Three-Dimensional Numerical Simulation of Mobile-Bed Hydrodynamics

by Miodrag Spasojevic, Forrest M. Holly, Jr., Iowa Institute of Hydraulic Research
Three-Dimensional Numerical Simulation of Mobile-Bed Hydrodynamics

by Miodrag Spasojevic, Forrest M. Holly, Jr.

Iowa Institute of Hydraulic Research
The University of Iowa
Iowa City, IA  52242

Final report
Approved for public release; distribution is unlimited

Prepared for U.S. Army Engineer District, New Orleans
P.O. Box 60267
New Orleans, LA 70160-0267

Monitored by U.S. Army Engineer Waterways Experiment Station
3909 Halls Ferry Road
Vicksburg, MS  39180-6199
Waterways Experiment Station Cataloging-in-Publication Data

Spasojevic, Miodrag.

Three-dimensional numerical simulation of mobile-bed hydrodynamics/by Miodrag Spasojevic, Forrest M. Holly, Jr.; prepared for U.S. Army Engineer District, New Orleans; monitored by U.S. Army Engineer Waterways Experiment Station.

163 p. : ill. ; 28 cm. -- (Contract report ; HL-94-2)

Includes bibliographical references.


TA7 W34c no.HL-94-2
Contents

Preface ........................................................................... v
Chapter I: Introduction ............................................... 1
Chapter II: Governing Equations ................................... 4
Chapter III: Numerical Solution ................................... 19
Chapter IV: Description of the Sediment-Operations Program Module ...... 40
Chapter V: Tests ........................................................... 56
Chapter VI: Conclusions and Suggestions for Further Development ...... 69
References ..................................................................... 71
Appendix A: Quickest Method ......................................... 73
Appendix B: Discretized Mass-Conservation Equation for Suspended Sediment .................................................. 89
Appendix C: Coefficients in Discretized Global Mass-Conservation Equation for Bed Sediment and Mass-Conservation Equations for Active-Layer Sediment ........................................... 133
Appendix D: CH3D Input Data Guide (May 1992) ................. 135
Appendix E: Model of the Mississippi River at the Old River Sample Input-Data Set .................................................. 156
SF 298
<table>
<thead>
<tr>
<th>Number</th>
<th>Figure Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Schematic representation of bed-material finite elemental volume</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>Stratum control volumes below active-layer elemental volume</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>Coordinate transformations</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>Computational grid</td>
<td>22</td>
</tr>
<tr>
<td>5</td>
<td>Summary block diagram of sediment-operations program module</td>
<td>41</td>
</tr>
<tr>
<td>6</td>
<td>Mississippi River at the Old River Control Structure complex: location map</td>
<td>58</td>
</tr>
<tr>
<td>7</td>
<td>The model domain and computational grid</td>
<td>60</td>
</tr>
<tr>
<td>8</td>
<td>Vertical concentration profiles for size class 2 at the model upstream boundary</td>
<td>62</td>
</tr>
</tbody>
</table>
Preface

This project was sponsored by the U.S. Army Engineer District, New Orleans, under Contract No. DACW39-91-K-0025. The study was conducted from August 1991 to December 1993 at the Iowa Institute of Hydraulic Research (IIHR), University of Iowa, Iowa City. The contract was monitored by the Hydraulics Laboratory (HL), U.S. Army Engineer Waterways Experiment Station (WES). Contracting Officer's Representative was Mr. Brad R. Hall, Math Modeling Branch, Waterways Division, HL.

Principal investigators were Drs. Miodrag Spasojevic and Forrest M. Holly, Jr., IIHR. The project benefitted from close collaboration and consultation with many individuals at HL. The support of Messrs. Michael J. Trawle, David D. Abraham, Ronald E. Heath, and Mr. Hall, all of the Math Modeling Branch, Dr. Billy H. Johnson, HL, and Dr. Kue W. Kim, Estuarine Simulation Branch, Estuaries Division, HL, was particularly appreciated. Special thanks go to Mrs. Twila Meder, IIHR, for her conscientious entry of the many equations in this report. This report was written by Drs. Spasojevic and Holly.

This project was conducted under the general supervision of Messrs. Frank A. Herrmann, Jr., Director, HL; Richard A. Sager, Assistant Director, HL; Marden B. Boyd, Chief, Waterways Division; and Mr. Trawle, Chief, Math Modeling Branch.

At the time of publication of this report, Director of WES was Dr. Robert W. Whalin. Commander was COL Bruce K. Howard, EN.

The contents of this report are not to be used for advertising, publication, or promotional purposes. Citation of trade names does not constitute an official endorsement or approval of the use of such commercial products.
CHAPTER I

INTRODUCTION

The objective of this research is to generalize innovative two-dimensional mobile-bed modelling techniques, recently developed at the Iowa Institute of Hydraulic Research, and to merge these techniques with the CH3D three-dimensional hydrodynamic simulation code, thus generalizing CH3D to include mobile-bed processes (such as aggradation and scour, bed-material sorting, and movement of both bedload and suspended load of nonuniform sediment mixtures).

During the past several years research efforts at the Iowa Institute of Hydraulic Research (IIHR) have been devoted to development of a new generation of two-dimensional mobile-bed modelling. The distinguishing technical features of this new generation include the following features:

1. - The fact that the same sediment particle can move either in suspension or as bedload, depending on local flow conditions, is explicitly recognized. This removes the need to assume any specific transport mode in advance.

2. - Criteria for distinguishing between bedload and suspended-sediment transport, as well as mechanisms defining exchange between the two, are incorporated.

3. - A sediment mixture in a natural watercourse is represented through a suitable number of size classes (with most mathematical relations for sediment written for a particular size class).

4. - The global set of sediment equations for all size classes, taken as a whole and solved simultaneously, describes the behavior of a nonuniform sediment, including natural phenomena such as differential settling, armoring and hydraulic sorting.

5. - The governing sediment equations have a clear analytical form which provides for the possibility to analyze their mathematical character and choose proper boundary conditions as well as the most appropriate numerical solution for each of them.

6. - The procedure directly accounts for the effects of change in the sediment size distribution at the bed surface, and change in bed elevation, on the flow field through iterative coupling of the water and sediment equations.

---

1 The numerical model referred to in this report as CH3D was developed from the CH3D-WES numerical model developed at the U.S. Army Engineer Waterways Experiment Station as described in Johnson, B. H., et al. (1991). "User's guide for a three-dimensional numerical hydrodynamic, salinity, and temperature model of Chesapeake Bay," Technical Report HL-91-20, U.S. Army Engineer Waterways Experiment Station, Vicksburg, MS.
7. - The tensor form of the governing water and sediment equations is written for an orthogonal curvilinear system, permitting ready representation of the boundaries of natural watercourses.

8. - The new concept of sediment-transport and bed-evolution processes is inspired by current qualitative understanding of particular aspects of sediment-flow interaction. The derived sediment equations contain several so-called auxiliary relations - terms that need to be evaluated by using empirical relations. However, the global concept and its associated numerical solution are structured so as to avoid use of any particular empirical relation until the very end of the derivations. Therefore, the overall structure of the new computational procedures remains independent of particular empirical expressions used to evaluate the auxiliary relations.

These features, taken together, have made it possible to perform two-dimensional mobile-bed simulation which is not subject to many of the traditional limitations, and which incorporates a more realistic conceptualization of the governing physics than has heretofore been possible. The proven hydrodynamic framework of the CH3D code for unsteady, three-dimensional fixed-bed simulation provides an opportunity to implement the above mobile-bed concepts in the three-dimensional environment.

Generalization of the IIHR two-dimensional mobile-bed modelling techniques, in preparation for their implementation in CH3D, means not only performing a three-dimensional generalization of the governing sediment equations, but also ensuring their compatibility with the overall CH3D environment. Generalization thus comprises the following specific tasks:

1. - Redefine the governing sediment equations in vector form and in standard Cartesian coordinates. For example, compatibility with the CH3D environment requires that the non-constant density of the water-sediment mixture be taken into account; that the three-dimensional suspended-sediment equation include a 'fall-velocity' term; that mechanisms for exchange between bed sediment and suspended sediment be redefined; and that several new auxiliary relations be introduced (e.g. mass-diffusion coefficient, density of a mixture containing water and suspended sediment); etc.

2. - Rederive the governing sediment equations in σ-stretched coordinates (partial coordinate transformation in the vertical direction).

3. - Nondimensionalize the governing sediment equations.

4. - Rederive the dimensionless sediment equations in general (nonorthogonal) horizontal curvilinear coordinates.
It does not appear to be necessary to anticipate direct coupling of the hydrodynamic and sediment processes in CH3D. The partial explicitness of CH3D precludes the use of large time steps at the present stage of development, so that the error involved in water-sediment uncoupling at the scale of one time step (the order of several minutes) is not judged to be serious. Furthermore, a small time step offers the possibility to couple suspended- and bed-sediment computations in an iterative manner.

One of the implications of short-term uncoupling of sediment and hydrodynamics operations is that a separate program module can be dedicated to the sediment operations. The implication of iterative coupling of the suspended-sediment and bed-sediment operations is that different numerical methods can be used for, and separate program submodules dedicated to, the suspended-sediment and bed-sediment computations.

Numerical solution of the suspended-sediment equations is based upon the QUICKEST numerical method (Leonard (1979)), in order to ensure compatibility with the existing CH3D salinity and temperature computations. The original QUICKEST method is generalized to accommodate the appropriate terms in the three-dimensional suspended-sediment equation in general curvilinear coordinates.

Numerical procedures for the bed-sediment equations are based on the previously-developed two-dimensional numerical-solution algorithm for sediment equations. This original IIHR two-dimensional solution was developed for both suspended- and bed-sediment operations (depth-averaged) expressed in dimensional equations in orthogonal curvilinear coordinates. The IIHR method is redefined herein to be used for bed-sediment operations only, in the context of dimensionless equations in general curvilinear coordinates.

Even though they are contained in a separate program module, the sediment-operations developed for CH3D fully communicate with the rest of the CH3D code. The hydrodynamics operations in CH3D provide all the necessary hydrodynamic input required by the sediment module (velocities, depths, etc.). The sediment module, in turn, communicates changes in bed elevations (i.e. depths), bed-surface size-distributions (i.e. friction coefficients) and density (due to the presence of suspended sediment) back to the CH3D hydrodynamics operations.

Detailed documentation of the coding and use of mobile-bed capability in CH3D is included herein, as well as results of tests performed on the Mississippi River near the Old River Control Structure complex.
CHAPTER II

GOVERNING EQUATIONS

Concept of Sediment Transport and Bed Evolution

The concept of sediment transport and bed evolution, and the appropriate mathematical formulation, described herein, accounts for the following important aspects of sediment-flow interaction in natural watercourses: suspended-sediment transport, bedload transport, and interaction between the two; bed level changes; differential settling; hydraulic sorting and armoring; interaction between the flow and changes in bed elevation and bed surface size distribution; and washload transport.

Bedload Transport and Bed Evolution

Since it is difficult to account for the exact position and size of each sediment particle being entrained from the bed or ending its trajectory at a certain spot on the bed surface (Fig. 1), a uniform size distribution is assumed inside a finite elemental area of bed surface. This elemental area must satisfy the condition that its dimension $\Delta l$ is not less than the maximum average saltation length. If this requirement is satisfied, then the bedload flux (taken as parallel to the bed surface) represents bedload exchange between two neighboring elemental areas.

Figure 1. Schematic Representation of Bed-Material Finite Elemental Volume.
The notion of an active layer is introduced to obviate the need to account for the position and size of each sediment particle below the bed surface that may, during erosion, become exposed to the flow and become part of the bed surface. The active layer is defined as an upper layer of the bed, including the bed surface, having a uniform size distribution over its depth. It is assumed that all sediment particles of a given size class inside the active layer are equally exposed to the flow irrespective of their location in the layer.

A finite elemental volume $\Delta V$ is defined as having dimension $\Delta l$ and a thickness $E_m$ (Fig. 1). For a fixed active-layer floor elevation, the mass-conservation equation for one particular size class of sediment in the active-layer elemental volume is written as follows:

$$\rho_s(1-p) \frac{\partial (\beta E_m)}{\partial t} + \nabla \cdot \bar{q}_b + S_e - S_d = 0$$

where $p =$ porosity of the bed material, assumed to be constant; $\rho_s =$ density of sediment, assumed to be constant; $\beta =$ active-layer size fraction, defined as a ratio of the mass of particles of one particular size class inside the active-layer elemental volume $\Delta V$ to the mass $\rho_s(1-p)\Delta V$ of all sediment particles contained in $\Delta V$; $S_e =$ suspended-sediment 'erosion' source, representing the entrainment of sediment particles from the bed into suspension; $S_d =$ suspended-sediment 'deposition' source, representing gravitational settling of suspended sediment particles onto the bed.

Subsurface material below the active-layer elemental volume is discretized into a sequence of control volumes, one below the other, called herein stratum control volumes (Fig. 2). Each stratum control volume has the same dimension $\Delta l$ as the active-layer elemental volume above it. The bed material inside one stratum control volume is assumed to have uniform size distribution.

The stratum control volume immediately below the active-layer elemental volume is called the active-stratum control volume. It is possible, indeed likely, that the active-layer elemental volume and active-stratum elemental volume have different size distributions. The active-layer floor, which is at the same time an active-stratum ceiling, descends or rises whenever the bed elevation changes due to deposition or erosion occurring in the active-layer elemental volume. If, for example, the active-layer floor descends, some of the material that belonged to the active-stratum control volume becomes part of the active-layer elemental volume, whose homogeneous size distribution thus may change.
In order to represent the exchange of sediment particles between the active-layer elemental volume and the active-stratum control volume due to active-layer floor movement, another 'source' term is introduced, called herein the active-layer floor 'source' $S_F$, again specific to one particular size class of particles. The mass-conservation equation for a size class of sediment particles in the active-layer elemental volume then reads:

$$
\rho_s (1 - p) \frac{\partial (\beta E_m)}{\partial t} + \nabla \cdot \bar{q}_b + S_e - S_d - S_F = 0 \quad (2)
$$

Since the bedload flux is a vector parallel to the bed surface, the vector Eq.(2) is essentially two-dimensional.

The mass of a particular size class in the active-stratum control volume may change only due to active-layer floor movement, i.e. due to exchange of material between the active layer and active stratum, while the active-stratum floor elevation remains unchanged. This is expressed by a mass-conservation equation written for each size class in the active-stratum control volume:
\[ \rho_s(1-p) \frac{\partial}{\partial t} \left[ \beta_s(z_b-E_m) \right] + S_F = 0 \]  

(3)

where \( z_b \) = bed-surface level (bed elevation); \( \beta_s \) = active-stratum size fraction; and \( (z_b-E_m) \) = active-layer floor elevation, i.e. active-stratum ceiling.

Summation of the mass-conservation equations for all size classes in the active-layer elemental volume and use of the basic constraint:

\[ \Sigma \beta = 1 \]  

(4)

where \( \Sigma \) represents summation over all size classes, leads to the global mass-conservation equation for the active-layer elemental volume:

\[ \rho_s(1-p) \frac{\partial E_m}{\partial t} + \Sigma (\nabla \cdot \tilde{q}_b + S_e - S_d - S_F) = 0 \]  

(5)

A similar equation can be obtained for the active-stratum control volume:

\[ \rho_s(1-p) \frac{\partial (z_b-E_m)}{\partial t} + \Sigma S_F = 0 \]  

(6)

where again Eq.(4) is invoked. Summation of Eqs.(5) and (6) gives the global mass-conservation equation for bed sediment:

\[ \rho_s(1-p) \frac{\partial Z_b}{\partial t} + \Sigma (\nabla \cdot \tilde{q}_b + S_e - S_d) = 0 \]  

(7)

which can be recognized as the familiar Exner equation with the addition of suspended-sediment source terms. This last equation is again essentially two-dimensional, for the bedload-flux vector is parallel to the bed surface.

When the overall bed slope is small, the mass-conservation equation for a particular size class of active-layer sediment and global mass-conservation equation for bed sediment, (Eqs.(2) and (7)), could be written in Cartesian coordinates as follows:

\[ \rho_s(1-p) \frac{\partial (\beta E_m)}{\partial t} + \frac{\partial q_{bx}}{\partial x} + \frac{\partial q_{by}}{\partial y} + S_e - S_d - S_F = 0 \]  

(8)
\( \rho_s(1-p) \frac{\partial z_b}{\partial t} + \sum \left( \frac{\partial q_{b_x}}{\partial x} + \frac{\partial q_{b_y}}{\partial y} + S_e - S_d \right) = 0 \) \quad (9)

where \( q_{b_x}, q_{b_y} = x- \) and \( y- \) direction components of the bedload flux.

**Suspended-Sediment Transport**

Under the assumption that the suspended sediment particles are advected essentially by the local water velocity, except for the downward gravitational settling expressed through the fall velocity, the mass-conservation equation for one particular size class of suspended sediment in the elemental volume has the following form in Cartesian coordinates:

\[
\frac{\partial (\rho C)}{\partial t} + \frac{\partial}{\partial x} (\rho C u) + \frac{\partial}{\partial y} (\rho C v) + \frac{\partial}{\partial z} (\rho C w) - \frac{\partial}{\partial z} (\rho C w_f) = \frac{\partial}{\partial x} \left( D_H \frac{\partial (\rho C)}{\partial x} \right) + \frac{\partial}{\partial y} \left( D_H \frac{\partial (\rho C)}{\partial y} \right) + \frac{\partial}{\partial z} \left( D_H \frac{\partial (\rho C)}{\partial z} \right) \]

where \( \rho = \) density of a mixture of water and suspended sediment (all size classes); \( C = \) dimensionless concentration, i.e. ratio of the mass \( \rho C dV \) of the particular size-class particles contained in elemental volume \( dV \) to the total mass of the elemental volume; \( w_f = \) fall velocity for suspended-sediment particles of a particular size class; \( u, v, w = \) water-velocity components; \( D_H = \) horizontal mass-diffusivity coefficient; \( D_v = \) vertical mass-diffusivity coefficient.

**Suspended-Sediment Source Terms**

The suspended-sediment 'erosion' source represents entrainment of active-layer and bedload sediment particles into suspension. It is generally accepted that the entrainment of near-bed sediment particles into suspension can be modeled as an upward near-bed mass diffusion flux modified by \( \beta \) to reflect the availability of the particular size class in the active-layer control volume:
\[ S_e = -\beta D_v \left[ \frac{\partial (\rho C)}{\partial z} \right]_a \]  

Subscript 'a' denotes that the mass-diffusion flux is evaluated at a near-bed point some distance \( a \) above the bed surface.

The suspended-sediment 'erosion' source is further modeled as:

\[ S_e = -\beta D_v \left[ (\rho C)_{a+\Delta a} - (\rho C)_a \right] / \Delta a \]  

where \( C_a \) is a near-bed concentration estimated in a way to reflect the action of near-bed flow on the active-layer and bedload particles at a certain bed-surface location. Concentration \( C_{a+\Delta a} \) is a near-bed concentration extrapolated from the suspended-sediment computations.

The suspended-sediment 'deposition' source represents gravitational settling of sediment particles already in suspension, i.e. particles that have been entrained into suspension elsewhere, or entered the model through a boundary, and transported as suspended sediment until reaching the vicinity of a certain location on the bed surface. The suspended-sediment 'deposition' source is modeled as a downward near-bed fall velocity flux:

\[ S_d = (w_f \rho C)_{a+\Delta a} \]  

where concentration \( C_{a+\Delta a} \) is extrapolated from the suspended-sediment computations. Subscript ' \( a + \Delta a \)' denotes that concentration \( C_{a+\Delta a} \) is evaluated at some distance \( a + \Delta a \) above the bed surface.

One should note that when Eq. (10) is integrated over the elemental volume next to the bed, it must contain the same terms as described by Eqs. (11)-(13), i.e. downward near-bed fall-velocity flux and upward near-bed mass diffusion flux modified by \( \beta \).

**Primary Sediment Unknowns and Auxiliary Relations**

If the sediment mixture in a natural watercourse is represented by a total of \( KS \) sediment size classes, then the following sediment variables are considered primary sediment unknowns: (1) bed-surface level \( z_b \) and \( KS \) active-layer size fractions \( \beta \) for each
active-layer elemental volume; and (2) KS suspended-sediment concentrations \( C \) for each elemental volume containing a mixture of water and suspended sediment.

The near-bed concentration \( C_a \), bedload flux \( q_b \), active-layer thickness \( E_m \), active-layer floor 'source' \( S_F \), fall velocity \( w_f \), mass-diffusion coefficient \( D \), and density of mixture containing water and suspended sediment \( \rho \) are in general functions of flow variables and primary sediment unknowns and are treated as auxiliary relations.

The basic nature of the auxiliary relations is described next. Beforehand, it is worth mentioning that the present work establishes a reliable computational framework for solving the relevant conservation laws as correctly as possible, incorporating the best available empirical information, that may be even site-specific for a particular application. The numerical procedure for solution of the sediment equations is formulated without reference to the specific empirical relations that ultimately must be invoked to evaluate the auxiliary relations. This allows for use of any suitable empirical relation when evaluating a particular auxiliary relation, and renders the formal numerical procedure independent of any specific empirical relation. Details of empirical relations currently used to quantify the auxiliary relations are presented in Chapter III.

The near-bed concentration \( C_a \) (for a particular size class of sediment) generally depends on the near-bed flow characteristics and it is evaluated by using an appropriate empirical relation, for example that of van Rijn (1984a).

The net bedload flux is represented herein as:

\[
q_b = (1 - \gamma)\zeta_h \beta q_b^t
\]  

(14)

where \( q_b^t \) = theoretical bedload capacity for a bed containing only sediment of the particular size class, evaluated using an appropriate bedload predictor such as proposed by van Rijn (1984a). This load is adjusted by \( \zeta_h \), a so called hiding factor accounting for the reduction or increase in a particular size class transport rate when it is part of a mixture. Empirical relations such as those proposed by Karim and Kennedy (1982) or Shen and Lu (1983) can be used to evaluate \( \zeta_h \). The adjusted load is modified by \( \beta \) to reflect the availability of the particular size class in the active-layer elemental volume. Finally, the load is modified by \( (1 - \gamma) \) to reflect the fact that some fraction \( \gamma \) of the particular size-class particles is transported as suspended load.

The active-layer thickness \( E_m \) is evaluated by an appropriate empirical concept of the depth of bed material that supplies material for bedload transport and suspended-sedi-
ment entrainment. Examples are the concepts of Karim and Kennedy (1982), Bennet and Nordin (1977), or Borah et al. (1982).

The active-layer floor 'source' \( S_F \) for a particular size class of sediment is derived from the mass-conservation equation for that particular size class in the active-stratum control volume. When the active-layer floor (active-stratum ceiling) descends, then:

\[
S_F = -\rho_s (1 - p) \frac{\partial}{\partial t} [\beta_s (z_b - E_m)]
\]  
(15)
gives the mass of the particular size class, formerly comprising size fraction \( \beta_s \) of the active-stratum control volume, which becomes part of the active-layer elemental volume. When the active-layer floor (active-stratum ceiling) rises, then:

\[
S_F = -\rho_s (1 - p) \frac{\partial}{\partial t} [\beta (z_b - E_m)]
\]  
(16)
gives the mass of the particular size class, formerly comprising size fraction \( \beta \) of the active-layer elemental volume, which becomes part of the active stratum control volume.

Depending on sediment-particle size, different experimental relations can be used to compute particle fall velocity, as described by van Rijn (1984b).

The mass-diffusion coefficient \( D \) is obtained by modifying the momentum-diffusion coefficient \( A \) to reflect the difference in the diffusion of a discrete sediment particle and the diffusion of a fluid 'particle' (or small coherent fluid structure), and also to reflect the damping of the fluid turbulence by the sediment particles, as suggested by van Rijn (1984b).

The local density of water with suspended sediment is modified to reflect the influence of local suspended-sediment concentration by using an appropriate empirical relation such as proposed by Zhou and McCorquodale (1992) or Holly and Rahuel (1989).

**Dimensional Equations in Cartesian Coordinates**

The dimensional governing sediment equations in Cartesian coordinates are summarized below, for convenient reference:

mass-conservation equation for one particular size class of active-layer sediment:
\[
\rho_s(1-p) \frac{\partial (\beta E_m)}{\partial t} + \frac{\partial q_{b,x}}{\partial x} + \frac{\partial q_{b,y}}{\partial y} - \beta \left( D_v \frac{\partial (\rho C)}{\partial z} \right)_a - (w_f \rho C)_{a+\Delta a} - S_F = 0
\]  
(17)

Global mass-conservation equation for bed sediment:

\[
\rho_s(1-p) \frac{\partial z_b}{\partial t} + \sum \left[ \frac{\partial q_{b,x}}{\partial x} + \frac{\partial q_{b,y}}{\partial y} - \beta \left( D_v \frac{\partial (\rho C)}{\partial z} \right)_a - (w_f \rho C)_{a+\Delta a} \right] = 0
\]  
(18)

Mass-conservation equation for a particular size class of suspended sediment:

\[
\begin{aligned}
\frac{\partial (\rho C)}{\partial t} + \frac{\partial}{\partial x} (\rho Cu) + \frac{\partial}{\partial y} (\rho Cv) + \frac{\partial}{\partial z} (\rho Cw) - \frac{\partial}{\partial z} (\rho Cw_f) \\
= \frac{\partial}{\partial x} \left( D_H \frac{\partial (\rho C)}{\partial x} \right) + \frac{\partial}{\partial y} \left( D_H \frac{\partial (\rho C)}{\partial y} \right) + \frac{\partial}{\partial z} \left( D_v \frac{\partial (\rho C)}{\partial z} \right)
\end{aligned}
\]  
(19)

In order to maintain compatibility with the rest of the CH3D code, the governing sediment equations are further manipulated the same way as the governing flow, salinity and temperature equations of CH3D. First, the governing sediment equations are transformed from Cartesian x,y,z coordinates into so-called σ-stretched coordinates x',y', σ (from now on called Cartesian σ-stretched coordinates), then made dimensionless, and, finally, transformed from Cartesian σ-stretched coordinates x',y', σ into curvilinear σ-stretched coordinates ξ, η, σ with a complete transformation used for nonorthogonal curvilinear coordinates ξ, η while the σ coordinate remains unchanged.

**Dimensional Equations in Cartesian σ-stretched Coordinates**

Cartesian σ-stretched coordinates x',y', σ are defined as follows:

\[
x' = x
\]
\[
y' = y
\]
\[
\sigma = \frac{z - \zeta}{\zeta + h} = \frac{z - \zeta}{H}
\]  
(20)
where \( \zeta \) = free-surface elevation, i.e. free-surface displacement (Fig. 3); \( h \) = distance between bed elevation and reference level \( z = 0 \); \( H = \zeta + h \) = water depth.

So-called \( \sigma \)-stretching is a partial transformation of the equations. Components of the position vector (independent variable) are easily transformed by using rules of partial transformation, and keep the same dimensions as in original equations, except for the dimensionless vertical coordinate \( \sigma \). Vector components of dependent variables remain untransformed i.e. aligned with the Cartesian coordinate directions and keep the same dimensions as in original equations, except for the newly defined vertical velocity component \( \omega = \frac{d\sigma}{dt} \). The physical domain in vertical plane (Fig. 3a) transforms into rectangular computational domain (Fig. 3b) with upper boundary \( \sigma = 0 \) (obtained from Eq. (20) for \( z = \zeta \)) and lower boundary \( \sigma = -1 \) (obtained from Eq. (20) for \( z = -h \)). More details on \( \sigma \)-stretching could be found in Sheng and Lick (1980) or Sheng (1983).

The dimensional equations in \( \sigma \)-stretched coordinates read:

mass-conservation equation for one particular size class of active-layer sediment:

\[
\rho_s (1 - p) \frac{\partial (\beta E_m)}{\partial t} + \frac{\partial q_{bx}}{\partial x} + \frac{\partial q_{by}}{\partial y} - \frac{\beta}{H} \left( D_v \frac{\partial (\rho C)}{\partial \sigma} \right)_{\sigma_a} - (w_f \rho C)_{\sigma_a+\Delta\sigma} - S_F = 0 \tag{21}
\]

global mass-conservation equation for bed sediment:

\[
\rho_s (1 - p) \frac{\partial z_b}{\partial t} + \sum \left[ \frac{\partial q_{bx}}{\partial x} + \frac{\partial q_{by}}{\partial y} - \frac{\beta}{H} \left( D_v \frac{\partial (\rho C)}{\partial \sigma} \right)_{\sigma_a} - (w_f \rho C)_{\sigma_a+\Delta\sigma} \right] = 0 \tag{22}
\]

mass-conservation equation for a particular size class of suspended sediment:

\[
\frac{1}{H} \frac{\partial}{\partial t} (H \rho C) + \frac{1}{H} \left[ \frac{\partial}{\partial x} (H \rho C u) + \frac{\partial}{\partial y} (H \rho C v) + \frac{\partial}{\partial \sigma} (H \rho C \omega) + H.O.T. - \frac{\partial}{\partial \sigma} (\rho C w_f) \right] \]

\[
= \frac{\partial}{\partial x} \left( D_H \frac{\partial (\rho C)}{\partial x} \right) + \frac{\partial}{\partial y} \left( D_H \frac{\partial (\rho C)}{\partial y} \right) + H.O.T. + \frac{1}{H^2} \frac{\partial}{\partial \sigma} \left( D_v \frac{\partial (\rho C)}{\partial \sigma} \right) \tag{23}
\]

where H.O.T. are the higher order terms that will be dropped in all that follows.
Figure 3. Coordinate Transformations.
Dimensionless Equations in $\sigma$-stretched Coordinates

In order to non-dimensionalize the equations, first a set of reference variables is defined, then the sediment variables are made dimensionless, and dimensionless numbers are defined.

Reference Variables:

\[
\begin{align*}
    f & = \text{frequency} \\
    x_r & = \text{reference length} \\
    z_r & = \text{reference depth} \\
    u_r & = \text{reference water velocity,} \\
    D_{H_r} & = \text{reference lateral mass-diffusivity coefficient,} \\
    D_{v_r} & = \text{reference vertical mass-diffusivity coefficient,} \\
    A_{H_r} & = \text{reference lateral momentum-diffusivity coefficient,} \\
    A_{v_r} & = \text{reference vertical momentum-diffusivity coefficient,} \\
    w_r & = \text{reference fall velocity (computed with maximum sediment-particle diameter and reference water velocity),} \\
    \rho_r & = \text{reference density of water-sediment mixture,}
\end{align*}
\]

Dimensionless Variables

\[
\begin{align*}
    t^* &= t \cdot f & \text{= time} \\
    x^* &= \frac{x}{x_r} & \text{= x coordinate} \\
    y^* &= \frac{y}{x_r} & \text{= y coordinate} \\
    \sigma^* &= \sigma & \text{= \sigma coordinate} \\
    u^* &= \frac{u}{u_r} & \text{= x-direction velocity component} \\
    v^* &= \frac{v}{u_r} & \text{= y-direction velocity component} \\
    w^* &= \frac{w}{u_r} & \text{= \sigma-direction velocity component} \\
    H^* &= \frac{H}{z_r} & \text{= depth} \\
    D_{H^*} &= \frac{D_{H}}{D_{H_r}} & \text{= lateral mass-diffusivity coefficient}
\end{align*}
\]
\[ D_v^* = \frac{D_v}{D_{vr}} \]  
= vertical mass-diffusivity coefficient

\[ z_b^* = \frac{z_b}{z_t} \]  
= bed elevation

\[ \beta^* = \beta \]  
= active-layer size fraction

\[ C^* = C \]  
= suspended-sediment concentration

\[ \rho_s^* = \frac{\rho_s}{\rho_r} \]  
= sediment-particle density

\[ p^* = p \]  
= bed-material porosity

\[ E_m^* = \frac{E_m}{z_t} \]  
= active-layer thickness

\[ q_b^* = \frac{q_b}{\rho_r z_t x_f} \]  
= bedload flux

\[ S_F^* = \frac{S_F}{\rho_r z_t x_f} \]  
= active-layer floor 'source'

\[ \rho^* = \frac{\rho}{\rho_r} \]  
= density of water-sediment mixture

\[ w_f^* = \frac{w_f}{w_{fr}} \]  
= fall velocity

**Dimensionless Numbers**

\[ R_o = \frac{u_r}{f x_r} \]  
= Rossby number

\[ R_o f = \frac{w_{fr}}{f z_r} \]  
= fall-velocity Rossby number

\[ E_{KH} = \frac{A_{Hr}}{f x_r^2} \]  
= lateral Ekman number

\[ E_{Kv} = \frac{A_{vr}}{f z_r^2} \]  
= vertical Ekman number

\[ S_{CH} = \frac{A_{Hr}}{D_{Hr}} \]  
= lateral Schmidt number

\[ S_{Cv} = \frac{A_{vr}}{D_{vr}} \]  
= vertical Schmidt number
Dimensionless Equations

When the '*' is omitted, the dimensionless equations read:

mass-conservation equation for one particular size class of active-layer sediment:

$$\rho_s (1 - p) \frac{\partial (\beta E_m)}{\partial t} + \frac{\partial q_{bx}}{\partial x} + \frac{\partial q_{by}}{\partial y} - \frac{E_{k_v} \beta}{S_{c_v}} \frac{D_v}{H} \frac{\partial (\rho C)}{\partial \sigma} \bigg|_{\sigma_a} - R_{o_1} (w_f \rho C)_{\sigma_a+\Delta} - S_F = 0 \quad (24)$$

global mass-conservation equation for bed sediment:

$$\rho_s (1 - p) \frac{\partial z_b}{\partial t} + \Sigma \left[ \frac{\partial q_{bx}}{\partial x} + \frac{\partial q_{by}}{\partial y} - \frac{E_{k_v} \beta}{S_{c_v}} \frac{D_v}{H} \frac{\partial (\rho C)}{\partial \sigma} \bigg|_{\sigma_a} - R_{o_1} (w_f \rho C)_{\sigma_a+\Delta} \right] = 0 \quad (25)$$

mass-conservation equation for a particular size class of suspended sediment:

$$\frac{1}{H} \frac{\partial}{\partial t} (H \rho C) + \frac{R_o}{H} \left[ \frac{\partial}{\partial x} (H \rho C_u) + \frac{\partial}{\partial y} (H \rho C_v) + \frac{\partial}{\partial \sigma} (H \rho C_w) \right] - \frac{R_{o_1}}{H} \frac{\partial}{\partial \sigma} (\rho C_w_f)$$

$$= \frac{E_{k_H}}{S_{c_H}} \left[ \frac{\partial}{\partial x} \left( H \frac{\partial (\rho C)}{\partial x} \right) + \frac{\partial}{\partial y} \left( H \frac{\partial (\rho C)}{\partial y} \right) \right] + \frac{E_{k_v}}{S_{c_v}} \frac{1}{H^2} \frac{\partial}{\partial \sigma} \left( D_v \frac{\partial (\rho C)}{\partial \sigma} \right) \quad (26)$$

Dimensionless Equations in Curvilinear \(\sigma\)-stretched Coordinates

As previously stated, the governing sediment equations are transformed from Cartesian \(\sigma\)-stretched coordinates \(x', y'\), \(\sigma\) into curvilinear \(\sigma\)-stretched coordinates \(\xi, \eta, \sigma\) with a complete transformation used for the nonorthogonal curvilinear coordinates \(\xi, \eta\) while the \(\sigma\) coordinate remains unchanged. A complete transformation means that all vector components, not only those representing independent variables, but also those representing dependent variables, formerly aligned with Cartesian coordinate directions \(x,y\), are now aligned with base vectors tangential to curvilinear coordinate lines \(\xi\) and \(\eta\). The physical domain in \((x,y)\) plane (Fig. 3c) transforms into a rectangular computational domain with a unit-square grid (Fig. 3d). Vector components aligned with the vertical \(\sigma\) direction remain unchanged.

Starting from a Cartesian tensor form of the governing equations, one can easily write general curvilinear tensor equations. The general curvilinear tensor form of the equa-
tions looks like a generalization of the Cartesian tensor form with partial derivatives replaced by general covariant derivatives, velocity expressed as a contravariant vector, and other vectors respecting the appropriate covariant/contravariant order. Finally, the working form of the governing sediment equations is obtained by expanding the tensor equations. The expanded form of the dimensionless sediment equations in curvilinear $\sigma$-stretched coordinates reads:

mass-conservation equation for one particular size class of active-layer sediment:

$$\rho_s(1-p) \frac{\partial(BE_m)}{\partial t} + \frac{1}{J} \left[ \frac{\partial}{\partial \xi} \left( Jq_{b_\xi} \right) + \frac{\partial}{\partial \eta} \left( Jq_{b_\eta} \right) \right] - \frac{E_{kv}}{S_{cv}} \left( \frac{D_v \partial (\rho C)}{\partial \sigma} \right)_{\sigma_a} - R_{of} (w_f \rho C)_{\sigma_a + \Delta \sigma} - S_F = 0$$  \hspace{1cm} (27)

global mass-conservation equation for bed sediment:

$$\rho_s(1-p) \frac{\partial z_b}{\partial t} + \frac{1}{J} \left[ \frac{\partial}{\partial \xi} \left( Jq_{b_\xi} \right) + \frac{\partial}{\partial \eta} \left( Jq_{b_\eta} \right) \right] - \frac{E_{kv}}{S_{cv}} \left( \frac{D_v \partial (\rho C)}{\partial \sigma} \right)_{\sigma_a} - R_{of} (w_f \rho C)_{\sigma_a + \Delta \sigma} = 0$$  \hspace{1cm} (28)

mass-conservation equation for a particular size class of suspended sediment:

$$\frac{1}{H} \frac{\partial}{\partial t} (HpC) + \frac{R_{of}}{H} \left[ \frac{1}{J} \frac{\partial}{\partial \xi} \left( JH \rho C_v \right) + \frac{1}{J} \frac{\partial}{\partial \eta} \left( JH \rho C_v \right) + \frac{\partial}{\partial \sigma} \left( JH \rho C_{\omega} \right) \right]$$

$$- \frac{R_{of}}{H} \frac{\partial}{\partial \sigma} \left( \rho C_{w_f} \right) = \frac{E_{kh}}{S_{ch}} \frac{1}{J} \left[ \frac{\partial}{\partial \xi} \left( D_H \frac{g_{22}}{J} \frac{\partial (\rho C)}{\partial \xi} \right) - \frac{\partial}{\partial \xi} \left( D_H \frac{g_{12}}{J} \frac{\partial (\rho C)}{\partial \eta} \right) \right]$$

$$- \frac{\partial}{\partial \eta} \left( D_H \frac{g_{12}}{J} \frac{\partial (\rho C)}{\partial \xi} \right) + \frac{\partial}{\partial \eta} \left( D_H \frac{g_{11}}{J} \frac{\partial (\rho C)}{\partial \eta} \right)$$

$$+ \frac{E_{kv}}{S_{cv}} \frac{1}{H^2} \frac{\partial}{\partial \sigma} \left( D_v \frac{\partial (\rho C)}{\partial \sigma} \right)$$  \hspace{1cm} (29)

where $q_{b_\xi}, q_{b_\eta}$ = contravariant bedload-flux components in $\xi$ and $\eta$ directions, respectively; $u, v, \omega$ = contravariant velocity components in $\xi, \eta$ and $\sigma$ directions, respectively;
\( g_{11}, g_{12}, g_{22} = \text{metric coefficients, and } J = \text{square root of the appropriate Jacobian-matrix determinant:} \)

\[
\begin{align*}
    g_{11} &= \left( \frac{\partial x}{\partial \xi} \right)^2 + \left( \frac{\partial y}{\partial \eta} \right)^2 \\
    g_{12} &= \frac{\partial x}{\partial \xi} \frac{\partial x}{\partial \eta} + \frac{\partial y}{\partial \xi} \frac{\partial y}{\partial \eta} \\
    g_{22} &= \left( \frac{\partial x}{\partial \eta} \right)^2 + \left( \frac{\partial y}{\partial \eta} \right)^2 \\
    J &= \sqrt{g_{11}g_{22} - g_{12}^2} = \frac{\partial x}{\partial \xi} \frac{\partial y}{\partial \eta} - \frac{\partial x}{\partial \eta} \frac{\partial y}{\partial \xi}
\end{align*}
\]

The principles of complete transformation of the basic fluid mechanics equations in general curvilinear coordinates are presented by Sokolnikoff (1951), Aris (1962) or, more recently, by Richmond et al. (1986). More details on sediment equations in curvilinear coordinates can be found in Spasojevic (1988).

