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A user-defined subroutine has been developed to implement special 2-D and 3-D layered continuum elements in the commercial finite element program ABAQUS. These elements are specially configured to accurately predict interface stresses in adhesively bonded joints and are formulated using the hybrid stress technique to explicitly enforce stress equilibrium throughout the element domain and stress continuity conditions at layer interfaces. This report details the use of developed special ‘adhesive elements’ in ABAQUS for elastostatic analysis together with several numerical examples which demonstrate an improved accuracy and convergence behavior over conventional displacement-based elements. Sample input and output datasets and the complete FORTRAN subroutine which performs all element computations are presented in separate appendices.
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1 Introduction

The widespread application of adhesively bonded joints has necessitated the development of methodology to predict ultimate static joint strength and service life under cyclic loading. Due to the complexity of mathematically modelling adhesive joint response, analytical treatments are limited to highly idealized joint configurations and simplified assumed stress states, applied loading and material behavior. To overcome these limitations, a specialized finite element-based numerical approach is advocated to provide a versatile approach to analyze actual bonded joint concepts with complex geometries, load paths and support conditions. To enhance a finite-element based methodology, special 2-D and 3-D layered elements have been developed in Reference [1] to improve the computational efficiency and accuracy of determining stresses in adhesive joints. The special 2-D and 3-D layered continuum elements are formulated using the hybrid stress technique to explicitly enforce stress equilibrium throughout the element domain and stress continuity conditions at layer interfaces. In an extensive investigation presented in Reference [1], optimum element configurations have been determined and demonstrate improved performance compared to standard displacement-based finite elements in predicting joint stresses.

This report details the use of several special 2-D and 3-D continuum elements in the commercial finite element code ABAQUS through a developed user-defined subroutine. The elements are specialized for the analysis of adhesive joints by incorporating a layered hybrid formulation to accurately model the adhesive/adherend interface and are, thus, referred to as 'adhesive elements'. The adhesive elements are currently restricted to linear elastic behavior and a geometric constraint is imposed which requires that all element layers are rectangular. To permit the representation of composite laminate adherends and property variation through the adhesive layer, material properties are input as orthotropic laminae within each element layer. In addition, 2-D adhesive elements are supported for arbitrary orientation in the global X-Y plane and 3-D elements may be arbitrarily oriented in space. Element stress and strain output may be selected in either global or local coordinate systems.

A brief description of the support of user-defined elements in ABAQUS is presented in the next section followed by a description of the input format established for the adhesive elements. The basic element library is discussed in subsequent sections detailing element configuration, coordinate system convention and comments on their use. Two illustrative numerical examples are presented demonstrating the use of selected 2-D and 3-D adhesive elements. Sample input and output datasets together with the complete FORTRAN source code performing all element computations are presented in separate appendices.

2 User-Defined Elements in ABAQUS

New finite elements may be used with ABAQUS via a subroutine denoted *UEL* (for UserElement) which performs the necessary element computations and interfaces with the main ABAQUS program through a standardized parameter list in the subroutine call statement. The **USER SUBROUTINE** statement in the input deck alerts ABAQUS to the presence of user-defined subroutines which either immediately follow this data entry or are contained in a separate file. These subroutines are then compiled and linked to the main ABAQUS executable prior to job execution. A complete description of this and other user-defined capabilities in ABAQUS may be found in [2]. Shown in Figure 1 is the basic format of the *UEL* subroutine with the argument list used by ABAQUS to pass into the user-defined subroutine all necessary information needed to compute element stiffness matrices. Once computed, these matrices are then passed back to ABAQUS for global assembly and problem solution. In static analysis, data recovery is performed during a second pass through the user-defined subroutine after the solution for global displacements has been obtained. During this phase, ABAQUS passes in the nodal displacements for the current element from which all element stresses and strains can be computed. In addition to linear static analysis, the information passed into the *UEL* subroutine is sufficient to support material and geometric nonlinear analysis.

The complete source code supporting linear static analysis for the special adhesive elements in ABAQUS is listed in Appendix A.
SUBROUTINE UEL(RHS,AMATRX,SVARS,ENERGY,NDOFEL,NRHS,NSVARS,
  PROPS,NPROPS,COORDS,MCRD,NNODE,U,DU,V,A,
  JTYPE,TIME,DTIME,KSTEP,KINC,JELEM,PARAMS,
  NDLOAD,JDLTYP,ADLMAG,PREDEF,NPREDF,LFLAGS,
  MLVARX,DDLMA,DMDLOAD,PNEWDT)
C
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C
VARIABLE AND ARRAY DECLARATIONS FOR UEL/ABAQUS INTERFACE
C
DIMENSION RIIS(MLVARX,*),AMATRX(NDOFEL,NDOFEL),PROPS(*),
  SVARS(1),ENERGY(8),COORDS(MCRD,NNODE),U(NDOFEL),
  DU(MLVARX),V(NDOFEL),A(NDOFEL),TIME(2),
  PARAMS(*),JDLTYP(MDLOAD,*),ADLMAG(MDLOAD,*),
  DDLMA(MDLOAD,*),PREDEF(2,NPREDF,NNODE),LFLAGS(5)
*********** *5**5**5*5*** *5****** ***** *5** ***
*** Source code for user-defined elements ***
*********** *5**5**5*5*** *5****** ***** *5** ***
RETURN
END

Figure 1. Format of the user-defined subroutine UEL supporting the special adhesive elements in ABAQUS.

3 Use of Special Adhesive Elements in ABAQUS

Three sets of statements are used to describe the adhesive elements in the ABAQUS input deck. Each of these statements may be identified in the sample input files presented in Appendices B and C. The first set, *USER ELEMENT*, defines the basic parameters of the user elements. All parameters are mandatory and the user must set the \( N \), \( n \) and \( M \) values as described below.

**Statement set 1:**

(i) *USER ELEMENT, NODES = N, TYPE = Un, PROPERTIES = M

(ii) \( n_1, n_2, \ldots \)

where the various input parameters are:

**Card (i):** \( NODES = N \) specifies the total number of nodes present in the adhesive element selected. 
\( TYPE = Un \) specifies the internal designation of the element through setting the value of \( n \). The element type designation, \( n \), will be given in the discussions of the various elements in following sections.
\( PROPERTIES = M \) specifies that a user-defined property list of length \( M \) is to established for each element as explained below.

**Card (ii):** This entry indicates the active degrees of freedom at each node in the element.
For 2-D elements, this list is: 1,2; for 3-D elements, this list is: 1,2,3.
The second entry, *UEL PROPERTY, is the primary list of input data used to compute element quantities for each adhesive element. The material input has been made general to allow the input of composite laminate material for the adherends, or to specify property variations in the adhesive layer. The size of this list is determined by the user as function of the total number of plies in each of the element layers. Only a single ply would be specified for a homogeneous material whereas any number may be specified to define laminate properties. ABAQUS requires for each line in the property list that all quantities be expressed as real numbers in free format with up to eight entries per line - missing entries are simply treated as zeros. In the basic format of the property list shown below, the total length of the property list is calculated as

\[ M = 8(1 + \sum_{i}^{k}[2NLAY_i + 1]) \]

where \( k \) is equal to the number of layers in the element and \( NLAY_i \) is the number of plies used within the \( i^{th} \) layer. This length is then entered as a parameter on the *USER ELEMENT entry.

Statement set II

(i) *UEL PROPERTY, ELSET = NM
(ii) NVER, IPLANE, OUTPUT, NSIDE
(iii) NPLY1, WDTH1
(iv) PTHK1, \( \Theta_1 \), \( E_1 \), \( E_2 \), \( E_3 \), \( \mu_{12} \), \( \mu_{23} \), \( \mu_{13} \)
(v) \( G_{12} \), \( G_{23} \), \( G_{31} \)
(vi) NPLY2, WDTH2
(vii) PTHK2, \( \Theta_2 \), \( E_1 \), \( E_2 \), \( E_3 \), \( \mu_{12} \), \( \mu_{23} \), \( \mu_{13} \)
(viii) \( G_{12} \), \( G_{23} \), \( G_{31} \)
(ix) NPLY3, WDTH3
(x) PTHK3, \( \Theta_3 \), \( E_1 \), \( E_2 \), \( E_3 \), \( \mu_{12} \), \( \mu_{23} \), \( \mu_{13} \)
(xi) \( G_{12} \), \( G_{23} \), \( G_{31} \)

where the input parameters are defined by:

Card(i): \( NM \) is the set Id of the adhesive element for which the following properties are to be used.
Card(ii): \( NVER \) designates a particular version of an element type.
IPLANE is used to select plane stress/plane strain assumptions in the use of 2-D elements.
For 3-D elements, this field is ignored.
IPLANE = 1 for plane stress.
IPLANE = 2 for plane strain.
OUTPUT is the element output control flag.
OUTPUT = 0 for suppression of element data output.
OUTPUT = 1 for output of stresses and strains in local element coordinates.
OUTPUT = 2 for output of stresses and strains in global coordinates.
NSIDE indicates the face on which zero tractions are explicitly enforced. This property is only recognized by the element types which support this option.
Card(iii): \( NPLY_1 \) is the number of plies in layer 1.
WDTH1 is the width of layer number 1 in 2-D elements. The width dimension is defined as normal to the element plane. This entry is left blank in 3-D elements.
Card(iv): PTHK1 is the ply thickness for the first ply in layer 1.
\( \Theta_1 \) is the orientation of the first ply in layer 1.
\( E_1 - \mu_{12} \) are layer Youngs moduli and Poisson ratios.
Card(v): \( G_{12} - G_{31} \) are layer shear moduli.

Cards (iv) and (v) are repeated for each ply specified. The data block represented by cards (vi) through (viii) follow the same format. The data block represented by cards (ix) through (xi) are used only if a third element layer is present in the element.
The last entry is the *USER SUBROUTINE statement. As stated above, this alerts ABAQUS to the presence of source code which is to be included together with the main executable code prior to running the requested job. This data statement is given by

**Statement set III:**

(i) *USER SUBROUTINE, INPUT = uel_hybrid.f

where the optional parameter, INPUT, specifies the name of an external file containing the source code for the user-defined adhesive elements. If this parameter is omitted, ABAQUS assumes that the source code immediately follows this statement.

The library of adhesive elements is discussed below.

4 Special Adhesive Elements

The element library presented herein contains several 2-D and 3-D special adhesive elements for general use in the analysis of bonded joint stresses. These elements are assumed to be used specifically for the numerical representation of the local region encompassing the adhesive bond with standard elements representing all other regions of the joint adherends. The use of 2-layer and 3-layer elements in modelling the bond layer is depicted in figures 2 and 3. Specific details of the specialized adhesive elements are described below.

![Figure 2](image)

Figure 2. Use of the 2-layered elements in modelling an adhesive layer.
4.1 2-D Adhesive Elements

Several 2-D special adhesive elements have been incorporated in the user-defined subroutine. The elements differ in number of layers, element order, assumed order of stress expansions and applied stress field constraints. An account of their performance in predicting bondline stresses is extensively examined in Reference [1]. Details of the 2-D elements, designated H2L6N, H2L10N, H3L8N and H2L13N, are discussed in the following subsections.

4.1.1 The H2L6N Element

The configuration of the H2L6N element is depicted in Figure 4. As shown, a local element coordinate system is defined at each layer centroid with the local $\xi$ and $\eta$ axes parallel to adjacent sides of the layer.
The H2L6N element is designated as $TYPE = U1$ and two versions are available incorporating complete linear and quadratic stress expansions. These versions are selected by setting the element version parameter as $NVER = 11$ and $NVER = 12$, respectively. The H2L6N element is also supported for use as an end-element in which zero traction conditions are enforced in the $\tau_{yy}$ stress component. This version is selected as $NVER = 13$ and the input parameter $NSIDE$ is used to select the traction-free element side by setting the property parameter $NSIDE = i$ where $i$ is indicated by the $Fi$ designation in the above figure.

This element has demonstrated excellent convergence properties in a study of bondline stress prediction in single-lap joints. The linear field used in version 11 yields good convergence behavior but the quadratic field used in version 12 should be selected if a coarse mesh is used along the bond axis. The increase in computational cost is minimal.

4.1.2 The H2L10N Element

The configuration of the H2L10N element is depicted in Figure 5. As shown, a local element coordinate system is defined at each layer centroid with the local $\xi$ and $\eta$ axes parallel to adjacent sides of the layer.

Figure 5. H2L10N element node configuration and local layer coordinate system.

The H2L10N element is designated as $TYPE = U2$ and two versions are available incorporating complete quadratic and cubic stress expansions. These versions are selected by setting the parameter $NVER = 11$ and $NVER = 12$, respectively.

The H2L10N element formulation has a higher-order strain field representation in the normal bondline direction. However, this selective increase in the degree of freedom representation in the bond thickness direction has not demonstrated an overall improvement in bond stress prediction in the single-lap joint case over the H2L6N element. It is maintained in the element library for a further assessment in analyzing other joint configurations.
4.1.3 The H2L13N Element

The configuration of the H2L12N element together with local layer coordinate system convention is depicted in Figure 6.

Figure 6. H2L13N element node configuration and local layer coordinate system.

The H2L13N element is designated as \( \text{TYPE} = U3 \) and two versions are contained in the element library. A difficulty was encountered in Reference [1] in the formulation of this element. It was found that adopting a higher-order displacement field and strictly enforcing all stress field constraints inevitably leads to spurious kinematic deformation modes in the resulting element stiffness matrix. Therefore, selective relaxation of some constraints were made in the two versions of this element. One version, selected using \( NVER = 11 \), incorporates a complete cubic stress field with the addition of two quartic terms in the shear stress expansion which are not constrained to enforce continuity at the element layer interface. These two terms are added to suppress zero energy modes which result from using complete expansions satisfying all equilibrium and continuity constraints. A second version, designated \( NVER = 12 \), is formulated using a complete quadratic field with only stress continuity conditions applied at the layer interface.

The performance of these versions in the analysis of a single-lap joint configuration has shown that both demonstrate accurate stress predictions with element version 12 showing a faster rate of convergence and a highly accurate recovery of bondline stresses. The violation of strict continuity enforcement in element version 11 has been shown to be of minimal consequence due to the high-order of the unconstrained stress expansion terms.
4.1.4 The H3L8N Element

The configuration of the H2L8N element and local coordinate system is depicted in Figure 7.

![Figure 7](image)

**Figure 7.** H3L8N element node configuration and local layer coordinate system.

The H3L8N element is designated as $\text{TYPE} = U4$ and two versions are available incorporating complete quadratic and cubic stress expansions. These versions are denoted as $NVER = 11$ and $NVER = 12$, respectively. The H3L8N element is also supported for use as an end-element in which zero traction conditions are enforced in the $\tau_{xy}$ stress component. This version is selected by setting $NVER = 13$ and the $\text{NSIDE}$ input parameter is set to $i$ from the $Fi$ designations shown above to select the traction-free element face.

The performance of H3L8N has been shown to be accurate in the prediction of joint stresses in single-lap configurations with faster convergence rates obtained by using the higher-order cubic stress field in coarse mesh models.

4.2 3-D Adhesive Elements

All 2-D elements developed have a theoretical counterpart in a 3-D formulation, however, from the study of 2-D element behavior, a single 3-D solid element has been developed and incorporated into the user-element library.
4.3 The H2L12N Element

The H2L12N element configuration and local coordinate system are depicted in Figure 8.

![Diagram of H2L12N element and local layer coordinate system.]

Figure 8. H2L12N element and local layer coordinate system.

The H2L12N element, designated as TYPE = U5, permits a general adhesive joint planform to be modelled and is available in two versions incorporating complete linear and quadratic stress fields. These versions are designated as NVER = 11 and NVER = 12, respectively.

Studies have shown that, as in the case of the 2-D H2L6N element, the higher-order quadratic expansion yields improved coarse mesh performance - for finer levels of discretization along the bondline the distinction between the performance of the two versions vanish.

5 Demonstration Problems

The analysis of two single-lap joints are presented in this section. Results are taken from Reference [1] and used to demonstrate the use of two representative adhesive elements, namely, the 2-D H2L6N and 3-D H2L12N elements. The material properties selected are given by:

- **Adherend**: \( E = 69000.0 \quad \mu = 0.32 \)
- **Adhesive**: \( E = 3000.0 \quad \mu = 0.36 \)

All stresses are normalized as \( \sigma_{ij}^* = \sigma_{ij}/\sigma_{ref} \) where \( \sigma_{ref} = P/A \) in which \( P \) is an uniformly applied tensile load and \( A \) is the cross-sectional area of the adherend end.

5.1 Problem I: 2-D Single-Lap Joint

Figures 9 and 10 show the geometry and boundary conditions, respectively, of a 2-D single-lap joint. A state of plane strain is assumed to exist in the joint and H2L6N elements are used to model the adhesive and locally adjacent regions of the adhesive.
Figures 11 and 12 show the convergence of models incorporating 10, 25, 50 and 100 H2L6N elements along the bondline in comparison to a reference solution. Details of the model discretization is presented in Reference [1]. Stress predictions were made for the $\sigma_{yy}$ and $\tau_{xy}$ stress components along the adhesive/adherend interface. Element version 12, incorporating complete quadratic stress fields, was selected in generating these results. To show the improvement in element performance over standard displacement-based elements, the same models were used in which the layered adhesive elements were each replaced by two 4-node plane-strain elements (CPE4) from the ABAQUS library. As can be seen in Figures 13 and 14, the purely displacement-based solutions actually converge away from the reference solution which validates the improvement in element efficiency afforded by the layered hybrid formulation. The ABAQUS input deck and selected output associated with the refined model using the H2L6N element is presented in Appendix B.
Figure 11. H2L6N prediction of $\sigma_{yy}$ distribution along the bondline.

Figure 12. H2L6N prediction of $\tau_{xy}$ distribution along the bondline.
Figure 13. CPE4 Prediction of $\sigma_{yy}$ distribution along the bondline.

Figure 14. CPE4 prediction of $\tau_{xy}$ distribution along the bondline.
5.2 Problem II: 3-D Single-Lap Joint

A rectangular 3-D single lap joint is analyzed using H2L12N elements to model the bond region along the adhesive/adherend interface. The geometry is depicted in Figure 15 and the applied boundary conditions are identical to those presented above in figure 10.

![3-D Single-lap joint geometry.](image)

The convergence behavior is identical to that shown in the 2-D lap joint example presented above. For clarity, 3-D solutions are depicted for a model incorporating 100 elements along the bondline showing comparisons between the special layered hybrid adhesive element and standard displacement-based elements with a reference solution. In generating the displacement-based solution, the same model was used in which the layered H2L12N elements were each replaced by two 8-node brick elements (C3D8) from the ABAQUS library. Figures 16 and 17 show predictions for $\sigma_{zz}$ and $\tau_{yz}$ over the bond interface using the H2L12N element. The $\tau_{yz}$ shear stress component is essentially zero for this particular joint problem and is, therefore, not shown. The purely displacement-based element solution is shown in figures 18 and 19 and demonstrates a convergence away from the reference solution. The ABAQUS input deck and selected output is presented in Appendix C.
Figure 16. H2L12N prediction of $\sigma_{zz}$ distribution along the bondline.

Figure 17. H2L12N prediction of $\tau_{xz}$ distribution along the bondline.
Figure 18. C3D8 prediction of $\sigma_{zz}$ distribution along the bondline.

Figure 19. C3D8 prediction of $\tau_{zz}$ distribution along the bondline.
6 Conclusion

A variety of 2-D and 3-D special layered hybrid element formulations have been developed for the analysis of bondline stresses in adhesive joints. The hybrid stress method was selected to allow the explicit enforcement of layer domain equilibrium and interface continuity constraints. In addition, for the H2L6N and H3L8N elements, stress fields have been derived to enforce zero traction conditions along element sides. The elements demonstrate improved efficiency over similar displacement-based elements and are fully supported for use in the commercial finite element code ABAQUS through the development of a user-defined subroutine. The required input format has been detailed and element performance demonstrated in two example problems. Sample input and output datasets together with the complete source code performing all element computations have been included in separate appendices. The developed special adhesive elements provide an ideal basis for further enhancements such as the incorporation of nonlinear material and geometric capabilities to accurately model bondline stresses in complex adhesive joint designs.
References


APPENDIX A

Source code listing of subroutine UEL supporting special adhesive elements in ABAQUS.
** User Defined Element Subroutine UEL for the ABAQUS Code **
** Incorporating Special Layered Element Formulations for **
** The Analysis of Adhesive Joints. **

*********************************************************************************

Subroutine UEL(RHS, AMATRX, SVARS, ENERGY, NDOFEL, NRHS, NSVARS,
1    PROPS, NPREF, COORDS, MCRD, NNODE, U, DU, V, A,
2    JTYPE, TIME, DTIME, KSTEP, KINC, JELEM, PARAMS,
3    NDLoad, JDLTYP, ADLMAG, PREDEF, NPRED, LFLAGS,
4    MLVARX, DDLmag, MDLoad, PNEWDT)

Implicit Double Precision (A-H,O-Z)

Variable and Array Declaration for UEL/ABAQUS Interface

Dimension RHS(MVARX,*), AMATRAX(NDOFEL,NDOFEL), PROPS(*),
1    SVARS(1), ENERGY(8), COORDS(MCRD,NNODE), U(NDOFEL),
2    DU(MVARX), V(NDOFEL), A(NDOFEL), TIME(2),
3    PARAMS(*), JDLTYP(MDLoad,*), ADLMAG(MDLoad,*),
4    DDLmag(MDLoad,*), PREDEF(2,NPRED,NNODE), LFLAGS(5)

Dimension HINV(100,100), GMAT(100,36)

Data EPS / 1.0D-8 /

Test if ABAQUS is in the Element Data Recovery
Phase by Checking Displacements

Test = 0.0
Do I = 1, NDOFEL
   Test = Test + ABS(DU(I))
End Do

If (Test .lt. EPS) Then
   Compute Element Stiffness Matrix

   Call HSTIFF (AMATRX, PROPS, COORDS, HINV, GMAT, TEST, NPREF,
1      NDOFEL, MCRD, NNODE, JTYPE, JELEM, NBVAL )

   Set RHS Vector to Zero

   Call MXINT (RHS(i,1), NDOFEL, 1, 0.0D0)

Else If (Test .gt. EPS) Then
   Perform Requested Element Data Recovery

   Compute Element G and H Matrix

   Call HSTIFF (AMATRX, PROPS, COORDS, HINV, GMAT, TEST, NPREF,
1      NDOFEL, MCRD, NNODE, JTYPE, JELEM, NBVAL )

   Call RECOV (COORDS, PROPS, DU, HINV, GMAT, MLVAX, NDOFEL,
1      NPREFS, MCRD, NNODE, JTYPE, JELEM, NBVAL )

End If

Return

End

Subroutine HSTIFF (AMATRX, PROPS, COORDS, HINV, GSBL, TEST, NPREF,
1      NDOFEL, MCRD, NNODE, JTYPE, JELEM, NBVAL )

*********************************************************************************
** ADHESIVE ELEMENT STIFFNESS GENERATION **

IMPLICIT DOUBLE PRECISION (A-H, O-Z)

DIMENSION AMATRX(NDOFEL,NDOFEL), PROPS(*), COORDS(MCRD, NNODE)

DIMENSION HMAT(100, 100), HINV(100, 100), AJINV(3, 3),
1 XH(20), YH(20), ZH(20), TMP1(100, 100), TMP2(100, 100),
2 XSI(20), DETA(20), DCSEE(20), BMATS(6, 60)

DIMENSION SMAT(6, 6), PMAT(6, 100), THETA(100),
1 GMAT(100, 36), INDX1(36), GSMBL(100, 36), HSMBL(100, 100),
2 PTHK(100), E1(100), E2(100), E3(100), V12(100),
3 V23(100), V13(100), G12(100), G23(100), G31(100)

DIMENSION WDT(3), NLAY(3), THK(3), ETRN(6, 6), STRN(6, 6), TRI(3, 3),
1 CTRN(36, 36)

DATA EPS / 1.0D-8 /

READ IN ELEMENT DATA FROM PROPS ARRAY AND SET ELEMENT PARAMETERS

CALL ELDATA( PROPS, PTHK, THETA, E1, E2, E3, V12, V23, V13, G12, G23, G31,
1 WDT, THK, NORD, IPLANE, IOTYPE, NLDIM, NDOFN, NDOFK, 2,
1 NODNUM, JTYPE, NSIDE, NDV, INTDOF, NDOFL, MORD, NLAY, 3,
3 NVER)

CALL MXINT( GSMBL, 100, 36, 0.0D0 )
CALL MXINT( HSMBL, 100, 100, 0.0D0 )

CHECK GEOMETRY OF ELEMENTS ON INITIAL PAST

IF ( TEST .LT. EPS ) CALL VCHECK( COORDS, MCRD, NNODE, JTYPE, JELEM )

OBTAIN TRANSFORMATION MATRIX BETWEEN GLOBAL AND LOCAL ELEMENT COORDINATES

CALL TRANS( COORDS, CTRN, STRN, ETRN, TRI, JTYPE, MCRD, NNODE )

LOOP OVER AND ASSEMBLE ALL ELEMENT LAYERS

DO K = 1, NLAYR

LAYER = K
TFAC = 1.0
IF ( NLDIM .EQ. 2 ) TFAC = WDT(K)

N = (NODNUM-INTNOD) *(K-1)
DO I = 1, NODNUM
  AX = COORDS(1, I+N)
  BY = COORDS(2, I+N)
  CZ = COORDS(3, I+N)

TRANSFORM TO LOCAL COORDINATES

  XH(I) = TRI(1,1) *AX+TRI(1,2) *BY+TRI(1,3) *CZ
  YH(I) = TRI(2,1) *AX+TRI(2,2) *BY+TRI(2,3) *CZ
  ZH(I) = TRI(3,1) *AX+TRI(3,2) *BY+TRI(3,3) *CZ
END DO

