EROSION EFFECTS ON
TVC VANE HEAT
TRANSFER CHARACTERISTICS

by

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Thesis Advisor: Morris Driels

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Erosion Effects on TVC Vane Heat Transfer Characteristics

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This work describes the effects of erosion on the heat transfer characteristics on thrust vector control vanes exposed to aluminized propellant exhaust flows. This was accomplished using an inverse heat transfer parameter identification of quarter scale models. The model is based on a four node lumped parameter system with two heat energy inputs. The erosion is modeled as decreasing the geometric dimensions linearly as a function of time and percentage of aluminum in the propellant. Excellent agreement was found between experimental and model temperature profiles. The heat transfer coefficients of the vanes were found to decrease with increasing erosion rates.
Erosion Effects on TVC Vane
Heat Transfer Characteristics

by

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ABSTRACT

This work describes the effects of erosion on the heat transfer characteristics on thrust vector control vanes exposed to aluminized propellant exhaust flows. This was accomplished using an inverse heat transfer parameter identification of quarter scale models. The model is based on a four node lumped parameter system with two heat energy inputs. The erosion is modeled as decreasing the geometric dimensions linearly as a function of time and the percentage of aluminum in the propellant. Excellent agreement was found between experimental and model temperature profiles. The heat transfer coefficients of the vanes were found to decrease with increasing erosion rates.
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I. INTRODUCTION

This thesis is a continuation of work done by the Naval Air Warfare Center Weapons Division (NAWCWPNS) and thesis work at the Naval Postgraduate School (NPS) to provide a better understanding of the heat transfer characteristics of jet vanes used for thrust vector control (TVC) of vertical launch missiles. This is accomplished using an inverse heat transfer parameter identification of quarter scale replicas which can be used to find full scale results.

Thrust vector control is a process by which jet vanes are inserted into the exhaust plume of a missile to control the flight path prior to the missile obtaining the required velocity for the external control surfaces to take effect. [Ref. 1] A schematic for the TVC system is shown in Figure 1.
Due to the harsh thermal environment that the vanes are exposed to, a better understanding of the heat transfer processes which take place will help in the improved design of jet vanes. This will lead to longer operation and the ability to use propellants that burn hotter and use a higher percentage of aluminum for greater momentum flux and better performance. (Ref. 2:p. 1)

There are five basic steps in determining the heat transfer characteristics of the vane:

1. Develop a mathematical model of the heat transfer processes which take place in the vane. It is expressed in terms of a number of physical constants, some of which are known, some of which are to be determined. (Ref. 3:p. 1)

2. Gather experimental data in the form of temperature-time data at selected locations on the vane.

3. Compare the predicted and experimental temperature-time data.

4. Use the differences between the simulated and actual temperatures to drive a systematic adjustment of unknown model parameters in an optimization routine. The process is repeated until the experimental and theoretical data differences are minimized in a least-squares sense. (Ref. 3:p. 2)

5. Calculate the heat transfer parameters of the system using the physical parameters of the model which give the best estimate of the actual behavior.
Previous work has concentrated on using parametric system identification to validate the use of full and quarter scale models to predict the heat transfer characteristics for full scale vanes in a non-erosive environment. The research in this report extends the quarter scale model to an erosive environment.
II. THEORY

A. BACKGROUND

1. Physical Description

The main pieces of equipment used in the experimental tests are the rocket motor and the jet vanes. The rocket motor is set up to provide a constant thrust-time profile. The propellants used in the motor are aluminized hydroxyl-terminated polybutadiene (HTPB) with either 0%, 9%, or 18% Al by weight. The jet vanes are made from pressed and sintered tungsten powder that is infiltrated by 10% copper by weight. There are four vanes for each motor. The experimental setup is shown in Figure 2. (Ref. 2:p. 1,2)

![Figure 2 Vane and Motor Assembly for Experimental Tests](image-url)
The experimental tests are conducted as either full or quarter scale. The quarter scale tests have several advantages. Most important is the cost savings over a full scale test. The reduced size of the motor, vane and test equipment account for much of the savings. [Ref. 4:p. 15,16] The biggest disadvantage of the quarter scale vane comes in the placement of the thermocouples. Whereas in the full scale vane the thermocouples can be placed inside the vane, for the quarter scale vane the thermocouples must be placed on the vane shaft. The thermocouple placement is contrasted in Figure 3.

**Figure 3** Thermocouple Placement for Full and Quarter Scale

### 3. Basic Modeling

In order to predict the thermal response of the jet vane, a simple model had to be developed. The model had to
consider the physical characteristics of the vane and the heat transfer processes that were taking place.

The physical quantities can be broken into two categories: material and geometric. The material properties considered were the vane density, thermal conductivity, and specific heat. The geometric properties considered were the conductive lengths, cross sectional and surface areas and volume.

The heat transfer processes considered were convection at the surface of the vane and conduction of heat through the vane.

![Figure 4: Thermal Energy Node Model](image)

Heat transfer in the vane is modeled by applying the law of conservation of energy. Energy balance equations can be derived using a model consisting of thermal resistances and
capacitances driven by the temperature difference between the nodes. The energy balance for Figure 4 is,

$$\dot{Q} = \frac{T_1 - T_0}{C_1 R_1} - \frac{T_1 - T_0}{C_1 R_0} + \frac{T_0}{C_1 R_0}$$

(1)

where $T_1 > T_0$. The convective resistance is found by,

$$R = \frac{1}{h A_s}$$

(2)

where $h$ is the convective heat transfer coefficient and $A_s$ is the surface area. The conductive resistance is found by,

$$R = \frac{L}{k A_x}$$

(3)

where $L$ is the conductive length, $k$ is the thermal conductivity and $A_x$ is the cross sectional area. The thermal capacitance is given by,

$$C = \rho V C_p$$

(4)

where $\rho$ is the material density, $V$ is the volume and $C_p$ is the material specific heat.

3. Lumped Parameter

The nodes of the basic model lend itself to dividing the vane into different sections, or lumps. For the full
scale model, the vane was geometrically divided into three separate sections: the tip, fin and shaft. A node is located at the center of each section. The sections are defined as shown in Figure 5. For the quarter scale model, a fourth node was added at the mount to account for the different thermocouple placement.

4. PSI Process

A simple model was needed that could easily be changed for different materials, geometries and exhaust conditions. This led to parametric system identification (PSI). PSI is a computer-based procedure where the parameters of a model are changed until a best fit approximation in a least squares sense to experimental data is obtained. [Ref. 5: p. 6]

Parameter identification has several advantages over the other modeling choice, computational fluid dynamics (CFD). Creating a mathematical model of the vane using CFD is almost impossible due to the complexity of the exhaust flow. The jet vane must operate in a high temperature, three-dimensional, turbulent, compressible supersonic flow [Ref. 5: p. 3, 4] PSI ignores these complexities and focuses on the end result. This makes PSI not only simpler, but the information that comes out of the PSI model can easily be used in improving the design. PSI also handles nonlinear conditions such as ablation. [Ref. 6: p. 2]
The simple three node model shown in Figure 6 can serve as baseline for other models. The model is driven by two heat sources, represented by temperatures $T_n$ and $T_h$, which are the stagnation and free stream recovery temperatures, respectively. The temperature $T_n$ heats the vane through convective heat transfer at node one through the thermal
resistance $R_n$, and stores the energy as a thermal capacitance in $C_1$. The same process occurs at node two with recovery temperature $T_n$, thermal resistance $R_n$, and thermal capacitance $C_1$. Node three stores energy in thermal capacitance $C_3$ and is connected to ground through thermal resistance $R_{30}$. All nodes are coupled by conductive resistances. [Ref. 6:p. 4] Applying the law of conservation of energy to the system leads to the following equations:

![Figure 6 Three Node Jet Vane Model](image-url)
\[
\dot{T}_1 = -\frac{T_1}{C_1 R_{F1}} - \frac{T_1}{C_1 R_{12}} + \frac{T_2}{C_1 R_{12}} + \frac{T_{R1}}{C_1 R_{F1}} \quad (5)
\]

\[
\dot{T}_2 = \frac{T_1}{C_2 R_{12}} - \frac{T_2}{C_2 R_{F2}} - \frac{T_2}{C_2 R_{12}} - \frac{T_3}{C_2 R_{23}} + \frac{T_3}{C_2 R_{F2}} + \frac{T_{R2}}{C_2 R_{F2}} \quad (6)
\]

\[
\dot{T}_3 = \frac{T_2}{C_3 R_{23}} - \frac{T_3}{C_3 R_{23}} - \frac{T_3}{C_3 R_{3g}} \quad (7)
\]

letting,

\[
a_{12} = \frac{1}{C_1 R_{12}} \quad a_{21} = -\frac{1}{C_2 R_{12}} \quad (8)
\]

\[
a_{23} = \frac{1}{C_2 R_{23}} \quad a_{32} = -\frac{1}{C_3 R_{23}} \quad (9)
\]

\[
a_{33} = \frac{1}{C_3 R_{3g}} \quad b_{11} = -\frac{1}{C_1 R_{F1}} \quad (10)
\]

\[
b_{22} = \frac{1}{C_2 R_{F2}} \quad (11)
\]

Combining coefficients at the same temperatures gives,

\[
a_{11} = a_{12} + b_{11} \quad (12)
\]

\[
a_{22} = a_{21} + a_{23} + b_{22} \quad (13)
\]

\[
a_{33} = a_{32} + a_{33} \quad (14)
\]

Rewriting the equations,

\[
\dot{T}_1 = -a_{11} T_1 + a_{12} T_2 + b_{11} T_{R1} \quad (15)
\]

11
\[ \dot{T}_2 = a_{21} T_1 - a_{22} T_2 + a_{23} T_3 + b_{22} T_{R2} \]  
[16]

\[ \dot{T}_3 = a_{32} T_2 - a_{33} T_3 \]  
[17]

Rewriting into state-space form, \( T = AT + Bu \), or

\[
\begin{bmatrix}
\dot{T}_1 \\
\dot{T}_2 \\
\dot{T}_3
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{21} & 0 \\
a_{21} & -a_{22} & a_{23} \\
0 & a_{32} & -a_{33}
\end{bmatrix}
\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} +
\begin{bmatrix} b_{11} & 0 & 0 \\
0 & b_{22} & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} T_{R1} \\ T_{R2} \end{bmatrix}
\]  
[18]

The energy balance equations are a set of linear, ordinary differential equations which can be readily solved on a computer. This was done in a Fortran program using an IMSL subroutine called DIVPRK. DIVPRK solves a double precision initial value problem for ordinary differential equations using fifth-order and sixth-order Runge-Kutta-Verner methods. DIVPRK requires a user supplied subroutine called FCN which defines the set of equations to be solved.

The main program containing DIVPRK and FCN is called SIM.FOR, and simulates the temperatures of the three node model. The model is driven by an input vector \( u \) which is the product of the recovery temperatures \( T_{R1} \) and \( T_{R2} \) and a step function simulating the thrust. Physical and geometric data was used to calculate the internal thermal conductive resistances and capacitances which lead to coefficients in the \( A \) matrix. Since the inputs at nodes one and two from
convection and node three from ground are unknown, values for these resulting coefficients must be guessed. The output of the program is temperature-time data which is written to a data file called TEMP.MAT. This data can then be read into MATLAB and plotted. The purpose is to try to match calculated temperatures with known experimental temperature data at node two and validate the numerical approach. The results are shown in Figure 7. Although the node two temperatures are close, they are not identical. By extending the program to include an optimizer that could adjust the unknown A and B coefficients, a closer approximation could be found.

This was done in a Fortran program called NODE3.FOR. It is in this parameter identification, or PID, program that the differential equations are set up and solved. First, physical and geometric data is read in from a data file called INPUT.DAT. This information is used to calculate the internal thermal conductive resistances and capacities which lead to coefficients in the A matrix. Since the inputs at nodes one and two from convection and node three from ground are unknown, the resulting unknown coefficients from the A and B matrices are sent to the optimizer as variables to be found. The optimizer used is an IMSL routine called DBCLSF which uses a modified Levenberg-Marquardt method and an active set strategy to minimize an error in a least-squares sense subject to simple constraints placed on the variables by the user. DBCLSF calls a user written subroutine called TEMP that
Figure 7 Simulated and Experimental Node Temperatures

calculates the temperature-time history using the current parameters supplied by DBCLSF called from the PID program. It does this through DIVPRK and FCN. Once the temperature-time history is calculated, an error function is returned to DBCLSF based on the differences between predicted and experimental temperature-time histories. The optimizer then adjusts the
unknown parameters and the process repeats until certain convergence criteria is met.

B. PREVIOUS MODELS

Work on the jet vane thermal model began at the Naval Postgraduate School (NPS) by Nunn and Kelleher [Ref. 7] in 1986. Further development of the model was continued by Nunn [Ref. 5] and Hatzenbuehler. [Ref. 8] Hatzenbuehler was able to create a four node quarter scale model using PSI procedures and a computer software package called Matrix X. Reno [Ref. 13] followed Hatzenbuehler and refined the four node model and attempted to compare the quarter scale results to full scale vanes, but was unsuccessful. More recent work has been done by Parker [Ref. 4]. He obtained good results using a full scale model of the jet vane. He also looked more closely at the scaling of the models and the applicability of quarter scale results to full scale vanes. He also found that existing quarter scale models did not provide an accurate picture of the heat transfer processes in the full scale vanes.

C. THREE NODE FULL SCALE MODEL

Parker's five node full scale model was reduced to a three node full scale model to investigate whether the three fin nodes could be reduced to one node and obtain the same
results. Parker's five node full scale model is shown in Figure 8.