CHAPTER III

NUMERICAL SOLUTION

Strategy for Approximate Solution

Fluid flow, sediment transport and bed evolution are elements of a coupled process. Sediment particles are being entrained, moving, and being deposited due to the action of fluid flow, causing bed evolution which is a direct consequence of sediment transport. On the other hand, change of the bed-surface level changes the flow domain, and thus the flow. Also, change in the size distribution of sediment particles at the bed surface changes the roughness of the bed surface, which in turn directly influences the bed shear stress. The suspended sediment changes the density of the water-sediment mixture.

On the other hand, one should note that fluid flow and sediment processes have quite different time scales. Bed-sediment transport and bed evolution are defined without following each sediment particle separately, but by considering sediment transport 'en masse'. It is impossible, therefore, to account for the changes at the bed due to movement.
of a single sediment particle, changes that have the same time scale as the flow process.
Instead, one focuses on global changes in bed level and bed-surface size distribution, with
their much larger time scale. Changes in bed level during a time step appropriate for flow
computations are usually too small to change the flow domain and flow field significantly.
Similarly, changes in bed-surface size distribution i.e. roughness or friction, during a time
step appropriate for flow computations, are insufficient to influence the flow field significa-
tantly. Only suspended-sediment transport has the same time scale as fluid flow.
However, in most cases suspended-sediment concentrations in natural watercourses do not
change abruptly with time, which suggests that changes in the density of the water and
sediment mixture during a time step appropriate for flow computations, are also generally
insufficient to influence the flow field significantly.

Given the above discussion, it would appear possible to uncouple water and sedi-
ment computations at the scale of one time step.

Furthermore, the small time step required by the numerical techniques of CH3D,
offers the possibility to couple suspended- and bed-sediment computations in an iterative
manner, as described below.

The suspended-sediment source terms are the link relating suspended and bed
sediment computations. The suspended-sediment 'deposition' source term depends on the
suspended-sediment concentrations, while the suspended-sediment 'erosion' source term
depends both on the suspended-sediment concentrations and on the active-layer size frac-
tions. The mass-conservation equations for each size class of active-layer sediment and the
global mass-conservation equation for bed sediment, with the appropriate suspended-sedi-
tment source terms, are solved in a coupled manner, for one iteration of one computational
time step, by assuming the suspended-sediment concentrations to be known from the pre-
vious iteration. An improved estimate of active-layer size fractions and the bed-surface ele-
vation is thus obtained. The mass-conservation equations for each size fraction of sus-
pended sediment are then solved for the same computational time step, by assuming the
active-layer size fractions to be known from the bed-sediment computations. The whole
procedure is repeated iteratively until a convergence criterion is satisfied.

However, in the present work a single global iteration is employed in each time
step, in recognition of the fact that numerical techniques of CH3D code require time step of
order of minutes and that changes in bed level, bed surface composition, and the density of
the water-sediment mixture are generally small during such a small time step, although
changes may accumulate to significant amounts over several time steps.

Figure 4 shows a portion of the computational domain. P denotes a computational point under consideration at the bed, while C denotes a suspended-sediment computational point under consideration. Computational points neighboring C (i.e. P) in the ξ direction are denoted as E (east), FE (far east), W (west) and FW (far west). Computational points neighboring C in the η direction are denoted as N (north), FN (far north), S (south) and FS (far south). Computational points neighboring C (i.e. P) in the σ direction are denoted as T (top) and B (bottom). Similarly, faces of control volume built around C are denoted as e (east), w (west), n (north), s (south), t (top), and b (bottom). Indexes i,j,k denote computational points along the ξ-, η- and σ-direction coordinate lines, respectively.

As previously stated, \( C_{\sigma_{a+\Delta a}} \) is a near-bed concentration extrapolated from the suspended sediment computations. A simple linear extrapolation gives:

\[
(pC)_{\sigma_{a+\Delta a}} = (1-c)(pC)_{k=2} + c(pC)_{k=1} \quad (31)
\]

where \( c = \) extrapolation coefficient; subscript \( k \) defines computational points along the vertical σ-direction line (Fig. 4b i.e. 4c).

Taking into account Eq. (12) and (31), Eqs. (27) and (28) are discretized by integrating them over the time step and the control volume built around a main computational point P (Fig. 4a):

global mass-conservation equation for bed-sediment

\[
\rho_s(1-p)J_p \frac{(z_b)_{p}^{n+1} - (z_b)_{p}^{n}}{\Delta t} + \sum \left[ \theta \frac{J_e(q_{b_{\xi}})_{e}^{n+1} - J_w(q_{b_{\xi}})_{w}^{n+1}}{\Delta \xi} + (1-\theta) \frac{J_e(q_{b_{\xi}})_{e}^{n} - J_w(q_{b_{\xi}})_{w}^{n}}{\Delta \xi} \right] \\
+ \sum \left[ \theta \frac{J_n(q_{b_{n}})_{n}^{n+1} - J_s(q_{b_{n}})_{s}^{n+1}}{\Delta \eta} + (1-\theta) \frac{J_n(q_{b_{n}})_{n}^{n} - J_s(q_{b_{n}})_{s}^{n}}{\Delta \eta} \right] \\
- \frac{E_{k_{\nu}}}{S_{c_{\nu}}} J_p \theta \frac{\beta_{p}^{n+1} D_{p}^{n+1} P_{v_{a}}^{n+1}}{H_{p}^{n+1} D_{v_{a}}} \left[ (1-c)(pC)_{k=2}^{n+1} + c(pC)_{k=1}^{n+1} - (pC)_{a_{\Delta a}}^{n+1} \right] \frac{1}{\Delta \sigma_{\Delta a}}
\]
Figure 4. Computational Grid.
Figure 4. Continued.
mass-conservation equation for a particular size class of active-layer sediment

\[
\rho_s (1 - p) J_p \left( \frac{\beta E_m n^{n+1}}{\Delta t} - \frac{(\beta E_m)^n}{\Delta t} \right) + \\
\theta \left( \frac{J_e (q_{b_e})_{n+1}}{\Delta \xi} - J_w (q_{b_w})_{n+1} \right) + (1 - \theta) \left( \frac{J_e (q_{b_e})_n}{\Delta \xi} - J_w (q_{b_w})_n \right) \\
\theta \left( \frac{J_n (q_{b_n})_{n+1}}{\Delta \eta} - J_s (q_{b_s})_{n+1} \right) + (1 - \theta) \left( \frac{J_n (q_{b_n})_n}{\Delta \eta} - J_s (q_{b_s})_n \right) \\
- \frac{E_{k_v} J_p (1 - \theta) \frac{\beta_p n^{n+1}}{H_p^{n+1}} D_{v_s}^{n+1} (1 - c)(\rho C)_{k=2}^{n+1} + c(\rho C)_{k=1}^{n+1} - (\rho C)_{s_a}^{n+1}}{\Delta \sigma_{\Delta a}} \\
- \frac{E_{k_v} J_p (1 - \theta) \frac{\beta_p n^p}{H_p^n} D_{v_s}^n (1 - c)(\rho C)_{k=2}^n + c(\rho C)_{k=1}^n - (\rho C)_{s_a}^n}{\Delta \sigma_{\Delta a}} \\
- J_p R_{of} w_f \theta \left[ (1 - c)(\rho C)_{k=2}^{n+1} + c(\rho C)_{k=1}^{n+1} \right] \\
- J_p R_{of} w_f (1 - \theta) \left[ (1 - c)(\rho C)_{k=2}^n + c(\rho C)_{k=1}^n \right] - J_c S_F = 0
\]  

where subscripts e,w,n,s denote east, west, north and south control-volume faces in the \((\xi, \eta)\) plane, respectively (Fig. 4a); subscript \(P\) denotes a main, i.e. cell-centered, computational point; superscript \(n\) denotes the time level; and \(\theta\) = weighting (implicitation) factor.

Further treatment of the bedload fluxes is inspired by their physical character. Bedload flux acting through a control-volume face is governed by the staggered velocity component perpendicular to the particular control-volume face. A control volume is assumed to have a uniform size distribution of sediment particles at the bed. Consequently it is appropriate to suppose that the fluid flowing through a control-volume face transports bedload-sediment particles contained in the control volume on the upstream side. The bedload flux through a control-volume face knows nothing about the active-layer size frac-
tions in the control volume towards which it is heading, but carries the full legacy of the control volume from which it is coming. This treatment of bedload flux resembles the 'tank-and-tube' model of Gosman at al (1969), used for heat-transfer problems, and is also the essence of an upwind scheme.

The bedload flux at time level \( n+1 \) is evaluated by using flow variables at time level \( n+1 \), obtained from the flow computations and related to the control-volume face. The sediment variables (primarily the active-layer size fraction) needed to evaluate the bedload flux, are related to the 'upwind' control volume and expressed explicitly (using their values at time level \( n \)).

Evaluation of the bedload flux in the manner described above means that all bedload-flux components are expressed explicitly in terms of bed elevation and active-layer size fractions. Also, due to iterative coupling of the bed (i.e. active-layer sediment) computations and suspended-sediment computations, suspended-sediment source terms are expressed explicitly in terms of suspended-sediment concentrations. It is useful to point out those elements of the discretized equations that are expressed explicitly in terms of sediment variables by introducing a special notation for them.

The discretized global mass-conservation equation for bed sediment (Eq. (32)) can be written as:

\[
\rho_s(1 - p)J_p \frac{(z_b)_p^{n+1} - (z_b)_p^n}{\Delta t} + \sum \theta (\text{div} q_b)_p^{n+1} + (1 - \theta)(\text{div} q_b)_p^n + \theta \beta_p^{n+1}(S_e)_p^{n+1} + (1 - \theta)(S_e)_p^n - \theta(S_d)_p^{n+1} - (1 - \theta)(S_d)_p^n = 0
\]  

(34)

while the discretized mass-conservation equation for one size class of active-layer sediment (Eq. (33) is written as:

\[
\rho_s(1 - p)J_p \frac{(\beta E_m)_p^{n+1} - (\beta E_m)_p^n}{\Delta t} + \theta(\text{div} q_b)_p^{n+1} + (1 - \theta)(\text{div} q_b)_p^n
\]

(35)
\[ + \theta p^{n+1}(S_e)_p^n + (1 - \theta)(S_e)_p^n \]

\[- \theta(S_d)_p^{n+1} - (1 - \theta)(S_d)_p^n - J_p S_F = 0 \]

where:

\[ (\text{div}q_b)_p^{n+1} = \frac{J_e(q_{b_x})_e^{n+1} - J_w(q_{b_z})_w^n + J_n(q_{b_S})_n^{n+1} - J_s(q_{b_S})_s^{n+1}}{\Delta \xi} + \frac{J_n(q_{b_S})_n^n - J_s(q_{b_S})_s^n}{\Delta \eta} \]  

(36)

stands for the divergence of the bedload flux vector, evaluated by using flow variables at time level \((n+1)\) and bed i.e. active-layer sediment variables at time level \(n\).

\[ (\text{div}q_b)_p^n = \frac{J_e(q_{b_x})_e^n - J_w(q_{b_z})_w^n + J_n(q_{b_S})_n^n - J_s(q_{b_S})_s^n}{\Delta \xi} + \frac{J_n(q_{b_S})_n^n - J_s(q_{b_S})_s^n}{\Delta \eta} \]  

(37)

stands for the divergence of the bedload flux vector, evaluated by using flow and sediment variables at time level \(n\).

\[ (S_e)_p^{n+1} = -\frac{E_{k_v}}{S_{c_S}} J_p \frac{D_{v_S}^{n+1}(1-c)(\rho C)_{k=2}^{n+1} + c(\rho C)_{k=1}^{n+1} - (\rho C)_{s_k}^{n+1}}{\Delta \sigma_{\Delta a}} \]  

(38)

stands for the 'theoretical' suspended-sediment 'erosion' source term ('theoretical' in the since that it is not reduced by \(\beta\), i.e. it does not account for the availability of a particular active-layer size fraction), evaluated using flow variables at time level \((n+1)\) and previous-iteration suspended-sediment concentrations at time level \((n+1)\).

\[ (S_e)_p^n = -\frac{E_{k_v}}{S_{c_S}} J_p \frac{D_{v_S}^n(1-c)(\rho C)_{k=2}^n + c(\rho C)_{k=1}^n - (\rho C)_{s_k}^n}{\Delta \sigma_{\Delta a}} \]  

(39)

stands for the suspended-sediment 'erosion' source term, evaluated by using flow and sediment variables at time level \(n\).
stands for the suspended-sediment 'deposition' source term, evaluated by using flow variables at time level \((n+1)\) and previous-iteration suspended-sediment concentrations at time level \((n+1)\).

\[
(S_d)_p^n = J_p R_o f \left[(1 - c)(\rho C)_k^{n=2} + c(\rho C)_k^{n=1}\right]
\]  

(41)

stands for the suspended-sediment 'deposition' source term, evaluated by using flow and sediment variables at time level \(n\).

The discretized equations (one Eq.(34) and KS Eqs.(35), written for a particular main computational point \(P\), form an 'explicit' system of algebraic equations at \(P\). 'Explicit' in this context means that all unknown sediment variables (bed elevation and KS active-layer size fractions at time level \(n+1\)) are located at point \(P\) only. Sediment variables located at neighboring computational points appear explicitly, i.e. as known values.

Even though the solution of an 'explicit' system of algebraic equations at the main computational point \(P\) does not depend on neighboring points, it still requires boundary conditions at inflow boundaries in order to evaluate bedload fluxes through the appropriate inflow control-volume faces. It is enough to assign known active-layer size fractions at inflow boundaries, requiring the basic constraint (sum of all fractions equal to unity) to be satisfied. The latter requirement actually replaces a known bed-surface elevation as an inflow boundary condition, even though the bed elevation itself appears to be unnecessary for evaluation of the bedload flux through an inflow-boundary face.

**Numerical Method for Mass-Conservation Equation for Suspended-Sediment**

In the original CH3D temperature and salinity computations, all but the vertical diffusion terms are discretized by using a numerical method called QUICKEST (Quadratic Upstream Interpolation for Convective Kinematics with Estimated Streaming Terms), developed by Leonard (1979). In order to be compatible with the rest of CH3D code, the same strategy is used herein for the mass-conservation equation for suspended sediment (Eq. (29)). Except for the fall-velocity and vertical-diffusion terms, all other terms in the mass-conservation equation for suspended-sediment are discretized by using a generalized version of the QUICKEST method. The fall-velocity term is discretized by using an upwind finite difference scheme, while the vertical-diffusion term is discretized by using a time weighted central differencing. The basic principles of Leonard's (1979) QUICKEST method, applied to a one-dimensional advection-diffusion equation in Cartesian coordinates.
(as in the original paper), are briefly outlined in Appendix A. Generalization of the original QUICKEST method to accommodate appropriate terms in the three-dimensional suspended-sediment equation in general curvilinear coordinates is presented in Appendix B. Appendix B also contains all details on discretization of the mass-conservation equation for suspended sediment (Eq. (29)) with the appropriate boundary conditions.

Equation (B90) (Appendix B) is a discretized mass-conservation equation for suspended sediment with special notation introduced to point out those elements of the equation that are expressed explicitly in terms of sediment variables:

\[
J_c H_c^{n+1} (pC)_c^{n+1} - J_c H_c^n (pC)_c^n = (\text{adv})_w - (\text{adv})_e + (\text{dif})_e - (\text{dif})_w \\
(1) \quad (2) \quad (3) \quad (4) \quad (5) \quad (6)
\]

\[
+ (\text{adv})_s - (\text{adv})_n + (\text{dif})_n - (\text{dif})_s \\
(7) \quad (8) \quad (9) \quad (10)
\]

\[
+ (\text{adv})_b - (\text{adv})_t \\
(11) \quad (12)
\]

\[
+ \theta R_{of} J_c w_f (pC)_c^{n+1} \frac{\Delta t}{\Delta \sigma} + (1 - \theta)(\text{fall})_t^n \frac{\Delta t}{\Delta \sigma} \\
(13) \quad (14)
\]

\[
- \theta R_{of} J_c w_f (pC)_c^{n+1} \frac{\Delta t}{\Delta \sigma} - (1 - \theta)(\text{fall})_b^n \frac{\Delta t}{\Delta \sigma} \\
(15) \quad (16)
\]

\[
+ \frac{E_k}{S_c} J_c \frac{D_v^{n+1}}{H_c^{n+1}} \frac{(pC)_e^{n+1} - (pC)_c^{n+1}}{\Delta \sigma} \frac{\Delta t}{\Delta \sigma} + (1 - \theta)(\text{dif})_e^n \frac{\Delta t}{\Delta \sigma} \\
(17) \quad (18)
\]

\[
- \frac{E_k}{S_c} J_c \frac{D_v^{n+1}}{H_c^{n+1}} \frac{(pC)_c^{n+1} - (pC)_b^{n+1}}{\Delta \sigma} \frac{\Delta t}{\Delta \sigma} - (1 - \theta)(\text{dif})_b^n \frac{\Delta t}{\Delta \sigma} \\
(19) \quad (20)
\]

where subscripts e, w, n, s, t, b, denote the east, west, north, south, top and bottom faces of the control volume built around a main computational point C (Fig. 4), respectively; subscripts T and B denote main computational points neighboring C at the top and bottom (Fig. 4b or 4c), respectively.
The exact form of all terms in Eq. (42) can be found in Appendix B, i.e. in Eqs. (B35), (B55) and (B79). The general meaning of the terms is briefly explained herein. Terms (1) and (2) originate in the discretization of the local rate-of-change term (Eq. (29)). Terms (3), (4), (7), (8), (11) and (12) represent advection fluxes through the west, east, south, north, bottom and top faces of the control volume built around the main computational point C (Fig. 4), respectively, and originate in the discretization of the appropriate advection terms. Terms (5), (6), (9) and (10) represent diffusion fluxes through the east, west, north and south faces of the control volume, respectively, and come from the discretization of the appropriate diffusion terms. Terms (1) to (12) are discretized by using the generalized QUICKEST method (Appendix B). Terms (13) and (15) represent fall-velocity fluxes, at the current time level (n+1), through the top and bottom faces of the control volume built around C, respectively. Terms (14) and (16) represent fall-velocity fluxes through the top and bottom faces at the known, previous, time level n, respectively. Terms (13) to (16) originate in the discretization of the fall-velocity term (using an upwind differencing). Terms (17) and (19) represent diffusion fluxes, at the current time level (n+1), through the top and bottom faces of the control volume built around C, respectively. Terms (18) and (20) represent diffusion fluxes through the top and bottom faces at the known, previous, time level n, respectively. Terms (17) to (20) originate in the discretization of the vertical diffusion term (using central differencing).

Terms (2) to (12) as well as (14), (16), (18) and (20) in Eq. (42) are all expressed explicitly in terms of sediment variables. Thus, Eq. (42) can be rewritten as:

\[ a(pC)^{n+1}_B + b(pC)^{n+1}_C + c(pC)^{n+1}_T = d \]  

where a, b, c, d are known coefficients presented in Appendix B. The sole unknowns in Eq. (43) are the volumetric suspended-sediment concentrations (pC).

The boundary conditions in the \( \xi \)- and \( \eta \)-coordinate directions (presented in detail in Appendix B) do not influence the overall solution algorithm, for all advection and diffusion fluxes in \( \xi \) and \( \eta \) directions are expressed explicitly in terms of sediment variables. Boundary conditions at the bed and free surface (also presented in detail in Appendix B) are briefly discussed below.

For a point C next to the free surface, advection, fall-velocity and diffusion fluxes through the free surface (top face of the control volume around C) are equal to zero. Therefore, the terms (12), (13), (14), (17) and (18) in Eq. (42) are equal to zero. Equation (42), written for a point next to the free surface, is easily rewritten in the form of Eq. (43), where a, b, c, d, are known coefficients (Appendix B).
For a point C next to the bed, the advection flux through the bed surface (bottom face of the control volume around C) is equal to zero. Thus term (11) in Eq. (42) is equal to zero. The fall-velocity flux through the bottom face of the control volume actually represents deposition of suspended-sediment particles onto the bed. Terms (15) and (16) in Eq. (42) are treated as 'deposition' source terms (Eqs. (40) and (41)) and rewritten as:

\[ \theta R_t J_c w_f \left[ (1-c)(\rho C)_T^{n+1} + c(\rho C)_c^{n+1} \right] \frac{\Delta t}{\Delta \sigma} \]

and

\[ (1-\theta)(S_d)_p^n \frac{\Delta t}{\Delta \sigma} \]

respectively. The diffusion flux through the bottom face of the control volume actually represents entrainment of sediment particles from the bed into suspension. Terms (19) and (20) in Eq. (42) are treated as 'erosion' source terms (Eqs. (38) and (39)) and rewritten as:

\[ \frac{E_k}{S_c} J_c \beta_p^{n+1} \frac{D_v h_b^{n+1}}{H_c^{n+1}} \frac{(1-c)(\rho C)_T^{n+1} + c(\rho C)_c^{n+1} - (\rho C)^{n+1}}{\Delta \sigma} \Delta \sigma \]

and

\[ (1-\theta)(S_e)_p^n \frac{\Delta t}{\Delta \sigma} \]

respectively. It should be noted that, due to the iterative coupling of bed (i.e. active-layer) sediment computations and suspended-sediment computations, the active-layer size fraction at time (n+1) appears as an explicit value, known from the previous iteration, in the diffusion-flux term (19) when Equation (42) is written for a point next to the bed. Therefore, Equation (42) written for points next to the bed can also be rewritten in form of the Eq. (43), where a, b, c, d, are known coefficients (Appendix B).

The discretized mass-conservation equation for a particular size class of suspended sediment (Eq. (43)) is implicit in the vertical direction. When a particular size class Eq. (43) is written for each computational point C along a single \( \sigma \)-direction line, the result is a system of K algebraic equations with K unknown volumetric concentrations of the particular suspended-sediment size class (where K is the number of computational points C along a \( \sigma \)-direction line - Fig. 4b or 4c).

There are KS Eqs. (43) at each computational point C, one for each size class of suspended sediment. Equations (43) written for the same computational point C but different size classes of suspended sediment are theoretically coupled through the 'erosion'
source term containing the active-layer size fraction. Active-layer size fractions are mutually dependent due to the basic constraint (Eq. (4)) built into the appropriate mass-conservation equations. However, due to the iterative coupling of bed (i.e. active layer) computations and suspended-sediment computations, Eqs. (43), written for the same computational point C but different size fractions, become mutually independent during one iteration.

Algorithm for Solution of Discretized Sediment Equations

The global steps of the solution algorithm for the discretized sediment equations are as follows:

(1) One discretized global mass-conservation equation for bed sediment (Eq. (34)) and KS discretized mass-conservation equations for active-layer sediment (Eq. (35)) are solved simultaneously at the bed point P (Fig. 4b i.e. 4c). Computed are the following primary sediment unknowns: one bed-surface elevation and KS active-layer size fractions. Suspended-sediment concentrations, appearing in the source terms, are considered to be known from previous iteration.

(2) The system of K discretized mass-conservation equations for a particular size class of suspended sediment, one Eq. (43) for each computational point C along the σ-direction line above the same bed point P as in step (1) (Fig. 4b i.e. 4c), is solved. Computed are the volumetric concentrations, for a particular size class of suspended sediment, at all computational points C along the σ-direction line above the same bed point P as in step (1). The appropriate size class active-layer size fraction, computed in step (1), is used to evaluate the coefficients in Eq. (43) for a computational point C next to the bed.

(3) Step (2) is repeated for each suspended-sediment size class.

(4) Steps (1) to (3) are repeated until the appropriate convergence criteria are satisfied.

(5) Steps (1) to (4) are repeated throughout the computational domain, for each bed point P and all points C along the σ-direction line above that particular bed point P.

Solution of Discretized Global Mass-Conservation Equation for Bed Sediment and Mass-Conservation Equations for Active-Layer Sediment at the Point

The discretized equations to be solved simultaneously for the same point at the bed, are: (1) one discretized global mass-conservation equation for bed sediment (Eq. (34)) and
(2) KS discretized mass-conservation equations for active-layer sediment (Eq. (35)). The primary sediment unknowns are: (1) one bed-surface elevation and (2) KS active-layer size fractions.

Unknown sediment variables at a main computational point P can be thought of as being components of a sediment-variables vector $\mathbf{s}$ at point P:

$$\mathbf{s}^{n+1} = \left( z_b^{n+1}, \beta_1^{n+1}, ..., \beta_{KS}^{n+1}, ..., \beta_{KS}^{n+1} \right)$$  \hspace{1cm} (44)

or, in more compact notation:

$$\mathbf{s}^{n+1} = (s_1, s_{KS+1}) \hspace{1cm} ks = 1, KS$$  \hspace{1cm} (45)

where subscript $ks$ is introduced to denote the $ks$-th size class of the sediment mixture under consideration.

Equation (34), the discretized global mass-conservation equation for bed sediment, can be symbolically written as:

$$F_1(\mathbf{s}^{n+1}) = 0$$  \hspace{1cm} (46)

The discretized mass conservation equations for active-layer sediment, one Eq.(35) for each of KS size classes, are written as:

$$F_{k, s+1}(\mathbf{s}^{n+1}) = 0 \hspace{1cm} ks = 1, KS$$  \hspace{1cm} (47)

Equations (46) and (47) form a system of nonlinear algebraic equations which can be linearized and solved iteratively by using a Newton-Raphson algorithm. The resulting system of linear algebraic equations can be written as:

$$\left[ \frac{\partial F_1}{\partial \mathbf{s}} \right] \Delta \mathbf{s} = -F_1(\mathbf{m}_s^{n+1})$$  \hspace{1cm} (48)

$$\left[ \frac{\partial F_{k, s+1}}{\partial \mathbf{s}} \right] \Delta \mathbf{s} = -F_{k, s+1}(\mathbf{m}_s^{n+1}) \hspace{1cm} ks = 1, KS$$  \hspace{1cm} (49)
in which \( \frac{\partial F}{\partial \bar{s}} \), evaluated with previous-iteration values of the sediment-variables vector \( m \bar{s}^{n+1} \), represents one row of the Jacobian matrix of coefficients. Superscript \( m \) denotes the Newton-Raphson iteration level.

The unknown vector of sediment-variable corrections \( \Delta \bar{s} \) can be written as:

\[
\Delta \bar{s} = (\Delta s_1, \Delta s_{ks+1}) \quad ks = 1, KS
\]  

or:

\[
\Delta \bar{s} = (\Delta z_b, \Delta \beta_1, ..., \Delta \beta_{KS}, ..., \Delta \beta_{KS})
\]

Coefficients in the Jacobian matrix are presented in more detail in Appendix C. The inverse matrix is computed by using a maximum pivot strategy (Carnahan, Luther and Wilkes (1969) for example).

When the vector of sediment-variable corrections at point \( P \) is obtained, the current-iteration value of the sediment-variable vector at \( P \) is computed as:

\[
m^{+1} \bar{s}^{n+1} = m \bar{s}^{n+1} + \Delta \bar{s}
\]

Iterations continue until the following convergence criterion is satisfied:

\[
\Delta \bar{s} \leq \bar{\varepsilon}
\]

where \( \bar{\varepsilon} \) is a convergence-criterion vector.

Solution of Discretized Mass-Conservation Equations for One Size Class of Suspended Sediment along \( \sigma \)-Direction Line

The system of \( K \) discretized mass-conservation equations for one size class of suspended sediment (one Eq. (43) for each computational point \( C \) along the \( \sigma \)-direction line) is to be solved in order to compute its primary sediment unknowns: \( K \) volumetric concentrations for a particular size class of suspended sediment, one concentration at each computational point \( C \) along the \( \sigma \)-direction line.

The system has a tridiagonal matrix of coefficients and it is easily solved by using a standard double-sweep procedure, described in e.g. Carnahan, Luther and Wilkes (1969).
Auxiliary Relations and Active-Layer Considerations

One should note that the numerical procedure for solution of the sediment equations is formulated above without reference to the specific empirical relations which ultimately must be invoked to evaluate the auxiliary relations. This allows for use of any suitable empirical relation when evaluating a particular auxiliary relation, and renders the formal numerical procedure independent of any specific empirical relation.

Auxiliary Relations

The empirical relations adopted herein to evaluate near-bed concentration $C_a$, components of bedload flux $q_b$, active-layer thickness $E_m$, active-layer floor 'source' $S_F$, fall velocity $w_f$, mass-diffusion coefficient $D$ and density of mixture containing water and suspended sediment $p$, are the same ones that are used as examples in the discussion of the nature of auxiliary relations in Chapter II.

Near-Bed Concentration

The near-bed concentration $C_a$ is evaluated using Van Rijn's (1984b) expression:

$$C_{a_{ks}} = 0.015 \frac{D_{ks} T_{ks}^{1.5}}{a D_{*_{ks}}^{0.3}}$$

(54)

in which

$$D_{*_{ks}} = D_{ks} \left[ \frac{(s-1)g}{\nu^2} \right]^{1/3} = \text{dimensionless particle diameter;}$$

$$T_{ks} = \frac{u_*^2 - (u_{*c})^2}{(u_{*c})_{ks}^2} = \text{transport-stage parameter;}$$

$$u_* = \frac{u \sqrt{g}}{C} = \text{effective bed-shear velocity;}$$
C = 18 \log \left( \frac{12d}{3D_{90}} \right) = \text{grain Chezy coefficient;}

s = \frac{\rho_s}{\rho} = \text{dimensionless sediment density;}

and \( a \)=height above the bed, defined as one half of the average dune height, \( g \)=gravity; \( \nu \)=kinematic viscosity; \( d \)=depth; \( D_{90} \)=characteristic particle diameter for sediment mixture; \( u_* \)=critical shear velocity evaluated from Shields diagram; \( u \)=depth-averaged velocity component.

**Components of Bedload Flux \( q \)**

The theoretical bedload capacity is computed by using the empirical relations proposed by Van Rijn (1984a). A component of the theoretical (i.e. equilibrium, or capacity) bedload flux in, for example, the \( \xi \)-direction can be expressed, for the \( k_s \)-th size class, as:

\[
(q_b)_{k_s} = 0.053 \cdot \rho_s \cdot \sqrt{(s-1)gD_{ks}D_{ks}} \frac{T_{k_s}^{2.1}}{D_{k_s}^{0.3}}
\]

(55)

where all variables have the same meaning as in Eq.(54).

Van Rijn's original expression, representing volume flux, is multiplied herein by the sediment density in order to obtain mass flux, to be consistent with the other terms in the sediment equations.

**Hiding Factor**

Hiding effects are taken into account through the Karim, Holly and Yang (1987) empirical relation for the hiding factor:

\[
(q_h)_{k_s} = \left( \frac{D_{k_s}}{D_{50}} \right)^{0.85}
\]

(56)
Transport-Mode Allocation Parameter

Van Rijn (1984b) presented theoretical-experimental curves relating the ratio of suspended load to total load to the ratio of bed-shear velocity to fall velocity. The ratio of suspended load to total load is adopted herein as the transport-mode allocation parameter $\gamma$ for the $k_s$-th size-class particles. The graphical results of Van Rijn (Van Rijn (1984b), Figure 18) are approximated by:

$$\gamma_{k_s} = \left( \frac{q_s}{q_t} \right)_{k_s} = 0.25 + 0.325 \ln \left( \frac{u_*}{w_{f_k}} \right) \quad 0.4 < \frac{u_*}{w_{f_k}} < 10 \quad (57)$$

where $\gamma$=transport-mode allocation parameter, $q_s$=suspended load, $q_t$=total load, $w_f$=fall velocity, and $u_*$=bed-shear velocity.

Active-Layer Considerations

It is pointed out in Chapter II that there is a marked difference in the treatment of the active layer during erosion as distinguished from deposition. During persistent erosion, the active layer is described as a mixing layer. As the bed elevation descends, erosion proceeds through the active stratum (underlying the active layer) with the active-layer floor changing its elevation. The active-layer thickness during erosion is defined, according to the conceptualization of Bennett and Nordin (1977), as proportional to the erosion that occurs over the current computational time step:

$$E_m = -c(1 + 1) - c(1 + 1)$$

where $c$ is a parameter.

As the bed surface approaches an armored condition, then Eq. (58) leads to a zero active-layer thickness. In such situations Borah's (Borah et al, 1982) armored-layer thickness can be used as a limiting value for the active-layer thickness:

$$E_m = -c(1 + 1) + \frac{1}{1 - p} \sum_{k_s=m}^{D_m} D_m$$

where $D_m$ is the smallest nonmoving size-class.
Movement of the active-layer floor \((z_b - E_m)\) generates the active-layer floor 'source' \(S_F\). A consequence of Eq. (58) is that the active-layer floor may be descending either more quickly or more slowly than the bed elevation, depending on the current erosion rate, and that in principle the active-layer floor may even happen to rise during erosion.

If the active-layer floor descends during erosion (which is the usual case) then the active-layer floor 'source' \((S_F)_{ks}\) for the ks-th size class, when discretized over the time step, has the following form:

\[
S_{F,ks} = -\frac{\rho_s (1 - \rho)}{\Delta t} \left[ \left( z_b^{n+1} - E_m^{n+1} \right) - \left( z_b^n - E_m^n \right) \right] (\beta_s)_{ks}
\] (60)

where \((\beta_s)_{ks}\) is the ks-th size-class fractional representation in the active stratum. One should note that the active-stratum size distribution remains unchanged in this case.

On rare occasions, when the erosion rate undergoes an extremely sharp decrease over the computational time step, the active-layer floor may actually rise during erosion. If this happens, then the active-layer floor 'source', discretized over the time step, has the form:

\[
S_{F,ks} = -\frac{\rho_s (1 - \rho)}{\Delta t} \left[ \left( z_b^{n+1} - E_m^{n+1} \right) - \left( z_b^n - E_m^n \right) \right] \frac{\beta_{ks}^{n+1} + \beta_{ks}^n}{2}
\] (61)

where \(\beta_{ks}\) is the ks-th size-class fractional representation in the active layer. In such a case the active-stratum size distribution changes, as active-layer material is released to it, and in principle this calls for inclusion of the active-stratum mass-conservation equation in the simultaneous solution of sediment equations. However since such situations are expected to be rare, they are handled explicitly herein, by updating the active-stratum size-distribution at the end of computational time step, when necessary.

During persistent deposition, the active layer is described as a deposition layer. Newly deposited material is added to the existing material in the active layer and assumed to be fully mixed with it. The active-layer floor elevation is assumed to remain constant during deposition and the active-layer thickness is defined as:

\[
E_m^{n+1} = E_m^n + \left( z_b^{n+1} - z_b^n \right)
\] (62)

while the active-layer floor 'source' is equal to zero:

\[37\]
The definition of the active layer, evaluation of the active-layer thickness and treatment of the active-layer floor 'source' depend on whether erosion or deposition occurs during the current computational time step. However, the tendency toward erosion or deposition is itself a part of the solution for the computational step. To resolve this ambiguity, one can make use of an iterative solution of the sediment equations. At the beginning of each iteration either erosion or deposition is assumed, based on whether erosion or deposition occurred during the previous iteration. Then one can easily compute the derivatives of the active-layer thickness \( E_m \) and active-layer floor 'source' \((S_F)_{ks} \) with respect to sediment variables, evaluated for previous-iteration values of sediment variables. These derivatives are then used to determine the coefficients in the discretized sediment equations.

**Fall Velocity**

In a clear still fluid the particle fall velocity \( w_f \) of a solitary sand particle smaller than about 100 \( \mu \text{m} \) (Stokes range) can be described by:

\[
wf_{ks} = \frac{1}{18} \frac{(s-1)gD^2_{ks}}{v}
\]  

For suspended-sand particles in the range 100-1000 \( \mu \text{m} \), the following equation, as proposed by Zanke (1977), is used:

\[
wf_{ks} = 10 \frac{v}{D_{ks}} \left[ \sqrt{1 + \frac{0.01(s-1)gD^3_{ks}}{v^2}} - 1 \right]
\]  

For particles larger than about 1000 \( \mu \text{m} \) the following equation, as proposed by Van Rijn (1982), is used:

\[
w_{f_{ks}} = 1.1 \sqrt{(s-1)gD^2_{ks}}
\]
Vertical Mass-Diffusion Coefficient \( D_v \)

As described by van Rijn (1984b), the vertical mass-diffusion coefficient \( D_v \) is related to the diffusion of fluid momentum by:

\[
D_v = \beta_d \phi A_v \tag{67}
\]

The \( \beta_d \)-factor describes the difference in the diffusion of a discrete sediment particle and the diffusion of a fluid 'particle' (or small coherent fluid structure) and is assumed to be constant over the depth:

\[
\beta_d = 1 + 2 \left( \frac{w_{f,kr}}{u_*} \right)^2 \quad 0.1 < \frac{w_{f,kr}}{u_*} < 1 \tag{68}
\]

The \( \phi \)-factor expresses the damping of the fluid turbulence by the sediment particles and is assumed to be dependent on the local sediment concentration:

\[
\phi = 1 + \left( \frac{C}{C_0} \right)^{0.8} - 2 \left( \frac{C}{C_0} \right)^{0.4} \tag{69}
\]

where \( C_0 = 0.65 \) = maximum volumetric near-bed concentration; \( C \) = total volumetric concentration.

The horizontal diffusivities in CH3D are assigned values.

Density of Mixture Containing Water and Suspended Sediment

According to Zhou and McCorquodale (1992), local fluid density is related to the local values of suspended-sediment concentration as follows:

\[
\rho = \rho_r + C \left( 1 - \frac{1}{s-1} \right) \tag{70}
\]

where \( \rho_r \) = the reference density of clear water, possibly influenced by temperature and/or salinity.
CHAPTER IV

DESCRIPTION OF THE SEDIMENT-OPERATIONS PROGRAM MODULE

Introduction

A separate sediment-operations program module, based on physical and numerical principles described in Chapters II and III, has been developed as an integral part of the CH3D code. Dedication of a separate program module to sediment operations is made possible by the short-term uncoupling of hydrodynamics and sediment operations. However, the sediment module fully communicates with the rest of the CH3D code. Hydrodynamics operations provide all necessary hydrodynamic input required by the sediment module (velocities, depths, etc.). The sediment module, in turn, provides changes in bed elevations (i.e. depths), distribution of active-layer size fractions (i.e. friction coefficients) and density (due to the presence of suspended sediment) back to the CH3D hydrodynamics operations.

A block diagram for the sediment-operations program module is shown in Figure 5. The sediment module comprises a main subroutine for sediment operations, SMAIN, and 51 other subroutines. Subroutine RHOCON (called from subroutine CH3DDE of the original CH3D code) is the only subroutine outside of the sediment module that is also part of the sediment operations. The basic function of SMAIN is to manage all other sediment subroutines (except for subroutine RHOCON).

The sediment module has two major blocks: preparatory operations and sediment computations. The division between the blocks, however, is not sharp for a few subroutines used in both of them.

Data defining initial and boundary conditions are read in dimensional form and made nondimensional in the same subroutine that reads the particular data. All computations are made with dimensionless variables. The only exception is the evaluation of auxiliary relations based on empirical expressions. Empirical expressions not only require dimensional variables, but may also be dimensionally inconsistent. Thus, all necessary variables (both hydrodynamic and sediment) are made dimensional upon entry to a subroutine that evaluates a particular auxiliary relation, and the computed auxiliary relation is then made dimensionless upon return from the same subroutine.

Sediment boundary conditions are associated with control-volume faces, rather than cell-centered computational points. Hydrodynamics boundary conditions are also mostly associated with control-volume faces (impermeable or river boundaries, for example),
Figure 5. Summary Block Diagram of Sediment-Operations Program Module.
except for tidal boundaries, that are associated with cell-centered points. If a sediment boundary coincides with a hydrodynamics boundary assigned to a control-volume face (e.g. a river boundary with the discharge assigned at the control-volume face), the actual position of both boundaries is the same. However, if a sediment boundary coincides with a hydrodynamic tidal boundary with a tidal condition assigned at a cell-centered point, the inflow sediment boundary is then associated with an 'inner' face (pointing towards the inside of the computational domain) of the control volume built around the tidal point. Thus, the sediment computational domain coincides with the hydrodynamics computational domain except at tidal points, which are excluded from the sediment computations.