COMPUTE ELEMENT MATERIAL PROPERTY MATRICES

CALL MATPROP( PTHK, THETA, E1, E2, E3, G12, G23, G31, V12, V23, V13,
1 SMAT, NLDIM, IPLANE, MORD, LAYER, NLAY)
CALL MXINT(HMAT, 100, 100, 0.0DO)
CALL MXINT(GMAT, 100, 36, 0.0DO)

STRAIN VECTOR CONVENTION: (EX, EY, EZ, TYZ, TXZ, TXY)

NRDZ = NORD
IF (NELDIM .EQ. 2) NRDZ = 1

DO IXSI = 1, NORD
   DO JETA = 1, NORD
      DO KCEE = 1, NRDZ

OBTAIN GAUSS POINTS AND WEIGHTS
CALL GAUSS(NORD, NELDIM, IXSI, JETA, KCEE, XSI, ETA, CEE, WEIGHT)

COMPUTE SHAPE FUNCTIONS AND THEIR DERIVATIVES AT
THE CURRENT GAUSS POINT
CALL SHAPE(NODNUM, NELDIM, XSI, ETA, CEE, DXSI, DETA, DCEE)

COMPUTE JACOBIAN MATRIX, ITS DETERMINANT AND
INVERSE
CALL JACOB(NODNUM, NELDIM, XH, YH, ZH, DXSI, DETA, DCEE,
            AJINV, DETJ)

COMPUTE MATRIX OF ASSUMED STRESS FUNCTIONS AT
CURRENT GAUSS POINT
CALL ASTRSS(XSI, ETA, CEE, JTYPE, LAYER, NELDIM, NODNUM,
             PMAT, THK, NBVAL, XH, YH, ZH, NSIDE, NVER)

COMPUTE THE STRAIN-DISPLACEMENT MATRIX BMATS
CALL BMAT(NELDIM, MORD, NODNUM, NDOFL, AJINV, DXSI, DETA,
            DCEE, BMATS)

FORM G AND H MATRICES
CALL MXATB(PMAT, BMATS, TMP2, 6, 6, 100, NBVAL, MORD, NDOFL)

INTEGRATE GMAT COEFFICIENTS
DO II = 1, NBVAL
   DO JJ = 1, NDOFL
      GMAT(II, JJ) = GMAT(II, JJ) + DETJ*WEIGHT*TFAC*TMP2(II, JJ)
   END DO
END DO

CALL MXMUL(SMAT, PMAT, TMP1, 6, 6, 100, MORD, MORD, NBVAL)
CALL MXATB(PMAT, TMP1, TMP2, 6, 100, 100, NBVAL, MORD, NBVAL)

INTEGRATE HMAT COEFFICIENTS
DO II = 1, NBVAL
   DO JJ = 1, NBVAL
      HMAT(II, JJ) = HMAT(II, JJ) + DETJ*WEIGHT*TFAC*TMP2(II, JJ)
   END DO
END DO

END DO
END DO

ASSEMBLE GMAT AND HMAT
N = (NODNUM-INTNOD) *(K-1) *NDOFN
DO I = 1, NBVAL
  DO J = 1, NDOFL
    GSMBL(I,J+N) = GSMBL(I,J+N) + GMAT(I,J)
  END DO
  DO J = 1, NBVAL
    HSMBL(I,J) = HSMBL(I,J) + HMAT(I,J)
  END DO
END DO

COMPUTE THE INVERSE OF HSMBL

NDIM = 100
CALL INVERS(HSMBL,HINV,INDX1,NDIM,NBVAL)

IF ( TEST .GT. EPS ) RETURN

COMPUTE STIFFNESS COEFFICIENTS

CALL MXMUL(HINV,GSMBL,TMP1,100,100,100,NBVAL,NBVAL,NDOFT)
CALL MXATB(GSMBL,TMP1,AMATRX,100,100,NDOFT,NDOFT,NBVAL,NDOFT)

TRANSFORM ELEMENT STIFFNESSES TO GLOBAL COORDINATES

CALL MXMUL(AMATRX,CTRN,TMP1,NDOFEL,36,100,NDOFEL,NDOFEL,NDOFEL)
CALL MXATB(CTRN,TMP1,AMATRX,36,100,NDOFEL,NDOFEL,NDOFEL)

RETURN

SUBROUTINE RECOV ( COORDS, PROPS, DU, HINV, GMAT, MLVARX, NDOFEL,
  NPROPS, MCRD, NNODE, JTYPE, JELEM, NEVAL )

******************************************************************************
** **
** ** PERFORM REQUESTED ELEMENT DATA RECOVERY **
** **
******************************************************************************

IMPLICIT DOUBLE PRECISION (A-H,O-Z)

DIMENSION PROPS(*), COORDS(MCRD,NNODE), DU(MLVARX)

DIMENSION HINV(100,100), TMP2(100,100), TMP1(100,100), AJINV(3,3),
  STRN(6,6), ETRN(6,6), XH(20), YH(20), ZH(20),
  ULI(36), UL(36), DXSI(20), DETA(20), DCEE(20)

DIMENSION PMAT(6,100), BMATS(6,60), GMAT(100,36), BETA(100),
  THETA(100), THK(3), PTHK(100), E1(100),
  E2(100), E3(100), V12(100), V23(100), V13(100),
  G12(100), G23(100), G31(100), TRI(3,3)

DIMENSION CTRN(36,36), SCOMP(27,6), ECOMP(27,6), NLAY(3)

CALL ELDATA( PROPS, PTHK, THETA, E1, E2, E3, V12, V23, V13, G12, G23, G31,
  WDT, THK, NORD, IPLANE, IOTYPE, NLAYR, NLDIM, NDOFN, INTNOD,
  NODNUM, JTYPE, NSIDE, HDV, INTDOF, NDOFT, NDOFL, MORD, NLAY,
  NVER )

IF ( IOTYPE .NE. 0 ) RETURN

WRITE(6,6794) JELEM

OBTAIN ORTHOGONAL AND TFNSORIAL TRANSFORMATION MATRICES FOR
DISPLACEMENTS, STRESSES AND STRAINS

CALL TRANS( COORDS, CTRN, STRN, ETRN, TRI, JTYPE, MCRD, NNODE )
TRANSFORM GLOBAL DISPLACEMENTS INTO LOCAL SYSTEM
CALL MXMUL (CTRN, DU, ULI, 36, MLVARX, 36, NDOFEL, NDOFEL, 1)

COMPUTE BETA VALUES FOR STRESS RECOVERY
CALL MXMUL (GMAT, ULI, TMP1, 100, 36, 100, NBVAL, NDOFEL, 1)
CALL MXMUL (HINV, TMP1, BETA, 100, 100, 100, NBVAL, NBVAL, 1)

LOOP OVER ALL ELEMENT LAYERS
DO K = 1, NLAG

CALL MXINT (SCOMP, 27, 6, 0.0D0)
CALL MXINT (ECOMP, 27, 6, 0.0D0)

LAYER = K

COMPUTE THE ELEMENT STRESS RECOVERY MATRICES
N = (NODNUM–INTNOD) * (K–1)
DO I = 1, NODNUM
  AX = COORDS (1, I+N)
  BY = COORDS (2, I+N)
  CZ = COORDS (3, I+N)

TRANSFORM TO LOCAL COORDINATES

XH (I) = TRI (1, 1) * AX + TRI (1, 2) * BY + TRI (1, 3) * CZ
YH (I) = TRI (2, 1) * AX + TRI (2, 2) * BY + TRI (2, 3) * CZ
ZH (I) = TRI (3, 1) * AX + TRI (3, 2) * BY + TRI (3, 3) * CZ
END DO

EXTRACT DISPLACEMENT SET FOR THE CURRENT LAYER
N = (NODNUM–INTNOD) * (K–1) * NBVAL
DO I = 1, NDOFL
  TL (I) = ULI (I+N)
END DO

COMPUTE STRESSES AND STRAINS AT SELECTED ELEMENT COORDINATES
IET = 0
CALL IOPNTS (IET, NTPS, JTYPE, NELDIM, NORD, LAYER, XSI, ETA, CEE)

DO IET = 1, NTPS

CALL IOPNTS (IET, NTPS, JTYPE, NELDIM, NORD, LAYER, XSI, ETA, CEE)

COMPUTE SHAPE FUNCTIONS AND THEIR DERIVATIVES AT THE CURRENT RECOVERY POINT
CALL SHAPE (NODNUM, NELDIM, XSI, ETA, CEE, DXSI, DETA, DCEE)

COMPUTE JACOBIAN MATRIX, ITS DETERMINANT AND INVERSE
CALL JACOB (NODNUM, NELDIM, XH, YH, ZH, DXSI, DETA, DCEE, AJINV, DETJ)

COMPUTE MATRIX OF ASSUMED STRESS FUNCTIONS AT CURRENT GAUSS POINT
CALL ASTRSS (XSI, ETA, CEE, JTYPE, LAYER, NELDIM, NODNUM, PMAT, THK, NBVAL, XH, YH, ZH, NSIDE, NVER)
COMPUTE THE STRAIN-DISPLACEMENT MATRIX BMATS

CALL BMAT (NELDIM, MORD, NODNUM, NDOFL, AJINV, DXSI, DETA, DCEE, BMATS)

COMPUTE ELEMENT STRAINS AT OUTPUT POINT

CALL MXMUL (BMATS, UL, TMP1, 6, 36, 100, MORD, NDOFL, 1)

DO IS = 1, MORD
  IF ( IOTYPE .EQ. 2) THEN
    TRANSFORM STRAINS IN LOCAL ELEMENT COORDINATES TO
    GLOBAL SYSTEM
    DO IT = 1, MORD
      ECOMP (IET, IS) = ECOMP (IET, IS) + STRN (IT, IS) * TMP1 (IT, 1)
    END DO
  ELSE
    ECOMP (IET, IS) = TMP1 (IS, 1)
  END IF
END DO

CALL MXMUL (PMAT, BETA, TMP2, 6, 100, 100, MORD, NBVAL, 1)

DO IS = 1, MORD
  IF ( IOTYPE .EQ. 2) THEN
    TRANSFORM STRESSES IN LOCAL ELEMENT COORDINATES TO
    GLOBAL SYSTEM
    DO IT = 1, MORD
      SCOMP (IET, IS) = SCOMP (IET, IS) + ETRN (IT, IS) * TMP2 (IT, 1)
    END DO
  ELSE
    SCOMP (IET, IS) = TMP2 (IS, 1)
  END IF
END DO

END DO

OUTPUT ELEMENT STRESSES AND STRAINS AT SELECTED POINTS

IF ( NELDIM .EQ. 2 ) THEN
  IF ( IOTYPE .EQ. 1 ) WRITE (6, 892) LAYER
  IF ( IOTYPE .EQ. 2 ) WRITE (6, 893) LAYER

  OUTPUT LAYER STRESSES
  DO IET = 1, NTPS
    CALL IOPNTS (IET, NTPS, JTYPE, NELDIM, NORD, LAYER,
    XSI, ETA, CEE)
    WRITE (6, 994) XSI, ETA, (SCOMP (IET, I), ECOMP (IET, I), I=1, 3)
  END DO
ELSE IF ( NELDIM .EQ. 3 ) THEN
  OUTPUT ELEMENT STRESSES AND STRAINS AT SELECTED POINTS
  IF ( IOTYPE .EQ. 1 ) WRITE (6, 894) LAYER
  IF ( IOTYPE .EQ. 2 ) WRITE (6, 895) LAYER

  OUTPUT LAYER STRESSES
DO IET = 1, NTPS
  CALL IOPNTS(IET, NTPS, JTYPE, NELDIM, NORD, LAYER, XSI, ETA, CEE)
  WRITE(6,995) XSI, ETA, CEE, (SCOMP(IET, I), I=1, 6)
END DO

IF ( IOTYPE .EQ. 1 ) WRITE(6,896) LAYER
IF ( IOTYPE .EQ. 2 ) WRITE(6,897) LAYER

OUTPUT LAYER STRAINS
DO IET = 1, NTPS
  CALL IOPNTS(IET, NTPS, JTYPE, NELDIM, NORD, LAYER, XSI, ETA, CEE)
  WRITE(6,995) XSI, ETA, CEE, (SCOMP(IET, I), I=1, 6)
END DO
END IF
END DO

FORMAT STATEMENTS FOR HYBRID ELEMENT OUTPUT

855 FORMAT(///,45X,'H Y B R I D E L E M E N T D A T A ',///)
848 FORMAT(///,'E L E M E N T I D: ',I5,///)
800 FORMAT(///,20X,'H Y B R I D S T I F F N E S S M A T R I X:',///)
801 FORMAT(///,10X11)
815 FORMAT(1X,I3,2X,10(E9.3,2X))
         1,I5,///,2X,'R E C O V E R Y P O I N T S',24X,'S T R E S S / S T R A I N',
         2'C O M P O N E N T S',//,3X,'C', CJ, 'SXX', 'SYY', 'SZZ',
         3'EXX', 'EYY', 'EZZ', 'SXY', 'SZZ', 'SXY')
         1,I5,///,2X,'R E C O V E R Y P O I N T S',24X,'S T R E S S / S T R A I N',
         2'C O M P O N E N T S',//,3X,'C', CJ, 'SXX', 'SYY', 'SZZ',
         3'EXX', 'EYY', 'EZZ', 'SXY', 'SZZ', 'SXY')
         1,9X,'R E C O V E R Y P O I N T S',29X,'S T R E S S C O M P O N E N T S',
         1,23X,'C', CJ, 'SXX', 'SYY', 'SZZ',
         3'EXX', 'EYY', 'EZZ', 'SXY', 'SZZ', 'SXY')
         1,9X,'R E C O V E R Y P O I N T S',29X,'S T R E S S C O M P O N E N T S',
         1,23X,'C', CJ, 'SXX', 'SYY', 'SZZ',
         3'EXX', 'EYY', 'EZZ', 'SXY', 'SZZ', 'SXY')
         1,9X,'R E C O V E R Y P O I N T S',29X,'S T R A I N C O M P O N E N T S',
         1,23X,'C', CJ, 'SXX', 'SYY', 'SZZ',
         3'EXX', 'EYY', 'EZZ', 'SXY', 'SZZ', 'SXY')
         1,9X,'R E C O V E R Y P O I N T S',29X,'S T R A I N C O M P O N E N T S',
         1,23X,'C', CJ, 'SXX', 'SYY', 'SZZ',
         3'EXX', 'EYY', 'EZZ', 'SXY', 'SZZ', 'SXY')
994 FORMAT(2(F7.4,2X),2X,6(E9.3,2X))
995 FORMAT(3(F7.4,2X),2X,12(E9.3,2X))
6794 FORMAT(/,'E L E M E N T I D ',I5,/)
SUBROUTINE VCHECK (COORDS, MCRD, NNODE, JTYPE, JELEM)

IMPLICIT DOUBLE PRECISION (A-H, O-Z)

DIMENSION COORDS(MCRD, NNODE), X(20), Y(20), Z(20)

DATA NONE /'0'/

TEST FOR IRREGULAR ELEMENT GEOMETRY BY CHECKING INTERNAL ANGLES IN ELEMENT LAYERS

DO I = 1, NNODE
   X(I) = COORDS(1, I)
   Y(I) = COORDS(2, I)
   Z(I) = COORDS(3, I)
END DO

INITIALIZE LAYER ERROR FLAGS

NERRL1 = 0
NERRL2 = 0
NERRL3 = 0

IF ( JTYPE .EQ. 1 ) THEN
   CALL ANGLE (X, Y, Z, 1, 2, 3, JELEM, NERRL1)
   CALL ANGLE (X, Y, Z, 2, 3, 4, JELEM, NERRL1)
   CALL ANGLE (X, Y, Z, 3, 4, 5, JELEM, NERRL1)
   CALL ANGLE (X, Y, Z, 6, 5, 4, JELEM, NERRL1)
ELSE IF ( JTYPE .EQ. 2 ) THEN
   CALL ANGLE (X, Y, Z, 1, 2, 5, JELEM, NERRL1)
   CALL ANGLE (X, Y, Z, 2, 6, 5, JELEM, NERRL1)
   CALL ANGLE (X, Y, Z, 6, 5, 10, JELEM, NERRL1)
   CALL ANGLE (X, Y, Z, 5, 6, 10, JELEM, NERRL1)
ELSE IF ( JTYPE .EQ. 3 ) THEN
   CALL ANGLE (X, Y, Z, 1, 3, 6, JELEM, NERRL1)
   CALL ANGLE (X, Y, Z, 3, 6, 8, JELEM, NERRL1)
   CALL ANGLE (X, Y, Z, 6, 8, 13, JELEM, NERRL1)
   CALL ANGLE (X, Y, Z, 8, 13, 11, JELEM, NERRL1)
ELSE IF ( JTYPE .EQ. 4 ) THEN
   CALL ANGLE (X, Y, Z, 1, 2, 3, JELEM, NERRL1)
   CALL ANGLE (X, Y, Z, 2, 3, 4, JELEM, NERRL1)
   CALL ANGLE (X, Y, Z, 3, 4, 5, JELEM, NERRL1)
   CALL ANGLE (X, Y, Z, 5, 6, 7, JELEM, NERRL1)
   CALL ANGLE (X, Y, Z, 6, 7, 8, JELEM, NERRL1)
ELSE IF ( JTYPE .EQ. 5 ) THEN
   CALL ANGLE (X, Y, Z, 1, 2, 5, JELEM, NERRL1)
   CALL ANGLE (X, Y, Z, 2, 5, 6, JELEM, NERRL1)
   CALL ANGLE (X, Y, Z, 5, 6, 2, JELEM, NERRL1)
   CALL ANGLE (X, Y, Z, 6, 2, 1, JELEM, NERRL1)
   CALL ANGLE (X, Y, Z, 2, 1, 5, JELEM, NERRL1)
   CALL ANGLE (X, Y, Z, 3, 5, 6, JELEM, NERRL1)
   CALL ANGLE (X, Y, Z, 5, 6, 3, JELEM, NERRL1)
   CALL ANGLE (X, Y, Z, 6, 3, 2, JELEM, NERRL1)
   CALL ANGLE (X, Y, Z, 3, 2, 4, JELEM, NERRL1)
   CALL ANGLE (X, Y, Z, 2, 4, 1, JELEM, NERRL1)
   CALL ANGLE (X, Y, Z, 4, 1, 8, JELEM, NERRL1)
   CALL ANGLE (X, Y, Z, 1, 8, 7, JELEM, NERRL1)
   CALL ANGLE (X, Y, Z, 8, 7, 5, JELEM, NERRL1)
   CALL ANGLE (X, Y, Z, 5, 7, 6, JELEM, NERRL1)
   CALL ANGLE (X, Y, Z, 7, 6, 1, JELEM, NERRL1)
END SUBROUTINE VCHECK
CALL ANGLE ( X, Y, Z, 12, 11, 8, JELEM, NERRL2 )
CALL ANGLE ( X, Y, Z, 5, 12, JELEM, NERRL2 )
CALL ANGLE ( X, Y, Z, 9, 12, 5, JELEM, NERRL2 )
END IF

IF ( NERRL1 .EQ. 1 .OR. NERRL2 .EQ. 1 .OR. NERRL3 .EQ. 1 ) THEN
  IF ( NONE .EQ. 0 ) THEN
    WRITE(6,100)
    NONE = 1
  END IF
END IF

IF ( NERRL1 .NE. 0 ) WRITE(6,10) JELEM
IF ( NERRL2 .NE. 0 ) WRITE(6,20) JELEM
IF ( NERRL3 .NE. 0 ) WRITE(6,30) JELEM

C

10 FORMAT(' ERROR - ELEMENT #',110,' IS DEFORMED IN LAYER 1')
20 FORMAT(' ERROR - ELEMENT #',110,' IS DEFORMED IN LAYER 2')
30 FORMAT(' ERROR - ELEMENT #',110,' IS DEFORMED IN LAYER 3')
100 FORMAT('///, ELEMENT LAYERS MUST BE OF RECTANGULAR GEOMETRY',///)

RETURN
END

SUBROUTINE ANGLE ( X, Y, Z, N1, N2, N3, JELEM, NERR )

IMPLICIT DOUBLE PRECISION (A-H,O-Z)

DIMENSION X(20), Y(20), Z(20)

PI = ACOS(-1.D0)

V1 = X(N2) - X(N1)
V2 = Y(N2) - Y(N1)
V3 = Z(N2) - Z(N1)
V4 = X(N3) - X(N1)
V5 = Y(N3) - Y(N1)
V6 = Z(N3) - Z(N1)

DOT = V1*V4 + V2*V5 + V3*V6
ADA = (V1*V1 + V2*V2 + V3*V3)**0.5
BDB = (V4*V4 + V5*V5 + V6*V6)**0.5
THETA = ABS(180*ACOS(DOT/(ADA*BDB))/PI)

IF ( THETA .GE. 95.0 .OR. THETA .LT. 85.0 ) NERR = 1
RETURN
END

SUBROUTINE ELDATA( PROPS,PTHK,THETA,E1,E2,E3,V12,V23,V13,G12,
1       G23,G31,MDT,THK,NORD,IPLANE,IOTYPE,NLAYR,
2       NELDIM,NDOFN,INTNOD,NODNUM,JTYPE,NSIDE,NDV,
3       INTDOF,NDOFT,NDOFL,MORD,NLAY,NVER )

IMPLICIT DOUBLE PRECISION (A-H,O-Z)

DIMENSION E1(100),E2(100),E3(100),V12(100),V23(100),V13(100),G12(100),
1       G23(100),G31(100),V12(100),V23(100),V13(100),PTHK(100),THETA(100),
2       PROPS(1),MDT(3),NLAY(3),THK(3)

NOTE: ALL PROPERTY VALUES MUST BE INPUTTED AS REAL NUMBERS ON THE UEL PROPERTY INPUT BLOCK
ELEMENT PARAMETERS:

- `NLAYR = N`: Number of layers in element
- `NELDIM = N`: Element dimension
- `NDOFN = N`: Number of degrees of freedom per node
- `INTNOD = N`: Number of nodes along layer interface
- `NODNUM = N`: Number of nodes per layer
- `INTDOF = N`: Total number of DOF along interface ( = INTNOD * NDOFN )
- `NDOF = N`: Total number of DOF per element ( = NDOFEL )
- `NDOFL = N`: Total number of DOF per layer ( = NODNUM * NDOFN )
- `NDV = NDOPN * NELDIM`: Dimension of isoparametric transformation matrix
- `NSIDE = N`: For zero traction condition on prescribed face N

CALL MXINT ( THK, 3, 1, 0.0D0 )

IF ( JTYPE .EQ. 1 ) THEN

***************
* H2L6N ELEMENT *
***************

LOCAL LAYER COORDINATE SYSTEM CONVENTION:

NLAYR = 2
NELDIM = 2
NDOFN = 2
INTNOD = 2
NODNUM = 4
NDV = NDOF * NELDIM
INTDOF = INTNOD * NDOFN
NDOF = 12
NDOFL = NODNUM * NDOFN
MORD = 3

ELEMENT INPUT PROPERTIES:

- `MVER`: Element version designation
- `IPLANE`: 0 for plane stress
- `1 for plane strain`
- `IOTYPE`: 0 to suppress output of element data
- `1 for element output in local coordinates
- `2 for element output in global coordinates`
- `NSIDE`: Element side designation for zero tractions
- `NLAY1`: Number of plies in element layer 1
- `NLAY2`: Number of plies in element layer 2
- `WDT (1)`: Depth dimension (width) of layer 1
- `WDT (2)`: Depth dimension (width) of layer 2
- `THK (1)`: Thickness of layer 1 (calculated from ply thickness)
- `THK (2)`: Thickness of layer 2 (calculated from ply thickness)
- `PTHK`: Ply thickness
THETA - PLY ORIENTATION
E1,E2,E3 - NORMAL MATERIAL MODULII
G12,G23,G31 - SHEAR MODULII
V12,V23,V13 - POISSON RATIOS

PROPERTY LIST FORMAT:
1) NVER, IPLANE, IOTYPE, NSIDE
2) NLAY1, WDT1,
3) PLTH, THETA, E1, E2, E3, V12, V23, V13
4) G12, G23, G31
   . REPEAT FOR EACH PLY IN LAYER 1
I) NLAY2, WDT2
J) PLTH, THETA, E1, E2, E3, V12, V23, V13
K) G12, G23, G31
   . REPEAT FOR EACH PLY IN LAYER 2

EXTRACT ELEMENT DATA OFF PROPS ARRAY

NVER = INT(PROPS(1))
IPLANE = INT(PROPS(2))
IOTYPE = INT(PROPS(3))
NSIDE = INT(PROPS(4))
NLAY(1) = INT(PROPS(9))
WDT(1) = PROPS(10)
DO I = 1, NLAY(1)
   N = I*16
   PTHK(I) = PROPS(N+1)
   THK(1) = THK(1) + PTHK(I)
   THETA(I) = PROPS(N+2)
   E1(I) = PROPS(N+3)
   E2(I) = PROPS(N+4)
   E3(I) = PROPS(N+5)
   V12(I) = PROPS(N+6)
   V23(I) = PROPS(N+7)
   V13(I) = PROPS(N+8)
   G12(I) = PROPS(N+9)
   G23(I) = PROPS(N+10)
   G31(I) = PROPS(N+11)
   END
DO
NLAY(2) = INT(PROPS(16*NLAY(1)+17))
WDT(2) = PROPS(16*NLAY(1)+18)
M = NLAY(1)
DO I = 1, NLAY(2)
   N = 24+16*(M+I-1)
   PTHK(I+M) = PROPS(N+1)
   THK(2) = THK(2) + PTHK(I+M)
   THETA(I+M) = PROPS(N+2)
   E1(I+M) = PROPS(N+3)
   E2(I+M) = PROPS(N+4)
   E3(I+M) = PROPS(N+5)
   V12(I+M) = PROPS(N+6)
   V23(I+M) = PROPS(N+7)
   V13(I+M) = PROPS(N+8)
   G12(I+M) = PROPS(N+9)
   G23(I+M) = PROPS(N+10)
   G31(I+M) = PROPS(N+11)
   END

