AN EXPERIMENTAL DETERMINATION OF METEOR DAILY ARRIVAL RATE VARIATION

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This report addresses the day-to-day variability in the meteor arrival rate over a two-month period on the PL/GP High Latitude Meteor Scatter Test Bed and the PL Mid Latitude Meteor Scatter Test bed. In particular, we present evidence that the meteor arrival variability fits an exponential distribution, thus the duration of the system testing period to estimate the meteor arrival variability can be calculated. The arrival rate data from two different sites show that the day-to-day percent change in the arrival rate, at least in the short term, can be modeled as an exponential distribution with a mean of near 10 percent. This suggests meteor scatter communication testing to evaluate performance should be conducted for 9 to 12 days as a minimum to approximate medium term, 30 to 90 days, day-to-day arrival variation.
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An Experimental Determination of
Meteor Daily Arrival Rate Variation

1. INTRODUCTION

Meteor arrival is an inhomogeneous Poisson process. The arrival rate varies during the day, month, year and solar cycle. Good models of the meteor scatter process can predict reliably to within a factor of four.\(^1\) Reasons for the wide range in modelling results range from the granularity of the meteor radiant maps to the uncertainty in some constants. The only way to get a better "prediction" is to measure the system performance in the field. The system is usually tested when the meteor arrival rate is at a minimum, typically in February or March in the northern hemisphere. This insures that the system will meet the required performance level during the meteor minimum. Even then, the arrival rate may vary considerably from day to day. If a field test is necessary, then we want to know how long the system should be tested to realistically assess the performance.

This analysis of the day-to-day arrival rate variation shows that long term testing is required to get any meaningful statistics since large variations in arrival rate from one day to the next can be expected. Data from the PL high-and mid-latitude links show that the day-to-day arrival rate variation ranges from near 0 to 45 percent during the two month test period. Data from the two links at widely separated latitudes, illustrated in Figure 1, were used in the analysis of the day-to-day arrival rate variation.

2. HIGH LATITUDE METEOR SCATTER LINK DESCRIPTION

The high latitude link acquires data on each of six frequencies, 35, 45, 65, 85, 104 and 147 MHz, for 10 to 20 minutes every two hours on each frequency. The transmitter is located at Sondrestromfjord, Greenland and the receiver is located at Thule Air Base, Greenland. Table 1 shows the geographical parameters of both links.

The wideband transmitter, operating at a nominal 900 W, uses a narrow band FM signal with a 400 Hz tone for identification. The transmit antennas are five-element horizontally polarized...
Figure 1. Location of the High Latitude and Mid Latitude Links
Yagi antennas mounted 1.5 wavelengths above the ground. The antenna foreground at the transmitter site is not flat. There is a steep ridge about 3 km to the north. All radiation below 1.8 degrees is blocked; the highest sections of the ridge extend to 2.2 degrees. The near foreground has a small hill that eliminates reflections at elevations up to 20 degrees.2

For each frequency, the receive antenna is a five-element vertically polarized Yagi antenna and five-element horizontally polarized Yagi antenna on a common boom. The antennas are mounted 1.5 wavelengths above the ground for optimum spatial coverage. The antenna foreground is essentially flat, with terrain blockage that limits the low elevation radiation to angles 1.1 to 1.7 degrees.2

The receiver is six-frequency, dual-channel built specifically for the test bed. Each of the six RF channels has an input from the horizontal and vertical antenna. The effective receiver bandwidth is 100 Hz on all the frequency bands. The site is limited by galactic noise on all six frequencies. Between each acquisition, noise measurements are taken for one minute. The waveforms and noise measurements are stored on magnetic tape and are processed on a mainframe computer.3

3. MID LATITUDE METEOR SCATTER LINK DESCRIPTION

The mid latitude system is a single frequency link operating at 47 MHz. Data are gathered for 30 minutes each hour, 24 hours a day, and noise measurements are taken between each acquisition. The data are processed on site, and returned to PL for inspection and validation. The transmitter is located at Glendive, Montana and the receiver at Springer, New Mexico.