Sediment variables are associated with cell-centered computational points and stored in appropriate arrays accordingly. Thus, sediment boundary conditions, even though related to control-volume faces, are in the code stored at the 'outer' points (where 'outer' means the closest neighboring point outside the sediment computational domain).

Preparatory Operations

Preparatory operations serve to define the initial and boundary conditions for sediment computations.

Subroutine INISED reads general sediment parameters and manages the group of subroutines that define initial conditions. INICON reads initial suspended-sediment concentrations, expressed as mass of sediment particles with respect to the total mass of the mixture of water and suspended sediment. At this point subroutine ROCONC is called, within a loop on all computational points, to initialize the density of the water and suspended sediment mixture at the particular computational point. Suspended-sediment concentrations, read by INICON, are multiplied by the density of water and suspended-sediment mixture, computed in ROCONC, to yield volumetric concentrations, expressed as mass of sediment particles with respect to the volume of the water and suspended-sediment mixture. Subroutine FALVEL computes fall velocities for each sediment size class. Subroutine NODIML computes dimensionless numbers. Subroutine INIBED reads the initial number of bed material strata below the active layer. It also reads initial depths of the active layer and all the strata as well as their initial size-fraction distributions. Finally, subroutines DCHAR and CFRICT are called, within the loop on all bed computational points, to initialize the characteristic grain diameters D50 and D90, and the friction coefficient, respectively, at the particular computational point.

Subroutine SBINFO defines the sediment boundary conditions. SBINFO first reads boundary-condition information for sediment computations: position of the boundary
point and the type of boundary (inflow, impermeable, or outflow boundary). Any inflow-boundary point requires a known distribution of active-layer size fractions and suspended-sediment concentrations for each size class, all as a function of time. Therefore, for each inflow boundary point SBINFO also reads sequence numbers of specific time functions used as boundary conditions, while the time functions themselves are read at the end of all input data (subroutine TIMEFU) in order to optimize memory use. Subroutine SBINFO also calls subroutines DCHAR and CFRICT to initialize the characteristic grain diameters $D_{50}$ and $D_{90}$, and the friction coefficient, respectively, at the particular inflow boundary point.

At this point subroutine PLTOUT is called, as a part of preparatory operations, to store computational-point Cartesian coordinates and initial bed elevations to be used within an eventual plot output.

Sediment Computations

While the preparatory-operations block is called only once, prior to the beginning of sediment computations, the sediment-computation block is called for each time step within the CH3D time loop. Within the sediment-computations block, SMAIN calls subroutines TIMEFU and INFLSB to evaluate boundary conditions, subroutines ACDER and HORQS to evaluate auxiliary and explicit terms in the suspended-sediment equations, subroutine SEDCOM to perform sediment computations, subroutine ROCONC to evaluate the new density of the water and suspended-sediment mixture, and subroutines PLTOUT and PRTOUT to manage plot and print output.

At the beginning of each time step, subroutine TIMEFU reads and interpolates time functions used as boundary conditions for sediment computations. Subroutine INFLSB evaluates active-layer size fractions and suspended-sediment concentrations at the current time, for inflow-boundary points. INFLSB also evaluates secondary sediment unknowns, characteristic grain diameters $D_{50}$ and $D_{90}$, friction coefficients, and the density of the water and suspended-sediment mixture, by using DCHAR, CFRICT and ROCONC, respectively, at the current time, for inflow-boundary points. Again, concentrations evaluated at inflow boundary points are multiplied by density, computed in ROCONC, to yield volumetric concentrations.

Subroutine ACDER evaluates the so-called auxiliary concentration derivatives (to be used in further computations of concentration derivatives, as in Eqs. (B26) to (B28), for example) for all computational points at the previous time and for inflow-boundary points at the current time. Subroutine HORQS manages subroutines WFACEB, WFACE,
EFACEB, SFACEB, SFACE, and NFACEB, that compute and store (explicit) horizontal advection and diffusion fluxes (terms (3) to (10) in Eq. (42)) through the west and south faces of the control volumes built around computational points within the entire computational domain, for all size classes of suspended sediment. Flux through the west (south) face for one computational point is a flux through the east (north) face for the appropriate neighboring point. Subroutine WFACEB is called if the west face of the control volume is a boundary, while subroutine WFACE is called if it is not, and subroutine EFACEB is called if the east face of the control volume (west face for the appropriate neighboring point) is a boundary. Similarly, subroutine SFACEB is called if the south face of the control volume is a boundary, while subroutine SFACE is called if it is not, and subroutine NFACEB is called if the north face of the control volume (south face for the appropriate neighboring point) is a boundary.

Sediment computations are managed by subroutine SEDCOM. First, SEDCOM manages the loop on all locations (i,j) in a horizontal plane (except for tidal boundaries), with the group of subroutines DIMVEL, EXTRAP, NBCONC, MASDVB and SOURCE, called to compute source term; subroutine BEDSED called to manage bed-sediment computations (solution of the discretized global mass-conservation equation for bed sediment and the mass-conservation equations for active-layer sediment at the bed point); and subroutine SUSSED called to manage suspended-sediment computations (solution of the discretized mass-conservation equations for suspended sediment along a vertical-direction line above the same bed point).

After the sediment computations are completed, a zero-gradient condition is applied to tidal outflow boundary points, with subroutines DCHAR and CFRICT used to evaluate characteristic grain diameters D50, D90, and friction coefficient, respectively. Finally, subroutine SBACK is called to evaluate the sediment-computation feedback, i.e. to compute new depths and Manning friction coefficients to be used in the hydrodynamics operations of CH3D.

Source-Term Computations

Subroutine DIMVEL computes the dimensional physical components of depth-averaged velocities, to be used in empirical relations. Subroutine EXTRAP computes the extrapolation coefficient used in the 'deposition' source computations (Eq. (31)). MASDVB computes mass-diffusion coefficients at the bed, used in the 'erosion' source computations. Finally, subroutine SOURCE computes the 'deposition' and 'erosion' source terms.
Bed-Sediment Computations

Subroutine QBDIV computes the divergence of the bedload flux vector. QBFLUX computes the bedload flux for one sediment size class. TMALOC computes the transport-mode allocation parameter, HIDFAC evaluates the hiding factor and TQBUNI computes the theoretical bedload flux (or transport capacity) for uniform-size sediment.

Subroutines SJACOB, EQZBED, EQBETA, SIMUL and ACTLAY are part of the Newton-Raphson iterative solution for the system of bed-sediment equations at a bed point (one global mass-conservation equation for bed sediment, and KS mass-conservation equations for active-layer sediment). Subroutine SJACOB loads the Jacobian matrix of the system of discretized and linearized sediment equations. EQZBED computes coefficients in the global mass-conservation equation for bed sediment ($z_b$-equation). EQBETA computes coefficients in the mass-conservation equation for the $ks$-th size class of active-layer sediment ($\beta$-equation). All coefficients are presented in Appendix C. Subroutine SIMUL solves the system of linear equations.

Subroutine ACTLAY manages the active-layer operations. If degradation (i.e. erosion) is anticipated during iterations, then INCIP defines the incipient motion, i.e. identifies the smallest non-moving grain size. DEGLAY then computes the active-layer thickness, active-layer floor source and their derivatives with respect to sediment variables. If aggradation (deposition) is assumed during iterations, the active layer thickness, active-layer floor source and their derivatives are computed by AGGLAY. Subroutine ACTLAY (together with AGGLAY or INCIP and DEGLAY) is also called prior to the Newton-Raphson iterations to evaluate the active layer thickness, active-layer floor source and their derivatives at the zero-iteration level.

Subroutines SDIM and SDIML are used to test the bed-sediment computations; they do not influence the computations in any way. Subroutine SDIM makes all necessary sediment variables dimensional, so that bed-sediment computations could also be performed in a dimensional environment. After the bed computations are completed, subroutine SDIML makes all sediment variables dimensionless again.

After the Newton-Raphson iterative solution is obtained at a point, subroutine ACTSTR is used to update the active-stratum thickness and size-fraction distribution if appropriate. DCHAR computes the characteristic grain size $D_{50}$. It also computes the sediment size $D_{90}$ used by CFRICT to evaluate the friction coefficient related to grain roughness.
An important feature of the sediment-module structure is that whenever an empirical expression is used (e.g. to compute bedload flux, hiding factor etc.) its evaluation is restricted to a single subroutine independent of the rest of the code. This makes it possible to incorporate the best available empirical information, even site-specific, without changing the rest of the code.

**Suspended-Sediment Computations**

Subroutine MASDV computes and stores the vertical mass-diffusion coefficients at the top faces of control volumes built around computational points along a vertical-direction line, for all size classes of suspended sediment.

Subroutine TFACE computes and stores the (explicit) vertical advection fluxes (terms (11) and (12) in Eq. (42)) through the top faces of control volumes built around computational points along a vertical-direction line, for all size classes of suspended sediment. Flux through the top face for one computational point is a flux through the bottom face for the appropriate neighboring point.

Subroutine SWPSIG is called from SUSSED for each size class of suspended sediment. SWPSIG evaluates coefficients a, b, c, and d, in the discretized mass-conservation equations for one size class of suspended sediment (Eq. (43)) and performs the double-sweep solution procedure along a vertical-direction line.

The last two subroutines, used repeatedly in the sediment module, are ERRWAR, which prints error and warning messages, and MTROUT, which prints matrix-form output.

Subroutine RHOCON (called from subroutine CH3DDE of the original CH3D code) computes the density of the water and suspended-sediment mixture to be used in the hydrodynamic computations of CH3D. Subroutine CH3DDE evaluates the 'initial' density of water (with or without temperature and/or salinity effects). At this point subroutine RHOCON is called to compute the density of the water and suspended-sediment mixture. The mixture density is nondimensionalized differently in the hydrodynamics and sediment equations. Thus, the dimensionless densities of the water and suspended-sediment mixture are evaluated by two slightly different subroutines, RHOCON and ROCONC, and stored in two different arrays.

**Memory and Time Requirements**

The required memory size for the sediment module is obtained by adding the following array sizes, multiplied by the appropriate number of arrays:
<table>
<thead>
<tr>
<th>Array size (bytes)</th>
<th>Number of arrays</th>
</tr>
</thead>
<tbody>
<tr>
<td>(IM+1)<em>(JM+1)</em>(KM+1)*(KSMAX)*8</td>
<td>2</td>
</tr>
<tr>
<td>(IM+1)<em>(JM+1)</em>(K)<em>KSMAX</em>8</td>
<td>8</td>
</tr>
<tr>
<td>(IM+1)<em>(JM+1)</em>(LMAX)*(KSMAX)*8</td>
<td>3</td>
</tr>
<tr>
<td>(IM+1)<em>(JM+1)</em>(KM+1)*8</td>
<td>1</td>
</tr>
<tr>
<td>(IM+1)<em>(JM+1)</em>(KM+1)*4</td>
<td>1</td>
</tr>
<tr>
<td>(MAXBP)<em>(KM+1)</em>(KSMAX)*8</td>
<td>4</td>
</tr>
<tr>
<td>(IM+1)<em>(JM+1)</em>(LMAX)*8</td>
<td>1</td>
</tr>
<tr>
<td>(IM+1)*(JM+1)*8</td>
<td>8</td>
</tr>
<tr>
<td>(IM+1)*(JM+1)*4</td>
<td>3</td>
</tr>
<tr>
<td>(KM+1)*(KSMAX)*8</td>
<td>3</td>
</tr>
<tr>
<td>(KSMAX)*8</td>
<td>2</td>
</tr>
<tr>
<td>(KSMAX)*(NSVAR)*8</td>
<td>3</td>
</tr>
<tr>
<td>(NSVAR)*(NSVAR)*8</td>
<td>1</td>
</tr>
<tr>
<td>(NSVAR)*8</td>
<td>1</td>
</tr>
<tr>
<td>(NSVAR)*4</td>
<td>23</td>
</tr>
<tr>
<td>(LMAX)*8</td>
<td>1</td>
</tr>
<tr>
<td>(LMAX)*4</td>
<td>1</td>
</tr>
<tr>
<td>(NSBINF)*4</td>
<td>1</td>
</tr>
<tr>
<td>(NSVAR)*8</td>
<td>2</td>
</tr>
<tr>
<td>(NSVAR)*4</td>
<td>3</td>
</tr>
</tbody>
</table>

where:

IM = ICELLS + 2  ICELLS = Exact number of computational cells in the longest KSI-direction computational row

JM = JCELLS + 2  JCELLS = Exact number of computational cells in the longest ETA-direction computational column

KM = Exact number of computational points in the vertical direction

KSMAX = Exact number of sediment size classes

LMAX = Maximum number of bed strata

MAXBP = 2*(IM+JM)  Maximum number of boundary points

NSBINF = 2*MAXBP  Maximum number of sediment boundary data identifiers (sequence numbers of sediment boundary data in the list of time-dependent data) stored in the INFOSB array

NSVAR = I+KSMAX  Number of bed-sediment unknowns at a point

NSVAR1 = NSVAR + 1  Dimension of the bed-sediment Jacobian matrix

The sediment-module memory size for the Old River model (ICELLS=51, JCELLS=24), with 3 sediment size classes and a maximum of 10 bed strata, is about 4.5 Mb. More than 80% of that memory is used by the four-dimensional arrays.
The CPU time required is highly dependent on the structure of the model data and the type of computer and compiler being used. The CPU time required for the runs of the Old River model on the University of Iowa's HP 750 computer is reported along with the sediment-module tests in Chapter V of this report.

**Sediment Module Input Data Guide (May 1993)**

This sediment module input-data guide is limited essentially to the formal structure and requirements of the input data; the user may need to refer to the rest of this report for further guidance on recommended parameter values, etc.

In the data guide, "rec" refers to the logical record number in the input data stream, and "var" refers to the variable name as it is used in the code itself. The name of the subroutine in which particular records are read is given as a banner entry preceding the usual record description.

An input-data guide for the remainder of the CH3D code is presented in Appendix D. The only data related to sediment operations read outside the sediment module are two parameters: ISCOM and NTSED0. These parameters are read in the "timestep info" record (record 2 in subroutine CH3DIR), shown in Appendix D. The "Timestep info" record is repeated here for convenient reference.

**Input Data Guide**

<table>
<thead>
<tr>
<th>rec</th>
<th>var</th>
<th>Format (columns)</th>
<th>Variable description and remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td><strong>SUBROUTINE CH3DIR</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Note: &quot;Timestep info&quot; record data are read from the main input file (file 4: main.inp)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td><strong>Timestep info:</strong></td>
</tr>
<tr>
<td></td>
<td>DUMMY</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>IT1</td>
<td>I8 (1-8)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IT2</td>
<td>I8 (9-16)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>DT</td>
<td>F8.0 (17-24)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ISTART</td>
<td>I8 (25-32)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ITEST</td>
<td>I8 (33-40)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ITSALT</td>
<td>I8 (41-48)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ISCOM</td>
<td>I8 (49-56)</td>
<td>=0 No sediment computations</td>
</tr>
<tr>
<td></td>
<td>NTSED0</td>
<td>I8 (57-64)</td>
<td>=1 Sediment computations are performed</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Number of time steps before sediment computations are initiated</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Note: All following data are read from sediment-computation input file (file 80: sed.inp)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td><strong>SUBROUTINE SMAIN</strong></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>NBUGS</td>
<td></td>
</tr>
<tr>
<td>Variable</td>
<td>Value</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>-------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>NBUGE</td>
<td>5(2I8)(1-80)</td>
<td>Pairs of debug-output flags 5 pairs per record 100 pairs total</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Debug output for subroutine M is written</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>if(NT.ge.NBUGS(M).and.NT.le.NBUGE(M))</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Subroutine debug identifications M:</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>M=1-SMAIN 13-TIMEFU 36-ERRWAR</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2-SEDCOM 14-INFLSB 37-MTROUT</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3-INISED 15-EXTRAP 38-PRTOUT</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4-INICON 16-NBCONC 39-PLTOUT</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>5-ROCONC 17-MASDV 40-ACDER</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>6-FALVEL 18-SOURCE 41-HORQS</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>7-NODIML 19-BEDSED 42-WFACEB</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>8-INIBED 20-QBDIV 43-WFACE</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>9-DIMVEL 21-QBFLUX 44-EFACEB</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>10-DCHAR 22-HIDFAC 45-SFACEB</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>11-CFRICT 23-TMALOC 46-SFACE</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>12-SBINFO 24-TQBUNI 47-NFACEB</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>25-ACTLAY 48-SUSSED</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>26-INCIP 49-MASDV</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>27-DEGLAY 50-TFACE</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>28-AGGLAY 51-SWPSIG</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>29-SDIML 52-SBACK</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>30-SJACOB 53-RHOCON</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>31-EQZBED</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>32-EQFBETA</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>33-SIMUL</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>34-SDIM</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>35-ACTSTR</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 NPRSED I8 (1-8) Time-step frequency for printing sediment-computation results (file 81: prtsed.out)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>NDIAGS I8 (9-16) Time-step frequency for printing sediment-computation diagnostics (max. errors, number of iterations etc.)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>NPLOT I8 (17-24) Plot output file (FORT ref num) (filename: pltsed.out) NPLT=0 : no plot output NPLT&gt;0 : short output (bed-surface elevation and active-layer size fractions) NPLT&lt;0 : long output (all of the above plus: cell-centered depths, all three cell-centered Cartesian velocity components, suspended-sediment concentrations and suspended-sediment source terms)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>NPLT I8 (25-32) Time-step frequency for NPLOT output</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3 IDENS I2 (1-2) Option for printing: density of water &amp; suspended mixt! (1 = print ; 0 = no print)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>ICONC I2 (3-4) suspended-sedim. concentrations</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>IZBED I2 (5-6) bed elevation</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>IDZBED I2 (7-8) bed-elev change over time step</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>IDZBO I2 (9-10) cumulative bed-elev change</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>IEM I2 (11-12) active-layer depth</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>IBETA I2 (13-14) active-layer size fractions</td>
<td></td>
</tr>
</tbody>
</table>
ILSTR  I2 (15-16)  number of strata
ISTRDP I2 (17-18)  depths of strata
IBSTR  I2 (19-20)  strata size fractions
ID50   I2 (21-22)  characteristic grain diameter
ICFRIC I2 (23-24)  friction coefficient
ISORCD I2 (25-26)  susp-sed 'deposition' source
ISORCE I2 (27-28)  susp-sed 'erosion' source
ISBTYP I2 (29-30)  sediment-comput boundary types

4    IBED   I2 (1-2)  Option for including/excluding: complete bed-sed computations
                          (1 = include ; 0 = exclude)

ISUS   I2 (3-4)  complete sus-sed computations
IADV   I2 (5-6)  i-direction sus-sed advection
IDIF   I2 (7-8)  i-direction sus-sed diffusion
JADV   I2 (9-10) j-direction sus-sed advection
JDEF   I2 (11-12) j-direction sus-sed diffusion
KADV   I2 (13-14) k-direction sus-sed advection

*SUBROUTINE INISED

1    ISDBG  I8 (1-8)  1 : sediment debug output at chosen points
                   0 : no sediment debug
IBSDIM I8 (9-16)  1 : dimensional bed-sed comp
                   0 : dimensionless bed-sed comp

2    IB,JB,KB 318 (1-16) Position of the points chosen for sediment debug output
                    Last record must have IB<0
                    Records 2 omitted if ISDBG=0

3    SDIAM  5F16.0 (1-80) Standard sediment sizes (m) defining size intervals 5 values per record
                    KSMAX values altogether
                    (KSMAX=number of size classes)

4    VISCOS F16.0 (1-16) Kinematic viscosity of water (m2/s)
5    SDENS  F8.0 (1-8)  Sediment particle density (t/m3)
6    POROS  F8.0 (9-16) Porosity of sediment mixture in the bed
7    THETAS F8.0 (1-8)  Implicitation factor used in sediment computations

STRMAX F8.0 (9-16)  Maximum depth of active stratum (m)
ALMIN  F8.0 (17-24)  Minimum depth of active layer (m), criterion for closing a deposition layer
ABED   F8.0 (25-32)  Near-bed position 'a' (m)
DABED  F8.0 (33-40)  Near-bed position increment (m)

7    EPSZB  F16.0 (1-16)  Treshold value (m) of bed-elevation changes for terminating iterations during sediment comput at a point
EPSBET F16.0 (16-32)  Treshold value (-) of active-layer size-fraction changes for terminating iterations during sediment comput at a point

8    MAXITS I8 (1-8)  Maximum number of iterations during bed-sed computations at a bed point
9    DH     F8.0 (1-8)  Horizontal mass diffusivity (m2/s)

*SUBROUTINE INICON

50
Note: suspended-sediment concentrations are initially assigned a negative value (=-0.001) at all computational points, in order to distinguish points inside the flow domain (where concentration is computed) from points outside the flow domain (conc remains equal to -0.001)

<table>
<thead>
<tr>
<th>1</th>
<th>IPAR</th>
<th>18 (1-8)</th>
<th>Parameter indicating chosen option for furnishing initial data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>IPAR=0 : only record 2 is furnished, records 3-5 are omitted</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IPAR=1 : records 2-4 are furnished, records 5 are omitted</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IPAR=-1 : records 5 are furnished, records 2-4 are omitted</td>
</tr>
</tbody>
</table>

Record 2 is repeated KMAX times (where KMAX is the number of computational points along the vertical direction) starting with the point k=KMAX next to the free surface, until the entire set of vertical concentration profiles is defined for all size classes

<table>
<thead>
<tr>
<th>2</th>
<th>CREAD</th>
<th>10F8.0 (1-80)</th>
<th>Default suspended-sediment concentrations (-), for all size classes at a point, KSMAX values (KSMAX=number of size classes)</th>
</tr>
</thead>
</table>

Record 3 is repeated KMAX times (where KMAX is the number of computational points along the vertical direction) starting with the point k=KMAX next to the free surface, until the entire set of vertical concentration profiles is defined for all size classes

Each set of vertical concentration profiles, for all size classes, defined by KMAX records 3, is followed by one or more records 4

Record(s) 4 specify points to which set of vertical concentration profiles, for all size classes, defined by KMAX records 3, applies

IAXES<0 : end of records 4 related to one particular set of records 3

IJ<0 : end of records 3 and 4 related to initial suspended-sediment concentrations

<table>
<thead>
<tr>
<th>3</th>
<th>CREAD</th>
<th>10F8.0 (1-80)</th>
<th>Other-than-default suspended-sediment concentrations (-) for all size classes at a point KSMAX values (KSMAX=number of size classes)</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>4</th>
<th>IAXES</th>
<th>18 (1-8)</th>
<th>[IAXES]=1 : XI coordinate line</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IJ</td>
<td>18 (9-16)</td>
<td>[IAXES]=2 : ETA coordinate line</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IJ specifies particular</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>XI or ETA coordinate line</td>
</tr>
<tr>
<td></td>
<td>IJS</td>
<td>18 (17-24)</td>
<td>Record 3 applies to points IJS</td>
</tr>
<tr>
<td></td>
<td>IJE</td>
<td>18 (25-32)</td>
<td>to IJE along the particular</td>
</tr>
</tbody>
</table>
Record 5 is repeated KMAX times
(where KMAX is number of computational points along the vertical direction) starting with the point k=KMAX next to the free surface, until the entire set of vertical concentration profiles is defined for all size classes
Set of KMAX records 5 is repeated for each vertical direction line within the flow domain, row-by-row starting from the first row inside the domain

*SUBROUTINE INIBED*

Note: bed elevations, active-layer depth and size fractions, number of strata, and each stratum depth and size fractions, are all initially set to zero at all points

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>PCONC</td>
<td>10F8.0 (1-80)</td>
</tr>
</tbody>
</table>

coordinate line
Initial suspended-sediment concentrations (-), for all size classes at a point KSMAX values (KSMAX=number of size classes)

### TYPICAL RECORDS

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>REL0</td>
<td>F8.0 (1-8)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Relative elevation used to evaluate initial bed-surface elevations (m)</td>
</tr>
</tbody>
</table>

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>IPAR</td>
<td>I8 (1-8)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Parameter indicating chosen option for furnishing initial data</td>
</tr>
<tr>
<td>IPAR=0</td>
<td>records 3 and 4 are furnished, records 5-9 are omitted</td>
<td></td>
</tr>
<tr>
<td>IPAR=1</td>
<td>records 3-7 are furnished, records 8-9 are omitted</td>
<td></td>
</tr>
<tr>
<td>IPAR=-1</td>
<td>records 8-9 are furnished, records 3-7 are omitted</td>
<td></td>
</tr>
</tbody>
</table>

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>ITYPBA</td>
<td>I8 (1-8)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Type of size-fraction distribution for active layer (default value)</td>
</tr>
</tbody>
</table>

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>LSTRAT</td>
<td>I8 (9-16)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Number of strata below the active layer at the point (default value)</td>
<td></td>
</tr>
</tbody>
</table>

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>ITYPBS</td>
<td>8I8 (17-80)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Types of size-fraction distributions for strata LSTRAT values (default) starting from active stratum</td>
<td></td>
</tr>
</tbody>
</table>

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>EMINIT</td>
<td>I8 (1-8)</td>
</tr>
<tr>
<td></td>
<td>Initial active-layer depth (m) (default) Not used</td>
<td></td>
</tr>
</tbody>
</table>

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>STRDEP</td>
<td>I8 (9-16)</td>
<td></td>
</tr>
<tr>
<td>I8 (9-16)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8F8.0 (17-80)</td>
<td>Initial strata depths (m) LSTRAT values (default) starting from active stratum</td>
<td></td>
</tr>
</tbody>
</table>
Records 5,6 are followed by one or more records 7
Record(s) 7 specify points to which records 5,6 apply
IAXES<0 : end of records 7 related to particular
records 5,6
I<0 : end of records 5,6 and 7 related to
initial bed-material data

<table>
<thead>
<tr>
<th></th>
<th>ITYPBA</th>
<th>18 (1-8)</th>
<th>Other-than-default type of size-fraction distribution for active layer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LSTRAT</td>
<td>18 (9-16)</td>
<td>Other-than-default number of strata below the active layer at the point</td>
</tr>
<tr>
<td></td>
<td>ITYPBS</td>
<td>8I8 (17-80)</td>
<td>Other-than-default types of size-fraction distributions for strata</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>LSTRAT values starting from active stratum</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IAXES I=1 : XI coordinate line</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IAXES I=2 : ETA coordinate line</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>I specifies particular XI or ETA coordinate line</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IJS specifies particular coordinate line</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IJE specifies particular coordinate line</td>
</tr>
</tbody>
</table>

Records 5,6 are followed by one or more records 7
Record(s) 7 specify points to which records 5,6 apply
IAXES<0 : end of records 7 related to particular
records 5,6
I<0 : end of records 5,6 and 7 related to
initial bed-material data

<table>
<thead>
<tr>
<th></th>
<th>ITYPBA</th>
<th>18 (1-8)</th>
<th>Other-than-default type of size-fraction distribution for active layer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LSTRAT</td>
<td>18 (9-16)</td>
<td>Other-than-default number of strata below the active layer at the point</td>
</tr>
<tr>
<td></td>
<td>ITYPBS</td>
<td>8I8 (17-80)</td>
<td>Other-than-default types of size-fraction distributions for strata</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>LSTRAT values starting from active stratum</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IAXES I=1 : XI coordinate line</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IAXES I=2 : ETA coordinate line</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>I specifies particular XI or ETA coordinate line</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IJS specifies particular coordinate line</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IJE specifies particular coordinate line</td>
</tr>
</tbody>
</table>

Records 8 and 9 are repeated for each point at the bed, within the flow domain, row-by-row, starting from the first row inside the domain

<table>
<thead>
<tr>
<th></th>
<th>ITYPB</th>
<th>18 (1-8)</th>
<th>Type of size-fraction distribution for active layer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LSTRAT</td>
<td>18 (9-16)</td>
<td>Number of strata below the active layer at the point</td>
</tr>
<tr>
<td></td>
<td>ITYPBS</td>
<td>8I8 (17-80)</td>
<td>Types of size-fraction distributions for strata</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>LSTRAT values starting from active stratum</td>
</tr>
</tbody>
</table>

Records 8 and 9 are repeated for each point at the bed, within the flow domain, row-by-row, starting from the first row inside the domain

<table>
<thead>
<tr>
<th></th>
<th>ITYPB</th>
<th>18 (1-8)</th>
<th>Type of size-fraction distribution type, must agree with one of types associated with active layer or strata at computational points</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BREAD</td>
<td>9F8.0 (9-80)</td>
<td>Size-fraction distribution (-)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>KSMAX values</td>
</tr>
</tbody>
</table>

53
The following record is read:
FIRST for all points along ETA-direction boundary line(s)
THEN for all points along XI-direction boundary line(s)

1. I 18(1-8) \( \text{II} \) : XI-direction index of the sediment-computation boundary point
   Last record 1 must have I<0

2. J 18(9-16) \( \text{I} \) : ETA-direction index of the sediment-computation boundary point

3. ITYP 18(17-24) Boundary-condition type associated with the sediment-computation boundary point
   ITYP=1 : inflow boundary
   ITYP=-1 : outflow boundary, NSEQB, NSEQC not used
   ITYP=0 : assigned automatically to impermeable boundary

4. NSEQB 18(25-32) Sequence number of the active-layer size-fraction distribution in list of time-dependent data

5. NSEQC 18(33-40) Sequence number of the set of suspended-sediment vertical concentration profiles in list of time-dependent data

The total number of records 2 must be equal to total number of different active-layer size-fraction distributions assigned in SBINFO

6. IDAYR 18(1-8) Day and hour defining time of reading time-dependent data

7. I HOUR 18(9-16) Sequence number (1,2,.....)

8. NOBT 116(1-16) Active-layer size-fraction distribution (-), its sequence number NOBT consistent with the NSEQB sequence numbers assigned to sediment-computation boundary points with ITYP=1 (subroutine SBINFO, record 1)

9. CTR 18F8.0 (17-80) Set of suspended-sediment concentration profiles

(KSMAX=number of size classes)
concentrations (-), at point k = KMAX next to the free surface, for all size classes, its sequence number NOCT consistent with the NSEQC sequence numbers assigned to sediment-computation boundary points with ITYP = 1 (SBINFO, record 1) Not used

Set of suspended-sediment concentrations (-), at point k-1, for all size classes

Record 4 is repeated KMAX-1 times, until the entire set of vertical concentration profiles, for all size classes, and sequence number NOCT, is defined
Total number of concentration-profile sets, defined by records 3 and 4, must be equal to the total number of different sets of suspended-sediment concentrations assigned in SBINFO

Structure of Sediment Plot Output
(pltsed.out loaded in subroutine P)

Record Contents

<table>
<thead>
<tr>
<th>Heading Records:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1 NPLOT, IM, JM, KM, KSMAX</td>
<td></td>
</tr>
<tr>
<td>2 XCT(IC1,JCl)</td>
<td></td>
</tr>
<tr>
<td>3 YCT(IC1,JCl)</td>
<td></td>
</tr>
<tr>
<td>4 PZBED(0:IM,0:JM)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time-Dependent Records:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5 IT</td>
<td></td>
</tr>
<tr>
<td>6 ZBED(0:IM,0:JM)</td>
<td></td>
</tr>
<tr>
<td>7 BETAA(0:IM,0:JM,KSMAX)</td>
<td></td>
</tr>
<tr>
<td>8 AHSS(0:IM,0:JM)</td>
<td></td>
</tr>
<tr>
<td>9 U(0:IM,0:JM,KM)</td>
<td></td>
</tr>
<tr>
<td>10 V(0:IM,0:JM,KM)</td>
<td></td>
</tr>
<tr>
<td>11 W(0:IM,0:JM,KM)</td>
<td></td>
</tr>
<tr>
<td>12 CONC(0:IM,0:JM,0:KM,KSMAX)</td>
<td></td>
</tr>
<tr>
<td>13 PSORCD(0:IM,0:JM,KSMAX)</td>
<td></td>
</tr>
<tr>
<td>14 PSORCE(0:IM,0:JM,KSMAX)</td>
<td></td>
</tr>
</tbody>
</table>

Short output: records 5-7
Long output: records 5-14
In contrast to man-made watercourses, such as channels, natural watercourses are characterized by continuous irregularity. Irregular contours of the flow domain, irregular shape of the bed surface, etc. are rather the rule than the exception in natural water bodies. Such a variety of shapes is impossible to find in man-made watercourses. Therefore, the application of the CH3D code with its new sediment-operations program module to a natural prototype case is imperative in order to validate all the procedures and to demonstrate the capabilities of the sediment module, which was developed to simulate mobile-bed phenomena in natural water-bodies.

The Mississippi River at the Old River Control Structure complex was used as a prototype case in order to test and validate the new mobile-bed numerical procedures in the CH3D code.

Model of the Mississippi River at the Old River

An approximately 12-mile long stretch of the Mississippi River (between River Mile 307 and River Mile 319) in the vicinity of the Old River Control Structure complex is modelled (Fig. 6). The entrance channel of the Sidney A. Murray Hydropower facility is located at River Mile 316.5. The Low Sill Structure is located at River Mile 315 and the Auxiliary Structure is located at River Mile 312. The Sidney A. Murray Hydropower facility is operated by the City of Vidalia, Louisiana. The Low Sill and Auxiliary Structures are operated by the U.S. Army Engineer District, New Orleans.

The model domain and computational grid (Fig. 7), as well as all data required for the CH3D hydrodynamics computations have been provided by Waterways Experiment Station personnel. Boundary (1) (Fig. 7) is an upstream inflow boundary. Boundary (2) is a downstream boundary. The entrance channel of the Sidney A. Murray Hydropower facility (boundary (3)) and the Low Sill and Auxiliary Structures (boundary (4)) are treated as closed during hydrodynamic computations. Thus, boundaries (3) and (4) could be treated as impermeable boundaries, just as for the other closed domain boundaries. The hydrodynamics simulations, performed by WES personnel before the sediment-operations program module had been built into the CH3D code, are briefly described in the following paragraph.
The hydrodynamic simulations used a computational time step of 30 seconds, and the simulated time period was 30 days. The imposed condition along the upstream inflow boundary (1) (Fig. 7) was a so-called 'river' condition (imposed unit discharges across the river), while the condition along the downstream boundary (2) was a so-called 'tidal' condition (imposed free-surface elevations across the river). The remaining domain boundaries were treated as impermeable. So called 'zero-flow' initial conditions (i.e. horizontal free-surface elevation and zero velocity field) were used. The chosen combination of initial and boundary conditions (downstream free-surface elevation and non-zero upstream discharge imposed on initially still water) is known to produce a disturbance (wave) that propagates back and forth throughout the flow domain. A so-called 'stabilization period' is required to allow the disturbance to eventually die out.

The sediment module was tested by repeating the simulation described in the previous paragraph, but with all or some of the sediment operations switched on.

The sediment-computation tests were made with a computational time step of 30 seconds, and the simulated period was 5 days and 10 hours. Initial and boundary conditions for the hydrodynamics computations were as previously described. Sediment computations were initiated 10 hours after the beginning of the hydrodynamics computations. A stabilization period of 10 hours proved to be sufficient for the dissipation of the most severe wave propagation in the domain.

The main source of sediment data was the Hall and Fagerburg (1990) paper. The paper describes the sampling techniques and provides some results of two field surveys of the hydrodynamics and suspended-sediment transport of the Mississippi River at the Old River Control Structure complex, completed in February and June 1990.

Six ranges were established across the Mississippi River in the vicinity of the Old River Control Structure complex, from River Mile 310 to River Mile 320. One range was also established parallel to the bank of the Mississippi River across the entrance channels of the Low Sill and Auxiliary Structures. An additional range was established parallel to the bank of the Mississippi River across the entrance channel of the Sidney A. Murray Hydropower facility. The location of the ranges is shown in Figure 6.

According to Hall and Fagerburg (1990), bed material was very uniform throughout the study reach, with predominant bed-material grain diameters varying between 0.125 mm and 0.5 mm. Hall and Fagerburg (1990) classified the suspended sediment as suspended silt and clay (with grain diameter less than 0.0625 mm), and suspended sand (with grain diameter greater than or equal to 0.0625 mm). The suspended silt and clay concentration was described as nearly uniform in the vertical direction and across the range, at each range, and the measured concentration of silt and clay was approximately 250 mg/l.
Figure 6. Mississippi River at the Old River Control Structure Complex: Location Map.
The suspended sand concentration was described as varying significantly in the vertical direction, as well as varying in maximum concentration along the channel bed laterally across the range. The only measured vertical suspended-sand concentration profiles are shown in Figure 3 of the Hall and Fagerburg (1990) paper. The Figure shows suspended-sand concentration at the right bank line, channel center line, and left bank line measured during the June 1990 survey at range 2, located at the upstream end of the study reach.

The data presented by Hall and Fagerburg (1990) are not sufficiently detailed for a definitive numerical study of the Mississippi River near the Old River Control Structure complex. A definitive numerical study would require more detailed measured sediment data in order to better define initial and boundary conditions for the sediment computations, and to permit comparison of computed results and measured data. Detailed measured hydrodynamic data would also be necessary, in order to detect any significant discrepancies between measured and computed velocities and depths that may influence the sediment computations.

However, the goal of the tests reported herein is not to do a detailed numerical study, but to test and verify the new numerical procedures, and to demonstrate the capabilities of the newly developed sediment-operations program module. The data presented by Hall and Fagerburg (1990) are sufficient to permit construction of an appropriate sediment data set that describes the Mississippi River near the Old River closely enough for verification and demonstration purposes. Indeed, a data set used for a detailed numerical study may not always be suitable for test purposes. For example, even though the initial size-fraction distribution is in reality different at each bed point, demonstration of the sediment module's capability to compute changes in size-fraction distribution at different bed points over time is much easier followed if the same size-fraction distribution is initially assigned to all bed points.

Three size classes were chosen to simulate the natural sediment mixture in the Mississippi River near the Old River complex. Size class 1, with an equivalent diameter of 0.01 mm, represents the fine sediment mainly moving in suspension without having much contact with the bed, and corresponds to suspended clay and silt as defined by Hall and Fagerburg (1990). Size class 2, with an equivalent diameter of 0.1 mm, represents sand that may move in suspension, but also has extensive contact with the bed and is an important component of the sediment mixture on the bed. Size class 3, with an equivalent diameter of 0.5 mm, remains mainly on or near the bed.

Boundaries (1) and (2) (Fig. 7) are defined as sediment inflow and outflow boundaries, respectively, while the remaining domain boundaries are treated as impermeable.
Figure 7. The Model Domain and Computational Grid.
Boundary conditions for sediment computations are required along inflow boundary (1) only.

An active-layer size-fraction distribution with 0% of size class 1, 50% of size class 2, and 50% of size class 3, was assigned at all points along inflow boundary (1).

Also, at the inflow boundary (1) vertical suspended-sediment concentration profiles were assigned for all three size classes. For size class 1, a constant vertical concentration profile of 250 ppm was assigned to all inflow boundary points. The initial suspended-sand concentration at the right bank line, channel center line, and left bank line measured during the June 1990 survey at range 2 (located at the upstream end of the study reach) (Hall and Fagerburg (1990), Fig.3) inspired the construction of the three different vertical concentration profiles for size class 2 (Figure 8). Concentration profile 1 (Fig. 8) is assigned to inflow boundary points close to the right bank. Concentration profile 2 (Fig. 8) is assigned to the inflow boundary points around the centerline, while concentration profile 3 (Fig. 8) is assigned to points next to the left bank. Finally, for size class 3, a constant vertical concentration profile, with concentration equal to zero, is assigned to all inflow boundary points.

The described inflow sediment boundary conditions are kept constant for the duration of the simulation.