SET GAUSSIAN INTEGRATION ORDER

IF ( NVER .EQ. 11 ) NORD = 2
IF ( NVER .EQ. 12 ) NORD = 3
IF ( NVER .EQ. 13 ) NORD = 3
ELSE IF ( JTYPE .EQ. 2 ) THEN

***************
* H2L10N ELEMENT *
***************

LOCAL LAYER COORDINATE SYSTEM CONVENTION:

9--------Y--------10
| ^ | |
| 7 |--> X 8 |
| | |
5--------Y--------6
| ^ | |
| 3 |--> X 4 |
| | |
1--------2

NLAYR = 2
NLDIM = 2
NDOFN = 2
INTNOD = 2
NODNUM = 6
NDDV = NDOFN*NLDIM
INTDOF = INTNOD*NDOFN
NDOTT = 20
NDOFL = NODNUM*NDOFN
MORD = 3

ELEMENT INPUT PROPERTIES:

NVER - ELEMENT VERSION DESIGNATION
IPLANE - 0 FOR PLANE STRESS
1 FOR PLANE STRAIN
IOTYPE - 0 TO SUPPRESS OUTPUT OF ELEMENT DATA
1 FOR ELEMENT OUTPUT IN LOCAL COORDINATES
2 FOR ELEMENT OUTPUT IN GLOBAL COORDINATES
NLAY1 - NUMBER OF PLIES IN ELEMENT LAYER 1
NLAY2 - NUMBER OF PLIES IN ELEMENT LAYER 2
WDT(1) - DEPTH DIMENSION (WIDTH) OF LAYER 1
WDT(2) - DEPTH DIMENSION (WIDTH) OF LAYER 2
THK(1) - THICKNESS OF LAYER 1 (CALCULATED FROM PLY THICKNESS)
THK(2) - THICKNESS OF LAYER 2 (CALCULATED FROM PLY THICKNESS)
PHTK - PLY THICKNESS
THETA - PLY ORIENTATION
E1,E2,E3 - NORMAL MATERIAL MODULII
G12,G23,G31 - SHEAR MODULII
V12,V23,V13 - POISSON RATIOS

PROPERTY LIST FORMAT:

1) NVER, IPLANE, IOTYPE
2) NLAY1, WDT1,
3) PHTK, THETA, E1, E2, E3, V12, V23, V13
4) G12, G23, G31
   . REPEAT FOR EACH PLY IN LAYER 1
6) NLAY2, WDT2
7) PHTK, THETA, E1, E2, E3, V12, V23, V13

31
K) G12, G23, G31

. REPEAT FOR EACH PLY IN LAYER 2

EXTRACT ELEMENT DATA OFF PROPS ARRAY

NVER = INT(PROPS(1))
IPLANE = INT(PROPS(2))
IOTYPE = INT(PROPS(3))
NLAY(1) = INT(PROPS(9))
WDT(1) = PROPS(10)
DO I = 1, NLAY(1)
   N = I^16
   PTHK(I) = PROPS(N+1)
   THK(I) = THK(1) + PTHK(I)
   THETA(I) = PROPS(N+2)
   E1(I) = PROPS(N+3)
   E2(I) = PROPS(N+4)
   E3(I) = PROPS(N+5)
   V12(I) = PROPS(N+6)
   V23(I) = PROPS(N+7)
   V13(I) = PROPS(N+8)
   G12(I) = PROPS(N+9)
   G23(I) = PROPS(N+10)
   G31(I) = PROPS(N+11)
END DO

NLAY(2) = INT(PROPS(16*NLAY(1)+17))
WDT(2) = PROPS(16*NLAY(1)+18)
M = NLAY(1)
DO I = 1, NLAY(2)
   N = 24+16*(M+I-1)
   PTHK(I+M) = PROPS(N+1)
   THK(2) = THK(2) + PTHK(I+M)
   THETA(I+M) = PROPS(N+2)
   E1(I+M) = PROPS(N+3)
   E2(I+M) = PROPS(N+4)
   E3(I+M) = PROPS(N+5)
   V12(I+M) = PROPS(N+6)
   V23(I+M) = PROPS(N+7)
   V13(I+M) = PROPS(N+8)
   G12(I+M) = PROPS(N+9)
   G23(I+M) = PROPS(N+10)
   G31(I+M) = PROPS(N+11)
END DO

SET GAUSSIAN INTEGRATION ORDER

IF ( NVER .EQ. 11 ) NORD = 4
IF ( NVER .EQ. 12 ) NORD = 5
ELSE IF ( JTYPE .EQ. 3 ) THEN

***********************
* H2L13N ELEMENT *
***********************

LOCAL LAYER COORDINATE SYSTEM CONVENTION:

11--------12--------13
| Y |
|  |
|  |
|  |
9 ----> X 10
ELEMENT INPUT PROPERTIES:

NVER - ELEMENT VERSION DESIGNATION
IPLANE - 0 FOR PLANE STRESS
         1 FOR PLANE STRAIN
IOTYPE - 0 TO SUPPRESS OUTPUT OF ELEMENT DATA
         1 FOR ELEMENT OUTPUT IN LOCAL COORDINATES
         2 FOR ELEMENT OUTPUT IN GLOBAL COORDINATES
NLAY1 - NUMBER OF PLIES IN ELEMENT LAYER 1
NLAY2 - NUMBER OF PLIES IN ELEMENT LAYER 2
WDT(1) - DEPTH DIMENSION (WIDTH) OF LAYER 1
WDT(2) - DEPTH DIMENSION (WIDTH) OF LAYER 2
THK(1) - THICKNESS OF LAYER 1 (CALCULATED FROM PLY THICKNESS)
THK(2) - THICKNESS OF LAYER 2 (CALCULATED FROM PLY THICKNESS)

FTHK - PLY THICKNESS
THETA - PLY ORIENTATION
E1,E2,E3 - NORMAL MATERIAL MODULII
G12,G23,G31 - SHEAR MODULII
V12,V23,V13 - POISSON RATIOS

PROPERTY LIST FORMAT:

1) NVER,IPLANE,IOTYPE
2) NLAY1,WDT1,
3) PHTK,THETA,E1,E2,E3,V12,V23,V13
4) G12,G23,G31
   . REPEAT FOR EACH PLY IN LAYER 1
   
I) NLAY2,WDT2
J) PHTK,THETA,E1,E2,E3,V12,V23,V13
K) G12,G23,G31
   . REPEAT FOR EACH PLY IN LAYER 2

EXTRACT ELEMENT DATA OFF PROPS ARRAY

NVER = INT(PROPS(1))
IPLANE = INT(PROPS(2))
IOTYPE = INT(PROPS(3))
NSIDE = INT(PROPS(4))
NLAY(1) = INT(PROPS(9))
WDT(1) = PROPS(10)
DO I = 1, NLAY(1)
N = I*16
PTHK(I) = PROPS(N+1)
THK(1) = THK(1) + PTHK(I)
THETA(I) = PROPS(N+2)
E1(I) = PROPS(N+3)
E2(I) = PROPS(N+4)
E3(I) = PROPS(N+5)
V12(I) = PROPS(N+6)
V23(I) = PROPS(N+7)
V13(I) = PROPS(N+8)
G12(I) = PROPS(N+9)
G23(I) = PROPS(N+10)
G31(I) = PROPS(N+11)
END DO
NLAY(2) = INT(PROPS(16*NLAY(1)+17))
WDT(2) = PROPS(16*NLAY(1)+18)
M = NLAY(1)
DO I = 1, NLAY(2)
N = 24+16*(M+I-1)
PTHK(I+M) = PROPS(N+1)
THK(2) = THK(2) + PTHK(I+M)
THETA(I+M) = PROPS(N+2)
E1(I+M) = PROPS(N+3)
E2(I+M) = PROPS(N+4)
E3(I+M) = PROPS(N+5)
V12(I+M) = PROPS(N+6)
V23(I+M) = PROPS(N+7)
V13(I+M) = PROPS(N+8)
G12(I+M) = PROPS(N+9)
G23(I+M) = PROPS(N+10)
G31(I+M) = PROPS(N+11)
END DO

SET GAUSSIAN INTEGRATION ORDER

IF ( NVER .EQ. 11 ) NORD = 4
IF ( NVER .EQ. 12 ) NORD = 5
ELSE IF ( JTYPE .EQ. 4 ) THEN

***************
* H3L8N ELEMENT *
***************

LOCAL LAYER COORDINATE SYSTEM CONVENTION:

```
7--------Y--------8
 |       ^
 | 1-----> X

5--------Y--------6
 |       ^
 | 1-----> X F2

F1

3--------Y--------4
 |       ^
 | 1-----> X

1--------------------2
```
NLAYR  =  3
NELDIM =  2
NDOFN =  2
INTNOD =  2
NODNUM =  4
NDV  =  NDOFN*NELDIM
INTDOF = INTNOD*NDOFN
NDOFT =  16
NDOFL =  NODNUM*NDOFN
MORD =  3

ELEMENT INPUT PROPERTIES:

NVER - ELEMENT VERSION DESIGNATION
IPLANE - 0 FOR PLANE STRESS
         1 FOR PLANE STRAIN
IOTYPE - 0 TO SUPPRESS OUTPUT OF ELEMENT DATA
         1 FOR ELEMENT OUTPUT IN LOCAL COORDINATES
         2 FOR ELEMENT OUTPUT IN GLOBAL COORDINATES
NSIDE - ELEMENT SIDE DESIGNATION FOR ZERO TRACTIONS
NLAY1 - NUMBER OF PLIES IN ELEMENT LAYER 1
NLAY2 - NUMBER OF PLIES IN ELEMENT LAYER 2
NLAY3 - NUMBER OF PLIES IN ELEMENT LAYER 3
WDT(1) - DEPTH DIMENSION (WIDTH) OF LAYER 1
WDT(2) - DEPTH DIMENSION (WIDTH) OF LAYER 2
WDT(3) - DEPTH DIMENSION (WIDTH) OF LAYER 3
THK(1) - THICKNESS OF LAYER 1 (CALCULATED FROM PLY THICKNESS)
THK(2) - THICKNESS OF LAYER 2 (CALCULATED FROM PLY THICKNESS)
THK(3) - THICKNESS OF LAYER 3 (CALCULATED FROM PLY THICKNESS)

PTHK  - PLY THICKNESS
THETA - PLY ORIENTATION
E1,E2,E3 - NORMAL MATERIAL MODULII
G12,G23,G31 - SHEAR MODULII
V12,V23,V13 - POISSON RATIOS

PROPERTY LIST FORMAT:

1) NTYPE,IPLANE,IOTYPE
2) NLAY1,WDT1,
3) PHTK,THETA,E1,E2,E3,V12,V23,V13
4) G12,G23,G31
   : REPEAT FOR EACH PLY IN LAYER 1

I) NLAY2,WDT2
J) PHTK,THETA,E1,E2,E3,V12,V23,V13
K) G12,G23,G31
   : REPEAT FOR EACH PLY IN LAYER 2

L) NLAY3,WDT3
M) PHTK,THETA,E1,E2,E3,V12,V23,V13
N) G12,G23,G31
   : REPEAT FOR EACH PLY IN LAYER 3

EXTRACT ELEMENT DATA OFF PROPS ARRAY

NVER  = INT(PROPS(1))
IPLANE = INT(PROPS(2))
IOTYPE = INT(PROPS(3))
NSIDE = INT(PROPS(4))
NLAY(1) = INT(PROPS(9))
WDT(1) = PROPS(10)
DO I = 1, NLAY(1)
    N = I*16
PTHK(1) = PROPS(N+1)
THK(1) = THK(1)' + PTHK(1)
THETA(I) = PROPS(N+2)
E1(I) = PROPS(N+3)
E2(I) = PROPS(N+4)
E3(I) = PROPS(N+5)
V12(I) = PROPS(N+6)
V23(I) = PROPS(N+7)
V13(I) = PROPS(N+8)
G12(I) = PROPS(N+9)
G23(I) = PROPS(N+10)
G31(I) = PROPS(N+11)

END DO
NLAY(2) = INT(PROPS(16*NLAY(1)+17))
WDT(2) = PROPS(16*NLAY(1)+18)
M = NLAY(1)
DO I = 1, NLAY(2)
   N = 24+16*(M+I-1)
   PTHK(I+M) = PROPS(N+1)
   THK(2) = THK(2) + PTHK(I+M)
   THETA(I+M) = PROPS(N+2)
   E1(I+M) = PROPS(N+3)
   E2(I+M) = PROPS(N+4)
   E3(I+M) = PROPS(N+5)
   V12(I+M) = PROPS(N+6)
   V23(I+M) = PROPS(N+7)
   V13(I+M) = PROPS(N+8)
   G12(I+M) = PROPS(N+9)
   G23(I+M) = PROPS(N+10)
   G31(I+M) = PROPS(N+11)
END DO

NLAY(3) = INT(PROPS(16*NLAY(1)+NLAY(2)+25))
WDT(3) = PROPS(16*NLAY(1)+NLAY(2)+26)
M = NLAY(1)+NLAY(2)
DO I = 1, NLAY(3)
   N = 32+16*(M+I-1)
   PTHK(I+M) = PROPS(N+1)
   THK(3) = THK(3) + PTHK(I+M)
   THETA(I+M) = PROPS(N+2)
   E1(I+M) = PROPS(N+3)
   E2(I+M) = PROPS(N+4)
   E3(I+M) = PROPS(N+5)
   V12(I+M) = PROPS(N+6)
   V23(I+M) = PROPS(N+7)
   V13(I+M) = PROPS(N+8)
   G12(I+M) = PROPS(N+9)
   G23(I+M) = PROPS(N+10)
   G31(I+M) = PROPS(N+11)
END DO

SET GAUSSIAN INTEGRATION ORDER

IF ( NVER .EQ. 11 ) NORD = 3
IF ( NVER .EQ. 12 ) NORD = 4
ELSE IF ( JTYPE .EQ. 5 ) THEN

***************
* H2L12N ELEMENT *
***************

NODE NUMBERING CONVENTION:

   12--------11
   |    /|
   |   / |
   |  /  |
   9--------10
   |    /|
   |   / |
   |  /  |

36
LOCAL LAYER COORDINATE SYSTEM CONVENTION:

\[ \begin{align*}
Z, W & \quad Y, V \\
I+7 & \quad I+6 \\
I+4 & \quad I+5 \\
I+3 & \quad \rightarrow X, U \\
I+2 & \quad \\
i & \quad I+1 \\
\end{align*} \]

NLAYR = 2
NELDIM = 3
NDOFN = 3
INTNOD = 4
NODNUM = 8
NDV = NDOFN*NELDIM
INTDOF = INTNOD*NDOFN
NDOFL = NODNUM*NDOFN
NDOFT = 36
MORD = 6

ELEMENT INPUT PROPERTIES:

NVER - ELEMENT VERSION DESIGNATION
IOTYPE - 0 TO SUPPRESS OUTPUT OF ELEMENT DATA
1 FOR ELEMENT OUTPUT IN LOCAL COORDINATES
2 FOR ELEMENT OUTPUT IN GLOBAL COORDINATES
NLAY1 - NUMBER OF PLYS IN ELEMENT LAYER 1
NLAY2 - NUMBER OF PLYS IN ELEMENT LAYER 2
THK(1) - THICKNESS OF LAYER 1 (CALCULATED FROM PLY THICKNESS)
THK(2) - THICKNESS OF LAYER 2 (CALCULATED FROM PLY THICKNESS)

PTHK - PLY THICKNESS
THETA - PLY ORIENTATION
E1, E2, E3 - NORMAL MATERIAL MODULII
G12, G23, G31 - SHEAR MODULII
V12, V23, V13 - POISSON RATIOS

PROPERTY LIST FORMAT:

1) NVER, IOTYPE
2) NLAY1
3) PTHK, THETA, E1, E2, E3, V12, V23, V13
4) G12, G23, G31
   : REPEAT FOR EACH PLY IN LAYER 1
5) NLAY2
6) PTHK, THETA, E1, E2, E3, V12, V23, V13
7) G12, G23, G31
. REPEAT FOR EACH PLY IN LAYER 2

EXTRACT ELEMENT DATA OFF PROPS ARRAY

NVER = INT(PROPS(1))
IOTYPE = INT(PROPS(2))
NLAY(1) = INT(PROPS(9))
DO I = 1, NLAY(1)
   N = I*16
   PTHK(I) = PROPS(N+1)
   THK(1) = THK(1) + PTHK(I)
   THETA(I) = PROPS(N+2)
   E1(I) = PROPS(N+3)
   E2(I) = PROPS(N+4)
   E3(I) = PROPS(N+5)
   V12(I) = PROPS(N+6)
   V23(I) = PROPS(N+7)
   V13(I) = PROPS(N+8)
   G12(I) = PROPS(N+9)
   G23(I) = PROPS(N+10)
   G31(I) = PROPS(N+11)
END DO
NLAY(2) = INT(PROPS(16*NLAY(1)+17))
M = NLAY(1)
DO I = 1, NLAY(2)
   N = 24+16*(M+I-1)
   PTHK(I+M) = PROPS(N+1)
   THK(2) = THK(2) + PTHK(I+M)
   THETA(I+M) = PROPS(N+2)
   E1(I+M) = PROPS(N+3)
   E2(I+M) = PROPS(N+4)
   E3(I+M) = PROPS(N+5)
   V12(I+M) = PROPS(N+6)
   V23(I+M) = PROPS(N+7)
   V13(I+M) = PROPS(N+8)
   G12(I+M) = PROPS(N+9)
   G23(I+M) = PROPS(N+10)
   G31(I+M) = PROPS(N+11)
END DO

SET GAUSSIAN INTEGRATION ORDER

IF ( NVER .EQ. 11 ) NORD = 2
IF ( NVER .EQ. 12 ) NORD = 3
END IF
RETURN
END

SUBROUTINE B MAT (ND, MORD, NODNUM, NDOFL, AJINV, DXSI, DETA,
1   DCEE, BMATS)

IMPLICIT DOUBLE PRECISION (A-H, O-Z)

DIMENSION AJINV(3,3), DXSI(20), DETA(20), DCEE(20), BMATS(6,60)
DIMENSION TMP1(36,36), HMATS(6,9), TMAT(9,9), UMAT(9,60)

CALL MXINT( HMATS, 6, 9, 0.0D0)
CALL MXINT( TMAT, 9, 9, 0.0D0)
CALL MXINT( UMAT, 6, 60, 0.0D0)

IF ( ND .EQ. 2 ) THEN
   HMATS(1,1) = 1.0
HMACS(2,4) = 1.0
HMACS(3,2) = 1.0
HMACS(3,3) = 1.0

ELSE IF ( ND = 3 ) THEN

HMACS(1,1) = 1.0
HMACS(2,5) = 1.0
HMACS(3,9) = 1.0
HMACS(4,6) = 1.0
HMACS(4,8) = 1.0
HMACS(5,3) = 1.0
HMACS(5,7) = 1.0
HMACS(6,2) = 1.0
HMACS(6,4) = 1.0

END IF

DO I = 1, ND
  DO J = 1, ND
    IF ( ND = 2 ) THEN
      TMAT(I,J) = AJINV(I,J)
      TMAT(I+ND,J+ND) = AJINV(I,J)
    ELSE IF ( ND = 3 ) THEN
      TMAT(I,J) = AJINV(I,J)
      TMAT(I+ND,J+ND) = AJINV(I,J)
      TMAT(I+2*ND,J+2*ND) = AJINV(I,J)
    END IF
  END DO
END DO

COMPUTE THE TRANSFORMATION MATRIX UMAT

IF ( ND = 2 ) THEN
  DO J = 1, NODNUM
    UMAT(1,2*(J-1)+1) = DCSI(J)
    UMAT(2,2*(J-1)+1) = DETA(J)
    UMAT(3,2*(J-1)+2) = DCSI(J)
    UMAT(4,2*(J-1)+2) = DETA(J)
  END DO
ELSE IF ( ND = 3 ) THEN
  DO J = 1, NODNUM
    UMAT(1,3*(J-1)+1) = DCSI(J)
    UMAT(2,3*(J-1)+1) = DETA(J)
    UMAT(3,3*(J-1)+1) = DCE(J)
    UMAT(4,3*(J-1)+2) = DCSI(J)
    UMAT(5,3*(J-1)+2) = DETA(J)
    UMAT(6,3*(J-1)+2) = DCE(J)
    UMAT(7,3*(J-1)+3) = DCSI(J)
    UMAT(8,3*(J-1)+3) = DETA(J)
    UMAT(9,3*(J-1)+3) = DCE(J)
  END DO
END IF

NDV = ND**2
CALL MDXNL(TMNT,UMAT,TMP1,9,9,36,NDV,NDV,NDOFL)
CALL MDXNL(HMACS,TMP1,BMATS,6,36,6,MORD,NDV,NDOFL)
RETURN
END

SUBROUTINE TRANS ( COORDS,CTR,N,T,T,TRI,JTYPE,MCD,NODE )
IMPLICIT REAL*8 (A-H,O-Z)
CALCULATE TRANSFORMATION MATRICES FOR CONVERTING QUANTITIES
BETWEEN ELEMENT AND GLOBAL COORDINATE SYSTEMS

DIMENSION X(20), Y(20), Z(20), EO(3, 3),
          EP(3, 3), TRI(3, 3), STRN(6, 6),
          ETRN(6, 6), CTRN(36, 36), COORDS(MCRD, NNODE)

CALL MXINT ( CTRN, 36, 36, 0.0D0 )
CALL MXINT ( STRN, 6, 6, 0.0D0 )
CALL MXINT ( ETRN, 6, 6, 0.0D0 )

DO I = 1, NNODE
    X(I) = COORDS(1, I)
    Y(I) = COORDS(2, I)
    Z(I) = COORDS(3, I)
END DO

UNIT VECTORS IN GLOBAL SYSTEM

EO(1, 1) = 1.0
EO(1, 2) = 0.0
EO(1, 3) = 0.0
EO(2, 1) = 0.0
EO(2, 2) = 1.0
EO(2, 3) = 0.0
EO(3, 1) = 0.0
EO(3, 2) = 0.0
EO(3, 3) = 1.0

DETERMINE ELEMENT COORDINATE VECTORS

IF ( JTYPE .EQ. 1 .OR. JTYPE .EQ. 2 .OR.
1               JTYPE .EQ. 4 ) THEN

    AL = SQRT( (X(2)-X(1))**2+(Y(2)-Y(1))**2 )
    EP(1, 1) = (X(2)-X(1))/AL
    EP(1, 2) = (Y(2)-Y(1))/AL
    AL = SQRT( (X(3)-X(1))**2+(Y(3)-Y(1))**2 )
    EP(2, 1) = (X(3)-X(1))/AL
    EP(2, 2) = (Y(3)-Y(1))/AL

    NDIM = 2

ELSE IF ( JTYPE .EQ. 3 ) THEN

    AL = SQRT( (X(3)-X(1))**2+(Y(3)-Y(1))**2 )
    EP(1, 1) = (X(3)-X(1))/AL
    EP(1, 2) = (Y(3)-Y(1))/AL
    AL = SQRT( (X(6)-X(1))**2+(Y(6)-Y(1))**2 )
    EP(2, 1) = (X(6)-X(1))/AL
    EP(2, 2) = (Y(6)-Y(1))/AL

    NDIM = 2

ELSE IF ( JTYPE .EQ. 5 ) THEN

    AL = SQRT( (X(2)-X(1))**2+(Y(2)-Y(1))**2+(Z(2)-Z(1))**2 )
    EP(1, 1) = (X(2)-X(1))/AL
    EP(1, 2) = (Y(2)-Y(1))/AL
    EP(1, 3) = (Z(2)-Z(1))/AL
    AL = SQRT( (X(4)-X(1))**2+(Y(4)-Y(1))**2+(Z(4)-Z(1))**2 )
    EP(2, 1) = (X(4)-X(1))/AL
    EP(2, 2) = (Y(4)-Y(1))/AL
    EP(2, 3) = (Z(4)-Z(1))/AL
    AL = SQRT( (X(5)-X(1))**2+(Y(5)-Y(1))**2+(Z(5)-Z(1))**2 )
    EP(3, 1) = (X(5)-X(1))/AL
    EP(3, 2) = (Y(5)-Y(1))/AL
    EP(3, 3) = (Z(5)-Z(1))/AL

    C
NDIM = 3

END IF

CONSTRUCT COORDINATE TRANSFORMATION MATRIX

\[ \{x'\} = [\text{TRI}]\{x\}; \quad \{x\} = [\text{TRI}]\{x'\} \]

DO I = 1, NDIM
  DO J = 1, NDIM
    TRI(I,J) = 0.0
  DO K = 1, NDIM
    TRI(I,J) = TRI(I,J) + EP(I,K)*EO(J,K)
  END DO
END DO

END DO

DO I = 1, NNODE
  NF = NDIM*(I-1)
  DO J = 1, NDIM
    CTRN(NF+J,NF+K) = TRI(J,K)
  END DO
END DO

END DO

CONSTRUCT STRESS AND STRAIN TRANSFORMATION MATRICES
TO CONVERT BETWEEN ELEMENT COORDINATE SYSTEM AND
GLOBAL SYSTEM

[STRN] = STRESS TRANSFORMATION
[ETRN] = STRAIN TRANSFORMATION

\[ \{\epsilon'\} = [\text{ETRN}]\{\epsilon\}; \quad \{\sigma'\} = [\text{STRN}]\{\sigma\} \]

IF ( NDIM .EQ. 2 ) THEN

[sx,sy,sxy]

STRESS TRANSFORMATION MATRIX, [STRN]:

\[
\begin{align*}
\text{STRN}(1,1) &= \text{TRI}(1,1)^2 \\
\text{STRN}(1,2) &= \text{TRI}(2,1)^2 \\
\text{STRN}(1,3) &= 2 \times \text{TRI}(1,1) \times \text{TRI}(2,1) \\
\text{STRN}(2,1) &= \text{TRI}(1,2)^2 \\
\text{STRN}(2,2) &= \text{TRI}(2,2)^2 \\
\text{STRN}(2,3) &= 2 \times \text{TRI}(1,2) \times \text{TRI}(2,2) \\
\text{STRN}(3,1) &= \text{TRI}(1,1) \times \text{TRI}(1,2) \\
\text{STRN}(3,2) &= \text{TRI}(2,1) \times \text{TRI}(2,2) \\
\text{STRN}(3,3) &= \text{TRI}(1,1) \times \text{TRI}(2,2) + \text{TRI}(2,1) \times \text{TRI}(1,2)
\end{align*}
\]