Figure 8 Parker Five Node Full Scale Model

The three node model was driven using the geometric data given in Table 1 and the following material data: \( \rho = 18310 \) \( \text{kg/m}^3 \), \( k = 173 \) \( \text{W/mK} \), and \( C_v = 146 \) \( \text{J/kgK} \). The recovery temperatures used to drive the system were \( T_{m} = 2670 \) K and \( T_{u} = 2570 \) K. [Ref. 6:p. 7,8] These temperatures were contained in the input vector \( u \), whose values were the product of the recovery temperature and a step function simulating the thrust function.
Table I  GEOMETRIC DATA FOR FULL SCALE VANES

<table>
<thead>
<tr>
<th></th>
<th>tip to vane</th>
<th>vane to shaft</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$, cm$^3$</td>
<td>2.6</td>
<td>52.0</td>
</tr>
<tr>
<td>$A'$, cm$^2$</td>
<td>5.9</td>
<td>5.2</td>
</tr>
<tr>
<td>$A''$, cm$^3$</td>
<td>4.35</td>
<td>112.16</td>
</tr>
<tr>
<td>$L$, cm</td>
<td>5.0</td>
<td>6.0</td>
</tr>
</tbody>
</table>

The program found the values for $b'' = 1.0029$ and $b' = 0.0809$. This corresponds to the convection heat transfer coefficients of 16,025 W/m²K and 1003 W/m²K at the tip and fin respectively. The ground resistance was found to be 0.0001. These values were found to be reasonably close to those from Parker's five node model. He found $b'' = 1.3787$, $b' = 0.0862$, and the ground resistance to be 0.0001, while the convection heat transfer coefficients were 22027.5 W/m²K and 1057 W/m²K at the tip and fin respectively. (Ref. 4:p. 63) The temperature-time histories for both models are shown in Figure 9.
Figure 9  Temperature-time Histories for Three and Five Node Full Scale Models
III. ABLATION EFFECTS

A. ABLATION MODELING

There was erosion in the quarter scale vanes exposed to aluminized propellant exhaust flows. For the 0% aluminized case, only 1% of the vane's mass was lost. But for the 9% and 18% aluminized cases, the loss became much more substantial. For the 9% case, 8% of the vane's mass was lost. For the 18% aluminized case, 50% of the vane's mass was lost. Vane mass loss was found to be nonlinear with the percentage of Al in the propellant. The relationship using an exponential function by an empirical fit was found to be

\[ \% \text{mass loss} = 1.042e^{0.2173(\% \text{Al})} \]  

(19)

Vane erosion profiles for the three cases are shown in Figure 10. [Ref. 2:p. 6,7] At least part of this erosion was likely caused by ablation. Ablation is due to the melting of the surface of the vane [Ref. 9:p. 122].

A short FORTRAN program, COEF.FOR, was written to see how the known A matrix coefficients were affected by the mass loss. The geometric dimensions of length, area, and volume were modeled as decreasing linearly as a function of time and percent mass loss. The results for the 9% Al and 18% Al cases are shown in Figure 11.
Several trends in Figure 11 are worth noting. The dominant coefficient in both cases is $a_{12}$. This is expected since

$$a_{12} = \frac{1}{C_1 R_{12}}$$

and $C_1$ is small due to the small volume at the tip of the vane. Also note that the coefficients are nonlinear over time and the nonlinearity increases with increased mass loss.

B. FOUR NODE QUARTER SCALE MODEL

The erosion present in the quarter scale vanes when the aluminized propellant was used needed to be investigated. A four node quarter scale model had already been derived by
Figure 11: Effect of Erosion on A Matrix Coefficients in the 9% Al and 18% Al Cases
Application of the law of conservation of energy led to the following equations:

\[ \dot{T}_1 = -a_{11}T_1 + a_{12}T_2 + b_{11}T_{RI} \]  (21)

\[ \dot{T}_2 = a_{21}T_1 - a_{22}T_2 + a_{23}T_3 + b_{22}T_{R2} \]  (22)

\[ \dot{T}_3 = a_{32}T_2 - a_{33}T_3 + a_{34}T_4 \]  (23)

\[ \dot{T}_4 = a_{43}T_3 - a_{44}T_4 \]  (24)

These equations needed to be modified though, since they did not include the effects of erosion. Erosion of the vane caused the geometric dimensions of the vane to change, while the material properties of density, thermal conductivity and specific heat remained constant. The program COEF.FOR modeled the changing geometric dimensions with time. All that was needed was to attach COEF.FOR to the main PID program as a subprogram.

The other aspect of interest in the cases with aluminized propellant was whether the convective heat transfer coefficients were time variant. Once the values of \( b_{11} \) and \( b_{22} \) are found in the PID program, the program COEF.FOR can be modified so that the heat transfer coefficients can be calculated at every time step since

\[ h_t = \frac{1}{R_{PTA_{st}}} \quad h_t' = \frac{1}{R_{P2A_{st}}} \]  (25)
\[ R_{r1} = \frac{1}{b_{11}c_1} \quad R_{r2} = \frac{1}{b_{22}c_2} \]  

and \( C_1, C_2, A_n, \) and \( A_m \) are all time dependant.

C. CONVERGING QUARTER SCALE MODEL WITH ABLATION

1. Case 1: 0% Al in Propellant

For case 1, data was taken for three seconds before thrust began to tail off. This allowed for 61 temperature-time data points to be taken, or 20 per second. The data points on the vane corresponded to nodes three and four of the model. This data was read into the PID program NODE40.FOR along with the geometric data and the recovery temperatures. In the subroutine FCN, a delay of 0.3 seconds was used to account for the time before the thrust reached its steady state value. The results obtained were excellent; the square root of the sum of the squares of the difference between experimental and model temperatures at nodes three and four was only 1.19 degrees Kelvin. A plot of the experimental and model temperatures is shown in Figure 12.

The values obtained for the unknown variables were \( a_m = 0.5376, a_o = 0.1528, a_n = -0.1651, b_{11} = 7.6511, \) and \( b_2 = 0.0722. \) These variables led to resistance values of \( R_{r1} = 1.2221, \) \( R_{r2} = 6.4795, \) and \( R_{r0} = -1.8206. \) The negative value obtained for \( R_{r0} \) indicates heating of the vane from ground. The convection heat transfer coefficients were calculated to be 30405.43
CASE 1. 0% Al in Exhaust

\[ T4 \]

\[ \text{T4 EXP. CAL} \]

\[ \text{T3} \]

\[ \text{EXP. CAL} \]

Figure 12 Case 1: Experimental and Model Temperatures Vs. Time

W/m²K and 222.43 W/m²K at the tip and fin respectively.

2. Case 2: 9% Al in Propellant

The same procedure was done for case 2. Temperature-time data was only taken for two seconds before thrust tailoff. A delay of 0.7 seconds was used to account for the time before the thrust reached its steady state value. Again, the results were excellent: the sum of the squares difference was only 0.73 degrees Kelvin. A plot of the experimental and model temperatures is shown in Figure 13.

The values obtained for the unknown variables were \( a_u = -0.2000, \ a_o = 3.5088, \ a_w = 100.00, \ b_n = 3.4427, \) and \( b_w = 0.0837. \)
The variables lead to resistance values of $R_n = 2.9135$, $R_m = 5.9930$, and $R_o = -0.1989$. Again, the negative resistance of $R_o$ indicates heating of the vane from ground. The convection heat transfer coefficients were calculated to be 13354.40 and 251.80 at the tip and fin respectively.

The values for $b_n$ and $b_m$ found from NODE49.FOR were added to the geometric and material data in COEF.FOR in order to calculate the convective heat transfer coefficients at every time step. The heat transfer coefficient at the tip decreased from an initial value of 13742.78 to the final value of 13354.40. The heat transfer coefficient for the fin decreased from an initial value of 259.17 to the final value
of 251.80. In both cases, there was only a three percent decrease. The coefficients are plotted versus time in Figure 14.

![Figure 14 Convective Heat Transfer Coefficients Plotted Vs. Time For Case 2.](image)

3. Case 3: 18% Al in Propellant

The same procedure was done for case 3. Temperature-time data was only taken for 1.6 seconds before the severity of the erosion caused direct plume impingment to the vane shaft. [Ref. 2; p.9] A delay of 0.1 seconds was used to account for the time before the thrust reached its steady state value. Again the results were excellent: the sum of
the squares difference was only 1.52 degrees Kelvin. A plot of the experimental and model temperatures is shown in Figure 15.

![Figure 15 Case 3: Experimental and Model Temperatures Vs. Time](image)

The values obtained for the unknown variables were $a_s = 0.2000$, $a_c = 5.9382$, $a_0 = 100.00$, $b_{11} = 1.6236$, and $b_{22} = 0.0500$. The variables lead to resistance values of $R_{11} = 11.7085$, $R_{22} = 19.0253$, and $R_{00} = -0.6380$. Again, the negative resistance of $R_{00}$ indicating heating from ground. The values of the convection heat transfer coefficients were calculated to be 4786.95 W/m$^2$K and 114.26 W/m$^2$K at the tip and fin respectively.

The values for $b_{11}$ and $b_{22}$ found from NODE418.FOR were added to the geometric and material data in COEF.FOR to again
find the convective heat transfer coefficients at every time step. At the tip, the heat transfer coefficient decreased from an initial value of 6451.38 to the final value of 4786.95. The heat transfer coefficient for the fin also decreased, from an initial value of 154.11 to the final value of 114.26. There was a 26% decrease at both the tip and fin, with the tip showing nonlinearities. The coefficients are plotted in Figure 16.

![Figure 16 Convective Heat Transfer Coefficients Plotted Vs. Time For Case 3.](image)
D. Erosion Front Modeling

An energy balance equation can be written between the leading edge erosion heat flux, \( q/A_0 \), and the heat required to maintain the vane leading edge ablation rate, or

\[
\frac{Q}{A_0} = S_T \rho_a U_a C_p (T_{AW} - T_w) = S \rho_{LE} F \left[ 1 + \frac{C(T_M - T_w)}{F} \right]
\]  (27)

where \( S_T \) is the Stanton number, \( T_{AW} \) is the leading edge recovery temperature, \( T_w \) is the vane leading edge temperature, \( T_M \) is the melting temperature of the vane material, \( F \) is the heat of fusion for tungsten, and \( C \) is the heat capacity of tungsten. Also note that

\[
S_T \rho_a U_a C_p = H_{LE}
\]  (28)

where \( H_{LE} \), the leading edge convection heat transfer coefficient, is found by a parameter identification program like one of those previously described. [Ref. 10:p. 2,3]

A theoretical erosion rate can be found by manipulating equations (27) and (28)

\[
\dot{S} = \frac{H_{LE} (T_{AW} - T_w)}{\rho_{LE} F \left[ 1 + \frac{C(T_M - T_w)}{F} \right]}
\]  (29)

\( T_w \) can be estimated by running a four node simulation model and using the node one temperatures at each time step.
Equation (27) is based upon ablation of the vane, which requires that $T_w > T_m$. Therefore the erosion rate was set equal to zero until $T_w$ reaches $T_m$. The melting temperature for the vane, which is a 90% tungsten-10% copper alloy by weight, is 3513K. This temperature is higher than $T_w$ for both the 9% and 18% cases, and therefore theoretically the vane should not erode. Since the vane does erode, $T_m$ for the vane was taken as the melting temperature of copper, 1358K. This seemed reasonable since the melting point of copper is lower than that of tungsten.

Once the erosion rate is found, it can then be integrated over the time of the firing to find a theoretical length of the vane eroded. This was done in both the 9% and 18% cases and is shown in Figures 17 and 18. The length of the vane eroded using this method is estimated as 1.1 cm for the 9% case and 2.3 cm for the 18% case. Although the 1.1 cm found for the 9% case is high compared to the 0.4 cm found experimentally, the 2.3 cm found for the 18% case is very close to the 2.5 cm found experimentally.

Equation (29) can also be used to try and validate the use of the melting temperature of copper for $T_m$. This was done by plotting the vane temperatures found in the simulation programs as a function of the length between nodes one and two, then using the known total length eroded from the experiment to find the apparent melting temperature. The plots for the 9% and 18% cases are shown in Figures 19 and 20.
Figure 17: Erosion Rate and Total Length Eroded for the 9% Case
Figure 18 Erosion Rate and Total Length Eroded for the 18% Case
The melting temperature found for the 9% case was 1732K while the melting temperature found for the 18% case was 1580K. Although both of these are higher than the melting temperature of copper, they are fairly close. The reason for the melting temperature of the vane being higher than predicted is due to the presence of tungsten which has a melting temperature of 3683K.

![Figure 19 Temperature Profiles Between Nodes One and Two For the 9% Case](image)
Figure 20: Temperature Profiles Between Nodes One and Two for the 18% Case
IV. DISCUSSION OF RESULTS

The full scale three node model attempted to show that the three fin nodes of the Parker five node full scale model could be reduced to one node. This was done, obtaining similar results for the convection heat transfer coefficients at the tip and fin. This validated the use of only one fin node in Reno’s four node quarter scale model.

The erosion effects of aluminized propellant on the quarter scale vanes had to be investigated. There were three postulates considered of how the heat transfer coefficients changed:

(1) the heat transfer coefficients were independent of erosion rate and time,
(2) the heat transfer coefficients were dependent upon erosion rate, but given a fixed erosion rate, were time independent, and
(3) the heat transfer coefficients were dependent upon erosion rate and were time variant.

The first postulate was investigated by A. Danielson in [Ref. 2]. He found that as the percentage of aluminum in the exhaust and the erosion rate increased, nonlinear factors began to have a larger impact and show the limitations of the linear model [Ref. 2:p. 9].
To investigate the remaining two postulates, a model for the erosion of the vanes had to be developed. The erosion of the vanes was modeled as a linear decrease of the geometric dimensions as a function of time and mass loss percentage. This was done in the subprogram COEF.

For the second and third postulates, the coefficients in the PID subprogram COEF were set to the appropriate values for cases two and three, thereby allowing the geometric dimensions to vary. This led to excellent results which remained fairly constant even as the percentage of aluminum in the propellant increased. The sum of the squares error was only 1.19 for the 0% Al case, 0.73 for the 9% Al case, and 1.52 for the 18% Al case. This seemed to link the erosion rate to the heat transfer coefficients.

To determine whether the heat transfer coefficients were time dependent, the program COEF.FOR was modified to calculate the heat transfer coefficients as a function of time. Although the heat transfer coefficients remained fairly constant at the fin, they decreased over time at the tip.

An equation based on ablation of the vane was used to try to predict the erosion rate. The erosion rate was then integrated over the time of the motor firing to obtain the theoretical length of the vane which eroded. Although the 9% case predicted an eroded length which was more than double the experimental value, the 18% case was very close. Two of the reasons the 9% case was off can be explained by the simplicity
of the model and the assumption that ablation would being occurring at the melting temperature of copper instead of the tungsten-copper alloy which the vane was composed of.

To find a closer value to the melting temperature of the vane, the simulated vane temperature was plotted as a function of length between nodes one and two. By using the known length of vane eroded, a theoretical melting temperature could be found. The melting temperatures found were 1732K and 1580K for the 9% and 18% cases respectively. This was much closer to the 1358K for the melting temperature of copper than the 3513K for the tungsten-copper alloy of the vane.
CONCLUSIONS

- The five node full scale model can be reduced to a three node full scale model by removing two of the three fin nodes and produce comparable convective heat transfer coefficients.