The antenna foreground at the transmitter site is flat for at least 100 m, with a 2 m chain link fence 65 m from the base of the antenna tower. The transmitter radiates at a nominal 1000 W with a narrow band 400 Hz FM signal for identification. A commercial timer cycles the transmitter on and off according to a programmed schedule. The output RF level is set by an exciter that has a leveling circuit to maintain a constant 1000 W of output power. It also monitors the incident and reflected power to prevent damage to the transmitter in case of high VSWR. The antenna is a five-element horizontally polarized Yagi antenna mounted 1.5 wavelengths above the ground.

The receiving station antenna has a flat foreground for at least 100 m, with no obstructions in sight. The radio noise environment is quiet, limited by galactic noise, except a few minutes each day when a nearby farm operates its equipment. The diurnal galactic noise variation during the test period can be seen in Figures 3 and 5. The receiving system includes a receiver, similar to the receiver at the Thule, Greenland site, except that only one RF channel, 47 MHz, is used. The antennas are similar to those used in the high latitude link. The boom is mounted 1.5 wavelengths above the ground.

The data gathering has been modified to perform most of the analysis on site using a PC. After data have been gathered for 30 minutes, the scattering mechanism (underdense or overdense meteor scatter, sporadic E scatter or unknown type scatter) is determined.
A database for the 30 minutes is constructed that includes statistics for meteor durations and arrival rates. At midnight, the classified waveforms and databases are transferred to a WORM drive. The AUTOCLASS program\textsuperscript{4} that determines the type of scatter has been ported from VAX FORTRAN to a version that runs on the system PC controller.

Table 1. Geographical Parameters of the Links.

<table>
<thead>
<tr>
<th></th>
<th>Receive Thule AB</th>
<th>Transmit Sondrestromfjord</th>
<th>Receive Springer</th>
<th>Transmit Glendive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latitude</td>
<td>76° 33'N</td>
<td>66° 59'N</td>
<td>36° 22'N</td>
<td>47° 00'N</td>
</tr>
<tr>
<td>Longitude</td>
<td>68° 40'W</td>
<td>50° 39'W</td>
<td>104° 38'W</td>
<td>104° 53'W</td>
</tr>
<tr>
<td>Azimuth</td>
<td>142°</td>
<td>338°</td>
<td>359°</td>
<td>179°</td>
</tr>
<tr>
<td>Terminal Altitude</td>
<td>240 m</td>
<td>330 m</td>
<td>1867 m</td>
<td>760 m</td>
</tr>
<tr>
<td>Horizon Blockage</td>
<td>1.1-1.7°</td>
<td>1.8-2.2°</td>
<td>--- Less Than 1°</td>
<td>---</td>
</tr>
<tr>
<td>Midpath Elevation at 100 km</td>
<td>6.5°</td>
<td></td>
<td>6.9°</td>
<td></td>
</tr>
<tr>
<td>Great Circle Distance</td>
<td>1210 km</td>
<td></td>
<td>1183 km</td>
<td></td>
</tr>
</tbody>
</table>

Figures 2 thru 13 show various data collected on the links during the test period. The figures show the diurnal noise curves, the extent of sporadic E, up to a 35 percent duty cycle, and the diurnal meteor arrival rate.

4. DAILY METEOR ARRIVAL RATE VARIATION

Data were collected for a period of 4 years on the PL High Latitude link. The data were entered into monthly databases. A subset of the databases, arrival rates at -126 dBm received signal level at 45 MHz from 5 April through 6 June 1992, was used. The data were further modified by removing 4 days data which was taken during an absorption event from 9 May thru 12 May. Any day that was missing one or more measurement periods was also removed from the analysis. On the high latitude link, data at 45 MHz are taken for a total of 120 minutes each day. This is only 8.3 percent of the whole day. If one period is missing, the total time drops to 110 minutes, 7.6 percent of the day, and, there is almost a 4-hour time block with no data. Of the 63 days of operation, a total of 33 days had all 12 periods, and of these, there were 20 occurrences of consecutive days. The data base stored the arrival rates as meteors per minute, so the daily average arrival rate was determined by averaging the 12 measurements taken over the day.