ELSE IF ( NDIM .EQ. 3 ) THEN

[sx,sy,sz,syx,szx,sxy]

STRESS TRANSFORMATION MATRIX, [STRN]:

\[
\begin{align*}
\text{STRN}(1,1) &= \text{TRI}(1,1)^2 \\
\text{STRN}(1,2) &= \text{TRI}(1,2)^2 \\
\text{STRN}(1,3) &= \text{TRI}(1,3)^2 \\
\text{STRN}(1,4) &= 2 \times \text{TRI}(1,2) \times \text{TRI}(1,3) \\
\text{STRN}(1,5) &= 2 \times \text{TRI}(1,3) \times \text{TRI}(1,1)
\end{align*}
\]
$TRY \text{(1, 6)} - 2 \text{TRI}(1,1) \text{TRI}(1,2)
STRN(2,1) = \text{TRI}(2,1) \text{TRI}(2,2)
STRN(2,2) = \text{TRI}(2,2) \text{TRI}(2,3)
STRN(2,3) = \text{TRI}(2,3) \text{TRI}(2,4)
STRN(2,4) = 2 \text{TRI}(2,2) \text{TRI}(2,3)
STRN(2,5) = 2 \text{TRI}(2,3) \text{TRI}(2,1)
STRN(2,6) = 2 \text{TRI}(2,1) \text{TRI}(2,2)
STRN(3,1) = \text{TRI}(3,1) \text{TRI}(3,2)
STRN(3,2) = \text{TRI}(3,2) \text{TRI}(3,3)
STRN(3,3) = \text{TRI}(3,3) \text{TRI}(3,4)
STRN(3,4) = 2 \text{TRI}(3,2) \text{TRI}(3,3)
STRN(3,5) = 2 \text{TRI}(3,3) \text{TRI}(3,1)
STRN(3,6) = 2 \text{TRI}(3,1) \text{TRI}(3,2)
STRN(4,1) = \text{TRI}(2,1) \text{TRI}(3,1)
STRN(4,2) = \text{TRI}(2,2) \text{TRI}(3,2)
STRN(4,3) = \text{TRI}(2,3) \text{TRI}(3,3)
STRN(4,4) = \text{TRI}(2,2) \text{TRI}(3,3) + \text{TRI}(3,2) \text{TRI}(2,3)
STRN(4,5) = \text{TRI}(2,3) \text{TRI}(3,1) + \text{TRI}(3,3) \text{TRI}(2,1)
STRN(4,6) = \text{TRI}(2,1) \text{TRI}(3,2) + \text{TRI}(3,3) \text{TRI}(2,2)
STRN(5,1) = \text{TRI}(3,1) \text{TRI}(1,1)
STRN(5,2) = \text{TRI}(3,2) \text{TRI}(1,2)
STRN(5,3) = \text{TRI}(3,3) \text{TRI}(1,3)
STRN(5,4) = \text{TRI}(3,2) \text{TRI}(1,3) + \text{TRI}(1,2) \text{TRI}(3,3)
STRN(5,5) = \text{TRI}(3,3) \text{TRI}(1,1) + \text{TRI}(1,3) \text{TRI}(3,1)
STRN(5,6) = \text{TRI}(3,1) \text{TRI}(1,2) + \text{TRI}(1,1) \text{TRI}(3,2)
STRN(6,1) = \text{TRI}(1,1) \text{TRI}(2,1)
STRN(6,2) = \text{TRI}(1,2) \text{TRI}(2,2)
STRN(6,3) = \text{TRI}(1,3) \text{TRI}(2,3)
STRN(6,4) = \text{TRI}(1,2) \text{TRI}(2,3) + \text{TRI}(2,2) \text{TRI}(1,3)
STRN(6,5) = \text{TRI}(1,3) \text{TRI}(2,1) + \text{TRI}(2,3) \text{TRI}(1,1)
STRN(6,6) = \text{TRI}(1,1) \text{TRI}(2,2) + \text{TRI}(2,1) \text{TRI}(1,2)

END IF

STRAIN TRANSFORMATION MATRIX, [ETRN]:

DO I = 1, NDIM
  DO J = 1, NDIM
    ETRN(I,J) = STRN(I,J)
    ETRN(I,J+NDIM) = STRN(I,J+NDIM) / 2.0
    ETRN(I+NDIM,J) = STRN(I+NDIM,J) / 0.5
    ETRN(I+NDIM,J+NDIM) = STRN(I+NDIM,J+NDIM)
  END DO
END DO

RETURN
END

SUBROUTINE MATPROP (PTHK, THETA, E1, E2, E3, G12, G23, G31, V12, V23, V13, F1, V21, V32, V31, SMAT, NELDIM, IPLANE, MORD, NLAY)

COMPUTATION OF ELEMENT MATERIAL PROPERTY MATRICES

IMPLICIT DOUBLE PRECISION (A-H, O-Z)

DIMENSION E1(100), E2(100), E3(100), V12(100), V23(100), V13(100),
1 G12(100), G23(100), G31(100), V21(100), V32(100), V31(100),
2 THETA(100), PTHK(100), DMAT(6,6), SMAT(6,6), QBAR(6,6),
3 CL(6,6), INDEX(6), NLAY(3)

FI = ACOS(-1.000)
TLM = 0.0
CALL MXINT(DMAT, 6, 6, 0.000)
CALL MXINT(QBAR, 6, 6, 0.000)

NOFF = 1
DO I = 1, K-1
  NOFF = NOFF + NLAY(I)
END DO
NL = NLAY(K) + NOFF - 1

IF ( NELDID .EQ. 2 ) THEN
  [EX, EY, GXY]
IF ( IPLANE .EQ. 0 ) THEN
  PLANE STRESS
  DO I = NOFF, NL
    PHI = THETA(I) * PI / 180.0
    C = COS(PHI)
    S = SIN(PHI)
    C2 = C*C
    S2 = S*S
    C3 = C2*C
    S3 = S2*S
    C4 = C2*C2
    S4 = S2*S2
    TLM = TLM + PTHK(I)
    V21(I) = V12(I)*E2(I)/E1(I)
    Q11 = E1(I)/(1.0-V12(I)*V21(I))
    Q12 = V21(I)*Q11
    Q22 = E2(I)/(1.0-V12(I)*V21(I))
    Q66 = G12(I)
    QBAR(1,1) = Q11*C4+2.*(Q12+2.*Q66)*C2*S2+Q22*S4
    QBAR(1,2) = (Q11+Q22-4.*Q66)*S2*C2+Q12*(C4+S4)
    QBAR(1,3) = (Q11-Q12-2.*Q66)*S*C3+(Q12-Q22+2.*Q66)*S3*C
    QBAR(2,1) = QBAR(1,2)
    QBAR(2,2) = Q11*S4+2.*(Q12+2.*Q66)*S2*C2+Q22*C4
    QBAR(2,3) = (Q11-Q12-2.*Q66)*S*C3+(Q12-Q22+2.*Q66)*S*C3
    QBAR(3,1) = QBAR(1,3)
    QBAR(3,2) = QBAR(2,3)
    QBAR(3,3) = (Q11+Q22-2.*Q12-2.*Q66)*S2*C2+Q66*(S4+C4)
  END DO
END DO
ELSE
  PLANE STRAIN
  DO I = NOFF, NL
    PHI = THETA(I) * PI / 180.0
    C = COS(PHI)
    S = SIN(PHI)
    C2 = C*C
    S2 = S*S
    C3 = C2*C
    S3 = S2*S
    DO L = 1, 3
      DO J = 1, 3
        DMAT(L,J) = DMAT(L,J) + QBAR(L,J) * PTHK(I)
      END DO
    END DO
  END DO
C4 = C2*C2
S4 = S2*S2

TLM = TLM + PTHK(I)
S11 = 1./E1(I)
S12 = -V12(I)/E1(I)
S13 = -V13(I)/E1(I)
S22 = 1./E2(I)
S23 = -V23(I)/E2(I)
S33 = 1./E3(I)
S66 = 1./G12(I)
R11 = S11 - S13**2/S33
R12 = S12 - S13*S23/S33
R22 = S22 - S23**2/S33
R33 = S66
Q11 = R22/(R11*R22-R12**2)
Q12 = -R12/(R11*R22-R12**2)
Q22 = R11/(R11*R22-R12**2)
Q66 = 1./R33
QBAR(1,1) = Q11*C4+2.*(Q12+2.*Q66) *C2*S24.Q22*S4
QBAR(1,2) = (Q11+Q22-4.*Q66) *S2*C2+Q12*C4+Q4
QBAR(1,3) = (Q11-Q12-2.*Q66) *S*C3+(Q12-Q22+2.*Q66) *S3*C
QBAR(2,1) = QBAR(1,2)
QBAR(2,2) = Q11*S4+2.*(Q12+2.*Q66) *S2*C2+Q22*C4
QBAR(2,3) = (Q11-Q12-2.*Q66) *S*C3+(Q12-Q22+2.*Q66) *S*C3
QBAR(3,1) = QBAR(1,3)
QBAR(3,2) = QBAR(2,3)
QBAR(3,3) = (Q11+Q22-2.*Q12-2.*Q66) *S2*C2+Q66*(S4+C4)

DO L = 1, 3
  DO J = 1, 3
    DMAT(L,J) = DMAT(L,J) + QBAR(L,J) * PTHK(I)
  END DO
END DO

ELSE IF ( NELDIM .EQ. 3 ) THEN

[EX, EY, EZ, GYZ, G2, GXY]

DO I = NOFF, NL
  V21(I) = V12(I) * (E2(I)/E1(I))
  V32(I) = V23(I) * (E3(I)/E2(I))
  V31(I) = V13(I) * (E3(I)/E1(I))
  DEL = -1.0-V12(I)*V21(I)-V23(I)*V32(I)-V13(I)*V31(I)-
       2.*V21(I)*V32(I)*V13(I))/(E1(I)*E2(I)*E3(I))

  CL(1,1) = ( 1.0000 - V23(I)*V32(I) ) / ( E2(I)*E3(I)*DEL )
  CL(1,2) = ( V12(I) + V32(I)*V13(I) ) / ( E1(I)*E3(I)*DEL )
  CL(1,3) = ( V13(I) + V12(I)*V23(I) ) / ( E1(I)*E2(I)*DEL )
  CL(2,1) = ( 1.0000 - V13(I)*V31(I) ) / ( E1(I)*E3(I)*DEL )
  CL(2,2) = ( V23(I) + V13(I)*V21(I) ) / ( E1(I)*E2(I)*DEL )
  CL(2,3) = ( V21(I) + V23(I)*V13(I) ) / ( E1(I)*E2(I)*DEL )
  CL(3,1) = CL(1,3)
  CL(3,2) = CL(2,3)
  CL(4,4) = G23(I)
  CL(5,5) = G31(I)

.END IF

END
CL(6, 6) = G12(I)

COMPUTE THE KTH REDUCED C MATRIX (PLATE COORDINATES)

C0 = COS(PHI)
C2 = C0*C0
C3 = C0*C0*C0
C4 = C2*C2
S0 = SIN(PHI)
S2 = S0*S0
S3 = S0*S0*S0
S4 = S2*S2
C2T = COS(2.*PHI)

QBAR(1,1) = CL(1,1)*C4 + CL(2,2)*S4 + 2*CL(1,2)*S2*C2 + 4*CL(6,6)*C2*S2
1 QBAR(2,2) = CL(1,1)*S4 + CL(2,2)*C4 + 2*(CL(1,2)*S2*C2
1 2*CL(6,6))*S2*C2
QBAR(3,3) = CL(3,3)
QBAR(4,4) = CL(4,4)*C2 + CL(5,5)*S2
QBAR(5,5) = CL(5,5)*C2 + CL(4,4)*S2
QBAR(6,6) = CL(6,6) + (CL(1,1) + CL(2,2) - 2*CL(1,2) - 4*CL(6,6))*S2*C2
1 QBAR(1,2) = CL(1,2) + (CL(1,1) + CL(2,2) - 2*CL(1,2) - 4*CL(6,6))*S2*C2
QBAR(2,1) = QBAR(1,2)
QBAR(1,3) = CL(1,3)*C2 + CL(2,3)*S2
QBAR(3,1) = QBAR(1,3)
QBAR(2,3) = CL(1,3)*S2 + CL(2,3)*C2
QBAR(3,2) = QBAR(2,3)
QBAR(1,6) = C0*S0*(CL(1,1)*C2 - CL(2,2)*S2 - C2T*(CL(1,2) + 2*CL(6,6)))
1 QBAR(6,1) = QBAR(1,6)
QBAR(2,6) = S0*C0*(CL(1,1)*S2 - CL(2,2)*C2 + C2T*(CL(1,2) + 2*CL(6,6)))
1

TLM = TLM + PTHK(I)

DO L = 1, 6
  DO J = 1, 6
    DMAT(L,J) = DMAT(L,J) + QBAR(L,J)*PTHK(I)
  END DO
END DO

DO L = 1, 6
  DO J = 1, 6
    DMAT(L,J) = DMAT(L,J)/TLM
  END DO
END DO

END IF

COMPUTE SMAT = DMAT

NDIM = 6
CALL INVERS(DMAT, SMAT, INDEX, NDIM, MORD)

RETURN
SUBROUTINE SHAPE ( NODNUM, NELDIM, XSI, ETA, CEE, DXSI, DETA, DCEE )
COMPUTATION OF SHAPE FUNCTIONS AND THEIR DERIVATIVES
IMPLIED DOUBLE PRECISION (A-H,O-Z)
DIMENSION SFUNC(20),DXSI(20),DETA(20),DCEE(20)
IF ( NELDIM .EQ. 2 ) THEN
  IF ( NODNUM .EQ. 4 ) THEN
    SFUNC(1) = 0.25*(1.-XSI)*(1.-ETA)
    SFUNC(2) = 0.25*(1.+XSI)*(1.-ETA)
    SFUNC(3) = 0.25*(1.-XSI)*(1.+ETA)
    SFUNC(4) = 0.25*(1.+XSI)*(1.+ETA)
    DXSI(1) = -0.25*(1.-ETA)
    DXSI(2) = 0.25*(1.-ETA)
    DXSI(3) = -0.25*(1.+ETA)
    DXSI(4) = 0.25*(1.+ETA)
    DETA(1) = -0.25*(1.-XSI)
    DETA(2) = -0.25*(1.+XSI)
    DETA(3) = 0.25*(1.-XSI)
    DETA(4) = 0.25*(1.+XSI)
  ELSE IF ( NODNUM .EQ. 6 ) THEN
    SFUNC(1) = 0.25*(1.-XSI)*(1.-ETA) -
               0.25*(1.-XSI)*(1.-ETA**2)
    SFUNC(2) = 0.25*(1.+XSI)*(1.-ETA) -
               0.25*(1.+XSI)*(1.-ETA**2)
    SFUNC(3) = 0.50*(1.-XSI)*(1.-ETA**2)
    SFUNC(4) = 0.50*(1.+XSI)*(1.-ETA**2)
    SFUNC(5) = 0.25*(1.-XSI)*(1.+ETA) -
               0.25*(1.-ETA**2)
    SFUNC(6) = 0.25*(1.+XSI)*(1.+ETA) -
               0.25*(1.+ETA**2)
    DXSI(1) = -0.25*(1.-ETA) + 0.25*(1.-ETA**2)
    DXSI(2) = 0.25*(1.-ETA) - 0.25*(1.-ETA**2)
    DXSI(3) = -0.50*(1.-ETA**2)
    DXSI(4) = 0.50*(1.-ETA**2)
    DXSI(5) = -0.25*(1.+ETA) + 0.25*(1.-ETA**2)
    DXSI(6) = 0.25*(1.+ETA) - 0.25*(1.-ETA**2)
    DETA(1) = -0.25*(1.-XSI) + 0.50*ETA*(1.-XSI)
    DETA(2) = -0.25*(1.+XSI) + 0.50*ETA*(1.+XSI)
    DETA(3) = -ETA*(1.-XSI)
    DETA(4) = -ETA*(1.+XSI)
    DETA(5) = 0.25*(1.-XSI) + 0.50*ETA*(1.-XSI)
    DETA(6) = 0.25*(1.+XSI) + 0.50*ETA*(1.+XSI)
  ELSE IF ( NODNUM .EQ. 8 ) THEN
    SFUNC(1) = 0.25*(1.-XSI)*(1.-ETA) -
               0.25*(1.-XSI)*(1.-ETA**2)
    SFUNC(2) = 0.25*(1.-XSI)*(1.-ETA**2)
    SFUNC(3) = 0.50*(1.-XSI**2)*(1.-ETA)
    SFUNC(4) = 0.50*(1.-XSI**2)*(1.-ETA)
    SFUNC(5) = 0.50*(1.+XSI)*(1.-ETA**2)
    SFUNC(6) = 0.25*(1.-XSI)*(1.+ETA) -
1
0.25*(1.-XSI)*(1.-ETA**2) -
0.25*(1.-XSI**2)*(1.+ETA)
SFUNC(7) = 0.50*(1.-XSI**2)*(1.+ETA)
SFUNC(8) = 0.25*(1.+XSI)*(1.+ETA) -
0.25*(1.-XSI**2)*(1.+ETA)
1
0.25*(1.+XSI)*(1.-ETA**2)
2

C

DXSI(1) = -0.25*(1.-ETA) + 0.5*XSI*(1.-ETA) +
0.25*(1.-ETA**2)
1
DXSI(2) = -XSI*(1.-ETA)
DXSI(3) = 0.25*(1.-ETA) + 0.5*XSI*(1.-ETA) -
1
0.25*(1.-ETA**2)
DXSI(4) = -0.50*(1.-ETA**2)
DXSI(5) = 0.50*(1.-ETA**2)
DXSI(6) = -0.25*(1.+ETA) + 0.25*(1.-ETA**2) +
1
0.50*XSI*(1.+ETA)
DXSI(7) = -XSI*(1.+ETA)
DXSI(8) = 0.25*(1.+ETA) + 0.5*XSI*(1.+ETA) -
1
0.25*(1.-ETA**2)

C

DETA(1) = -0.25*(1.-XSI) + 0.5*ETA*(1.-XSI) +
0.25*(1.-XSI**2)
1
DETA(2) = -0.50*(1.-XSI**2)
DETA(3) = -0.25*(1.+XSI) + 0.5*ETA*(1.+XSI) +
1
0.25*(1.-XSI**2)
DETA(4) = -ETA*(1.-XSI)
DETA(5) = -ETA*(1.+XSI)
DETA(6) = 0.25*(1.-XSI) - 0.25*(1.-XSI**2) +
1
0.50*ETA*(1.-XSI)
DETA(7) = 0.50*(1.-XSI**2)
DETA(8) = 0.25*(1.+XSI) + 0.5*ETA*(1.+XSI) -
1
0.25*(1.-XSI**2)

C

END IF

C
ELSE IF ( NELDIM .EQ. 3 ) THEN

C
SFUNC(1) = 0.125*(1.-XSI)*(1.-ETA)*(1.-CEE)
SFUNC(2) = 0.125*(1.+XSI)*(1.-ETA)*(1.-CEE)
SFUNC(3) = 0.125*(1.+XSI)*(1.+ETA)*(1.-CEE)
SFUNC(4) = 0.125*(1.-XSI)*(1.+ETA)*(1.-CEE)
SFUNC(5) = 0.125*(1.-XSI)*(1.-ETA)*(1.+CEE)
SFUNC(6) = 0.125*(1.+XSI)*(1.-ETA)*(1.+CEE)
SFUNC(7) = 0.125*(1.+XSI)*(1.+ETA)*(1.+CEE)
SFUNC(8) = 0.125*(1.-XSI)*(1.+ETA)*(1.+CEE)

C

DXSI(1) = -0.125*(1.-ETA)*(1.-CEE)
DXSI(2) = 0.125*(1.-ETA)*(1.-CEE)
DXSI(3) = 0.125*(1.+ETA)*(1.-CEE)
DXSI(4) = -0.125*(1.+ETA)*(1.-CEE)
DXSI(5) = -0.125*(1.-ETA)*(1.+CEE)
DXSI(6) = 0.125*(1.-ETA)*(1.+CEE)
DXSI(7) = 0.125*(1.+ETA)*(1.+CEE)
DXSI(8) = -0.125*(1.+ETA)*(1.+CEE)

C

DETA(1) = -0.125*(1.-XSI)*(1.-CEE)
DETA(2) = -0.125*(1.+XSI)*(1.-CEE)
DETA(3) = 0.125*(1.+XSI)*(1.-CEE)
DETA(4) = -0.125*(1.+XSI)*(1.-CEE)
DETA(5) = -0.125*(1.-XSI)*(1.+CEE)
DETA(6) = 0.125*(1.-XSI)*(1.+CEE)
DETA(7) = 0.125*(1.+XSI)*(1.+CEE)
DETA(8) = 0.125*(1.-XSI)*(1.+CEE)

C

DCEE(1) = -0.125*(1.-XSI)*(1.-ETA)
DCEE(2) = -0.125*(1.+XSI)*(1.-ETA)
DCEE(3) = -0.125*(1.+XSI)*(1.+ETA)
DCEE(4) = -0.125*(1.-XSI)*(1.+ETA)
SUBROUTINE JACOB(NODNUM, NELDIM, X, Y, Z, DXSI, DETA, DCEE, 
AJINV, DETJ)

COMPUTE JACOBIAN MATRIX, ITS DETERMINANT AND INVERSE

IMPLICIT DOUBLE PRECISION (A-H, O-Z)

DIMENSION X(20), Y(20), Z(20), DXSI(20), DETA(20), DCEE(20), 
AJMT(3,3), AJINV(3,3)

CALL MXINT(AJMT, 3, 3, 0.0D0)

IF (NELDIM .EQ. 2) THEN

DO I = 1, NODNUM

AJMT(1, 1) = AJMT(1, 1) + DXSI(I) * X(I)
AJMT(1, 2) = AJMT(1, 2) + DXSI(I) * Y(I)
AJMT(1, 3) = AJMT(1, 3) + DXSI(I) * Z(I)
AJMT(2, 1) = AJMT(2, 1) + DETA(I) * X(I)
AJMT(2, 2) = AJMT(2, 2) + DETA(I) * Y(I)
AJMT(2, 3) = AJMT(2, 3) + DETA(I) * Z(I)
AJMT(3, 1) = AJMT(3, 1) + DCEE(I) * X(I)
AJMT(3, 2) = AJMT(3, 2) + DCEE(I) * Y(I)
AJMT(3, 3) = AJMT(3, 3) + DCEE(I) * Z(I)

END DO

DETJ = AJMT(1, 1) * AJMT(2, 2) - AJMT(2, 1) * AJMT(1, 2)

ELSE IF (NELDIM .EQ. 3) THEN

DO I = 1, NODNUM

AJMT(1, 1) = AJMT(1, 1) + DXSI(I) * X(I)
AJMT(1, 2) = AJMT(1, 2) + DXSI(I) * Y(I)
AJMT(1, 3) = AJMT(1, 3) + DXSI(I) * Z(I)
AJMT(2, 1) = AJMT(2, 1) + DETA(I) * X(I)
AJMT(2, 2) = AJMT(2, 2) + DETA(I) * Y(I)
AJMT(2, 3) = AJMT(2, 3) + DETA(I) * Z(I)
AJMT(3, 1) = AJMT(3, 1) + DCEE(I) * X(I)
AJMT(3, 2) = AJMT(3, 2) + DCEE(I) * Y(I)
AJMT(3, 3) = AJMT(3, 3) + DCEE(I) * Z(I)

END DO

DETJ = AJMT(1, 1) * (AJMT(2, 2) * AJMT(3, 3) - AJMT(2, 3) * AJMT(3, 2)) - 1
AJMT(1, 2) * (AJMT(2, 1) * AJMT(3, 3) - AJMT(2, 3) * AJMT(3, 1)) +
2 AJMT(1, 3) * (AJMT(2, 1) * AJMT(3, 2) - AJMT(2, 2) * AJMT(3, 1))

END IF

RETURN
END
SUBROUTINE ASTRSS(XSI, ETA, CEE, JTYPE, LAYER, NELDIM, NODNUM, PMAT, THK, NBDVL, XE, YE, ZE, NSIDE, NVER)

IMPLICIT DOUBLE PRECISION (A-H, O-Z)

DIMENSION PMAT(6, 100), XE(20), YE(20), ZE(20), THK(3)

IF ( NELDIM .EQ. 2 ) THEN
    IF ( NODNUM .EQ. 4 ) THEN
        AC = (-XE(1)+XE(2)-XE(3)+XE(4))/4.0
        BC = (-YE(1)-YE(2)+YE(3)+YE(4))/4.0
        X = AC*XSI
        Y = BC*ETA
        A = (XE(2)-XE(1))/2.0
    ELSE IF ( NODNUM .EQ. 6 ) THEN
        AC = (-XE(3)+XE(4))/2.0
        BC = (-YE(1)-YE(2)+YE(5)+YE(6))/4.0
        X = AC*XSI
        Y = BC*ETA
    ELSE IF ( NODNUM .EQ. 8 ) THEN
        AC = (-XE(4)+XE(5))/2.0
        BC = (-YE(2)+YE(7))/2.0
        X = AC*XSI
        Y = BC*ETA
        A = (XE(2)-XE(1))/2.0
    END IF
ELSE IF ( NELDIM .EQ. 3 ) THEN
    IF ( NODNUM .EQ. 8 ) THEN
        AC = (-XE(1)+XE(2)+XE(3)-XE(4)-XE(5)+XE(6)+XE(7)-XE(8))/8.0
        BC = (-YE(1)-YE(2)+YE(3)+YE(4)-YE(5)-YE(6)+YE(7)+YE(8))/8.0
        CC = (-ZE(1)-ZE(2)-ZE(3)-ZE(4)+ZE(5)+ZE(6)+ZE(7)+ZE(8))/8.0
        X = AC*XSI
        Y = BC*ETA
        Z = CC*CEE
    END IF
CALL MXINT(PMAT, 6, 100, 0.0D0)
IF ( JTYPE .EQ. 1 ) THEN

2-D 2-LAYERED 6-NODE HYBRID ELEMENT.

