TERABIT NETWORK
CONCEPTS

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Rome Laboratory
Air Force Materiel Command
Griffiss Air Force Base, New York
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This program was intended to determine how single mode fiber optic technology can be used to significantly improve the robustness and ruggedness of communications networks. The thrust of the study was to utilize the known potential bandwidth of single mode fiber (at least 10 GHz). The report describes several topologies, and plots their minimum power delivered between any transmitter-receiver pair for several values of coupler losses and numbers of nodes and transceivers. The conclusion shows that in a network designed for a 50 dB loss budget, over 1024 users can be supported if the topology is designed properly, and low loss components are used.
1 Introduction

Looking to passive networks as a way of simplifying network control, or using less expensive components, we find ourselves confronted with the power division problem. In a fully-connected passive network (passive, except for transceivers), the only way to ensure that power from a given transmitter gets to a given receiver is to let the power reach every receiver. Thus, the intended receiver gets only a fraction of the input power. This being the case, we must distribute the power as evenly as possible, in order to maximize the minimum power delivered between any transmitter-receiver pair, and thus maximize the number of supportable users. [1]

Another concern is robustness, defined here as the maximum number of links that can be cut while maintaining full connectivity. In an active network, we can attain robustness by providing multiple paths through the network, routing a given message or packet over only one of these paths. We can use the strategy of employing multiple paths in passive networks, too. Of course, the broadcast nature of the passive network in combination with the multiple paths yields multiple receptions of a message at the receiver. To deal with this, we assume that all of the available bandwidth of the fiber (approximately $10^6$ GHz) is accessible, and we choose wideband signals such that if the differential delay between receptions exceeds the reciprocal
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bandwidth of the signal, then these receptions are resolvable. This principle works with lasers or LEDs. The design of a receiver to resolve receptions and add their power in such a passive fiber optic network is analogous to the work done on resolving receptions in multipath radio channels. [2]

Given that we desire to use such robust, passive fiber optic networks for local communications, we strive to understand how network parameters affect power distribution. This will enable us to design topologies to distribute power optimally, which entails distributing power as evenly as possible and with as little loss as possible. Since we are concerned with local communications, we neglect loss in the fiber. Section 2 describes the network components, while Section 3 explains the mathematical model. In Section 4 we describe several topologies. We plot and discuss computational results for the power distribution of these topologies in Section 5, drawing our conclusions in Section 6.

2 Network Components

Our networks are composed of active transceivers, each corresponding to a user, and passive nodes. A simple example is illustrated in Figure 1(a), which shows the logical layout of the network, not the physical layout. The transceivers always have a single neighboring node to which they transmit
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Figure 1: A Simple Passive Network

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power, and from which they receive power. Nodes can have an arbitrary number of neighboring nodes and transceivers. Our model for a node is a star coupler, which has total loss equal to the sum of the excess loss within the coupler and the splice loss, and which divides the rest of its input power evenly among its outputs. An example of how star couplers might actually be connected in such a network is shown in Figure 1(b). Henceforth, the combination of excess loss and splice loss will be referred to as "coupler loss," while loss due to dividing power will be referred to as "splitting loss."

3 An Analogic Mathematical Model

Clearly, the more receptions of a message that the receiver resolves, the more power it can collect. In practice there will be some limit to the number of receptions that are actually detected due to a desire to limit time delay. The precise implementation of such a scheme depends upon the impulse response of the network, which in turn depends upon the physical locations of transceivers and nodes. Since we do not assume a particular physical layout, we compare topologies in the ideal case, in which receivers can wait forever to collect all of the power that circulates through the network. This case, which entails taking into account power that comes to a receiver over
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infinitely many paths, provides an upper bound on the power that could be collected.

In order to deal analytically with the task of adding up power over infinitely many paths, we make an analogy to Markov chains. Since the total power that reaches the receivers plus the total power lost due to coupler loss equals the total power injected into the network, we consider the fraction of power collected by a receiver as the probability that a particular quantum of power is collected by that receiver. Likewise, we consider the fraction of power lost due to coupler loss as the probability that a particular quantum of power is lost. On a smaller scale, since the power into a node equals the power lost due to coupler loss plus the power sent to other nodes and receivers, we consider these fractions of power as the probabilities for that node that a particular quantum of input power is lost or sent to another node or receiver. In fact, if each node and each receiver is considered to be a state, and a loss state is included, then these probabilities become the transition probabilities of a Markov chain, and the normalized final power distribution becomes the final probability distribution of the Markov chain.