- Erosion of thrust vector control vanes can be adequately modeled by a linear decrease of the geometric properties as a function of time and the percentage of aluminum used in the propellant.

- The negative values found for $R_{m}$ indicate heating of the vane from the mount area.

- Both the tip and fin convective heat transfer coefficients were dependant upon erosion rate and were time variant.

- The erosion rate and therefore the length of the vane which will erode over the time of a motor firing can be adequately predicted using an energy balance equation based upon ablation of the vane.

- The melting temperature of the vane appears to be much closer to that of copper than the tungsten-copper alloy which is expected.
RECOMMENDATIONS FOR FURTHER STUDY

- The erosion front modeling needs to be investigated further to see if erosion mechanisms other than ablation can be modeled such as direct impingement of the aluminized particles on the vane.

- The G-law erosion algorithm explained in [Ref. 9] may provide a method to use results from a quarter scale model to predict full scale heat transfer characteristics.

- The quarter scale model needs to be modified to include the heating effects in the vane mount area.
APPENDIX A. SIMULATION PROGRAM

This appendix contains the FORTRAN code used in the program SIM.FOR, which is a forward model program to simulate the temperatures of a three node full scale model, and SIM4.FOR which is a forward model program to simulate the temperatures of the four node quarter scale models.
Program SIM

c This is a forward model program to simulate the
c temperatures of a three node full scale model.

integer maxparam, neq

parameter (maxparam=50, neq=3)

integer id0, istep, nout

real*8 t, tend, a(3, 3), b(3, 3), u(3), t2(61), y(3)
real*8 param(maxparam), fcn, float, a3g

intrinsic float

external fcn, divprk, sset

common/data1/a, b, u

c Open files for data input/output

open (9, name='sim3.nmat', status='new')
open (8, name='datam.dat', status='old')

c read in experimental data

do i=1, 61
   read(8, *) t2(i)
enddo

close(8)

c initialize matrices

do i=1, 3
   do j=1, 3
      a(i, j)= 0.0
      b(i, j)= 0.0
   enddo
enddo

c enter data for trial run

a(1, 2)= 0.2936
a(2, 1)= 0.0147
a(2, 3)= 0.0107
a(3, 2)= 0.0243
a3g=0.0001
b(1,1)=1.0000
b(2,2)=0.0500

a(1,1)=-(a(1,2)+b(1,1))
a(2,2)=-(a(2,1)+a(2,3)+b(2,2))
a(3,3)=-(a(3,2)+a3g)

u(1)=2670
u(2)=2570
u(3)=0.0

c set initial conditions

t=0.0
do i=1,3
   y(i)=0.0
enddo

tol=0.0005
call sset(maxparam, 0.0, param, 1)

id0=1

do istep=1,61
   tend=0.0768*float(istep)
call DIVPRK(id0,neq,fcn,t,tend,tol,param,y)
   write(9,9001) t,t2(istep),y
enddo

c final call to release workspace

id0=3
call DIVPRK(id0,neq,fcn,t,tend,tol,param,y)

9001 format(1f6.3,4f10.4)

close(9)
end

subroutine fcn(neq,t,y,yp)

integer neq
real*8 t,y(neq),yp(neq)
real*8 a(3,3),b(3,3),u(3),d

common/data1/a,b,u
c thrust profile simulation as step input

if (t.gt.0.2) then
  d=1.0
else
  d=0.0
end if

do i=1,neq
  yprime(i)=0.0
  do j=1,neq
    yprime(i)=yprime(i)+a(i,j)*y(j)+b(i,j)*u(j)*d
  enddo
enddo

return
end
Program SIM4

c  This is a forward model program to simulate the
  temperatures of a four node quarter scale model.

  integer maxparam, neq

  parameter (maxparam=50, neq=4)

  integer id0, istep, nout

  real*8 t, tend, a(4,4), b(4,4), u(4), y(4)
  real*8 param(maxparam), fcn, float, a4g

  intrinsic float

  external fcn, divprk, sset, coef

  common/data1/a, b, u

c  Open files for data input/output:

  open(9, name='sim49.mat', status='new')

c  Initialize matrices

  do i=1,4
    do j=1,4
      a(i,j)=0.0
      b(i,j)=0.0
    enddo
  enddo

  u(1)=2155
  u(2)=2061
  u(3)=0.0
  u(4)=0.0

c  Set initial conditions

  t=0.0
  do i=1,4
    y(i)=0.0
  enddo

  tol=0.0005
  call sset(maxparam, 0.0, param, 1)
id0=1
do istep=1,41
    tend=0.05*float(istep)
call coef(tend)
call DIVPRK(id0,neq,fcn,t,tend,tol,param,y)
write(9,9001) t,y
enddo

c final call to release workspace
id0=3
call DIVPRK(id0,neq,fcn,t,tend,tol,param,y)

9001 format(1f6.3,4f10.4)
close(9)
end

c -------------------------------------------------------
subroutine fcn(neq,t,y,yprime)
integer neq
real*8 t,y(neq),yprime(neq)
real*8 a(4,4),b(4,4),u(4),d
common/data1/a,b,u

c thrust profile simulation as step input
if (t.gt.0.7) then
    d=1.0
else
    d=0.0
end if

do i=1,neq
    yprime(i)=0.0
    do j=1,neq
        yprime(i)=yprime(i)+a(i,j)*y(j)+b(i,j)*u(j)*d
    enddo
enddo
return
end

C--------------------------------------------------------

subroutine coef(tend)

45
real*8 vt,vf,vs,atf,afs,ltf,lfs,rho,cp,k,sf
real*8 vt0,vf0,vs0,atf0,afs0,ltf0,lfs0,a12,a21,a23,a32
real*8 a(4,4),b(4,4),u(4),c1,c2,c3,r12,r23

common/data1/a,b,u

vt0=2.6
vf0=52.0
vs0=23.0

atf0=5.9
afs0=5.2

ltf0=5.0
lfs0=6.0

rho=18310.0
cp=146.0
k=173.0
sf=0.25

vt=vt0-0.0*tend
vf=vf0-0.0*tend
vs=vs0-0.0*tend

atf=atf0-0.0*tend
afs=afs0-0.0*tend

ltf=ltf0-0.0*tend
lfs=lfs0-0.0*tend

r12=100.0*ltf/(k*atf)
r23=100.0*lfs/(k*afs)
c1=rho*cp*vt*0.0000001
c2=rho*cp*vf*0.0000001
c3=rho*cp*vs*0.0000001

r12=r12/sf
r23=r23/sf
c1=c1*sf**3
c2=c2*sf**3
c3=c3*sf**3

a(1,2)=1/(c1*r12)
a(2,1)=1/(c2*r12)
a(2,3)=1/(c2*r23)
a(3,2)=1/(c3*r23)
a(3,4)=0.5376
a(4,3)=0.1528
a4g=-0.1651
b(1,1) = 7.6511
b(2,2) = 0.0722

a(1,1) = -(a(1, 2) + b(1, 1))
a(2,2) = -(a(2, 1) + a(2, 3) + b(2, 2))
a(3,3) = -(a(3, 2) + a(3, 4))
a(4,4) = -(a(4, 3) + a(4))

return
end
APPENDIX B. COEFFICIENT PROGRAM

This appendix contains the FORTRAN code used in the program COEF.FOR which calculated the effect of erosion on the known coefficients of the A matrix and the heat transfer coefficients.
program coef

integer i

real*8 vt,vf,vs,atf,afs,ltf,lfs,t,rho,cp,k,sf
real*8 vt0,vf0,vs0,atf0,afs0,ltf0,lfs0,a12,a21,a23,a32
real*8 asf0,asf,ast0,ast,b11,b22,ht,hf

intrinsic float

open(10,name='coef18.mat',status='new')
open(11,name='htc18.mat',status='new')

vt0=2.6
vf0=52.0
vs0=23.0

atf0=5.9
afs0=5.2

ast0=4.35
asf0=112.16

ltf0=5.0
lfs0=6.0

rho=18310.0
cp=146.0
k=173.0
sf=0.25

b11=1.6236
b22=0.05

do i=1,33

t=0.05*float(i)

vt=vt0-0.8125*t
vf=vf0-16.25*t
vs=vs0-7.1875*t

atf=atf0-0.0*t
afs=afs0-0.0*t

ast=ast0-0.90625*t
asf=asf0-23.367*t

ltf=ltf0-1.5625*t

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lfs=lfs0-1.875*t
r12=100.0*lfs/(k*atf)
r23=100.0*lfs/(k*afs)
c1=rho*cp*vt*0.000001
c2=rho*cp*vf*0.000001
c3=rho*cp*vs*0.000001

r12=r12/sf
r23=r23/sf
c1=c1*sf**3
c2=c2*sf**3
c3=c3*sf**3
ast=ast*sf**2
asf=asf*sf**2

a12=1/(c1*r12)
a21=1/(c2*r12)
a23=1/(c2*r23)
a32=1/(c3*r23)

rf1=1/(b11*c1)
rf2=1/(b22*c2)

ht=10000.0/(rf1*ast)
hf=10000.0/(rf2*asf)

9999  format(1f10.4,2f10.2)
9998  format(5f10.5)

write(10,9998)t,a12,a21,a23,a32
write(11,9999)t,ht,hf
end do
close(10)
close(11)

end
APPENDIX C. PID PROGRAMS

This appendix contains the PID programs for the three node full scale model (NODE3), and the four node quarter scale models for propellant with 0% Al (NODE40), 9% Al (NODE49) and 18% Al (NODE418).
This program is the PID program for the three node vane model.

external temp

integer m,n,iparm(6),ibtype,ldfjac

parameter (m=61,n=3,ldfjac=m)

real*8 rparm(7),x(n),f(m),xjac(m,n),xg(n),ssq,ub1,ub2
real*8 xlb(n),xub(n),xscale(n),fscale(m),float,ht,hf
real*8 a(3,3),b(3,3),u(3),t2(61),ys(3,61)
real*8 rho,k,cp,sf,c1,c2,c3,r12,r23,a3g
real*8 vt,vf,vs,atf,afs,ast,asf,ltf,lfs

Variables

m = number of functions
n = number of variables
iparm = list of parameters for DBCLSF setup
ibtype = type of bounds on variables
ldfjac = leading dimension of fjac
rparm = list of parameters for DBCLSF setup
x(n) = the pt where the function is evaluated
f(m) = the computed function at the point x
xjac(m,n) = matrix containing a finite difference approx Jacobian at the approx solution
xg(n) = initial guess of x
xlb(n) = x lower bound
xub(n) = x upper bound
xscale(n) = vector containing the scaling matrix for the variables
fscale(m) = vector containing the scaling matrix for the functions
ssq = sum of the squares
a(3,3) = a matrix
b(3,3) = b matrix
u(3) = [TR1, TR2, 0]
t2(61) = experimental temperatures
ys(3,61) = calculated temperatures
rho = density
k = conduction heat transfer coefficient
cp = specific heat
vt = volume of the tip
vf = volume of the fin
vs = volume of the shaft
atf = cross sectional area from tip to fin
cross sectional area from fin to shaft

surface area of the tip

surface area of the fin

length from tip to fin

length from fin to shaft

scale factor

stagnation temperature, TR1

free stream temperature, TR2

convection heat transfer coefficient at tip

convection heat transfer coefficient at fin

intrinsic float

common/data1/a,b,u,t2,ys

Open files for data input/output

open(10,name='result.dat', status='new')
open(9,name='temp.mat', status='new')
open(8,name='datam.dat', status='old')
open(7,name='input.dat', status='old')

read in experimental data

do i=1,61
   read(8,*) t2(i)
enddo
close(8)

read in input data

read(7,*)
read(7,*)
read(7,*)
read(7,*)
read(7,*) rho, k, cp
read(7,*)
read(7,*) vt, vf, vs
read(7,*)
read(7,*)
read(7,*) atf, afs
read(7,*)
read(7,*) ast, asf
read(7,*)
read(7,*) ltf, lfs
read(7,*)

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read(7,*)
read(7,*) sf, ub1, ub2
close(7)

c initial conditions
c full scale data
r12=100.0*ltf/(k*atf)
r23=100.0*lfs/(k*afs)
c1=rho*cp*vt*0.000001
c2=rho*cp*vf*0.000001
c3=rho*cp*vs*0.000001

c scaled data
r12=r12/sf
r23=r23/sf
c1=c1*sf**3
c2=c2*sf**3
c3=c3*sf**3

c initialize matrices to zero

do i=1,3
  u(i)=0.0
    do j=1,3
      a(i,j)=0.0
      b(i,j)=0.0
    enddo
  enddo

  a(1,2)=1/(c1*r12)
a(2,1)=1/(c2*r12)
a(2,3)=1/(c2*r23)
a(3,2)=1/(c3*r23)

  a3g=0.0
  b(1,1)=0.0
  b(2,2)=0.0

  a(1,1)=-(a(1,2)+b(1,1))
a(2,2)=-(a(2,1)+a(2,3)+b(2,2))
a(3,3)=-(a(3,2)+a3g)

  u(1)=ub1
  u(2)=ub2

  xg(1)=a3g
  xg(2)=b(1,1)
  xg(3)=b(2,2)
c set up parameters for DBCLSF call

do i=1,n
    xscale(i)=1.0
    xlb(i)=0.0001
    xub(i)=100.0
    xg(i)=0.01
    x(i)=0.0
end do

do i=1,m
    fscale(i)=1.0
end do

ibtype=0
call dbclsf(temp,m,n,xg,ibtype,xlb,xub,xscale,fscale,
    & iparm, rparm, x, f, xjac, ldfjac)

c calculate unknown resistances and convection heat transfer
c coefficients

a3g =x(1)
b(1,1)=x(2)
b(2,2)=x(3)

cl=rho*cp*vt*0.000001

c2=rho*cp*vf*0.000001

c3=rho*cp*vs*0.000001

c1=c1*sf**3
c2=c2*sf**3
c3=c3*sf**3

rf1 =1/(b(1,1)*cl)
rf2 =1/(b(2,2)*c2)
r3g =1/(a3g*c3)

ht =10000.0/(rf1*ast)
hf =10000.0/(rf2*asf)

c print and save results

write(6,*) 'a3g','b11','b22'
write(6,9000) x(1),x(2),x(3)

