Wideband Beamforming for a Sparse Nonuniformly Spaced Array

Jeff Krolik

Final Report to the
Office of Naval Research
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For the Period 8-30-92 - 8-29-93

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### Abstract (Maximum 200 words)

This project concerns the continued development and evaluation of a computationally-efficient wideband adaptive beamformer for a large nonuniformly-spaced sensor array which effectively extends its operation to wavelengths which are considerably smaller than the average inter-element spacing.

### Subject Terms

- Signal processing
- Sensor array
- Inter-element spacing
- Frequency-dependent bandwidths
Wideband Beamforming for a Sparse Nonuniformly Spaced Sensor Array

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Objective

This project concerns the continued development and evaluation of a computationally-efficient wideband adaptive beamformer for a large nonuniformly-spaced sensor array which effectively extends its operation to wavelengths which are considerably smaller than the average inter-element spacing.

Task Report

A requirement exists to extend the useful bandwidth of existing large nonuniformly spaced bottom-mounted arrays to include frequencies where the acoustic wavelength is much smaller than the average inter-element spacing. The performance of the existing fixed-aperture (FA) adaptive beamformer (FA-ABF) is seriously degraded at higher frequencies due to: large gaps in the azimuthal coverage provided by a fixed set of beams as a result of the decreasing mainlobe width of each beam with increasing frequency, a loss of multipath signal energy captured within a beam which results when the spread of multipath grazing angles exceeds the mainlobe width, and interferences which leak through beamformer grating plateaux causing both a reduction in array gain and masking of the signals of interest. In the reception of wideband transient signals, grating plateaux leakage could cause serious overlapping of emissions from different sources resulting in a highly cluttered environment for classification. In this project, an adaptive constant-mainlobe (CM) beamformer
was developed as an alternative means of extending the useful bandwidth of 
these arrays. The adaptive CM beamformer maintains a frequency-independent 
mainlobe response for each beam so as to ensure complete azimuthal coverage 
by a fixed set of beams over the entire frequency range of interest. Appropriate 
design of the frequency-independent mainlobe response was achieved which 
not only avoids gaps in azimuthal coverage, but also ensured that all multipath 
arrivals from a common source are captured within a single beam across the full 
receiver band. Although an approximately frequency-independent mainlobe 
response could have been achieved by simple aperture shading, i.e. using 
shorter subarrays at higher frequencies, the new design eschews this strategy 
since it does not facilitate suppression of grating plateaux interferences at higher 
frequencies where the aperture-shaded subarray is nearly uniformly spaced. 
Instead, the new method developed in this task achieves a constant mainlobe 
response by frequency-dependent linear constraints on the adaptive minimum 
variance solution to the beamformer array response. In the absence of 
interfering sources, the constraint-maintained adaptive CM beamformer 
response is close to that of an aperture-shaded array. However, since the 
constraint-maintained adaptive CM solution retains the ability to use the entire 
non-uniformly spaced array, in the presence of grating plateaux interferences it 
offers much better array gain and reduced signal masking. The array gain 
performance of the constraint-maintained adaptive CM beamformer has been 
evaluated in both diffuse ambient noise and multiple-target stationary 
environments. Theoretical comparison of the CM versus FA beamformers 
indicates that the CM beamformer provides a trade-off between main response 
axis (MRA) and off-MRA array gain. Results up to 400 Hz indicate that the CM 
beamformer gives as much as a 20 dB improvement over the worst-case 
response of the FA beamformer at the cost of less than a 5 dB loss relative to the 
best-case on-MRA FA beamformer gain. Real data provided by NRaD has been 
used to verify the theoretical array gain comparison up to approximately 180 Hz 
in diffuse ambient noise.