Figure 2 shows the Markov chain for the network in Figure 1. States A, B, and C represent nodes, while all other states but the one marked “loss” represent receivers. Transmitter states are not included, but it is assumed
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![Diagram](image)

Figure 2: Analogic Markov Chain

\[
M = \begin{bmatrix}
A & B & C & A_R & B_R & C_R^1 & C_R^2 & \text{loss} \\
A & 0 & .3 & .3 & .3 & 0 & 0 & 0 & .1 \\
B & .3 & 0 & .3 & 0 & .3 & 0 & 0 & .1 \\
C & .225 & .225 & 0 & 0 & 0 & .225 & .225 & .1 \\
A_R & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
B_R & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
C_R^1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
C_R^2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
\text{loss} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 
\end{bmatrix}
\]
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that, with probability one, all power that they transmit goes to the node to
which they are attached. The transition probabilities reflect the fact that
in this example, coupler loss is 0.4 dB.

Figure 2 also shows a transition matrix for the Markov chain. Each row
of the matrix shows the power distribution after the message has traveled
one link, given that it started at the node or receiver corresponding to that
row.

A transition matrix for a general network would be of the form

\[ M = \begin{pmatrix} S & R & L \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix}, \]

with node states listed first, followed by receiver states, and finally the loss
state. Thus the matrix blocks across the top row of \( M \), assuming \( n \) nodes
and \( m \) receivers, are described:

\( S: n \times n \) matrix representing transitions between nodes

\( R: n \times m \) matrix representing transitions from nodes to receivers

\( L: n \times 1 \) matrix representing power lost from nodes

0 and \( I \) are the zero matrix and the identity matrix, respectively.

Each row of the limiting matrix shows the normalized final power dis-
tribution after power has been collected over all possible paths, given that
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power started at the node or receiver corresponding to that row. The limiting matrix is: [1]

\[
\lim_{b \to \infty} M^b = M_\infty = \begin{pmatrix}
0 & (I - S)^{-1}R & (I - S)^{-1}L \\
0 & I & 0 \\
0 & 0 & I
\end{pmatrix}
\]

The i-jth element of \((I - S)^{-1}R\) represents the power, expressed as a fraction of the input power, delivered from any transmitter on node i to receiver j. One way to optimize the power distribution is to maximize, over topologies, the minimum element of \((I - S)^{-1}R\), given the number of users. Equivalently, we can maximize the number of users while maintaining the minimum element of \((I - S)^{-1}R\) above some threshold.

We are currently analyzing these matrices in detail in order to determine general desirable and undesirable characteristics for these topologies. [1] This paper contains a sample of both qualitative and quantitative results of this work.

4 Promising Topologies

Figure 3 illustrates the five topologies to be discussed. In all topologies except the ring, the links are bi-directional. In the ring they are uni-directional, because “ring” usually refers to a network in which power
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Figure 3: Examples of Topologies

a) Star

b) Ring

c) Wrap-Around Triangular Mesh
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Figure 3: Examples of Topologies
circulates without reversing direction. In part (a) of the figure, several transceivers are shown for clarity. In the rest of the figure the transceivers are not drawn, but we assume they are spread evenly among the nodes.

The star and ring are not robust, because if one link is cut anywhere, the network is no longer fully-connected. Of course, the other topologies also have single links at each transceiver, but these are more reliable than the transceiver links of the star topology, because they can be made shorter when there is a multiplicity of nodes.

We define several terms which we will use as we discuss the topologies. By “performance”, we mean the minimum normalized power delivered from any transmitter to any receiver. The “degree of connectivity” of a node is the number of nodes to which it outputs power. “Uniform connectivity” refers to the situation in which all nodes in a network have the same degree of connectivity. A node’s “neighboring nodes” and “neighboring transceivers” are those nodes and transceivers to which it outputs power. Finally, the length of a path between two nodes, or “path length,” is the number of links which comprise the path.

Figure 3(a) illustrates the star topology. Although it is not robust, it provides an upper bound on the performance of robust topologies, because it is the optimal power distributor overall. The star topology performs op-
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timally because no matter which transmitter transmits, there is loss from only one coupler, and the rest of the power is divided evenly, thus maximizing the minimum delivered power.

The ring, illustrated in part (b) of Figure 3 is included because it is a commonly used topology. Although fiber rings perform adequately with active nodes [3], [4] or amplifiers [5], we will see later that the ring topology is unacceptable for a passive fiber optic network.

Part (c) of the figure shows the wrap-around triangular mesh. The nodes and unbent links are arranged (logically) in a mesh with triangular cells. The bent links connect nodes at the edges of this mesh, hence the name "wrap-around" triangular mesh.

In the fully-connected mesh of part (d), every node is connected to every other node. We will see later that this topology performs very well, and since the optimum number of nodes for this topology is much smaller than the number of transceivers, it does not require much more fiber than the other topologies.