Initial conditions for sediment computations are required for all points inside the flow domain. An initial active-layer size-fraction distribution, with 0% of size class 1, 50% of size class 2, and 50% of size class 3, was assigned to all points. Also, initial vertical suspended-sediment concentration profiles were assigned for the three size classes, at all points. For size class 1, a constant vertical concentration profile of 250 ppm was assigned to all points. For size class 2, an initial vertical concentration profile identical to the centerline profile 2 (Fig. 8) is assigned to all points. Finally, for size class 3, a constant vertical concentration profile, with concentration equal to zero, is assigned to all points.

A sample input-data set used for the tests herein is presented in Appendix E.

Printed Output and Test Results

The sediment-computation program module has a built-in system for checking sediment input data and issuing error-warning messages. Error-warning messages contain several parameters identifying the location of error (such as time-step index and (i,j,k) indexes defining the location of the point), and a few parameters suggesting the nature of the error. In cases for which more information about the error/warning is needed, the messages also contain an error/warning message number identifying the subroutine where the
Figure 8. Vertical Concentration Profiles for Size Class 2 at the Model Upstream Boundary.
message originates. The last two digits to the right of the error/warning message number denote the error/warning sequential number in a specific subroutine (there is room for a maximum of 99 messages per subroutine). The remaining digits to the left of the error/warning message number denote the subroutine number (corresponding to the subroutine debug identification M listed in the Input Data Guide, Chapter IV).

The sediment-computation program module has the capability of including/excluding different computation steps, in order to permit separate testing of different procedures, using parameters IBED, ISUS, IADV, IDIF, JADV, JDIF, KADV (Input Data Guide, Chapter IV). First, the complete suspended-sediment computations were excluded in order to test only the bed-sediment computations (bedload transport and bed-evolution processes). Then the complete bed-sediment computations were excluded in order to test only the suspended-sediment transport. Furthermore, during testing of the suspended-sediment transport procedures, first some, and then all, of the advection and diffusion fluxes computed using the QUICKEST method were excluded, in order to separately test the implicit vertical-diffusion and fall-velocity procedures. Finally, the complete sediment-computation program module was tested. Only the test results for the complete sediment-computation module are reported herein.

The sediment-computations program module has number of built-in computation tests. Results of these tests for the particular run are designated 'sediment-computation diagnostics' in the printed output (file PRTSED.OUT). These diagnostics are printed with the chosen time-step frequency NDIAGS (see Input Data Guide, Chapter IV). The sediment-computation diagnostics are printed in the dimensionless form. The general structure of the sediment-computation diagnostics, as well as the specific diagnostics for the Old-River Complex model test runs, are briefly described below.

Sediment-computation diagnostics start with information concerning the number of Newton-Raphson iterations performed during bed-sediment computations. Reported are: maximum number of Newton-Raphson iterations (MITERS), for all bed points in the domain, during the current time step; indexes (IMITS, JMITS) defining the position (i,j) of the bed computational point with the maximum number of Newton-Raphson iterations; the average number of Newton-Raphson iterations (AVITS), for the entire domain during one time step; and the total number of Newton-Raphson iterations (ITSUM) for all bed points during one time step. The maximum number of Newton-Raphson iterations for the test runs was generally 5-6, while the average number of iterations was about 3.

Information concerning bed-sediment computation errors at the end of the Newton-Raphson iterations is reported next: the maximum change in bed elevation between two iterations (MAXDZB), for all bed points in the domain, during the current time step;
indexes (IMDZB,JMDZB) defining the position (i,j) of the bed computational point with the maximum change in bed elevation; the maximum change in active-layer size fraction between two iterations (MAXDB), for all bed points and all size fractions, during the current time step; and indexes (IMDB,JMDB,KSMDDB) defining the position (i,j) of the bed computational point and the size class (ks) of the active-layer size fraction with the maximum change. For the test runs, the number of Newton-Raphson iterations was always less than the maximum allowed number, meaning that the maximum changes in bed elevation and active-layer size fractions between the two last iterations were always less than the specified threshold values.

Negative values of active-layer size-class fractions have no physical meaning, and if they occur in the computations, they indicate possible computational anomalies. Therefore, the sediment-computation diagnostics also report: the total number of negative computed active-layer size fractions (NEGBET), within the entire domain and for all size classes; the minimum computed active-layer size fraction (MINBET), within the entire domain and for all size classes; and indexes (IMINB,JMINB,KSMINB) defining the position (i,j) of the bed computational point and the size class (ks) of the minimum computed active-layer size fraction. There were no negative computed active-layer size fractions during the test runs.

The basic constraint of the sum of all active-layer size fractions being equal to unity is also checked. The sediment-computation diagnostics report: the maximum error in the basic constraint (ERSUMB, departure from unity), for all bed computational points in the domain, during the current time step; and indexes (IERSB,JERSB) defining the position (i,j) of the point with the maximum departure from unity. The maximum departure from unity during the test runs was of the order of 1.E-10.

Mass-conservation errors during bed-sediment computations are computed and compared (where appropriate) to the corresponding total mass. The sediment-computation diagnostics report: the maximum mass-conservation error in the global mass-conservation equation for bed sediment (MZBED), for all bed computational points in the domain, during the current time step; indexes (IMZBED,JMZBED) defining the position (i,j) of the bed computational point with the maximum mass-conservation error in the global mass-conservation equation for bed sediment; the sum of all mass-conservation errors in the global mass-conservation equation for bed sediment (SZBED); the maximum mass-conservation error in the mass-conservation equation for one size class of active-layer sediment (MBETA), for all bed computational points in the domain and all size classes, during the current time step; indexes (IMBETA,JMBETA) defining the position (i,j) of the bed computational point with the maximum mass-conservation error in the mass-conservation equation for one size class of active-layer sediment; index (KMBETA) defining the size class
of the active-layer equation with the maximum mass-conservation error, the total mass (MASB) of sediment in active-layer control volume built around the bed computational point with the maximum mass-conservation error in the mass-conservation equation for one size class of active-layer sediment; the sum (SBETA) of all mass-conservation errors in mass-conservation equations for one size class of active-layer sediment, in the entire domain, during the current time step; and the total mass (SMASB) of active-layer sediment, in the entire domain, during the current time step.

During the test runs, the maximum mass-conservation error in the global mass-conservation equation for bed sediment (MZBED) was of the order of 1.E-13; the sum of all mass-conservation errors in the global mass-conservation equations for bed sediment (SZBED) was of the order of 1.E-12; the maximum mass-conservation error in the mass-conservation equation for one size class of active-layer sediment (MBETA) was of the order of 1.E-13; the total mass of sediment in the corresponding active-layer control volume (MASB) was of the order of 1.E-2; the sum of all mass-conservation errors in mass-conservation equations for one size class of active-layer sediment (SBETA) was of the order of 1.E-13 to 1.E-15, depending on the size class; and the total mass of active-layer sediment (SMASB) was of the order of 1.E-0.

Mass-conservation errors during the suspended-sediment computations are also computed and compared (where appropriate) to the corresponding total mass. The sediment-computation diagnostics report: the maximum mass-conservation error in the mass-conservation equation for one size class of suspended sediment (MCONC), for all computational points in the domain and for all size classes, during the current time step; indexes (IMCON,JMCON,KMCON) defining the position (i,j,k) of the computational point with the maximum mass-conservation error in the mass-conservation equation for one size class of suspended sediment; the index (KMSCON) defining the size class (ks) of suspended-sediment equation with the maximum mass-conservation error, the total mass (MASC) of suspended sediment in a control volume, computed for the computational point and size class that define the maximum mass-conservation error in the conservation equation; the sum (SCONC) of all mass-conservation errors in the mass-conservation equations for one size class of suspended sediment, in the entire domain, during the current time step; the total mass (SMASC) of suspended sediment, in the entire domain, during the current time step; the maximum relative (with respect to the appropriate total mass) mass-conservation error in the mass-conservation equation for one size class of suspended sediment (MCONR), for all computational points in the domain and for all size classes, during the current time step; the indexes (IMCR,JMCR,KMCR) defining the position (i,j,k) of the computational point with the maximum relative mass-conservation error in the mass-con-
ervation equation for one size class of suspended sediment; the index (KMSCR) defining the size class (ks) of the suspended-sediment equation with the maximum relative mass-conservation error; and the total mass (MASC) of suspended sediment in the control volume, computed for the computational point and size class that define the maximum relative mass-conservation error in the conservation equation. During the test runs, the maximum mass-conservation error in the mass-conservation equation for one size class of suspended sediment (MCONC) was of the order of 1.E-18; the total mass of sediment in the corresponding suspended-sediment control volume (MASC) was of the order of 1.E-5; the sum of all mass-conservation errors in the mass-conservation equations for one size class of suspended sediment (SCONC) was of the order of 1.E-17 to 1.E-18, depending on the size class; the sum of total mass of suspended sediment (SMASC) was of the order of 1.E-1; the maximum relative mass-conservation error in the mass-conservation equation for one size class of suspended sediment (MCONR) was of the order of 1.E-11; and the total mass (MASC) of suspended sediment in the corresponding control volume varied depending on the control-volume size.

It is known that the quadratic interpolations used in the QUICKEST method sometimes yield negative suspended-sediment concentrations. The sediment-computation diagnostics report: the total number of computed negative suspended-sediment concentrations (NEGCON), for all computational points in the domain, and for all size classes, during the current time step; the minimum computed negative suspended-sediment concentration (MINCON), for all computational points in the domain, and for all size classes, during the current time step; the indexes (IMINC,JMINC,KMINC) defining the position (i,j,k) of the computational point with the minimum computed concentration of suspended sediment; and the index (KSMINC) defining the size class (ks) of the minimum computed suspended-sediment concentration. During the test runs, the total number of computed negative suspended-sediment concentrations was as high as 2000, but the minimum computed concentration was of the order of -1.E-6.

The sediment-computation program module prints selected results (in the dimensional form) on the output file PRTSED.OUT with a selected time-step frequency NPERSED (Input Data Guide, Chapter IV). The available output matrices include: densities of water-sediment mixture, suspended-sediment concentrations, bed elevations, bed-elevation changes over a time step, cumulative bed-elevation changes, active-layer depths, active-layer size fractions, numbers of strata, depths of strata, strata size fractions, characteristic grain diameters, friction coefficients, suspended-sediment 'deposition' sources, suspended-sediment 'erosion' sources, and sediment-computation boundary types. These output matrices are selected by setting parameters IDENS, ICONC, IZBED, IDZBED,
Several basic sediment variables (suspended-sediment concentrations, cumulative bed-elevation changes (instead of bed elevations), and active-layer size fractions), at the end of a five-day simulation period for the Old-River complex test run are briefly described below.

The suspended-sediment concentration for the finest size class, with an equivalent diameter of 0.01 mm, (roughly corresponding to suspended clay and silt as defined by Hall and Fagerburg (1990)) varies both across the flow and along the Mississippi River reach, between 60-70 ppm and 300 ppm; but the vertical concentration profile remains roughly uniform. The suspended-sediment concentration for the medium size class, with an equivalent diameter of 0.1 mm, (roughly corresponding to suspended sand as defined by Hall and Fagerburg (1990)) varies not only across and along the flow, but also over the depth. Vertical concentration profiles are relatively steep, with mid-depth concentration being between 1 and 5 ppm. The coarsest size class, with an equivalent diameter of 0.5 mm, (roughly corresponding to the bed material of Hall and Fagerburg (1990)) is also entrained into suspension from place to place. However, where they exist, vertical concentration profiles are rather steep, with the third-depth concentration being close to 1 ppm. It is obvious that the suspended-sediment concentration for the coarsest size class must vary significantly across and along the flow.

Cumulative bed-elevation changes show the total erosion or deposition at the end of a simulated period. The maximum erosion occurs immediately upstream of the Auxiliary Structure. Erosion in that area varies between 30 and 90 cm. The highest computed erosion (90 cm) is probably unrealistic, occurring due to the schematic representation of initial (and boundary) data. For example, initial active-layer size-fraction distribution was simply assumed, due to the lack of data. The time history of the highest computed erosion (40 cm of erosion during the first simulated day and another 30 cm of erosion during the second simulated day, with the erosion rate of 6-7 cm for remaining three simulated days) also suggests an initial imbalance in input data. Maximum erosion in other parts of the domain does not exceed 20-30 cm. Maximum deposition is around 30 cm and occurs immediately downstream of the Auxiliary Structure, as well as immediately downstream of the inflow boundary of the domain. The deposition downstream of the Auxiliary Structure occurs next to the right bank, away from the main flow, due to the low velocities in that region. The deposition downstream of the inflow boundary occurs due to the constant inflow of suspended sediment, assigned as a boundary condition.
The finest size class generally plays a minor role in the sediment mixture at the bed. The only significant amount of the finest size class is computed in the low velocity region downstream of the Auxiliary Structure, next to the right bank. Significant amounts of medium size-class particles are found at the bed where deposition occurs. In regions where deposition occurs, the medium size class usually comprises between 30 and 60% of the sediment mixture at the bed, while the coarsest size class comprises the remainder. In regions where erosion occurs, the coarsest size class usually comprises over 90% of the sediment mixture at the bed.

The total simulated period, after 10 hours of stabilization, was 5 days. The CPU time required for that simulation was about 25 hours on an HP 750. The corresponding unit CPU time was about 6 seconds per time step, or 7 milliseconds per time step and per bed computational point and per each suspended-sediment computational point above a bed point.
CHAPTER VI

CONCLUSIONS AND SUGGESTIONS FOR FURTHER DEVELOPMENT

During the course of this research the innovative two-dimensional mobile-bed modelling techniques recently developed for depth-averaged, two-dimensional modelling at the Iowa Institute of Hydraulic Research, were generalized and merged with the CH3D three-dimensional hydrodynamic simulation code, thus generalizing CH3D to include mobile-bed processes (such as aggradation and scour, bed-material sorting, and movement of both bedload and suspended load of nonuniform sediment mixtures).

The original IIHR conceptualization of sediment transport and bed evolution is fully preserved:

1. - Sediment movement is identified as either suspended-sediment transport, bedload transport, or a combination of both. To describe suspended-sediment transport, the advection/diffusion (mass-conservation) equation is used. To describe bedload transport and bed evolution, a mass-conservation equation for active-layer sediment and a global mass-conservation equation for bed sediment are used. As defined herein, the active layer comprises sediment particles at (or immediately below) the bed surface and sediment particles moving as bedload close to the bed surface. The suspended-sediment 'erosion' and 'deposition' sources are defined as the exchange between bedload and suspended-sediment transport. Criteria for distinguishing between bedload and suspended-sediment transport are incorporated into expressions for the bedload flux.

2. - There is no need to seek any a priori recognition of washload and its possible interaction with the bed. This recognition occurs naturally and implicitly in the algorithm's distinctly different treatment of bedload and suspended-sediment transport.

3. - The sediment mixture in a natural watercourse is described by a number of discrete size classes. Mass-conservation equations for active-layer sediment and suspended sediment are written for each size class separately. The procedure assumes no restriction on the number of discrete size classes.

4. - The friction coefficient is determined by the distribution of active-layer size fractions (i.e. by the sediment size distribution at the bed surface).

The IIHR two-dimensional mobile-bed techniques were generalized to ensure their compatibility with the overall CH3D environment. First, the governing sediment equations (initially written in vector form and standard Cartesian coordinates) were redefined by (1) taking into account the non-constant density of the water-sediment mixture; (2) introducing
a 'fall-velocity' term into the three-dimensional suspended-sediment equation; (3) defining the suspended-sediment 'erosion' source as an upward near-bed diffusion flux; (4) defining the suspended-sediment 'deposition' source as a downward near-bed fall-velocity flux; (5) and introducing several new auxiliary relations (e.g. mass-diffusion coefficient, density of a mixture containing water and suspended sediment etc.). Then the governing sediment equations were rederived in $\sigma$-stretched coordinates, then nondimensionalized and rederived in general (nonorthogonal) horizontal curvilinear coordinates.

The described generalization improves the original IIHR mobile-bed techniques in several ways:

1. Three-dimensional computations allow actual computation of vertical concentration profiles, rather than computing depth-averaged concentrations and assuming theoretical self-similar concentration profiles, as is done in two-dimensional computations.

2. Accommodation of the non-constant density of the water-sediment mixture allows the numerical procedures to become more general, even offering the possibility to analyze density currents.

3. In the three-dimensional environment, both 'erosion' and 'deposition' suspended-sediment sources are defined more physically than is possible in the two-dimensional depth-averaged environment.

4. General curvilinear coordinates allow for better representation of natural-water-course complex geometry than do orthogonal curvilinear coordinates.

5. The non-constant density of the water-sediment mixture makes it possible to simulate how sediment-transport and bed-evolution processes influence the flow field through changes in the density of water-sediment mixture, in addition to the influences arising from changes in the friction coefficient and bed elevation.

The newly-developed procedure opens a number of possibilities for further research. The general computational framework makes it possible to isolate, study and possibly improve a number of specific aspects of sediment-flow interaction that rely on empirical relations. Specific experiments may even be designed to isolate particular details of sediment-flow interaction, and experimental results then might be compared to the appropriate computational results. Specific aspects of sediment-flow interaction to be studied in this way may include: the diffusion of suspended sediment, particularly the diffusion coefficient; the density of the water-sediment mixture; bedload transport etc.
REFERENCES


Hall, B.R., and Fagerburg, T.L., (1990), "Field Measurements of Hydrodynamics and Sediment Transport of the Mississippi River at Old River", U.S. Army Engineer Waterways Experimental Station, Vicksburg, Mississippi.


Spasojevic, M., (1988), "Numerical Simulation of Two-Dimensional (plan-view) Unsteady Water and Sediment Movement in Natural Watercourses", Theses presented to the University of Iowa, at Iowa City, Iowa, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.


APPENDIX A

QUICKEST METHOD

The basic concepts of Leonard's (1979) QUICKEST method are summarized herein. First, a simple one-dimensional transport (advection) equation is considered:

\[
\frac{\partial \phi}{\partial t} = -\frac{\partial (u\phi)}{\partial \xi} \tag{A1}
\]

Equation (A1) is integrated over a time step and a control volume built around main computational point C:

\[
\frac{\Delta \xi}{2} \int \phi^{n+1}_w \, dp - \frac{\Delta \xi}{2} \int \phi^n_w \, dp = \int_{E} u_w \phi_w \, d\tau - \int_{E} u_e \phi_e \, d\tau
\]

where: (1) and (2) are local rate-of-change terms; (3) and (4) are advection terms; \(\tau, p\) are local coordinates (Fig. A1); subscripts e, w denote east and west faces of the control volume built around the main computational point C (Fig. A1); subscripts E and W denote main computational points neighboring C (to the east and the west, respectively) (Fig. A1).

Figure A1.

73
Local rate-of-change terms are evaluated by representing \( \phi(p) \) as a quadratic function between W and E computational points (using main-point values at W, C, and E computational points), and then by integrating that function from 'w' to 'e':

\[
\frac{\Delta \xi}{2} \int_{-\frac{\Delta \xi}{2}}^{\frac{\Delta \xi}{2}} \phi(p)dp = \Delta \xi \left[ \phi_c + \frac{\Delta \xi^2}{24} \left( \frac{\partial^2 \phi}{\partial \xi^2} \right)_c \right]
\]

(A3)

where the following notation is invoked:

\[
\left( \frac{\partial^2 \phi}{\partial \xi^2} \right)_c = \frac{\phi_e - 2\phi_c + \phi_w}{\Delta \xi^2}
\]

Advection terms are treated by using a so-called 'upstream quadratic interpolation'. Consider, for example, the term \((3)\) in Eq. (A2).

When the continuity equation (mass-conservation for fluid flow) is invoked, Eq. (A1) becomes:

\[
\frac{\partial \phi}{\partial t} + u \frac{\partial \phi}{\partial \xi} = 0
\]

(A4)

When the Lagrangian approach is used, Equation (A4) reduces to:

\[
\frac{D\phi}{Dt} = 0 \quad \text{i.e.} \quad \phi = \text{const.}
\]

(A5)

valid along the trajectory of a fluid particle defined by:

\[
u = \frac{d\xi}{dt}
\]

(A6)

Figure A2 shows particle trajectories, with their 'departure' points (d) at time \( \tau = 0 \), and their 'arrival' points (a) on the control volume w-face at different times between 0 and \( \tau = \Delta t \).

If the trajectories are straight and parallel (\( u = \text{const} \)), as in Fig. A2, then it is fully justified to write:
\[
\Delta t \int u_w \phi_w d\tau = \int \phi^n dp \quad (A7)
\]

where:

\[
P_D = u_w \Delta t = \frac{u_w \Delta t}{\Delta \xi} \Delta \xi = C_{T_w} \Delta \xi
\]

and \(\tau, p\) are local coordinates as in Fig. (A2).

Figure A2.

The advection term is now evaluated by upstream quadratic interpolation: if the velocity direction is as in Fig. A2, \(\phi(p)\) is represented as a quadratic function between FW and C (using main-point values at FW, W, and C computational points), and then the function is integrated from 0 to \(p_D\):

\[
\int_{t^n}^{t^n+1} u_w \phi_w dt = \int_0^{p_D} \phi^n dp
\]

\[
= \bar{C}_{T_w} \Delta \xi \left[ \frac{\phi^n_c + \phi^n_w}{2} - \frac{\Delta \xi}{2} \bar{C}_{T_w} \left( \frac{\partial \phi}{\partial \xi} \right)_w^n + \frac{\Delta \xi^2}{6} \bar{C}_{T_w} \left( \frac{\partial^2 \phi}{\partial \xi^2} \right)_w^n - \frac{\Delta \xi^2}{8} \left( \frac{\partial^2 \phi}{\partial \xi^2} \right)_w^n \right] \quad (A8)
\]

where the following notation is invoked:
\[
\left( \frac{\partial^2 \phi}{\partial \xi^2} \right)_{w}^n = \frac{\phi_{e}^n - \phi_{W}^n}{\Delta \xi^2} \quad \text{for } \tilde{C}_{r_w} \geq 0
\]

\[
\left( \frac{\partial^2 \phi}{\partial \xi^2} \right)_{w}^n = \frac{\phi_{e}^n - 2\phi_{W}^n + \phi_{W}^n}{\Delta \xi^2} \quad \text{for } \tilde{C}_{r_w} < 0
\]

\[
\tilde{C}_{r_w} = \frac{\tilde{u}_w \Delta t}{\Delta \xi}
\]

\[
\tilde{u}_w = \frac{u_{w}^{n+1} + u_{w}^n}{2}
\]

If the velocity direction is the opposite to what is shown in Fig. A2, Eq. (A8) remains the same, except that:

\[
\left( \frac{\partial^2 \phi}{\partial \xi^2} \right)_{w}^n = \frac{\phi_{E}^n - 2\phi_{C}^n + \phi_{W}^n}{\Delta \xi^2} \quad \text{for } \tilde{C}_{r_e} < 0
\]

A similar expression is easily obtained for the advection flux through the e-face of control volume (Fig. A2):

\[
\int_{0}^{\Delta t} u_{e} \phi_{e} d\tau = \int_{0}^{\phi} d\rho
\]

\[
= \tilde{C}_{r_e} \Delta \xi \left[ \frac{\phi_{E}^n + \phi_{C}^n}{2} - \frac{\Delta \xi}{2} \tilde{C}_{r_e} \left( \frac{\partial \phi}{\partial \xi} \right)_{e}^n + \frac{\Delta \xi^2}{6} \tilde{C}_{r_e}^2 \left( \frac{\partial^2 \phi}{\partial \xi^2} \right)_{e}^n - \frac{\Delta \xi^2}{8} \left( \frac{\partial^2 \phi}{\partial \xi^2} \right)_{e}^n \right]
\]

where:

\[
\left( \frac{\partial \phi}{\partial \xi} \right)_{e}^n = \frac{\phi_{E}^n - \phi_{C}^n}{\Delta \xi}
\]

\[
\left( \frac{\partial^2 \phi}{\partial \xi^2} \right)_{e}^n = \frac{\phi_{E}^n - 2\phi_{C}^n + \phi_{W}^n}{\Delta \xi^2} \quad \text{for } \tilde{C}_{r_e} \geq 0
\]
\[
\left( \frac{\partial^2 \phi}{\partial \xi^2} \right)_e^n = \frac{\phi^{n+1}_c - 2\phi^n_c + \phi^n_c}{\Delta \xi^2} \quad \text{for} \quad \bar{C}_r \geq 0
\]

\[
\bar{C}_r = \frac{\bar{u}_c \Delta t}{\Delta \xi}
\]

\[
\bar{u}_e = \frac{u_e^{n+1} + u_e^n}{2}
\]

The local rate-of-change term:

\[
\frac{\Delta \xi}{2} \int \phi^{n+1}_c dp - \frac{\Delta \xi}{2} \int \phi^n_c dp = \Delta \xi \left( \phi^{n+1}_c - \phi^n_c \right) + \Delta \xi \frac{\Delta \xi^2}{24} \left[ \left( \frac{\partial^2 \phi}{\partial \xi^2} \right)^{n+1}_c - \left( \frac{\partial^2 \phi}{\partial \xi^2} \right)^n_c \right] \quad (A10)
\]

where \( p \) is a local coordinate as in Fig. A1, is further manipulated by using the governing Eq. (A1). Equation (A1) can be written as:

\[
\frac{\partial}{\partial t} \left( \frac{\partial^2 \phi}{\partial \xi^2} \right) = -\frac{\partial}{\partial \xi} \left[ \frac{\partial^2 (u \phi)}{\partial \xi^2} \right] \quad (A11)
\]

The left-hand-side of Eq. (A11) can be written as:

\[
\frac{\partial}{\partial t} \left( \frac{\partial^2 \phi}{\partial \xi^2} \right) = \frac{1}{\Delta t} \left[ \left( \frac{\partial^2 \phi}{\partial \xi^2} \right)^{n+1}_c - \left( \frac{\partial^2 \phi}{\partial \xi^2} \right)^n_c \right] \quad (A12)
\]

while the right-hand-side of Eq. (A11) is approximated, assuming a constant velocity, by:

\[
\frac{\partial}{\partial \xi} \left( \frac{\partial^2 (u \phi)}{\partial \xi^2} \right) = \frac{\partial}{\partial \xi} \left[ \frac{\partial^2 \phi}{\partial \xi^2} \right] = \frac{1}{\Delta \xi} \left[ \bar{u} e \left( \frac{\partial^2 \phi}{\partial \xi^2} \right)_e^n - \bar{u} w \left( \frac{\partial^2 \phi}{\partial \xi^2} \right)_w^n \right] \quad (A13)
\]

The final outcome is:
When Eq. (A14) is introduced into Eq. (A10), the local rate-of-change terms become:

\[
\begin{align*}
\frac{\Delta \xi}{2} \frac{2 \Delta \xi}{2} \left[ \phi \right]_{n+1}^{n} dp - \int \phi^{n} dp = \Delta \xi \left( \phi_{c}^{n+1} - \phi_{c}^{n} \right) + \frac{\Delta \xi}{2} \frac{\Delta \xi}{2} \left[ \frac{\partial^{2} \phi}{\partial \xi^{2}} \right]_{w}^{n} - \frac{\Delta \xi}{2} \frac{\Delta \xi}{2} \left[ \frac{\partial^{2} \phi}{\partial \xi^{2}} \right]_{e}^{n}
\end{align*}
\]  

By using Eqs. (A8), (A9) and (A15), the discretized Eq. (A12) reads:

\[
\phi^{n+1} - \phi^{n} = \tilde{C}_{w} \left[ \frac{\phi_{c} + \phi_{w}}{2} - \frac{\Delta \xi}{2} \frac{\Delta \xi}{2} \left( \frac{\partial \phi}{\partial \xi} \right)_{w}^{n} - \frac{\Delta \xi}{2} \frac{\Delta \xi}{2} \left( \frac{\partial \phi}{\partial \xi} \right)_{e}^{n} \right]
\]

A simple advection-diffusion equation is considered next:

\[
\frac{\partial \phi}{\partial t} = -u \frac{\partial \phi}{\partial \xi} + D \frac{\partial^{2} \phi}{\partial \xi^{2}}
\]  

Equation (A17) is integrated over a time step and a control volume built around a main computational point C (Fig. A1):

\[
\begin{align*}
\frac{\Delta \xi}{2} \frac{2 \Delta \xi}{2} \left[ \phi \right]_{n+1}^{n} dp - \int \phi^{n} dp = \int u_{w} \phi_{w} d\tau - \int u_{e} \phi_{e} d\tau + \int D_{w} \left( \frac{\partial \phi}{\partial \xi} \right)_{w} d\tau - \int D_{e} \left( \frac{\partial \phi}{\partial \xi} \right)_{e} d\tau
\end{align*}
\]  

The local rate-of-change terms (1) and (2), as well as the advection terms (3) and (4) are treated as previously described. The diffusion terms (5) and (6) are also treated
according to the same idea of 'upstream quadratic interpolation'. Consider, for example, the term (6):

\[
\int_0^{\Delta t} D_w \left( \frac{\partial \phi}{\partial \xi} \right)_w \, d\tau = \tilde{D}_w \left( \frac{\partial \phi}{\partial \xi} \right)_w \Delta t
\]  

(A19)

where

\[
\tilde{D}_w = \frac{D_w^{n+1} + D_w^n}{2}
\]

and

\[
\left( \frac{\partial \phi}{\partial \xi} \right)_w = \left( \frac{\partial \phi}{\partial \xi} \right)_w^{n+\frac{1}{2}}
\]

is the w-wall gradient of \( \phi \) evaluated at \( \frac{\Delta t}{2} \).

It is further assumed that the gradient of \( \phi \) is convected downstream essentially unchanged, along the constant-slope trajectory (Fig. A3). It remains to find the departure point \( D \) of the trajectory that arrives at the w-face of the control volume at \( -\frac{\Delta t}{2} \) and to evaluate the gradient at \( D \) by the upstream quadratic interpolation. If the w-wall velocity direction is as in Fig. A3 \( \phi \) is represented as a quadratic function between FW and C (using main-point values at FW, W, and C computational points), with the gradient of \( \phi \) readily derived from the function. The departure point of the trajectory is defined as:

\[
p_D = -\bar{u}_w \frac{\Delta t}{2} = -\bar{u}_w \Delta t \frac{\Delta \xi}{2} = -\frac{\Delta \xi}{2} \bar{C}_{r_w}
\]  

(A20)

where \( p_D \) is the position of the departure point in local coordinates (Fig. A3), and the gradient of \( \phi \) is evaluated at \( D \), leading to:
\[ \int_{0}^{\Delta t} D_w \left( \frac{\partial \phi}{\partial \xi} \right)_w \, d\tau = \bar{D}_w \left( \frac{\partial \phi}{\partial \xi} \right)_w \Delta t = \bar{\alpha}_w \Delta \xi \left[ \Delta \xi \left( \frac{\partial \phi}{\partial \xi} \right)_w^n - \frac{\Delta \xi^2}{2} \bar{C}_r \left( \frac{\partial^2 \phi}{\partial \xi^2} \right)_w^n \right] \] (A21)

Figure A3.

where:

\[ \bar{\alpha}_w = \frac{\bar{D}_w \Delta t}{\Delta \xi^2} \]

and the right-hand-side derivatives have the same meaning as in Eq. (A8).

A similar expression is easily obtained for the diffusion flux across the e-face of control volume:

\[ \int_{0}^{\Delta t} D_e \left( \frac{\partial \phi}{\partial \xi} \right)_e \, d\tau = \bar{D}_e \left( \frac{\partial \phi}{\partial \xi} \right)_e \Delta t = \bar{\alpha}_e \Delta \xi \left[ \Delta \xi \left( \frac{\partial \phi}{\partial \xi} \right)_e^n - \frac{\Delta \xi^2}{2} \bar{C}_r \left( \frac{\partial^2 \phi}{\partial \xi^2} \right)_e^n \right] \] (A22)

where

\[ \bar{\alpha}_e = \frac{\bar{D}_e \Delta t}{\Delta \xi^2} \]
and the right-hand-side derivatives have the same meaning as in Eq. (A9).

By using Eqs. (A8), (A9), (A15), (A21) and (A22), the discretized advection-diffusion equation (Eq. (A17) i.e. (A18)) reads:

\[
\begin{align*}
\phi_{c}^{n+1} - \phi_{c}^{n} &= \tilde{C}_{r,w} \left[ \frac{\phi_{c}^{n} + \phi_{w}^{n}}{2} - \frac{\Delta \xi}{2} \tilde{C}_{r,w} \left( \frac{\partial \phi}{\partial \xi} \right)_{w}^{n} \right. \\
&\left. - \frac{\Delta \xi^{2}}{6} \left( 1 - \tilde{C}_{r,w}^{2} \right) \left( \frac{\partial^{2} \phi}{\partial \xi^{2}} \right)_{w}^{n} \right] \\
- \tilde{C}_{r,e} \left[ \frac{\phi_{e}^{n} + \phi_{c}^{n}}{2} - \frac{\Delta \xi}{2} \tilde{C}_{r,e} \left( \frac{\partial \phi}{\partial \xi} \right)_{e}^{n} \right. \\
&\left. - \frac{\Delta \xi^{2}}{6} \left( 1 - \tilde{C}_{r,e}^{2} \right) \left( \frac{\partial^{2} \phi}{\partial \xi^{2}} \right)_{e}^{n} \right] \\
+ \tilde{A}_{e} \left[ \Delta \xi \left( \frac{\partial \phi}{\partial \xi} \right)_{e}^{n} - \frac{\Delta \xi^{2}}{2} \tilde{C}_{r,e} \left( \frac{\partial^{2} \phi}{\partial \xi^{2}} \right)_{e}^{n} \right] \\
- \tilde{A}_{w} \left[ \Delta \xi \left( \frac{\partial \phi}{\partial \xi} \right)_{w}^{n} - \frac{\Delta \xi^{2}}{2} \tilde{C}_{r,w} \left( \frac{\partial^{2} \phi}{\partial \xi^{2}} \right)_{w}^{n} \right]
\end{align*}
\]

(A23)

The discretized Eq. (A23) is explicit, meaning that the unknown scalar \( \phi \) is related to the main computational point \( C \) only. Scalars \( \phi \) related to the neighboring computational points appear in Eq. (A23) explicitly, i.e as known values.

Boundary conditions differ depending on the type of the boundary (impermeable, outflow or inflow). Following the recommendation of Leonard (1979), boundary conditions are specified to be control-volume wall values, rather than node values. Also, special interpolations are required at boundary points, or at points next to the boundary. Boundary conditions are summarized below.

First, in order to make further discussion of boundary conditions more clear, Eq. (A23) is rewritten as:

\[
\begin{align*}
\phi_{c}^{n+1} - \phi_{c}^{n} &= \text{(adv)}_{w} \left( \frac{\phi_{c}^{n} + \phi_{w}^{n}}{2} \right) - \text{(adv)}_{e} \left( \frac{\phi_{c}^{n} + \phi_{e}^{n}}{2} \right) + \text{(dif)}_{e} \left( \frac{\phi_{c}^{n} + \phi_{e}^{n}}{2} \right) - \text{(dif)}_{w} \left( \frac{\phi_{c}^{n} + \phi_{w}^{n}}{2} \right) \\
\text{(adv)}_{w} &= \tilde{C}_{r,w} \left[ \frac{\phi_{c}^{n} + \phi_{w}^{n}}{2} - \frac{\Delta \xi}{2} \tilde{C}_{r,w} \left( \frac{\partial \phi}{\partial \xi} \right)_{w}^{n} \right. \\
&\left. - \frac{\Delta \xi^{2}}{6} \left( 1 - \tilde{C}_{r,w}^{2} \right) \left( \frac{\partial^{2} \phi}{\partial \xi^{2}} \right)_{w}^{n} \right] \\
\text{(adv)}_{e} &= \tilde{C}_{r,e} \left[ \frac{\phi_{e}^{n} + \phi_{c}^{n}}{2} - \frac{\Delta \xi}{2} \tilde{C}_{r,e} \left( \frac{\partial \phi}{\partial \xi} \right)_{e}^{n} \right. \\
&\left. - \frac{\Delta \xi^{2}}{6} \left( 1 - \tilde{C}_{r,e}^{2} \right) \left( \frac{\partial^{2} \phi}{\partial \xi^{2}} \right)_{e}^{n} \right]
\end{align*}
\]

(A24)
where \((\text{adv})_w\) and \((\text{adv})_e\) stand for advection fluxes across the w-face and the e-face of the control volume, respectively, while \((\text{dif})_w\) and \((\text{dif})_e\) stand for diffusion fluxes across the w-face and the e-face of control volume, respectively:

**Outflow Boundary at w-Face of Control Volume**

The same procedure, as described by Eqs. (A1) to (A24), is applied to the computational point that has an outflow boundary at the w-face of control volume. The only difference is that the appropriate upstream quadratic interpolation, next to the w-face of control volume, uses known values at points E, C and the known outflow boundary value at \(B_w\), i.e. at the w-face of the control volume built around the computational point C.

As a result, advection and diffusion fluxes across the e-face of the the control volume, \((\text{adv})_e\) and \((\text{dif})_e\), have the same form as in Eq. (A24).

Advection and diffusion fluxes across the w-face of the control volume, \((\text{adv})_w\) and \((\text{dif})_w\), have the following form:

\[
(\text{adv})_w = \tilde{\alpha}_w \left[ \Delta \xi \left( \frac{\partial \phi}{\partial \xi} \right)_w - \frac{\Delta \xi^2}{2} \tilde{c}_{r,w} \left( \frac{\partial^2 \phi}{\partial \xi^2} \right)_w \right] \tag{A25}
\]

\[
(\text{dif})_w = \tilde{\alpha}_w \left[ \Delta \xi \left( \frac{\partial \phi}{\partial \xi} \right)_{B_w}^n - \frac{\Delta \xi^2}{2} \tilde{c}_{r,w} \left( \frac{\partial^2 \phi}{\partial \xi^2} \right)_{B_w}^n \right] \tag{A26}
\]

where

\[
(\text{adv})_e = \tilde{\alpha}_e \left[ \Delta \xi \left( \frac{\partial \phi}{\partial \xi} \right)_e - \frac{\Delta \xi^2}{2} \tilde{c}_{r,e} \left( \frac{\partial^2 \phi}{\partial \xi^2} \right)_e^n \right]
\]
The same procedure as described by Eqs. (A1) to (A24) is applied to a computational point that has an inflow boundary at the w-face of the control volume. The difference is that the appropriate upstream quadratic interpolation, next to the w-face of the control volume, uses known values at points E, C and a known outflow boundary value at Bw, i.e. at the w-face of the control volume built around computational point C. Also, the term (3) of Eq. (A2) (or Eq. (A18)) is evaluated by using straightforward integration, with the known inflow boundary value at Bw, i.e. at the w-face of the control volume:

\[
\int_0^{\Delta t} \tilde{u}_w \phi_w \, d\tau = \tilde{u}_w \phi_{Bw} \Delta t = \tilde{C}_{tw} \tilde{\phi}_{Bw} \Delta \xi \quad (A27)
\]

where

\[
\tilde{\phi}_{Bw} = \frac{\phi_{Bw}^{n+1} + \phi_{Bw}^n}{2}
\]

As a result, advection and diffusion fluxes across the e-face of the control volume, (adv)e and (dif)e, have the same form as in Eq. (A24).

Advection and diffusion fluxes across the w-face of the control volume, (adv)w and (dif)w, have the following form:

\[
(adv)_w = \tilde{C}_{tw} \left[ \frac{\Delta \xi^2}{24} \left( \frac{\partial^2 \phi}{\partial \xi^2} \right)_{Bw}^n \right] \quad (A28)
\]
\[(\text{dif})_w = \tilde{\alpha}_w \Delta \xi \left( \frac{\partial \phi}{\partial \xi} \right)^n_{B_w} \quad \text{(A29)}\]

where
\[
\left( \frac{\partial^2 \phi}{\partial \xi^2} \right)^n_{B_w} = \frac{4\phi^n_E - 12\phi^n_C + \phi^n_{B_w}}{3\Delta \xi^2} \\
\left( \frac{\partial \phi}{\partial \xi} \right)^n_{B_w} = \frac{-8\phi^n_{B_w} + 9\phi^n_C - \phi^n_E}{3\Delta \xi^2}
\]

are derivatives evaluated at the boundary \(B_w\), i.e. at the w-face of the control volume. The second right-hand-side term of Eq. (A28) originates from the appropriate rate-of-change term, as in Eq. (A15).