OPTIONAL ZERO TRACTION CONDITIONS MAY BE SPECIFIED ON DESIGNATED ELEMENT SIDES.

\[ \begin{align*}
&\text{Y} \\
&\quad | \\
&\quad | \\
&\quad 5------------6 \\
&\quad | \\
&\quad F1 \quad | \\
&\quad | \\
&\quad 3------------4 \\
&\quad | \\
&\quad F2 \quad | \\
&\quad | \\
&\quad 1------------2 \quad >X
\end{align*} \]

\[ \begin{align*}
T1 &= \text{THK}(1)/2. \\
T2 &= \text{THK}(2)/2.
\end{align*} \]

IF ( NVER .EQ. 11 ) THEN

LINEAR STRESS FIELD

\[ \text{NBVAL} = 10 \]

IF ( LAYER .EQ. 1 ) THEN

\[ \begin{align*}
\text{PMAT}(1,1) &= 1.0 \\
\text{PMAT}(1,2) &= \text{X} \\
\text{PMAT}(1,3) &= \text{Y} \\
\text{PMAT}(2,4) &= 1.0 \\
\text{PMAT}(2,5) &= \text{X} \\
\text{PMAT}(2,7) &= -\text{Y} \\
\text{PMAT}(3,6) &= 1.0 \\
\text{PMAT}(3,7) &= \text{X} \\
\text{PMAT}(3,2) &= -\text{Y}
\end{align*} \]

ELSE IF ( LAYER .EQ. 2 ) THEN

\[ \begin{align*}
\text{PMAT}(1,8) &= 1.0 \\
\text{PMAT}(1,9) &= \text{X} \\
\text{PMAT}(1,10) &= \text{Y} \\
\text{PMAT}(2,4) &= 1.0 \\
\text{PMAT}(2,5) &= \text{X} \\
\text{PMAT}(2,7) &= -(\text{T1}+\text{T2}+\text{Y}) \\
\text{PMAT}(3,6) &= 1.0 \\
\text{PMAT}(3,2) &= -\text{T1} \\
\text{PMAT}(3,7) &= \text{X} \\
\text{PMAT}(3,9) &= -(\text{T2}+\text{Y})
\end{align*} \]

END IF

ELSE IF ( NVER .EQ. 12 ) THEN

QUADRATIC STRESS FIELD

\[ \text{NBVAL} = 18 \]
IF (LAYER.EQ.1) THEN

PMAT(1,1) = 1.0
PMAT(1,2) = -X
PMAT(1,3) = Y
PMAT(1,4) = -2*X*Y
PMAT(1,5) = -0.5*X**2
PMAT(1,6) = Y**2
PMAT(2,7) = 1.0
PMAT(2,8) = -2*(T1+T2)*X - 2*X*Y
PMAT(2,9) = -X
PMAT(2,10) = T1*Y - 0.5*Y**2
PMAT(2,11) = -Y
PMAT(2,12) = -T2*Y
PMAT(2,13) = 1.0
PMAT(3,5) = -T1*X + X*Y
PMAT(3,10) = X
PMAT(3,11) = -2*T2*X
PMAT(3,2) = T1
PMAT(3,8) = X**2
PMAT(3,4) = Y**2

ELSE IF (LAYER.EQ.2) THEN

PMAT(1,14) = 1.0
PMAT(1,15) = -X
PMAT(1,16) = Y
PMAT(1,17) = -2*X*Y
PMAT(1,11) = -0.5*X**2
PMAT(1,18) = Y**2
PMAT(2,7) = 1.0
PMAT(2,5) = 0.5*T1**2
PMAT(2,10) = -(T1+T2) - Y
PMAT(2,11) = (T1*T2+0.5*T2**2) - 0.5*Y**2
PMAT(2,9) = X
PMAT(2,8) = -2*X*Y
PMAT(2,12) = X**2
PMAT(3,13) = 1.0
PMAT(3,2) = T1
PMAT(3,4) = T1**2
PMAT(3,15) = T2 + Y
PMAT(3,17) = -T2**2 + Y**2
PMAT(3,10) = X
PMAT(3,11) = X*Y
PMAT(3,8) = X**2

END IF

ELSE IF (NVER.EQ.13 .AND. NSIDE.EQ.1) THEN

TRACTIONS SX & TXY SET TO ZERO ON FACE F1

NBVAL = 15

IF (LAYER.EQ.1) THEN

PMAT(1,1) = 1.0
PMAT(1,2) = -X
PMAT(1,3) = Y
PMAT(1,4) = -2*X*Y
PMAT(1,5) = -0.5*X**2
PMAT(1,6) = Y**2
PMAT(2,5) = -0.5*T1**2 + T1*Y - 0.5*Y**2
PMAT(2,7) = -(T1+T2) + Y
PMAT(2,8) = 1.0
PMAT(2,9) = -T1*T2 + T2*Y
PMAT(2,10) = 2*T1*X - 2*X*Y
PMAT(2,11) = X
PMAT(2,12) = -T2*X
PMAT(2,13) = X**2
PMAT(3,2) = -T1 + Y
PMAT(3,4) = -T1**2 + Y**2
PMAT(3,7) = -A - X
PMAT(3,10) = -A**2 + X**2
PMAT(3,9) = -A*T2 - T2*X
PMAT(3,5) = -T1*X + X*Y

C
ELSE IF ( LAYER .EQ. 2 ) THEN
C
PMAT(1,9) = -A*(X+A)
PMAT(1,14) = (X**2-A**2)
PMAT(1,15) = Y*(X+A)
PMAT(2,8) = 1.0
PMAT(2,11) = X
PMAT(2,7) = Y
PMAT(2,12) = X*Y
PMAT(2,13) = X**2
PMAT(3,7) = -(X+A)
PMAT(3,10) = (X**2-A**2)
PMAT(3,9) = Y*(X+A)
C
END IF
C
ELSE IF ( NVER .EQ. 13 .AND. NSIDE .EQ. 2 ) THEN
C
TRACTIONS SX & TXY SET TO ZERO ON FACE F2
C
NBVAL = 15
C
IF ( LAYER .EQ. 1 ) THEN
C
PMAT(1,9) = -A*(X+A)
PMAT(1,14) = (X**2-A**2)
PMAT(1,15) = Y*(X+A)
PMAT(2,8) = 1.0
PMAT(2,11) = X
PMAT(2,7) = Y
PMAT(2,12) = X*Y
PMAT(2,13) = X**2
PMAT(3,7) = -(X+A)
PMAT(3,10) = (X**2-A**2)
PMAT(3,9) = Y*(X+A)
C
ELSE IF ( LAYER .EQ. 2 ) THEN
C
PMAT(1,1) = 1.0
PMAT(1,2) = -X
PMAT(1,3) = Y
PMAT(1,4) = -2*X*Y
PMAT(1,5) = -0.5*X**2
PMAT(1,6) = Y**2
PMAT(2,5) = -0.5*T2*X**2 - T2*Y - 0.5*Y**2
PMAT(2,7) = (T1+T2) + Y
PMAT(2,8) = 1.0
PMAT(2,9) = -T1*T2 - T1*Y
PMAT(2,10) = -2*T2*X - 2*X*Y
PMAT(2,11) = X
PMAT(2,12) = T1*X
PMAT(2,13) = X**2
PMAT(3,2) = T2 + Y
PMAT(3,4) = -T2**2 + Y**2
PMAT(3,7) = -A - X
PMAT(3,10) = -A**2 + X**2
PMAT(3,9) = A*T1 + T1*X
PMAT(3,5) = T2*X + X*Y

52
END IF

ELSE IF ( NVER .EQ. 13 .AND. NSIDE .EQ. 4 ) THEN

TRACCTIONS SX & TXY SET TO ZERO ON FACE F4

NBVAL = 15

IF ( LAYER .EQ. 1 ) THEN

PMAT(1,1) = 1.0
PMAT(1,2) = -X
PMAT(1,3) = Y
PMAT(1,4) = -2*X*Y
PMAT(1,5) = -0.5*X**2
PMAT(1,6) = Y**2
PMAT(2,5) = -0.5*1**2 + T1*Y - 0.5*Y**2
PMAT(2,7) = -(T1+T2) + Y
PMAT(2,8) = 1.0
PMAT(2,9) = -T1*T2 + T2*Y
PMAT(2,10) = 2*T1*X - 2*X*Y
PMAT(2,11) = X
PMAT(2,12) = -T2*X
PMAT(2,13) = X**2
PMAT(3,2) = -T1 + Y
PMAT(3,4) = -T1**2 + Y**2
PMAT(3,7) = A - X
PMAT(3,10) = -A**2 + X**2
PMAT(3,9) = A*T2 - T2*X
PMAT(3,5) = -T1*X + X*Y

ELSE IF ( LAYER .EQ. 2 ) THEN

PMAT(1,9) = A*(X-A)
PMAT(1,14) = (X**2-A**2)
PMAT(1,15) = Y*(X-A)
PMAT(2,8) = 1.0
PMAT(2,11) = X
PMAT(2,7) = Y
PMAT(2,12) = X*Y
PMAT(2,13) = X**2
PMAT(3,7) = -(X-A)
PMAT(3,10) = (X**2-A**2)
PMAT(3,9) = Y*(X-A)

END IF

ELSE IF ( NVER .EQ. 13 .AND. NSIDE .EQ. 3 ) THEN

TRACIONS SX & TXY SET TO ZERO ON FACE F3

NBVAL = 15

IF ( LAYER .EQ. 1 ) THEN

PMAT(1,9) = A*(X-A)
PMAT(1,14) = (X**2-A**2)
PMAT(1,15) = Y*(X-A)
PMAT(2,8) = 1.0
PMAT(2,11) = X
PMAT(2,7) = Y
PMAT(2,12) = X*Y
PMAT(2,13) = X**2
PMAT(3,7) = -(X-A)
PMAT(3,10) = (X**2-A**2)
PMAT(3,9) = Y*(X-A)
ELSE IF ( LAYER .EQ. 2 ) THEN

PMAT(1,1)  =  1.0
PMAT(1,2)  =  -X
PMAT(1,3)  =  Y
PMAT(1,4)  =  -2*X*Y
PMAT(1,5)  =  -0.5*X**2
PMAT(1,6)  =  Y**2
PMAT(2,5)  =  -0.5*T2**2 - T2*Y - 0.5*Y**2
PMAT(2,7)  =  (T1+T2) + Y
PMAT(2,8)  =  1.0
PMAT(2,9)  =  -T1*T2 - T1*Y
PMAT(2,10) =  -2*T2*X - 2*X*Y
PMAT(2,11) =  X
PMAT(2,12) =  T1*X
PMAT(2,13) =  X**2
PMAT(3,2)  =  T2 + Y
PMAT(3,4)  =  -T2**2 + Y**2
PMAT(3,7)  =  A - X
PMAT(3,10) =  -A**2 + X**2
PMAT(3,9)  =  -A*T1 + T1*X
PMAT(3,5)  =  T2*X + X*Y

END IF

ELSE IF ( JTYPE .EQ. 2 ) THEN

2-D 2-LAYERED 10-NODE HYBRID ELEMENT.

Y
 |
V
 |
9------------------------10
 |
7
 |
6
 |
5----------------------6
 |
4
 |
3
 |
2-------------------1
 |
1-------------------X

T1 = THK(1)/2.
T2 = THK(2)/2.

IF ( IVER .EQ. 11 ) THEN

QUADRATIC STRESS FIELD

NBVAL = 18

IF ( LAYER .EQ. 1 ) THEN

PMAT(1,1)  =  1.0
PMAT(1,2)  =  -X
PMAT(1,3)  =  Y
PMAT(1,4)  =  -2*X*Y
PMAT(1,5)  =  -0.5*X**2
PMAT(1,6)  =  Y**2
PMAT(2,7) = 1.0
PMAT(2,8) = 2*(T1+T2)*X - 2*X*Y
PMAT(2,9) = X
PMAT(2,5) = T1*Y - 0.5*Y**2
PMAT(2,10) = -Y
PMAT(2,11) = T2*Y
PMAT(2,12) = X**2
PMAT(3,13) = 1.0
PMAT(3,5) = -T1*X + X*Y
PMAT(3,10) = X
PMAT(3,11) = -T2*X
PMAT(3,2) = Y
PMAT(3,8) = X**2
PMAT(3,4) = Y**2

ELSE IF (LAYER .EQ. 2) THEN

PMAT(1,14) = 1.0
PMAT(1,15) = -X
PMAT(1,16) = Y
PMAT(1,17) = -2*X*Y
PMAT(1,11) = -0.5*X**2
PMAT(1,18) = Y**2
PMAT(2,7) = 1.0
PMAT(2,5) = 0.5*T1**2
PMAT(2,10) = -(T1+T2) - Y
PMAT(2,11) = (T1*T2+0.5*T2**2) - 0.5*Y**2
PMAT(2,9) = X
PMAT(2,12) = -2*X*Y
PMAT(2,12) = X**2
PMAT(3,2) = T1
PMAT(3,4) = T1**2
PMAT(3,15) = T2 + Y
PMAT(3,17) = -T2**2 + Y**2
PMAT(3,10) = X
PMAT(3,11) = X*Y
PMAT(3,8) = X**2

END IF

ELSE IF ( NVER .EQ. 12 ) THEN

CUBIC ORDER EXPANSION

NBVAL = 28

IF (LAYER .EQ. 1) THEN

PMAT(1,1) = 1.0
PMAT(1,2) = -X
PMAT(1,3) = Y
PMAT(1,4) = -2*X*Y
PMAT(1,5) = -0.5*X**2
PMAT(1,6) = Y**2
PMAT(1,7) = -3*X*Y**2
PMAT(1,8) = -Y**2
PMAT(1,9) = -X**3/3.
PMAT(1,10) = X**3
PMAT(2,5) = -T1**2/2 + T1*Y - 0.5*Y**2
PMAT(2,8) = -2*T1**3/3 + T1**2*Y - Y**3/3.
PMAT(2,13) = T1+T2 - Y
PMAT(2,12) = 1.0
PMAT(2,14) = -(T1*T2+0.5*T2**2) + T2*Y
PMAT(2,15) = (T1*T2**2+T2**3/3.) - T2**2*Y
PMAT(2,9) = -T1**2*X + 2*T1*X*Y - X*Y**2
PMAT(2,16) = X
PKAT(2,17) = 2*(T1+T2)*X - 2*X*Y
PKAT(2,18) = -(2*T1*T2+T2**2)*X + 2*T2*X*Y
PKAT(2,19) = 3*(T1+T2)*X**2 - 3*Y*X**2
PKAT(2,11) = X**2
PKAT(2,20) = X**3
PKAT(3,2) = -T1 + Y
PKAT(3,4) = -T1**2 + Y**2
PKAT(3,7) = -T1**3 + Y**3
PKAT(3,21) = X
PKAT(3,22) = -T2
PKAT(3,23) = T2**2
PKAT(3,24) = -T2**3
PKAT(3,5) = -T1*X + X*Y
PKAT(3,8) = -T1**2*X + X*Y**2
PKAT(3,13) = X
PKAT(3,14) = -T2*X
PKAT(3,15) = T2**2*X
PKAT(3,16) = -T1*X**2 + Y*X**2
PKAT(3,17) = X**2
PKAT(3,18) = -T2*X**2
PKAT(3,19) = X**3

C ELSE IF (LAYER .EQ. 2) THEN
C
PKAT(1,25) = 1.0
PKAT(1,22) = -X
PKAT(1,26) = Y
PKAT(1,23) = -2*X*Y
PKAT(1,14) = -0.5*X*Y**2
PKAT(1,27) = Y**2
PKAT(1,24) = -3*X*Y**2
PKAT(1,15) = -Y*X**2
PKAT(1,18) = -X**3/3.
PKAT(1,28) = Y**3
PKAT(2,12) = 1.0
PKAT(2,16) = X
PKAT(2,13) = -Y
PKAT(2,17) = -2*X*Y
PKAT(2,11) = X**2
PKAT(2,14) = -0.5*Y*X**2
PKAT(2,18) = -X*Y**2
PKAT(2,19) = -3*Y*X**2
PKAT(2,20) = X**3
PKAT(2,15) = -Y**3/3.
PKAT(3,21) = 1.0
PKAT(3,13) = X
PKAT(3,22) = Y
PKAT(3,14) = X*Y
PKAT(3,17) = X**2
PKAT(3,23) = Y**2
PKAT(3,15) = X*Y**2
PKAT(3,18) = Y*X**2
PKAT(3,19) = X**3
PKAT(3,24) = Y**3

C END IF
C END IF
C ELSE IF (JTYPE .EQ. 3) THEN

2-D 2-LAYERED 13-NODE HYBRID ELEMENT.

Y | 11-------12-------13
^ |
|
C

9   10

6-------7-------8

4   5

1-------2-------3-------x

T1 = THK(2)/2.
T2 = THK(3)/2.

IF ( NVER .EQ. 11 ) THEN

COMPLETE CUBIC EXPANSION

( CONTINUITY OF TXY IS VIOLATED BY THE BETA 26 & 27 TERMS )

NBVAL = 27

IF ( LAYER .EQ. 1 ) THEN

PMAT(1,1) = 1.0
PMAT(1,2) = X
PMAT(1,3) = Y
PMAT(1,4) = X**2
PMAT(1,5) = X*Y
PMAT(1,6) = Y**2
PMAT(1,7) = X**3
PMAT(1,8) = 3*X**2*Y
PMAT(1,9) = 3*X*Y**2
PMAT(2,4) = T1**2 - 2*T1*Y + Y**2
PMAT(2,8) = 2*T1**3 - 3*T1**2*Y + Y**3
PMAT(2,11) = 1.0
PMAT(2,16) = -(T1+T2) + Y
PMAT(2,17) = -(T2**2+2*T1*T2) - 2*T2*Y
PMAT(2,23) = -(T2**3+3*T1*T2**2) + 3*T2**2*Y
PMAT(2,7) = 3*T1**2*X - 6*T1*X*Y + 3*X*Y**2
PMAT(2,20) = -(T1+T2)*X + X*Y
PMAT(2,22) = (6*T1*T2+3*T2**2)*X - 6*T2*X*Y
PMAT(2,14) = X
PMAT(2,25) = -3*(T1+T2)*X**2 + 3*X**2*Y
PMAT(2,18) = X**2
PMAT(3,2) = T1 - Y
PMAT(3,5) = 0.5*T1**2 - 0.5*Y**2
PMAT(3,9) = T1**3 - Y**3
PMAT(3,12) = 1.0
PMAT(3,13) = T2
PMAT(3,19) = -0.5*T2**2
PMAT(3,24) = T2**3
PMAT(3,4) = 2*T1*X - 2*X*Y
PMAT(3,8) = 2*T1**2*X - 3*X*Y**2
PMAT(3,16) = -X
PMAT(3,17) = 2*T2*X
PMAT(3,23) = -3*T2**2*X
PMAT(3,7) = 3*T1*X**2 - 3*X**2*Y
PMAT(3,20) = -0.5*X**2
PMAT(3,22) = 3*T2*X**2
PMAT(3,25) = -X**3
PMAT(3,26) = X**4

ELSE IF ( LAYER .EQ. 2 ) THEN
PMAT(1,10) = 1.0
PMAT(1,13) = X
PMAT(1,15) = Y
PMAT(1,17) = X**2
PMAT(1,19) = X*Y
PMAT(1,21) = Y**2
PMAT(1,22) = X**3
PMAT(1,23) = 3*X**2*Y
PMAT(1,24) = 3*X*Y**2
PMAT(2,11) = 1.0
PMAT(2,14) = X
PMAT(2,16) = Y
PMAT(2,17) = Y**2
PMAT(2,18) = X**2
PMAT(2,20) = X*Y
PMAT(2,22) = 3*X*Y**2
PMAT(2,23) = Y**3
PMAT(2,25) = 3*X**2*Y
PMAT(3,12) = 1.0
PMAT(3,13) = -Y
PMAT(3,16) = -X
PMAT(3,17) = -2*X*Y
PMAT(3,19) = -0.5*Y**2
PMAT(3,20) = -0.5*X**2
PMAT(3,22) = -3*X**2*Y
PMAT(3,23) = -3*X*Y**2
PMAT(3,24) = -Y**3
PMAT(3,25) = -X**3
PMAT(3,27) = X**4

END IF

ELSE IF (NVER .EQ. 12) THEN

QUADRATIC STRESS FIELD

(ONLY STRESS CONTINUITY CONDITIONS AT INTERFACE ENFORCED)

NBVAL = 30

IF (LAYER .EQ. 1) THEN

PMAT(1,1) = Y**2
PMAT(1,2) = X**2
PMAT(1,3) = X*Y
PMAT(1,4) = Y
PMAT(1,5) = X
PMAT(1,6) = 1.0
PMAT(2,7) = X*Y-X*T1
PMAT(2,8) = Y
PMAT(2,9) = 1.0
PMAT(2,10) = X**2
PMAT(2,11) = -X*T2
PMAT(2,12) = X
PMAT(2,13) = Y**2
PMAT(3,14) = Y**2
PMAT(3,15) = X*Y
PMAT(3,16) = X**2
PMAT(3,17) = Y
PMAT(3,18) = X
PMAT(3,19) = 1.0

ELSE IF (LAYER .EQ. 2) THEN

PMAT(1,20) = 1.0
PMAT(1,21) = Y**2
PMAT(1,22) = X**2
PMAT(1,23) = X*Y
PMAT(1,24) = Y
PMAT(1,25) = X
PMAT(2,26) = Y**2-T2**2
PMAT(2,27) = T2+Y
PMAT(2,8) = T1
PMAT(2,9) = 1.0
PMAT(2,10) = X**2
PMAT(2,11) = X*Y
PMAT(2,12) = X
PMAT(2,13) = T1**2
PMAT(3,28) = Y**2-T2**2
PMAT(3,29) = X*Y+T2*X
PMAT(3,30) = T2+Y
PMAT(3,14) = T1**2
PMAT(3,15) = T1*X
PMAT(3,16) = X**2
PMAT(3,17) = T1
PMAT(3,18) = X
PMAT(3,19) = 1.0

C
END IF
C
END IF
C
ELSE IF ( JTYPE .EQ. 4 ) THEN

2-D 3-LAYERED 8-NODE HYBRID ELEMENT.

| Y |
|   |
| 7------------------- 8 |
|   |
|   |
|   |
|   |
| 5------------------- 6 |
| F1 | F2 |
|   |   |
| 3-------------- 4 |
|   |   |
|   |   |
|   |   |
| 1------------------2 ------->X |

T1 = THK(1)/2.
T2 = THK(2)/2.
T3 = THK(3)/2.