Three sizes of hypercube are shown in Figure 3(e) to illustrate that a $d$-dimensional hypercube is constructed from two $(d - 1)$-dimensional hypercubes joined appropriately.
5 Computational Results

The results plotted in this section were computed using the MATLAB software package on a DG20000 computer. In the plots, we assume coupler loss is uniform across couplers, even couplers of different sizes, therefore we model multimode networks better than single-mode networks. This is because the loss in a single-mode coupler depends on its size.

All of the figures show the normalized minimum power delivered in dB. Recall that this quantity takes into account power collected over infinitely many paths, and thus upperbounds the performances of practical networks. As long as this quantity exceeds the loss budget, all users are supported in this bounding case.

5.1 The Effect of the Number of Nodes on Distributed Power

Figure 4 shows the normalized minimum power delivered vs. the number of nodes for the topologies illustrated in Figure 3. Careful consideration of the plot, and an understanding of the structure of the topologies, reveal that for a given topology, number of transceivers, and coupler loss, there is an optimal number of nodes. As noted previously, when we compare topologies with identical coupler loss, the performance of the star topology
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Figure 4: The number of nodes affects power via the splitting loss at each node, the length of the shortest path between two nodes, and the number of paths of a given strength.
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provides an upper bound on the performances of the other topologies.

The ring performs the worst, because in the worst-case even the strongest reception of the message has circulated around the entire ring. Recall that every pass through a node means more loss. Data points for the minimum power delivered in a ring with more than four nodes fall below the range of this plot, therefore four is the optimal number of nodes for a ring with 512 users attached and 0.3 dB coupler loss.

The wrap-around triangular mesh has been arbitrarily defined to be equal to the fully-connected mesh for the case of four nodes. Let \( m \) be the number of transceivers, and \( n \) be the number of nodes. As the number of nodes increases, and the transceivers are spread out more thinly among them, the splitting loss, \( 10 \log (6 + m/n) \), decreases, so that we might expect the minimum power to increase. Instead it decreases, because as the number of nodes increases, the path lengths between worst-case transmitter-receiver pairs increase, causing the loss along each path to increase. For this topology, number of transceivers, and coupler loss, as the number of nodes increases, the effect of increasing path lengths is stronger than the effect of decreasing splitting loss. Again, the optimal number of nodes is four.

In the fully-connected mesh, the splitting loss at each node is \( 10 \log (n - 1 + m/n) \).
Because of the $m/n$ term inside the logarithm, the splitting loss attains a minimum at $n = \sqrt{m}$. Thus for $m = 512$, we find that the splitting loss decreases as $n$ increases to 22, and then increases as $n$ continues to increase.\textsuperscript{2} The splitting loss is the same for $n = 16$ and $n = 32$. Naturally, when the splitting loss decreases, the minimum power delivered increases, and vice-versa, as shown in the plot.

In this topology, every node has a single-link path to every other node, regardless of the number of nodes. Thus, increasing the number of nodes does not tend to increase path lengths. However, another effect of changing the number of nodes is to change the number of paths of a given length between any two transceivers. The reason that the minimum power is higher for $n = 32$ than for $n = 16$ in the fully-connected mesh is that although coupler loss and splitting loss are the same at every node for both the 16-node and 32-node topologies, there are more paths of any given length and strength in the 32-node fully-connected mesh. Therefore, for 512 users and 0.3 dB coupler loss, the optimal number of nodes for a fully-connected mesh is 32.

For the hypercube, we see an unexpected decrement in performance between the $n = 4$ case and the $n = 8$ case. The splitting loss at each node $^2$This is approximate when $m/n$ is not an integer. In such cases, some nodes would have to have more splitting loss than others.
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For this topology is \(10 \log \log_2 n + m/n\), so that we would expect an improvement in performance based on the splitting loss argument. However, path lengths increase as \(n\) increases. In jumping from \(n = 4\) to \(n = 8\), for 0.3 dB coupler loss, the effect of increasing path lengths dominates, causing performance to diminish, but in increasing the number of nodes beyond \(n = 8\), the splitting loss effect dominates. The splitting loss will continue to decrease until \(n = 354\). Thus for this coupler loss and number of transceivers, the optimal number of nodes for a hypercube (whose number of nodes must be a power of 2) is 256. This point is out of the domain of the plot in Figure 4.

Note that with a 50 dB loss budget, if the number of nodes were to be chosen correctly, any of the three robust topologies depicted could support 512 transceivers.