### Impermeable Boundary at w-Face of Control Volume

The same procedure as described by Eqs. (A1) to (A24) is applied to a computational point that has an impermeable boundary at the w-face of the control volume. The advection term (3) and the diffusion term (6) in Eq. (A18) are equal to zero. The velocity \(\tilde{u}_w\) i.e. the Courant number \(\tilde{C}_r_w\) are also equal to zero, which eliminates the appropriate term in Eq. (A15).

As a result, both advection and diffusion fluxes across the w-face of the control volume are equal to zero:

\[(\text{adv})_w = 0 \quad \text{(A30)}\]
\[(\text{dif})_w = 0 \quad \text{(A31)}\]

while the advection and diffusion fluxes across the e-face of the control volume \((\text{adv})_e\) and \((\text{dif})_e\), have the same form as in Eq. (A24).

### Outflow Boundary at e-Face of Control Volume

The same approach as for the outflow boundary at the w-face of the control volume is used. The appropriate upstream quadratic interpolation, next to the e-face of the control
volume, uses known values at points W, C and the known outflow boundary value at Be, i.e. at the e-face of the control volume built around C.

As a result, advection and diffusion fluxes across the w-face of the control volume, \((\text{adv})_w\) and \((\text{dif})_w\), have the same form as in Eq. (A24).

Advection and diffusion fluxes across the e-face of the control volume, \((\text{adv})_e\) and \((\text{dif})_e\), have the following form:

\[
(\text{adv})_e = C_e \left[ \phi^n_{Be} - \frac{\Delta \xi}{2} C_e \left( \frac{\partial \phi}{\partial \xi} \right)^n_{Be} - \frac{\Delta \xi^2}{6} \left( \frac{1}{4} - C_e^2 \left( \frac{\partial^2 \phi}{\partial \xi^2} \right)^n_{Be} \right) \right]
\]

\[(A32)\]

\[
(\text{dif})_e = \alpha_e \left[ \Delta \xi \left( \frac{\partial \phi}{\partial \xi} \right)^n_{Be} - \frac{\Delta \xi^2}{2} C_e \left( \frac{\partial^2 \phi}{\partial \xi^2} \right)^n_{Be} \right]
\]

\[(A33)\]

where

\[
\left( \frac{\partial \phi}{\partial \xi} \right)^n_{Be} = \frac{8 \phi^n_{Be} - 9 \phi^n_C + \phi^n_W}{3 \Delta \xi}
\]

\[
\left( \frac{\partial^2 \phi}{\partial \xi^2} \right)^n_{Be} = \frac{8 \phi^n_{Be} - 12 \phi^n_C + \phi^n_W}{3 \Delta \xi^2}
\]

are derivatives evaluated at the boundary, i.e. at the e-face of the control volume.

**Inflow Boundary at e-Face of Control Volume**

The same approach as for the outflow boundary at the w-face of the control volume is used. The appropriate upstream quadratic interpolation, next to the e-face of the control volume, uses known values at points W, C and the known outflow boundary value at Be, i.e. at the e-face of the control volume built around C. Term (4) of Eq. (A2) (or Eq. (A18)) is evaluated by using straightforward integration, with the known inflow boundary value at Be, i.e. at the e-face of the control volume built around C.

As a result, advection and diffusion fluxes across the w-face of the control volume, \((\text{adv})_w\) and \((\text{dif})_w\), have the same form as in Eq. (A24).
Advection and diffusion fluxes across the e-face of the control volume, \((\text{adv})_e\) and \((\text{dif})_e\), have the following form:

\[
(\text{adv})_e = \tilde{C}_e \left[ \phi_{Be} - \frac{\Delta \xi^2}{24} \left( \frac{\partial^2 \phi}{\partial \xi^2} \right)_{Be} \right] \quad (A34)
\]

\[
(\text{dif})_e = \bar{c}_e \Delta \xi \left( \frac{\partial \phi}{\partial \xi} \right)_{Be} \quad (A35)
\]

where

\[
\phi_{Be} = \frac{\phi_{Be}^{n+1} + \phi_{Be}^n}{2}
\]

\[
\frac{\partial \phi}{\partial \xi}_{Be}^n = \frac{8 \phi_{Be}^n - 9 \phi_c^n + \phi_w^n}{3 \Delta \xi}
\]

\[
\frac{\partial^2 \phi}{\partial \xi^2}_{Be}^n = \frac{8 \phi_{Be}^n - 12 \phi_c^n + 4 \phi_w^n}{3 \Delta \xi^2}
\]

are evaluated at the boundary, i.e. at the e-face of the control volume.

**Impermeable Boundary at e-Face of Control Volume**

Following the same arguments as for impermeable boundary at w-face of control volume, it is easy to conclude that both advection and diffusion fluxes across an impermeable e-face of the control volume are equal to zero:

\[
(\text{adv})_e = 0 \quad (A36)
\]

\[
(\text{dif})_e = 0 \quad (A37)
\]

while advection and diffusion fluxes across the w-face of the control volume, \((\text{adv})_w\) and \((\text{dif})_w\), have the same form as in Eq.(A24).
Interpolations for Points Next to the Boundary

Consider the w-face of the control volume built around the computational point C. First assume that the direction of the velocity is as in Fig. A4 and that the w-face of the control volume built around the computational point W is a boundary $B_{fw}$.

![Figure A4.](image)

The same procedure, as described by Eqs. (A1) to (A24), is applied to the computational point C. Advection and diffusion fluxes across the w-face, $(\text{adv})_w$ and $(\text{dif})_w$, of the control volume built around the computational point C have the same form as in Eq. (A24), the only difference being that the appropriate upstream quadratic interpolation uses known values at points C, W, and boundary $B_{fw}$, so that:

$$
\left( \frac{\partial^2 \phi}{\partial \xi^2} \right)^n_w = \frac{4\phi^n_C - 12\phi^n_W + 8\phi^n_{B_{fw}}}{3\Delta\xi^2}
$$

(A38)

Now, assume that the direction of the velocity is as in Fig. A5 and that the e-face of the control volume built around the computational point C is a boundary $B_{e}$.

The same procedure as described by Eqs. (A1) to (A24) is applied to the computational point C. Advection and diffusion fluxes across the w-face, $(\text{adv})_w$ and $(\text{dif})_w$, of the control volume built around the computational point C have the same form as in Eq. (A24), with one difference, namely that the appropriate upstream quadratic interpolation uses known values at points W, C, and boundary $B_{e}$, so that:
Figure A5.
First, the mass-conservation equation for suspended sediment (Eq. 29) is rewritten as follows:

\[
\frac{\partial}{\partial t} (JH\rho C) + R_o \left[ \frac{\partial}{\partial \xi} (JH\rho Cu) + \frac{\partial}{\partial \eta} (JH\rho Cv) + \frac{\partial}{\partial \sigma} (JH\rho Cw) \right] - R_{of} \frac{\partial}{\partial \sigma} (J\rho Cw_f) =
\]

\[
= \frac{E_{k_H} H}{S_{c_H}} \left[ \frac{\partial}{\partial \xi} \left( \frac{D_H}{J} g_{22} \frac{\partial (\rho C)}{\partial \xi} \right) - \frac{\partial}{\partial \eta} \left( \frac{D_H}{J} g_{12} \frac{\partial (\rho C)}{\partial \eta} \right) \right] - \frac{\partial}{\partial \eta} \left( \frac{D_H}{J} g_{12} \frac{\partial (\rho C)}{\partial \eta} \right) + \frac{E_{k_v} J}{H} \frac{\partial}{\partial \sigma} \left( \frac{D_v}{H} \frac{\partial (\rho C)}{\partial \sigma} \right) \tag{B1}
\]

Then, a sort of split-operator approach is used to split Eq. (B1) into three parts. The local rate of change due to the action of advection and diffusion terms in the \(\xi\)-coordinate direction, denoted as \(\left[ \frac{\partial}{\partial t} (JH\rho C) \right]^{\xi}\), can be expressed as:

\[
\left[ \frac{\partial}{\partial t} (JH\rho C) \right]^{\xi} = -R_o \frac{\partial}{\partial \xi} (JH\rho Cu) + \frac{E_{k_H} H}{S_{c_H}} \frac{\partial}{\partial \eta} \left( \frac{D_H}{J} g_{22} \frac{\partial (\rho C)}{\partial \xi} - \frac{D_H}{J} g_{12} \frac{\partial (\rho C)}{\partial \eta} \right) \tag{B2}
\]

In a similar notation, the local rate of change due to the action of advection and diffusion terms in the \(\eta\)-coordinate direction, added to the action of advection and diffusion terms in the \(\xi\)-coordinate direction, is expressed as:

\[
\left[ \frac{\partial}{\partial t} (JH\rho C) \right]^{\eta} = \left[ \frac{\partial}{\partial t} (JH\rho C) \right]^{\xi}
\]

\[
- R_o \frac{\partial}{\partial \eta} (JH\rho Cv) + \frac{E_{k_H} H}{S_{c_H}} \frac{\partial}{\partial \eta} \left( \frac{D_H}{J} g_{11} \frac{\partial (\rho C)}{\partial \eta} - \frac{D_H}{J} g_{12} \frac{\partial (\rho C)}{\partial \xi} \right) \tag{B3}
\]
Finally, the local rate of change due to the action of advection, fall-velocity and diffusion terms in the \( \sigma \)-coordinate direction, added to the combined action of advection and diffusion terms in the \( \xi \)- and \( \eta \)-coordinate directions, is expressed as:

\[
\left( \frac{\partial}{\partial t} (JH\rho C) \right)^{\sigma} = \left( \frac{\partial}{\partial t} (JH\rho C) \right)^{\eta}
\]

\[
= -R_o \frac{\partial}{\partial \sigma} (JH\rho C) + \frac{E_{k_v}}{S_{c_v}} \frac{J}{H} \frac{\partial}{\partial \sigma} \left( D_v \frac{\partial (\rho C)}{\partial \sigma} \right)
\]  

The mass-conservation equation for suspended sediment (Eq. (34)) is discretized in three successive steps: (1) \( \xi \)-direction step i.e discretization of Eq. (B2) by using a generalized version of the QUICKEST method; (2) \( \eta \)-direction step i.e. discretization of Eq. (B3), also by using a generalized QUICKEST method; (3) \( \sigma \)-direction step i.e. discretization of Eq. (B4) by using the QUICKEST method for local rate-of-change and advection terms, an upwind finite-difference scheme for the fall-velocity term, and a time weighted central differencing for the diffusion term. A complete discretized mass-conservation equation for suspended sediment is obtained by adding the results of the three successive steps.

\( \xi \)-direction step

The original Leonard's (1979) QUICKEST method, briefly outlined in Appendix A for the one-dimensional advection-diffusion equation in Cartesian coordinates, is generalized herein in order to accommodate the appropriate suspended-sediment equation terms in a three-dimensional curvilinear context.

First, a simplified equation is considered:

\[
\frac{\partial}{\partial t} (JH\rho C) = -R_o \frac{\partial}{\partial \xi} (JH\rho C)
\]  

Equation (B5) is integrated over a time step and a control volume built around a main computational point C:
\[
\frac{\Delta k}{2} \int J H^{n+1} (pC)^{n+1} dp - \frac{\Delta k}{2} \int J H^n (pC)^n dp
\]
\[
\frac{\Delta k}{2}
\]

(1) \hspace{1cm} (2) \hspace{1cm} \text{(B6)}

\[= R_o \int_0^{\Delta t} Jw H_w (pC)_w u_w d\tau - R_o \int_0^{\Delta t} J_e H_e (pC)_e u_e d\tau \]

(3) \hspace{2cm} (4)

where: (1) and (2) are local rate-of-change terms; (3) and (4) are advection terms; \(\tau, p\) are local coordinates (Fig. B1); subscripts e, w denote east and west faces of the control volume built around the main computational point C (Fig. B1).

Fig. B1.

The integral Equation (B6) is further simplified, to facilitate the application of the QUICKEST method:
Local rate-of-change terms are evaluated by representing \((\rho C)\) as a quadratic function between \(W\) and \(E\) (using known values at \(W\), \(C\) and \(E\)), and then by integrating that function from 'w' to 'e':

\[
\frac{\Delta \xi}{2} J_c H_{C} \int \frac{(\rho C)_n + 1}{2} dp - J_c H_{C} \int \frac{(\rho C)_n}{2} dp
\]

(1) \hspace{1cm} (2)

\[
= R_o J_w \tilde{H}_w \int (\rho C)_w u_w dt - R_o J_e \tilde{H}_e \int (\rho C)_e u_e dt
\]

(3) \hspace{1cm} (4)

where

\[
\tilde{H} = \frac{H_{n+1} + H_n}{2}
\]

Advection terms are treated by using a so-called 'upstream quadratic interpolation'. Consider, for example, term (3) in Eq. (B7). When the continuity equation (mass conservation for fluid flow) is invoked, Eq. (B5) becomes:

\[
\frac{\partial (\rho C)}{\partial t} + R_o u \frac{\partial (\rho C)}{\partial \xi} = 0
\]

(B9)
When the Lagrangian approach is used, Eq. (B9) reduces to:

$$\frac{D(pC)}{Dt} = 0 \quad \text{i.e.} \quad pC = \text{const.} \quad (B10)$$

valid along the trajectory of a fluid particle defined by:

$$R_0 u = \frac{d\xi}{dt} \quad (B11)$$

Figure B2 shows particle trajectories, with their 'departure' points (d) at time and their 'arrival' points (a) on the w-face of the control volume at different times between 0 and $\Delta t$.

If the trajectories are straight and parallel ($u=\text{const}$), as in Fig. B2, then it is fully justified to write:

$$R_0 \int_{t_n}^{t_{n+1}} (pC)_w u_w dt = \int_{t_n}^{t_{n+1}} (pC)^n dp \quad (B12)$$

where:
\[ p_D = R_0 u_w \Delta t = R_0 \frac{u_w \Delta t}{\Delta \xi} \Delta \xi = C_{rw} \Delta \xi \]

\[ \tilde{C}_{rw} = R_0 \tilde{u}_w \Delta t \]

and \( \tau, p \) are local coordinates as in Fig. B2.

The advection term is now evaluated by upstream quadratic interpolation: if the velocity direction is as in Fig. B2, \((\rho C)\) is represented as a quadratic function between FW and C (using main-point values at FW, W, and C), and then the function is integrated from 0 to \( p_D \):

\[
J_w \tilde{H}_w \left( \int_0 (\rho C)^n \, dp \right) = J_w \tilde{H}_w \tilde{C}_{rw} \Delta \xi \left[ \frac{(\rho C)_c + (\rho C)_w}{2} \right] 
\]

\[
- \frac{\Delta \xi}{2} \left( \frac{\partial (\rho C)}{\partial \xi} \right)_w^n + \frac{\Delta \xi^2}{6} \tilde{C}_{rw} \left( \frac{\partial^2 (\rho C)}{\partial \xi^2} \right)_w^n - \frac{\Delta \xi^2}{8} \left( \frac{\partial^2 (\rho C)}{\partial \xi^2} \right)_w^n \right] 
\]

where the following notation is invoked:

\[
\left( \frac{\partial (\rho C)}{\partial \xi} \right)_w^n = \frac{(\rho C)_c^n - (\rho C)_W^n}{\Delta \xi} 
\]

\[
\left( \frac{\partial^2 (\rho C)}{\partial \xi^2} \right)_w^n = \frac{(\rho C)_c^n - 2(\rho C)_W^n + (\rho C)_FW^n}{\Delta \xi^2} \quad \text{for} \quad \tilde{C}_{rw} \geq 0
\]

\[
\tilde{C}_{rw} = R_0 \frac{\tilde{u}_w \Delta t}{\Delta \xi} 
\]

\[
\tilde{u}_w = \frac{\tilde{u}_w^{n+1} + u_w^n}{2}
\]

If the velocity direction is the opposite to what is shown in Fig. B2, Eq. (B13) remains the same, except that:
\[
\left( \frac{\partial^2 (\rho C)}{\partial \xi^2} \right)^n_w = \frac{(\rho C)^n_E - 2(\rho C)^n_c + (\rho C)^n_W}{\Delta \xi^2} \quad \text{for } \tilde{C}_{t_e} < 0
\]

A similar expression is easily obtained for the advection flux through the e-face of the control volume (Fig. B2):

\[
J_e \tilde{H}_e \int_0^{\tilde{C}_{t_e} \Delta \xi} (\rho C)^n dp = J_e \tilde{H}_e \Delta \xi \left[ \frac{(\rho C)^n_E + (\rho C)^n_c}{2} - \frac{\Delta \xi}{2} \left( \frac{\partial (\rho C)}{\partial \xi} \right)^n_e \right] \\
+ \frac{\Delta \xi^2}{6} \tilde{C}_{t_e}^2 \left( \frac{\partial^2 (\rho C)}{\partial \xi^2} \right)^n_e - \frac{\Delta \xi^2}{8} \left( \frac{\partial^2 (\rho C)}{\partial \xi^2} \right)^n_e
\]

(B14)

where:

\[
\left( \frac{\partial (\rho C)}{\partial \xi} \right)^n_e = \frac{(\rho C)^n_E - (\rho C)^n_c}{\Delta \xi}
\]

\[
\left( \frac{\partial^2 (\rho C)}{\partial \xi^2} \right)^n_e = \frac{(\rho C)^n_E - 2(\rho C)^n_c + (\rho C)^n_W}{\Delta \xi^2} \quad \text{for } \tilde{C}_{t_e} \geq 0
\]

\[
\left( \frac{\partial^2 (\rho C)}{\partial \xi^2} \right)^n_e = \frac{(\rho C)^n_E - 2(\rho C)^n_c + (\rho C)^n_c}{\Delta \xi^2} \quad \text{for } \tilde{C}_{t_e} < 0
\]

\[
\tilde{C}_{t_e} = R_0 \frac{\bar{u}_e \Delta t}{\Delta \xi}
\]

\[
\bar{u}_e = \frac{u_{e}^{n+1} + u_{e}^n}{2}
\]

The local rate-of-change term:

\[
J_e \tilde{H}_e^{n+1} \int \frac{2}{\Delta \xi} (\rho C)^{n+1} dp - J_e \tilde{H}_e^n \int \frac{2}{\Delta \xi} (\rho C)^n dp
\]
\[ a_2 \begin{pmatrix} \frac{\partial}{\partial \xi^2} (JH \rho C) \end{pmatrix} = \frac{-R_o}{\Delta \xi} \left[ J_c H_c^{n+1} \left( \frac{\partial^2 (\rho C)}{\partial \xi^2} \right)_c^n - J_c H_c^n \left( \frac{\partial^2 (\rho C)}{\partial \xi^2} \right)_c^n \right] \] (B15)

where \( p \) is a local coordinate as in Fig. B1, is further manipulated by using the governing Eq. (B5). Equation (B5) can be written as:

\[ \frac{\partial}{\partial t} \left[ \frac{\partial^2}{\partial \xi^2} (JH \rho C) \right] = -R_o \frac{\partial}{\partial \xi} \left[ \frac{\partial^2}{\partial \xi^2} (JH \rho C) \right] \] (B16)

The left-hand-side of Eq. (B16) is approximated by:

\[ \frac{\partial}{\partial t} \left[ \frac{\partial^2}{\partial \xi^2} (JH \rho C) \right] = \frac{\partial}{\partial t} \left[ JH \frac{\partial^2 (\rho C)}{\partial \xi^2} \right] = \frac{1}{\Delta t} \left[ J_c H_c^{n+1} \frac{\partial^2 (\rho C)}{\partial \xi^2} \right] \] (B17)

while the right-hand-side of Eq. (B16) is approximated by:

\[ -R_o \frac{\partial}{\partial \xi} \left[ \frac{\partial^2}{\partial \xi^2} (JH \rho C) \right] = R_o \frac{\partial}{\partial \xi} \left[ JH \frac{\partial^2 (\rho C)}{\partial \xi^2} \right] = \frac{R_o}{\Delta \xi} \left[ J_c H_c u_c \left( \frac{\partial^2 (\rho C)}{\partial \xi^2} \right)_e^n - J_w H_w u_w \left( \frac{\partial^2 (\rho C)}{\partial \xi^2} \right)_w^n \right] \] (B18)

The final outcome is:

\[ J_c H_c^{n+1} \left( \frac{\partial^2 (\rho C)}{\partial \xi^2} \right)_c^n - J_c H_c^n \left( \frac{\partial^2 (\rho C)}{\partial \xi^2} \right)_c^n \] (B19)

\[ = J_w \tilde{H}_w \tilde{C}_w \left( \frac{\partial^2 (\rho C)}{\partial \xi^2} \right)_w^n - J_e \tilde{H}_e \tilde{C}_e \left( \frac{\partial^2 (\rho C)}{\partial \xi^2} \right)_e^n \]
When Eq. (B19) is introduced into Eq. (B15), the local rate-of-change terms become:

\[
\begin{align*}
\frac{\Delta \xi}{2} & J_{c}H_{c}^{n+1} \int (pC)^{n+1} dp - \frac{\Delta \xi}{2} J_{c}H_{c}^{n} \int (pC)^{n} dp \\
= & \Delta \xi \left[ J_{c}H_{c}^{n+1}(pC)_{c}^{n+1} - J_{c}H_{c}^{n}(pC)_{c}^{n} \right] \\
& + \Delta \xi \frac{\Delta \xi^{2}}{24} \left[ J_{w}H_{w}C_{w} \left( \frac{\partial^{2}(pC)}{\partial \xi^2} \right)_{w}^{n} - J_{e}H_{e}C_{e} \left( \frac{\partial^{2}(pC)}{\partial \xi^2} \right)_{e}^{n} \right]
\end{align*}
\]

(B20)

By using Eqs. (B13), (B14) and (B20), the discretized Eq. (B7) reads:

\[
\begin{align*}
J_{c}H_{c}^{n+1}(pC)_{c}^{n+1} - J_{c}H_{c}^{n}(pC)_{c}^{n} \\
= J_{w}H_{w}C_{w} \left[ \frac{(pC)_{w}^{n} + (pC)_{w}^{n}}{2} - \frac{\Delta \xi}{2} \left( \frac{\partial(pC)}{\partial \xi} \right)_{w}^{n} - \frac{\Delta \xi^{2}}{24} \left( 1 - C_{w}^{2} \right) \left( \frac{\partial^{2}(pC)}{\partial \xi^2} \right)_{w}^{n} \right] \\
& + J_{e}H_{e}C_{e} \left[ \frac{(pC)_{e}^{n} + (pC)_{e}^{n}}{2} - \frac{\Delta \xi}{2} \left( \frac{\partial(pC)}{\partial \xi} \right)_{e}^{n} - \frac{\Delta \xi^{2}}{24} \left( 1 - C_{e}^{2} \right) \left( \frac{\partial^{2}(pC)}{\partial \xi^2} \right)_{e}^{n} \right]
\end{align*}
\]

(B21)

A complete Eq. (B2) is considered next. Equation (B2) is integrated over a time step and the control volume built around a main computational point C (Fig. B1):

\[
\begin{align*}
\frac{\Delta \xi}{2} \int J_{H_{c}}^{n+1}(pC)^{n+1} dp & - \frac{\Delta \xi}{2} \int J_{H_{c}}^{n}(pC)^{n} dp \\
= R_{o} \int J_{w}H_{w}(pC)_{w}u_{w}d\tau & - R_{o} \int J_{e}H_{e}(pC)e_{e}d\tau
\end{align*}
\]

(1) (2) (3) (4)
The local rate-of-change terms (1) and (2), as well as the advection terms (3) and (4) are treated as previously described. The diffusion terms (5) and (6) are also treated according to the same idea of 'upstream quadratic interpolation'. Consider, for example, term (6):

\[
\frac{E_{kH} H_c}{S_{ch}} \int_0^{\Delta t} \left[ g_{22} \left( \frac{\partial (pC)}{\partial \xi} \right)_c - g_{12} \left( \frac{\partial (pC)}{\partial \eta} \right)_c \right] d\tau
\]

(5)

\[
\frac{E_{kH} H_c}{S_{ch}} \int_0^{\Delta t} \left[ g_{22} \left( \frac{\partial (pC)}{\partial \xi} \right)_w - g_{12} \left( \frac{\partial (pC)}{\partial \eta} \right)_w \right] d\tau
\]

(6)

\[
\frac{E_{kH} H_c}{S_{ch}} \int_0^{\Delta t} \left[ g_{22} \left( \frac{\partial (pC)}{\partial \xi} \right)_w - g_{12} \left( \frac{\partial (pC)}{\partial \eta} \right)_w \right] d\tau =
\]

(B22)

(B23)

where

\[
\tilde{D}_{Hw} = \frac{D_{Hw}^{n+1} + D_{Hw}^n}{2}
\]

and

\[
\left( \frac{\partial (pC)}{\partial \xi} \right)_w = \left( \frac{\partial (pC)}{\partial \xi} \right)_w^{n+1/2}
\]

\[
\left( \frac{\partial (pC)}{\partial \eta} \right)_w = \left( \frac{\partial (pC)}{\partial \eta} \right)_w^{n+1/2}
\]

are w-wall gradients of \((pC)\) evaluated at \(\frac{\Delta t}{2}\).
It is further assumed that the gradients of \((\rho C)\) are advected downstream essentially unchanged, along the constant-slope trajectory (Fig. B3). It remains to find the departure point \(D\) of the trajectory that arrives at the \(w\)-face of the control volume at \(\frac{\Delta t}{2}\) and to evaluate gradients at \(D\) by upstream quadratic interpolation. The departure point of the trajectory is defined as:

\[
p_D = -R_0 \tilde{u}_w \frac{\Delta t}{2} = -R_0 \frac{\tilde{u}_w \Delta t}{\Delta \xi^2} = -\tilde{C}_{\tau w} \frac{\Delta \xi^2}{2} \tag{B24}
\]

where \(p_D\) is the position of the departure point in local coordinates (Fig. B3).

Figure B3.

Further treatment of gradients \(\left(\frac{\partial (\rho C)}{\partial \xi}\right)\) and \(\left(\frac{\partial (\rho C)}{\partial \eta}\right)\) slightly differs. If the \(w\)-wall velocity direction is as in Fig. B3, \((\rho C)\) is represented as a quadratic function between \(FW\) and \(C\) (using main-point values at \(FW\), \(W\), and \(C\), with the gradient \(\left(\frac{\partial (\rho C)}{\partial \xi}\right)\) readily derived from the function. The gradient \(\left(\frac{\partial (\rho C)}{\partial \xi}\right)^n\) evaluated at \(D\), reads:

\[
\left(\frac{\partial (\rho C)}{\partial \xi}\right)_w^n = \left(\frac{\partial (\rho C)}{\partial \xi}\right)_D^n = \Delta \xi \frac{1}{\Delta \xi^2} \left[ \Delta \xi \left(\frac{\partial (\rho C)}{\partial \xi}\right)_w^n - \frac{\Delta \xi^2}{2} \tilde{C}_{\tau w} \left(\frac{\partial^2 (\rho C)}{\partial \xi^2}\right)_w^n \right] \tag{B25}
\]
and the right-hand-side derivatives have the same meaning as in Eq. (B13). If the w-wall velocity has the opposite direction from what is shown in Fig. B3, the right-hand-side derivatives again have the same meaning as in Eq. (B13).

Since it is not possible to derive the gradient \( \frac{\partial (pC)}{\partial \eta} \) from a function defined along the \( \xi \) direction, the solution is found in defining previous-time-level auxiliary derivatives \( \left( \frac{\partial (pC)}{\partial \eta} \right)_n \) at main computational points (by simple central differencing) and evaluating the gradients \( \left( \frac{\partial (pC)}{\partial \eta} \right)_D \) by upstream linear interpolation. Linear interpolation is justified for gradients of a quadratic function. The gradient \( \left( \frac{\partial (pC)}{\partial \eta} \right)_n \) evaluated at \( D \) reads:

\[
\left( \frac{\partial (pC)}{\partial \eta} \right)_w = \left( \frac{\partial (pC)}{\partial \eta} \right)_D = \frac{1 - \tilde{C}_{rw}}{2} \left( \frac{\partial (pC)}{\partial \eta} \right)_c + \frac{1 + \tilde{C}_{rw}}{2} \left( \frac{\partial (pC)}{\partial \eta} \right)_w \quad \text{for } |\tilde{C}_{rw}| < 1 \quad (B26)
\]

i.e.

\[
\left( \frac{\partial (pC)}{\partial \eta} \right)_w = \left( \frac{\partial (pC)}{\partial \eta} \right)_D = \frac{3 - \tilde{C}_{rw}}{2} \left( \frac{\partial (pC)}{\partial \eta} \right)_c + \frac{\tilde{C}_{rw} - 1}{2} \left( \frac{\partial (pC)}{\partial \eta} \right)_w \quad \text{for } \tilde{C}_{rw} > 1 \quad (B27)
\]

i.e.

\[
\left( \frac{\partial (pC)}{\partial \eta} \right)_w = \left( \frac{\partial (pC)}{\partial \eta} \right)_D = \frac{3 + \tilde{C}_{rw}}{2} \left( \frac{\partial (pC)}{\partial \eta} \right)_c - \frac{1 - \tilde{C}_{rw}}{2} \left( \frac{\partial (pC)}{\partial \eta} \right)_E \quad \text{for } \tilde{C}_{rw} < -1 \quad (B28)
\]

where the Courant-type number \( \tilde{C}_{rw} \) shows not only the w-wall velocity direction, but also the interpolation interval.

Similar expressions are easily obtained for the diffusion flux across the e-face of the control volume:
\[
\frac{E_{kH}}{S_{cH}} \hat{H}_e \int_o^{\Delta t} \left[ g_{22e} \left( \frac{\partial (pC)}{\partial \xi} \right)_e - g_{12e} \left( \frac{\partial (pC)}{\partial \eta} \right)_e \right] \, d\tau = \\
= \frac{L_{kH_1}}{S_{cH}} \hat{H}_e \int_o^{\Delta t} \left[ g_{22e} \left( \frac{\partial (\sim pC)}{\partial \xi} \right)_e - g_{12e} \left( \frac{\partial (\sim pC)}{\partial \eta} \right)_e \right] \, d\tau
\] 

where

\[
\hat{D}_{H_e} = \frac{D_{H_e}^{n+1} + D_{H_e}^n}{2}
\]

The gradient \( \left( \frac{\partial (\sim pC)}{\partial \xi} \right)_e \) reads:

\[
\left( \frac{\partial (\sim pC)}{\partial \xi} \right)_e = \left( \frac{\partial (pC)}{\partial \xi} \right)_D^n + \frac{1}{\Delta \xi} \left[ \Delta \xi \left( \frac{\partial (pC)}{\partial \xi} \right)_e^n - \frac{\Delta \xi^2}{2} C_{re} \left( \frac{\partial^2 (pC)}{\partial \xi^2} \right)_e^n \right]
\]

where the right-hand-side derivatives have the same meaning as in Eq. (B14).

The gradient \( \left( \frac{\partial (pC)}{\partial \eta} \right)_e \) reads:

\[
\left( \frac{\partial (\sim pC)}{\partial \eta} \right)_e = \left( \frac{\partial (pC)}{\partial \eta} \right)_D^n + \frac{1 - C_{re}}{2} \left( \frac{\partial (pC)}{\partial \eta} \right)_E^n + \frac{1 + C_{re}}{2} \left( \frac{\partial (pC)}{\partial \eta} \right)_C^n \quad \text{for } \left| C_{re} \right| < 1
\]

i.e.

\[
\left( \frac{\partial (\sim pC)}{\partial \eta} \right)_e = \left( \frac{\partial (pC)}{\partial \eta} \right)_D^n + \frac{3 - C_{re}}{2} \left( \frac{\partial (pC)}{\partial \eta} \right)_C^n + \frac{C_{re} - 1}{2} \left( \frac{\partial (pC)}{\partial \eta} \right)_W^n \quad \text{for } C_{re} > 1
\]

i.e.
\[
\left( \frac{\partial (\rho C)}{\partial \eta} \right)_e = \left( \frac{\partial (\rho C)}{\partial \eta} \right)_D - \frac{1}{2} \frac{\tilde{C}_r e}{\tilde{C}_r e} \left( \frac{\partial (\rho C)}{\partial \eta} \right)_E + \frac{3}{2} \frac{\tilde{C}_r e}{\tilde{C}_r e} \left( \frac{\partial (\rho C)}{\partial \eta} \right)_E \text{ for } \tilde{C}_r e < -1 \quad (B33)
\]

where the Courant-type number \( \tilde{C}_r e \) shows not only the e-wall velocity direction, but also the interpolation interval.

Finally, the discretized Equation (B2), i.e. (B22), reads:

\[
J_{cH}^{n+1} (\rho C)_c^{n+1} - J_{cH}^n (\rho C)^n_c
= J_w \tilde{H}_w \tilde{C}_r w \left[ \frac{(\rho C)_c^n + (\rho C)_w^n}{2} - \frac{\Delta \xi^2}{2} \left( \frac{\partial (\rho C)}{\partial \xi} \right)_w^n - \frac{\Delta \xi^2}{6} (1 - \tilde{C}_r w) \left( \frac{\partial^2 (\rho C)}{\partial \xi^2} \right)_w^n \right] \\
- J_c \tilde{H}_e \tilde{C}_r e \left[ \frac{(\rho C)_e^n + (\rho C)_c^n}{2} - \frac{\Delta \xi^2}{2} \left( \frac{\partial (\rho C)}{\partial \xi} \right)_e^n - \frac{\Delta \xi^2}{6} (1 - \tilde{C}_r e) \left( \frac{\partial^2 (\rho C)}{\partial \xi^2} \right)_e^n \right] \\
+ \tilde{\alpha}_{H_e} \tilde{H}_e \left\{ g_{22 e} \left[ \Delta \xi \left( \frac{\partial (\rho C)}{\partial \xi} \right)_e^n - \frac{\Delta \xi^2}{2} \tilde{C}_r e \left( \frac{\partial^2 (\rho C)}{\partial \xi^2} \right)_e^n \right] - g_{12 e} \Delta \xi \left( \frac{\partial C}{\partial \eta} \right)_e^n \right\} \\
- \tilde{\alpha}_w \tilde{H}_w \left\{ g_{22 w} \left[ \Delta \xi \left( \frac{\partial (\rho C)}{\partial \xi} \right)_w^n - \frac{\Delta \xi^2}{2} \tilde{C}_r w \left( \frac{\partial^2 (\rho C)}{\partial \xi^2} \right)_w^n \right] - g_{12 w} \Delta \xi \left( \frac{\partial C}{\partial \eta} \right)_w^n \right\}
\]
\]
\[
(B34)
\]

where:

\[
\tilde{\alpha}_{H_w} = \frac{E_{kH}}{S_{cH}} \frac{\tilde{H}_w \Delta t}{\Delta \xi^2} \quad \tilde{\alpha}_{H_e} = \frac{E_{kH}}{S_{cH}} \frac{\tilde{H}_e \Delta t}{\Delta \xi^2}
\]

The discretized Eq. (B34) is explicit, meaning that the unknown suspended-sediment concentration is related to the computational point C only. Suspended-sediment concentrations related to neighboring computational points appear in Eq. (B34) explicitly, i.e. as known values. It is useful to point out those elements of the discretized equation that are expressed explicitly in terms of sediment variables by introducing a special notation for them. Therefore, Equation (B34) is rewritten as:

\[
J_{cH}^x_c (\rho C)_c^x - J_{cH}^b_c (\rho C)_c^b = (\text{adv})_w - (\text{adv})_e + (\text{dif})_e - (\text{dif})_w
\]
\[
(B35)
\]
\[(\text{adv})_e = J_w \hat{H}_w \hat{C}_{t_w} \left[ \frac{(pC)_e^n + (pC)_w^n}{2} - \frac{\Delta \xi}{2} \frac{\left( \partial(pC) \right)_w^n}{\xi} - \frac{\Delta \xi^2}{6} \left( 1 - \hat{C}_{t_w} \right) \frac{\left( \partial^2(pC) \right)_w^n}{\xi^2} \right] \]

\[(\text{adv})_w = J_w \hat{H}_w \hat{C}_{t_w} \left[ \frac{(pC)_e^n + (pC)_w^n}{2} - \frac{\Delta \xi}{2} \frac{\left( \partial(pC) \right)_w^n}{\xi} - \frac{\Delta \xi^2}{6} \left( 1 - \hat{C}_{t_e} \right) \frac{\left( \partial^2(pC) \right)_e^n}{\xi^2} \right] \]

\[(\text{dif})_w = \tilde{a}_H \frac{\hat{H}_w}{J_w} \left[ g_{22w} \left[ \Delta \xi \frac{\left( \partial(pC) \right)_w^n}{\xi} - \frac{\Delta \xi^2}{2} \hat{C}_{t_w} \frac{\left( \partial^2(pC) \right)_w^n}{\xi^2} \right] - g_{12w} \Delta \xi \frac{\left( \partial(pC) \right)_w^n}{\eta} \right] \]

\[(\text{dif})_e = \tilde{a}_H \frac{\hat{H}_e}{J_e} \left[ g_{22e} \left[ \Delta \xi \frac{\left( \partial(pC) \right)_e^n}{\xi} - \frac{\Delta \xi^2}{2} \hat{C}_{t_e} \frac{\left( \partial^2(pC) \right)_e^n}{\xi^2} \right] - g_{12e} \Delta \xi \frac{\left( \partial(pC) \right)_e^n}{\eta} \right] \]

where \((\text{adv})_w\) and \((\text{adv})_e\) stand for advection fluxes across the w-face and the e-face of the control volume, respectively, while \((\text{dif})_e\) and \((\text{dif})_w\) stand for diffusion fluxes across the w-face and the e-face of the control volume, respectively:

**Boundary Conditions**

Boundary conditions for Eq. (B34) (i.e. Eq. (B35)) differ depending on the type of the boundary (impermeable, outflow or inflow). Following the recommendation of Leonard (1979), boundary conditions are specified to be control-volume wall values, rather than node values. Also, special interpolations are required at boundary points, or at points next to the boundary. Boundary conditions are summarized below.

**Outflow Boundary at w-Face of Control Volume**

The same procedure as described by Eqs. (B5) to (B35) is applied to a computational point that has an outflow boundary at the w-face of the control volume. The only difference is that the appropriate upstream quadratic interpolation, next to the w-face of the control volume, uses known values at points E, C and known outflow boundary value at \(B_w\), i.e at the w-face of the control volume built around point C.

