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UTILIZATION OF SHORAN FOR MAPPING

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SUMMARY

Subject. This interim report covers an analysis of errors in the Shoran-controlled mapping photography obtained to date, and contains conclusions regarding sources of error and operational use.

Conclusions. The report concludes that: The principal sources of instrumental errors which affect internal map accuracy are timing non-linearity and pip alignment; the principal sources of instrumental errors which affect the positioning of the map as a whole are equipment delays and signal intensity; position errors introduced by errors in flying height determination are of least consequence when the Shoran aircraft operates at distances greater than 50 miles from the ground stations; the multiplex EZ curve method of establishing photo tilts is sufficiently accurate for use with the present Shoran equipment; Shoran-controlled photogrammetric maps can be compiled by the multiplex method with an internal map accuracy such that 90 percent of the features will have a relative accuracy of approximately ±104 feet; when no existing identifiable horizontal control is available, a "shift" of the entire map sheet as much as 200 feet with respect to the horizontal datum is possible; improved map accuracy is possible when Shoran instrumental errors are decreased and operational procedures are perfected; an improved method of recovering photo plumb points is required if random Shoran instrumental errors are decreased; the Shoran system, in its present state of development, is ready for use in positioning photography whenever the accuracy obtainable with this new tool satisfies mapping requirements.

Recommendations. It is recommended that:

1. Future studies of possible equipment refinement give prime consideration to errors caused by pip alignment, signal intensity, delays, and timing non-linearity.

2. Shoran photographic missions be planned so that the operating distance from the aircraft to either ground station is always in excess of 50 miles.

3. The Air Force be requested to continue the vigorous prosecution of current studies and tests of stabilized camera mounts and methods of recovering photo tilts.

4. The Shoran system be adopted immediately for use in controlling photography of areas where the obtainable accuracy will satisfy the mapping requirements.
I. INTRODUCTION

1. Subject. This interim report covers an analysis of errors in the Shoran-controlled mapping photography obtained to date, and contains conclusions regarding operational use and possible methods of gaining improved accuracy.

2. Authority. This project was authorized by a lst indorsement from the Chief of Engineers to the Engineer Board (Engineer Research and Development Laboratories) dated 13 August 1947, EMIE (12 Jun 47), subject: Request for Research Project, Utilization of Shoran for Mapping, MP 828. The project number was later changed to 8-35-05-001 to conform to the index and classification of research and development projects as established on 20 August 1947 by the Research and Development Division, General Staff, U. S. Army. The Research and Development Project Card is included in Appendix A.

3. Background and Previous Investigations. The use of Shoran to position aircraft during bombing operations immediately revealed sufficient accuracy to warrant an investigation into its possible use for mapping. In January 1944, preliminary tests to determine the accuracy of positioning the airplane at the instant of aerial camera exposures were made at Boca Raton, Florida. The results were encouraging, but extensive tests were postponed long enough to permit construction of a recorder that would eliminate the uncertainty and possible errors resulting from manual tabulation of the Shoran readings. At approximately the same time, a proposal was made to use the equipment for triangulation by measuring the side lengths of appropriate figures. Accordingly, new airborne tests were scheduled in three parts: Phase I to cover an investigation of the use of Shoran in controlling aerial photography; Phase II to investigate the "line crossing" method of triangulation; and Phase III to be an operational test of Shoran triangulation between Florida, Cuba, and the Bahamas. The 7th Geodetic Squadron, 311th Air Division, Reconnaissance (formerly 311th Reconnaissance Wing) was designated as the testing agency. It was jointly agreed that results of the photographic tests would be reported by the Corps of Engineers and that triangulation tests would be covered by Air Force reports. Photogrammetric tests by the Engineer Research and Development Laboratories were first authorized by a lst indorsement from the Chief of Engineers, dated 18 August 1945, subject: Service Project...
for Investigation of Shoran for Mapping (MPS 673) and later converted to research project MP 828 when it was found that a large portion of the investigation would be in the nature of research.

Photography for Phase I was accomplished during the period of August to October 1945 in the vicinity of Denver, Colorado. The newly designed photographic recorder contained distance counters duplicating those of the Shoran instrument and was operated from the aerial camera intervalometer to assure exposure synchronization. Other than for replacement of the bombing computer with the photographic recorder, the equipment used was that constructed for bombardment operations. As the tests progressed, it became increasingly evident that various "delay constants" furnished by the manufacturer were inadequate and were introducing rather large constant errors in all readings. For this reason, tests were halted to permit a complete readjustment and calibration of the equipment being used. A second group of tests were then run in the same area. Distinction between the groups is made by reference to photography accomplished with "calibrated" or with "uncalibrated" equipment.

Additional sources of error were revealed during subsequent operations in connection with Phases II and III, which have now been completed. The equipment was modified and operational procedures were developed to eliminate or reduce many of these errors during the line crossing work but, in many cases, similar procedures are not applicable to photographic operations. Additional photographic tests, scheduled for later this year, are being designed to establish refinements in the Shoran-photographic procedures and to accomplish equipment modifications that will further improve mapping accuracy.

When the first few photogrammetric tests had been completed it was apparent that the Shoran equipment, as then used, would be suitable for establishing photographic control in areas where geodetic control is unavailable, or where the time required for establishing geodetic control to gain increased accuracy is unwarranted. Because of the urgent need for an instrument of this type in carrying out the current U. S. Army mapping program, the first Engineer Research and Development Laboratories reports were concerned primarily with obtainable mapping accuracy and with the dissemination of information needed in applying this new kind of control to existing mapping methods. The following ERDL reports have been published to date:


An analysis of line crossing errors, together with a discussion of equipment modifications and procedures developed during Phases II and III are covered in the Air Proving Ground Command report of Project No. 8-46-7, Tactical Test of Shoran Control for Mapping and Charting, dated 15 October 1947.

4. Personnel. The work on this project was accomplished during the period August 1946 through June 1948 by personnel of the Shoran Section, under the direction of G. G. Lorenz, Chief, Photogrammetric Branch, Engineer Research and Development Laboratories.

II. INVESTIGATION

5. The Shoran System. Shoran measures the distance between the aircraft and each ground station in terms of time required for a pulse of electromagnetic energy to travel the round trip path. A complete technical description of the equipment is given in the Air Proving Ground Command report of Project No. 8-46-7. A few basic facts concerning the timing unit are summarized in the following paragraphs.

The airborne timing unit in its simplest form may be thought of as a stop watch for measuring the time required for round trip travel of the pulse. The signal pulses, produced in a pulse generator, are fed to both the transmitter and the cathode ray oscilloscope at a repetition rate that is controlled by a crystal oscillator. The frequency of the oscillator also controls the speed of the "bright spot" as it travels around the face of the cathode ray tube. This bright spot is merely a tiny point of light that moves in a circular path on the face of the oscilloscope. The persistence of vision, the persistence of the oscilloscope, and the rapid rate at which the spot travels, create the impression of a stationary fluorescent ring of light called the "sweep circle." Application of Shoran pulses to the oscilloscope momentarily deflects the bright spot from its circular path and forms the "pips" used in measuring pulse travel time.

The part of the pulse that is fed directly to the cathode ray tube causes a stationary pip, called the "marker pip," to appear in a fixed position on the sweep circle. The portion of the pulse that is fed to the transmitter, however, must travel from the aircraft to the ground station and back and, in addition, is further delayed in traveling through the circuits of the airborne and ground station receivers and transmitters. Therefore, this "echo"
signal will appear at a different place on the sweep circle. The distance between the pips is the distance that the cathode ray bright spot has traveled during the time which has elapsed between the formation of the marker pip and the arrival of the echo pip. A certain fraction of the distance indicated on the oscilloscope is caused by the "delays" in ground and airborne sets, but the greatest portion is caused by the time lost in traveling the round trip path to the ground station. Any errors resulting from time delays in the various components must be determined by calibration and subtracted from the total time before the true time interval can be determined.

The actual timing, then, depends upon the speed of the bright spot which is controlled, in turn, by the oscillator. In order to convert from time to distance readings it also becomes necessary to assume a constant wave velocity. In this conversion, Anderson's determination of the speed of light at sea level and under standard atmospheric conditions (186,219 miles per second) is used. Since Shoran measures the loop travel, it is convenient to use a velocity of 186,219 divided by 2 or 93,109.5 miles per second. Tuning the oscillator to a frequency of exactly 93,109.5 cycles per second drives the bright spot once around the sweep circle in the length of time required for the signal pulse to travel 1 loop mile, giving a sweep circle scale of 1 mile per revolution. Equipment design is such that this basic frequency can be divided by 10 and 100 to give outputs of 9,310.95 kilocycles or 931.095 cycles per second as well as the basic value of 93,109.5 kilocycles and, therefore, furnish sweep circle scales of 10 miles or 100 miles, in addition to 1 mile per revolution. The desired scale is chosen by means of a selector switch on the face of the timing unit.

The purpose of this elementary explanation is to furnish a clear concept of the possible errors in the timing system. In operation, this method is modified in an important way. Actually, instead of using the distance between the pulses on the cathode ray tube as an indication distance, a "timing advance system" is used to produce transmitted pulses sufficiently earlier than the corresponding marker pulses, so that a signal returning from its round trip arrives just in time to meet the corresponding pulse at the oscilloscope. The amount of time advance or headway given the outgoing pulse is variable, and is controlled by knobs attached to the front cover of the recording unit. These knobs, in turn, are connected to the mileage counters that have been calibrated to read the distance to the ground station. In order to obtain distances from two points, simultaneous indications from both ground stations are received by sending pulses from the airplane in alternate groups or trains, first to one, then to the other, and so on. Separate timing advance systems are used so that the two Shoran mileages can be set and read separately. The front cover of the recorder contains
two sets of control knobs to drive the two timing advance systems. A single cathode ray tube is used as an indicator. The marker pip appears at the top of the screen of this tube with the two received pipes arranged so that one projects toward, and the other away from, the sweep circle center. When the control knobs have been set so as to move the two received pipes into coincidence with the marker pip, the resulting readings on the two sets of mileage counters are the distances of the airplane from each ground station. Initial setting of the two distances is made by aligning the pipes with the sweep circle scale set first at the 100-mile scale, then at 10 miles, and finally at the 1-mile scale. Once this initial setting has been made, however, the scale setting is kept at 1 mile per revolution and, so long as pip alignment is maintained, the correct total distances continually appear on the mileage counters.