5.2 The Effect of the Number of Transceivers on Distributed Power

Figures 5 and 6 plot the normalized minimum power delivered vs. the number of transceivers. Figure 5 shows results for the robust topologies with 128 nodes, while Figure 6 shows results for the wrap-around triangular mesh and the ring for four nodes, since these two topologies perform best
Figure 5: The number of transceivers affects power by determining the overall power division required, and the splitting loss at each node.
Figure 6: The number of transceivers affects power by determining the overall power division required, and the splitting loss at each node.
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with fewer nodes. The results confirm our intuition, since the more transceivers there are, the less power each receives. This is due to two effects, the first being that the more transceivers there are, the more ways we must divide up the power to support all of the transceivers. The second effect is that the more transceivers there are, the bigger the splitting loss at each node.

Notice that performance does not diminish as fast for the fully-connected mesh as for the other topologies, as the number of transceivers increases. For the numbers plotted, each node of the fully-connected mesh has so many neighboring nodes (127), that increasing the number of neighboring transceivers does not change the splitting loss much. On the other hand, the nodes of the other topologies have so few neighboring nodes (7 for hypercube, 6 for 128-node wrap-around triangular mesh, 3 for 4-node wrap-around triangular mesh, 1 for ring), that the increase in the number of transceivers noticeably affects the splitting loss at each node.

Figure 5 indicates that if our loss budget were 50 dB, then the fully-connected mesh could support at least 1024 users, and the hypercube could support at least 640 users. As mentioned previously, we expect a 256-node hypercube to do even better.
5.3 The Effect of Coupler Loss on Distributed Power

Figure 7 shows normalized minimum power delivered vs. number of nodes for all topologies and three different values of coupler loss. This figure illustrates that too much coupler loss degrades the performance of a network and decreases the number of supportable users. This effect is more pronounced for networks with more nodes, due to the increased path lengths and the increased number of paths of a given length. The precise dependence of power distribution on coupler loss depends on the topology and size of a network.

Notice that even with more lossy couplers and a 50 dB loss budget, we can still support at least 512 users with robust topologies.

6 Conclusion

We have demonstrated that robust passive fiber optic networks with multiple paths, in which receptions of signal over the multiple paths are resolved, and the power from these receptions is added, can distribute enough power to support over one-thousand users, if the topology is designed correctly, and low-loss components are used. The topology should be designed to distribute power evenly, and the number of nodes should be chosen so as
Figure 7: The degree to which coupler loss affects power distribution depends upon the topology and size of the network.
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to balance the effects of splitting loss, path lengths, and number of paths of a given strength.

Based on the data we have collected, we conclude that of the robust topologies that we have studied, the fully-connected mesh provides the best power distribution, and therefore supports the most users. The other robust topologies can also be used in many situations. The plots of the previous section support our conclusion.

We noted before, based on Figure 4, that the robust topologies can support at least 512 users. Whatever topology we decided to implement, we would choose the optimum number of nodes (depending on coupler loss and number of transceivers) in order to maintain the largest possible power margin (in case we would wish to add more transceivers later).

Suppose we wanted to build a robust passive fiber optic network for 512 users with 0.3 dB coupler loss. Every topology would entail 512 node-to-transceiver fibers. If we chose the fully-connected mesh, we would use 32 nodes, and $32 \times 31 = 992$ node-to-node fibers. We would use 256 nodes and $256 \times \log_2(256) = 2048$ node-to-node fibers in the hypercube. For the wrap-around triangular mesh we could use 4 nodes and 12 node-to-node fibers, but then our node-to-transceiver fibers would be longer and less reliable. Recall that the 4-node wrap-around triangular mesh is actually a
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fully-connected mesh.

Since we cannot tell from the plots how good the 256-node hypercube is, we might choose to use 128 nodes and 896 node-to-node fibers. But Figure 4 shows that we can gain over 10 dB in power margin by using 96 more fibers and 96 fewer couplers, and implementing the 32-node fully-connected mesh.

Extrapolation from the plot of Figure 5 indicates that within a 50 dB loss budget, only the fully-connected mesh could support well over 1024 users with 0.3 dB coupler loss. Figure 7 shows that for the same loss budget, only the fully-connected mesh can support at least 512 users when couplers with 0.9 dB coupler loss are used. Therefore, in order to maintain a large power margin, we recommend the use of the fully-connected mesh to support more than 512 users. For fewer users, Figures 5 and 6 suggest that other robust topologies will also perform adequately.

7 Acknowledgements

The ideas presented in the introduction are central to the research of the Local Communication Networks Group at MIT's Laboratory for Information and Decision Systems. I would like to thank Professors Robert S. Kennedy and Pierre A. Humblet and the group as a whole for their in-
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sightful questions and comments during several of my presentations of this work.

References


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