As a result, advection and diffusion fluxes across the e-face of the control volume, \((\text{adv})_e\) and \((\text{dif})_e\), have the same form as in Eq. (B35).
Advection and diffusion fluxes across the w-face of the control volume, \((\text{adv})_w\) and \((\text{dif})_w\), have the following form:

\[
(\text{adv})_w = J_w \tilde{H}_w \tilde{C}_w \left[ (\rho C)_B^w \frac{\Delta \xi}{\xi} \tilde{C}_r \left( \frac{\partial (\rho C)}{\partial \xi} \right)_B^w - \frac{\Delta \xi^2}{6} \left( \frac{\partial (\rho C)}{\partial \xi^2} \right)_B^w \right] \tag{B36}
\]

\[
(\text{dif})_w = \tilde{H}_w \frac{\tilde{C}_w}{J_w} \left[ g_{22} \frac{\Delta \xi}{\xi} \left( \frac{\partial (\rho C)}{\partial \xi} \right)_B^w - \frac{\Delta \xi^2}{2} \tilde{C}_r \left( \frac{\partial^2 (\rho C)}{\partial \xi^2} \right)_B^w \right] - g_{12} \frac{\Delta \xi}{\xi} \left( \frac{\tilde{\rho} (\rho C)}{\partial \eta} \right)_B^w \right] \tag{B37}
\]

where

\[
\left( \frac{\partial (\rho C)}{\partial \xi} \right)_B^w = -\frac{8(\rho C)_B^w + g(\rho C)_B^w - (\rho C)_E^w}{3\Delta \xi}
\]

\[
\left( \frac{\partial^2 (\rho C)}{\partial \xi^2} \right)_B^w = \frac{4(\rho C)_B^w - 12(\rho C)_C^w + 8(\rho C)_B^w}{3\Delta \xi^2}
\]

\[
\left( \frac{\tilde{\rho} (\rho C)}{\partial \eta} \right)_B^w = \left( 1 + \tilde{C}_r \right) \left( \frac{\partial (\rho C)}{\partial \eta} \right)_B^w - \tilde{C}_r \left( \frac{\partial (\rho C)}{\partial \eta} \right)_C^w \quad \text{for} \quad -1 < \tilde{C}_r < 0
\]

i.e.

\[
\left( \frac{\tilde{\rho} (\rho C)}{\partial \eta} \right)_B^w = \frac{3 + \tilde{C}_r}{2} \left( \frac{\partial (\rho C)}{\partial \eta} \right)_C^w - \frac{1 + \tilde{C}_r}{2} \left( \frac{\partial (\rho C)}{\partial \eta} \right)_E^w \quad \text{for} \quad \tilde{C}_r < -1
\]

are derivatives evaluated at the boundary \(B_w\), i.e. at the w-face of the control volume built around point C.
Inflow Boundary at w-Face of Control Volume

The same procedure as described by Eqs. (B5) to (B35) is applied to a computational point that has an inflow boundary at the w-face of the control volume. The difference is that the appropriate upstream quadratic interpolation, next to the w-face of the control volume, uses known values at points E, C and the known outflow boundary value at B_w, i.e. at the w-face of the control volume built around C. Also, the term (3) of Eq. (B6) (or Eq. (B22)) is evaluated by using straightforward integration, with the known inflow boundary value at B_w, i.e. at the w-face of the control volume built around C:

\[
R_0 J_w \tilde{H}_w \int_0^{\Delta t} (\rho C)_w u_w \, dt = R_0 J_w \tilde{H}_w \tilde{u}_w (\frac{\tilde{\rho}_C}{B_w}) \Delta t = J_w \tilde{H}_w \tilde{C}_r \left(\frac{\tilde{\rho}_C}{B_w}\right) \Delta \xi \tag{B38}
\]

where

\[
\left(\frac{\tilde{\rho}_C}{B_w}\right) = \frac{(\rho C)_{B_w}^{n+1} + (\rho C)_{B_w}^{n}}{2}
\]

As a result, advection and diffusion fluxes across the e-face of the control volume, \((\text{adv})_e\) and \((\text{dif})_e\), have the same form as in Eq. (B35).

Advection and diffusion fluxes across the w-face of control volume, \((\text{adv})_w\) and \((\text{dif})_w\), have the following form:

\[
(\text{adv})_w = J_w \tilde{H}_w \tilde{C}_r = \left[ \left(\frac{\tilde{\rho}_C}{B_w}\right) - \frac{\Delta \xi^2}{24} \left(\frac{\partial^2 (\rho C)}{\partial \xi^2}\right)_{B_w} \right] \tag{B39}
\]

\[
(\text{dif})_w = \tilde{H}_w \frac{\tilde{C}_r}{J_w} \left[ g_{22w} \Delta \xi \left(\frac{\partial (\rho C)}{\partial \xi}\right)_{B_w} - g_{12w} \Delta \xi \left(\frac{\partial (\rho C)}{\partial \eta}\right)_{B_w} \right] \tag{B40}
\]

where

\[
\left(\frac{\partial^2 (\rho C)}{\partial \xi^2}\right)_{B_w} = \frac{4(\rho C)_{B_w}^n - 12(\rho C)_{B_w}^n + (\rho C)_{B_w}^n}{3\Delta \xi^2}
\]
are derivatives evaluated at the boundary, i.e. at the w-face of the control volume. The second right-hand-side term of Eq. (B39) originates from the appropriate rate-of-change term, as in Eq. (B20).

**Impermeable Boundary at w-Face of Control Volume**

The same procedure as described by Eqs. (B5) to (B35) is applied to a computational point that has an impermeable boundary at the w-face of the control volume. The advection term (3) in Eq. (B22) is equal to zero. The velocity \( \tilde{u}_w \), i.e. the Courant number \( \tilde{C}_r_w \), are also equal to zero, which eliminates the appropriate term in Eq. (B20). The first derivative across the impermeable boundary is also set to zero.

As a result, advection and diffusion fluxes across the w-face of the control volume read:

\[
(\text{adv})_w = 0 \tag{B41}
\]

\[
(\text{dif})_w = \tilde{a}_w \frac{\tilde{H}_w}{J_w} \left[ g_{22_w} \Delta \xi \left( \frac{\tilde{\left( pC \right)}}{\xi} \right)_w - g_{12_w} \Delta \eta \left( \frac{\tilde{\left( pC \right)}}{\eta} \right)_w \right] \tag{B42}
\]

where:

\[
\left( \frac{\tilde{\left( pC \right)}}{\xi} \right)_w = 0
\]

\[
\left( \frac{\tilde{\left( pC \right)}}{\eta} \right)_w = \left( \frac{\left( pC \right)}{\eta} \right)_w^n
\]
are derivatives evaluated at the boundary, i.e. at the w-face of the control volume built around point C.

Advection and diffusion fluxes across the e-face of the control volume, \((\text{adv})_e\) and \((\text{dif})_e\), have the same form as in Eq. (B35).

**Outflow Boundary at e-Face of Control Volume**

The same approach as for an outflow boundary at the w-face of the control volume is used. The appropriate upstream quadratic interpolation, next to the e-face of the control volume, uses known values at points W, C and a known outflow boundary value at \(B_e\), i.e. at the e-face of the control volume built around point C.

As a result, advection and diffusion fluxes across the w-face of the control volume, \((\text{adv})_w\) and \((\text{dif})_w\), have the same form as in Eq. (B35).

Advection and diffusion fluxes across the e-face of control volume, \((\text{adv})_e\) and \((\text{dif})_e\), have the following form:

\[
\begin{align*}
(\text{adv})_e &= J_e \tilde{H}_e \tilde{C}_e \left[ \rho C_B^n - \frac{\Delta \xi}{2} \tilde{C}_e \left( \frac{\partial (\rho C)}{\partial \xi} \right)^n_{B_e} - \frac{\Delta \xi^2}{6} \left( \frac{1}{4} - \tilde{C}_e^2 \right) \left( \frac{\partial^2 (\rho C)}{\partial \xi^2} \right)^n_{B_e} \right] \\
(\text{dif})_e &= \alpha H_e \frac{\tilde{H}_e}{J_e} \left[ g_{22} \left( \frac{\partial (\rho C)}{\partial \xi} \right)^n_{B_e} - \frac{\Delta \xi^2}{2} \tilde{C}_e \left( \frac{\partial^2 (\rho C)}{\partial \xi^2} \right)^n_{B_e} \right] \\
&\quad - g_{12} \Delta \xi \left( \frac{\partial (\rho C)}{\partial \eta} \right)^n_{B_e} \right] \\
\end{align*}
\]

(B43)

(B44)

where

\[
\frac{\partial (\rho C)}{\partial \xi}^n_{B_e} = \frac{-8(\rho C)^n_{B_e} - 9(\rho C)^n_C + (\rho C)^n_W}{3\Delta \xi} \\
\frac{\partial^2 (\rho C)}{\partial \xi^2}^n_{B_e} = \frac{8(\rho C)^n_{B_e} - 12(\rho C)^n_C + 4(\rho C)^n_W}{3\Delta \xi^2}
\]
\[
\left( \frac{\partial (pC)}{\partial \eta} \right)_{Be} = \left( 1 - \tilde{C}_{re} \right) \left( \frac{\partial (pC)}{\partial \eta} \right)_{C}^{n} + \tilde{C}_{re} \left( \frac{\partial (pC)}{\partial \eta} \right)_{C}^{n} \quad \text{for} \quad 0 < C_{re} < 1
\]

i.e.
\[
\left( \frac{\partial (pC)}{\partial \eta} \right)_{Be} = \frac{3 - \tilde{C}_{re}}{2} \left( \frac{\partial (pC)}{\partial \eta} \right)_{C}^{n} + \frac{\tilde{C}_{re} - 1}{2} \left( \frac{\partial (pC)}{\partial \eta} \right)_{W}^{n} \quad \text{for} \quad C_{re} > 1
\]

are derivatives evaluated at the boundary, i.e. at e-face of control volume built around C.

**Inflow Boundary at e-Face of Control Volume**

The same approach as for an inflow boundary at the w-face of the control volume is used. The appropriate upstream quadratic interpolation, next to the e-face of the control volume, uses known values at points W, C and a known outflow boundary value at Be, i.e. at the e-face of the control volume built around point C. Term (4) of Eq. (B6) (or Eq. (B22)) is evaluated by using straightforward integration, with a known inflow boundary value at Be, i.e. at the e-face of the control volume.

As a result, advection and diffusion fluxes across the w-face of the control volume, \((\text{adv})_{w}\) and \((\text{dif})_{w}\), have the same form as in Eq. (B35).

Advection and diffusion fluxes across the e-face of the control volume, \((\text{adv})_{e}\) and \((\text{dif})_{e}\), have the following form:

\[
(\text{adv})_{e} = J_{e} \tilde{H}_{e} \tilde{C}_{re} \left[ \left( \frac{\partial (pC)}{\partial \eta} \right)_{Be} - \frac{\Delta \xi^{2}}{24} \left( \frac{\partial^{2} (pC)}{\partial \xi^{2}} \right)_{Be}^{n} \right] \quad \text{(B45)}
\]

\[
(\text{dif})_{e} = \tilde{H}_{e} \tilde{H}_{e} \left\{ g_{12} \Delta \xi^{2} \left( \frac{\partial (pC)}{\partial \xi} \right)_{Be}^{n} - g_{12} \Delta \xi \left( \frac{\partial (pC)}{\partial \eta} \right)_{Be}^{n} \right\} \quad \text{(B46)}
\]

where

\[
\left( \frac{\partial^{2} (pC)}{\partial \xi^{2}} \right)_{Be}^{n} = \frac{8(pC)_{Be}^{n} - 12(pC)_{C}^{n} + 4(pC)_{W}^{n}}{3\Delta \xi^{2}}
\]

108
\[
\left( \frac{\partial(pC)}{\partial \xi} \right)_e^n = \frac{8(pC)_B^n}{3\Delta \xi} - 9(pC)_C^n + (pC)_W^n
\]

\[
\left( \frac{\partial(pC)}{\partial \eta} \right)_e^n = \frac{1}{2} \left[ \left( \frac{\partial(pC)}{\partial \eta} \right)_e^{n+1} + \left( \frac{\partial(pC)}{\partial \eta} \right)_e^n \right]
\]

\[
\left( \rho_C \right)_e^n = \frac{1}{2} \left[ (pC)_B^{n+1} + (pC)_e^n \right]
\]

are evaluated at the boundary, i.e. at the e-face of the control volume built around C.

**Impermeable Boundary at e-Face of Control Volume**

Following the same arguments as for an impermeable boundary at the w-face of the control volume, it is easy to derive expressions for advection and diffusion fluxes across an impermeable e-face of the control volume:

\[
(\text{adv})_e = 0 \tag{B47}
\]

\[
(\text{dif})_e = \bar{\Omega}_e \frac{\vec{H}_e}{J_e} \left\{ g_{22_e} \Delta \xi \left( \frac{\partial(pC)}{\partial \xi} \right)_e^{n+1} - g_{12_e} \Delta \xi \left( \frac{\partial(pC)}{\partial \eta} \right)_e^n \right\} \tag{B48}
\]

where

\[
\left( \frac{\partial(pC)}{\partial \xi} \right)_e^n = 0
\]

\[
\left( \frac{\partial(pC)}{\partial \eta} \right)_e^n = \left( \frac{\partial(pC)}{\partial \eta} \right)_e^n
\]

while advection and diffusion fluxes across the w-face of the control volume, \((\text{adv})_w\) and \((\text{dif})_w\), have the same form as in Eq. (B35).
Interpolations for Points Next to the Boundary

Consider the w-face of a control volume built around a computational point C.

First assume that the direction of the velocity is as in Fig. B4 and that the w-face of the control volume built around the computational point W is a boundary \( B_{fw} \).

![Figure B4.](image)

The same procedure as described by Eqs. (B5) to (B35) is applied to the computational point C. Advection and diffusion fluxes across the w-face of the control volume built around the computational point C, \((\text{adv})_w\) and \((\text{dif})_w\), have the same form as in Eq. (B35), the only difference being that the appropriate upstream quadratic interpolations use known values at points C, W and boundary \( B_{fw} \), so that:

\[
\left( \frac{\partial^2 (\rho C)}{\partial \xi^2} \right)_w^n = \frac{4(\rho C)_C^n - 12(\rho C)_W^n + 8(\rho C)_{B_{fw}}^n}{3\Delta \xi^2} \tag{B49}
\]

\[
\left( \frac{\partial (\rho C)}{\partial \eta} \right)_w^n = \left( 2 - \tilde{C}_{r_w} \right) \left( \frac{\partial (\rho C)}{\partial \eta} \right)_W^n + \left( \tilde{C}_{r_w - 1} \right) \left( \frac{\partial (\rho C)}{\partial \eta} \right)_{B_{fw}}^n \quad \text{for} \quad \tilde{C}_{r_w} > 1 \tag{B50}
\]

Now, assume that the direction of the velocity is as in Fig. B5 and that the e-face of the control volume built around the computational point C is a boundary \( B_{ce} \).
The same procedure as described by Eqs. (B5) to (B35) is applied to the computational point C. Advection and diffusion fluxes across the w-face of the control volume built around the computational point C, \((\text{adv})_w\) and \((\text{dif})_w\), have the same form as in Eq. (B35), with one difference, namely that the appropriate different upstream interpolations use known values at points W, C and boundary \(B_e\), so that:

\[
\left( \frac{\partial^2 (pC)}{\partial \xi^2} \right)_w^n = \frac{8(pC)_{B_e}^n - 12(pC)_C^n + 4(pC)_W^n}{3\Delta \xi^2}
\]

\[
\left( \frac{\tilde{\partial}(pC)}{\partial \eta} \right)_w = (2 + \tilde{C}_{rw}\left( \frac{\partial(pC)}{\partial \eta} \right)_C^n - (1 - \tilde{C}_{rw})\left( \frac{\partial(pC)}{\partial \eta} \right)_{B_e}^n \quad \text{for} \quad \tilde{C}_{rw} < -1
\]

\[\text{\eta-direction step}\]

Equation (B3) is discretized by using the same adapted version of the QUICKEST method as in the \(\xi\)-direction step. The only difference is that the local rate-of-change due to the action of the \(\xi\)-direction terms \(\frac{\partial}{\partial t} (JHpC) \) is integrated from the south to the north face of the control volume, assuming that cell-centered variables are representative of the entire integration interval:
\[
\begin{align*}
\frac{\Delta t}{2} \int J H^{\xi} (\rho C)^{\xi} d\eta - \frac{\Delta t}{2} \int J H^{n} (\rho C)^{n} d\eta &= \Delta \eta J_c H_c^{\xi} (\rho C)_c^{\xi} - \Delta \eta J_c H_c^n (\rho C)_c^n \\
\text{(B53)}
\end{align*}
\]

The discretized Equation (B3) reads:

\[
J_c H_c^n (\rho C)_c^n - J_c H_c^n (\rho C)_c^n = J_c H_c^{\xi} (\rho C)_c^{\xi} - J_c H_c^n (\rho C)_c^n
\]

\[
+ J_s H_s \tilde{C}_{rs} \left[ \frac{(\rho C)_c^n + (\rho C)_s^n}{2} - \frac{\Delta \eta}{2} \tilde{C}_{rs} \left( \frac{\partial (\rho C)}{\partial \eta} \right)_s^n - \frac{\Delta \eta^2}{6} \left( 1 - \tilde{C}_{rs} \right) \left( \frac{\partial^2 (\rho C)}{\partial \eta^2} \right)_s^n \right]
\]

\[
- J_n \tilde{H}_n \tilde{C}_{tn} \left[ \frac{J_c(\rho C)_c^n + (\rho C)_n^n}{2} - \frac{\Delta \eta}{2} \tilde{C}_{tn} \left( \frac{\partial (\rho C)}{\partial \eta} \right)_n^n - \frac{\Delta \eta^2}{6} \left( 1 - \tilde{C}_{tn} \right) \left( \frac{\partial^2 (\rho C)}{\partial \eta^2} \right)_n^n \right]
\]

\[
+ \alpha_{H_n} \tilde{H}_{n} \left\{ \xi_{11n} \left[ \Delta \eta \left( \frac{\partial (\rho C)}{\partial \eta} \right)_n^n - \frac{\Delta \eta^2}{2} \tilde{C}_{tn} \left( \frac{\partial^2 (\rho C)}{\partial \eta^2} \right)_n^n \right] - \xi_{12n} \Delta \eta \left( \frac{\partial (\rho C)}{\partial \xi} \right)_n^n \right\}
\]

\[
- \alpha_{H_s} \tilde{H}_{s} \left\{ \xi_{11s} \left[ \Delta \eta \left( \frac{\partial (\rho C)}{\partial \eta} \right)_s^n - \frac{\Delta \eta^2}{2} \tilde{C}_{rs} \left( \frac{\partial^2 (\rho C)}{\partial \eta^2} \right)_s^n \right] - \xi_{12s} \Delta \eta \left( \frac{\partial (\rho C)}{\partial \xi} \right)_s^n \right\}
\]

\[
\text{(B54)}
\]

where superscripts \(\xi\) and \(\eta\) define variables at time \((n+1)\) after the \(\xi\)- and \(\eta\)-direction steps, respectively; subscripts \(n, s\) define the north and south faces of the control volume built around a main computational point \(C\), respectively, (Fig. B6); subscripts \(N\) and \(S\) define main computational points north and south from \(C\), respectively; subscripts \(FN\) and \(FS\) define main computational points far north and far south from \(C\) (Fig. B6); and where:

\[
\text{Figure B6.}
\]

112
\[ \tilde{H}_s = \frac{H_{n+1} + H_{n}}{2} \quad \tilde{H}_n = \frac{H_{n+1} + H_{n}}{2} \quad \tilde{H}_c = \frac{H_{n+1} + H_{n}}{2} \]

\[ \bar{C}_r = R_o \frac{\bar{v}_r \Delta t}{\Delta \eta} \quad \bar{C}_r = R_o \frac{\bar{v}_r \Delta t}{\Delta \eta} \]

\[ \bar{v}_s = \frac{v_{n+1} + v_{n}}{2} \quad \bar{v}_n = \frac{v_{n+1} + v_{n}}{2} \]

\[ \tilde{\alpha}_H = \frac{E_k H \bar{D}_{H_n} \Delta t}{S_{cH} \Delta \eta^2} \quad \tilde{\alpha}_H = \frac{E_k H \bar{D}_{H_n} \Delta t}{S_{cH} \Delta \eta^2} \]

\[ \tilde{D}_H = \frac{D_{n+1}^H + D_{n}^H}{2} \quad \tilde{D}_H = \frac{D_{n+1}^H + D_{n}^H}{2} \]

\[ \left( \frac{\partial (pC)}{\partial \eta} \right)_s^n = \left( \frac{(pC)^n_c - (pC)^n_s}{\Delta \eta} \right) \]

\[ \left( \frac{\partial^2 (pC)}{\partial \eta^2} \right)_s^n = \left( \frac{(pC)^n_c - 2(pC)^n_s + (pC)^n_{fs}}{\Delta \eta^2} \right) \text{ if } \bar{C}_r \geq 0 \]

\[ \left( \frac{\partial^2 (pC)}{\partial \eta^2} \right)_s^n = \left( \frac{4(pC)^n_c - 12(pC)^n_s + 8(pC)^n_{fs}}{3\Delta \eta^2} \right) \text{ if } \bar{C}_r \geq 0 \text{ and } s (\text{Fig. B6}) \text{ is a boundary } B_{fs} \]

\[ \left( \frac{\partial^2 (pC)}{\partial \eta^2} \right)_s^n = \left( \frac{(pC)^n_N - 2(pC)^n_c + (pC)^n_s}{\Delta \eta^2} \right) \text{ if } \bar{C}_r < 0 \]

\[ \left( \frac{\partial^2 (pC)}{\partial \eta^2} \right)_s^n = \left( \frac{8(pC)^n_c - 12(pC)^n_n + 4(pC)^n_{fs}}{3\Delta \eta^2} \right) \text{ if } \bar{C}_r > 0 \text{ and } n (\text{Fig. B6}) \text{ is a boundary } B_{n} \]

\[ \left( \frac{\partial(pC)}{\partial \xi} \right)_s^\wedge = \frac{1 - \bar{C}_r}{2} \left( \frac{\partial(pC)}{\partial \xi} \right)_c^n + \frac{1 + \bar{C}_r}{2} \left( \frac{\partial(pC)}{\partial \xi} \right)_s^n \text{ if } |\bar{C}_r| < 1 \]

\[ \left( \frac{\partial(pC)}{\partial \xi} \right)_s^\wedge = \frac{3 - \bar{C}_r}{2} \left( \frac{\partial(pC)}{\partial \xi} \right)_s^n - \frac{\bar{C}_r - 1}{2} \left( \frac{\partial(pC)}{\partial \xi} \right)_{fs}^n \text{ if } \bar{C}_r > 1 \]
\[
\left( \frac{\partial (pC) \tilde{C}}{\partial \xi} \right)_s = \left( 2 - \tilde{C}_{r_s} \right) \left( \frac{\partial (pC)}{\partial \xi} \right)_s^n - \left( \tilde{C}_{r_s} - 1 \right) \left( \frac{\partial (pC)}{\partial \xi} \right)_B^n
\]

if \( \tilde{C}_{r_s} > 1 \) and \( s \) (Fig.B6) is a boundary \( B_s \)

\[
\left( \frac{\partial (pC) \tilde{C}}{\partial \xi} \right)_s = \frac{3 + \tilde{C}_{r_s}}{2} \left( \frac{\partial (pC)}{\partial \xi} \right)_c^n - \frac{1 - \tilde{C}_{r_s}}{2} \left( \frac{\partial (pC)}{\partial \xi} \right)_n^n \quad \text{if} \quad \tilde{C}_{r_s} < -1
\]

\[
\left( \frac{\partial (pC) \tilde{C}}{\partial \xi} \right)_s = \left( 2 + \tilde{C}_{r_s} \right) \left( \frac{\partial (pC)}{\partial \xi} \right)_c^n - \left( 1 + \tilde{C}_{r_s} \right) \left( \frac{\partial (pC)}{\partial \xi} \right)_B^n
\]

if \( \tilde{C}_{r_s} < -1 \) and \( n \) (Fig.B6) is a boundary \( B_n \)

All terms in Eq. (B54) are fully equivalent to the appropriate terms in the discretized \( \xi \)-direction equation (Eq. B34), but reorganized to be ready for coding.

The discretized Eq. (B54) is explicit, meaning that the unknown suspended-sediment concentration is related to the computational point \( C \) only. Suspended-sediment concentrations related to neighboring computational points appear in Eq. (B54) explicitly, i.e as known values. Again, it is useful to point out those elements of the discretized equation that are expressed explicitly in terms of sediment variables by introducing a special notation for them. Therefore, Equation (B54) is rewritten as:

\[
J_C H^n_C (pC)_C^n - J_C H^n_C (pC)_C^n = J_C H^s_C (pC)_C^s - J_C H^n_C (pC)_C^n + (\text{adv})_s - (\text{adv})_n + (\text{dif})_n - (\text{dif})_s
\]

where \( (\text{adv})_s \) and \( (\text{adv})_n \) stand for advection fluxes across the s-face and n-face of the control volume, respectively, while \( (\text{dif})_s \) and \( (\text{dif})_n \) stand for diffusion fluxes across the s-face and n-face of the control volume, respectively:

\[
(\text{adv})_s = J_s H_s \tilde{C}_{r_s} \left[ \frac{(pC)_s^n + (pC)_s^b}{2} - \frac{\Delta \eta}{2} \tilde{C}_{r_s} \left( \frac{\partial (pC)}{\partial \eta} \right)_s^n - \frac{\Delta \eta^2}{6} \left( 1 - \tilde{C}_{r_s} \right) \left( \frac{\partial^2 (pC)}{\partial \eta^2} \right)_s^n \right]
\]
Boundary Conditions

Boundary conditions for Eq. (B54) (i.e. Eq. (B55)) differ depending on the type of the boundary (impermeable, outflow or inflow). Following the recommendation of Leonard (1979), boundary conditions are specified to be control-volume wall values, rather than node values. Boundary conditions, fully equivalent to those for the \( \xi \)-direction equation, are summarized below.

Impermeable boundary at \( s \)-face of control volume:

\[
(\text{adv})_s = 0 \quad (B56)
\]

\[
(\text{dif})_s = \bar{a}_{H_s} \left[ \frac{\partial (\rho C)}{\partial \eta} \right]_{B_s} - \bar{g} \left( \frac{\partial (\rho C)}{\partial \xi} \right)_{B_s} \quad (B57)
\]

where:

\[
\left( \frac{\partial (\rho C)}{\partial \eta} \right)_{B_s} = 0
\]

\[
\left( \frac{\partial (\rho C)}{\partial \xi} \right)_{B_s} = \left( \frac{\partial (\rho C)}{\partial \eta} \right)_{B_s}^n
\]

Outflow boundary at \( s \)-face of control volume:
(adv)\(_s\) = \(J_s \tilde{H}_s \tilde{C}_{rs} \left[ (\rho C)_{B_s}^n - \frac{\Delta \eta}{2} \tilde{C}_{rs} \left( \frac{\partial (\rho C)}{\partial \eta} \right)_{B_s}^n - \frac{\Delta \eta^2}{2} \left( \frac{1}{4} - \tilde{C}_{rs}^2 \right) \left( \frac{\partial^2 (\rho C)}{\partial \eta^2} \right)_{B_s}^n \right] \) \tag{B58}

(dif)\(_s\) = \(\tilde{\alpha}_H \tilde{H}_s \tilde{C}_{rs} \left\{ g_{11s} \left[ \Delta \eta \left( \frac{\partial (\rho C)}{\partial \eta} \right)_{B_s}^n - \frac{\Delta \eta^2}{2} \tilde{C}_{rs} \left( \frac{\partial^2 (\rho C)}{\partial \eta^2} \right)_{B_s}^n \right] - g_{12s} \Delta \eta \left( \frac{\partial (\rho C)}{\partial \eta} \right)_{B_s}^n \right\} \) \tag{B59}

where:

\[
\left( \frac{\partial (\rho C)}{\partial \xi} \right)_{B_s}^n = \left( 1 + \tilde{C}_{rs} \right) \left( \frac{\partial (\rho C)}{\partial \xi} \right)_{c}^n - \tilde{C}_{rs} \left( \frac{\partial (\rho C)}{\partial \xi} \right)_{c}^n \quad \text{for} \quad -1 < \tilde{C}_{rs} < 0
\]

\[
\left( \frac{\partial (\rho C)}{\partial \xi} \right)_{B_s}^n = \frac{3 + \tilde{C}_{rs}}{2} \left( \frac{\partial (\rho C)}{\partial \xi} \right)_{c}^n - \frac{1 + \tilde{C}_{rs}}{2} \left( \frac{\partial (\rho C)}{\partial \xi} \right)_{N}^n \quad \text{for} \quad \tilde{C}_{rs} < -1
\]

Inflow boundary at s-face of control volume:

(adv)\(_s\) = \(J_s \tilde{H}_s \tilde{C}_{rs} \left[ (\rho C)_{B_s} - \frac{\Delta \eta^2}{24} \left( \frac{\partial^2 (\rho C)}{\partial \eta^2} \right)_{B_s}^n \right] \) \tag{B60}

(dif)\(_s\) = \(\tilde{\alpha}_H \tilde{H}_s \tilde{C}_{rs} \left\{ g_{11s} \Delta \eta \left( \frac{\partial (\rho C)}{\partial \eta} \right)_{B_s}^n - g_{12s} \Delta \eta \left( \frac{\partial (\rho C)}{\partial \eta} \right)_{B_s}^n \right\} \) \tag{B61}

where:

\[
\left( \tilde{\rho} (\rho C) \right)_{B_s} = \frac{(\rho C)_{B_s}^{n+1} + (\rho C)_{B_s}^n}{2}
\]

\[
\left( \frac{\partial (\rho C)}{\partial \xi} \right)_{B_s} = \frac{1}{2} \left[ \left( \frac{\partial (\rho C)}{\partial \xi} \right)_{B_s}^{n+1} + \left( \frac{\partial (\rho C)}{\partial \xi} \right)_{B_s}^n \right]
\]

and:

116
\[ \left( \frac{\partial (\rho C)}{\partial \eta} \right)_B^n = \frac{-8(\rho C)_B^n + 9(\rho C)_C^n - (\rho C)_N^n}{3\Delta \eta} \]

\[ \left( \frac{\partial^2 (\rho C)}{\partial \eta^2} \right)_B^n = \frac{4(\rho C)_N^n - 12(\rho C)_C^n + 8(\rho C)_B^n}{3\Delta \eta^2} \]

with the last two terms being the same for both outflow and inflow boundary conditions.

Impermeable boundary at \( n \)-face of control volume:

\[ (\text{adv})_n = 0 \quad (\text{B62}) \]

\[ (\text{dif})_n = \alpha_H \frac{H^n}{J_n} \left\{ \varepsilon_{11n} \Delta \eta \left( \frac{\partial (\rho C)}{\partial \eta} \right)_B^n - \varepsilon_{12n} \Delta \eta \left( \frac{\partial (\rho C)}{\partial \xi} \right)_B^n \right\} \quad (\text{B63}) \]

where:

\[ \left( \frac{\partial (\rho C)}{\partial \eta} \right)_B^n = 0 \]

\[ \left( \frac{\partial (\rho C)}{\partial \xi} \right)_B^n = \left( \frac{\partial (\rho C)}{\partial \xi} \right)_B^n \]

Outflow boundary at \( n \)-face of control volume:

\[ (\text{adv})_n = J_n \tilde{H}_n \tilde{C}_r \left[ (\rho C)_B^n - \frac{\Delta \eta \tilde{C}_r (\partial (\rho C))_B^n}{2} - \frac{\Delta \eta^2}{6} \left( \frac{1}{4} - \tilde{C}_r^2 \right) \left( \frac{\partial^2 (\rho C)}{\partial \eta^2} \right)_B^n \right] \quad (\text{B64}) \]
(dif)_{n} = \tilde{\alpha}_{H_{n}} \frac{\tilde{H}_c}{J_{n}} \left\{ g_{11_{n}} \Delta \eta \left( \frac{\partial (\rho C)}{\partial \eta} \right)_{B_{n}}^{n} - \frac{\Delta \eta^{2}}{2} \tilde{C}_{r_{n}} \left( \frac{\partial^{2} (\rho C)}{\partial \eta^{2}} \right)_{B_{n}}^{n} ight\} \tag{B65}

where:

\begin{align*}
\left( \frac{\partial (\rho C)}{\partial \xi} \right)_{B_{n}}^{n} &= \left( 1 - \tilde{C}_{r_{n}} \right) \left( \frac{\partial (\rho C)}{\partial \xi} \right)_{B_{n}}^{n} + \tilde{C}_{r_{n}} \left( \frac{\partial (\rho C)}{\partial \xi} \right)_{c}^{n} \quad \text{for} \quad 0 < C_{r_{n}} < 1 \\
\left( \frac{\partial (\rho C)}{\partial \xi} \right)_{B_{n}}^{n} &= \frac{3}{2} \tilde{C}_{r_{n}} \left( \frac{\partial (\rho C)}{\partial \xi} \right)_{c}^{n} + \tilde{C}_{r_{n}} - 1 \left( \frac{\partial (\rho C)}{\partial \xi} \right)_{s}^{n} \quad \text{for} \quad \tilde{C}_{r_{n}} > 1
\end{align*}

Inflow boundary at n-face of control volume:

(adv)_{n} = J_{n} \tilde{H}_c \tilde{C}_{r_{n}} \left\{ \left( \frac{\partial (\rho C)}{\partial \eta} \right)_{B_{n}}^{n} - \Delta \eta^{2} \left( \frac{\partial^{2} (\rho C)}{\partial \eta^{2}} \right)_{B_{n}}^{n} \right\} \tag{B66}

(dif)_{n} = \tilde{\alpha}_{H_{n}} \frac{\tilde{H}_c}{J_{n}} \left\{ g_{11_{n}} \Delta \eta \left( \frac{\partial (\rho C)}{\partial \eta} \right)_{B_{n}}^{n} - g_{12_{n}} \Delta \eta \left( \frac{\partial (\rho C)}{\partial \xi} \right)_{B_{n}}^{n} \right\} \tag{B67}

where:

\begin{align*}
\left( \frac{\partial (\rho C)}{\partial \eta} \right)_{B_{n}}^{n} &= \frac{(\rho C)_{B_{n}}^{n+1} + (\rho C)_{B_{n}}^{n}}{2} \\
\left( \frac{\partial (\rho C)}{\partial \xi} \right)_{B_{n}}^{n} &= \frac{1}{2} \left[ \left( \frac{\partial (\rho C)}{\partial \xi} \right)_{B_{n}}^{n+1} + \left( \frac{\partial (\rho C)}{\partial \xi} \right)_{B_{n}}^{n} \right]
\end{align*}

and:

118
\[
\left( \frac{\partial^2 (p C)}{\partial \eta^2} \right)_{B_n}^n = \frac{8(p C)_B^n - 9(p C)_c^n + (p C)_S^n}{3\Delta \eta} \\
\left( \frac{\partial^2 (p C)}{\partial \eta^2} \right)_{B_n}^n = \frac{8(p C)_B^n - 12(p C)_c^n + 4(p C)_S^n}{3\Delta \eta^2}
\]

with the last two terms being the same for both outflow and inflow boundary conditions.

\textbf{σ-direction step}

Equation (B4) is integrated over the time step and a control volume built around a main computational point C:

\[
\frac{\Delta \sigma}{2} \int J H^{n+1} (p C)^{n+1} dp - \frac{\Delta \sigma}{2} \int J H^n (p C)^n dp = \frac{\Delta \sigma}{2} \int J H^n (p C)^n dp - \frac{\Delta \sigma}{2} \int J H^n (p C)^n dp
\]

(1) (2) (1') (2')

\[
+ R_{o} \int J_b H_b (p C)_b \omega_b d\tau - R_{o} \int J_t H_t (p C)_t \omega_t d\tau
\]

(3) (4)

\[
+ R_{o} \int J_t (p C)_t w_f d\tau - R_{o} \int J_b (p C)_b w_f d\tau
\]

(5) (6)

\[
+ \frac{E_k_v}{S_{c_v}} \int J_t \left( D_v \frac{\partial (p C)}{\partial \sigma} \right)_t d\tau - \frac{E_k_v}{S_{c_v}} \int J_b \left( D_v \frac{\partial (p C)}{\partial \sigma} \right)_b d\tau
\]

(7) (8)

(B68)

where: (1) and (2) are local rate-of-change terms due to the action of σ-direction terms added to the combined action of ξ- and η-direction terms; (1') and (2') are local rate-of-change terms due to the action of ξ- and η-direction terms; (3) and (4) are advection terms in the σ direction; (5) and (6) are fall-velocity terms in the σ direction; (7) and (8) are
diffusion terms in the $\sigma$ direction; $r,p$ are local coordinates (Fig. B7); subscripts $t,b$ denote top and bottom faces of the control volume built around the main computational point $C$ (Fig. B7); superscript $\eta$ denotes variables at time $(n+1)$ after the $\eta$-direction step; superscript $(n+1)$ defines variables at time $(n+1)$ after the last, i.e. $\sigma$-direction, step.

Figure B7.

Taking into account that $J_c (= J_b = J_t)$ does not depend on time and the $\sigma$-coordinate direction, that $H_c (= H_b = H_t)$ does not depend on the $\sigma$-coordinate direction, and further simplifying the local rate-of-change and advection terms (to make application of the QUICKEST method easier), integral Eq. (B66) now reads:

$$\frac{\Delta \sigma}{2} \int \frac{1}{2} (\rho C)^{n+1} dp - \frac{\Delta \sigma}{2} (\rho C)^{n} dp = \frac{\Delta \sigma}{2} (\rho C)^{\eta} dp - \frac{\Delta \sigma}{2} (\rho C)^{(n+1)} dp$$

(1) \hspace{1cm} (2) \hspace{1cm} (1') \hspace{1cm} (2')

$$+ R_0^\sigma J_c \hat{H}_c \int (\rho C)_b \omega_b d\tau - R_0^\sigma J_c \hat{H}_c \int (\rho C)_t \omega_t d\tau$$

(3) \hspace{1cm} (4)
\[
\begin{align*}
+ R_{\alpha t} \int_{0}^{\Delta t} (\rho C)_{t} w_{f} d\tau - R_{\alpha t} \int_{0}^{\Delta t} (\rho C)_{b} w_{f} d\tau \\
+ \frac{E_{k x}}{S_{c_v}} \int_{0}^{\Delta t} \left( D_{v} \frac{\partial (\rho C)}{\partial \sigma} \right)_{t} d\tau - \frac{E_{k x}}{S_{c_v}} \int_{0}^{\Delta t} \left( D_{v} \frac{\partial (\rho C)}{\partial \sigma} \right)_{b} d\tau
\end{align*}
\]

The local rate-of-change terms (1') and (2') are integrated by assuming that cell-center values are representative of the entire integration interval:

\[
J_{c} \frac{\Delta \sigma}{2} \int_{0}^{\Delta \sigma} (\rho C)^{n} dp - J_{c} \frac{\Delta \sigma}{2} \int_{0}^{\Delta \sigma} (\rho C)^{n} dp = \Delta \sigma J_{c} (\rho C)^{n}_{c} - \Delta \sigma J_{c} (\rho C)^{n}_{c}
\]

The local rate-of-change terms (1) and (2) as well as the advection terms (3) and (4), are discretized by using the same adapted version of the QUICKEST method as in the \(\zeta\)- and \(\eta\)-direction steps:

\[
J_{c} \frac{\Delta \sigma}{2} \int_{0}^{\Delta \sigma} (\rho C)^{n+1} dp - J_{c} \frac{\Delta \sigma}{2} \int_{0}^{\Delta \sigma} (\rho C)^{n} dp \equiv \Delta \sigma \left[ J_{c} (\rho C)^{n+1} - J_{c} (\rho C)^{n} \right]
\]

\[
+ \Delta \sigma \frac{\Delta \sigma^2}{24} J_{c} \tilde{C}_{t b} \left( \frac{\partial^2 (\rho C)}{\partial \sigma^2} \right)_{b}^{n} - \tilde{C}_{t b} \left( \frac{\partial^2 (\rho C)}{\partial \sigma^2} \right)_{t}^{n}
\]

\[
R_{0} J_{c} \frac{\Delta t}{2} (\rho C)_{b} \omega_{b} d\tau = R_{0} J_{c} \tilde{C}_{t b} \frac{(\rho C)^{n}_{c} + (\rho C)^{n}_{b}}{2} - \frac{\Delta \sigma}{2} \tilde{C}_{t b} \left( \frac{\partial (\rho C)}{\partial \sigma} \right)_{b}^{n}
\]

121
\[
R_0 J_c \int_0^{\Delta t} \omega_t \, d\tau = R_0 J_c \tilde{H}_c \tilde{C}_{r_1} \Delta \sigma \left[ \frac{(pC)_{T}^{n} + (pC)_{C}^{n}}{2} - \frac{\Delta \sigma}{2} \tilde{C}_{r_1} \left( \frac{\partial (pC)}{\partial \sigma} \right)_{T}^{n} \right. \\
- \frac{\Delta \sigma^2}{6} \tilde{C}_{r_1}^2 \left( \frac{\partial^2 (pC)}{\partial \sigma^2} \right)_{T}^{n} \left. - \frac{\Delta \sigma^2}{8} \left( \frac{\partial^2 (pC)}{\partial \sigma^2} \right)_{T}^{n} \right]
\]

where:

\[
\tilde{C}_{r_1} = R_0 \frac{\tilde{\omega}_t \Delta t}{\Delta \sigma}
\]

\[
\tilde{\omega}_t = \frac{\omega_t^{n+1} + \omega_t^{n}}{2}
\]

\[
\left( \frac{\partial (pC)}{\partial \sigma} \right)_{T}^{n} = \frac{(pC)_{T}^{n} - (pC)_{C}^{n}}{\Delta \sigma}
\]

\[
\left( \frac{\partial^2 (pC)}{\partial \sigma^2} \right)_{T}^{n} = \frac{(pC)_{T}^{n} - 2(pC)_{C}^{n} + (pC)_{B}^{n}}{\Delta \sigma^2} \quad \text{for} \quad \tilde{C}_{r_1} \geq 0
\]

\[
\left( \frac{\partial^2 (pC)}{\partial \sigma^2} \right)_{T}^{n} = \frac{(pC)_{FT}^{n} - 2(pC)_{T}^{n} + (pC)_{C}^{n}}{\Delta \sigma^2} \quad \text{for} \quad \tilde{C}_{r_1} < 0
\]

and subscripts T and B denote computational points top and bottom from C, respectively (Fig. B7); subscripts FT and FB denote computational points far top and far bottom from C, respectively.