9000  format(3f12.4)
9003  format(2f12.4)

write(10,*) 'a3g','b(1,1)','b(2,2)'
write(10,9000) x(1),x(2),x(3)
write(10,*)
write(10,*)
write(10,*),’    rf1     rf2    r3g’
write(10,9000) rf1,rf2,r3g
write(10,*)
write(10,*),’    ht     hf’
write(10,9003) ht,hf

c  write the temp-time data for MATLAB analysis

do i=1,61
   tt=0.0768*float(i)
   write(9,9001) tt,ys(2,i),t2(i)
endo!
9001  format(1f6.2,2f10.3)

close(10)
close(9)
cend

c--------------------------------------------------------

Subroutine TEMP (m,n,x,f)

c This calculates the temperature-time history using the
c current parameters supplied by DBCLS F called from PID. It
c calculates an error function returned to DBCLS F based on
the differences between predicted and observed temperature
c histories.

integer maxparam, neq
parameter(maxparam=50, neq=3)

integer ido,istep,nout,m,n

real*8 t,tend,y(3),tol,fcn,float,param(maxparam),
real*8 x(3),f(61),coef
real*8 a(3,3),b(3,3),u(3),t2(61),ys(3,61)
real*8 rho,k,cp,sf,c1,c2,c3,r12,r23,a3g
real*8 vt,vf,vs,atf,afs,ast,asf,ltf,lfs

intrinsic float

external fcn,divprk,sset

common/data1/a,b,u,t2,ys

open(12,name=’incoming.dat’,status=’new’)

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a3g = x(1)
b(1,1)=x(2)
b(2,2)=x(3)

write(6,8000)a3g,b(1,1),b(2,2)

8000 format(3f12.4)

a(1,1)=-(a(1,2)+b(1,1))
a(2,2)=-(a(2,1)+a(2,3)+b(2,2))
a(3,3)=-(a(3,2)+a3g)

c Set initial conditions

t=0.0
do i=1,neq
   y(i)=0.0
   do j=1,61
      ys(i,j)=0.0
   enddo
enddo
tol=0.0005

call sset (maxparm, 0.0, param, 1)

id0=1
do istep=1,61
   tend=0.0768*float(istep)
   CALL DIVPRK (id0, neq, fcn, t, tend, tol, param, y)
   do i=1,3
      ys(i,istep)=y(i)
   enddo
enddo
c Final call to release workspace

id0=3
call divprk (id0,neq,fcn,t,tend,tol,param,y)

c calculate error functions

do i=1,61
   f(i)=ys(2,i)-t2(i)
enddo
c calculate error functions

do i=1,61
   ssqr=ssqr+f(i)*f(i)
enddo
ssqr=ssqr/61
C------------------------------------------------------------------------

subroutine fcn(neq,t,y,yprime)

integer neq

real*8 t,y(neq),yprime(neq)
real*8 a(3,3),b(3,3),u(3),d,ys(3,61)

common/data1/a,b,u,t2,ys

c thrust profile simulation as step input

if (t.gt.0.2) then
  d=1.0
else
  d=0.0
end if

do i=1,neq
  yprime(i)=0.0
  do j=1,neq
    yprime(i)=yprime(i)+a(i,j)*y(j)+b(i,j)*u(j)*d
  enddo
enddo

return
end
Program NODE40

c This program is the PID program for the four node vane
c model with ablation from exhaust with 0% Al.

external temp

ingeger m,n,iparm(6),ibtype,ldfjac

parameter (m=122,n=5,ldfjac=m)

real*8 rparm(7),x(n),f(m),xjac(m,n),xg(n),ssq,ub1,ub2
real*8 xlb(n),xub(n),xscale(n),fscale(m),float,ht,hf
real*8 a(4,4),b(4,4),u(4),t3(61),t4(61),ys(4,61)
real*8 rho,k,cp,sf,ci,c1,c2,c3,c4,r12,r23,a4g
real*8 vt0,vf0,vs0,atf0,afs0,ast0,asf0,ltf0,lf0
real*8 vt,vf,vs,atf,afs,ast,asf,ltf,lfs

Variables

m = number of functions
n = number of variables
iparm = list of parameters for DBCLSF setup
ibtype = type of bounds on variables
ldfjac = leading dimension of fjac
rparm = list of parameters for DBCLSF setup
x(n) = the pt where the function is evaluated
f(m) = the computed function at the point x
xjac(m,n) = matrix containing a finite difference approx Jacobian at the approx solution
xg(n) = initial guess of x
xlb(n) = x lower bound
xub(n) = x upper bound
xscale(n) = vector containing the scaling matrix for the variables
fscale(m) = vector containing the scaling matrix for the functions
ssq = sum of the squares
a(neq,neq) = a matrix
b(neq,neq) = b matrix
u(neq) = [TR1, TR2, 0, 0]
t3(61) = experimental temperatures at node 3
t4(61) = experimental temperatures at node 4
ys(neq,61) = calculated temperatures
rho = density
k = conduction heat transfer coefficient
cp = specific heat
vt = volume of the tip
vf = volume of the fin
c vs     = volume of the shaft
c atf    = cross sectional area from tip to fin
c afs    = cross sectional area from fin to shaft


инаst    = surface area of the tip

c asf    = surface area of the fin

c ltf    = length from tip to fin

c lfs    = length from fin to shaft

c sf     = scale factor

c ub1    = stagnation temperature, TR1

c ub2    = free stream temperature, TR2

c ht     = convection heat transfer coefficient at
tip

c hf     = convection heat transfer coefficient at
fin

intrinsic float

common/data1/a,b,u,t3,t4,ys
common/data2/rho,k,cp,sf,c1,c2,c3
common/data3/vt0,vf0,vs0,atf0,afs0,ast0,asf0,ltf0,lfs0
common/data4/vt,vf,vs,atf,afa,ast,asf,ltf,lfs

c Open files for data input/output

open(10,name='result0.dat', status='new')
open(9,name='temp0.mat', status='new')
open(8,name='data00.dat', status='old')
open(7,name='input.dat', status='old')

c read in experimental data

do i=1,61
   read(8,*) t3(i)
endo
do i=1,61
   read(8,*) t4(i)
endo
close(8)

c read in input data

read(7,*)
read(7,*)
read(7,*)
read(7,*)
read(7,*) rho,k,cp
read(7,*)
read(7,*)
read(7,*) vt0, vf0, vs0
read(7,*)
read(7,*)
read(7,*) atf0, afs0
read(7,*)
read(7,*)
read(7,*) ast0, asf0
read(7,*)
read(7,*)
read(7,*) ltf0, lfs0
read(7,*)
read(7,*)
read(7,*) sf, ub1, ub2
close(7)

C initial conditions

t=0
 tend=0
call coef(x,tend)

u(1)=ub1
u(2)=ub2
u(3)=0.0
u(4)=0.0

C set up parameters for DBCLSF call

do i=1,n

   xscale(i)=1.0
   xlb(i)=-0.2
   xub(i)=100.0
   xg(i)=0.1
   x(i)=0.0
end do

do i=1,m

   fscale(i)=1.0
end do

ibtype=0

call dbclsf(temp,m,n,xg,ibtype,xlb,xub,xscale,fscale, &
               iparm, rparm,x,f,xjac,ldfjac)

C calculate unknown resistances and convection heat transfer
C coefficients

a(3,4)=x(1)
a(4,3)=x(2)
a4g =x(3)
b(1,1)=x(4)
b(2,2)=x(5)
c1 = \rho \cdot c_p \cdot v_t \cdot 0.000001
\newline c2 = \rho \cdot c_p \cdot v_f \cdot 0.00001
\newline c3 = \rho \cdot c_p \cdot v_s \cdot 0.00001
\newline
\newline cl = cl \cdot s_f^3
\newline c2 = c2 \cdot s_f^3
\newline c3 = c3 \cdot s_f^3
\newline
rf_1 = 1 / (b(1,1) \cdot c_1)
rf_2 = 1 / (b(2,2) \cdot c_2)
rf_34 = 1 / (a(3,4) \cdot c_3)
rf_4g = 1 / (a(4,3) \cdot r_34)
\newline
ht = 10000.0 / (rf_1 \cdot ast \cdot s_f^2)
hf = 10000.0 / (rf_2 \cdot asf \cdot s_f^2)
\newline
c print and save results
\newline
write(6,*) ' a34 a43 a4g b11 b22'
write(6,9000) x(1), x(2), x(3), x(4), x(5)
\newline
9000 format(5f10.4)
9003 format(2f11.4)
\newline
write(10,*) ' a34 a43 a4g b11 b22'
write(10,9000) x(1), x(2), x(3), x(4), x(5)
write(10,*)
write(10,*) ' rf1 rf2 r4g'
write(10,9000) rf_1, rf_2, rf_4g
write(10,*)
write(10,*) ' ht hf'
write(10,9003) ht, hf
\newline
c write the temp-time data for MATLAB analysis.
\newline
do i=1,61
\quad tt = 0.05 \cdot float(i)
\quad write(9,9001) tt, ys(3,i), ys(4,i), t_3(i), t_4(i)
\quad enddo
\newline
9001 format(2x,1f6.4,4f10.4)
\newline
close(10)
close(9)
\newline
c---------------------------------------------------------------
Subroutine TEMP (m,n,x,f)

This calculates the temperature-time history using the current parameters supplied by DBCLSF called from PID. It calculates an error function returned to DBCLSF based on the differences between predicted and observed temperature histories.

integer maxparam, neq

parameter(maxparam=50, neq=4)

integer id0, istep, nout, m, n

real*8 t, tend, y(4), tol, fcn, float, param(maxparam),
real*8 x(n), f(m), coef
real*8 a(4,4), b(4,4), u(4), t3(61), t4(61), y(4,61)
real*8 rho, k, cp, sf, c1, c2, c3, c4, r12, r23, a4g
real*8 vt0, vf0, vs0, atf0, afs0, ast0, asf0, ltf0, lfs0
real*8 vt, vf, vs, atf, afs, ast, asf, ltf, lfs

intrinsic float

external fcn, divprk, sset, coef

common/data1/a,b,u,t3,t4,ys
common/data2/rho,k,cp,sf,c1,c2,c3
common/data3/vt0,vf0,vs0,atf0,afs0,ast0,asf0,ltf0,lfs0
common/data4/vt,vf,vs,atf,afs,ast,asf,ltf,lfs

open(12,name='incoming.dat',status='new')

a(3,4)=x(1)
a(4,3)=x(2)
a4g=x(3)
b(1,1)=x(4)
b(2,2)=x(5)

write(6,8000)a(3,4),a(4,3),a4g,b(1,1),b(2,2)

8000 format(5f10.4)

a(1,1)=-(a(1,2)+b(1,1))
a(2,2)=-(a(2,1)+a(2,3)+b(2,2))
a(3,3)=-(a(3,2)+a(3,4))
a(4,4)=-(a(4,3)+a4g)

c Set initial conditions

t=0.0
do i=1,neq
y(i)=0.0
  do j=1,61
    ys(i,j)=0.0
  enddo
dt
  t(0)=0.0005
call sset (maxparm, 0.0, param, 1)
enddo
  tol=0.0005
call divprk (id0, neq, fcn, t, tend, tol, param, y)
calleq(1,4)
  y(i)=ys(i,istep)=y(i)
enddo
c Final call to release workspace
  id0=3
call divprk (id0,neq,fcn,t,tend,tol,param,y)
c calculate error functions
  do i=1,61
    f(i)=ys(3,i)-t3(i)
    f(i+61)=ys(4,i)-t4(i)
  enddo
c print out rms error
  ssqr=0.0
  do i=1,m
    ssqr=ssqr+f(i)*f(i)
  enddo
  ssqr=ssqr/m
  xer=sqrt(ssqr)
  write(6,*) xer
  write(12,*) xer
return
end
C---------------------------------------------------------------
subroutine fcn(neq,t,y,yprime)
ineteq
real*8 t,y(neq),yprime(neq)
real*8 a(4,4),b(4,4),u(4),d,ys(4,61)

common/data1/a,b,u,t3,t4,ys
common/data2/rho,k,cp,sf,c1,c2,c3
common/data3/vt0,vf0,vs0,atf0,afs0,ast0,asf0,ltf0,lfs0
common/data4/vt,vf,vs,atf,afs,ast,asf,ltf,lfs

c thrust profile simulation as step input

if (t.gt.0.3) then
  d=1.0
else
  d=0.0
end if

do i=1,neq
  yprime(i)=0.0
  do j=1,neq
    yprime(i)=yprime(i)+a(i,j)*y(j)+b(i,j)*u(j)*d
  enddo
enddo

return
end

C--------------------------------------------------------

subroutine coef(x,tend)
integer i,j

real*8 tend,x(5)
real*8 a(4,4),b(4,4),u(4)
real*8 rho,k,cp,sf,c1,c2,c3,c4,r12,r23,a4g
real*8 vt0,vf0,vs0,atf0,afs0,ast0,asf0,ltf0,lfs0
real*8 vt,vf,vs,atf,afs,ast,asf,ltf,lfs

common/data1/a,b,u,t3,t4,ys
common/data2/rho,k,cp,sf,c1,c2,c3
common/data3/vt0,vf0,vs0,atf0,afs0,ast0,asf0,ltf0,lfs0
common/data4/vt,vf,vs,atf,afs,ast,asf,ltf,lfs

c a,b matrix modification due to ablation effects

c full scale data

vt=vt0-0.013*tend
vf=vf0-0.26*tend
vs=vs0-0.115*tend
atf=atf0-0.0*tend
afs=afs0-0.0*tend
ast=ast0-0.0145*tend
asf=asf0-0.374*tend
ltf=ltf0-0.025*tend
lfs=lfs0-0.03*tend
r12=100.0*ltf/(k*atf)
r23=100.0*lfs/(k*afs)
c1=rho*cp*vt*0.000001
c2=rho*cp*vf*0.000001
c3=rho*cp*vs*0.000001

c  scaled data
r12=r12/sf
r23=r23/sf
c1=c1*sf**3
c2=c2*sf**3
c3=c3*sf**3

a(1,2)=1/(c1*r12)
a(2,1)=1/(c2*r12)
a(2,3)=1/(c2*r23)
a(3,2)=1/(c3*r23)

a(3,4)=x(1)
a(4,3)=x(2)
a4g=x(3)
b(1,1)=x(4)
b(2,2)=x(5)

a(1,1)=-(a(1,2)+b(1,1))
a(2,2)=-(a(2,1)+a(2,3)+b(2,2))
a(3,3)=-(a(3,2)+a(3,4))
a(4,4)=-(a(4,3)+a4g)

return
end
Program NODE49

c This program is the PID program for the four node vane model with erosion from exhaust with 9% Al.