6. Test Procedures. The photography of Phase I covered two areas in Colorado for which existing horizontal and vertical control was available. An area just east of Denver was chosen because of the small amount of relief, and the other area was selected in the mountains southeast of Denver. The map included as Appendix B, Exhibit 1, indicates the location of the two areas and shows the position of the Shoran ground stations. Photography was accomplished with a T-5 aerial camera from approximately 20,000 feet above mean ground elevation. The Shoran recorder exposures were synchronized with those of the aerial camera and furnished the information necessary to compute the horizontal position of each exposure station.

a. The Plains Area. The area of little relief is referred to in this report as the Plains area. It extends from 104°37.30’ to 105°00.00’ West longitude and from 39°30.00’ to 40°00.00’ North latitude. The Shoran ground stations located at Cheyenne, Wyoming, and Imperial, Nebraska, were about 100 and 175 miles, respectively, from the area center. A wealth of existing horizontal and vertical control within the area furnished all the information necessary to make a complete check of the accuracy obtained. One complete mission containing six parallel flights of normal overlapping mapping photography controlled by uncalibrated Shoran equipment and two complete missions controlled by calibrated equipment were available for test.

b. The Mountain Area. This area extends from 105°045’ to 106°15’ West longitude and from 39°15’ to 39°43’ North latitude. The center of the area was approximately 135 and 65 miles, respectively, from controlling ground stations at Cheyenne, Wyoming, and Pikes Peak, Colorado. Only one complete photo coverage mission is available for test and this was made with the uncalibrated equipment. Existing control in the area is not dense and, during the photogrammetric tests, proved difficult to reconcile to a common datum. This may have been due to misidentification of points on
the photographs or to errors in the surveying over this very rugged terrain but, in either case, it detracts from the weight that can be placed on the results of tests with this Mountain area photography.

7. Possible Sources of Instrument Error. A study of the equipment during all phases of the tests has revealed the following possible sources of error:

a. Delay Errors. The original setting of the bombing equipment assumed that the combined delay time for any combination of a ground-airborne station pair was equal to a measured distance of 0.180 mile. For this reason, all airborne sets were adjusted to read 99.820 when receiving their own transmitted signal (i.e., measuring a zero distance). It was later found that the delays varied considerably and, therefore, had to be determined for each combination. Determination of this constant during the Phase I tests represents the major difference between photography with calibrated or with uncalibrated equipment as discussed in the present report. The delay of the ground stations is established by measuring lines of known length on a ground calibration range. The "ground station delay correction" is the difference between the true delay and the theoretical value of 0.180 mile. Actually, the ground station delay can be varied in steps of about 0.004 mile by means of a control knob on the face of the instrument. Any operational error in the setting of this control knob to the same "click position" for which the equipment was calibrated will introduce errors in the distance readings. It has also been found that the delay is subject to drift and that signal intensity at the time of calibration has an appreciable effect upon the determined constant. In fact, a full realization of the many complexities involved in the problem of ground station delay was not apparent until after a thorough investigation was made during later phases of the project. For this reason, it is believed that even the photography with calibrated equipment may still contain errors attributable to ground station delay. The airborne delay is established by tuning the airborne set so as to receive its own signal. The "zero correction" is the difference between 99.820 miles and the actual "zero observation." Tests during Phases II and III indicated that the observed zero does not remain constant, but tends to drift, possibly because of changes in pressure and temperature. During the line crossing work this error source was eliminated by making a zero observation at the start of each run. No similar correction was available for use in the photogrammetric tests.

b. Timing Non-linearity. This error source is caused by slight irregularities in the functioning of the 1-mile goniometers which form a constituent part of each timing advance system. The resulting error, when plotted against distance from the ground
station, is of a sinusoidal nature, repeating at 1-mile intervals. Equipment tests following Phase I indicate that this source may have introduced as much as 0.006 mile (±30 feet) error in the photographic tests. Subsequent instrument modification has reduced the error to about 0.003 mile, and further study may even produce a method of calibration that will permit corrective terms to be added in the computation step.

c. Oscillator Frequency. The actual timing of the pulse is controlled by the airborne oscillator. This oscillator, however, is subject to wide temperature and pressure variations and so, in operation, is tuned to a highly stable, temperature-controlled oscillator in the ground equipment which, in turn, has been carefully calibrated against WWV, the radio station of the National Bureau of Standards which broadcasts time signals. When tuned to a frequency of exactly 93,109.5 cycles per second, correct distance readings in miles are obtained. Small deviations from this nominal value introduce errors in the mileage counter readings but corrections are easily computed when the true frequency is known. Oscillator errors result from the following causes:

(1) Drift of the ground station oscillator during the time interval between calibration and operation.

(2) Drift of the airborne oscillator during the time interval between tuning to the ground station and operation.

(3) Error in calibrating the ground station oscillator.

d. Pip Alignment. The two echo pips must be aligned with the marker pip in order for correct readings to each ground station to be correctly registered in the Shoran recorder. Adjustment of each pip must be made by regulation of dual rate and displacement knobs placed one behind the other to permit operation of both with the same hand. Complete alignment, then, requires continuous manipulation of two sets of control knobs (one set for each ground station) during the mission and this is a tiring operation, especially at photographic altitudes. With present equipment no method is available for photographing the pips or otherwise determining the extent of error from this source. Discussions with Air Force personnel familiar with this phase of the work indicate that random distance errors of as much as 35 feet may be attributable to pip alignment.

e. Intensity Errors. The Shoran signals received at the photographic airplane are of varying intensity in different sections of the field of coverage. This is because of the fact
that one wave path travels directly from ground antenna to aircraft, while a second travels from the ground antenna to the ground and is then reflected to the airborne set. The difference in the lengths of the two paths is too small to form two echo pulses on the scope. The only effect is that the intensity of the signal is increased or decreased, depending on whether the two signals are in or out of phase. During the latter part of the triangulation tests it was found that signal intensity directly affected the distance readings. Investigations revealed that considerable improvement in the similarity of repeated measurements resulted from reading the intensity at the time of making each line crossing and then applying a distance correction as determined from calibration data gathered on lines of known lengths. The effect of these signal intensity errors when plotted against distance from the ground station will be of a sinusoidal nature but will be repeated at large intervals. In fact, in controlling photography of a 30-minute quadrangle where the range of readings from either ground station will not exceed about 45 miles, the error may well appear as one increasing or decreasing with distance from the ground station, or possibly as a constant error, independent of distance. No provision was made for considering this source of error in the photographic tests.

8. Sources of Propagation Errors. As previously noted, the Shoran instrument converts from time measurement to distance readings by assuming a constant velocity for the pulse. However, the velocity of electromagnetic waves actually varies over a range of about 55 miles per second, from the minimum of 186,219 miles per second, at a barometric pressure of 29.92 inches of mercury to a maximum of 186,274 miles per second in a vacuum. The variation in propagation speed along the Shoran ray path is a function of the dielectric constant which can be computed from measurements of humidity, temperature, and pressure. For maximum accuracy it becomes necessary to fly a weather observing airplane along the approximate ray path to take meteorological soundings at the time the Shoran measurements are being made. From this meteorological data the dielectric constant is computed at regular intervals, and the total "velocity correction" determined by a rather involved numerical integration process. Errors of the method are dependent upon the accuracy of reading the weather instruments and upon the accuracy to which the actual height of the Shoran path is known. Many phenomena concerning electromagnetic propagation are not as yet fully understood and, therefore, may introduce errors in the determination of path height. This, in turn, will introduce small errors in applying the differential velocity corrections along the path.

In the photographic tests, velocity corrections based on an "average atmosphere" were used. This average atmosphere was developed by using the N.A.C.A. standard dry atmosphere modified to
Fig. 1. Velocity correction curves for various atmospheric conditions in the Caribbean area.
include average moisture conditions at about 40 degrees latitude in the United States. This atmosphere is often referred to in Shoran work as the "standard N.A.C.A. moist atmosphere." Use of an empirical atmosphere, such as this, greatly simplifies velocity correction determination since it permits the construction of tables or nomograms. Errors in the simplified method are dependent upon the difference between the "average atmosphere" and the true atmospheric condition at the time of photography. Development of the velocity correction procedures is covered in an unpublished report by D. A. Rice of the U. S. Coast and Geodetic Survey and is discussed at length in the previously mentioned Air Proving Ground Command report.

In order to check the possible effects on Shoran measurements, the 311th Air Division, Reconnaissance gathered considerable weather data in the Caribbean area prior to their line crossing tests. A total of five more or less characteristic types of atmospheric conditions were encountered. Their effect on Shoran observations from a flying height of 20,000 feet is shown in Fig. 1. This figure also includes curves showing the corresponding corrections for the standard N.A.C.A. dry atmosphere and the standard N.A.C.A. moist atmosphere.

It is probable that the range of weather conditions shown in Fig. 1 represents about the maximum that could be expected at any time or place in the United States. This means that the greatest velocity correction error that can result from the use of tables based on standard N.A.C.A. moist atmosphere will amount to no more than 1 part in 20,000 where the operating altitude is 15,000 feet or higher. Of course, the method of obtaining weather data at the time of photography will greatly reduce this error.