Special cases of upstream quadratic interpolation occur for points next to boundaries. For point C next to the bed, k=1, Fig. B8:

\[
\left( \frac{\partial^2 (pC)}{\partial \sigma^2} \right)_{T}^{n} = 2 \left[ \frac{(pC)_{T}^{n} + (pC)_{C}^{n}}{\Delta \sigma_1 (\Delta \sigma_1 + \Delta \sigma_0)} - \frac{(pC)_{C}^{n}}{\Delta \sigma_1 \Delta \sigma_0} + \frac{(pC)_{C}^{n}}{\Delta \sigma_0 (\Delta \sigma_1 + \Delta \sigma_0)} \right] \quad \text{for} \quad \tilde{C}_{r_1} \geq 0, k = 1
\]

For point C next to free surface, k=K-1, Fig. B9:
When the zero-gradient condition next to the free surface is applied, the last expression reduces to:

\[
\left( \frac{\partial^2 (\rho C)}{\partial \sigma^2} \right)_{t}^{n} = \frac{8(\rho C)_{ft}^{n} - 12(\rho C)_{C}^{n} + 4(\rho C)_{c}^{n}}{3\Delta \sigma^2} \quad \text{for } \bar{C}_{r,t} < 0, k = K - 1
\]

\[
(\rho C)_{ft} = (\rho C)_{T}
\]

Similar terms are easily written for the b-face of the control volume built around the computational point C.

Integration of the fall-velocity terms (5) and (6) (Eq. (B69)) yields:

\[
R_{o}J_{c} \int_{0}^{\Delta t} \langle (\rho C)_{t} w_{f} \rangle dt = R_{o}J_{c} w_{f} \left[ \theta (\rho C)_{T}^{n+1} + (1 - \theta) (\rho C)_{C}^{n} \right] \Delta t \quad \text{(B74)}
\]
\[ R_o J_c \int_0^{\Delta t} (\rho C)_b w_f \, dt = R_o J_c w_f \left[ \theta (\rho C)^{n+1}_c + (1 - \theta) (\rho C)^n_c \right] \Delta t \] (B75)

where $\theta$ is a weighting factor. The same result could be obtained by applying an upwind finite-difference scheme.

Integration of the diffusion terms (7) and (8) of Eq. (B69) yields:

\[
\frac{E_k}{S_c} J_c \int_0^{\Delta t} \left( \frac{D_v}{H} \frac{\partial (\rho C)}{\partial \sigma} \right)_t \, dt
\]

\[= \frac{E_k}{S_c} J_c \left[ \theta \frac{D_v^{n+1}}{H^{n+1}} \frac{(\rho C)^{n+1}_c - (\rho C)^n_c}{\Delta \sigma} + (1 - \theta) \frac{D_v^n}{H^n} \frac{(\rho C)^n_c - (\rho C)^n_c}{\Delta \sigma} \right] \Delta t \] (B76)

\[
\frac{E_k}{S_c} J_c \int_0^{\Delta t} \left( \frac{D_v}{H} \frac{\partial (\rho C)}{\partial \sigma} \right)_b \, dt
\]

\[= \frac{E_k}{S_c} J_c \left[ \theta \frac{D_v^{n+1}}{H^{n+1}} \frac{(\rho C)^{n+1}_b - (\rho C)^n_b}{\Delta \sigma} + (1 - \theta) \frac{D_v^n}{H^n} \frac{(\rho C)^n_c - (\rho C)^n_c}{\Delta \sigma} \right] \Delta t \] (B77)

The same result could be obtained by applying a time weighted central differencing.

The discretized $\sigma$-direction equation now reads:

\[ J_c H^{n+1}_c (\rho C)^{n+1}_c - J_c H^n_c (\rho C)^n_c = J_c H^n_c (\rho C)^n_c - J_c H^{n}_c (\rho C)^n_c \]

\[+ J_c \tilde{H}_c \tilde{C}_{rb} \left[ \frac{(\rho C)^n_c + (\rho C)^n_b}{2} - \frac{\Delta \sigma}{2} \tilde{C}_{rb} \left( \frac{\partial (\rho C)}{\partial \sigma} \right)_b^n - \frac{\Delta \sigma^2}{6} \left(1 - \tilde{C}_{rb}^2\right) \left( \frac{\partial^2 (\rho C)}{\partial \sigma^2} \right)_b^n \right] \]

\[- J_c \tilde{H}_c \tilde{C}_{rt} \left[ \frac{(\rho C)^n_c + (\rho C)^n_t}{2} - \frac{\Delta \sigma}{2} \tilde{C}_{rt} \left( \frac{\partial (\rho C)}{\partial \sigma} \right)_t^n - \frac{\Delta \sigma^2}{6} \left(1 - \tilde{C}_{rt}^2\right) \left( \frac{\partial^2 (\rho C)}{\partial \sigma^2} \right)_t^n \right] \]

\[+ R_o J_c w_f \left[ \theta (\rho C)^{n+1}_t + (1 - \theta) (\rho C)^n_t \right] \frac{\Delta t}{\Delta \sigma} \]
The discretized \( \sigma \)-direction equation (Eq. (B78)) is implicit, meaning that the unknown suspended-sediment concentrations are related not only to the computational point \( C \), but also to the neighboring points \( T \) and \( B \). Still, it is useful to point out those elements of the discretized equation that are expressed explicitly in terms of sediment variables by introducing a special notation for them. Therefore, Equation (B78) is rewritten as:

\[
-J_c H^{n+1}_c (\rho C)^{n+1}_c - J_c H^{n}_c (\rho C)^{n}_c = J_c H^{n}_c (\rho C)^{n}_c - J_c H^{n}_c (\rho C)^{n}_c + (\text{adv})_b - (\text{adv})_t
\]

\[
+ R_{ot} J_c w_f (\rho C)^{n+1}_T \frac{\Delta t}{\Delta \sigma} + (1 - \theta)(\text{fall})^n_T \frac{\Delta t}{\Delta \sigma}
\]

\[
- R_{ot} J_c w_f (\rho C)^{n+1}_B \frac{\Delta t}{\Delta \sigma} - (1 - \theta)(\text{fall})^n_B \frac{\Delta t}{\Delta \sigma}
\]

\[
+ \frac{E_{k_v}}{S_{c_v}} J_c \theta \frac{D_{v_i}^{n+1}}{H^{n+1}_c} (\rho C)^{n+1}_T \frac{\Delta t}{\Delta \sigma} + (1 - \theta)(\text{dif})^n_T \frac{\Delta t}{\Delta \sigma}
\]

\[
- \frac{E_{k_v}}{S_{c_v}} J_c \theta \frac{D_{v_i}^{n+1}}{H^{n+1}_c} (\rho C)^{n+1}_B \frac{\Delta t}{\Delta \sigma} - (1 - \theta)(\text{dif})^n_B \frac{\Delta t}{\Delta \sigma}
\]

where \( \text{adv}_t \) and \( \text{adv}_b \) stand for advection fluxes across the \( t \)-face and the \( b \)-face of the control volume built around point \( C \), respectively; \( \text{fall}_t \) and \( \text{fall}_b \) stand for previous-time fall-velocity fluxes across the \( t \)-face and the \( b \)-face of the control volume built around point.
\( (\text{adv})_b = J_c \tilde{H}_c \tilde{C}_{rb} \left[ \frac{(\rho C)_b^n + (\rho C)_b^n}{2} - \frac{\Delta \sigma}{2} \tilde{C}_{rb} \left( \frac{\partial (\rho C)}{\partial \sigma} \right)_b^n - \frac{\Delta \sigma^2}{6} \left( 1 - \tilde{C}_{rb}^2 \right) \left( \frac{\partial^2 (\rho C)}{\partial \sigma^2} \right)_b^n \right] \)

\( (\text{adv})_t = J_c \tilde{H}_c \tilde{C}_{rt} \left[ \frac{(\rho C)_t^n + (\rho C)_c^n}{2} - \frac{\Delta \sigma}{2} \tilde{C}_{rt} \left( \frac{\partial (\rho C)}{\partial \sigma} \right)_t^n - \frac{\Delta \sigma^2}{6} \left( 1 - \tilde{C}_{rt}^2 \right) \left( \frac{\partial^2 (\rho C)}{\partial \sigma^2} \right)_t^n \right] \)

\( (\text{fall})_b^n = R_{of} J_c w_f (\rho C)_b^n \)

\( (\text{fall})_t^n = R_{of} J_c w_f (\rho C)_t^n \)

\( (\text{dif})_t^n = \frac{E_{kv} J_c D^n_{it} (\rho C)_t^n - (\rho C)_c^n}{S_{cv} \frac{H_c^n}{\Delta \sigma}} \)

\( (\text{dif})_b^n = \frac{E_{kv} J_c D^n_{vb} (\rho C)_c^n - (\rho C)_b^n}{S_{cv} \frac{H_c^n}{\Delta \sigma}} \)

**Boundary Conditions**

The discretized \( \sigma \)-direction equation is different for points next to the bed (\( k=1 \)) and points next to the free surface (\( k=K \)).

**Points Next to the Bed**

The advection flux across the \( b \)-face of the control volume is equal to zero:

\( (\text{adv})_b = 0 \) \hspace{1cm} (B80)

The fall-velocity flux across the \( b \)-face of the control volume actually represents deposition of suspended sediment onto the bed. It uses the near-bed concentration \( (\rho C)_{\sigma=a+\Delta \sigma} \), defined as in Eq. (31), i.e. evaluated at a near-bed location \( \sigma_{a+\Delta \sigma} \) by simple linear extrapolation:
(\rho C)_b = (\rho C)_{\sigma_{a+\Delta a}} = (1-c)(\rho C)_T + c(\rho C)_c \quad (B81)

where \( c \) is an extrapolation coefficient.

The fall-velocity flux across the b-face of the control volume reads:

\[
R_{0_t} J_c \int_{0}^{\Delta t} (\rho C)_b \ w_f \ dt = R_{0_t} J_c \ w_f \left\{ \theta \left[ (1-c)(\rho C)_T^{n+1} + c(\rho C)_c^{n+1} \right] \\
+ (1-\theta) \left[ (1-c)(\rho C)_T^n + c(\rho C)_c^n \right] \right\} \Delta t
\]

The diffusion flux across the b-face of the control volume actually represents erosion, i.e. entrainment of sediment particles from the bed into suspension and has to be modified by the active-layer size fraction to reflect the availability of a particular size class in the active-layer control volume. Furthermore, the derivative is defined as in the 'erosion' source term (Eq. (12)):

\[
\left( \frac{\partial (\rho C)}{\partial \sigma} \right)_b = \left( \frac{\partial (\rho C)}{\partial \sigma} \right)_{\sigma_a} = \frac{(\rho C)_{\sigma_{a+\Delta a}} - (\rho C)_{\sigma_a}}{\Delta \sigma_{\Delta a}} = \frac{(1-c)(\rho C)_T + c(\rho C)_c - (\rho C)_{\sigma_a}}{\Delta \sigma_{\Delta a}} \quad (B83)
\]

where \((\rho C)_{\sigma_{a+\Delta a}}\) is defined as in Eq. (B81), and \((\rho C)_{\sigma_a}\) is near-bed concentration defined by the appropriate empirical relation.

The diffusion flux across the b-face of the control volume, modified by \( \beta \), reads:

\[
\frac{E_{k_{y}} J_c \Delta t}{S_{c_{y}}} \int_0^{\Delta t} \beta \left( D_v \frac{\partial (\rho C)}{\partial \sigma} \right)_b \ dt = \frac{E_{k_{y}} J_c}{S_{c_{y}}} \left[ \theta \beta^{n+1} \frac{D_{v_b}^{n+1} (1-c)(\rho C)_T^{n+1} + c(\rho C)_c^{n+1} - (\rho C)_{\sigma_a}^{n+1}}{\Delta \sigma_{\Delta a}} \\
+ (1-\theta)\beta^n \frac{D_{v_b}^n (1-c)(\rho C)_T^n + c(\rho C)_c^n - (\rho C)_{\sigma_a}^n}{\Delta \sigma_{\Delta a}} \right] \Delta t
\]

127
The discretized $\sigma$-direction equation for a point next to the bed (with a special notation, as in Eq. (B79), used to point out those elements of the discretized equation that are expressed explicitly in terms of sediment variables) reads:

$$J_c H_c^{n+1} (\rho C)_c^{n+1} - J_c H_c^n (\rho C)_c^n = J_c H_c^n (\rho C)_c^n - J_c H_c^n (\rho C)_c^n - (\text{adv})_t$$

$$+ R_{o_t} J_c \theta w_f (\rho C)_T^n \frac{\Delta t}{\Delta \sigma} + (1 - \theta) (\text{fall})_t^n \frac{\Delta t}{\Delta \sigma}$$

$$- R_{o_t} J_c \theta w_f \left[ (1 - c)(\rho C)_T^{n+1} + c(\rho C)_c^{n+1} \right] \frac{\Delta t}{\Delta \sigma} - (1 - \theta) (\text{fall})_b^n \frac{\Delta t}{\Delta \sigma}$$

(B85)

$$+ \frac{E_{k_x}}{S_{c_v}} J_c \theta \frac{D_{v_b}^{n+1}}{H_c^{n+1}} (\rho C)_T^{n+1} + (\rho C)_c^{n+1} \frac{\Delta t}{\Delta \sigma} - (1 - \theta) (\text{dif})_t^n \frac{\Delta t}{\Delta \sigma}$$

$$- \frac{E_{k_x}}{S_{c_v}} J_c \theta \frac{D_{v_b}^{n+1}}{H_c^{n+1}} (1 - c)(\rho C)_T^{n+1} + c(\rho C)_c^{n+1} - (\rho C)_b^{n+1} \frac{\Delta t}{\Delta \sigma} - (1 - \theta)(\text{dif})_b^n \frac{\Delta t}{\Delta \sigma}$$

where $(\text{fall})_b^n$ and $(\text{dif})_b^n$ are the same as the 'deposition' and 'erosion' source terms in the discretized mass-conservation equation for active-layer sediment (Eq. (35)) and the global mass-conservation equation for bed sediment (Eq. (34)):

$$(\text{fall})_b^n = R_{o_t} J_c w_f \left[ (1 - c)(\rho C)_T^n + c(\rho C)_c^n \right] = (S_d)_b^n$$

$$k_v \frac{D_{v_b}^{n+1}}{H_c^{n+1}} (1 - c)(\rho C)_T^{n+1} + c(\rho C)_c^{n+1} - (\rho C)_b^{n+1} \frac{\Delta t}{\Delta \sigma} - (1 - \theta)(\text{dif})_b^n \frac{\Delta t}{\Delta \sigma}$$

and $(\text{adv})_t$, $(\text{fall})_t^n$ and $(\text{dif})_t^n$ are the same as in Eq. (B79).

**Point Next to the Free Surface**

Advection, fall-velocity, and diffusion fluxes across the free surface (t-face of the control volume) are equal to zero:
\[(\text{adv})_t = 0\]  
\[R_{of} J_c \int_0^{\Delta t} (\rho C)_t w_f dt = 0\]  
\[\frac{E_k}{S_{cv}} \int_0^{\Delta t} \frac{1}{H_c} D_v \frac{\partial (\rho C)}{\partial \sigma} dt = 0\]

The discretized \(\sigma\)-direction equation for a point next to the free surface (with a special notation, as in Eq. (B79), used to point out those elements of the discretized equation that are expressed explicitly in terms of sediment variables) reads:

\[J_c H_c^{n+1} (\rho C)_c^{n+1} - J_c H_c^n (\rho C)_c^n = J_c H_c^n (\rho C)_c^n - J_c H_c^n (\rho C)_c^n + (\text{adv})_b\]

\[-R_{of} J_c \theta w_f (\rho C)_c^{n+1} \frac{\Delta t}{\Delta \sigma} - (1 - \theta)(\text{fall})_b^n \frac{\Delta t}{\Delta \sigma}\]

\[-\frac{E_k}{S_{cv}} J_c \theta \frac{D_v}{H_c^{n+1}} (\rho C)_c^{n+1} - (\rho C)_b^{n+1} \frac{\Delta t}{\Delta \sigma} - (1 - \theta)(\text{dif})_b^n \frac{\Delta t}{\Delta \sigma}\]

where \((\text{fall})_b^n\), \((\text{adv})_b\) and \((\text{dif})_b^n\) are the same as in Eq. (B79).

**Complete Discretized Mass-Conservation Equation for Suspended Sediment**

A complete discretized mass-conservation equation for suspended sediment is obtained by adding the discretized \(\xi\)-, \(\eta\)- and \(\sigma\)-direction step equations (Eqs. (B35), (B55) and (B79)):

\[J_c H_c^{n+1} (\rho C)_c^{n+1} - J_c H_c^n (\rho C)_c^n = (\text{adv})_w - (\text{adv})_e + (\text{dif})_e - (\text{dif})_w\]

\[+(\text{adv})_s - (\text{adv})_n + (\text{dif})_n - (\text{dif})_s\]

\[+(\text{adv})_b - (\text{adv})_t\]
\[ +R_{of} J_c \theta w_f (\rho C)^{n+1}_f \frac{\Delta t}{\Delta \sigma} + (1 - \theta)(\text{fall})^n \frac{\Delta t}{\Delta \sigma} \]  
\[ -R_{of} J_c \theta w_f (\rho C)^{n+1}_c \frac{\Delta t}{\Delta \sigma} - (1 - \theta)(\text{fall})^n_b \frac{\Delta t}{\Delta \sigma} \]  
\[ + \frac{E_{kv}}{S_{cv}} J_c \theta \frac{D_{n+1}^{n+1}}{H_c^{n+1}} (\rho C)^{n+1}_i - (\rho C)^{n+1}_c \frac{\Delta t}{\Delta \sigma} + (1 - \theta)(\text{diff})^n \frac{\Delta t}{\Delta \sigma} \]  
\[ - \frac{E_{kv}}{S_{cv}} J_c \theta \frac{D_{n+1}^{n+1}}{H_c^{n+1}} (\rho C)^{n+1}_i - (\rho C)^{n+1}_c \frac{\Delta t}{\Delta \sigma} - (1 - \theta)(\text{diff})^n_b \frac{\Delta t}{\Delta \sigma} \]  

The discretized mass-conservation equation for suspended sediment (Eq. (B90)) is implicit in the \( \sigma \) direction only, and can be rewritten as:

\[ a(\rho C)^{n+1}_f + b(\rho C)^{n+1}_c + c(\rho C)^{n+1}_T = d \]  

(B91)

where

\[ a = -\frac{E_{kv}}{S_{cv}} J_c \theta \frac{D^{n+1}_b}{H_c^{n+1}} \frac{\Delta t}{\Delta \sigma^2} \]

\[ b = J_c H_c^{n+1} + \frac{E_{kv}}{S_{cv}} J_c \theta \frac{1}{H_c^{n+1}} (D^{n+1}_i + D^{n+1}_b) \frac{\Delta t}{\Delta \sigma^2} + R_{of} J_c \theta w_f \frac{\Delta t}{\Delta \sigma} \]

\[ c = -\frac{E_{kv}}{S_{cv}} J_c \theta \frac{D^{n+1}_i}{H_c^{n+1}} \frac{\Delta t}{\Delta \sigma^2} - R_{of} J_c \theta w_f \frac{\Delta t}{\Delta \sigma} \]
\[ d = J_c H_c^n (\rho C)_c^n \]
\[ + (\text{adv})_w - (\text{adv})_e + (\text{dif})_e - (\text{dif})_w \]
\[ + (\text{adv})_s - (\text{adv})_n + (\text{dif})_n - (\text{dif})_s \]
\[ + (\text{adv})_l - (\text{adv})_t \]
\[ + (1 - \theta) (\text{fall})_l^n \frac{\Delta t}{\Delta \sigma} - (1 - \theta) (\text{fall})_b^n \frac{\Delta t}{\Delta \sigma} \]
\[ + (1 - \theta) (\text{diff})_l^n \frac{\Delta t}{\Delta \sigma} - (1 - \theta) (\text{diff})_b^n \frac{\Delta t}{\Delta \sigma} \]

are known coefficients.

Similarly, by combining the discretized Eqs. (B35), (B55) and (B85), an equation for points next to the bed is obtained in the same form as Eq. (B91) where the appropriate coefficients are:

\[ a = 0 \]

\[ b = J_c H_c^{n+1} + R_o t J_c \theta w_f \frac{\Delta t}{\Delta \sigma} + \frac{E_{ky}}{S_c} J_c \frac{\theta}{H_c^{n+1}} \left( \frac{D_{vt}^{n+1}}{\Delta \sigma} + \beta^{n+1} \frac{D_{vb}^{n+1}}{\Delta \sigma \Delta a} \right) \frac{\Delta t}{\Delta \sigma} \]

\[ c = R_o t J_c \theta w_f \frac{\Delta t}{\Delta \sigma} + R_o t J_c \theta w_f (1 - c) \frac{\Delta t}{\Delta \sigma} - \frac{E_{ky}}{S_c} J_c \frac{\theta}{H_c^{n+1}} \left[ \frac{D_{vt}^{n+1}}{\Delta \sigma} - \beta^{n+1} \frac{D_{vb}^{n+1}}{H_c^{n+1} (1 - c)} \right] \frac{\Delta t}{\Delta \sigma} \]

\[ d = J_c H_c^n (\rho C)_c^n \]
\[ + (\text{adv})_w - (\text{adv})_e + (\text{dif})_e - (\text{dif})_w \]
\[ + (\text{adv})_s - (\text{adv})_n + (\text{dif})_n - (\text{dif})_s \]
\[ - (\text{adv})_l \]
\[ + (1 - \theta) (\text{fall})_l^n \frac{\Delta t}{\Delta \sigma} - (1 - \theta) (S_d)_p^n \frac{\Delta t}{\Delta \sigma} \]
\[ + (1 - \theta) (\text{diff})_l^n \frac{\Delta t}{\Delta \sigma} - (1 - \theta) (S_c)_p^n \frac{\Delta t}{\Delta \sigma} \]

\[ + \frac{E_{ky}}{S_c} J_c \theta \beta^{n+1} \frac{D_{vb}^{n+1} (\rho C)_c^{n+1}}{H_c^{n+1} \Delta \sigma \Delta a} \frac{\Delta t}{\Delta \sigma} \]

131
By combining the discretized Eqs. (B35), (B55) and (B89), an equation for points next to the free surface is obtained in the same form as Eq. (B91) where the appropriate coefficients are:

\[
a = -\frac{E_k}{S_{cv}} J_c \theta \frac{D_{v_b}^{n+1}}{H_c^{n+1}} \frac{\Delta t}{\Delta \sigma^2}
\]

\[
b = J_c H_c^{n+1} + R_{or} J_c \theta w_f \frac{\Delta t}{\Delta \sigma} + \frac{E_k}{S_{cv}} J_c \theta \frac{D_{v_b}^{n+1}}{H_c^{n+1}} \frac{\Delta t}{\Delta \sigma^2}
\]

\[c = 0\]

\[d = J_c H_c^n (\rho C)_c^n\]

\[+(\text{adv})_w - (\text{adv})_e + (\text{dif})_e - (\text{dif})_w\]

\[+(\text{adv})_s - (\text{adv})_n + (\text{dif})_n - (\text{dif})_s\]

\[+(\text{adv})_b\]

\[-(1 - \theta) (\text{fall})_b^n \frac{\Delta t}{\Delta \sigma} - (1 - \theta) (\text{dif})_b^n \frac{\Delta t}{\Delta \sigma}\]
APPENDIX C

COEFFICIENTS IN DISCRETIZED GLOBAL MASS-CONSERVATION EQUATION FOR BED SEDIMENT AND MASS-CONSERVATION EQUATIONS FOR ACTIVE-LAYER SEDIMENT

The discretized global mass-conservation equation for bed sediment, Eq. (34), which after linearization is written as Eq. (48), can be rewritten as:

\[ a_{1,1} \Delta s_1 + \sum_{kks=1}^{KKS} a_{1,kk+1,1} \Delta s_{kks+1} = b_1 \]  

(C.1)

where

\[ a_{11} = \frac{\partial F_1}{\partial s_1} = \frac{\partial F_1}{\partial z_b} \frac{\rho_s(1-p)J}{\Delta t} \]

\[ a_{1,kk+1} = \frac{\partial F_1}{\partial s_{kks+1}} = \frac{\partial F_1}{\partial \beta_{kks}} = \theta (S_e^t)_{kks}^{n+1} \]

\[ b_1 = -F_1 (m_s^n + 1) = -\rho_s(1-p)J \frac{m_{z_b}^{n+1} - z_b^n}{\Delta t} \]

\[ -\sum_{ks=1}^{KS} \left[ \theta (\text{div} q_b)_{kks}^{n+1} + (1-\theta)(\text{div} q_b)_{kks}^n \right. \]

\[ + \theta m_{\beta_{kks}}^{n+1} (S_e^t)_{kks}^{n+1} + (1-\theta)(S_e)_{kks}^n \]

\[ -\theta (S_d)_{kks}^{n+1} - (1-\theta)(S_d)_{kks}^n \]

Since all variables are evaluated at the same main computational point P, there is no need for a special subscript to denote the computational point. Subscript ks is introduced to denote a sediment size class, while subscript kks denotes a sediment variable. Superscript m denotes the iteration level. All other notation is as previously defined for Eq. (34).

The discretized mass-conservation equation for the ks-th size class of active-layer sediment, Eq. (35), after linearization as shown in Eq. (49), can be rewritten as:
\[ a_{ks+1,1} \Delta s_1 + \sum_{kks=1}^{KKS} a_{ks+1,kks+1} = b_{ks+1} \]  

(C2)

where

\[ a_{ks+1,1} = \frac{\partial F_{ks+1}}{\partial s_1} = \frac{\partial F_{ks+1}}{\partial z_b} = \rho_s (1-p)J \frac{\partial t}{\partial t} m_{p_{ks}^{n+1}} (E_m)_{kks}^{n+1} - J(S_F)_{kks}^{n+1} \]

\[ a_{ks+1,kks+1} = \frac{\partial F_{ks+1}}{\partial s_{kks+1}} = \frac{\partial F_{ks+1}}{\partial a_{kks}} = \rho_s (1-p)J \frac{\partial t}{\partial t} m_{p_{ks}^{n+1}} (E_m)_{kks}^{n+1} - J(S_F)_{kks}^{n+1} \]

\[ b_{ks+1} = -\rho_s (1-p)J \frac{\partial t}{\partial t} m_{p_{ks}^{n+1}} (E_m)_{kks}^{n+1} - \beta_{kks} \]

\[ \Delta s_1 \]

\[ \frac{\partial F_{ks+1}}{\partial s_{kks+1}} = \frac{\partial F_{ks+1}}{\partial a_{kks}} = \rho_s (1-p)J \frac{\partial t}{\partial t} m_{p_{ks}^{n+1}} (E_m)_{kks}^{n+1} - J(S_F)_{kks}^{n+1} \]

\[ b_{ks+1} = -\rho_s (1-p)J \frac{\partial t}{\partial t} m_{p_{ks}^{n+1}} (E_m)_{kks}^{n+1} - \beta_{kks} \]

Here \((E_m)'_{kks+1}\) denotes the derivative of active-layer thickness \(E_m\) with respect to sediment variable \(s_1 = z_b\); \((E_m)'_{kks+1}\) denotes the derivative of active-layer thickness \(E_m\) with respect to sediment variable \(s_{kks+1} = \beta_{kks}\); \((S_F)'_{kks,1}\) represents the derivative of active-layer floor "source" \((S_F)_{kks,1}\) for the \(ks\)-th size-class, with respect to sediment variable \(s_1 = z_b\); \((S_F)'_{kks,kks+1}\) represents the derivative of active-layer floor "source" \((S_F)_{kks}\) for the \(ks\)-th size-class, with respect to sediment variable \(s_{kks+1} = \beta_{kks}\). All other notation is as previously defined.
This input data guide was developed through analysis of the CH3D code by the authors of this report. Question marks indicate uncertainty in this analysis, and may require consultation with the developers of CH3D.

**DUMMY** indicates a blank data record (separator).

<table>
<thead>
<tr>
<th>rec</th>
<th>var</th>
<th>Format (columns)</th>
<th>Variable description and remarks</th>
</tr>
</thead>
</table>

**MAIN PROGRAM**

Note: Data are read from main input file (file 4: main.inp)

Debug-output flags

<table>
<thead>
<tr>
<th>NBUGS</th>
<th>NBUGE</th>
<th>5(2I8) (1-80)</th>
<th>Pairs of debug-output flags</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>5 pairs per record</td>
<td>100 pairs total</td>
</tr>
</tbody>
</table>

**SUBROUTINE CH3DIR**

Note: All data are read from main input file (file 4: main.inp) except for centered-cell depths (file 12: dpth.inp) and Manning's roughness coefficients (file 18: mann.inp)

Run descriptor

<table>
<thead>
<tr>
<th>DUMMY</th>
<th>TITLE</th>
<th>A80 (1-80)</th>
<th>Title of run</th>
</tr>
</thead>
</table>

Timestep info

<table>
<thead>
<tr>
<th>DUMMY</th>
<th>IT1</th>
<th>I8 (1-8)</th>
<th>Starting time-step number (=1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IT2</td>
<td>I8 (9-16)</td>
<td>Ending time-step number</td>
</tr>
<tr>
<td></td>
<td>DT</td>
<td>F8.0 (17-24)</td>
<td>Computational time step [s]</td>
</tr>
<tr>
<td></td>
<td>ISTART</td>
<td>I8 (25-32)</td>
<td>=0 Cold start</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(arrays initialized in CH3DIF)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>&gt;0 Hot start (not operational?)</td>
</tr>
<tr>
<td></td>
<td>ITEST</td>
<td>I8 (33-40)</td>
<td>=0 No diagnostic output</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>=3 Diagnostic output for XC, QXU, XU in subroutine CH3DTR</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>=4 Diagnostic output for GIJ,HIJ,DIJK in subroutine CH3DTR</td>
</tr>
<tr>
<td></td>
<td>ITSALT</td>
<td>I8 (41-48)</td>
<td>Number of time steps before salinity and temperature computations are</td>
</tr>
<tr>
<td>ISCOM</td>
<td>I8 (49-56)</td>
<td>initiated =0 No sediment computations =1 Sediment computations performed</td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td>------------</td>
<td>---------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>NTSED0</td>
<td>I8 (57-64)</td>
<td>Number of time steps before sediment computations are initiated</td>
<td></td>
</tr>
</tbody>
</table>

### Printout windows

<table>
<thead>
<tr>
<th>DUMMY</th>
</tr>
</thead>
<tbody>
<tr>
<td>WPRCRD</td>
</tr>
</tbody>
</table>

**Following record is repeated WPRCRD times**

Omitted if WPRCRD=0

| WXCEL1 | I8 (1-8) | Starting printout-window index in KSI direction |
| WXCEL2 | I8 (9-16) | Ending printout-window index in KSI direction |
| WYCEL1 | I8 (17-24) | Starting printout-window index in ETA direction |
| WYCEL2 | I8 (25-32) | Ending printout-window index in ETA direction |
| WZCEL1 | I8 (33-40) | Starting printout-window index in vertical direction |
| WZCEL2 | I8 (41-48) | Ending printout-window index in vertical direction |
| WPRINT | I8 (49-56) | Printout interval |
| WPRSTR | I8 (57-64) | Starting time-step number |
| WPREND | I8 (65-72) | Ending time-step number |
| WPRVAR | A8 (73-80) | Printout variables |
|        | E - water-surface fluctuations |
|        | X - X-direction unit flow rate |
|        | Y - Y-direction unit flow rate |
|        | U - X-direction velocity |
|        | V - Y-direction velocity |
|        | W - vertical-direction velocity |
|        | S - salinity |
|        | T - temperature |
|        | A - velocity magnitude and direction |

### Snapshots

<table>
<thead>
<tr>
<th>DUMMY</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNPCRD</td>
</tr>
</tbody>
</table>

**Following record is repeated SNPCRD times**

Omitted if SNPCRD=0

| SXCEL1 | I8 (1-8) | Starting index in KSI direction |
| SXCEL2 | I8 (9-16) | Ending ------------------------ |
| SYCEL1 | I8 (17-24) | Starting index in ETA direction |
| SYCEL2 | I8 (25-32) | Ending ------------------------ |

136
Flow rate ranges

**NRANG** I8 (1-8)
- >0 Number of flow-rate ranges
- =0 No flow-rate ranges
- Stop if NRANG>NRANGS

Following record is repeated NRANG times
Omitted if NRANG=0

**RANGDR** A1 (8)
- Direction of a flow
  - =X Flow is in KSI direction, therefore the range is along an ETA-direction coordinate line
  - =Y Flow is in ETA direction, therefore the range is along a KSI-direction coordinate line

**RPOS1** I8 (9-16)
- I index defining the appropriate ETA-direction range

**RPOS2** I8 (17-24)
- J1 starting J index of the range

**RPOS3** I8 (25-32)
- J2 ending J index of the range

**RRNAME** A45 (33-77)
- Range name

**IGI** I8 (1-8)
- =1 Printout arrays such as NS, MS, NR, MR, IROW, IU1, IU2, ISW etc.
- NBX, NBY
- 0 No printout

**IGH** I8 (9-16)
- =1 Printout all depth arrays HS, HU, HV
- 0 No printout

**IGT** I8 (17-24)
- =1 Print initial temperature arrays?
- 0 No printout

**IGS** I8 (25-32)
- =1 Printout initial surface elevations S, UI, VI
- 0 No printout

**IGU** I8 (33-40)
- =1 Printout mass flux and velocity in horizontal direction ??(Sheng)
- 0 No printout
=1 Print the initial wind-shear stress
0 No printout

=1 Print dimensional grid coordinates
0 No printout

=1 Print turbulent velocity? (Sheng)
0 No printout

=1 Save cell-centered depth to file 23
Save Cartesian coordinates of cell centered grid points to file 23
Save NS, MS arrays to file 23
Save FY1, FY12, FY21, FY22 arrays (transform. coeff.) on file 23
for snapshot plot
0 Does not save

Physical constants

<table>
<thead>
<tr>
<th>DUMMY</th>
<th>XREF</th>
<th>F8.0 (1-8)</th>
<th>Reference horizontal grid distance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Maximum horizontal dimension</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>divided by number of cells in</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>that direction in cm</td>
</tr>
<tr>
<td></td>
<td>ZREF</td>
<td>F8.0 (9-16)</td>
<td>Reference depth</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Average typical depth in cm</td>
</tr>
<tr>
<td></td>
<td>UREF</td>
<td>F8.0 (17-24)</td>
<td>Reference horizontal velocity</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Average velocity in cm/s</td>
</tr>
<tr>
<td></td>
<td>COR</td>
<td>F8.0 (25-32)</td>
<td>Coriolis parameter (1/s) - ref. 'time'</td>
</tr>
<tr>
<td></td>
<td>GR</td>
<td>F8.0 (33-40)</td>
<td>Gravitational acceleration (cm/s²)</td>
</tr>
<tr>
<td></td>
<td>ROO</td>
<td>F8.0 (41-48)</td>
<td>Minimum density expected (gr/cm³)</td>
</tr>
<tr>
<td></td>
<td>ROR</td>
<td>F8.0 (49-56)</td>
<td>Reference density</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Maximum expected (gr/cm³)</td>
</tr>
<tr>
<td></td>
<td>T0</td>
<td>F8.0 (57-64)</td>
<td>Minimum temperature (Celsius)</td>
</tr>
<tr>
<td></td>
<td>TR</td>
<td>F8.0 (65-72)</td>
<td>Reference temperature</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Maximum expected (Celsius)</td>
</tr>
<tr>
<td></td>
<td>SAR</td>
<td>F8.0 (1-8)</td>
<td>Reference salinity (Max expected)</td>
</tr>
<tr>
<td></td>
<td>SA0</td>
<td>F8.0 (9-16)</td>
<td>Minimum salinity ??</td>
</tr>
</tbody>
</table>

Water depths read from file 12 (dpth.inp)

| DUMMY | HS     | 16F5.0 | Water depths (ft) at the center of computational cell along a KSI-coordinate line, 16 values per record, ICELLS values altogether (read from file 12 (dpth.inp)) Previous records are repeated JCELLS times |

Manning's roughness coefficients n
(Read from file 18 (mann.inp))

| DUMMY | FMAN   | 20F4.0 | Manning's roughness coefficients n(m-1/3*s??) (divided by 0.001) |

138
at computational points along an ETA-coordinate line, 20 values per record, JCELLS values altogether (read from file 18 (mann.inp))

Previous records are repeated JCELLS times

**Time-level weighting coefficients**

<table>
<thead>
<tr>
<th>DUMMY</th>
<th>THETA</th>
<th>F8.0 (1-8)</th>
<th>Time-level weighting coefficient</th>
</tr>
</thead>
</table>

**Flags for computing inertia and diffusion**

<table>
<thead>
<tr>
<th>DUMMY</th>
<th>ITEMP</th>
<th>I8 (1-8)</th>
<th>=2 Compute temperature (use time-varying temperature as river boundary temperature)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>=1 Compute temperature (use daily equilibrium temperature as river boundary temperature)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>=0 No computation of temperature</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&lt;0</td>
<td>&lt;0 Update time-variable equilibrium temperature and coefficient of heat exchange at every time step</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DUMMY</th>
<th>ISALT</th>
<th>I8 (9-16)</th>
<th>=1 Compute salinity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>=0 No computation of salinity</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DUMMY</th>
<th>ICC</th>
<th>I8 (17-24)</th>
<th>=1 Compute dissolved species concen.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>=0 Do not compute concentration</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DUMMY</th>
<th>IFI</th>
<th>I8 (25-32)</th>
<th>=1 Compute nonlinear inertia terms in the momentum equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>=0 Do not compute inertia terms</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DUMMY</th>
<th>IFA</th>
<th>I8 (33-40)</th>
<th>=1 Include one group higher-order lateral diffusion terms ??</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>=0 Do not include</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DUMMY</th>
<th>IFB</th>
<th>I8 (41-48)</th>
<th>=1 Include another group higher-order diffusion terms ??</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>=0 Do not include</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DUMMY</th>
<th>IFC</th>
<th>I8 (49-56)</th>
<th>=1 Include another group higher-order diffusion terms ??</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>=0 Do not include</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DUMMY</th>
<th>IFD</th>
<th>I8 (57-64)</th>
<th>=1 Include diffusion terms in salinity and temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>=0 Do not include</td>
</tr>
</tbody>
</table>

**Temperature parameters**

<table>
<thead>
<tr>
<th>DUMMY</th>
<th>BVR</th>
<th>F8.0 (1-8)</th>
<th>Reference turbulent thermal eddy diffusivity??</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S1</td>
<td>F8.0 (9-16)</td>
<td>Constant in computation of variable vertical eddy viscosity [GA=GX(1+S1*Ri)**FM1] if first-order turbulence model is used</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>F8.0 (17-24)</td>
<td>Constant in computation of variable</td>
</tr>
</tbody>
</table>