external temp
integer m,n,iparm(6),ibtype,ldfjac

parameter (m=82,n=5,ldfjac=m)

real*8 rparm(7),x(n),f(m),xjac(m,n),xg(n),ssq,ub1,ub2
real*8 xlb(n),xub(n),xscale(n),fscale(m),float,ht,hf
real*8 a(4,4),b(4,4),u(4),t3(41),t4(41),ys(4,41)
real*8 rho,k,cp,sf,cl,c2,c3,c4,r12,r23,a4g
real*8 vt0,vf0,vsO,atfO,afsO,astO,asfO,ltf0,ltfs0
real*8 vt,vf,vs,atf,afs,ast,asf,ltf,ltfs

Variables

c m = number of functions
c n = number of variables
c iparm = list of parameters for DBCLSF setup
c ibtype = type of bounds on variables
c ldfjac = leading dimension of fjac
c rparm = list of parameters for DBCLSF setup
c x(n) = the pt where the function is evaluated
c f(m) = the computed function at the point x
c xjac(m,n) = matrix containing a finite difference approx Jacobian at the approx solution
c xg(n) = initial guess of x
c xlb(n) = x lower bound
c xub(n) = x upper bound
c xscale(n) = vector containing the scaling matrix for the variables
c fscale(m) = vector containing the scaling matrix for the functions
c ssq = sum of the squares

c a(neq,neq) = a matrix

c b(neq,neq) = b matrix

c u(neq) = [TR1, TR2, 0, 0]

c t3(41) = experimental temperatures at node 3

c t4(41) = experimental temperatures at node 4

c ys(neq,41) = calculated temperatures

c rho = density

c k = conduction heat transfer coefficient

c cp = specific heat

c vt = volume of the tip

c vf = volume of the fin

c vs = volume of the shaft

c atf = cross sectional area from tip to fin

c afs = cross sectional area from fin to shaft

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ast = surface area of the tip
asf = surface area of the fin
ltf = length from tip to fin
lfs = length from fin to shaft
sf = scale factor
ub1 = stagnation temperature, TR1
ub2 = free stream temperature, TR2
ht = convection heat transfer coefficient at tip
hf = convection heat transfer coefficient at fin

intrinsic float

common/data1/a,b,u,t3,t4,ys
common/data2/rho,k,cp,sf,c1,c2,c3
common/data3/vt0,vf0,vs0,atf0,afs0,ast0,asf0,ltf0,lfs0
common/data4/vt,vf,vs,atf,afs,ast,asf,ltf,lfs

Open files for data input/output

open(10,name='result9.dat', status='new')
onopen(9,name='temp9.mat', status='new')
onopen(8,name='datam9.dat', status='old')
onopen(7,name='input.dat', status='old')

read in experimental data

do i=1,41
   read(8,*) t3(i)
enddo

do i=1,41
   read(8,*) t4(i)
enddo

read in input data

read(7,*)
read(7,*)
read(7,*)
read(7,*)
read(7,*) rho,k,cp
read(7,*)
read(7,*)
read(7,*) vt0, vf0, vs0
read(7,*)
read(7,*)
read(7,*) atf0, afs0
read(7,*)
read(7,*)
read(7,*) ast0, asf0
read(7,*)
read(7,*)
read(7,*) ltf0, lfs0
read(7,*)
read(7,*)
read(7,*) sf, ub1, ub2
close(7)

c initial conditions

t=0
tend=0
call coef(x,tend)

u(1)=ub1
u(2)=ub2
u(3)=0.0
u(4)=0.0

c set up parameters for DBCLSF call

do i=1,n
   xscale(i)=1.0
   xlb(i)=-0.2
   xub(i)=100.0
   xg(i)=0.1
   x(i)=0.0
end do

do i=1,m
   fscale(i)=1.0
end do

ibtype=0

call dbclsf(temp,m,n,xg,ibtype,xlb,xub,xscale,fscale,
   & iparm,rparm,x,f,xjac,ldfjac)

c calculate unknown resistances and convection heat transfer
c coefficients

a(3,4)=x(1)
a(4,3)=x(2)
a4g =x(3)
b(1,1)=x(4)
b(2,2)=x(5)

c1=rho*cp*vt*0.000001
c2=rho*cp*vf*0.000001
c3=rho*cp*vs*0.000001
cl = c1*sf**3

c2 = c2*sf**3

c3 = c3*sf**3

rf1 = 1/(b(1,1)*c1)
rf2 = 1/(b(2,2)*c2)
r34 = 1/(a(3,4)*c3)
c4 = 1/(a(4,3)*r34)
r4g = 1/(a4g*c4)

ht = 10000.0/(rf1*(ast*sf**2))
hf = 10000.0/(rf2*(asf*sf**2))

c print and save results

write(6,*) ' a34 a43 a4g b11 b22'
write(6,9000) x(1),x(2),x(3),x(4),x(5)

9000 format(5f10.4)
9003 format(5x,2f10.4)

write(10,*)' a34 a43 a4g b(1,1) b(2,2)'
write(10,9000) x(1),x(2),x(3),x(4),x(5)
write(10,*)
write(10,*)
write(10,*)' rf1 rf2 r4g'
write(10,9000) rf1,rf2,r4g
write(10,*)
write(10,*)
write(10,*)' ht hf'
write(10,9003) ht,hf

c write the temp-time data for MATLAB analysis

do i=1,41
   tt=0.05*float(i)
   write(9,9001) tt,ys(3,i),ys(4,i),t3(i),t4(i)
endo

9001 format(2x,1f6.4,4f10.4)

clo(10)
close(9)

end

Subroutine TEMP (m,n,x,f)

c This calculates the temperature-time history using the
c current parameters supplied by DBCLSF called from PID. It
calculates an error function returned to DBCLSF based on the differences between predicted and observed temperature histories.

integer maxparam, neq

parameter(maxparam=50, neq=4)

integer id0, istep, nout, m, n

real*8 t, tend, y(4), tol, fcn, float, param(50), x(n), f(m), coef
real*8 a(4,4), b(4,4), u(4), t3(41), t4(41), y(4,41)
real*8 rho, k, cp, sf, c1, c2, c3, c4, r12, r23, a4g
real*8 vt0, vf0, vs0, atf0, afs0, ast0, asf0, ltf0, lfs0
real*8 vt, vf, vs, atf, afs, ast, asf, ltf, lfs

intrinsic float

external fcn, divprk, sset, coef

common/data1/a,b,u,t3,t4,ys
common/data2/rho,k,cp,sf,c1,c2,c3
common/data3/vt0,vf0,vs0,atf0,afs0,ast0,asf0,ltf0,lfs0
common/data4/vt,vf,vs,atf,afs,ast,asf,ltf,lfs

open(12,name='incoming9.dat',status='new')

a(3,4)=x(1)
a(4,3)=x(2)
a4g =x(3)
b(1,1)=x(4)
b(2,2)=x(5)

write(6,8000)a(3,4),a(4,3),a4g,b(1,1),b(2,2)

8000 format(5f10.4)

a(1,1)=-(a(1,2)+b(1,1))
a(2,2)=-(a(2,1)+a(2,3)+b(2,2))
a(3,3)=-(a(3,2)+a(3,4))
a(4,4)=-(a(4,3)+a4g)

Set initial conditions

t=0.0

do i=1,neq
    y(i)=0.0
    do j=1,41
        ys(i,j)=0.0
    enddo
enddo
tol=0.0005

call sset (maxparm, 0.0, param, 1)

id0=1

do istep=1,41
    tend=0.05*float(istep)
    call coef(x,tend)
    CALL DIVPRK (id0, neq, fcn, t, tend, tol, param, y)
    do i=1,4
        ys(i,istep)=y(i)
    enddo
enddo

c Final call to release workspace

id0=3

call divprk (id0,neq,fcn,t,tend,tol,param,y)

c calculate error functions

do i=1,41
    f(i)=ys(3,i)-t3(i)
    f(i+41)=ys(4,i)-t4(i)
enddo

c print out rms error

ssqr=0.0

do i=1,m
    ssqr=ssqr+f(i)*f(i)
enddo

ssqr=ssqr/m

xer=sqrt(ssqr)

write(6,*),xer

write(12,*),xer

return
end

C-----------------------------------------------------

subroutine fcn(neq,t,y,yprime)

integer neq

real*8 t,y(neq),yprime(neq)
real*8 a(4,4),b(4,4),u(4),d,ys(4,41)

common/data1/a,b,u,t3,t4,ys
common/data2/rho,k,cp,sf,c1,c2,c3
common/data3/vt0,vf0,vs0,atf0,afs0,ast0,asf0,ltf0,lfs0
common/data4/vt,vf,vs,atf,afs,ast,asf,ltf,lfs

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c thrust profile simulation as step input

    if (t.gt.0.7) then
        d=1.0
    else
        d=0.0
    end if

do i=1,neq
    yprime(i)=0.0
    do j=1,neq
        yprime(i)=yprime(i)+a(i,j)*y(j)+b(i,j)*u(j)*d
    enddo
endo
dreturn
dend

C------------------------------------------------------------------

subroutine coef(x,tend)

integer i,j

real*8 tend,x(5),
real*8 a(4,4),b(4,4),u(4),
real*8 rho,k,cp,sf,c1,c2,c3,c4,r12,r23,a4g,
real*8 vt0,vf0,vs0,atf0,afs0,ast0,asf0,ltf0,lfs0,
real*8 vt,vf,vs,atf,afs,ast,asf,ltf,lfs

common/data1/a,b,u,t3,t4,ys
common/data2/rho,k,cp,sf,c1,c2,c3
common/data3/vt0,vf0,vs0,atf0,afs0,ast0,asf0,ltf0,lfs0
common/data4/vt,vf,vs,atf,afs,ast,asf,ltf,lfs

c a,b matrix modification due to ablation effects

c full scale data

vt=vt0-0.0*tend
vf=vf0-0.0*tend
vs=vs0-0.0*tend
atf=atf0-0.0*tend
afs=afs0-0.0*tend
ast=ast0-0.0*tend
asf=asf0-0.0*tend
ltf=ltf0-0.0*tend
lfs=lfs0-0.0*tend
r12=100.0*l1f/(k*atf)
r23=100.0*l1f/(k*afs)
c1=rho*cp*vt*0.000001
c2=rho*cp*vf*0.000001
c3=rho*cp*vs*0.000001

c  scaled data

r12=r12/sf
r23=r23/sf
c1=c1*sf**3
c2=c2*sf**3
c3=c3*sf**3

a(1,2)=1/(c1*r12)
a(2,1)=1/(c2*r12)
a(2,3)=1/(c2*r23)
a(3,2)=1/(c3*r23)

a(3,4)=x(1)
a(4,3)=x(2)
a4g =x(3)
b(1,1)=x(4)
b(2,2)=x(5)

a(1,1)= (a(1,2)+b(1,1))
a(2,2)= -(a(2,1)+a(2,3)+b(2,2))
a(3,3)= -(a(3,2)+a(3,4))
a(4,4)= -(a(4,3)+a4g)

return
end
Program NODE418

This program is the PID program for the four node vane model with ablation from exhaust with 18% Al.

external temp

integer m,n,iparm(6),ibtype,ldfjac

parameter (m=66,n=5,ldfjac=m)

real*8 rparm(7),x(n),f(m),xjac(m,n),xg(n),ssq,ub1,ub2
real*8 xlb(n),xub(n),xscale(n),fscale(m),float,ht,hf
real*8 a(4,4),b(4,4),u(4),t3(33),t4(33),ys(4,33)
real*8 rho,k,cp,sf,cl,c2,c3,c4,r12,r23,a4g
real*8 vtO,vfO,vsO,atfO,afsO,.astO,asfO,ltfO,lfs0
real*8 vt,vf,vs,atf,afs,astasf,ltf,lfs

c Variables

c m = number of functions
c n = number of variables
c iparm = list of parameters for DBCLS setup
c ibtype = type of bounds on variables
c ldfjac = leading dimension of fjac

c rparm = list of parameters for DBCLS setup

c x(n) = the pt where the function is evaluated

c f(m) = the computed function at the point x

c xjac(m,n) = matrix containing a finite difference approx Jacobian at the approx solution

c xg(n) = initial guess of x

c xlb(n) = x lower bound

c xub(n) = x upper bound

c xscale(n) = vector containing the scaling matrix for the variables

c fscale(m) = vector containing the scaling matrix for the functions

c ssq = sum of the squares

c a(neq,neq) = a matrix

c b(neq,neq) = b matrix

c u(neq) = [TR1, TR2, 0, 0]

c t3(33) = experimental temperatures at node 3

c t4(33) = experimental temperatures at node 4

c ys(neq,33) = calculated temperatures

c rho = density

c k = conduction heat transfer coefficient

c cp = specific heat

c vt = volume of the tip

c vf = volume of the fin

c vs = volume of the shaft

c atf = cross sectional area from tip to fin
afs = cross sectional area from fin to shaft
ast = surface area of the tip
asf = surface area of the fin
ltf = length from tip to fin
lfs = length from fin to shaft
sf = scale factor
ub1 = stagnation temperature, TR1
ub2 = free stream temperature, TR2
ht = convection heat transfer coefficient at tip
hf = convection heat transfer coefficient at fin

intrinsic float

common/data1/a,b,u,t3,t4,ys
common/data2/rho,k,cp,sf,c1,c2,c3
common/data3/vt0,vf0,vs0,atf0,afs0,ast0,asf0,ltf0,lfs0
common/data4/vt,vf,vs,atf,afs,ast,asf,ltf,lfs

c Open files for data input/output

open(10,name='result18.dat', status='new')
open(9,name='templ8.mat', status='new')
open(8,name='datam181.dat', status='old')
open(7,name='input.dat', status='old')

c read in experimental data

do i=1,33
   read(8,*) t3(i)
enddo
do i=1,33
   read(8,*) t4(i)
enddo
close(8)
c read in input data

read(7,*)
read(7,*)
read(7,*)
read(7,*)
read(7,*) rho,k,cp
read(7,*)
read(7,*) vt0, vf0, vs0
read(7,*)
read(7,*) atf0, afs0
read(7,*)
read(7,*)
read(7,*) ast0, asf0
read(7,*)
read(7,*)
read(7,*) ltf0, lfs0
read(7,*)
read(7,*)
read(7,*) sf, ub1, ub2
close(7)

c initial conditions

t=0
tend=0
call coef(x,tend)

u(1)=ub1
u(2)=ub2
u(3)=0.0
u(4)=0.0

c set up parameters for DBCLSF call

do i=1,n
   xscale(i)=1.0
   xlb(i)=-0.2
   xub(i)=100.0
   xg(i)=0.1
   x(i)=0.0
end do

do i=1,m
   fscale(i)=1.0
end do

ibtype=0

call dbclsf(temp,m,n,xg,ibtype,xlb,xub,xscale,fscale,
&           iparm, rparm, x, f, xjac, ldfjac)

c calculate unknown resistances and convection heat transfer
c coefficients

a(3,4)=x(1)
a(4,3)=x(2)
a4g=x(3)
b(1,1)=x(4)
b(2,2)=x(5)

c1=rho*cp*vt*0.000001

c2=rho*cp*vf*0.000001
c3=rho*cp*vs*0.000001

c1=c1*sf**3

c2=c2*sf**3

c3=c3*sf**3

rf1 =1/(b(1,1)*c1)

rf2 =1/(b(2,2)*c2)

r34 =1/(a(3,4)*c3)

c4 =1/(a(4,3)*r34)

r4g =1/(a4g*c4)

ht =10000.0/(rf1*(ast*sf**2))

hf =10000.0/(rf2*(asf*sf**2))

c print and save results

write(6,*), a34, a43, a4g, b11, b22
write(6,9000) x(1), x(2), x(3), x(4), x(5)