9. Possible Errors in Use of the Reduction Formula. Once the Shoran reading has been corrected for instrument and velocity errors it must be reduced from distance along the ray path to equivalent distance on the sea level surface of the earth. This geodetic distance is given by the following formula:

\[ M = 2R_a \arcsin \sqrt{\frac{4r^2 \sin^2 \frac{S_1}{2r} - (H_1 - K_1)^2}{4(R_a + H_1)(R_a + K_1)}} \]

where:

- \( M \) = ground (map) distance in miles.
- \( R_a \) = mean radius in miles of earth along line being measured.
- \( r \) = mean radius in miles of Shoran ray path.
Shoran reading in miles after correction for instrument and velocity errors.

$H_1$ = height in miles of airborne set above sea level.

$K_1$ = height in miles of ground station antenna above sea level.

The most practical method of solving the above equation is obtained by making several assumptions, expanding the basic equation to a more usable form, and then modifying the resulting values by use of charts or tables that have been computed to correct for the assumptions. The equation can then be put in the following form:

$$M = S - A$$

$$A = 2.3920 \cdot \frac{S(H+K)}{10^8} + 1.7935 \cdot \frac{(H-K)^2}{S} - 0.24848 \cdot \frac{S^3}{10^8}$$

$$+ 1.6083 \cdot \frac{(H-K)^4}{S^3} + V-C + VII - VIII - IX + X - XI$$

where:

$M$ = map distance in miles.

$S$ = uncorrected Shoran distance reading in miles.

$A$ = total correction to Shoran reading in miles.

$H$ = height in feet of aircraft.

$K$ = height in feet of ground station antenna.

$V$ = Shoran velocity correction in miles.

$C$ = total electronic corrections in miles.

$VII$, $VIII$, $IX$, $X$, $XI$ = corrections for the various assumptions (values from tables or charts).

A complete discussion of the formula is given in the Air Proving Ground Command report. Only one of the corrections for the assumptions is significant in photogrammetric mapping. The others were not used in the Phase I investigation.

Though no significant error is introduced through the geometry of the reduction formula, it can be seen that errors in the determination of flying height will introduce errors in the
reduced Shoran distances. The magnitude of these errors can be determined by taking the differential of the correction formula, as follows:

\[ da = \left[ \frac{2.3920}{10^5} \cdot S + \frac{3.5870}{10^5} \cdot \frac{(H-K)}{S} + \frac{6.4332}{10^{-6}} \cdot \frac{(H-K)^3}{S^3} \right] dh \]

By substituting small values for \( dh \) (errors in flying height), a table of errors can be made for any particular value of \( H-K \). Table I shows the distance errors resulting from various errors in flying height, with the aircraft at 20,000 feet and the ground station at sea level (\( H-K = 20,000 \) feet).

**Table I. Errors in Distance Resulting from Errors in the Determination of True Flying Height When \( H-K = 20,000 \) Feet**

<table>
<thead>
<tr>
<th>Shoran Distance (ft)</th>
<th>Error in Shoran Distance (ft)</th>
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<tbody>
<tr>
<td>Flying S (mi)</td>
<td>Error Ht</td>
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<tr>
<td>50-ft</td>
<td>49</td>
</tr>
<tr>
<td>100-ft</td>
<td>20</td>
</tr>
<tr>
<td>150-ft</td>
<td>13</td>
</tr>
<tr>
<td>200-ft</td>
<td>8</td>
</tr>
<tr>
<td>250-ft</td>
<td>8</td>
</tr>
<tr>
<td>300-ft</td>
<td>3</td>
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In the photogrammetric application where one Shoran position corresponding to each photograph is needed, the computation work is materially reduced if a constant flying height for large blocks of photographs can be assumed. With the Phase I photography each mission was computed separately using a constant flying height equivalent to the mean of all photographs of the mission. This procedure introduces an error in each position if the mean flying height has been determined incorrectly and an additional error in photographs in which the exposure heights deviate from the mean. The magnitude of the distance error produced by an error in the mean flying height depends upon the accuracy with which this true flying height can be determined. Errors caused by deviations of individual exposures from the mean could be reduced by taking into account the variations indicated by the barometric altimeter reading appearing on each Shoran recording photograph. With good photography, however,
the value of this refinement is questionable since the resulting error will be small in comparison with other equipment errors.

10. Determination of True Flying Height. The following methods of determining true flying height for use in the reduction equation are under consideration:

a. Barometric altimeter
b. Radio altimeter
c. Photogrammetric

The face of a barometric altimeter indicating height to the nearest 20 feet appears in each recorder exposure. Altimeters of this type, however, are simply aneroid barometers which have been calibrated to read correct altitude only under more or less average atmospheric conditions. It is easily possible for the actual instrument readings to be in error as much as 1,000 to 2,000 feet with flying heights of 20,000 feet and over. If, however, complete meteorological data are available for the time and place of the photographic mission, corrections to greatly improve the readings can be computed. Differences in flying height between successive exposures as indicated by the altimeter readings are considerably more reliable.

Barometric altimeter flying heights were used in connection with Phase I photography. Corrections to the readings obtained during the missions with calibrated equipment were supplied by the Air Weather Service and computed from normal radiosonde data gathered daily by the U. S. Weather Bureau. Briefly, the method used consisted of computing the thickness of air layers between successive pressure levels as indicated by radiosonde readings of temperature, pressure, and humidity. By adding these thicknesses together, the altitude corresponding to any pressure can be determined. The mean altimeter reading of all exposures for the mission in question was then converted to its basic value of atmospheric pressure and the corresponding altitude was taken from the meteorological computations for the particular day. As normal radiosonde information is gathered only at 12-hour intervals and at a few observation stations scattered throughout the United States, it becomes necessary to prepare "contour maps" showing the elevation of the "pressure level" in question and covering the area in which the photography was accomplished. Interpolation between one map prepared from data obtained at the regular observation hour just preceding the mission and another for the regular hour following, permits flying height determination for the actual place and hour desired.
During the line crossing work of Phase III, barometric altimeter readings were corrected by gathering meteorological data with a separate airplane and at the actual time of the mission. The plane spiraled up or down between near sea level and the point of line crossing and, since the accuracy of weather instruments used exceeded that possible from radiosonde, it is believed that much improved values of flying height were obtained. In fact, the results were estimated to be accurate to within ±30 feet.

To test the accuracy of Phase I barometric altimeter values, the flying height of photography from two missions was carefully determined by multiplex methods. Models were oriented to ground control and all corrections, such as principal distance, slope of the projector Y slide, and film shrinkage were considered. It is estimated that values are accurate to within 1/500th to 1/1,000th of the flying height. The mean value for each mission was then compared with the corresponding mean flying heights as determined by the Air Weather Service. Table II shows the results.

Table II. Comparison of Corrected Altimeter Readings with True Flying Heights as Established by Multiplex

<table>
<thead>
<tr>
<th>Mission No.</th>
<th>Exposures Analyzed</th>
<th>Mean Flying Height (ft), Multiplex</th>
<th>Mean Flying Height (ft), Altimeter</th>
<th>Difference (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>44</td>
<td>25,503</td>
<td>25,377</td>
<td>-126</td>
</tr>
<tr>
<td>2</td>
<td>76</td>
<td>26,680</td>
<td>25,608</td>
<td>-72</td>
</tr>
</tbody>
</table>

Table III shows the error in flying height difference indicated by the same test.

Table III. Accuracy of the Barometric Altimeter in Recording Flying Height Differences

<table>
<thead>
<tr>
<th>Mission No.</th>
<th>Standard Deviation (ft) Within Which About 66% of Points Will Fall</th>
<th>2x Standard Deviation (ft) Within Which About 95% of Points Will Fail</th>
<th>3x Standard Deviation (ft) Within Which About 99 to 100% of Points Will Fall</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>±51</td>
<td>±102</td>
<td>±153</td>
</tr>
<tr>
<td>2</td>
<td>±42</td>
<td>±84</td>
<td>±126</td>
</tr>
</tbody>
</table>

These results indicate that barometric altimeter readings can probably be expected to furnish mean flying heights accurate to within
±75 to 150 feet when considerable meteorological data are available for use in applying the necessary corrections. It should be mentioned, however, that altimeter accuracy is dependent upon the method of installation of the static line, the roughness of the air, the type of aircraft, and the state of adjustment and calibration of the altimeter. The results also show that the altimeter is capable of indicating flying height differences to an accuracy of only about ±100 feet in 95 percent of the cases, and may have occasional errors of as much as 150 feet. Other more complete altimeter tests discussed in ERDL Report 973, Vertical Control for Aeronautical Charting and Mapping, indicate somewhat greater accuracy in the determination of flying height differences but still show possible errors of as much as 75 feet. For this reason, the assumption of a constant flying height in the position computations of large blocks of photographs is perfectly reasonable so long as altimeter readings indicate differences of no more than about 100 feet from the mean.

An investigation of the actual deviation from mean flying height of exposures obtained on the above-mentioned photographic missions gives the following results, as set forth in Table IV:

Table IV. Actual Deviation from Mean Flying Height of Exposures Obtained on Photographic Missions

<table>
<thead>
<tr>
<th>Mission No.</th>
<th>Standard Deviation (ft)</th>
<th>2x Standard Deviation (ft)</th>
<th>3x Standard Deviation (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Within Which</td>
<td>Within Which</td>
<td>Within Which</td>
</tr>
<tr>
<td></td>
<td>About 66% of</td>
<td>About 95% of</td>
<td>99 to 100% of</td>
</tr>
<tr>
<td></td>
<td>Exposures Can</td>
<td>Exposures Can</td>
<td>Exposures Can</td>
</tr>
<tr>
<td></td>
<td>be Expected to Fall</td>
<td>be Expected to Fall</td>
<td>be Expected to Fall</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Fall</td>
<td>Fall</td>
</tr>
<tr>
<td>1</td>
<td>±52</td>
<td>±104</td>
<td>±156</td>
</tr>
<tr>
<td>2</td>
<td>±38</td>
<td>±76</td>
<td>±114</td>
</tr>
</tbody>
</table>

A comparison of these values with those of altimeter accuracy shows that the pilot’s ability to hold a constant elevation was about equal to the ability of the Shoran altimeter to record the differences. This type of flying cannot be expected on operational missions over enemy terrain, but does show how careful flying can decrease the computational time required for other types of Shoran mapping projects.

