are determined. The buffer dynamics are plotted for the optimal path and show that no overflow or underflow occurs. For the example studied, the sub-optimal path also does not incur overflow or underflow. In comparison, the required constant rate channel to prevent buffer overflow is
inefficient, as the system is forced to stuff bits for a number of frames during the sequence.

8 Future Work

For the simulation described in this paper, the discrete transmission rates chosen were simply postulated. A natural question is what set of rates is optimum. One strategy might be to construct this question as a continuous optimization problem, and apply Lagrangian's method of undetermined multipliers. The number of rates also plays an important factor in the performance of a quantized variable bandwidth network. As the number of rates increases, it is conjectured that the performance behaves asymptotically. Perhaps this asymptotic behavior can be quantified. Furthermore, the buffer size is an important parameter in the performance of this system.

Switching rates in an ATM network may incur some penalty in the sense of increasing network management complexity. In addition, different rates may have varying associated costs, which may not be linear in relationship. The mechanisms on how a network will effectively multiplex VBR sources over quantized transmission rate channels is an open question. All of these considerations need to be incorporated in the analysis.

Clearly, many of the proposed ideas need to be experimented with real video data. Another important aspect will be the development of precise traffic descriptors for video sources. In comparison to a CBR channel, it is proposed that a VBR network will be able to accommodate VBR sources more efficiently. However, because the statistics of video sources are extremely unpredictable, perhaps a combination of the methods for VBR networks and CBR channels should be considered. That is, consider a network with quantized transmission rates in tandem with a buffer monitor that regulates video encoder bit output.

The effect of transmitting a VBR source at quantized transmission rates can be viewed as a form of traffic shaping. This function, is in some sense, low pass filtering the VBR source; smoothing out high frequency fluctuations. The output of the quantized rate can be interpreted as a process itself, which can be described by a Markov chain with the number of states equal the number of rates.
References