9000 format(5f10.4)

9003 format(2x,2f10.4)

write(10,*), a34, a43, a4g, b(1,1), b(2,2)
write(10,9000) x(1), x(2), x(3), x(4), x(5)
write(10,*)
write(10,*), rf1, rf2, r4g
write(10,9000) rf1, rf2, r4g
write(10,*)
write(10,*), ht, hf
write(10,9003) ht, hf

c write the temp-time data for MATLAB analysis

do i=1,33
    tt=0.05*float(i)
    write(9,9001) tt, ys(3,i), ys(4,i), t3(i), t4(i)
endo

9001 format(2x,1f6.4,4f10.4)

close(10)
close(9)

c---------------------------

Subroutine TEMP (m,n,x,f)
This calculates the temperature-time history using the current parameters supplied by DBCLSF called from PID. It calculates an error function returned to DBCLSF based on the differences between predicted and observed temperature histories.

```fortran
integer maxparam, neq

parameter(maxparam=50, neq=4)

integer ido, istep, nout, m, n

real*8 t, tend, y(4), tol, fcn, float, param(maxparam),
real*8 x(n), f(m), coef
real*8 a(4,4), b(4,4), u(4), t3(33), t4(33), y3(4,33)
real*8 rho, k, cp, sf, c1, c2, c3, c4, r12, r23, a4g
real*8 vt0, vf0, vs0, atf0, afs0, ast0, asf0, ltf0, lfs0
real*8 vt, vf, vs, atf, afs, ast, asf, ltf, lfs

intrinsic float

external fcn, divprk, sset, coef

common/data1/a, b, u, t3, t4, y3
common/data2/rho, k, cp, sf, c1, c2, c3
common/data3/vt0, vf0, vs0, atf0, afs0, ast0, asf0, ltf0, lfs0
common/data4/vt, vf, vs, atf, afs, ast, asf, ltf, lfs

open(12, name='incoming18.dat', status='new')

a(3, 4) = x(1)
a(4, 3) = x(2)
a4g = x(3)
b(1, 1) = x(4)
b(2, 2) = x(5)

write(6, 8000) a(3, 4), a(4, 3), a4g, b(1, 1), b(2, 2)

8000 format(5f10.4)

a(1, 1) = -(a(1, 2) + b(1, 1))
a(2, 2) = -(a(2, 1) + a(2, 3) + b(2, 2))
a(3, 3) = -(a(3, 2) + a(3, 4))
a(4, 4) = -(a(4, 3) + a4g)

c Set initial conditions

t = 0.0
do i = 1, neq
  y(i) = 0.0
  do j = 1, 33

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ys(i,j)=0.0
enddo
enddo
tol=0.0005
call sset (maxparm, 0.0, param, 1)

id0=1
do istep=1,33
  tend=0.05*float(istep)
call coef(x,tend)
call DIVPRK (id0, neq, fcn, t, tend, tol, param, y)
do i=1,4
  ys(i,istep)=y(i)
enddo
enddo

c Final call to release workspace

id0=3
call divprk (id0,neq,fcn,t,tend,tol,param,y)

c calculate error functions

do i=1,33
  f(i)=ys(3,i)-t3(i)
  f(i+33)=ys(4,i)-t4(i)
endo

c print out rms error
ssqr=0.0
do i=1,m
  ssqr=ssqr+f(i)*f(i)
endo
ssqr=ssqr/m
xer=sqrt(ssqr)
write(6,*) xer
write(12,*) xer
return
end

C--------------------------------------------

subroutine fcn(neq,t,y,yprime)

integer neq

real*8 t,y(neq),yprime(neq)
real*8 a(4,4),b(4,4),u(4),d,ys(4,33)
common/data1/a,b,u,t3,t4,ys
common/data2/rho,k,cp,sf,c1,c2,c3
common/data3/vt0,vf0,vs0,atf0,afs0,ast0,asf0,ltf0,lfs0
common/data4/vt,vf,vs,atf,afs,ast,asf,ltf,lfs

C thrust profile simulation as step input

if (t.gt.0.1) then
  d=1.0
else
  d=0.0
end if

do i=1,neq
  yprime(i)=0.0
  do j=1,neq
    yprime(i)=yprime(i)+a(i,j)*y(j)+b(i,j)*u(j)*d
  enddo
endo

return
end

C--------------------------------------------------

subroutine coef (x, tend)

integer i,j

real*8 tend,x(5)
real*8 a(4,4),b(4,4),u(4)
real*8 rho,k,cp,sf,c1,c2,c3,c4,r12,r23,a4g
real*8 vt0,vf0,vs0,atf0,afs0,ast0,asf0,ltf0,lfs0
real*8 vt,vf,vs,atf,afs,ast,asf,ltf,lfs

common/data1/a,b,u,t3,t4,ys
common/data2/rho,k,cp,sf,c1,c2,c3
common/data3/vt0,vf0,vs0,atf0,afs0,ast0,asf0,ltf0,lfs0
common/data4/vt,vf,vs,atf,afs,ast,asf,ltf,lfs

C a,b matrix modification due to ablation effects

C full scale data

vt=vt0-0.0*tend
vf=vf0-0.0*tend
vs=vs0-0.0*tend
atf=atf0-0.0*tend
afs=afs0-0.0*tend
ast=ast0-0.0*tend
asf=asf0-0.0*tend

ltf=ltf0-0.0*tend
lfs=lfs0-0.0*tend

r12=100.0*ltf/(k*atf)
r23=100.0*lfs/(k*aafs)
c1=rho*cp*vt*0.000001
c2=rho*cp*vf*0.000001
c3=rho*cp*vs*0.000001

c scaled data

r12=r12/sf
r23=r23/sf
c1=c1*sf**3

c2=c2*sf**3

c3=c3*sf**3

a(1,2)=1/(c1*r12)
a(2,1)=1/(c2*r12)
a(2,3)=1/(c2*r23)
a(3,2)=1/(c3*r23)
a(3,4)=x(1)
a(4,3)=x(2)
a4g=x(3)
b(1,1)=x(4)
b(2,2)=x(5)
a(1,1)=-(a(1,2)+b(1,1))
a(2,2)=-(a(2,1)+a(2,3)+b(2,2))
a(3,3)=-(a(3,2)+a(3,4))
a(4,4)=-(a(4,3)+a4g)

return
end
APPENDIX D. PHYSICAL DATA FILES

The physical data files for the PID programs which contains the geometric and material properties of the vanes and the recovery temperatures used in each case.
**NAWC INVERSE HEAT TRANSFER PROGRAM. INPUT DATA FOR NODE3.FOR**

Material properties:

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho ) (kg/m(^3))</td>
<td>18310.0</td>
</tr>
<tr>
<td>( k ) (W/m-deg K)</td>
<td>173.0</td>
</tr>
<tr>
<td>( C_p ) (J/kg deg K)</td>
<td>146.0</td>
</tr>
<tr>
<td>( \text{Vol (tip, cm}^3)</td>
<td>2.6</td>
</tr>
<tr>
<td>( \text{Vol (fin, cm}^3)</td>
<td>52.00</td>
</tr>
<tr>
<td>( \text{Vol (shaft, cm}^3)</td>
<td>23.0</td>
</tr>
<tr>
<td>( \text{X-section areas: tip-fin (cm}^2)</td>
<td>5.9</td>
</tr>
<tr>
<td>( \text{tip-shaft (cm}^2)</td>
<td>5.2</td>
</tr>
<tr>
<td>( \text{Surface areas: tip (cm}^2)</td>
<td>4.35</td>
</tr>
<tr>
<td>( \text{fin (cm}^2)</td>
<td>112.16</td>
</tr>
<tr>
<td>( \text{Conductive path: tip-fin (cm)}</td>
<td>5.0</td>
</tr>
<tr>
<td>( \text{fin-shaft (cm)}</td>
<td>6.0</td>
</tr>
<tr>
<td>( \text{Scale factor: TR1}</td>
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<tr>
<td>( \text{TR2}</td>
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</tr>
<tr>
<td></td>
<td>2570</td>
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</tbody>
</table>
NAWC INVERSE HEAT TRANSFER PROGRAM. INPUT DATA FOR NODE40.FOR

**Material properties:**

<table>
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<th>Property</th>
<th>Value</th>
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</thead>
<tbody>
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<td>18310.0</td>
</tr>
<tr>
<td>( k ) (w/m-deg k)</td>
<td>173.0</td>
</tr>
<tr>
<td>( C_p ) (J/kg deg k)</td>
<td>146.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Volume</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{tip} ) (cm(^3))</td>
<td>2.6</td>
</tr>
<tr>
<td>( V_{fin} ) (cm(^3))</td>
<td>52.00</td>
</tr>
<tr>
<td>( V_{shaft} ) (cm(^3))</td>
<td>23.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Section areas</th>
<th>Value</th>
</tr>
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<tbody>
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<td>Tip-fin</td>
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</tr>
<tr>
<td>Fin-shaft</td>
<td>5.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Surface areas</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tip</td>
<td>4.35</td>
</tr>
<tr>
<td>Fin</td>
<td>112.16</td>
</tr>
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</table>

<table>
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<th>Conductive path</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tip-fin</td>
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</tr>
<tr>
<td>Fin-shaft</td>
<td>6.0</td>
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</table>

<table>
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<tr>
<th>Scale factor</th>
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<tr>
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### NAWC INVERSE HEAT TRANSFER PROGRAM. INPUT DATA FOR NODE49FOR

Material properties:

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<th>Property</th>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
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<tr>
<td>k (W/m-deg K)</td>
<td>173.0</td>
<td></td>
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<tr>
<td>Cp (J/kg deg K)</td>
<td>146.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vol (tip, cm³)</td>
<td>2.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vol (fin, cm³)</td>
<td>52.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vol (shaft, cm³)</td>
<td>23.0</td>
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<td>X-section areas:</td>
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<tr>
<td>tip-fin (cm²)</td>
<td>5.9</td>
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<td></td>
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<tr>
<td>fin-shaft (cm²)</td>
<td>5.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surface areas:</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>tip (cm²)</td>
<td>4.35</td>
<td></td>
<td></td>
</tr>
<tr>
<td>fin (cm²)</td>
<td>112.16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conductive path</td>
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<td></td>
</tr>
<tr>
<td>tip-fin (cm)</td>
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<td></td>
<td></td>
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<tr>
<td>fin-shaft (cm)</td>
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NAWC INVERSE HEAT TRANSFER PROGRAM. INPUT DATA FOR NODE418.

Material properties:

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<tr>
<th>Property</th>
<th>Value 1</th>
<th>Value 2</th>
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<tr>
<td>( \rho ) (kg/m(^3))</td>
<td>18310.0</td>
<td>173.0</td>
</tr>
<tr>
<td>( k ) (W/m-deg k)</td>
<td>146.0</td>
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</tr>
<tr>
<td>( \rho ) (cm(^3))</td>
<td>2.6</td>
<td>52.00</td>
</tr>
<tr>
<td>( V ) (cm(^3))</td>
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<td>23.0</td>
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<td>X-section areas:</td>
<td>tip-fin (cm(^2))</td>
<td>fin-shaft (cm(^2))</td>
</tr>
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<td></td>
<td>5.9</td>
<td>5.2</td>
</tr>
<tr>
<td>Surface areas:</td>
<td>tip (cm(^2))</td>
<td>fin (cm(^2))</td>
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<tr>
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<td>112.16</td>
</tr>
<tr>
<td>Conductive path</td>
<td>tip-fin (cm)</td>
<td>fin-shaft (cm)</td>
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<tr>
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APPENDIX E. IMSL ROUTINES

A description of the IMSL routines DBCLSF, DIVPRK, and SSET used in the PID and simulation programs.
Purpose: Solve a nonlinear least squares problem subject to bounds on the variables using a modified Levenberg-Marquardt algorithm and a finite-difference Jacobian.

Usage: CALL BCLSF (FCN, M, N, XGUESS, IBTYPE, XLB, XUB, XSCALE, FSCALE, IPARA, IPARAM, X, FVEC, FJAC, LDFJAC)

Arguments

FCN - User-supplied SUBROUTINE to evaluate the function to be minimized. The usage is
CALL FCN (M, N, X, F), where
M - Length of F. (Input)
N - Length of X. (Input)
X - The point at which the function is evaluated. (Input)
F - The computed function at the point X. (Output)
FCN must be declared EXTERNAL in the calling program.

M - Number of functions. (Input)
N - Number of variables. (Input)

XGUESS - Vector of length N containing the initial guess. (Input)

IBTYPE - Scalar indicating the types of bounds on variables. (Input)

IBTYPE Action
0 User will supply all the bounds.
1 All variables are nonnegative.
2 All variables are nonpositive.
3 User supplies only the bounds on 1st variable, all other variables will have the same bounds.

XLB - Vector of length N containing the lower bounds on variables. (Input, if IBTYPE = 0; output, if IBTYPE = 1 or 2; input/output, if IBTYPE = 3)

XUB - Vector of length N containing the upper bounds on variables. (Input, if IBTYPE = 0; output, if IBTYPE = 1 or 2; input/output, if IBTYPE = 3)

XSCALE - Vector of length N containing the diagonal scaling matrix for the variables. (Input)
In the absence of other information, set all entries to 1.0.

FSCALE - Vector of length N containing the diagonal scaling matrix

BCLSF/DBCLSF IMSL MATH/LIBRARY
for the functions. (Input)
In the absence of other information, set all entries to
1.0.

**IPARAM** - Parameter vector of length 6. (Input/Output)
See Remarks.

**DPARAM** - Parameters vector of length 7. (Input/Output)
See Remarks.

**X** - Vector of length N containing the approximate solution.
(Output)

**FVEC** - Vector of length M containing the residuals at the
approximate solution. (Output)

**FJAC** - M by N matrix containing a finite difference approximate
Jacobian at the approximate solution. (Output)

**LDFJAC** - Leading dimension of FJAC exactly as specified in the
dimension statement of the calling program. (Input)

**Remarks**

1. Automatic workspace usage is

   **BCLS** 14*N + 2*M - 1 units, or
   **DBCLS** 26*N + 4*M - 2 units.

   Workspace may be explicitly provided, if desired, by use of
   **B2LSF/DB2LSF**. The reference is

   ```
   CALL B2LSF (FCN, M, N, XGUESS, IBTYPE, XLE, XUB, XSCALE,
               FSCL, IPARAM, DPARAM, X, FVEC, FJAC,
               LDFJAC, W, IWK)
   ```

   The additional arguments are as follows:

   **WK** - Work vector of length 12*N + 2*M - 1. WK contains
   the following information on output:
   The second N locations contain the last step taken.
   The third N locations contain the last Gauss-Newton step.
   The fourth N locations contain an estimate of the
   gradient at the solution.

   **IWK** - Work vector of length 2*M containing the
   permutations used in the QR factorization of the Jacobian
   at the solution.

2. **Informational errors**

   **Type Code**
   3 1 Both the actual and predicted relative reductions in the
      function are less than or equal to the relative function
      convergence tolerance.
   4 2 The iterates appear to be converging to a noncritical
      point.
   4 3 Maximum number of iterations exceeded.

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**BCLS**/**DBCLS**
Maximum number of function evaluations exceeded. Five consecutive steps have been taken with the maximum step length.

3. The first stopping criterion for BCLSF occurs when the norm of the function is less than the absolute function tolerance. The second stopping criterion occurs when the norm of the scaled gradient is less than the given gradient tolerance. The third stopping criterion for BCLSF occurs when the scaled distance between the last two steps is less than the step tolerance.

4. If nondefault parameters are desired for IPARAM or RPARAM, then U4LSF is called and the corresponding parameters are set to the desired value before calling the optimization program. Otherwise, if the default parameters are desired, then set IPARAM(1) to zero and call the optimization program omitting the call to U4LSF. The call to U4LSF would be as follows:

```
CALL U4LSF (IPARAM, RPARAM).
```

The following is a list of the parameters and the default values:

**IPARAM** - Integer vector of length 6.
- **IPARAM(1)** = Initialization flag. (0)
- **IPARAM(2)** = Number of good digits in the function.
  (Machine dependent)
- **IPARAM(3)** = Maximum number of iterations. (100)
- **IPARAM(4)** = Maximum number of function evaluations. (400)
- **IPARAM(5)** = Maximum number of Jacobian evaluations. (100)
  (Not used in BCLSF.)
- **IPARAM(6)** = Internal variable scaling flag. (1)
  If IPARAM(6) = 1 the values for XSCALE are set internally.

**RPARAM** - Real vector of length 7.
- **RPARAM(1)** = Scaled gradient tolerance.
  (SQRAT(eps) in single precision)
  (eps=(1/3) in double precision)
- **RPARAM(2)** = Scaled step tolerance. (eps=(2/3))
- **RPARAM(3)** = Relative function tolerance.
  (MAX(1.0E-10, eps=(2/3)) in single precision)
  (MAX(1.0D-20, eps=(2/3)) in double precision)
- **RPARAM(4)** = Absolute function tolerance.
  (MAX(1.0E-20, eps=2) in single precision)
  (MAX(1.0D-40, eps=2) in double precision)
- **RPARAM(5)** = False convergence tolerance. (100-eps)
- **RPARAM(6)** = Maximum allowable step size.
(1000=MAX(TOL1,TOL2)) where,
TOL1 = SQRT(sum of (XSCALE(I)*XGUESS(I))^2)
for I = 1,...,N
TOL2 = 2-norm of XSCALE.
RPARAM(7) = Size of initial trust region radius.
(Based on the initial scaled Cauchy step)
eps is machine epsilon.
If double precision is desired, then DU4LSF is called and RPARAM
is declared double precision.

Keywords: Levenberg-Marquardt; Trust region

Algorithm

BCLS uses a modified Levenberg-Marquardt method and an active set strategy to
solve nonlinear least squares problems subject to simple bounds on the variables.
The problem is stated as follows:

$$\min_{x \in \mathbb{R}^n} \frac{1}{2} F(x)^T F(x) = \frac{1}{2} \sum_{i=1}^m f_i(x)^2$$

subject to $l \leq x \leq u$.

where $m \geq n$, $F : \mathbb{R}^n \rightarrow \mathbb{R}^m$, and $f_i(x)$ is the $i$-th component function of $F(x)$.

From a given starting point, an active set IA, which contains the indices of the
variables at their bounds, is built. A variable is called a 'free variable' if it is not in
the active set. The routine then computes the search direction for the free variables
according to the formula

$$d = -(J^T J + \mu J)^{-1} J^T F,$$

where $\mu$ is the Levenberg-Marquardt parameter, $F = F(x)$, and $J$ is the Jacobian
with respect to the free variables. The search direction for the variables in IA is
set to zero. The trust region approach discussed by Dennis and Schnabel (1983)
is used to find the new point. Finally, the optimality conditions are checked. The
conditions are:

$$n \leq n \leq u$$
$$g(x_i) < 0, \quad z_i = u_i$$
$$g(x_i) > 0, \quad z_i = l_i.$$

where $\epsilon$ is a gradient tolerance. This process is repeated until the optimality criterion
is achieved.

The active set is changed only when a free variable hits its bounds during an
iteration, or the optimality condition is met for the free variables but not for all
variables in IA, the active set. In the latter case, a variable which violates the
optimality condition will be dropped out of IA. For more detail on the Levenberg-Marquardt method, see Levenberg (1944), or Marquardt (1963). For more detailed information on active set strategy, see Gill and Murray (1976).

Since a finite-difference method is used to estimate the Jacobian, for some single precision calculations, an inaccurate estimate of the Jacobian may cause the algorithm to terminate at a nocaritical point. In such cases high precision arithmetic is recommended. Also, whenever the exact Jacobian can be easily provided, IMSL routine BCLSJ should be used instead.

Example

The nonlinear least squares problem

$$\min_{x \in \mathbb{R}^n} \frac{1}{2} \sum_{i=1}^{2} f_i(x)^2$$

subject to 
- $-2 \leq x_1 \leq 0.5$
- $-1 \leq x_2 \leq 2.$

where $f_1(x) = 10(x_2 - x_1^2)$, and $f_2(x) = (1 - x_1)$ is solved with an initial guess $(-1.2, 1.0)$ and default values for parameters.

C Declaration of variables

INTEGER LDFJAC, M, N
PARAMETER (LDFJAC=2, M=2, N=2)

INTEGER IPARAM(7), ITP, KOUT
REAL FJAC(LDFJAC,M), FSCALE(M), FVEC(N), ROSBCK,
& IPARAM(7), XP(M), XGUESS(M), XLB(M), XU(M), ISUB(N)
EXTERNAL BCLSF, ROSBCK, UMACH

C Compute the least squares for the
C Housbrock function.
DATA XGUESS/-1.2E0, 1.0E0/, XS/2*1.0E0/, FSCALE/2*1.0E0/
DATA XLB/-2.0E0, -1.0E0/, XUB/0.5E0, 2.0E0/

C ITP = 0
C Default parameters are used

C IPARM(1) = 0
C
C CALL BCLSF (ROSBCK, M, N, XGUESS, ITP, XLB, XUB, XS, FSCALE,
& IPARM, IPARM, X, FVEC, FJAC, LDFJAC)
C
C CALL UMACH (2, KOUT)
C WRITE (KOUT,99999) X, FVEC, IPARM(3), IPARM(4)

99999 FORMAT (' The solution is ', 2F9.4, ' ', 'The function ','
& 'evaluated at the solution is ', /, 16X, 2F9.4, ' ',
& 'The number of iterations is ', 10X, 13X, 'The ')

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Output

The solution is 0.5000 0.2500
The function evaluated at the solution is
.0000 .5000
The number of iterations is 15
The number of function evaluations is 22

References


Purpose: Solve an initial-value problem for ordinary differential equations using the Runge-Kutta-Verner fifth-order and sixth-order method.

Usage: CALL IVPRK (IDO, NEQ, FCN, X, XEND, TOL, PARA, Y)

Arguments

IDO - Flag indicating the state of the computation. (Input/Output)
1 Initial entry
2 Normal reentry
3 Final call to release workspace
4 Return because of interrupt 1
5 Return because of interrupt 2 with step accepted
6 Return because of interrupt 2 with step rejected

Normally, the initial call is made with IDO=1. The routine then sets IDO=2 and this value is then used for all but the last call which is made with IDO=3. This final call is only used to release workspace, which was automatically allocated by the initial call with IDO=1. See Remark 3 for a description of the interrupts.

NEQ - Number of differential equations. (Input)

FCN - User-supplied SUBROUTINE to evaluate functions. The usage is
CALL FCN (NEQ, X, Y, YPRIME), where

NEQ - Number of equations. (Input)
X - Independent variable. (Input)
Y - Array of length NEQ containing the dependent variable values. (Input)
YPRIME - Array of length NEQ containing the values of dy/dx at (X,Y). (Output)

FCN must be declared EXTERNAL in the calling program.

X - Independent variable. (Input/Output)
On input, X supplies the initial value.
On output, X is replaced by XEND unless error conditions arise. See IDO for details.

XEND - Value of X at which the solution is desired. (Input)
XEND may be less than the initial value of X.

TOL - Tolerance for error control. (Input)
An attempt is made to control the norm of the local error such that the global error is proportional to TOL.
More than one run, with different values of TOL, can be
used to estimate the global error. Generally, it should not be greater than 0.001.

PARAM - Vector of length 50 containing optional parameters.
(Input/Output)

If a parameter is zero then a default value is used.

The following parameters must be set by the user.

<table>
<thead>
<tr>
<th>PARAM</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 HI.IT</td>
<td>Initial value of the step size $H$. Default: See Algorithm section.</td>
</tr>
<tr>
<td>2 HMIN</td>
<td>Minimum value of the step size $H$. Default: 0.0</td>
</tr>
<tr>
<td>3 HMAX</td>
<td>Maximum value of the step size $H$. Default: No limit is imposed on the step size.</td>
</tr>
<tr>
<td>4 NXSTEP</td>
<td>Maximum number of steps allowed. Default: 500</td>
</tr>
<tr>
<td>5 NXFCN</td>
<td>Maximum number of function evaluations allowed. Default: No limit</td>
</tr>
<tr>
<td>6</td>
<td>Not used.</td>
</tr>
<tr>
<td>7 INTRP1</td>
<td>If nonzero then return with IDO=4. before each step. See Remark 3.</td>
</tr>
<tr>
<td></td>
<td>Default: 0.</td>
</tr>
<tr>
<td>8 INTRP2</td>
<td>If nonzero then return with IDO=5. after every successful step and with IDO=6 after every unsuccessful step. See Remark 3.</td>
</tr>
<tr>
<td></td>
<td>Default: 0.</td>
</tr>
<tr>
<td>9 SCALE</td>
<td>A measure of the scale of the problem. such as an approximation to the average value of a norm of the Jacobian along the trajectory. Default: 1.0</td>
</tr>
<tr>
<td>10 INORM</td>
<td>Switch determining error norm. In the following $E_i$ is the absolute value of an estimate of the error in $Y_i$ (called $Y_i$ here). 0: $\min(\text{absolute error, relative error}) = \max(E_i/W_i), i=1,2,...,\text{SEQ}$, where $W_i = \max(\text{abs}(Y_i), 1.0)$. 1: absolute error $= \max(E_i), i=1,2,...$ 2: $\max(E_i/W_i), i=1,2,...$, where $W_i = \max(\text{abs}(Y_i), \text{floor})$, and $\text{FLOOR}$ is $\text{PARAM}(11)$.</td>
</tr>
</tbody>
</table>
3 - Euclidean norm scaled by $Y_{\text{MAX}}$
   $\sqrt{\sum (E_i^2/W_i^2)}$, where
   $E_i = \max(\text{abs}(Y_i), 1.0)$, for $Y_{\text{MAX}}$.
   see Remark 1.

11 FLOOR - Used in the norm computation.
   Default: 1.0

12-30 - Not used.

The following entries in PARAM are set by the program:
31 NTRIAL - Current trial step size.
32 NWINC - Computed minimum step size allowed.
33 NHMAX - Computed maximum step size allowed.
34 NSTEP - Number of steps taken.
35 NFCN - Number of function evaluations used.
36-50 - Not used.

$Y$ - Vector of length $\text{NEQ}$ of dependent variables.
   (Input/Output)
   On input, $Y$ contains the initial values. On output, $Y$ contains the approximate solution.

Remarks

1. Automatic workspace usage is
   IVPRK 10-$\text{NEQ}$ units, or
   DIVPRK 20-$\text{NEQ}$ units.
   Workspace may be explicitly provided, if desired, by use of
   I2PRK/DI2PRK. The reference is
   CALL I2PRK (IDO, $\text{NEQ}$, FCN, X, XEND, TOL, PARAM, Y, VCNRM, WK).

The additional arguments are as follows:

VCNRM - User-supplied SUBROUTINE to compute the norm of the error. (Input)
   The routine may be provided by the user, or the IMSL
   routine I3PRK/DI3PRK may be used.
   The usage is
   CALL VCNRM ($\text{NEQ}$, V, Y, $Y_{\text{MAX}}$, ENORM), where
   $\text{NEQ}$ - Number of equations. (Input)
   V - Vector of length $\text{NEQ}$ containing the vector whose
       norm is to be computed. (Input)
   Y - Vector of length $\text{NEQ}$ containing the values of
       the dependent variable. (Input)
   $Y_{\text{MAX}}$ - Vector of length $\text{NEQ}$ containing the maximum $Y$
       values computed so far. (Input)
   ENORM - Norm of the vector V. (Output)
   VCNRM must be declared EXTERNAL in the calling program.
   WK - Work array of length 10-$\text{NEQ}$. WK must not be changed

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from the first call with IDO=1 until after the final call with IDO=3.

2. Informational errors

Type Code
4 1 Cannot satisfy error condition. TOL may be too small.
4 2 Too many function evaluations needed.
4 3 Too many steps needed. The problem may be stiff.

3. If PARAN(7) is nonzero, the subroutine returns with IDO = 4, and will resume calculation at the point of interruption if reentered with IDO = 4. If PARAN(8) is nonzero, the subroutine will interrupt the calculations immediately after it decides whether or not to accept the result of the most recent trial step. IDO = 5 if the routine plans to accept, or IDO = 6 if it plans to reject. IDO may be changed by the user in order to force acceptance of a step (by changing IDO from 6 to 5) that would otherwise be rejected, or vice versa.

Relevant parameters to observe after return from an interrupt are IDO, HTRIAL, NSTEP, NFCN, and Y. Y is the newly computed trial value, accepted or not.

Algorithm

IVPRK finds an approximation to the solution of a system of first-order differential equations of the form \( y' = f(x, y) \) with initial conditions. The routine attempts to keep the global error proportional to a user-specified tolerance. The proportionality depends on the differential equation and the range of integration.

IVPRK is efficient for nonstiff systems where the derivative evaluations are not expensive and where the solution is not required at a large number of finely spaced points (as might be required for graphical output). IVPRK is based on a code designed by T. E. Hull, W. H. Enright and K. R. Jackson (1976, 1977). It uses Runge-Kutta formulas of order five and six developed by J. H. Verner.

Example

Consider a predator-prey problem with rabbits and foxes. Let \( r \) be the density of rabbits and let \( f \) be the density of foxes. In the absence of any predator-prey interaction the rabbits would increase at a rate proportional to their number and the foxes would die of starvation at a rate proportional to their number. Mathematically,

\[
\begin{align*}
    r' &= 2r, \\
    f' &= -f.
\end{align*}
\]

IVPRK, DIVPRK

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The rate at which the rabbits are eaten by the foxes is \(2rf\) and the rate at which the foxes increase, because they are eating the rabbits, is \(rf\). So the model to be solved is

\[
\begin{align*}
    r' &= 2r - 2rf \\
    f' &= -f + rf.
\end{align*}
\]

The initial conditions are \(r(0) = 1\) and \(f(0) = 3\) over the interval \(0 \leq t \leq 10\).

In the program \(Y(1) = r\) and \(Y(2) = f\). Note that the parameter vector is first set to zero (using IMSL routine SSET) and then absolute error control is selected by setting \(\text{PARAM}(10) = 1.0\).

The last call to IVPRK with \(\text{IDO} = 3\) releases IMSL workspace, that was reserved on the first call to IVPRK. It is not necessary to release the workspace in this example, because the program ends after solving a single problem. The call to release workspace is made as a model of what would be needed if the program included further calls to IMSL routines.

The following plots are the result of using IVPRK with more closely spaced output than what is printed. (The program which does the plotting is not shown.) The second plot is a phase diagram for this system and clearly shows the periodic nature of the solution.