The radio altimeter measures distance from airplane to ground by means of radio pulse timing. It showed considerable promise in the Phase III work since many of the line crossings were made over sea water and, therefore, permitted a direct reading of elevation above sea level. It would not be of much value over land
of unknown elevation since Shoran computations require a knowledge of flying height above a sea level datum. One other disadvantage of the instrument is that it measures distance from the aircraft to the nearest point covered by a beam of several degrees. Over rough terrain, then, the elevation above some high peak is likely to be measured instead of the elevation above the point directly beneath the aircraft. A narrow beam radio altimeter, which is now under development, will at least partially overcome this latter objection. The instrument, however, will not be suitable for measuring flying height differences since undulations of the ground over which the plane flies will not be known. Where the plane can fly over large flat areas, such as large bodies of water, of known elevation, the recordings will probably furnish reliable information. One suggested application has been to use the instrument in conjunction with the barometric altimeter. In this proposed method both altimeters would be recorded as the plane flies over an area of known elevation. The radio altimeter reading plus the elevation of the datum area would give the elevation above sea level. Thereafter, the barometric altimeter would furnish information concerning flying height difference. Upon completion of the mission the plane would again fly over the datum area to permit readings of both instruments and thereby provide an adjustment of the barometric values for temporal variation of the pressure level. Use of the radio altimeter in this manner would avoid the necessity of applying meteorological corrections to the barometric readings. Though this "radio altimeter carry" method may have some practical application in specific cases, no tests that would permit an evaluation of the results have been made, as yet.

Photogrammetric procedures may also be used to determine the height of the exposure stations above the ground. As in the case of the radio altimeter, however, a reference point of known elevation must appear in the photography if flying height above sea level is to be determined. Multiplex methods present the simplest solution to this problem. When multiplex models have been correctly oriented they represent an accurate, true-to-scale miniature of conditions at the instant of the aerial exposures. Flying height can be determined by measuring the vertical distance from any point in the model to the emergent node of the projector lens. The scale of this measurement is the same as the horizontal scale of the model. Small corrections for film shrinkage, etc., are added where utmost accuracy is desired. Since the Shoran positions are available to furnish horizontal scale, the photogrammetric method seems to offer the easiest and simplest solution to the height determination problem, provided one or two identifiable points of known elevation are available within the photographed area. The proposed procedure would require computation of a few Shoran positions, first using a flying height as near correct as can be estimated from the barometric altimeter readings. Approximate horizontalization can be made by the
Fig. 2. Effect of station angle variation on Shoran position accuracy.

BZ curve method. Three or four multiplex models would then be oriented to the preliminary positions and a more reliable value for flying height determined. Though the process could then be repeated to obtain a more accurate value, this second determination probably would not be necessary unless the original estimate of flying height was greatly in error. As previously mentioned, multiplex methods, together with accurate ground control and the use of careful laboratory methods will probably furnish values accurate to within 1/500th to 1/1,000th of the flying height. When less accurate Shoran positions are used, the accuracy of these values will probably be about 1/200th to 1/400th of the flying height.

11. Sources of Error in Applying Shoran to Position Determination. The present paragraph deals with errors in the positioning of individual photographs. As evidenced in later discussion, some of this error can be eliminated during mapping operations by scaling several photographs to the best average fit of all points involved. The accuracy of the Shoran measurement from the aircraft to either ground station is a function of the errors just discussed. The accuracy of position determination is further dependent upon the station angle, the accuracy with which the effective point of position can be projected and identified on an aerial photograph, and the accuracy with which the distance between the ground stations is known.

a. In Shoran work, the angle subtended at the airplane by the ground stations is called the station angle. The effect of station angle variation on position accuracy is illustrated in Fig. 2.
In each case point 1 is the true position and point 2 represents the apparent location as determined by Shoran measurements, all of which are assumed to be in error by the same amount, E. If both distance measurements are either too long or too short, position error is a maximum when station angles are large and a minimum when the angles are small. Where distance measurements contain errors of opposite sign, position errors are maximum when station angles are small. Since errors are just as likely to be plus as they are minus, the strongest position determinations will usually occur with station angles of 90 degrees and the greater the departure therefrom, the greater is the region of uncertainty. Station angles of from 60 to 120 degrees are considered acceptable for mapping photography. Under these conditions the position error may vary from 1.4E with 90 degree station angles to 2E with station angles of 60 or 120 degrees. Here E represents an error of equal magnitude in the measurement of distances to each ground station.

b. The Shoran position represents the point directly beneath the aircraft. On an aerial photograph the corresponding position is the plumb point, which can be determined only if the tilt of the aerial camera can be recovered. Since normal "vertical" photographs may contain tilts up to 3 degrees or more, use of the principal points, which assumes a zero tilt, will introduce considerable error. Table V shows the plumb point displacement introduced by errors in the recovery of photographic tilts from various flying heights.

<table>
<thead>
<tr>
<th>Tilt</th>
<th>Error in Plumb Point Position (ft)</th>
<th>5,000-ft</th>
<th>10,000-ft</th>
<th>20,000-ft</th>
<th>30,000-ft</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Error</td>
<td>Flying</td>
<td>Flying</td>
<td>Flying</td>
<td>Flying</td>
</tr>
<tr>
<td></td>
<td>Height</td>
<td>Height</td>
<td>Height</td>
<td>Height</td>
<td>Height</td>
</tr>
<tr>
<td></td>
<td>0° 05'</td>
<td>7</td>
<td>15</td>
<td>29</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td>0° 10'</td>
<td>15</td>
<td>29</td>
<td>58</td>
<td>87</td>
</tr>
<tr>
<td></td>
<td>0° 30'</td>
<td>44</td>
<td>87</td>
<td>175</td>
<td>262</td>
</tr>
<tr>
<td></td>
<td>1° 00'</td>
<td>87</td>
<td>174</td>
<td>349</td>
<td>524</td>
</tr>
<tr>
<td></td>
<td>2° 00'</td>
<td>175</td>
<td>349</td>
<td>698</td>
<td>1048</td>
</tr>
<tr>
<td></td>
<td>3° 00'</td>
<td>262</td>
<td>524</td>
<td>1048</td>
<td>1572</td>
</tr>
</tbody>
</table>

A study of this table indicates the large emphasis that must be placed on the tilt determination problem. Unless some method of at least approximate plumb point location is used, the error introduced by tilt may greatly exceed the errors attributable to the equipment. Though work to develop both a stabilized camera mount and a tilt indicator is now in progress, the photogrammetrist must presently look to other methods for establishing plumb points.
Two types of camera installations may be encountered. In most cases, a photographer using a level bubble for reference will be available to level the camera, as near as possible, just prior to each exposure. It is to be expected that, where a large number of exposures are made in this manner, the deviations of the camera will be first one way and then another so that the mean tilt for the mission will be zero. Of course, if the level bubble has not been oriented correctly with respect to the camera axis, the mean tilt of all exposures on one mission will probably be equal to the amount of this missetting.

Occasionally, a fixed camera mount will be used. In this case, the camera is adjusted before take-off so that it will point as near vertical as possible when the plane reaches the predetermined flying height. Since this presetting of the camera for attitude of the aircraft at flying height is dependent upon the loading of the airplane and many other variables, it often contains considerable error. The mean tilt of a group of photographs can then be expected to equal any error in setting the camera. Tilt of the individual exposures will be controlled by this mean error plus the ability of the pilot to maintain a "level" airplane. Position errors introduced by missetting for attitude (or for level bubble setting) will be constant in amount and direction for any one flight line but in opposite directions between flights since the airplane will normally be headed in opposite directions on successive flights.

c. The error in position introduced by an error in the distance between ground stations can best be understood by reference to Fig. 3. Here AB is the base and P represents the position of the photographic airplane. Location is established by solving for the apex point, P, in terms of the base length, W, and the two Shoran distances, M1 and M2. In the mapping application, positions are computed for use with a rectangular grid based on an X axis through the ground stations and with the origin of coordinates at A (the Westernmost ground station). Computation of coordinates is a simple operation and the use of a rectangular system greatly simplifies point plotting. It has also been found that discrepancies between maps compiled on the Shoran grid and on other projections, such as the Polyconic, are negligible over the relatively small area covered from two ground stations.

Coordinates of point P (Fig. 3), are found by the following formulas:

\[
X = \frac{M_1 - M_2^2 + W^2}{2 W} \\
Y = \sqrt{M_1^2 - x^2}
\]
The effect of small errors in base length may be found by taking the differential of both formulas and solving for the resultant, as follows:

\[ dx = \frac{w^2 - M_1^2 + M_2^2}{2w^2} \cdot dw \]

\[ dy = \frac{-x}{\sqrt{M_1^2 - x^2}} \cdot dx \]

Position error = \( dP = \sqrt{dx^2 + dy^2} \)

The pattern of these errors is illustrated in Fig. 4. The error "contours" show lines of constant factors by which the base length error must be multiplied in order to find position error. Direction of the errors is always normal to a line from station A. The arrows in the figure show the direction for errors caused by a base that has been measured too long. For a base line measurement that is too short, directions will be reversed.