139
<table>
<thead>
<tr>
<th>Variable</th>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PR</td>
<td>F8.0</td>
<td>Turbulent Prandtl number</td>
</tr>
<tr>
<td>PRV</td>
<td>F8.0</td>
<td>Vertical turbulent Prandtl number</td>
</tr>
<tr>
<td>TWE</td>
<td>F8.0</td>
<td>Temperature in the epilimnion (for the initial condition)</td>
</tr>
<tr>
<td>TWH</td>
<td>F8.0</td>
<td>Temperature in the hypolimnion (for the initial condition)</td>
</tr>
<tr>
<td>FKB</td>
<td>F8.0</td>
<td>Vertical grid index of the initial thermocline location (boundary between epilimnion and hypolimnion)</td>
</tr>
<tr>
<td>TQO</td>
<td>F8.0</td>
<td>Initial surface heat flux TQ (cal/cm/cm/s)</td>
</tr>
</tbody>
</table>

Concentration parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IVER</td>
<td>I8</td>
<td>=1 Explicit vertical diffusion term for water quality equations</td>
</tr>
<tr>
<td>ICON</td>
<td>I8</td>
<td>=0 Do not compute advection terms for water quality equations</td>
</tr>
<tr>
<td>ICON</td>
<td>I8</td>
<td>=1 Compute advection terms in conservative form with central differencing</td>
</tr>
<tr>
<td>ICON</td>
<td>I8</td>
<td>=3 Compute advection terms in conservative form with second upwind differencing scheme</td>
</tr>
<tr>
<td>ICON</td>
<td>I8</td>
<td>=4 Compute advection terms in conservative form with combined central and upwind differencing</td>
</tr>
<tr>
<td>IUBO</td>
<td>I8</td>
<td>Bottom orbital velocity flag (=0)</td>
</tr>
<tr>
<td>IBL</td>
<td>I8</td>
<td>Concentration computation does not have to be performed for the entire domain. Instead, it can be done for a window which covers an area from I=IBL to I=IBR and from J=JBM to J=JBP. The window will change in time.</td>
</tr>
<tr>
<td>ICMAX</td>
<td>F8.0</td>
<td>Maximum concentration allowed by the code (halts if exceeded)</td>
</tr>
<tr>
<td>C0</td>
<td>F8.0</td>
<td>Initial concentration</td>
</tr>
<tr>
<td>ICC1</td>
<td>I8</td>
<td>Initial concentration field may be specified to be zero everywhere in the computational domain except within two windows: the first one</td>
</tr>
</tbody>
</table>
ID1  I8 (33-40)  covers an area from I=ICC1 to I=ICC2
ID2  I8 (41-48)  and from J=JCC1 to J=JCC2
JD1  I8 (49-56)  the second one from I=ID1 to I=ID2
JD2  I8 (57-64)  and from J=JD1 to J=JD2

Turbulent parameters and eddy coefficients

<table>
<thead>
<tr>
<th>DUMMY</th>
<th>IEXP</th>
<th>I8 (1-8)</th>
<th>Vertical eddy-coefficient flag</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>0 Constant eddy coefficient</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(must also set ISPAC(9)=0 ?)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-1 Munk-Anderson type first order turbulence model</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(unstratified eddy coefficients are determined from mixing length theory)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Richardson-number-dependent eddy coefficient with length scale linearly increasing from the bottom and surface</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-2 Munk-Anderson type first order turbulence model</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Richardson-number dependent eddy coefficient with length scale linearly increasing from the bottom to the surface</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-3 Second-order turbulence model</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>IAV</th>
<th>I8 (9-16)</th>
<th>0 Input parameter AVR is used as reference eddy viscosity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>=1 Reference eddy viscosity is computed from AV1+TXY*AV2, where TXY is the total wind stress and AV1 and AV2 are input parameters</td>
</tr>
<tr>
<td>AVR</td>
<td>F8.0 (17-24)</td>
<td>Reference vertical eddy coeff. (cm²/s)</td>
</tr>
<tr>
<td>AV1</td>
<td>F8.0 (25-32)</td>
<td>Background vertical eddy viscosity when wind is zero</td>
</tr>
<tr>
<td>AV2</td>
<td>F8.0 (33-40)</td>
<td>If IAV=1 unstratified vertical eddy viscosity is computed from AV1+TXY*AV2</td>
</tr>
<tr>
<td>AVM</td>
<td>F8.0 (41-48)</td>
<td>Minimum vertical eddy coeff. (cm²/s)</td>
</tr>
<tr>
<td>AVM1</td>
<td>F8.0 (49-56)</td>
<td>Minimum vert. eddy diffusivity (cm²/s)</td>
</tr>
<tr>
<td>AHR</td>
<td>F8.0 (57-64)</td>
<td>Reference horizontal eddy viscosity or diffusivity (cm²/s)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DUMMY</th>
<th>FM1</th>
<th>F8.0 (1-8)</th>
<th>Parameter in Richardson-number dependent eddy viscosity (see definition of S1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FM2</td>
<td>F8.0 (9-16)</td>
<td>Parameter in Richardson-number dependent eddy diffusivity (see definition of S2)</td>
</tr>
<tr>
<td></td>
<td>ZTOP</td>
<td>F8.0 (17-24)</td>
<td>Distance between the top of computational domain and the free surface. Used in computing</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>--------</td>
<td>------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td><strong>SLMIN</strong></td>
<td>F8.0  (25-32)</td>
<td>turbulence length scale (cm)</td>
<td></td>
</tr>
<tr>
<td><strong>QQMIN</strong></td>
<td>F8.0  (33-40)</td>
<td>Minimum value of turbulence macro scale (cm)</td>
<td></td>
</tr>
<tr>
<td><strong>ICUT</strong></td>
<td>I8  (1-8)</td>
<td>Minimum value of turbulence kinetic energy (gr/cm/s²)</td>
<td></td>
</tr>
<tr>
<td><strong>KSMALL</strong></td>
<td>I8  (9-16)</td>
<td>=0 Eddy coefficients constant below halocline</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>=1 Eddy coefficients computed below halocline</td>
<td></td>
</tr>
<tr>
<td><strong>QCUT</strong></td>
<td>F8.0  (17-24)</td>
<td>Coefficient in second-order turbulence model (0.15-0.25)</td>
<td></td>
</tr>
<tr>
<td><strong>GAMAX</strong></td>
<td>F8.0  (25-32)</td>
<td>Max. value of eddy viscosity (cm²/s)</td>
<td></td>
</tr>
<tr>
<td><strong>GBMAX</strong></td>
<td>F8.0  (33-40)</td>
<td>Max. value of eddy diffusivity (cm²/s)</td>
<td></td>
</tr>
<tr>
<td><strong>FZS</strong></td>
<td>F8.0  (41-48)</td>
<td>Turbulence scale is not allowed to exceed the product of FZS and depth</td>
<td></td>
</tr>
</tbody>
</table>

**Wind parameters**

| **IWIN** | I8  (1-8) | =0 Steady and uniform wind stress specified by TAUX and TAUY |
|          |          | =1 Steady and uniform wind speed                                |
|          |          | =2 Steady and space variab. wind stress                        |
|          |          | =3 Steady and space variable wind speed                        |
|          |          | =4 Time variable and uniform wind stress                       |
|          |          | =5 Time variable and uniform wind speed                        |
|          |          | =6 Time and space variable wind stress                         |
|          |          | =7 Time and space variable wind speed                           |
| **TAUX(1,1)** | F8.0  (9-16) | Uniform wind stress in KSI direction if IWIN=0 |
|          |          | Uniform wind speed (m/s) in KSI direction if IWIN=1           |
| **TAUYY(1,1)** | F8.0  (17-24) | Uniform wind stress in ETA direction if IWIN=0 |
|          |          | Uniform wind speed (m/s) in ETA direction if IWIN=1           |

**Flags**

(Notes different meaning for ISPAC, JSPAC in Sheng’s report)

| **ISPAC(1)** | I8  (1-8) | ISPAC(1)=1 Use spatially variable Manning’s n in vertically averaged model |
|              |          | =0 Use constant Manning’s n in vertically averaged model         |
| **ISPAC(2)** | I8  (9-16) | ISPAC(2)=1 Bottom drag coefficients computed in 3D model         |
|              |          | =0 Bottom drag coefficients set to CBF(?) in 3D model            |
ISPAC(3) I 8 (17-24) ISPAC(3)=1 Coriolis ON
ISPAC(4) I 8 (25-32) ISPAC(4)=1 Variable bottom roughness height
ISPAC(5) I 8 (33-40) ISPAC(5)=1 Constant bottom roughness height given by BZ1
ISPAC(6) I 8 (41-48) ISPAC(6)=1 Not used
ISPAC(7) I 8 (49-56) ISPAC(7)=1 Not used
ISPAC(8) I 8 (57-64) ISPAC(8)=1 Not used
ISPAC(9) I 8 (65-72) ISPAC(9)=0 Constant vertical eddy coefficient
ISPAC(10) I 8 (73-80) ISPAC(10)=0 Not used

JSPAC(1) I 8 (1-8) JSPAC(1)=1 Save dimensional Cartesian unit flows U1 and V1 on a file
JSPAC(2) I 8 (9-16) JSPAC(2)=1 Save depth-averaged dimensional Cartesian U and V on a file
JSPAC(3) I 8 (17-24) JSPAC(3)=1 Save dimensional Cartesian 3D u and v at cell faces on a file
JSPAC(4) I 8 (25-32) JSPAC(4)=1 Save dimensional Cartesian 3D u and v at cell center on a file
JSPAC(5) I 8 (33-40) JSPAC(5)=1 Save nondimensional cartesian U and V on a file
JSPAC(6) I 8 (41-48) JSPAC(6)=1 Not used
JSPAC(7) I 8 (49-56) JSPAC(7)=1 Not used
JSPAC(8) I 8 (57-64) JSPAC(8)=1 Not used
JSPAC(9) I 8 (65-72) JSPAC(9)=1 Not used
JSPAC(10) I 8 (73-80) JSPAC(10)=1 Not used

DUMMY
RSPAC(1) F8.0 (1-8) RSPAC(1)= Manning's n in c.g.s. units
RSPAC(2) F8.0 (9-16) RSPAC(2)= Not used
RSPAC(3) F8.0 (17-24) RSPAC(3)= An infinitesimal number used in checking the convergence to steady state (0.0001)
RSPAC(4) F8.0 (25-32) RSPAC(4)= An infinitesimal number used in checking the convergence to steady state (0.0001)
RSPAC(5) F8.0 (33-40) RSPAC(5) = Not used
RSPAC(6) F8.0 (41-48) RSPAC(6) = Not used
RSPAC(7) F8.0 (49-56) RSPAC(7) = Depth below which the bottom friction coefficient follows a ramp function (see subs CH2DXY, CH3DXYZ)
RSPAC(8) F8.0 (57-64) RSPAC(8) = Not used
RSPAC(9) F8.0 (65-72) RSPAC(9) = Coefficient for the spatial smoother (0.25)
RSPAC(10) F8.0 (73-80) RSPAC(10) = Coefficient for the curvature check of the spatial smoother (4)

Depth flags and constants

<table>
<thead>
<tr>
<th>DUMMY</th>
<th>I8 (1-8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IBTM</td>
<td>I8 (1-8)</td>
</tr>
<tr>
<td>HADD</td>
<td>F8.0 (9-16)</td>
</tr>
<tr>
<td>NMIN</td>
<td>F8.0 (17-24)</td>
</tr>
<tr>
<td>H1</td>
<td>F8.0 (25-32)</td>
</tr>
<tr>
<td>H2</td>
<td>F8.0 (33-40)</td>
</tr>
<tr>
<td>SSS0</td>
<td>F8.0 (41-48)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DUMMY</th>
<th>I8 (1-8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISMALL</td>
<td>I8 (1-8)</td>
</tr>
<tr>
<td>ISF</td>
<td>I8 (9-16)</td>
</tr>
<tr>
<td>ITB</td>
<td>I8 (17-24)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DUMMY</th>
<th>I8 (1-8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>XMAP</td>
<td>F8.0 (1-8)</td>
</tr>
</tbody>
</table>

Variable mapping for computational grid

<table>
<thead>
<tr>
<th>DUMMY</th>
<th>I8 (1-8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>XMAP</td>
<td>F8.0 (1-8)</td>
</tr>
</tbody>
</table>

Constant datum added to the depth at all locations (0.0)
Minimum depth
=0 No adjustment of the depth data
>0 Depth cannot be less than HMIN
H=max(H,NMIN)
Depth along one boundary
Depth along the opposing boundary
Initial water-surface elevation relative to initial water depth
Scaling factor in reference surface elevation computations (sub CH3DND)
Do not compute current at the free surface
Compute free-surface current from linear formula
Linear bottom friction for internal mode
Quadratic bottom friction for internal mode
Reference height above bottom (cm)
Constant bottom drag coefficient (typical value of 0.003)
Bottom roughness height (cm)
Reference height at the top
Constant surface roughness height
Factor that scales the (x,y) coordinates created by grid generation codes to the real world
| ALXREF | F8.0 (9-16) | Reference length in the X direction of the computational domain |
| ALYREF | F8.0 (17-24) | Reference length in the Y direction of the computational domain |

<table>
<thead>
<tr>
<th><strong>Transformation parameters</strong></th>
</tr>
</thead>
</table>
| ITRAN | I8 (1-8) | =0 Compute grid coordinates for equidistant Cartesian grid
| | | =1 Read grid coordinates (file created by WESCOR?)
| | | =2 Read grid coordinates and corner (grid with spline evaluated depths (file created by WESCOR?) transformation coefficients ?)

| IBD | I8 (9-16) | Spline boundary conditions ? |

<table>
<thead>
<tr>
<th><strong>Timebreaks for storing snapshot data</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>ITBRK</td>
</tr>
</tbody>
</table>

|**Timefile gage stations**|

<table>
<thead>
<tr>
<th><strong>Current stations</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>NSTA</td>
</tr>
<tr>
<td>NFREQ</td>
</tr>
<tr>
<td>NSTART</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Tide stations</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>NSTAS</td>
</tr>
</tbody>
</table>
Elevations are saved for time series plots.

- **NFREQS**: I8 (9-16) - Time-step number frequency for saving surface elevations.
- **NSTRTS**: I8 (17-24) - Beginning time-step number for saving water-surface elevations.

Following record is repeated NSTAS times.

- **ISTS**: I4 (1-4) - Coordinate (I,J) of a station where water-surface elevations are saved.
- **JSTS**: I4 (5-8) - Station descriptor.

Salinity and temperature gaging stations.

- **MSTA**: I8 (1-8) - Number of stations where salinity, temperature, etc., are saved for time series plots.
- **MFREQ**: I8 (9-16) - Time-step number frequency for saving information.
- **MSTART**: I8 (17-24) - Beginning time-step number for saving information.

Following record is repeated MSTA times.

- **ISTSA**: I4 (1-4) - Coordinate (I,J) of a station where salinities etc. are saved.
- **JSTSA**: I4 (5-8) - Station descriptor.

Salinity and temperature gaging stations.

River information.

- **NRIVER**: I8 (1-8) - Number of river-type boundaries (assigned inflow).
  - NRIVER = 0 No river-type boundaries.
  - <0 River inflows are steady.
  - >0 Time variable inflows read from file 13: rivr.inp in subroutine CH3DRI, CH5DRV.

- **IJRDIR**: I8 (1-8) - River boundary is on left (west), bottom (south), right (east), or top (north).

If NRIVER = 0, use the following records:

If NRIVER > 0, use the following records:

146
| **IJRROW** | I8 (9-16) | Index of the row (J) or column (I) of the river boundary |
| **IJRSTR** | F8.0 (17-24) | Starting I or J index of the river boundary |
| **IJREND** | F8.0 (25-32) | Ending I or J index of the river boundary |

If previous record is repeated NRIVER times

**DUMMY**

If NRIVER<0, use the following records:

| **IJRDIR** | I8 (1-8) | =1 River boundary is on left (west) |
|            |        | =2 River boundary is on bottom (south) |
|            |        | =3 River boundary is on right (east) |
|            |        | =4 River boundary is on top (north) |

| **IJRROW** | I8 (9-16) | Index of the row (J) or column (I) of the river boundary |
| **IJRSTR** | F8.0 (17-24) | Starting I or J index of the river boundary |
| **IJREND** | F8.0 (25-32) | Ending I or J index of the river boundary |

| I | I8 (1-8) | Coordinate (I,J) of a cell (vertical) where QRIVER is prescribed |
| J | I8 (9-16) | |
| **QRIVER** | F8.0 (17-24) | Steady river inflow (cfs) |

Previous record is repeated for each cell, from IJRSTR to IJREND, along a specific river boundary defined by IJRDIR. Procedure is repeated abs(NRIVER) times

Thin-wall barriers

| **DUMMY** |
| **NBAR** | I8 (1-8) | Number of interior barriers |

Stop if NBAR>NBARRS

| **NBARU** | I8 (9-16) |
| **KU** | I8 (17-24) |
| **NBARV** | I8 (25-32) |
| **KV** | I8 (33-40) |

| **DUMMY** |

If NBAR=0 following record is omitted

Otherwise, record is repeated NBAR times

| **IJBDIR** | I8 (1-8) | =1 barrier is horizontal (KSI direction) |
|            |        | =2 barrier is vertical (ETA direction) |
| **IJBROW** | I8 (9-16) | Index of row (J) or column (I) of interior barrier |
| **IJBSTR** | F8.0 (17-24) | Starting I or J index of interior barrier |
| **IJBEND** | F8.0 (25-32) | Ending I or J index of interior barrier |

If NBARU=0 following record is omitted

Otherwise, record is repeated NBARU times

Initially BARU(I,J,K)=1.0 for all I,J,K

| I | I8 (1-8) | I,J point where BARU(I,J,K)=0.0 |

147
For K=1,KU
If NBARV=0 following record is omitted
Otherwise, record is repeated NBARV times
Initially BARV(I,J,K)=1.0 for all I,J,K

If NBARV=0 following record is omitted
Otherwise, record is repeated NBARV times
Initially BARV(I,J,K)=1.0 for all I,J,K

Tidal boundary conditions

| DUMMY |
| TIDFNO | I8 (1-8) | Number of tidal-elevation tables entered as an input |
| TIDBND | I8 (9-16) | Number of tidal elevation boundaries |
| DUMMY |

If TIDFNO=0 following record is omitted
Otherwise, record is repeated TIDFNO times

TIDSTR | 10I8 (1-80) | The entry number in each tidal-elevation table corresponding to the starting time of the simulation From 1 to TIDFNO, maximum 10 values |

DUMMY

If TIDBND=0 following record is omitted
Otherwise, record is repeated TIDBND times

IJDIR | I8 (1-8) | I=1 Tidal boundary is on left (west) I=2 Tidal boundary is on bottom (south) I=3 Tidal boundary is on right (east) I=4 Tidal boundary is on top (north) |
IJROW | I8 (9-16) | Index of row (J) or column (I) of the tidal boundary |
IJSTRT | I8 (17-24) | Starting I or J of the tidal boundary |
IJEND | I8 (25-32) | Ending I or J of the tidal boundary |
TIDTYP | A8 (33-40) | 'CONSTANT' Constant tidal elevation between IJSTRT and IJEND 'INTERP' Linear interpolation of tidal elevation at IJSTRT and IJEND |
TIDFN1 | I8 (41-48) | Number of the tidal-elevation table for CONSTANT or INTERP type of boundaries |
TIDFN2 | I8 (49-56) | Number of second tidal-elevation table used for interpolation on INTERP type boundary |

*SUBROUTINE CH3DTR

Note: Data are read from file 15 : grid.inp
If ITRAN=1 read following records:

<table>
<thead>
<tr>
<th>FIELD</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>FILENM</td>
<td>A80 (1-80)</td>
</tr>
<tr>
<td>NX</td>
<td>I12 (1-12)</td>
</tr>
<tr>
<td>NY</td>
<td>I12 (13-24)</td>
</tr>
<tr>
<td>XCT</td>
<td>unformatted</td>
</tr>
<tr>
<td>YCT</td>
<td>unformatted</td>
</tr>
</tbody>
</table>

FILENM: Data-file descriptor

NX: Number of cell-centered grid points along a KSI-direction coord. line

NY: Number of cell-centered grid points along an ETA-direction coord. line

XCT: Cartesian X coordinates of cell-centered grid points, created by the grid generation code WESCOR. Coordinates are read along a KSI-direction coordinate lines, starting at the first KSI line (ETA=1). ICl*JCl points altogether

YCT: Cartesian Y coordinates of cell-centered grid points, created by the grid generation code WESCOR. Coordinates are read along a KSI-direction coordinate lines, starting at the first KSI line (ETA=1). ICl*JCl points altogether

If ITRAN=2 read following records:

<table>
<thead>
<tr>
<th>FIELD</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>FILENM</td>
<td>A80 (1-80)</td>
</tr>
<tr>
<td>NX</td>
<td>unformatted</td>
</tr>
<tr>
<td>NY</td>
<td>unformatted</td>
</tr>
<tr>
<td>XCT</td>
<td>unformatted</td>
</tr>
<tr>
<td>YCT</td>
<td>unformatted</td>
</tr>
<tr>
<td>HC</td>
<td>unformatted</td>
</tr>
<tr>
<td>LSLIT</td>
<td>unformatted</td>
</tr>
</tbody>
</table>

FILENM: Data-file descriptor

NX: Number of cell-centered grid points along a KSI-direction coord. line

NY: Number of cell-centered grid points along an ETA-direction coord. line

XCT: Cartesian X coordinate of a cell-centered grid point, created by the grid generation code WESCORA

YCT: Cartesian Y coordinate of a cell-centered grid point, created by the grid generation code WESCORA

HC: Corner-point depth

LSLIT: Previous record is read point by point along KSI-coordinate lines, starting at the first KSI line (ETA=1). NX*NY (ICl*JCl) points altogether

Cartesian X and Y coordinates, created by the grid generation codes WESCOR or WESCORA, are first multiplied by XMAP to scale them to the real world, and then divided by XREF to make them nondimensional.
*SUBROUTINE BJINTR

Note: Data are read from file 4: main.inp

** DUMMY **

I unformatted  
J unformatted  

Point (I,J) where the cell-centered depth HS has been set to zero

** DUMMY **

I unformatted  
J unformatted  

Point (I,J) where the U-face depth HU has been set to zero

** DUMMY **

I unformatted  
J unformatted  

Point (I,J) where the V-face depth HV has been set to zero

** DUMMY **

I unformatted  
J unformatted  
RDEPHT unformatted  

Point (I,J) where the cell-centered depth HS has been reset to the given value RDEPHT (ft)

** DUMMY **

I unformatted  
J unformatted  
AREAM unformatted  

Point (I,J) where the storage area has been assigned a non-zero value AREAM (m2)

IWIND<2, following records read in CH3DWS, CH3DWT are omitted

*SUBROUTINE CH3DWS, ENTRY CH3DWT

Note: Wind data are read from file 14: wind.inp

IWIND=2: (steady and space variable ?? wind stress)

TX1(1,1) F8.0 (1-8)  
TY1(1,1) F8.0 (9-16)  

X-direction component of wind stress  
Y-direction component of wind stress

Subroutine CH3DWS is called, prior to the main computational time loop, to read wind-stress components and to assign them to all computational points

150
IWIND=3: (steady and space variable wind speed)
Same as for IWIND=2, except that TX1,TY1 are pairs of X- and Y-direction components of wind speed

IWIND=4: (time-variable and uniform wind stress)
File 14 (wind.inp) contains wind stress time table
| IDAY1 | I5 (1-5) | Day and hour defining the first time level in the time table
| IHOUR1 | I5 (6-10) | Level in the time table
| WNDX1 | 6F10.0 (21-80) | Pairs of X- and Y-direction components of time-varying wind stress
| WNDY1 | 6F10.0 (21-80) | Total of NWINDS pairs

Previous record is repeated until the last time level in the time table exceeds the end-of-computation time.

Subroutine CH3DWS is called, prior to the main computational time loop, to read wind-stress components for the first two time levels (called time levels 1 and 2, defined by IDAY1,IHOUR1 and IDAY2,IHOUR2) in a wind stress time table.
and to evaluate initial wind stress by interpolation, between time levels 1 and 2 read from the table.
Entry CH3DWT is called, during the main computational loop, to update the wind-stress components i.e. to read the next time level from the wind stress time table if the current computational time exceeds the time level 2, last read from the table.

IWIND=5: (time-variable and uniform wind speed)
File 14 (wind.inp) contains wind speed time table
Same as for IWIND=4, except that WNDX,WNDY are pairs of X- and Y-direction components of time-varying wind speed

IWIND=6: (time and space variable wind stress)
File 14 (wind.inp) contains wind stress time table
| IDAY1 | I5 (1-5) | Day and hour defining the first time level in the time table
| IHOUR1 | I5 (6-10) | Level in the time table
| TX1(1,1) | F8.0 (1-8) | X-direction component of time-varying wind stress
| TY1(1,1) | F8.0 (9-16) | Y-direction component of time-varying wind stress

Previous two records are repeated until the last time level in the time table exceeds the end-of-computation time

Subroutine CH3DWS is called, prior to the main computational time loop, to read wind-stress components for the first two time levels (called time levels 1 and 2, defined by IDAY1,IHOUR1 and IDAY2,IHOUR2) in a wind stress time table.
to evaluate initial wind stress by interpolation, between time levels 1 and 2 read from the time table.
and to assign them to all computational points.
Entry CH3DWT is called, during the main computational loop, to update the wind-stress components i.e. to read the next time level from the wind stress time table if the current computational time exceeds the time level 2, last read from the table.
if IWIND=7: (time and space variable wind speed)
File 14 (wind.inp) contains wind speed time table
Same as for IWIND=6, except that TX1,TY1 are pairs of X- and
Y-direction components of time-varying wind speed

*ITEMP=0, records read in CH3DTK are omitted
*ITEMP.GE.0.OR.NRIVER.EQ.0 records read in CH3DTB are omitted
*SUBROUTINE CH3DTK, ENTRY CH3DTB

Note: Time-varying equilibrium temperature (TEP) and
time-varying coefficient of surface heat exchange (TEK)
are read from file 19: heat.inp

| IDAYT1 | I5 (1-5) | Day and hour defining the first time level in the TEP/TEK time table |
| IHRT1  | I5 (6-10) | Equilibrium temperature (Celsius) |
| TEP1   | F10.0 (11-20) | Coefficient of surface heat exchange (cm³/s³) |
| TEK1   | E12.5 (21-32) |

Previous record is repeated until the last time level in the time table exceeds the end-of-computation time

Subroutine CH3DTK is called, prior to the main computational time loop, to read TEP and TEK for the first two time levels (called time levels 1 and 2, defined by IDAYT1,IHRT1 and IDAYT2,IHRT2) in a TEP/TEK time table.

Entry CH3DTB is called, during the main computational loop, to update TEP and TEK, i.e. to read the next time level from the TEP/TEK time table if the current computational time exceeds the time level 2, last read from the table to evaluate TEP and TEK by interpolation, between time levels 1 and 2 read from the time table and to assign them to all computational points.

*NRIVER=0, no river boundaries
*NRIVER<0, records read in CH3DRI are omitted
(steady QRIVER read in CH3DIR)
*SUBROUTINE CH3DRI, ENTRY CH3DRV

Note: Time-varying river inflows are read from file 13: rivr.inp

| IDAYA | I8 (1-8) | Day and hour defining the time level in the river inflow time table |
| HOURA | I8 (9-16) |
| I     | I8 (1-8) | Coordinate (I,J) of a cell (vertical) |
J I8 (9-16) where the river inflow is prescribed
QRIVRA F8.0 (17-24) Boundary river inflow (cfs)

Previous record is repeated for each cell, from IJRSTR to IJREND, along a specific river boundary defined by IJRDIR, IJRROW.
The same is done for each of NRIVER river boundaries.

Subroutine CH3DRI is called, prior to the main computational time loop, to read boundary river inflows for the first two time levels (called time levels A and B, defined by IDAYA, IHOURLA and IDAYB, IHOURLB) in a river inflow time table.
Entry CH3DRV is called, during the main computational loop, to update boundary river inflow, i.e. to read the next time level from the river inflow time table if the current computational time exceeds the time level B, last read from the table and to evaluate boundary river inflows by interpolation, between time levels A and B read from the time table.

*SUBROUTINE CH3DTI, ENTRY CH3DTD

Note: Tidal-elevation time tables read from file 16: tide.inp

TIDTIT A80 (1-80) Tidal-data descriptor
IMO0 I2 (1-2) Reference month, day, year, hour and minute defining reference time
IDY0 I3 (4-6) level for the data read in the tide elevations time table
IYR0 I3 (7-9) IMN0 I2 (13-14)
IM0 I2 (1-2) IMO I2 (1-2)
IDY I3 (4-6) IDY I3 (4-6)
IYR I3 (7-9) hour and minute
IHR I2 (11-12) defining time level
IMN I2 (13-14) in the tidal elevations time table
TIDELV 8F8.0 (17-80) Tidal elevations

Previous record is repeated until the last time level in the time table exceeds the end-of-computation time

Subroutine CH3DTI is called, prior to the main computational time loop, to read complete tidal-elevation time table.
Entry CH3DRV is called, during the main computational loop, to load tidal elevations at boundaries, by interpolating between points in tidal-elevations time table.

*ISALT.NE.-2, records read in CH3DSAI, CH3DSAV are omitted
*SUBROUTINE CH3DSAI, ENTRY CH3DSAV
Note: Time table containing vertical salinity and temperature profiles along tidal boundaries are read from file 76: tidesate.inp

Following records are repeated until the last time level in the time table exceeds the end-of-computation time

<table>
<thead>
<tr>
<th>IDAYS1</th>
<th>I5 (1-5)</th>
<th>Day and hour defining the time level in the time table containing vertical salinity and temperature profiles along tidal boundaries</th>
</tr>
</thead>
<tbody>
<tr>
<td>IHRS1</td>
<td>I5 (6-10)</td>
<td>IJ</td>
</tr>
<tr>
<td>I</td>
<td>I5 (1-5)</td>
<td>IJ</td>
</tr>
<tr>
<td>J</td>
<td>I5 (6-10)</td>
<td>IJ</td>
</tr>
<tr>
<td>SAl1</td>
<td>11F5.0  (11-65)</td>
<td>Vertical salinity profile Salinities are prescribed at all points along the vertical direction, point-by-point, starting from the point at the bed</td>
</tr>
<tr>
<td>TE1</td>
<td>11F5.0  (11-65)</td>
<td>Vertical temperature profile Temperatures are prescribed at all points along the vertical direction, point-by-point, starting from the point at the bed</td>
</tr>
</tbody>
</table>

Previous two records are repeated for each cell, from IJSTRT to IJEND, along a specific tidal boundary defined by IJDIR and IJROW.
The same is done for each of TIDBND=IJLINE tidal boundaries.

Subroutine CH3DSAI is called, prior to the main computational time loop, to read tidal-boundary salinities and temperatures for the first two time levels (called time levels 1 and 2, defined by IDAYS1,IHRS1 and IDAYS2,IHRS2) in the time table containing vertical salinity and temperature profiles along tidal boundaries.

Entry CH3DSAV is called, during the main computational loop, to update tidal-boundary salinities and temperatures, i.e. to read the next time level from the time table containing vertical salinity and temperature profiles along tidal boundaries if the current computational time exceeds the time level 2, last read from the table and to evaluate tidal-boundary salinities and temperatures by interpolation, between time levels 1 and 2 read from the time table.

*NRIVER.EQ.0.OR.ISALT.NE.-2, records read in CH3DTEI,CH3DTEV are omitted
*SUBROUTINE CH3DTEI, ENTRY CH3DTEV
Note: Vertical temperature profiles along river-type boundaries are read from file 78: rivrte.inp

| IDYTE1 | I5 (1-5) | Day and hour defining time level in the time table containing vertical temperature profiles along river boundaries |
| IHRTE1 | I5 (6-10) | |
| I | I5 (1-5) | Coordinate (I,J) of a cell (vertical) on a river boundary where the vertical temperature profile is prescribed |
| J | I5 (6-10) | |
| TE3 | 11F5.0 (11-65) | Vertical temperature profile Temperatures are prescribed at all points along the vertical direction, point-by-point, starting from the point at the bed |

Previous record is repeated for each cell, from IJRSTR to IJREND, along a specific river boundary defined by IJRDIR and IJRRAS. The same is done for each of abs(NRIVER) river boundaries.

NRIVER≤0:
Subroutine CH3DTE1 is called, prior to the main computational time loop, to read constant vertical temperature profiles along river boundaries. The time table with vertical temperature profiles along river boundaries contains only one time level.

NRIVER>0:
Subroutine CH3DTE1 is called, prior to the main computational time loop, to read river-boundary temperatures for the first two time levels (called time levels 1 and 2, defined by IDYTE1,IHRTE1 and IDYTE2,IHRTE2) in the time table containing vertical temperature profiles along river boundaries. Entry CH3DTEV is called, during the main computational loop, to update river-boundary temperatures, i.e. to read the next time level from the time table containing vertical temperature profiles along river boundaries if the current computational time exceeds the time level 2, last read from the table and to evaluate river-boundary temperatures by interpolation, between time levels 1 and 2 read from the time table.
APPENDIX E

MODEL OF THE MISSISSIPPI RIVER AT THE OLD RIVER SAMPLE INPUT-DATA SET

99999 99999 99999 99999 99999 99999 99999 99999 99999 99999
99999 99999 99999 99999 99999 99999 99999 99999 99999 99999
99999 99999 99999 99999 99999 99999 99999 99999 99999 99999
99999 99999 99999 99999 99999 99999 99999 99999 99999 99999
99999 99999 99999 99999 99999 99999 99999 99999 99999 99999
99999 99999 99999 99999 99999 99999 99999 99999 99999 99999
99999 99999 99999 99999 99999 99999 99999 99999 99999 99999
99999 99999 99999 99999 99999 99999 99999 99999 99999 99999
99999 99999 99999 99999 99999 99999 99999 99999 99999 99999
99999 99999 99999 99999 99999 99999 99999 99999 99999 99999
99999 99999 99999 99999 99999 99999 99999 99999 99999 99999
99999 99999 99999 99999 99999 99999 99999 99999 99999 99999
99999 99999 99999 99999 99999 99999 99999 99999 99999 99999
99999 99999 99999 99999 99999 99999 99999 99999 99999 99999
99999 99999 99999 99999 99999 99999 99999 99999 99999 99999
99999 99999 99999 99999 99999 99999 99999 99999 99999 99999
99999 99999 99999 99999 99999 99999 99999 99999 99999 99999
99999 99999 99999 99999 99999 99999 99999 99999 99999 99999
99999 99999 99999 99999 99999 99999 99999 99999 99999 99999
0 1 0 1 1 1 1 1 1 1 1 0 1 0 0 1 0 0 0 0 0
1 1 1 1 1 1 1
0 0
0.00001 0.0001 0.0005
0.00000114
2.65 0.4
0.50 0.3 0.1 0.1 0.01
0.0000001 0.00000001
30
0.1
0
0.000250 .000015 0.0
.000250 .000020 0.0
.000250 .000025 0.0
.000250 .000035 0.0
.000250 .000055 0.0
.000250 .000075 0.0
.000250 .000100 0.0
.000250 .000130 0.0
.000250 .000165 0.0
.000250 .000220 0.0
1000.
0
1 1 1
0.05 100.0
-1 0.0 0.5 0.5
1 3 -1
1 4 -1

156
<table>
<thead>
<tr>
<th>1</th>
<th>5</th>
<th>-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>11</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>12</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>13</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>14</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>15</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>16</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>17</td>
<td>-1</td>
</tr>
<tr>
<td>S1</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>S1</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>S1</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>S1</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>S1</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>S1</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>S1</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>S1</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>S1</td>
<td>11</td>
<td>1</td>
</tr>
<tr>
<td>S1</td>
<td>12</td>
<td>1</td>
</tr>
<tr>
<td>S1</td>
<td>13</td>
<td>1</td>
</tr>
<tr>
<td>S1</td>
<td>14</td>
<td>1</td>
</tr>
<tr>
<td>S1</td>
<td>15</td>
<td>1</td>
</tr>
<tr>
<td>S1</td>
<td>16</td>
<td>1</td>
</tr>
<tr>
<td>-S1</td>
<td>17</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1</th>
<th>0.0</th>
<th>0.5</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.000250</td>
<td>.000025</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>.000250</td>
<td>.000035</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>.000250</td>
<td>.000045</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>.000250</td>
<td>.000065</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>.000250</td>
<td>.000085</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>.000250</td>
<td>.000110</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>.000250</td>
<td>.000140</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>.000250</td>
<td>.000180</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>.000250</td>
<td>.000225</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>.000250</td>
<td>.000280</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>.000250</td>
<td>.000015</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>.000250</td>
<td>.000020</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>.000250</td>
<td>.000025</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>.000250</td>
<td>.000035</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>.000250</td>
<td>.000055</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>.000250</td>
<td>.000075</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>.000250</td>
<td>.000100</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>.000250</td>
<td>.000130</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>.000250</td>
<td>.000165</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>.000250</td>
<td>.000220</td>
<td>0.0</td>
</tr>
<tr>
<td>3</td>
<td>.000250</td>
<td>.000000</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>.000250</td>
<td>.000000</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>.000250</td>
<td>.000005</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>.000250</td>
<td>.000010</td>
<td>0.0</td>
</tr>
</tbody>
</table>

157
<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.0</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>1</td>
<td>0.000250</td>
<td>0.000025</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.000250</td>
<td>0.000035</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.000250</td>
<td>0.000045</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.000250</td>
<td>0.000065</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.000250</td>
<td>0.000085</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.000250</td>
<td>0.000110</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.000250</td>
<td>0.000140</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.000250</td>
<td>0.000180</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.000250</td>
<td>0.000225</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.000250</td>
<td>0.000280</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>0.000250</td>
<td>0.000015</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.000250</td>
<td>0.000020</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.000250</td>
<td>0.000025</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.000250</td>
<td>0.000035</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.000250</td>
<td>0.000055</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.000250</td>
<td>0.000075</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.000250</td>
<td>0.000100</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.000250</td>
<td>0.000130</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.000250</td>
<td>0.000165</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.000250</td>
<td>0.000220</td>
<td>0.0</td>
</tr>
<tr>
<td>3</td>
<td>0.000250</td>
<td>0.000000</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.000250</td>
<td>0.000000</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.000250</td>
<td>0.000005</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.000250</td>
<td>0.000010</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.000250</td>
<td>0.000015</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.000250</td>
<td>0.000020</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.000250</td>
<td>0.000030</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.000250</td>
<td>0.000045</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.000250</td>
<td>0.000065</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.000250</td>
<td>0.000090</td>
<td>0.0</td>
</tr>
</tbody>
</table>
This report describes the theoretical principles of three-dimensional sediment transport and bed evolution processes and numerical solution of the appropriate governing equations. It also includes technical documentation and user’s instructions for the sediment-operations program module developed as an integral part of the CH3D-WES code.

The generalized CH3D-WES code provides numerical simulation of three-dimensional unsteady water flow, sediment transport, and bed evolution in natural watercourses. Sediment mixtures are represented through a suitable and unlimited number of discrete size classes, any of which may be subject to either suspended load or bed load transport (or both) depending on prevailing local hydrodynamic conditions. The governing dimensionless sediment equations are SIGMA-stretched in the vertical direction and transformed into general (nonorthogonal) curvilinear coordinates in the other two directions. The generalized CH3D-WES code includes the feedback between the flow field and changes in bed elevation, bed-surface size distribution, and the density of the mixture containing water and suspended sediment.

(Continued)
15. (Concluded).

This report contains detailed descriptions of the sediment-operations program module, memory and time requirements, and an input-data guide for the module.

The new mobile-bed numerical procedures are tested by applying the CH3D-WES code with its new sediment-operations program module to a natural prototype case: the Mississippi River at the Old River Control Structure complex.

14. (Concluded).

<table>
<thead>
<tr>
<th>Alluvial channels</th>
<th>Sedimentation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bed load transport</td>
<td>Suspended sediment transport</td>
</tr>
<tr>
<td>Numerical modeling</td>
<td></td>
</tr>
</tbody>
</table>