```c
INTEGER MXPARN, KED 
PARAMETER (MXPARN=50, KED=2)
C
INTEGER IDO, ISTEP, MOUT 
REAL FCN, FLOAT, PARAM(MXPARN), T, TEND, TOL, Y(MEX)
INTRINSIC FLOAT 
EXTERNAL FCN, IVPRK, SSET, UNACH
C
CALL UNACH (2, MOUT) 
C
      Set initial conditions 
      T = 0.0 
      Y(1) = 1.0 
      Y(2) = 3.0 
C
      Set error tolerance 
      TOL = 0.0005 
C
      Set PARAM to default 
      CALL SSET (MXPARN, 0.0, PARAM, 1) 
C
      Select absolute error control 
      PARAM(10) = 1.0 
C
      Print header 
      WRITE (MOUT, 'I99999') 
      I99999 FORMAT (4X, 'ID0', 5X, 'STEP', 5X, 'Time', 5X, 'Y1', 11X, 'Y2') 
      ID0 = 1 
      DO 10 ISTEP=1, 10 
         TEND = FLOAT (ISTEP) 
         CALL IVPRK (IDO, KED, FCN, T, TEND, TOL, PARAM, Y) 
         WRITE (MOUT, '(16.3F12.3') ISTEP, T, Y 
   10 CONTINUE
```

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IVPRK/DIVPRK
C Final call to release workspace

IDO = 3
CALL IVPRK (IDO, NEQ, FCN, T, TEND, TOL, PARAM, Y)
END

SUBROUTINE FCN (NEQ, T, Y, YPRIME)
INTEGER NEQ
REAL T, Y(NEQ), YPRIME(NEQ)

C

YPRIME(1) = 2.0-Y(1) - 2.0*Y(1)*Y(2)
YPRIME(2) = -Y(2) - Y(1)*Y(2)
RETURN
END

Output

<table>
<thead>
<tr>
<th>ISTEP</th>
<th>Time</th>
<th>Y1</th>
<th>Y2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>0.078</td>
<td>1.465</td>
</tr>
<tr>
<td>2</td>
<td>2.000</td>
<td>0.085</td>
<td>0.578</td>
</tr>
<tr>
<td>3</td>
<td>3.000</td>
<td>0.292</td>
<td>0.250</td>
</tr>
<tr>
<td>4</td>
<td>4.000</td>
<td>1.440</td>
<td>1.187</td>
</tr>
<tr>
<td>5</td>
<td>5.000</td>
<td>4.040</td>
<td>1.444</td>
</tr>
<tr>
<td>6</td>
<td>6.000</td>
<td>3.178</td>
<td>2.256</td>
</tr>
<tr>
<td>7</td>
<td>7.000</td>
<td>0.055</td>
<td>0.008</td>
</tr>
<tr>
<td>8</td>
<td>8.000</td>
<td>0.145</td>
<td>0.367</td>
</tr>
<tr>
<td>9</td>
<td>9.000</td>
<td>0.635</td>
<td>0.168</td>
</tr>
<tr>
<td>10</td>
<td>10.000</td>
<td>3.157</td>
<td>0.352</td>
</tr>
</tbody>
</table>

IVPRK-DIVPRK

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References


Basic Linear Algebra Subprograms

The basic linear algebra subprograms, usually called the BLAS, are routines for low-level vector operations such as dot products. Lawson et al. (1979) developed the original set of 38 BLAS routines. The IMSL BLAS collection includes these original 38 routines plus additional routines. The original BLAS are marked with an * in the descriptions.

Programming Notes

The BLAS do not follow the usual IMSL naming conventions. Instead the names consist of a prefix of one or more of the letters 'I', 'S', 'D', 'C' and 'Z', a root name, and sometimes a suffix. For subprograms involving a mixture of data types the output type is indicated by the first prefix letter. The suffix denotes a variant algorithm. The prefix denotes the type of the operation according to the following table:

- I Integer
- S Real
- D Double
- SD Single and double
- DQ Double and quadruple
- C Complex
- CZ Single and double complex
- ZQ Double and quadruple complex

Vector arguments have an increment parameter which specifies the storage space between elements. The correspondence between the vector \( z \) and the arguments \( SX \) and \( INCZ \) is

\[
x_i = \begin{cases} 
  SX((I-1)\cdot INCX+1) & \text{if } INCX \geq 0 \\
  SX((I-N)\cdot INCX+1) & \text{if } INCX < 0.
\end{cases}
\]

Only positive values of \( INCX \) are allowed for operations which have a single vector argument.

The loops in all of the BLAS routines process the vector arguments in order of increasing \( i \). For \( INCX < 0 \), this implies processing in reverse storage order.

With the definitions,

\[
NX = \max(1, (N-1)|INCX|) \\
NY = \max(1, (N-1)|INCY|) \\
NZ = \max(1, (N-1)|INCY|)
\]

the routine descriptions assume the following FORTRAN declarations:

- `IMPLICIT INTEGER (I-N)`
- `IMPLICIT REAL S`
- `IMPLICIT DOUBLE PRECISION D`
- `IMPLICIT COMPLEX C`
- `IMPLICIT DOUBLE COMPLEX Z`
- `INTEGER IX(NX)`
- `REAL SX(NX), SY(NY), SZ(NZ), SPARAN(S)`
- `SH(LDH,=)`

BLAS

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### Basic Linear Algebra Subprograms

<table>
<thead>
<tr>
<th>( x_i = a )</th>
<th>Integer</th>
<th>Real</th>
<th>Double</th>
<th>Complex</th>
<th>Double Complex</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_i = a )</td>
<td>ISET</td>
<td>SSEI</td>
<td>DSET</td>
<td>CSET</td>
<td>ZSET</td>
<td>1026</td>
</tr>
<tr>
<td>( y_i = x_i )</td>
<td>ICOPY</td>
<td>SCOPY</td>
<td>DCOPY</td>
<td>CCOPY</td>
<td>ZCOPY</td>
<td>1026</td>
</tr>
<tr>
<td>( x_i = x_i )</td>
<td>SSCAL</td>
<td>DSCAL</td>
<td>CSCAL</td>
<td>ZSCAL</td>
<td>1026</td>
<td></td>
</tr>
<tr>
<td>( a \in \mathbb{R} )</td>
<td>SVCAL</td>
<td>DVCAI</td>
<td>CVCAI</td>
<td>ZVCAI</td>
<td>1027</td>
<td></td>
</tr>
<tr>
<td>( y_i = x_i )</td>
<td>SSET</td>
<td>OSET</td>
<td>CSET</td>
<td>ZSET</td>
<td>1027</td>
<td></td>
</tr>
<tr>
<td>( x_i = x_i )</td>
<td>ICOPY</td>
<td>SCOPY</td>
<td>DCOPY</td>
<td>CCOPY</td>
<td>ZCOPY</td>
<td>1027</td>
</tr>
<tr>
<td>( y_i = x_i )</td>
<td>ISUB</td>
<td>SSUB</td>
<td>DSUB</td>
<td>CSUB</td>
<td>ZSUB</td>
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</tr>
<tr>
<td>( y_i = x_i )</td>
<td>ICOPY</td>
<td>SCOPY</td>
<td>DCOPY</td>
<td>CCOPY</td>
<td>ZCOPY</td>
<td>1027</td>
</tr>
<tr>
<td>( y_i = x_i )</td>
<td>ISWAP</td>
<td>SSWAP</td>
<td>DSWAP</td>
<td>CSWAP</td>
<td>ZSWAP</td>
<td>1028</td>
</tr>
<tr>
<td>( x \cdot y )</td>
<td>SDOT</td>
<td>DDOT</td>
<td>CDOTU</td>
<td>ZDOTU</td>
<td>1028</td>
<td></td>
</tr>
<tr>
<td>( x \cdot y )</td>
<td>SDOT</td>
<td>DDOT</td>
<td>CDOTU</td>
<td>ZDOTU</td>
<td>1028</td>
<td></td>
</tr>
<tr>
<td>( x \cdot y )</td>
<td>DSDOT</td>
<td>CDOTU</td>
<td>ZDOTU</td>
<td>1028</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x \cdot y )</td>
<td>DSDOT</td>
<td>CDOTU</td>
<td>ZDOTU</td>
<td>1028</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( a + x \cdot y )</td>
<td>SDSDDOT</td>
<td>CDODOT</td>
<td>ZDODOT</td>
<td>1026</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( a + z \cdot y )</td>
<td>SDSDDOT</td>
<td>CDODOT</td>
<td>ZDODOT</td>
<td>1026</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b + x \cdot y )</td>
<td>SDSDDOT</td>
<td>CDODOT</td>
<td>ZDODOT</td>
<td>1026</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( A(x + b - x \cdot y) )</td>
<td>SDDOTI</td>
<td>DDDOTI</td>
<td>CDDOTI</td>
<td>ZDDOTI</td>
<td>1029</td>
<td></td>
</tr>
<tr>
<td>( z_i = x_i )</td>
<td>SHPROD</td>
<td>DHPROD</td>
<td>1029</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( z_i = x_i )</td>
<td>SHPROD</td>
<td>DHPROD</td>
<td>1029</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( | z_i |_2 )</td>
<td>SARS2</td>
<td>DWRN2</td>
<td>SCRR2</td>
<td>ZWRN2</td>
<td>1030</td>
<td></td>
</tr>
<tr>
<td>( | z_i |_\infty )</td>
<td>SPMCT</td>
<td>DPMCT</td>
<td>1030</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( | |_2 )</td>
<td>SPMCT</td>
<td>DPMCT</td>
<td>1030</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Higher precision accumulation used.

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**BLAS**

103
DOUBLE PRECISION DX(MX), DY(MY), DZ(NZ), DPARAM(5), 
  DH(LDH,*), DACC(2), DZACC(4), 
COMPLEX CX(NX), CY(NY), 
DOUBLE COMPLEX ZX(NX), ZY(NY)

Since FORTRAN 77 does not include the type DOUBLE COMPLEX routines with
DOUBLE COMPLEX arguments are not available for all systems. Some systems use
the declaration COMPLEX*16 instead of DOUBLE COMPLEX.

The set of BLAS routines are summarized by the table on page 1025. Routines
marked with a dagger (†) in the table use higher precision accumulation.

Set a Vector to a Constant Value

CALL ISET (N, IA, IX, INCX)
CALL SSET (N, SA, SX, INCX)
CALL DSET (N, DA, DX, INCX)
CALL CSET (N, CA, CX, INCX)
CALL ZSET (N, ZA, ZX, INCX)

These subroutines set \( x_i = a \) for \( i = 1, 2, \ldots, N \). If \( N \leq 0 \) then the routines
return immediately.

Copy a Vector

CALL ICOPY (N, IX, INCX, IY, INCY)
CALL SCOPY (N, SX, INCX, SY, INCY)
CALL DCOPY (N, DX, INCX, DY, INCY)
CALL CCOPY (N, CX, INCX, CY, INCY)
CALL ZCOPY (N, ZX, INCX, ZY, INCY)

These subroutines set \( y_i = x_i \) for \( i = 1, 2, \ldots, N \). If \( N \leq 0 \) then the routines
return immediately.

Scale a Vector

CALL SSCL (N, SA, SX, INCX)
CALL DSCL (N, DA, DX, INCX)
CALL CSCL (N, CA, CX, INCX)
CALL ZSCL (N, ZA, ZX, INCX)
CALL ZSSCL (N, SA, CX, INCX)
CALL ZDSCL (N, DA, ZX, INCX)

These subroutines set \( x_i = ax_i \) for \( i = 1, 2, \ldots, N \). If \( N \leq 0 \) then the routines
return immediately.
LIST OF REFERENCES


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