Where possible, Shoran ground stations will normally be placed at, or tied by, surveys to existing triangulation or traverse stations, and their separation determined by making an inverse computation between the two positions. The accuracy of the resulting distance, then, will depend upon the order of accuracy of the control networks within which the ground stations are located. Since 150 miles is approximately the correct ground station separation to give maximum area coverage for 20,000-foot photography, Table VI was computed as follows:

<table>
<thead>
<tr>
<th>Order of Survey</th>
<th>Probable Distance Errors</th>
<th>Probable Error (ft) in 150 Mile Line</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>1/75,000</td>
<td>10</td>
</tr>
<tr>
<td>Second</td>
<td>1/25,000</td>
<td>30</td>
</tr>
<tr>
<td>Third</td>
<td>1/5000 to 1/10,000</td>
<td>150 to 75</td>
</tr>
</tbody>
</table>

12. Determination of Photo Tilts. The establishment of photo tilts and subsequent recovery of plumb points is a straightforward operation in areas where considerable vertical control is available. It is probable, however, that one of the major uses of the Shoran equipment will be for mapping enemy terrain or in other areas with little or no control, either vertical or horizontal. For this
Fig. 3. Method of computing rectangular coordinates for use with Shoran grid.

\[
x = \frac{W^2 - W_1^2 + W_2^2}{2W}
\]
\[
y = \sqrt{W^2 - x^2}
\]

Fig. 4. Position errors introduced by errors in length of Shoran base.
reason, a method of recovering photo tilts by multiplex methods and without the use of ground control was investigated. A so-called "BZ curve method" that was tried, requires the use of barometric altimeter readings coincident with the aerial exposures and the use of a special flight plan similar to that shown in Fig. 5. The application of the method to photogrammetric mapping is covered in ERDL Report 987, Second Interim Report, Application of Shoran to Mapping. The present report is only concerned with the possible accuracy of recovering tilts so that an evaluation of errors can be made.

The true camera tilt of the 59 exposures of one mission were carefully determined by measuring the multiplex projector tip and tilt with models set to plentiful ground control. Past tests have indicated the method to be accurate to within $\pm 0.05$ minutes of arc. The values were then compared with tip and tilt as established in orienting the same photography by the BZ curve method. The regular parallel flights (flights 1-6 of Fig. 5) were set up in units of 6 or 7 models, leveled in the "tip" (X) direction by use of BZ curves, and leveled in the "tilt" (Y) direction using the lines of relative elevations established with the cross flight photography. Table VII shows the indicated accuracy of this BZ curve method.

Table VII. Accuracy of Establishing Photo Tilts Using Cross Flights and the BZ Curve Method

<table>
<thead>
<tr>
<th>Mean</th>
<th>2x Standard Deviation (Accuracy within Which Tilt on 2/3 of Photographs Can Be Recovered)</th>
<th>3x Standard Deviation (Accuracy within Which Tilt on 99% of Photographs Can Be Recovered)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tip</td>
<td>$\pm 0^\circ 04'$</td>
<td>$\pm 0^\circ 08'$</td>
</tr>
<tr>
<td>Tilt</td>
<td>$\pm 0^\circ 04'$</td>
<td>$\pm 0^\circ 08'$</td>
</tr>
<tr>
<td>Resultant</td>
<td>$\pm 0^\circ 06'$</td>
<td>$\pm 0^\circ 11'$</td>
</tr>
</tbody>
</table>

A comparison of these values with Table V, paragraph 11, shows the effect tilt errors of this magnitude will have on Shoran-controlled maps.

As an indication of the amount of tilt that can be expected, even on comparatively easy missions accomplished under peacetime conditions, an analysis was made of the actual photo tilts on one Plains area mission of 59 exposures. Table VIII indicates the results. A comparison with Table VII shows the great
1. 2, 3, 4, 5, 6 Represent % of regular Shoran controlled mapping flights.

7, 8, 9 Represent % of auxiliary flights used in areas where existing vertical control is insufficient to permit absolute orientation in normal manner.

Fig. 5. Flight plan for use in Shoran multiplex mapping of areas with little or no vertical control.

Improvement in projector orientation that can be expected through use of the BZ curve method.

Table VIII. Deviation from Vertical of Peacetime Photography As Indicated from One Plains Area Mission

<table>
<thead>
<tr>
<th>Mean</th>
<th>2x Standard Deviation (Range from Vertical within Which Which Tilt on 2/3 of Photos Will Fall)</th>
<th>5x Standard Deviation (Range from Vertical within Which Tilt on 99% of Photos Will Fall)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tip</td>
<td>±0° 06'</td>
<td>±0° 32'</td>
</tr>
<tr>
<td>Tilt</td>
<td>-0° 09'</td>
<td>±0° 52'</td>
</tr>
<tr>
<td>Resultant</td>
<td>0° 09'</td>
<td>±1° 05'</td>
</tr>
</tbody>
</table>
20,000 feet above mean ground elevation will consist of six parallel north-south flights of about 16 exposures each.

a. The individual Shoran measurements from one exposure to either ground station are subject to four different types of errors that are distinguished as follows:

(1) Constant errors.
(2) Systematic errors.
(3) Cyclical errors.
(4) Random errors.

A constant error is one that has the same effect on all observations in the same series of measurements. For instance, an error in determination of ground station delay will introduce errors of equal magnitude and direction in all readings to the ground station in question.

A systematic error is one in which the algebraic sign and the magnitude bear a fixed relation to some condition. The simplest type is illustrated by an error that is proportional to the distance measured, such as the effect of an error in determination of the oscillator frequency. In this case, an error of 1 cycle per second would produce a distance error of about 1 part in 93,000. Another example of systematic errors is the velocity errors introduced by variations in meteorological conditions between missions. In this case, the error can be reduced considerably by computing corrections based on the measurement of actual atmospheric conditions at the time of the mission.

A cyclical error is a specific type of systematic error, but is listed separately because of the nature of its effect on Shoran photography. This type of error is illustrated by the timing non-linearity error which is of a sinusoidal nature with respect to distance and completes 1 cycle per mile.

Random errors are not constant from observation to observation and are just as likely to be positive as negative. Errors of pip alignment are of this class. In general, they will follow the exponential law of errors as represented by the normal probability curve.

b. Of prime importance here, is the way in which the various types of distance errors from both ground stations combine to produce position errors in the individual exposures of a mission. The effect of constant distance errors is illustrated in Fig. 6.
Points A, B, and C represent the true position of three photographs having acceptable station angle characteristics with respect to the ground stations. Since each distance reading from Imperial is too long by a constant amount, and each distance from Cheyenne is too short, the Shoran positions will be plotted at A', B', and C'. The magnitude and direction of the position error, then, not only depends upon the magnitude and direction of the distance errors, but also upon the location of the position point in relation to the ground stations. Since A and B are relatively close together, however, the position error is almost the same at both points. In fact, the difference in azimuth and length of a line from A to B and one from A' to B' is smaller than the error likely to be introduced in establishing the intermediate scale by multiplex or slotted templet methods. In a comparatively small area, therefore, constant errors in each of the distances will produce Shoran positions which, for all practical purposes, are in error by a constant amount both in magnitude and in direction. The resulting map will have good internal accuracy as far as relative location of the features is concerned, but the entire area will be shifted with respect to the parallels and meridians.

Systematic distance errors will affect the positions in a manner somewhat similar to constant errors. Since internal scale accuracy needs to be no better than about 1 part in 3,000, even for multiplex mapping, any systematic distance error of less
than about 1 part in 5,000 will be of little consequence as far as relative accuracy between points within the area to be mapped. However, if the distance from the ground stations is large, even a small systematic error may cause a noticeable shift of the entire area with respect to the parallels and meridians. For example, an error of 1 part in 25,000 in the measurement of distances from the test area to station Imperial (see Fig. 6) would cause the map to be displaced approximately 37 feet.

Random errors will vary from point to point and may, therefore, act in any direction. Since the map is controlled by scaling to Shoran positions corresponding to the 50 to 100 photographs of the area, random errors will be evidenced as localized scale and azimuth errors within the map sheet. There will be little, if any, displacement of the entire area since the net effect of random errors tends to balance out when a large number of points are used. As in the case of all errors, the position error is dependent upon the magnitude and direction (plus or minus) of the error in distance to both ground stations and upon the station angle. The magnitude of this error is given by the formula:

\[ E = \text{csc} \alpha \sqrt{E_1^2 + E_2^2 - 2E_1E_2 \cos \alpha} \]

where

- \( E \) = error of position
- \( E_1 \) and \( E_2 \) = distance errors in measurement of lines from ground stations A and B
- \( \alpha \) = station angle

The direction of the position error is further dependent upon the location of the Shoran point with respect to the ground stations.

A cyclical error such as the timing non-linearity error which repeats every time the distance from one ground station varies by 1 mile will show up in the finished map in much the same manner as a random error. The cyclical error introduced by signal intensity is dependent upon the heights of aircraft and ground station, and the height and electrical properties of the ground at the point where the auxiliary reflection occurs. The cycle repeats at such large distances that it is possible the resultant effect on a mission will be equivalent to a simple systematic or even a constant error.

c. The various known errors of the Shoran system and their possible effect on photogrammetric maps are listed below:
Possible Sources of Error

Instrument Error
- Delay errors
- Timing non-linearity
- Oscillator frequency
- Pip alignment
- Signal intensity

Type of Error
- Constant
- Cyclical
- Systematic
- Random

Propagation Errors
- Errors in measuring and applying meteorological data (velocity correction)
- Use of an "average" atmosphere in all reduction computations

Type of Error
- Systematic
- Systematic

Errors of Flying Height
- Determination of mean flying height
- Determination of flying height difference

Type of Error
- Systematic
- Random

Geometric Errors in Establishing Position
- Errors in base line determination

Type of Error
- Systematic

Errors of Tilt Determination
- Errors in camera level bubble or in setting up of fixed mount camera
- Errors in leveling camera (or airplane) for each individual exposure; or errors in recovery of plumb points by photogrammetric means

Type of Error
- Random
- Random

Though an error in the camera level bubble will be constant for all photographs of one flight line, it is shown as a random error in the above chart since its effect in the mapping of an area with several flights will be only that of internal scale distortion.