IF ( NVER .EQ. 11 ) THEN

QUADRATIC FIELD EXPANSION

NBVAL = 24

IF ( LAYER .EQ. 1 ) THEN

PMAT(1,1) = 1.0
PMAT(1,2) = X
PMAT(1,3) = Y
PMAT(1,4) = X*Y
PMAT(1,5) = -0.5*X**2
PMAT(1,6) = Y**2
PMAT(2,5) = -0.5*T1**2 + T1*Y - 0.5*Y**2
PMAT(2,7) = -(T1+T2) + Y
PMAT(2,8) = -(T1*T2+0.5*T2**2) + T2*Y
PMAT(2,9) = 1.0
PMAT(2,10) = X
PMAT(2,11) = -(T1+T2)*X + X*Y
PMAT(2,12) = X**2
PMAT(3,2) = T1 - Y
PMAT(3,4) = 0.5*T1**2 - 0.5*Y**2
PMAT(3,13) = 1.0
PMAT(3,14) = T2
PMAT(3,15) = -0.5*T2**2
PMAT(3,5) = -T1*X + X*Y
PMAT(3,7) = -X
PMAT(3,8) = -T2*X
PMAT(3,11) = -0.5*X**2

C
ELSE IF (LAYER.EQ.2) THEN
C
PMAT(1,16) = 1.0
PMAT(1,14) = X
PMAT(1,17) = Y
PMAT(1,15) = X*Y
PMAT(1,18) = -0.5*X**2
PMAT(1,19) = Y**2
PMAT(2,9) = 1.0
PMAT(2,10) = X
PMAT(2,11) = X*Y
PMAT(2,12) = X**2
PMAT(2,8) = -0.5*Y**2
PMAT(3,13) = 1.0
PMAT(3,7) = -X
PMAT(3,14) = -Y
PMAT(3,8) = X*Y
PMAT(3,11) = -0.5*X**2
PMAT(3,15) = -0.5*Y**2
C
ELSE IF (LAYER.EQ.3) THEN
C
PMAT(1,19) = 1.0
PMAT(1,20) = X
PMAT(1,21) = Y
PMAT(1,22) = X*Y
PMAT(1,23) = -0.5*X**2
PMAT(1,24) = Y**2
PMAT(2,9) = 1.0
PMAT(2,7) = (T2+T3) + Y
PMAT(2,8) = -(T2*T3+0.5*T2**2) - T2*Y
PMAT(2,23) = -0.5*T3**2 - T3*Y - 0.5*Y**2
PMAT(2,10) = X
PMAT(2,11) = (T2+T3)*X + X*Y
PMAT(2,12) = X**2
PMAT(3,13) = 1.0
PMAT(3,14) = -T2
PMAT(3,15) = -0.5*T2**2
PMAT(3,20) = -T3 - Y
PMAT(3,22) = 0.5*T3**2 - 0.5*Y**2
PMAT(3,7) = -X
PMAT(3,8) = T2*X
PMAT(3,23) = T3*X + X*Y
PMAT(3,11) = -0.5*X**2
C
END IF
C
ELSE IF (NVER.EQ.12) THEN
C
CUBIC FIELD EXPANSION

NBVAL = 38

IF ( LAYER .EQ. 1 ) THEN

PMAT(1,1) = 1.0
PMAT(1,2) = X
PMAT(1,3) = Y
PMAT(1,4) = XY
PMAT(1,5) = -X**2/2.
PMAT(1,6) = Y**2
PMAT(1,7) = -3.X*Y**2
PMAT(1,8) = -Y*X**2
PMAT(1,9) = -X**3/3.
PMAT(1,10) = Y**3
PMAT(2,5) = -0.5*T1**2 + T1*Y - 0.5*Y**2
PMAT(2,8) = -2*T1**3/3 + T1**2*Y - Y**3/3.
PMAT(2,21) = 1.0
PMAT(2,23) = T1+T2 - Y
PMAT(2,24) = 2*(T1+T2)*X - 2*X*Y
PMAT(2,19) = -(2*T1*T2+T2**2)*X + 2*T2*X*Y
PMAT(2,26) = 3*(T1+T2)*X**2 - 3*Y*X**2
PMAT(2,25) = X**2
PMAT(2,27) = X**3
PMAT(3,2) = T1 - Y
PMAT(3,4) = 0.5*T1**2 - 0.5*Y**2
PMAT(3,7) = -T1**3 + Y**3
PMAT(3,28) = 1.0
PMAT(3,12) = T2
PMAT(3,14) = -0.5*T2**2
PMAT(3,17) = -T2**3
PMAT(3,5) = -T1*X + X*Y
PMAT(3,8) = -T1**2*X + X*Y**2
PMAT(3,23) = X
PMAT(3,15) = -T2*X
PMAT(3,18) = T2**2*X
PMAT(3,9) = -T1*X**2 + Y*X**2
PMAT(3,24) = X**2
PMAT(3,19) = -T2*X**2
PMAT(3,26) = X**3

ELSE IF ( LAYER .EQ. 2 ) THEN

PMAT(1,11) = 1.0
PMAT(1,12) = X
PMAT(1,13) = Y
PMAT(1,14) = XY
PMAT(1,15) = -0.5*X**2
PMAT(1,16) = Y**2
PMAT(1,17) = -3*X*Y**2
PMAT(1,18) = -Y*X**2
PMAT(1,19) = -X**3/3.
PMAT(1,20) = Y**3
PMAT(2,21) = 1.0
PMAT(2,22) = X
PMAT(2,23) = -Y
PMAT(2,24) = -2*X*Y
PMAT(2,25) = X**2
PMAT(2,15) = -0.5*X**2
PMAT(2,19) = -X*Y**2
PMAT(2,26) = -3*Y*X**2
PMAT(2,27) = X**3
PMAT(2,18) = -Y**3/3.
C
ELSE IF ( LAYER .EQ. 3 ) THEN

PMAT(1, 29) = 1.0
PMAT(1, 30) = X
PMAT(1, 31) = Y
PMAT(1, 32) = X*Y
PMAT(1, 33) = -0.5*X**2
PMAT(1, 34) = Y**2
PMAT(1, 35) = -3*X*Y**2
PMAT(1, 36) = -Y*X**2
PMAT(1, 37) = -X**3/3.
PMAT(1, 38) = Y**3
PMAT(2, 23) = -Y - (T2+T3)
PMAT(2, 15) = -(T2*T3+0.5*T2**2) - T2*Y
PMAT(2, 18) = -(T3*T2**2+T2**3/3.) - T2**2*Y
PMAT(2, 33) = -0.5*T3**2 - T3*Y - 0.5*Y**2
PMAT(2, 21) = 1.0
PMAT(2, 24) = -2*(T2+T3)*X - 2*X*Y
PMAT(2, 19) = -2*(T2*T3+0.5*T2**2)*X - 2*T2*X*Y
PMAT(2, 37) = (T3**2-2*T2*T3)*X - 2*T2*X*Y - X*Y**2
PMAT(2, 22) = X
PMAT(2, 25) = X**2
PMAT(2, 26) = -3*(T2+T3)*X**2 - 3*Y*X**2
PMAT(2, 27) = X**3
PMAT(3, 12) = -T2
PMAT(3, 14) = -0.5*T2**2
PMAT(3, 17) = T2**3
PMAT(3, 28) = 1.0
PMAT(3, 15) = T2*X
PMAT(3, 18) = T2**2*X
PMAT(3, 33) = T3*X + X*Y
PMAT(3, 36) = -T3**2*X + X*Y**2
PMAT(3, 24) = X**2
PMAT(3, 19) = T2*X**2
PMAT(3, 37) = T3*X**2 + Y*X**2
PMAT(3, 26) = X**3

C
END IF
C
ELSE IF ( NVER .EQ. 13 .AND. NSIDE .EQ. 1 ) THEN

ZERO TRACTION CONDITION OF SXX AND TXY
IMPOSED ON ELEMENT SIDE F1

NBVAL = 21

IF ( LAYER .EQ. 1 ) THEN

PMAT(1, 1) = 1.0
PMAT(1, 2) = -X
PMAT(1, 3) = Y
PMAT(1, 4) = -2*X*Y

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PMAT(1,5) = -X**2
PMAT(1,6) = Y**2
PMAT(2,5) = -0.5*T1**2 + Y*T1 - 0.5*Y**2
PMAT(2,7) = -(T1+T2) + Y
PMAT(2,8) = 1.0
PMAT(2,9) = -T1*T2 + Y*T2
PMAT(2,10) = 2*X*T1 - 2*X*Y
PMAT(2,11) = X
PMAT(2,12) = -X*T2
PMAT(2,13) = X**2
PMAT(3,2) = -T1 + Y
PMAT(3,4) = -T1**2 + Y**2
PMAT(3,7) = -A - X
PMAT(3,10) = -A**2 + X**2
PMAT(3,9) = -T2*A - X*T2
PMAT(3,5) = -X*T1 + X*Y

ELSE IF ( LAYER .EQ. 2 ) THEN

PMAT(1,9) = -A*(X+A)
PMAT(1,14) = (X**2-A**2)
PMAT(1,15) = Y*(X+A)
PMAT(2,8) = 1.0
PMAT(2,11) = X
PMAT(2,7) = Y
PMAT(2,12) = X*Y
PMAT(2,13) = X**2
PMAT(3,7) = -(X+A)
PMAT(3,10) = (X**2-A**2)
PMAT(3,9) = Y*(X+A)

ELSE IF ( LAYER .EQ. 3 ) THEN

PMAT(1,16) = 1.0
PMAT(1,17) = -X
PMAT(1,18) = Y
PMAT(1,19) = -X*Y
PMAT(1,20) = -0.5*X**2
PMAT(1,21) = Y**2
PMAT(2,20) = -0.5*T3**2 - T3*Y - 0.5*Y**2
PMAT(2,7) = (T2+T3) + Y
PMAT(2,8) = 1.0
PMAT(2,9) = -T2*T3 - T2*Y
PMAT(2,10) = -2*T3*X - 2*X*Y
PMAT(2,11) = X
PMAT(2,12) = T2*X
PMAT(2,13) = X**2
PMAT(3,17) = T3 + Y
PMAT(3,19) = -T3**2 + Y**2
PMAT(3,7) = -A - X
PMAT(3,10) = -A**2 + X**2
PMAT(3,9) = T2*A + T2*X
PMAT(3,20) = T3*X + X*Y

END IF

ELSE IF ( NVER .EQ. 13 .AND. NSIDE .EQ. 2 ) THEN

ZERO TRACTION CONDITION OF SXX AND TXY
IMPOSED ON FACE F2

NBVAL = 21

IF ( LAYER .EQ. 1 ) THEN

PMAT(1,1) = 1.0
PMAT(1,2) = -X
PMAT(1,3) = Y

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```
PMA{1,4} = -2*X*Y
PMA{1,5} = -X**2
PMA{1,6} = Y**2
PMA{2,5} = -0.5*T1**2 + Y*T1 - 0.5*Y**2
PMA{2,7} = -(T1+T2) + Y
PMA{2,8} = 1.0
PMA{2,9} = -T1*T2 + Y*T2
PMA{2,10} = 2*X*T1 - 2*X*Y
PMA{2,11} = X
PMA{2,12} = -X*T2
PMA{2,13} = X**2
PMA{3,2} = -T1 + Y
PMA{3,4} = -T1**2 + Y**2
PMA{3,7} = A - X
PMA{3,10} = -A**2 + X**2
PMA{3,9} = -X*T1 + X*Y

C ELSE IF ( LAYER .EQ. 2 ) THEN

C
PMA{1,9} = A*(X-A)
PMA{1,14} = (X**2-A**2)
PMA{1,15} = Y*(X-A)
PMA{2,8} = 1.0
PMA{2,11} = X
PMA{2,7} = Y
PMA{2,12} = X*Y
PMA{2,13} = X**2
PMA{3,7} = -(X-A)
PMA{3,10} = (X**2-A**2)
PMA{3,9} = Y*(X-A)

C ELSE IF ( LAYER .EQ. 3 ) THEN

C
PMA{1,16} = 1.0
PMA{1,17} = -X
PMA{1,18} = Y
PMA{1,19} = -X*Y
PMA{1,20} = -0.5*X**2
PMA{1,21} = Y**2
PMA{2,20} = -0.5*T3**2 - T3*Y - 0.5*Y**2
PMA{2,7} = (T2+T3) + Y
PMA{2,8} = 1.0
PMA{2,9} = -T2*T3 - T2*Y
PMA{2,10} = -2*T3*X - 2*X*Y
PMA{2,11} = X
PMA{2,12} = T2*X
PMA{2,13} = X**2
PMA{3,17} = T3 + Y
PMA{3,19} = -T3**2 + Y**2
PMA{3,7} = A - X
PMA{3,10} = -A**2 + X**2
PMA{3,9} = -T2*A + T2*X
PMA{3,20} = T3*X + X*Y

C END IF

C ELSE IF ( JTYPE .EQ. 5 ) THEN

2-LAYER 3-D HYBRID ELEMENT.

2,W
Y,V
```

```
T1 = THK(1)/2.
T2 = THK(2)/2.

IF ( NVER .EQ. 11 ) THEN
  TRILINEAR STRESS FIELD
  NBVAL = 48
  IF ( LAYER .EQ. 1 ) THEN
    PMAT(1, 1) = 1.0
    PMAT(1, 2) = -X
    PMAT(1, 3) = -X
    PMAT(1, 4) = Y
    PMAT(1, 5) = Z
    PMAT(1, 6) = -X*Y
    PMAT(1, 7) = -X*Z
    PMAT(1, 8) = Z*Y
    PMAT(2, 9) = 1.0
    PMAT(2,10) = X
    PMAT(2,11) = Y
    PMAT(2,12) = Z
    PMAT(2,13) = X*Y
    PMAT(2,14) = X*Z
    PMAT(2,15) = -Z*Y
    PMAT(3,16) = T1+T2 - Z
    PMAT(3,17) = T1+T2 - Z
    PMAT(3,18) = 1.0
    PMAT(3,19) = (T1+T2)*X - X*Z
    PMAT(3,20) = X
    PMAT(3,21) = (T1+T2)*Y - Z*Y
    PMAT(3,22) = Y
    PMAT(3,23) = X*Y
    PMAT(4,25) = -T1 + Z
    PMAT(4,27) = 1.0
    PMAT(4,28) = -T2
    PMAT(4,13) = T1*X - X*Z
    PMAT(4,29) = X
    PMAT(4,30) = T2*X
    PMAT(4,17) = Y
    PMAT(4,19) = X*Y
    PMAT(5,3) = -T1 + Z
    PMAT(5,31) = 1.0
    PMAT(5,32) = -T2
    PMAT(5,16) = X
    PMAT(5,6) = -T1*Y + Z*Y
    PMAT(5,33) = Y
    PMAT(5,34) = -T2*Y
    PMAT(5,21) = X*Y
    PMAT(6,24) = 1.0
    PMAT(6,11) = -X
    PMAT(6,25) = -X
    PMAT(6,2) = Y
PMAT(6,26) = Z
PMAT(6,15) = X*Z
PMAT(6,7) = Z*Y

ELSE IF ( LAYER .EQ. 2 ) THEN

PMAT(1,35) = 1.0
PMAT(1,36) = -X
PMAT(1,32) = -X
PMAT(1,37) = Y
PMAT(1,38) = Z
PMAT(1,34) = -X*Y
PMAT(1,39) = -X*Z
PMAT(1,40) = Y*Z
PMAT(2,41) = 1.0
PMAT(2,42) = X
PMAT(2,47) = Y
PMAT(2,43) = Z
PMAT(2,30) = X*Y
PMAT(2,44) = X*Z
PMAT(2,45) = -Z*Y
PMAT(3,18) = 1.0
PMAT(3,20) = X
PMAT(3,22) = Y
PMAT(3,16) = -Z
PMAT(3,17) = -Z
PMAT(3,23) = X*Y
PMAT(3,19) = -X*Z
PMAT(3,21) = -Z*Y
PMAT(4,27) = 1.0
PMAT(4,29) = X
PMAT(4,17) = Y
PMAT(4,28) = Z
PMAT(4,19) = X*Y
PMAT(4,30) = -X*Z
PMAT(5,31) = 1.0
PMAT(5,16) = X
PMAT(5,33) = Y
PMAT(5,32) = Z
PMAT(5,21) = X*Y
PMAT(5,34) = Y*Z
PMAT(6,46) = 1.0
PMAT(6,47) = -X
PMAT(6,28) = -X
PMAT(6,36) = Y
PMAT(6,48) = Z
PMAT(6,45) = X*Z
PMAT(6,39) = Z*Y

END IF

ELSE IF ( NVER .EQ. 12 ) THEN

QUADRATIC STRESS FIELD

NBVAL = 78

IF ( LAYER .EQ. 1 ) THEN

PMAT(1,1) = Y*Z
PMAT(1,2) = X*Z
PMAT(1,3) = Z
PMAT(1,4) = Y**2
PMAT(1,5) = X**2
PMAT(1,6) = X*Y
PMAT(1,7) = Y
PMAT(1,8) = X
PMAT(1,9) = Z**2
ELSE IF (LAYER .EQ. 2) THEN

        PMAT(1,50) = Y*Z
        PMAT(1,51) = X*Z
        PMAT(1,52) = Z
        PMAT(1,53) = -2*X*Y
        PMAT(1,54) = Y**2
        PMAT(1,55) = X**2
        PMAT(1,56) = Y
        PMAT(1,57) = X
        PMAT(1,58) = 1.0
        PMAT(1,59) = Z**2
        PMAT(1,60) = -X*Y
        PMAT(2,61) = Y*Z
        PMAT(2,62) = X*Z
        PMAT(2,63) = Z
        PMAT(2,64) = Y**2
        PMAT(2,65) = X**2
        PMAT(2,66) = X*Y
        PMAT(2,67) = Y
        PMAT(2,68) = X
        PMAT(2,69) = 1.0
        PMAT(2,70) = Z**2
        PMAT(3,31) = (-T2**2+Z**2-2*T1*T2)
        PMAT(3,22) = (Y*(T2+T1)+Y*Z)
        PMAT(3,23) = (X*(T2+T1)+X*Z)
        PMAT(3,24) = (T2+T1+Z)
        PMAT(3,25) = Y**2
        PMAT(3,26) = X**2
        PMAT(3,27) = X*Y
        PMAT(3,28) = Y
        PMAT(3,29) = X
        PMAT(3,30) = 1.0
        PMAT(3,31) = -T1**2
        PMAT(4,77) = (Z**2-T2**2)
        PMAT(4,55) = (Y**2+T2*X)
        PMAT(4,67) = (-T2-Z)
        PMAT(4,31) = (T2*X-Y*Z)
        PMAT(4,64) = (-Y**2+T2*X)
        PMAT(4,66) = (-X**2+T2*X)
        PMAT(4,73) = (-T2-Z)
        PMAT(4,72) = (-2*X*Z-2*T2*X)
        PMAT(4,23) = -X*Y
        PMAT(4,24) = -Y
        PMAT(4,14) = -T1*Y
        PMAT(4,34) = -2*T1*X
        PMAT(4,16) = -T1*X
        PMAT(4,17) = -T1
        PMAT(4,41) = -Y
        PMAT(4,42) = X
        PMAT(4,36) = -T1
        PMAT(4,43) = 1.0
        PMAT(4,39) = T1**2
        PMAT(4,5) = T1*Y
        PMAT(4,21) = T1*Y
        PMAT(4,44) = Y**2
        PMAT(4,45) = -2*X*Y
        PMAT(4,46) = X**2
        PMAT(5,76) = (Z**2-T2**2)
        PMAT(5,60) = (Y**2+T2*X)
        PMAT(5,64) = (X**2+T2*X)
        PMAT(5,55) = (-X**2-T2*X)
        PMAT(5,31) = (-X**2-T2*X)
        PMAT(5,75) = (T2+Z)
        PMAT(5,22) = -X*Y
        PMAT(5,33) = -2*T1*Y
        PMAT(5,14) = T1*X

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PMAT(5,47) = Y
PMAT(5,35) = -T1
PMAT(5,41) = X
PMAT(5,48) = 1.0
PMAT(5,38) = T1*X
PMAT(5,5) = -T1*X
PMAT(5,6) = -T1*Y
PMAT(5,21) = -T1*X
PMAT(5,8) = -T1
PMAT(5,49) = Y*X
PMAT(5,44) = -2*X*Y
PMAT(5,45) = X*X
PMAT(6,61) = -X*Z
PMAT(6,51) = -Y*Z
PMAT(6,71) = Z
PMAT(6,53) = Y*Z
PMAT(6,64) = -X*Y
PMAT(6,72) = X*Z
PMAT(6,55) = -X*Y
PMAT(6,73) = X
PMAT(6,57) = -Y
PMAT(6,74) = 1.0
PMAT(6,31) = X*Y
PMAT(6,75) = -Y
PMAT(6,76) = -2*Y*Z
PMAT(6,77) = -2*X*Z
PMAT(6,78) = Z*Z

END IF

END IF

END IF

RETURN
END

SUBROUTINE IOPNTS (IET, NTPS, JTYPE, NELDIM, NORD, LAYER, 1 XSI, ETA, CEE

IMPLICIT REAL*8(A-H,O-Z)

IN FIRST PASS SET NUMBER OF OUTPUT POINTS

IF ( IET .EQ. 0 ) THEN

IF ( JTYPE .NE. 5 ) NTPS = 9
IF ( JTYPE .EQ. 5 ) NTPS = 17

RETURN

END IF

SELECT SPECIFIC OUTPUT POINTS AT ELEMENT
CORNERS AND AT GAUSS (N-2) POINTS

IF ( JTYPE .NE. 5 ) THEN

2-D ADHESIVE ELEMENTS

IF ( IET .EQ. 1 ) THEN

XSI = -1.0
ETA = -1.0

ELSE IF ( IET .EQ. 2 ) THEN

69
XSI = -1.0
ETA = 1.0

ELSE IF ( IET .EQ. 3 ) THEN
XSI = 1.0
ETA = -1.0

ELSE IF ( IET .EQ. 4 ) THEN
XSI = 1.0
ETA = 1.0

ELSE IF ( IET .EQ. 5 ) THEN
XSI = 0.0
ETA = 0.0

ELSE IF ( IET .EQ. 6 ) THEN
XSI = -0.577350269189626
ETA = -0.577350269189626

ELSE IF ( IET .EQ. 7 ) THEN
XSI = 0.577350269189626
ETA = 0.577350269189626

ELSE IF ( IET .EQ. 8 ) THEN
XSI = -0.577350269189626
ETA = 0.577350269189626

ELSE IF ( IET .EQ. 9 ) THEN
XSI = 0.577350269189626
ETA = 0.577350269189626

END IF

ELSE IF ( JTYPE .EQ. 5 ) THEN

3-D ADHESIVE ELEMENT

IF ( IET .EQ. 1 ) THEN
XSI = -1.0
ETA = -1.0
CEE = -1.0

ELSE IF ( IET .EQ. 2 ) THEN
XSI = -1.0
ETA = -1.0
CEE = 1.0

ELSE IF ( IET .EQ. 3 ) THEN
XSI = -1.0
ETA = 1.0
CEE = -1.0

ELSE IF ( IET .EQ. 4 ) THEN
XSI = -1.0
ETA = 1.0
CEE = 1.0
ELSE IF ( IET .EQ. 5 ) THEN
  XSI = 1.0
  ETA = -1.0
  CEE = -1.0

ELSE IF ( IET .EQ. 6 ) THEN
  XSI = 1.0
  ETA = -1.0
  CEE = 1.0

ELSE IF ( IET .EQ. 7 ) THEN
  XSI = 1.0
  ETA = 1.0
  CEE = -1.0

ELSE IF ( IET .EQ. 8 ) THEN
  XSI = 1.0
  ETA = 1.0
  CEE = 1.0

ELSE IF ( IET .EQ. 9 ) THEN
  XSI = 0.0
  ETA = 0.0
  CEE = 0.0

ELSE IF ( IET .EQ. 10 ) THEN
  XSI = -0.577350269189626
  ETA = -0.577350269189626
  CEE = -0.577350269189626

ELSE IF ( IET .EQ. 11 ) THEN
  XSI = 0.577350269189626
  ETA = -0.577350269189626
  CEE = -0.577350269189626

ELSE IF ( IET .EQ. 12 ) THEN
  XSI = -0.577350269189626
  ETA = 0.577350269189626
  CEE = -0.577350269189626

ELSE IF ( IET .EQ. 13 ) THEN
  XSI = 0.577350269189626
  ETA = 0.577350269189626
  CEE = -0.577350269189626

ELSE IF ( IET .EQ. 14 ) THEN
  XSI = -0.577350269189626
  ETA = -0.577350269189626
  CEE = 0.577350269189626

ELSE IF ( IET .EQ. 15 ) THEN
  XSI = 0.577350269189626
  ETA = -0.577350269189626
  CEE = 0.577350269189626

ELSE IF ( IET .EQ. 16 ) THEN

XSI = -0.577350269189626
ETA = 0.577350269189626
CEE = 0.577350269189626

ELSE IF (IET .EQ. 17) THEN
XSI = 0.577350269189626
ETA = 0.577350269189626
CEE = 0.577350269189626
END IF

RETURN

SUBROUTINE INVERS(A, B, INDEX, NDIM, NFT)
IMPLICIT DOUBLE PRECISION (A-H, O-Z)
DIMENSION A(NDIM, NDIM), B(NDIM, NDIM), INDEX(NDIM)

DO 12 I = 1, NFT
  DO 11 J = 1, NFT
    B(I, J) = 0.0
  CONTINUE
  B(I, I) = 1.0
12 CONTINUE
CALL LUDMC(A, NFT, NDIM, INDEX, D)
DO 13 J = 1, NFT
  CALL LUBKS(A, NFT, NDIM, INDEX, B(1, J))
13 CONTINUE
RETURN

SUBROUTINE LUDMC(A, N, NP, INDX, D)
IMPLICIT DOUBLE PRECISION (A-H, O-Z)
PARAMETER (NMAX=100, TINY=1.0E-20)
DIMENSION A(NP, NP), INDX(NP), VV(NMAX)
D=1.