In addition to achieving improved wideband performance, the design of a 
computationally-efficient implementation of the constraint-maintained adaptive 
CM beamformer has been achieved. At the higher sampling rates imposed by 
larger receiver bandwidths, a conventional frequency-domain beamformer 
implementation is precluded due to the very long FFT lengths and/or large 
percent overlap required to handle endfire signal propagation delays. For 
improved efficiency, an implementation which performs coarse nearest-sample 
pre-steering of the sensor outputs has been developed. By dividing the full 
bearing space into sectors and then coarse pre-steering for each sector, the 
maximum signal propagation delay is reduced to that of an arrival coming from 
the sector edge. This reduction in the maximum signal propagation delay means 
that implementation of the beamformer can be achieved with shorter FFT's 
and/or smaller percent overlap, thus reducing both the computational and 
memory requirements of the design.
Evaluation and development of the CM adaptive beamformer has been limited to quasi-stationary signal and noise environments. In particular, the constraint-maintained adaptive CM beamformer was compared with an aperture-shaded adaptive beamformer in an interference-dominated environment. Array gain and source location ambiguity performance was evaluated for both approaches. Results indicate that for quasi-stationary interference, superior array gain was achieved at higher interference-to-noise ratios by the constraint-maintained CM beamformer due to its ability to more effectively suppress grating plateaux interference. The design of a recursive algorithm for updating the constraint-maintained adaptive CM beamformer weights was performed and is shown in the block diagram of Figure 1. The criteria of both computational efficiency and convergence rate were used to optimize the design. To ensure statistically stable adaptive beamformer weight solutions, partitioning of the entire receiver frequency band for adaptation in sub-bands was used. Sub-band selection involves a trade-off between asymptotic array gain and misadjustment of the adaptive beamformer weights. For a fixed convergence rate, large sub-bands result in less variability in the beamformer weights and thus lower misadjustment. However decorrelation of the interference over larger sub-bands results in a reduction in asymptotic array gain. Optimal selection of the sub-bands may require some a priori knowledge of both typical signal and interference auto-spectra as well as field dynamics. Extensive consulting services to NRaD were provided to facilitate the implementation of the constraint-maintained adaptive CM beamformer in MATLAB code which runs on both SUN workstations and the CONVEX mainframe at NRaD. This extensive software library is in the possession of Dr. David Schwartz, Code 73, NRaD.
FIGURE 1: Block Diagram of Wideband Beamforming for a Sparse Nonuniformly-Spaced Array

For 2fs system with N=90 phones (A/D conversion not shown).

Coarse presteering achieved by inserting nearest integer delays to align signal from center of angular sector. Number of sectors = \( N_{s} \) (equispaced in \( \cos(\text{bearing}) \)).

FFT is 2K overlapped 50% from each 1K processing frame to the next. Data window is "flat-top" with ones in the middle (1K plus \( 2fsL/N_c \) where \( L \) is array length and \( c \) is sound-speed) and linear skirts chosen to minimize sidelobe leakage.

Inverse FFT is 2K with middle 1K real output time-series samples from each frame retained and concatenated. Input: beamformed complex frequency bins across band (filter to zero at DC and above 0.864fs).

\[ y(n, p, k) = N x I \] complex sensor data corresponding to \( k^{th} \) frequency bin, \( n^{th} \) frame, and \( p^{th} \) coarse steered sector.

\[ Y(n, p, k) = [y(n, p, l_1), ..., y(n, p, h_k)] \quad = N \times b_k \] complex matrix of sensor data corresponding to correlation band, of width \( b_k = (h_k - l_k + 1) \) bins, associated with bin \( k \). Correlation bands overlap with frequency-dependent bandwidths.
\[ \mathbf{\varphi}_i(n, p, k) = \mathbf{Y}(n, p, k)^H \mathbf{w}_i(n, p, k) \] is the correlation-band gradient estimate for beam \( k \) and weight vector at frequency bin \( k \).

\[ \mathbf{w}_i(n, p, k) = \mathbf{g}_i(p, k) + \mathbf{v}_i(n, p, k) \] is an \( N \times 1 \) beamformer weight vector for beam \( i \), sector \( p \), and frame \( n \). Consisting of data-independent “quiescent” weights, \( \mathbf{g}_i(p, k) \), plus data-dependent adaptive weights, \( \mathbf{v}_i(n, p, k) \).

**LMS update:**
\[ \mathbf{v}_i(n+1, p, k) = \mathbf{v}_i(n, p, k) - \frac{\mu_k}{\mathbf{\sigma}^2(n, p, k)} \mathbf{\varphi}_i(n, p, k) \]

**Projection onto orthogonal subspace of mainlobe constraint vectors:**
\[ \mathbf{\tilde{v}}_i(n+1, p, k) = \mathbf{v}_i(n+1, p, k) - \mathbf{C}(p, k, i) \mathbf{C}(p, k, i)^H \mathbf{\varphi}_i(n+1, p, k) \]

**Weight vector scaling to impose white noise gain constraint for robustness to signal mismatch** (cf. H. Cox et.al., IEEE ASSP Trans., October, 1987)
\[ \mathbf{v}_i(n+1, p, k) = \begin{cases} \mathbf{\tilde{v}}_i(n+1, p, k) & \text{for } |\mathbf{\tilde{v}}| \leq \beta_k \\ \beta_k |\mathbf{\tilde{v}}_i(n+1, p, k)| & \text{for } |\mathbf{\tilde{v}}| > \beta_k \end{cases} \]

**Correlation band power estimate update:**
\[ \mathbf{\sigma}^2(n+1, p, k) = (1 - \lambda) \mathbf{\sigma}^2(n, p, k) + \lambda \left\{ \frac{1}{B_k} \sum_{j=1}^{B_k} |\mathbf{y}(n, p, j)|^2 \right\} \]
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