This tabulation lists only the errors that have been revealed through a study of the equipment and an analysis of the computational and photogrammetric procedures. Other deviations in the finished map may possibly result from equipment errors of, as yet, undiscovered origins and from such human errors as those of interpolation of distance readings and of drafting. Undiscovered errors may prove to be of a constant, systematic, cyclical, or random nature. Human errors will be present in any type of mapping and will usually be of a random nature.


a. Eight missions containing a total of 390 exposures were used in the analysis of accuracy to be expected from the Shoran
Table IX. Accuracy of Shoran Positions and Individual Distance Measurements As Established from Phase I Photography

<table>
<thead>
<tr>
<th>Test Number</th>
<th>Equipment Used</th>
<th>Type of Mission</th>
<th>Number of Points Analyzed</th>
<th>Position Error Mean Error</th>
<th>Error of Distance Measurements Mean Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td>Y</td>
</tr>
<tr>
<td>1</td>
<td>Uncalibrated</td>
<td>Area Coverage Contact Navigation Plains Area</td>
<td>91</td>
<td>-3'</td>
<td>+113'</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Uncalibrated</td>
<td>Perimeter Photog. Plains Area</td>
<td>55</td>
<td>+92'</td>
<td>+115'</td>
</tr>
<tr>
<td>3</td>
<td>Uncalibrated</td>
<td>Area Coverage Precom. Coor. Navigation Mountain Area</td>
<td>60</td>
<td>-64'</td>
<td>+16'</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Calibrated</td>
<td>Area Coverage Contact Navigation Plains Area</td>
<td>41</td>
<td>-18'</td>
<td>+28'</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Calibrated</td>
<td>Area Coverage Arc Navigation Plains Area</td>
<td>43</td>
<td>-23'</td>
<td>-25'</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Calibrated</td>
<td>Area Coverage Pre. Coor. Navigation Plains Area</td>
<td>61</td>
<td>-16'</td>
<td>+35'</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Calibrated</td>
<td>Control Point Photography Plains Area</td>
<td>23</td>
<td>-46'</td>
<td>-3'</td>
</tr>
<tr>
<td>8</td>
<td>Calibrated</td>
<td>Control Point Photography Plains Area</td>
<td>16</td>
<td>-26'</td>
<td>+33'</td>
</tr>
</tbody>
</table>

Mean of all Points

\[
\text{Mean} = 74' \quad 49'
\]

* S_1 = Distances from Cheyenne; S_2 = Distances from Imperial

** In the Mountains area tests S_1 = Distances from Pikes Peak; S_2 = Distances from Cheyenne.
equipment. True plumb point positions, established by multiplex methods, were used as a standard for comparison with the computed Shoran positions. In the Plains area it was possible to fix these standard positions by scaling the photography to known horizontal positions in small units of not over four multiplex models and then leveling each model individually to at least four well-distributed elevation points. Control was not this plentiful in the Mountain area, though even here, it is believed that the positions established are sufficiently accurate to permit a fair evaluation. Table IX shows the result of the tests. Also shown is a tabulation of the accuracy of the individual distance measurements as obtained by comparing the reduced Shoran distances to each ground station with the corresponding distances as computed from the true plumb point positions. Since these reference positions were scaled from multiplex plotting sheets prepared at a scale of 1:17,000, the tabulated results cannot be considered significant to any closer than about five to eight feet. The improvement in mean position resulting from equipment calibration is readily apparent in this tabulation. A study of the table also shows that the random errors, as indicated by the values of standard deviation, showed very little significant improvement after calibration.

b. The test results can best be evaluated if the errors are found to follow a normal distribution. Fig. 7 shows the theoretical probability curve as based on an infinitely large number of observations. If the frequency of occurrence of similar errors is plotted against magnitude this "normal" curve results. The axis of symmetry is the arithmetic mean of all errors and its displacement from the zero axis is the "mean error." In Shoran photography the magnitude of this mean error is dependent upon the degree of control that can be placed on the sources producing both constant and systematic errors. The following basic laws concerning random errors are also indicated by the curve:

1. Plus and minus errors are equally numerous.
2. Small errors are much more numerous than large ones.
3. Very large errors seldom occur.

Another statistic used in this report is the "standard deviation" which is computed by the formula $\sqrt{\frac{\Sigma v^2}{n}}$ where $v$ is the deviation of the individual observations from the mean and $n$ is the number of observations. The area under the normal curve between perpendiculars at one standard deviation on either side of the mean encompasses two-thirds of the total area under the curve. For this reason, plus or minus one standard deviation represents the range of errors.
within which about two-thirds of the observations can be expected to fall. Two standard deviations (2\times \text{the value of the standard deviation}) represent the range of errors within which about 95 percent of the observations can be expected to fall and three standard deviations represent the error range of about 99 to 100 percent of the observations.

In order to validate use of functions of the normal curve in predicting Shoran accuracy, a frequency distribution curve was prepared from the 592 distance observations that have been analyzed in the course of the tests. Mean errors in the individual missions that may have been caused by failure to correct fully for meteorological conditions, delay errors, etc., were subtracted before tabulation so that only the random errors would be left. A theoretical frequency curve was then fitted to the data by the ordinate method and a comparison was made. Table X shows the computation and Fig. 8 shows the two curves as plotted from the results. The similarity is obvious and indicates that Shoran measurement errors can be assumed to follow a normal distribution. Further verification for this assumption is given by the Chi square test shown in Appendix B, Exhibit 2. The Chi square test is a statistical tool often used in testing the goodness of fit of sample data to the normal curve.

c. The tabulation of errors shows a standard deviation of \( \frac{4}{7} \) feet in the established Shoran positions. This means, then, that in establishing the positions of the 100 or so photographs needed for mapping a representative area, 67 percent of the Shoran points can be expected to contain random errors of between 0 and 74 feet, 28 percent (95-67=28) will probably contain errors of 74 to 148 feet, and 5 percent may contain errors of 148 to 222 feet. Maps compiled by holding rigidly to this control will incorporate all these errors plus additional errors such as those introduced in drafting or by failure to recover true plumb points. Since, however, the accuracy of relative orientation between successive multiplex models is considerably better than the relative accuracy of the Shoran positions, it is obvious that improved map accuracy can be obtained by orienting in groups of several photographs and then scaling the entire unit to the best mean fit of the corresponding Shoran positions. The optimum number of photographs to be used in each unit will depend upon the point at which errors attributable to multiplex bridging begin to exceed those possible from the Shoran positions. Since plumb points are the scale points, the ability of the multiplex to hold relative orientation in "tip" and "tilt" was found to be of greater consequence than its accuracy in holding horizontal scale. Tests previously reported in Engineer Board (ERDL) Report No. 987 indicate that orientation in units of about 6 to 7 models produces the best results. This fact further facilitates operations in that this number of projectors fits nicely on a standard multiplex single frame.
Fig. 7. Normal probability curve.

Fig. 8. A comparison of the distribution of Shoran distance errors with the normal probability curve.
Table I. Fitting of Normal Curve - Ordinate Method

Distribution of Shoran Distance Errors from the Mean Value for Each Mission

<table>
<thead>
<tr>
<th>Deviation from Mean in Meters (X)</th>
<th>Number of Measurements (N)</th>
<th>Deviation from Mean of Standard Deviation ((\sigma))</th>
<th>Percent of from Mean Ordinate</th>
<th>Theoretical Ordinate Frequency</th>
<th>Fitted Ordinate Frequency (F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-64 to -55</td>
<td>-60</td>
<td>0</td>
<td>-3.33</td>
<td>0.30</td>
<td>0.47</td>
</tr>
<tr>
<td>-54 to -45</td>
<td>-50</td>
<td>3</td>
<td>2500</td>
<td>7500</td>
<td>-2.67</td>
</tr>
<tr>
<td>-44 to -35</td>
<td>-40</td>
<td>4</td>
<td>1600</td>
<td>6400</td>
<td>-2.00</td>
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<tr>
<td>-34 to -25</td>
<td>-30</td>
<td>19</td>
<td>900</td>
<td>17100</td>
<td>1.33</td>
</tr>
<tr>
<td>-24 to -15</td>
<td>-20</td>
<td>62</td>
<td>400</td>
<td>24800</td>
<td>0.67</td>
</tr>
<tr>
<td>-14 to -5</td>
<td>-10</td>
<td>141</td>
<td>100</td>
<td>14100</td>
<td>-0.67</td>
</tr>
<tr>
<td>-4 to +5</td>
<td>0</td>
<td>173</td>
<td>0</td>
<td>0</td>
<td>0.67</td>
</tr>
<tr>
<td>+6 to +15</td>
<td>+10</td>
<td>108</td>
<td>100</td>
<td>10800</td>
<td>+0.67</td>
</tr>
<tr>
<td>+16 to +25</td>
<td>+20</td>
<td>54</td>
<td>400</td>
<td>21600</td>
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<td>+26 to +35</td>
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<td>20</td>
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<td>18000</td>
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<tr>
<td>+36 to +45</td>
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<td>8</td>
<td>1600</td>
<td>12800</td>
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<td>+50</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+3.33</td>
</tr>
</tbody>
</table>

Total 592 133100

Standard Deviation = \(\sigma = \sqrt{\frac{\sum F_0X^2}{N}} = \sqrt{\frac{133,100}{592}} = \pm 15\) meters \(49\) ft

\(\sigma\) (Class interval units) = \(\frac{+15}{10} = \pm 1.5\)

Theoretical Maximum Ordinate = \(\frac{N}{2.506628\sigma} = \frac{592}{2.506628(1.5)} = 157.449\)
d. As a test of the multiplex mapping accuracy to be expected from this type of control, four missions of about five parallel flights and 60 exposures each were scaled in units of 6 or 7 models to the best mean fit of the Shoran position. Since it was assumed that no ground vertical control was available, cross flights and the \( BZ \) curve method were used in establishing projector orientation. Upon completion of the scaling of each mission, the individual units were graphically adjusted to remove any internal position disagreement evidenced by non-coincidence of pass points plotted along the edges of each unit. A number of identifiable points of known position were then plotted and their positions compared with those of the known points. Table XI shows the results. Here again, the ability to scale coordinates from 1:17,000 plotting sheets limits the significance of the accuracy figures to about five to eight feet. It is interesting to note that the average internal map accuracy is actually better than that of the Shoran control even though errors of drafting and tilt uncertainties are present in the compilation stage. This fact proves the value of scaling to a mean fit of the Shoran positions and of using the \( BZ \) curve method for establishing absolute orientation.