DO 12 I=1, N
  AAMAX=0.
  DO 11 J=1, N
    IF (ABS(A(I, J)) .GT. AAMAX) AAMAX=ABS(A(I, J))
    CONTINUE
    IF (AAMAX.EQ.0.) PAUSE 'Singular matrix.'
    VV(I)=1./AAMAX
  CONTINUE
DO 19 J=1, N
  IF (J.GT.1) THEN
    DO 14 I=1, J-1
      SUM=A(I, J)
      IF (I.GT.1) THEN
        DO 13 K=1, I-1
          SUM=SUM-A(I, K)*A(K, J)
        CONTINUE
        A(I, J)=SUM
      ENDIF
    14 CONTINUE
  19 CONTINUE
72
ENDIF
AAMAX=0.
DO 16 I=J,N
   SUM=A(I,J)
   IF (J.GT.1) THEN
      DO 15 K=1,J-1
         SUM=SUM-A(I,K)*A(K,J)
      CONTINUE
      A(I,J)=SUM
   ENDF
   DUM=VV(I)*ABS(SUM)
   IF (DUM.GE.AAMAX) THEN
      IMAX=I
      AAMAX=DUM
   ENDF
16 CONTINUE
IF (J.NE.IMAX) THEN
   DO 17 K=1,N
      DUM=A(IMAX,K)
      A(IMAX,K)=A(J,K)
      A(J,K)=DUM
   CONTINUE
   VV(IMAX)=VV(J)
ENDIF
INDX(J)=IMAX
IF (J.NE.N) THEN
   IF (A(J,J) .EQ.O.) THEN
      DO 13 J=1,N
         SUM=A(I,J)*B(J)
      CONTINUE
      II=I
   ELSE IF (SUM.NE.0.) THEN
      II=I
   ENDIF
   B(I)=SUM
ENDIF
12 CONTINUE
DO 14 I=1,N,1
   SUM=B(I)
   IF (I.LT.N) THEN
      DO 13 J=I+1,N
         SUM=SUM-A(I,J)*B(J)
      CONTINUE
      ENDIF
   B(I)=SUM/A(I,I)
14 CONTINUE
SUBROUTINE GAUSS (NORD, NELDIM, IXSI, JETA, KCEE, XSI, ETA, CEE, WEIGHT)

IMPLICIT DOUBLE PRECISION (A-H,O-Z)

RETURN GAUSS POINTS AND WEIGHTS FOR NUMERICAL QUADRATURE

IF ( NELDIM .EQ. 2 ) THEN

CEE = 1.0

IF ( NORD .EQ. 1 ) THEN

XSI = 0.0
ETA = 0.0

WEIGHT = 4.0

ELSE IF ( NORD .EQ. 2 ) THEN

WEIGHT = 1.000
XSI = 0.577350269189626
ETA = 0.577350269189626
IF ( IXSI .EQ. 2 ) XSI = -.577350269189626
IF ( JETA .EQ. 2 ) ETA = -.577350269189626

ELSE IF ( NORD .EQ. 3 ) THEN

IF ( IXSI .EQ. 1 ) THEN
XSI = 0.774596669241483
WX = 0.555555555555556
ELSE IF ( IXSI .EQ. 2 ) THEN
XSI = 0.000000000000000
WX = 0.888888888888889
ELSE IF ( IXSI .EQ. 3 ) THEN
XSI = -.774596669241483
WX = 0.555555555555556
END IF

IF ( JETA .EQ. 1 ) THEN
ETA = 0.774596669241483
WE = 0.555555555555556
ELSE IF ( JETA .EQ. 2 ) THEN
ETA = 0.000000000000000
WE = 0.888888888888889
ELSE IF ( JETA .EQ. 3 ) THEN
ETA = -.774596669241483
WE = 0.555555555555556
END IF

WEIGHT = WX*WE

ELSE IF ( NORD .EQ. 4 ) THEN

IF ( IXSI .EQ. 1 ) THEN
XSI = 0.861136311594053
WX = 0.347854845137454
ELSE IF ( IXSI .EQ. 2 ) THEN
XSI = 0.339981043584856
WX = 0.652145154862546
ELSE IF ( IXSI .EQ. 3 ) THEN
XSI = -.339981043584856
WX = 0.652145154862546
ELSE IF ( IXSI .EQ. 4 ) THEN
XSI = -.861136311594053
WX = 0.347854845137454
END IF

IF ( JETA .EQ. 1 ) THEN
ETA = 0.861136311594053
WE = 0.347854845137454
ELSE IF ( JETA .EQ. 2 ) THEN
ETA = 0.339981043584856
WE = 0.6521451546862546
ELSE IF ( JETA .EQ. 3 ) THEN
ETA = -0.339981043584856
WE = 0.6521451546862546
ELSE IF ( JETA .EQ. 4 ) THEN
ETA = -0.861136311594053
WE = 0.347854845137454
END IF

WEIGHT = WX*WE

ELSE IF ( NORD .EQ. 5 ) THEN
IF ( IXSI .EQ. 1 ) THEN
XSI = 0.906179845938664
WX = 0.236926885056189
ELSE IF ( IXSI .EQ. 2 ) THEN
XSI = 0.538469310105683
WX = 0.478628670499366
ELSE IF ( IXSI .EQ. 3 ) THEN
XSI = 0.000000000000000
WX = 0.568888888888889
ELSE IF ( IXSI .EQ. 4 ) THEN
XSI = -0.538469310105683
WX = 0.478628670499366
ELSE IF ( IXSI .EQ. 5 ) THEN
XSI = -0.906179845938664
WX = 0.236926885056189
END IF
ELSE IF ( JETA .EQ. 1 ) THEN
ETA = 0.906179845938664
WE = 0.236926885056189
ELSE IF ( JETA .EQ. 2 ) THEN
ETA = 0.538469310105683
WE = 0.478628670499366
ELSE IF ( JETA .EQ. 3 ) THEN
ETA = 0.000000000000000
WE = 0.568888888888889
ELSE IF ( JETA .EQ. 4 ) THEN
ETA = -0.538469310105683
WE = 0.478628670499366
ELSE IF ( JETA .EQ. 5 ) THEN
ETA = -0.906179845938664
WE = 0.236926885056189
END IF
ELSE IF ( NELDIM .EQ. 3 ) THEN
IF ( NORD .EQ. 1 ) THEN
XSI = 0.0
ETA = 0.0
CEE = 0.0
WEIGHT = 8.0
ELSE IF ( NORD .EQ. 2 ) THEN
WEIGHT = 1.000
XSI = 0.577350269189626
ETA = 0.577350269189626
CEE = 0.577350269189626
IF ( IXSI .EQ. 2 ) XSI = -0.577350269189626
75
IF ( JETA .EQ. 2 ) ETA = -.577350269189626
IF ( KCEE .EQ. 2 ) CEE = -.577350269189626

ELSE IF ( NORD .EQ. 3 ) THEN

IF ( IXSI .EQ. 1 ) THEN
XSI = 0.774596669241483
WX = 0.555555555555556
ELSE IF ( IXSI .EQ. 2 ) THEN
XSI = 0.000000000000000
WX = 0.888888888888889
ELSE IF ( IXSI .EQ. 3 ) THEN
XSI = -.774596669241483
WX = 0.555555555555556
END IF

IF ( JETA .EQ. 1 ) THEN
ETA = 0.774596669241483
WE = 0.555555555555556
ELSE IF ( JETA .EQ. 2 ) THEN
ETA = 0.000000000000000
WE = 0.888888888888889
ELSE IF ( JETA .EQ. 3 ) THEN
ETA = -.774596669241483
WE = 0.555555555555556
END IF

IF ( KCEE .EQ. 1 ) THEN
CEE = 0.774596669241483
WC = 0.555555555555556
ELSE IF ( KCEE .EQ. 2 ) THEN
CEE = 0.000000000000000
WC = 0.888888888888889
ELSE IF ( KCEE .EQ. 3 ) THEN
CEE = -.774596669241483
WC = 0.555555555555556
END IF

WEIGHT = WX*WE*WC

ELSE IF ( NORD .EQ. 4 ) THEN

IF ( IXSI .EQ. 1 ) THEN
XSI = 0.861136311594053
WX = 0.347854845137454
ELSE IF ( IXSI .EQ. 2 ) THEN
XSI = 0.339981043584856
WX = 0.652145154862546
ELSE IF ( IXSI .EQ. 3 ) THEN
XSI = -.339981043584856
WX = 0.652145154862546
ELSE IF ( IXSI .EQ. 4 ) THEN
XSI = -.861136311594053
WX = 0.347854845137454
END IF

IF ( JETA .EQ. 1 ) THEN
ETA = 0.861136311594053
WE = 0.347854845137454
ELSE IF ( JETA .EQ. 2 ) THEN
ETA = 0.339981043584856
WE = 0.652145154862546
ELSE IF ( JETA .EQ. 3 ) THEN
ETA = -.339981043584856
WE = 0.652145154862546
ELSE IF ( JETA .EQ. 4 ) THEN
ETA = -.861136311594053
WE = 0.347854845137454
END IF

IF ( KCEE .EQ. 1 ) THEN
CEE = 0.861136311594053
WC = 0.347854845137454

76
ELSE IF ( KCEE .EQ. 2 ) THEN
CEE = 0.339981043584856
WC = 0.652145154862546
ELSE IF ( KCEE .EQ. 3 ) THEN
CEE = -0.339981043584856
WC = 0.652145154862546
ELSE IF ( KCEE .EQ. 4 ) THEN
CEE = -0.861136311594053
WC = 0.347854845137454
END IF
WEIGHT = WX*WE*WC
END IF
RETURN
END

SUBROUTINE MXMUL (A, B, C, IDIM, JDIM, KDIM, IROW, JCOL, KCOL)
MATRIX (A) TIMES (B)
IMPLICIT DOUBLE PRECISION (A-H, O-Z)
DIMENSION A(IDIM,1), B(JDIM,1), C(KDIM,1)
DO I = 1, IROW
DO K = 1, KCOL
SUM = 0.0
DO J = 1, JCOL
SUM = SUM + A(I,J)*B(J,K)
END DO
C(I,K) = SUM
END DO
END DO
RETURN
END

SUBROUTINE MXATB (A, B, C, IDIM, JDIM, KDIM, IROW, JCOL, KCOL)
MATRIX (A) TRANSPOSE TIMES (B)
IMPLICIT DOUBLE PRECISION (A-H, O-Z)
DIMENSION A(IDIM,1), B(JDIM,1), C(KDIM,1)
DO I = 1, IROW
DO K = 1, KCOL
SUM = 0.0
DO J = 1, JCOL
SUM = SUM + A(J,I)*B(J,K)
END DO
C(I,K) = SUM
END DO
END DO
RETURN
END

SUBROUTINE MXINT (A, IDIM, JDIM, VAL)
77
MATRIX INITIALIZATION

IMPLICIT DOUBLE PRECISION (A-H, O-Z)

DIMENSION A(IDIM, JDIM)

DO I = 1, IDIM
   DO J = 1, JDIM
      A(I, J) = VAL
   END DO
END DO

RETURN
END

SUBROUTINE MXADD(A, B, IDIM, JDIM, IROW, JCOL, COEFF)

MATRIX ADDITION: A = A + COEFF*B

IMPLICIT DOUBLE PRECISION (A-H, O-Z)

DIMENSION A(IDIM, JDIM), B(IDIM, JDIM)

DO I = 1, IROW
   DO J = 1, JCOL
      A(I, J) = A(I, J) + COEFF * B(I, J)
   END DO
END DO

RETURN
END
APPENDIX B

Demonstration problem I: 2-D analysis of a single-lap joint.
ABAQUS INPUT FILE

*HEADING
2-D SINGLE-LAP JOINT. 100 H2L6N ELEMENTS ALONG BONDLINE.
**

*NODE
1, 0.0, 0.0
51, 63.5, 0.0
151, 76.2, 0.0
605, 0.0, 1.6
655, 63.5, 1.6
755, 76.2, 1.6
1001, 63.5, 1.75
1101, 76.2, 1.75
1151, 139.7, 1.75
1605, 63.5, 3.35
1705, 76.2, 3.35
1755, 139.7, 3.35
2001, 63.5, 1.675
2101, 76.2, 1.675
**

*NGEN, NSET=BL
1, 605, 151
*NGEN, NSET=BM
51, 655, 151
*NGEN, NSET=BR
151, 755, 151
*NGEN, NSET=TL
1001, 1605, 151
*NGEN, NSET=TM
1101, 1705, 151
*NGEN, NSET=TR
1151, 1755, 151
*NGEN, NSET=MIDDLE
2001, 2101, 1
**

*NFILL
BL, BM, 50, 1
BM, BR, 100, 1
TL, TM, 100, 1
TM, TR, 50, 1
*ELEMENT, TYPE=CPE4
1, 1, 2, 153, 152
451, 454, 455, 606, 605
1101, 1102, 1253, 1252
1151, 1152, 1153, 1304, 1303
**

** DEFINE ADHESIVE ELEMENT H2L6N
**

*USER ELEMENT, NODES=6, TYPE=U1, PROPERTIES=56
1,2
*ELEMENT, TYPE=U1
2001, 504, 505, 655, 656, 2001, 2002
2101, 2001, 2002, 1001, 1002, 1152, 1153
*ELGEN, ELSET=TOP
1101, 50, 1, 1, 1
1151, 150, 1, 1, 3, 151, 150
*ELGEN, ELSET=BOT
1, 150, 1, 1, 3, 151, 150
451, 50, 1, 1, 1
*ELGEN, ELSET=MID4
2001, 100, 1, 1, 1

80
*ELGEN, ELSET=MID5
2101, 100, 1, 1, 1
*ELSET, ELSET=ONE
1
**
** USER DEFINED SUBROUTINE:
**
*USER SUBROUTINE, INPUT=uel_hybrid.f
**
** ELEMENT PROPERTIES
**
*SOLID SECTION, ELSET=TOP, MATERIAL=MID1
*SOLID SECTION, ELSET=BOT, MATERIAL=MID3
*MATERIAL, NAME=MID1
*ELASTIC, TYPE=ISO
0.69000E+05, 0.32E+00, 0.000000000E+00
**
**
*MATERIAL, NAME=MID3
*ELASTIC, TYPE=ISO
0.69000E+05, 0.32E+00, 0.000000000E+00
**
** USER DEFINED ELEMENT PROPERTY LIST:
**
** BOTTOM ROW
**
*UEL PROPERTY, ELSET=MID4
11.0, 1.0, 1.0
1.0, 1.0
0.4, 0.0, 69000.0, 69000.0, 69000.0, 0.32, 0.32, 0.32
26136.3636, 26136.3636, 26136.3636
1.0, 1.0
0.15, 0.0, 3000.0, 3000.0, 3000.0, 0.36, 0.36, 0.36
1102.9412, 1102.9412, 1102.9412,
**
** TOP ROW
**
*UEL PROPERTY, ELSET=MID5
11.0, 1.0, 1.0
1.0, 1.0
0.15, 0.0, 3000.0, 3000.0, 3000.0, 0.36, 0.36, 0.36
1102.9412, 1102.9412, 1102.9412,
1.0, 1.0
0.4, 0.0, 69000.0, 69000.0, 69000.0, 0.32, 0.32, 0.32
26136.3636, 26136.3636, 26136.3636
**
**
*NSET, NSET=HOLD
1,152,303,454,605
*ELSET, ELSET=PULL
1150,1300,1450,1600
*ELSET, ELSET=NAVE
500
*NGEN, NSET=ROLLER
2, 5, 1
606, 609, 1
1147, 1151, 1
1751, 1755, 1
**
**
*BOUNDARY
HOLD, 1, 2
ROLLER, 2
**
**
*STEP, PERTURBATION
*STATIC
**
** LOAD CASE SPECIFICATION:
**
*DLOAD, OP=NEW
PULL, P2, -93.75
**
**
*NODE PRINT
U
RF
*EL PRINT, ELSET=NAVE, POSITION=AVERAGED AT NODES
S
*END STEP
ABAQUS OUTPUT FILE

```

```

THIS PROGRAM HAS BEEN DEVELOPED BY
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1080 MAIN STREET
PAWTUCKET, R.I. 02860

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***************
" NOTICE "
***************

THIS IS ABAQUS VERSION 5.3.

PLEASE MAKE SURE YOU ARE USING VERSION 5.3 MANUALS
PLUS THE NOTES ACCOMPANYING THIS RELEASE. THESE NOTES
CAN BE OBTAINED BY USING THE INFORMATION OPTION ON THE
ABAQUS COMMAND LINE.

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ABAQUS INPUT ECHO

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```
*NGEN, NSET-BL
1, 605, 151
*NGEN, NSET-BM
51, 655, 151
*NGEN, NSET-BR
151, 755, 151
*NGEN, NSET-TL
1001, 1605, 151
*NGEN, NSET-TM
1101, 1705, 151
*NGEN, NSET-TR
1151, 1755, 151
*NGEN, NSET-MIDDLE
CARD 10
1001, 76.2, 1.675
CARD 15
2001, 63.5, 1.675
CARD 20
51, 655, 151
*NGEN, NSET-BM
151, 755, 151
*NGEN, NSET-TL
1001, 1605, 151
*NGEN, NSET-TM
1101, 1705, 151
*NGEN, NSET-TR
1151, 1755, 151
*NGEN, NSET-MIDDLE
CARD 25
1101, 1253, 1252
CARD 30
1151, 1152, 1153, 1304, 1303
*USEf(ELEMENT,NODES=6,TYPE=U1, PROPERTIES-24,COORDINATES=3, VARIABLES=1
CARD 35
1, 150, 1, 1, 3, 151, 150
*ELGEN, ELSET-BOTTOM
2001, 100, 1, 1, 1
*ELGEN, ELSET-MID4
2101, 100, 1, 1, 1
*ELGEN, ELSET-MID5
CARD 40
1, 150, 1, 1, 3, 151, 150
*ELGEN, ELSET-ONE
2101, 100, 1, 1, 1
** USER DEFINED SUBROUTINE:
CARD 60
** USER SUBROUTINE, INPUT=uel_hybrid.f
**
CARD 65
** ELEMENT PROPERTIES
**
*SOLID SECTION,ELSET-TOP ,MATERIAL-MID1
*SOLID SECTION,ELSET-BOTTOM ,MATERIAL-MID3
*MATERIAL, NAME-MID1
*ELASTIC, TYPE-ISO
CARD 70
0.69000E+05, 0.32E+00, 0.000000000E+00
**
**
*MATERIAL, NAME-MID3
*ELASTIC, TYPE-ISO
CARD 75
0.69000E+05, 0.32E+00, 0.000000000E+00
**
** USER DEFINED ELEMENT PROPERTY LIST:
**
CARD 80
*UEL PROPERTY,ELSET-MID4
0.69E5, 0.32E+0.0 0.3E4,0.36,1.0
6.0,1.0,3.0,0.0
** TOP ROW
CARD 85
*UEL PROPERTY,ELSET-MID5
0.3E4,0.36,1.0
6.0,1.0,3.0,0.0
*NSET, NSET=HOLD
CARD 90
1, 152, 303, 454, 605
*ESSET, ESSET-PULL
1150, 1300, 1450, 1600
*ESSET, ESSET-NAVE
500

CARD 95
*NGEN, NSET-ROLLER
2, 5, 1
606, 609, 1
1147, 1151, 1
1751, 1755, 1

CARD 100
**
**
*BOUNDARY
HOLD, 1, 2
ROLLER, 2

CARD 105
*STEP, PERTURBATION
*STATIC
**
**

CARD 110
*DLOAD, OP-NEW
PULL, P2, -93.75
*NODE PRINT
U
RF

CARD 115
*EL PRINT, ESSET-NAVE, POSITION=AVERAGED AT NODES
S
*END STEP

5 10 15 20 25 30 35 40 45 50 55 60 65 70 75

OPTIONS BEING PROCESSED

*HEADING
*NODE
*NGEN, NSET-BL
*NGEN, NSET-BM
*NGEN, NSET-BR
*NGEN, NSET-TL
*NGEN, NSET-TM
*NGEN, NSET-TR
*NGEN, NSET-MIDDLE
*NFILL

THE FOLLOWING NODES WILL BE USED IN THE NFILL GENERATION

BOUND 1 1 152 303 454 605
BOUND 2 51 202 353 504 655

THE FOLLOWING NODES WILL BE USED IN THE NFILL GENERATION

BOUND 1 51 202 353 504 655
BOUND 2 151 302 453 604 755

THE FOLLOWING NODES WILL BE USED IN THE NFILL GENERATION

BOUND 1 1001 1152 1303 1454 1605
BOUND 2 1101 1252 1403 1554 1705

THE FOLLOWING NODES WILL BE USED IN THE NFILL GENERATION

BOUND 1 1101 1252 1403 1554 1705
BOUND 2 1151 1302 1453 1604 1755

*ELEMENT, TYPE=CPE4
*USER ELEMENT, NODES=6, TYPE=U1, PROPERTIES=24, COORDINATES=3, VARIABLES=1
*ELEMENT, TYPE=U1
*ELGEN, ESSET=TOP
*ELGEN, ESSET=BOTTOM
*ELGEN, ESSET=MID4
*ELGEN, ESSET=MID5
*ELSET, ESSET=ONE

85
**ELEMENT DEFINITIONS**

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<tr>
<th>NUMBER</th>
<th>TYPE</th>
<th>PROPERTY</th>
<th>NODES FORMING ELEMENT</th>
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<td>10 11 162 161</td>
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</tbody>
</table>

**USER ELEMENTS**

**ELEMENT TYPE** u1  
**NUMBER OF NODES** 6
NUMBER OF COORDINATES 3  
NUMBER OF PROPERTIES 24  
NUMBER OF VARIABLES 1  

DEGREES OF FREEDOM  
NODE D.O.F.  
1 1 2  
2 1 2  
3 1 2  
4 1 2  
5 1 2  
6 1 2  

SOLID SECTION  

PROPERTY NUMBER 1  
MATERIAL NAME MIDI  
ATTRIBUTES 1.0000 .00000E+00 .00000E+00  

HOURGLASS CONTROL STIFFNESS PARAMETER 130.68  
(USED WITH LOWER ORDER REDUCED INTEGRATED SOLID ELEMENTS LIKE CPS4R,CPE4RH,C3DBR)  

PROPERTY NUMBER 2  
MATERIAL NAME MID3  
ATTRIBUTES 1.0000 .00000E+00 .00000E+00  

HOURGLASS CONTROL STIFFNESS PARAMETER 130.68  
(USED WITH LOWER ORDER REDUCED INTEGRATED SOLID ELEMENTS LIKE CPS4R,CPE4RH,C3DBR)  

USER ELEMENT PROPERTY  

PROPERTY NUMBER 3  
PROPERTIES  
11.00 1.000 1.000 .00000E+00 .00000E+00 .00000E+00 .00000E+00 .00000E+00  
1.000 1.000 .00000E+00 .00000E+00 .00000E+00 .00000E+00 .00000E+00 .00000E+00  
.4000 .00000E+00 6.90000E+04 6.90000E+04 6.90000E+04 .3200 .3200 .3200  
2.6136E+04 2.6136E+04 2.6136E+04 .00000E+00 .00000E+00 .00000E+00 .00000E+00 .00000E+00  
1.000 1.000 .00000E+00 .00000E+00 .00000E+00 .00000E+00 .00000E+00 .00000E+00  
.1500 .00000E+00 3000. 3000. 3000. .3600 .3600 .3600  
1103. 1103. 1103. .00000E+00 .00000E+00 .00000E+00 .00000E+00 .00000E+00  

PROPERTY NUMBER 4  
PROPERTIES  
11.00 1.000 1.000 .00000E+00 .00000E+00 .00000E+00 .00000E+00 .00000E+00  
1.000 1.000 .00000E+00 .00000E+00 .00000E+00 .00000E+00 .00000E+00 .00000E+00  
.1500 .00000E+00 3000. 3000. 3000. .3600 .3600 .3600  
1103. 1103. 1103. .00000E+00 .00000E+00 .00000E+00 .00000E+00 .00000E+00  
1.000 1.000 .00000E+00 .00000E+00 .00000E+00 .00000E+00 .00000E+00 .00000E+00  
.1500 .00000E+00 6.90000E+04 6.90000E+04 6.90000E+04 .3200 .3200 .3200  
2.6136E+04 2.6136E+04 2.6136E+04 .00000E+00 .00000E+00 .00000E+00 .00000E+00 .00000E+00  

MATERIAL DESCRIPTION  

MATERIAL NAME: MIDI  
ELASTIC YOUNG'S POISSON'S  
MODULUS RATIO  
69000. .32000  

MATERIAL NAME: MID3  
ELASTIC YOUNG'S POISSON'S  
MODULUS RATIO  
69000. .32000  

ELEMENT SETS  

SET TOP MEMBERS  
1101 1102 1103 1104 1105 1106 1107 1108 11  
1113 1114 1115 1116 1117 1118 1119 1120 11  
1125 1126 1127 1128 1129 1130 1131 1132 11  
1581 1582 1583 1584 1585 1586 1587 1588 15  
1593 1594 1595 1596 1597 1598 1599 1600 16  

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## NODE SETS

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<thead>
<tr>
<th>Set Name</th>
<th>Nodes</th>
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<tr>
<td>BOTTOM</td>
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<tr>
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<td>2001-2008</td>
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<td>MID5</td>
<td>2101-2108</td>
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<td>ONE</td>
<td>2001-2008</td>
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<tr>
<td>PULL</td>
<td>1150-1300</td>
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<td>NAVE</td>
<td>500</td>
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<tr>
<td>BL</td>
<td>1-152</td>
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<tr>
<td>BM</td>
<td>51-202</td>
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<td>BR</td>
<td>151-302</td>
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<tr>
<td>TL</td>
<td>1001-1152</td>
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<tr>
<td>TM</td>
<td>1101-1152</td>
</tr>
<tr>
<td>TR</td>
<td>1151-1152</td>
</tr>
<tr>
<td>MIDDLE</td>
<td>2001-2008</td>
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<tr>
<td>HOLD</td>
<td>1-152</td>
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<tr>
<td>ROLLER</td>
<td>1151-1152</td>
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## NODE DEFINITIONS

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<th>Node Number</th>
<th>Coordinates</th>
<th>Single Point Constraints</th>
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<td>6.3500</td>
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<td>2101</td>
<td>76.200</td>
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STEP 1 STATIC ANALYSIS

FIXED TIME INCREMENTS
TIME INCREMENT IS 2.220E-16
TIME PERIOD IS 2.220E-16

THIS IS A LINEAR PERTURBATION STEP.
ALL LOADS ARE DEFINED AS CHANGE IN LOAD TO THE REFERENCE STATE
EXTRAPOLATION WILL NOT BE USED
CHARACTERISTIC ELEMENT LENGTH .492
PRINT OF INCREMENT NUMBER, TIME, ETC., EVERY 1 INCREMENTS

ELEMENT PRINT
SUMMARIES WILL BE PRINTED WHERE APPLICABLE

TABLE 1 S11 S22 S33 S12

NODE PRINT

THE FOLLOWING TABLE IS PRINTED FOR ALL NODES AT EVERY 1 INCREMENT
SUMMARIES WILL BE PRINTED

TABLE 1 U1 U2

THE FOLLOWING TABLE IS PRINTED FOR ALL NODES AT EVERY 1 INCREMENT
SUMMARIES WILL BE PRINTED

TABLE 2 RF1 RF2

DISTRIBUTED LOADS

ELEMENT LOAD AMP. MAGNITUDE ELEMENT LOAD AMP. MAGNITUDE
TYPE REF. TYPE REF.
1150 P2 -93.750 1300 P2 -93.750
1600 P2 -93.750 1450 P2 -93.750

BOUNDARY CONDITIONS

NODE DOF AMP. MAGNITUDE NODE DOF AMP. MAGNITUDE
REF. REF.
1 1 (RAMP) .00000E+00 1 2 (RAMP) .00000E+00
2 2 (RAMP) .00000E+00 3 2 (RAMP) .00000E+00
4 2 (RAMP) .00000E+00 5 2 (RAMP) .00000E+00
152 1 (RAMP) .00000E+00 152 2 (RAMP) .00000E+00
303 1 (RAMP) .00000E+00 303 2 (RAMP) .00000E+00
454 1 (RAMP) .00000E+00 454 2 (RAMP) .00000E+00
605 1 (RAMP) .00000E+00 605 2 (RAMP) .00000E+00
606 2 (RAMP) .00000E+00 607 2 (RAMP) .00000E+00
608 2 (RAMP) .00000E+00 609 2 (RAMP) .00000E+00
1147 2 (RAMP) .00000E+00 1148 2 (RAMP) .00000E+00
1149 2 (RAMP) .00000E+00 1150 2 (RAMP) .00000E+00
1151 2 (RAMP) .00000E+00 1152 2 (RAMP) .00000E+00
1752 2 (RAMP) .00000E+00 1753 2 (RAMP) .00000E+00
1754 2 (RAMP) .00000E+00 1755 2 (RAMP) .00000E+00

- (RAMP) OR (STEP) - INDICATE USE OF DEFAULT AMPITUDES ASSOCIATED WITH THE STEP

WAVEFRONT MINIMIZATION

WAVEFRONT MINIMIZATION METHOD 1 WILL BE USED.
NUMBER OF NODES 1611
NUMBER OF ELEMENTS 1200
ORIGINAL MAXIMUM D.O.F WAVEFRONT ESTIMATED AS 714
ORIGINAL RMS D.O.F WAVEFRONT ESTIMATED AS 429
PERIPHERAL DIAMETER IS DEFINED BY NODES 1 1151
WAVEFRONT OPTIMIZED BY CHOOSING 1151 AS THE STARTING NODE

MINIMUM WAVEFRONT OBTAINED USING METHOD 1. USE
*WAVEFRONT MINIMIZATION, NODES, METHOD=1
1 1151
TO REDUCE THE CPU TIME ON SUBSEQUENT JOBS USING THIS SAME MESH.