Table XI. Accuracy Test of Multiplex Maps Compiled with Shoran Control

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Equipment Used</th>
<th>No. of Known Positions Compared</th>
<th>Mean Error (ft)</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td>Y</td>
</tr>
<tr>
<td>*1</td>
<td>Uncalibrated</td>
<td>35</td>
<td>+18</td>
<td>+130</td>
</tr>
<tr>
<td>3</td>
<td>Uncalibrated</td>
<td>22</td>
<td>-37</td>
<td>+17</td>
</tr>
<tr>
<td>5</td>
<td>Calibrated</td>
<td>35</td>
<td>-11</td>
<td>-2</td>
</tr>
<tr>
<td>6</td>
<td>Calibrated</td>
<td>44</td>
<td>+5</td>
<td>-2</td>
</tr>
</tbody>
</table>

Mean of all Points

\( \pm 63 \)

* Test numbers have been made to correspond with the photography of Table IX so as to permit a comparison of Shoran position accuracy with resulting map accuracy.
III. DISCUSSION

15. Accuracy of Shoran in Photogrammetric Mapping. Since the analysis of Shoran accuracy is based on 390 position observations and 592 distance observations, it is believed that sufficient information is available to permit a reliable evaluation of this new type of mapping control. It should also be pointed out that these tests were Phase I of the project and, therefore, were accomplished before personnel concerned with the tests were fully cognizant of the many instrument idiosyncrasies and error sources that have been revealed throughout all phases. In fact, there is little doubt but that future Shoran photogrammetric missions can produce greater map accuracy than that indicated by the current tests.

a. Shoran map accuracy must be considered in two separate parts: (1) the accuracy with which the finished map sheet is located with respect to the ground stations; and (2) the internal, or relative, accuracy of positioning detail within the map. As a comparison of the accuracy with which map sheet positioning should be accomplished in the ideal case, the following paragraph is quoted from the U. S. Coast and Geodetic Survey Manual of First-order Triangulation:

"According to the plan adopted by the Federal Board of Surveys and Maps for the completion of the 1-inch-to-the-mile standard topographic map of the United States, first-order triangulation or traverse would be executed in belts about 100 miles apart. Second-order triangulation or traverse would subdivide the intervening areas until no considerable area would be farther than 25 miles from a horizontal-control point of the first or second order. Horizontal control of the third order would then be established, with a density of distribution of points depending upon the requirements of the topography."

In the ideal case, then, a map sheet should be located to at least second-order accuracy with respect to the datum used. In the case of Shoran maps, where the map sheet may be as much as 100 to 200 miles from the ground stations, first-order accuracy would be required. The results shown in Table IX indicate that the mean error is too large to meet this first-order requirement. Even during Phase III in which every precaution was taken to eliminate all possible errors, Shoran produced only low second-order accuracy. Of course, if one or preferably two identifiable ground control points appear within the Shoran photography, this positioning problem could be overcome. Even third-order positions resulting from fairly short survey ties between first- or second-order positions would be sufficient. In this case, the multiplex
Shoran map would first be compiled in the normal manner and the entire sheet then "slipped" into position as indicated by the survey points. Where several adjacent sheets are to be compiled, survey points in every second or third sheet would probably be sufficient.

b. The internal map accuracy is a function of the random errors. Table IX shows the standard deviation of Shoran positions to be \( \pm 74 \) feet. However, by using multiplex methods to "average out" part of the error, the standard deviation of plotted map points was found to be \( \pm 63 \) feet. The Army Map Service requirements for position accuracy of 1:25,000 and 1:50,000 scale maps, as outlined in Technical Instructions No. 41, dated 15 May 1947, are that 90 percent of all well-defined features shall be located within 0.02 inch of true position at the publication scale. This means that for 1:50,000 scale maps, 90 percent of the features must be positioned within \( \pm 83 \) feet. By the same rule, 1-inch-to-the-mile maps must have 90 percent of the features located to within \( \pm 106 \) feet. From the laws of the probability curve, the range of deviations within which 90 percent of the observations will fall is found by multiplying the standard deviation by 1.65. Therefore, the indicated internal accuracy of Shoran maps compiled by multiplex methods is that 90 percent of the features will have a relative accuracy of \( \pm 104 \) feet (63 x 1.65). This shows that except for positioning of the map sheets, even the present Shoran equipment produces sufficient horizontal accuracy for compilation of 1-inch-to-the-mile maps and comes very close to meeting peacetime requirements for 1:50,000 scale maps. For military mapping where occupation of the ground is often impossible, this degree of accuracy will usually be superior to that obtainable by any other method.

16. Instrument and Propagation Errors. The various error sources and their effect on Shoran distances are listed in paragraph 13c. Constant or systematic errors are evidenced as "mean errors" in the accuracy tabulation of Table IX, paragraph 14. Constant errors in the system can be of different magnitude and sign in measurement of distances to either ground station, will remain the same during any one mission, but may change between missions. Systematic errors, such as those introduced by a change in oscillator frequency, would cause errors proportional to distance and of like sign in the measurements to both ground stations. Errors in applying the velocity corrections would also show up as proportional errors of like sign provided meteorological conditions were about the same along the radio paths to both ground stations. This can be safely assumed in these tests since atmospheric conditions are very stable in the Denver area. If any consequential errors of a systematic nature had been present in the Phase I Plains tests, it would be expected that errors in the distances from Imperial (S2) would always be considerably greater than those in the Cheyenne (S1) distance since Imperial is approximately 175 miles from the mapped area.
whereas Cheyenne is only about 100 miles. An examination of Table IX shows no indication that this was the trend since only in Test 4 do the mean errors follow this general pattern. In the present tests, then, sources producing constant errors would appear to be of much more concern than those producing systematic errors.

Equipment errors that could have introduced constant errors in the test photography are those caused by signal intensity and by delays. Instrument modifications designed to reduce these errors have been recommended in the previously mentioned Air Force report. It is also expected that further study of these two error sources can be made during the coming photographic tests.

Random errors in the equipment are caused by timing non-linearity and by pip alignment. The timing non-linearity error has already been reduced by modifications on some of the existing Shoran equipment, and it is possible that a way may be found to reduce its effect still further through calibration. Additional study of this problem should be made during the photographic tests. The absolute effect of pip alignment can only be estimated since no method is presently available for separating these errors from others of a random nature. Accuracy in this operation is largely a human factor dependent upon training and skill. In fact, the slight improvement in random error (as evidenced by the decreased value of standard deviation) between the tests with calibrated and with uncalibrated equipment probably results from the increased skill in pip alignment obtained as further experience in operations was gained. Photographic recording of the oscilloscope at the instant of each aerial exposure would permit the application of corrections for these errors and is one method being considered for future use. Consideration is also being given to automatic pip alignment. Some method of controlling these alignment errors is probably the most salient of all equipment requirements in connection with the operational use of Shoran in controlling photography.

17. Compilation Errors. For the purpose of this discussion, compilation errors will be considered to include errors of flying height, geometric errors, and errors of tilt determination. The determination of mean flying height and base lengths introduce systematic errors which, as previously mentioned, appear to be small in the present tests. From a study of Table I, paragraph 9, it can be seen that flying height errors are large only when the Shoran aircraft operates near one of the ground stations. So long as this operating distance is more than about 50 miles, an error of 100 to 150 feet in determination of the mean flying height of 20,000-foot photography will introduce smaller errors than those now possible from delays or signal intensity.
Fig. 4 shows the position errors that will be caused by errors in base length. In practically all of the area having acceptable station angles, this position error will be equal or less than the base line error. To be in keeping with other errors in the system, the base length distance should be known to second-order accuracy or better. Here again, however, if one or two ground control stations are available in the survey photography or if absolute position of the map is unimportant, base line distances of third-order accuracy will be satisfactory.

Table III, paragraph 10, indicates that the barometric altimeter will furnish values of flying height difference accurate to about 100 feet in 95 percent of the cases. From Table I, then, it can be seen that this will introduce random errors in the position computations of 20,000-foot photography of no more than about 15 feet (8 feet in each distance) so long as distance from the airplane to the ground stations is 50 miles or more. As this is less than other random errors in the system, the barometric altimeter appears satisfactory for use with present Shoran equipment. This accuracy also indicates that, so long as the operating distance of the aircraft from the ground stations is over 50 miles, Shoran positions for an entire mission can be computed using a constant flying height, with differential distance corrections applied only where the barometric altimeter indicates deviations from the mean of more than 100 feet.

As the Shoran airplane operates nearer to the ground stations, the errors of flying height difference have more and more effect on the Shoran positions. At operating distances of less than about 50 miles it will become necessary to determine the differences with greater precision than is now possible with the barometric altimeter; possibly by measurement of projector heights during a preliminary multiplex orientation. This condition will also increase the computation time since it will no longer be safe to assume a constant flying height for an entire mission.