PROBLEM SIZE

NUMBER OF ELEMENTS IS 1200
NUMBER OF NODES IS 1611
NUMBER OF NODES DEFINED BY THE USER 1611
NUMBER OF INTERNAL NODES GENERATED BY THE PROGRAM 0
TOTAL NUMBER OF VARIABLES IN THE MODEL 3222
(DEGREES OF FREEDOM PLUS ANY LAGRANGE MULTIPLIER VARIABLES)
MAXIMUM D.O.F. WAVEFRONT ESTIMATED AS 32
RMS WAVEFRONT ESTIMATED AS 24

FILE SIZES - THESE VALUES ARE IN WORDS AND ARE CONSERVATIVE UPPER BOUNDS

UNIT LENGTH
21 167400
22 167400

IF THE RESTART FILE IS WRITTEN ITS LENGTH WILL BE APPROXIMATELY
107747 WORDS WRITTEN IN THE PRE PROGRAM
PLUS 60720 WORDS WRITTEN AT THE BEGINNING OF EACH STEP
PLUS 223141 WORDS FOR EACH INCREMENT WRITTEN TO THE *RESTART FILE

ALLOCATED WORKSPACE 436165

*USER SUBROUTINE, INPUT=uel_hybrid.f

END OF USER INPUT PROCESSING

JOB TIME SUMMARY
CPU TIME (SEC) = 5.8000

STEP 1 STATIC ANALYSIS

FIXED TIME INCREMENTS
TIME INCREMENT IS 2.220E-16
TIME PERIOD IS 2.220E-16

THIS IS A LINEAR PERTURBATION STEP.
ALL LOADS ARE DEFINED AS CHANGE IN LOAD TO THE REFERENCE STATE

ELEMENT ID 2101

STRESS OUTPUT IN LOCAL COORDINATES FOR LAYER 1

STRESS POINTS STRESS COMPONENTS
CI CJC SXX SYX
-1.0000 -1.0000 -.797E+00 .426E+02 .247E+02
-1.0000 1.0000 .498E+01 .317E+02 .161E+02
1.0000 -1.0000 .693E+02 .426E+02 .247E+02
1.0000 1.0000 .317E+02 .161E+02 .247E+02
0.0000 1.0000 .159E+02 .531E+02 .330E+02
-.5774 -.5774 .616E+01 .462E+02 .297E+02
-.5774 -.5774 .962E+01 .633E+02 .364E+02
-.5774 -.5774 .178E+02 .412E+02 .263E+02
-.5774 -.5774 .258E+02 .594E+02 .370E+02

STRESS OUTPUT IN LOCAL COORDINATES FOR LAYER 2

STRESS POINTS STRESS COMPONENTS
CI CJC SXX SYX
-1.0000 -1.0000 -.265E+02 .317E+02 .161E+02
-1.0000 1.0000 -.111E+02 -.170E+01 .110E+02
1.0000 -1.0000 .528E+02 .619E+02 .438E+02
1.0000 1.0000 .203E+02 .242E+02 .772E+00
-.0000 1.0000 .158E+02 -.107E+01
-.5774 -.5774 .166E+00 .234E+02 -.907E+01
-.5774 -.5774 .122E+02 .384E+02 .354E+02

90
### ELEMENT STRAIN ENERGY = \(0.801\times10^{-2}\)

### ELEMENT ID 2001

#### STRESS OUTPUT IN LOCAL COORDINATES FOR LAYER 1

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#### STRESS OUTPUT IN LOCAL COORDINATES FOR LAYER 2

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<tr>
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<tr>
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</table>

### ELEMENT STRAIN ENERGY = \(0.476\times10^{-1}\)

### ELEMENT ID 2100

#### STRESS OUTPUT IN LOCAL COORDINATES FOR LAYER 1

<table>
<thead>
<tr>
<th>STRESS POINTS</th>
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</tr>
</thead>
<tbody>
<tr>
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</table>

#### STRESS OUTPUT IN LOCAL COORDINATES FOR LAYER 2

<table>
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<tr>
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</tr>
</thead>
<tbody>
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### ELEMENT ID 2200

#### STRESS OUTPUT IN LOCAL COORDINATES FOR LAYER 1

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</tr>
</thead>
<tbody>
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<td>CJ</td>
</tr>
<tr>
<td>-1.0000</td>
<td>-1.0000</td>
</tr>
<tr>
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</tr>
<tr>
<td>1.0000</td>
<td>-1.0000</td>
</tr>
</tbody>
</table>
### Stress Output in Local Coordinates for Layer 2

**Stress Points**

<table>
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<tr>
<th>CI</th>
<th>CJ</th>
<th>SXX</th>
<th>SYY</th>
<th>SXY</th>
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<tbody>
<tr>
<td>-1.0000 -1.0000</td>
<td>.368E+03</td>
<td>.509E+02</td>
<td>.512E+02</td>
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</tr>
<tr>
<td>-1.0000 1.0000</td>
<td>.169E+03</td>
<td>.177E+02</td>
<td>-.627E+01</td>
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</tr>
<tr>
<td>1.0000 -1.0000</td>
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<td>.115E+03</td>
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<tr>
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<td>-.459E+01</td>
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</tr>
<tr>
<td>.5774 - .5774</td>
<td>.353E+03</td>
<td>.645E+02</td>
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<tr>
<td>-.5774 - .5774</td>
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<tr>
<td>-.5774 -.5774</td>
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<tr>
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<td>.229E+03</td>
<td>.239E+02</td>
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</tr>
</tbody>
</table>

**Element Strain Energy** = .476E-01

**Increment 1 Summary**

**Time Increment Completed** 2.220E-16, Fraction of Step Completed 1.00

**Step Time Completed** 2.220E-16, Total Time Completed .000E+00

### Element Output

The following table is printed for all nodes.

**Node Footnote**

<table>
<thead>
<tr>
<th>Node</th>
<th>Foot</th>
<th>S11</th>
<th>S22</th>
<th>S33</th>
<th>S12</th>
</tr>
</thead>
<tbody>
<tr>
<td>503</td>
<td>248.5</td>
<td>22.90</td>
<td>86.84</td>
<td>-57.95</td>
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<tr>
<td>504</td>
<td>235.6</td>
<td>35.79</td>
<td>86.84</td>
<td>66.43</td>
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<tr>
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<td>274.8</td>
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<td>70.49</td>
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<tr>
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<td>86.84</td>
<td>70.49</td>
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</tbody>
</table>

**Maximum**

<table>
<thead>
<tr>
<th>Node</th>
<th>Foot</th>
<th>S11</th>
<th>S22</th>
<th>S33</th>
<th>S12</th>
</tr>
</thead>
<tbody>
<tr>
<td>235.6</td>
<td>-16.28</td>
<td>86.84</td>
<td>-57.95</td>
<td></td>
<td></td>
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</tbody>
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**Minimum**

<table>
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<tr>
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<th>Foot</th>
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<th>S33</th>
<th>S12</th>
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</thead>
<tbody>
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<td>35.79</td>
<td>86.84</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

**Node Output**

The following table is printed for all nodes.

<table>
<thead>
<tr>
<th>Node</th>
<th>Foot</th>
<th>U1</th>
<th>U2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
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<td>.0000E+00</td>
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</tr>
<tr>
<td>3</td>
<td>2.3850E-03</td>
<td>.0000E+00</td>
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<tr>
<td>4</td>
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</tr>
<tr>
<td>5</td>
<td>6.3518E-03</td>
<td>.0000E+00</td>
<td></td>
</tr>
<tr>
<td>6</td>
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</tr>
<tr>
<td>7</td>
<td>1.3104E-02</td>
<td>1.1474E-02</td>
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</tr>
<tr>
<td>8</td>
<td>1.6396E-02</td>
<td>2.1403E-02</td>
<td></td>
</tr>
<tr>
<td>9</td>
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<tr>
<td>10</td>
<td>2.2558E-02</td>
<td>4.8975E-02</td>
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</tbody>
</table>

**Maximum**

<table>
<thead>
<tr>
<th>Node</th>
<th>Foot</th>
<th>U1</th>
<th>U2</th>
</tr>
</thead>
<tbody>
<tr>
<td>.2527</td>
<td>.6578</td>
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</tbody>
</table>
The analysis has been completed.
APPENDIX C

Demonstration problem II: 3-D analysis of a single-lap joint.
ABAQUS INPUT FILE

*HEADING
3-D SINGLE-LAP JOINT. 100 H2L12N ELEMENTS ALONG BONDLINE.
*PREPRINT, ECHO = NO, HISTORY = NO, MODEL = NO
*NODE
1, 0.0, 0.0, 0.0
301, 63.5, 0.0, 0.0
401, 76.2, 0.0, 0.0
4001, 0.0, 0.0, 1.6
4301, 63.5, 0.0, 1.6
4401, 76.2, 0.0, 1.6
5001, 63.5, 0.0, 1.75
5101, 76.2, 0.0, 1.75
5401, 139.7, 0.0, 1.75
9001, 63.5, 0.0, 3.35
9101, 76.2, 0.0, 3.35
9401, 139.7, 0.0, 3.35
4701, 63.5, 0.0, 1.675
4801, 76.2, 0.0, 1.675
50001, 0.0, 1.0, 0.0
50301, 63.5, 1.0, 0.0
50401, 76.2, 1.0, 0.0
54001, 0.0, 1.0, 1.6
54301, 63.5, 1.0, 1.6
54401, 76.2, 1.0, 1.6
55001, 63.5, 1.0, 1.75
55101, 76.2, 1.0, 1.75
55401, 139.7, 1.0, 1.75
59001, 63.5, 1.0, 3.35
59101, 76.2, 1.0, 3.35
59401, 139.7, 1.0, 3.35
54701, 63.5, 1.0, 1.675
54801, 76.2, 1.0, 1.675
**
*MGEN, NSET=FBL
1, 4001, 1000
*MGEN, NSET=FBM
301, 4301, 1000
*MGEN, NSET=FBR
401, 4401, 1000
*MGEN, NSET=FTL
5001, 9001, 1000
*MGEN, NSET=FTM
5101, 9101, 1000
*MGEN, NSET=FTR
5401, 9401, 1000
*MGEN, NSET=FMIDDLE
4701, 4801, 1
*MGEN, NSET=BBL
50001, 54001, 1000
*MGEN, NSET=BBM
50301, 54301, 1000
*MGEN, NSET=BBR
50401, 54401, 1000
*MGEN, NSET=BTL
55001, 59001, 1000
*MGEN, NSET=BTM
55101, 59101, 1000
96
*NGEN, NSET=BTR
55401, 59401, 1000
*NGEN, NSET=BMIDDLE
54701, 54801, 1

**
*NFILL, NSET=FRONT
FBL, FBM, 300
FBM, FBR, 100
FTL, FTM, 100
FTM, FTR, 300
*NFILL, NSET=BACK
BBL, BBM, 300
BBM, BBR, 100
BTL, BTM, 100
BTM, BTR, 300
*NFILL
FRONT, BACK, 5, 10000
FMIDDLE, BMIDDLE, 5, 10000

**
*ELEMENT, TYPE=C3D8
1, 1, 3
151, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312

*ELEMENT, TYPE=U5
1, 2, 3
151, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312

**
*ELGEN, ELSET=TOP
5001, 100, 1, 1, 3, 1000, 1000, 5, 10000, 10000
6001, 100, 1, 1, 3, 1000, 1000, 5, 10000, 10000
7001, 100, 1, 1, 3, 1000, 1000, 5, 10000, 10000
8001, 100, 1, 1, 3, 1000, 1000, 5, 10000, 10000

**
** DEFINE ADHESIVE ELEMENT H2L12N
**
*USER ELEMENT, NODES=12, TYPE=U5, PROPERTIES=100, COORDINATES=3, VARIABLES=1
1, 2, 3

*ELEMENT, TYPE=U5
4701, 4702, 4703, 4704, 4705, 4706, 4707, 4708, 4709, 4710, 4711, 4712

**
** USER DEFINED SUBROUTINE:
**
*USER SUBROUTINE, INPUT=uel_report.f

**
** ELEMENT PROPERTIES
**
*SOLID SECTION, ELSET=TOP , MATERIAL=MID1
*MATERIAL, NAME=MID1
*ELASTIC, TYPE=ISO
69000.0, 0.32, 0.0

**
*SOLID SECTION, ELSET=BOT , MATERIAL=MID3
*MATERIAL, NAME=MID3
*ELASTIC, TYPE=ISO
69000.0, 0.32, 0.0

**
** USER DEFINED ELEMENT PROPERTY LIST:
**
** TOP ROW

97
**

*UEL PROPERTY, ELSET-ADHTOP
11.0, 1.0, 1.0
1.0
0.15, 0.0, 3000.0, 3000.0, 3000.0, 0.36, 0.36, 0.36
1102.9412, 1102.9412, 1102.9412
1.0
0.4, 0.0, 69000.0, 69000.0, 69000.0, 0.32, 0.32, 0.32
26136.3636, 26136.3636, 26136.3636

**

** BOTTOM ROW
**

*UEL PROPERTY, ELSET-ADHBOT
11.0, 1.0, 1.0
1.0
0.4, 0.0, 69000.0, 69000.0, 69000.0, 0.32, 0.32, 0.32
26136.3636, 26136.3636, 26136.3636
1.0
0.15, 0.0, 3000.0, 3000.0, 3000.0, 0.36, 0.36, 0.36
1102.9412, 1102.9412, 1102.9412

*NSET, NSET-L, GENERATE
10001, 14001, 1000
20001, 24001, 1000
30001, 34001, 1000
40001, 44001, 1000

*NSET, NSET-LSIDE
L, FBL, BBL

*NSET, NSET-ROLLB, GENERATE
1, 24, 1
10001, 10024, 1
20001, 20024, 1
30001, 30024, 1
40001, 40024, 1
50001, 50024, 1
4001, 4024, 1
14001, 14024, 1
24001, 24024, 1
34001, 34024, 1
44001, 44024, 1
54001, 54024, 1

*NSET, NSET-ROLLE, GENERATE
5378, 5401, 1
15378, 15401, 1
25378, 25401, 1
35378, 35401, 1
45378, 45401, 1
55378, 55401, 1
9378, 9401, 1
19378, 19401, 1
29378, 29401, 1
39378, 39401, 1
49378, 49401, 1
59378, 59401, 1

*ELSET, ELSET-PULL
5150, 6250, 7250, 8250
15150, 16250, 17250, 18250
25150, 26250, 27250, 28250
35150, 36250, 37250, 38250
45150, 46250, 47250, 48250

*ELSET, ELSET-ONE
1

**

** BOUNDARY CONDITIONS:
**

*BOUNDARY
LSIDE, 1, 3
ROLLE, 3

98
ROLLB, 3
*STEP, PERTURBATION
*STATIC
**
** LOAD CASE SPECIFICATION:
**
*DLOAD, OP=NEW
FULL, P4, -93.75
*NODE PRINT
U
*EL PRINT, ELSET=ONE
 MISES,
*END STEP
ABAQUS OUTPUT FILE

*PREPRINT, ECHO = NO, HISTORY = NO, MODEL = NO

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<tr>
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<th>BBBB BBBB</th>
<th>AAAA</th>
<th>QQQQQQQ</th>
<th>U</th>
<th>U</th>
<th>SSSSSSSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>A A B B</td>
<td>A A A Q</td>
<td>Q</td>
<td>U</td>
<td>U</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>A A B B</td>
<td>A A A Q</td>
<td>Q</td>
<td>U</td>
<td>U</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>AAAAAAAA</td>
<td>BBBB BBBB</td>
<td>AAAAAA</td>
<td>Q</td>
<td>Q</td>
<td>U</td>
<td>U</td>
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<tr>
<td>A A B B</td>
<td>A A A Q</td>
<td>Q</td>
<td>Q</td>
<td>U</td>
<td>U</td>
<td>S</td>
</tr>
<tr>
<td>A A B B</td>
<td>A A A Q</td>
<td>Q</td>
<td>Q</td>
<td>U</td>
<td>U</td>
<td>S</td>
</tr>
<tr>
<td>A A BBB BBBB</td>
<td>A A</td>
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<td>UUUUUUU</td>
<td>SSSSSSSS</td>
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<td></td>
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</tbody>
</table>

OPTIONS BEING PROCESSED

*MODE
*NGEN, NSET=FB1
*NGEN, NSET=F83
*NGEN, NSET=F82
*NGEN, NSET=F71
*NGEN, NSET=F7M
*NGEN, NSET=FTR
*NGEN, NSET=FMIDDLE
*NGEN, NSET=FB1L
*NGEN, NSET=FB1M
*HEADING
3-D SINGLE-LAP JOINT. 100 H2L2M ELEMENTS ALONG BONDLINE.
**NGEN, NSET=HMIDDLE**

**NFILL, NSET=FRONT**

The following nodes will be used in the NFILL generation

3-D single-lap joint. 100 H2L12N elements along bondline.

<table>
<thead>
<tr>
<th>Bound</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1001</td>
<td>2001</td>
<td>3001</td>
<td>4001</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Bound 2

| 301   | 1301 | 2301  | 3301  | 4301  |       |       |       |       |       |

The following nodes will be used in the NFILL generation

**NFILL, NSET=FRONT**

The following nodes will be used in the NFILL generation

Bound 1

| 301   | 1301 | 2301  | 3301  | 4301  |       |       |       |       |       |

Bound 2

| 401   | 1401 | 2401  | 3401  | 4401  |       |       |       |       |       |

The following nodes will be used in the NFILL generation

Bound 1

| 5001  | 6001 | 7001  | 8001  | 9001  |       |       |       |       |       |

Bound 2

| 5101  | 6101 | 7101  | 8101  | 9101  |       |       |       |       |       |

The following nodes will be used in the NFILL generation

Bound 1

| 50001 | 60001 | 70001 | 80001 | 90001 |       |       |       |       |       |

Bound 2

| 50301 | 51301 | 52301 | 53301 | 54301 |       |       |       |       |       |

The following nodes will be used in the NFILL generation

Bound 1

| 50301 | 51301 | 52301 | 53301 | 54301 |       |       |       |       |       |

Bound 2

| 50401 | 51401 | 52401 | 53401 | 54401 |       |       |       |       |       |

The following nodes will be used in the NFILL generation

Bound 1

| 55001 | 56001 | 57001 | 58001 | 59001 |       |       |       |       |       |

Bound 2

| 55101 | 56101 | 57101 | 58101 | 59101 |       |       |       |       |       |

The following nodes will be used in the NFILL generation

3-D single-lap joint. 100 H2L12N elements along bondline.

Bound 1

| 55101 | 56101 | 57101 | 58101 | 59101 |       |       |       |       |       |

Bound 2

| 55401 | 56401 | 57401 | 58401 | 59401 |       |       |       |       |       |

The following nodes will be used in the NFILL generation

Bound 1

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<th>8</th>
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<tbody>
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<td>13</td>
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<td>16</td>
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</table>

Bound 2

<table>
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<tr>
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<th>50002</th>
<th>50003</th>
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<th>50005</th>
<th>50006</th>
<th>50007</th>
<th>50008</th>
<th>50009</th>
</tr>
</thead>
<tbody>
<tr>
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101
THE FOLLOWING NODES WILL BE USED IN THE NFILL GENERATION

BOUND 1
4701 4702 4703 4704 4705 4706 4707 4708 4709
4711 4712 4713 4714 4715 4716 4717 4718 4719
4721 4722 4723 4724 4725 4726 4727 4728 4729
4731 4732 4733 4734 4735 4736 4737 4738 4739
4741 4742 4743 4744 4745 4746 4747 4748 4749
4751 4752 4753 4754 4755 4756 4757 4758 4759
4761 4762 4763 4764 4765 4766 4767 4768 4769
4771 4772 4773 4774 4775 4776 4777 4778 4779
4781 4782 4783 4784 4785 4786 4787 4788 4789
4791 4792 4793 4794 4795 4796 4797 4798 4799
4801

BOUND 2
54701 54702 54703 54704 54705 54706 54707 54708 54709
54711 54712 54713 54714 54715 54716 54717 54718 54719
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54741 54742 54743 54744 54745 54746 54747 54748 54749
54751 54752 54753 54754 54755 54756 54757 54758 54759
54761 54762 54763 54764 54765 54766 54767 54768 54769
54771 54772 54773 54774 54775 54776 54777 54778 54779
54781 54782 54783 54784 54785 54786 54787 54788 54789
54791 54792 54793 54794 54795 54796 54797 54798 54799
54801

*ELEMENT, TYPE=C3D8
*ELGEN, ELSET=BOT
*ELGEN, ELSET=TOP
*USER ELEMENT, NODES=12, TYPE=US, PROPERTIES=100, COORDINATES=3, VARIABLES=1
*ELEMENT, TYPE=US
*ELGEN, ELSET=ADHDBOT
*ELGEN, ELSET=ADHTOP
*NSET, NSET=L, GENERATE
*NSET, NSET=LSIDE, GENERATE
*NSET, NSET=ROLLE, GENERATE
*NSET, NSET=ROLLB, GENERATE
*ELSET, ELSET=PULL
*ELSET, ELSET=ONE
*MATERIAL, NAME=MID1
*MATERIAL, NAME=MID3
*MATERIAL, NAME=MID3
*MATERIAL, NAME=MID1
*SOLID SECTION, ELSET=TOP, MATERIAL=MID1
*SOLID SECTION, ELSET=BOT, MATERIAL=MID3
*UEL PROPERTY, ELSET=ADHTOP
*UEL PROPERTY, ELSET=ADHDBOT
*STEP, PERTURBATION
*STATIC
*LOAD, OP=NEW
*EL PRINT, ELSET=ONE
*END STEP
*BOUNDARY
*STEP, PERTURBATION
*STATIC
*MODE PRINT
*END STEP

WAVEFRONT MINIMIZATION
WAVEFRONT MINIMIZATION METHOD 1 WILL BE USED.
NUMBER OF NODES 15666
NUMBER OF ELEMENTS 10000
ORIGINAL MAXIMUM D.O.F WAVEFRONT ESTIMATED AS 8592
ORIGINAL RMS D.O.F WAVEFRONT ESTIMATED AS 6967
PERIPHERAL DIAMETER IS DEFINED BY NODES 1 5401
WAVEFRONT OPTIMIZED BY CHOOSING 1 AS THE STARTING NODE
MINIMUM WAVEFRONT OBTAINED USING METHOD 2. USE
WAVEFRONT MINIMIZATION, NODES, METHOD=2
1, 5401
TO REDUCE THE CPU TIME ON SUBSEQUENT JOBS USING THIS SAME MESH.

PROBLEM SIZE
NUMBER OF ELEMENTS IS 10000
NUMBER OF NODES IS 15666
NUMBER OF NODES DEFINED BY THE USER 15666
NUMBER OF INTERNAL NODES GENERATED BY THE PROGRAM 0
TOTAL NUMBER OF VARIABLES IN THE MODEL 46998
(DEGREES OF FREEDOM PLUS ANY LAGRANGE MULTIPLIER VARIABLES)
MAXIMUM D.O.F. WAVEFRONT ESTIMATED AS 273
RMS WAVEFRONT ESTIMATED AS 172

FILE SIZES - THESE VALUES ARE IN WORDS AND ARE CONSERVATIVE UPPER BOUNDS

UNIT LENGTH

2 8815647
10 2282000
19 3328000
21 3320000
22 3320000
25 2220000

IF THE RESTART FILE IS WRITTEN ITS LENGTH WILL BE APPROXIMATELY
3323232 WORDS WRITTEN IN THE PRE PROGRAM
PLUS 2772000 WORDS WRITTEN AT THE BEGINNING OF EACH STEP
PLUS 4134521 WORDS FOR EACH INCREMENT WRITTEN TO THE RESTART FILE

ALLOCATED WORKSPACE 2002539

*USER SUBROUTINE, INPUT=uel_report.f

END OF USER INPUT PROCESSING

STEP 1 STATIC ANALYSIS

FIXED TIME INCREMENTS
TIME PERIOD IS 2.220E-16
THIS IS A LINEAR PERTURBATION STEP.
ALL LOADS ARE DEFINED AS CHANGE IN LOAD TO THE REFERENCE STATE

ELEMENT ID 44900

STRESS OUTPUT IN LOCAL COORDINATES FOR LAYER 1

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103
### Layer 1

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104
### STRAIN OUTPUT IN LOCAL COORDINATES FOR LAYER 2

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### ELEMENT OUTPUT

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**STEP TIME COMPLETED** 2.220E-16, **TOTAL TIME COMPLETED** 0.00E+00

**THE FOLLOWING TABLE IS PRINTED FOR ELSET ONE AND ELEMENT TYPE C3D8 AT THE INTEGRATION**

**INCREMENT SUMMARY**

- **STRAIN OUTPUT IN LOCAL COORDINATES FOR LAYER 1**
- **STRESS OUTPUT IN LOCAL COORDINATES FOR LAYER 2**
1 1 71.96
2 83.45
3 65.04
4 77.85
5 71.94
6 83.43
7 65.02
8 77.83

MAXIMUM 83.45
ELEMENT 1

MINIMUM 65.02
ELEMENT 1

NODE OUTPUT

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MINIMUM .0000E+00 -1.4846E-03 -.8600
AT NODE 1 54702 58177

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