The BZ curve method of recovering photo tilts without use of ground control produced an accuracy of 23 minutes of arc or less for deviations of 95 percent of the photographs. This means that with photography from 20,000 feet, the random deviation of the true from recovered plumb points will amount to 135 feet or less in 95 percent of the cases. The random deviation of the Shoran positions amounts to only 150 feet or less in 95 percent of the determinations and, therefore, indicates a relative Shoran accuracy approaching that of the ability of the BZ curve method to establish plumb points. Any substantial improvement of the random error in the Shoran system, as may be brought about by the recording of the oscilloscope, will unquestionably improve mapping accuracy, but will require a parallel improvement in the method of recovering tilts if maximum efficiency
is to be maintained. Since the BZ curve method appears to be about the most reliable method now available for use with photography of areas without vertical control this problem will offer a real challenge. Development of a stabilized aerial camera mount would, of course, be the most desirable solution. Other possibilities are the celestial tilt indicator, the use of horizon photographs, and a study of possibilities of improved accuracy in the BZ curve method through use of a statoscope to furnish more reliable flying height differences.

18. Utilization of Shoran for Photo Control. The tests show that 1:50,000 scale maps closely approaching peacetime standards of accuracy for relative position can be produced with present Shoran equipment. The error in positioning map sheets with respect to the ground stations will probably never exceed 100 to 200 feet and even this can be greatly reduced if identifiable first-, second-, or third-order positions are available at intervals of 50 to 75 miles in the area to be mapped. Availability of such control will also reduce the care with which meteorological conditions, flying height and other error sources of a constant and systematic nature must be determined. In many parts of the world, existing ground control of the required density can be recovered without additional field work. In other cases, it will be found that Shoran triangulation will meet the needed accuracy requirement. Where maps of 1:100,000 scale and smaller are needed, Shoran errors will usually be insignificant and, in fact, some simplified procedure such as the "control point" method discussed in Engineer Board (ERDL) Report 1012 will often suffice.

Service and operational tests of the equipment in Shoran triangulation have shown that field use is practical. Ground station equipment is air transportable and, in a few instances, has been back-packed several miles into position. Further operational use, as with any new tool, can be expected to increase the facility with which all steps of the operations can be performed and probably will point the way to further improvements in accuracy.

Position accuracy equal to, or better than, that obtained in the present tests can only be assured if personnel well-trained in the mapping application carry out the operations. Existing equipment was designed originally for bombing and requires careful operation, calibration, and adjustment if mapping accuracy is to be achieved. The degree of refinement needed can be appreciated when it is realized that a timing error of only one one-millionth of a second will introduce a distance error of 490 feet. This need for specialists will decrease somewhat as further experience reveals ways in which many of the operations can be reduced to more or less routine procedures.
IV. CONCLUSIONS

19. Conclusions. It is concluded that:

a. The principal sources of instrumental errors which affect internal map accuracy are timing non-linearity and pip alignment.

b. The principal sources of instrumental errors which affect the positioning of the map as a whole are equipment delays and signal intensity.

c. Position errors introduced by errors in flying height determination are of least consequence when the Shoran aircraft operates at distances greater than 50 miles from the ground stations, with the result that both the possibilities of map errors and the time required for computations are materially decreased.

d. The multiplex BZ curve method of establishing photo tilts is sufficiently accurate for use with the present Shoran equipment.

e. Shoran-controlled photogrammetric maps can be compiled by the multiplex method with an internal map accuracy such that 90 percent of the features will have a relative accuracy of approximately \( \pm 0.4 \) feet.

f. When no existing identifiable horizontal control is available, a "shift" of the entire map sheet as much as 200 feet with respect to the horizontal datum is possible.

g. Improved map accuracy is possible when Shoran instrumental errors are decreased and operational procedures are perfected.

h. An improved method of recovering photo plumb points is required if random Shoran instrumental errors are decreased.

i. The Shoran system, in its present state of development, is ready for use in positioning photography whenever the accuracy obtainable with this new tool satisfies mapping requirements.

V. RECOMMENDATIONS

20. Recommendations. It is recommended that:

a. Future studies of possible equipment refinement give prime consideration to errors caused by pip alignment, signal intensity, delays, and timing non-linearity.
b. Shoran photographic missions be planned so that the operating distance from the aircraft to either ground station is always in excess of 50 miles.

c. The Air Force be requested to continue the vigorous prosecution of current studies and tests of stabilized camera mounts and methods of recovering photo tilts.

d. The Shoran system be adopted immediately for use in controlling photography of areas where the obtainable accuracy will satisfy the mapping requirements.

Submitted by:

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Chief, Shoran Section

Forwarded by:

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Chief, Photogrammetric Branch

Approved 5 November 1948 by:

WILLIAM C. CUDE
Chief, Technical Department V
APPENDIX A

AUTHORITY

Research and Development Project Card (RDB Form 1A)
for Project 8-35-05-001
One of the most promising developments in the field of mapping and charting lies in the adaptation of Shoran to establishing horizontal control, eliminating the necessity for much of the ground survey now required. Procedures and techniques for utilizing Shoran data in photogrammetric mapping must be developed. This project will result in procedures required to supplement new concepts of warfare resulting from radically new weapons developed by great scientific advancements.

a. REFERENCES:
(1) Letter from the Engineer Board to Chief of Engineers, dated 11 August 1945, subject, "Service Project for Investigation of Shoran for Mapping (MPS 673)" with 2 inclosures and one indorsement.
(3) War Department Equipment Board Report (Secret) dated 29 May 1946, Section VII, paragraph 7, "General Equipment".
(4) War Department General Staff Disposition Form, Comment No. 1, dated 1 April 1947, file MID 906, subject, "Shoran", from Director of Intelligence, WDGS; to Director of Plans & Operations, WDGS; Director of Research & Development, WDGS; Commanding General, Army Air Forces; and Chief of Engineers; (In Turn); with 2 inclosures and Comments Nos. 2 to 5 inclusive.
(5) First partial report of the Army Ground Forces Board #2 on project No. 1146, subject; Military Characteristics of Surveying Equipment, dated 17 October 1946.
(6) Second partial report of the Army Ground Forces Board #2 on project No. 1146, subject; Military Characteristics of Surveying Equipment, dated 3 December 1946.

b. OBJECTIVE:
(1) Development of the necessary techniques and procedures for utilizing and adapting Shoran for photogrammetric mapping.

c. MILITARY CHARACTERISTICS:
(1) Not applicable
d. DISCUSSION:

(1) This project is designed to provide the necessary techniques for utilizing Shoran and similar electronic equipment to control aerial photography in conjunction with existing or new photogrammetric methods. Its scope is limited to research in photogrammetric mapping from one or more pairs of ground Shoran stations. Shoran measuring equipment, originally designed for navigation and bombardment, has now been developed to a point where it may be used to obtain accurate horizontal control for topographic maps. Considerable mapping research is required, however, to assure maximum utilization of this revolutionary mapping method.

(2) Investigation of Shoran accuracy and possible use in mapping, (Reference (1) has shown that Shoran is practical for military mapping and indicates the necessity for establishing standard procedures for use by military units (reference (2)). The ability to establish accurate horizontal control during the aerial photographic mission offers vastly changed concepts of military mapping with great promise of expediting these operations and accomplishing accurate mapping without access to the ground. The constant development of airborne electronic equipment and flying procedures requires parallel mapping research if maximum utilization of the equipment is to be realized. Research for determining mapping requirements and for developing mapping procedures and techniques will permit maximum exploitation of this new tool. This study is also of prime importance in the planning of fire control for guided missiles since absolute target location is required. The combination of electronics and photogrammetry appears to offer the best solution to this problem. Research in this field is needed, not only to prepare for possible war time mapping, but also to accomplish, as expeditiously possible, military peace time mapping, such as current mapping operations which are being conducted in the Caribbean and Pacific areas.

(3) Agencies interested in this project, in addition to the Office, Chief of Engineers, are Army Ground Forces and Department of the Air Force.

e. PROJECT PLAN:

(1) In view of the immediate requirements, a tentative manual of Shoran Mapping will be given prompt attention. Subsequent investigations will be directed toward development of photogrammetric procedures for use of Shoran or similar electronic methods at ranges in excess of that now obtainable and toward perfection of mapping procedures to take full advantage of improvements and changes in Shoran equipment and operating procedures.

(2) New mapping procedures will be investigated where necessary, and changed airborne procedures will be recommended for test if studies indicate possibilities for improved accuracy or efficiency.

(3) The successful prosecution of all contemplated phases will require continuous coordination with the Department of the Air Force. Necessary liaison will be maintained through personnel and facilities already provided by both services for this purpose.
### APPENDIX B

**TEST DATA AND RESULTS**

<table>
<thead>
<tr>
<th>Exhibit</th>
<th>Item</th>
<th>Page</th>
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<td>49</td>
</tr>
<tr>
<td>2</td>
<td>Chi Square Test</td>
<td>50</td>
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</tbody>
</table>
Exhibit 1

Location Map
Exhibit 2

Chi Square Test for Goodness of Fit of Shoran Distance Errors to Normal Curve

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<thead>
<tr>
<th>Deviation from Mean in Meters</th>
<th>Number of Measurements</th>
<th>Theoretical Frequency</th>
<th>( F_O - F )</th>
<th>( (F_O - F)^2 )</th>
<th>( \frac{(F_O - F)^2}{F} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-54 to -45</td>
<td>3</td>
<td>0.47</td>
<td>2.07</td>
<td>4.28</td>
<td>0.868</td>
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<tr>
<td>-44 to -35</td>
<td>4</td>
<td>4.46</td>
<td>-2.30</td>
<td>5.29</td>
<td>0.248</td>
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<td>-34 to -25</td>
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<td>21.30</td>
<td>-3.01</td>
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<td>0.139</td>
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<td>6.01</td>
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<td>125.80</td>
<td>15.55</td>
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<td>1.536</td>
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<td>157.45</td>
<td>-17.80</td>
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<td>65.01</td>
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<td>21.30</td>
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<td>+46 to +55</td>
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<td>0.47</td>
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\[ P \text{ (from tables)} = 0.31 \]