The data at the mid latitude link were collected from 5 April 1992 to 6 June 1992, a total of 63 days. Two consecutive days were removed from the data set because more than two periods were missing. On 10 days, one or two measurement periods were missing due to transmitter outages. These 10 days were used in this analysis, which resulted in a total of 60 occurrences of
consecutive days. Days which had one or two missing periods were used in this analysis because the measurements, even with the missing periods, still reflect a reasonable diurnal variation, and data are acquired for 11 hours over the day, which covers more than 40 percent of the day. The average daily arrival rate was calculated by dividing the number of meteor arrivals for the day by the time the system was operating.

The analysis relied exclusively on the computer's determination that the calculated meteor arrival rate was due to meteors and not other propagation modes. Weitzen\textsuperscript{5} has estimated AUTOCLASS to be about 98 percent reliable in determining meteoric scatter and the program tends to be conservative in its determination of meteoric scatter.

The analysis consists of comparing the day-to-day difference in the arrival rate at the two sites during 5 April 1992 through 6 June 1992. In particular, we show that the absolute value of the percent difference of the day-to-day arrival rate can be modeled as an exponential distribution. The day-to-day percent variation was determined as:

\[ x(n) = \frac{|r_{n+1} - r_n|}{r_n} \times 100 \]  

where

\[ r_n = \text{meteor arrival rate on day } n \]
\[ r_{n+1} = \text{meteor arrival rate on day } n+1 \]

The Kolmogorov-Smirnov (K-S) test is first used to test the hypothesis that both data sets have the same distribution. This test compares both cumulative data distributions at each of the data points. The statistic, \( D_n \), is the absolute value of the largest difference between the two cumulative distributions.\textsuperscript{6} The hypothesis was tested at a 0.05 significance level, where \( D_n \) should be less than 0.351 to be accepted. Figure 14 shows the cumulative distributions plotted for both data sets, with \( D_n \) equal to 0.17. Thus we can conclude at a 0.05 significance level, or 95 percent confidence level, that the observed data are drawn from the same distribution.

The results shown in Table 2 suggest that the underlying distribution fits an exponential distribution, where the mean and standard deviation are equal. The exponential hypothesis was tested on both data sets using the K-S test at a 0.05 significance level. The sample mean of each data set was used to test the exponential hypothesis. Normally, the K-S test requires that the hypothesis is not tested using the sample statistics. However Lilliefors,\textsuperscript{7} published tables in 1969 showing the K-S bounds for an exponential hypothesis tested with the sample mean. These tables were used to test the exponential hypothesis. The results in Figures 15 and 16 show the K-S statistic is well below the 95 percent bound of 0.234 for the Greenland data set and below 0.137 for the USA data set.\textsuperscript{8}

Since the arrival rate percent variation can be modeled as an exponential distribution, we can use that to determine some bounds on how long tests should be performed for stated confidence levels.
Table 2. Sample Statistics for the Absolute Day-to-Day Percent Change in Meteor Arrival Rates for 5 April to 6 June 1992

<table>
<thead>
<tr>
<th>Link</th>
<th>RSL (dBm)</th>
<th>n</th>
<th>Average</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Latitude 45 MHz</td>
<td>-126</td>
<td>20</td>
<td>9.63%</td>
<td>10.24%</td>
</tr>
<tr>
<td>Mid Latitude 47 MHz</td>
<td>-126</td>
<td>60</td>
<td>9.63%</td>
<td>8.50%</td>
</tr>
</tbody>
</table>

The confidence interval for the mean of an exponential distribution is given as:

\[ Pr\left\{ \frac{2n\bar{x}}{\chi^2_x \cdot a / 2n} < \theta < \frac{2n\bar{x}}{\chi^2_x \cdot a / 2n} \right\} = 1 - \alpha \]  

(2)

Where

- \( n = \text{number of samples} \)
- \( \bar{x} = \text{sample mean} \)
- \( \alpha = \text{significance level} \)
- \( \chi^2 = \text{Chi square distribution} \)
- \( \theta = \text{true mean} \)

By dividing the above inequality by the sample mean, \( \bar{x} \), the expression can be normalized to the ratio, \( \frac{\theta}{\bar{x}} \), of the true mean to the sample mean. The 90 percent and 95 percent confidence levels for \( \frac{\theta}{\bar{x}} \) are plotted as a function of the number of days tested in Figure 17. This can be used to estimate how close to true mean you can expect to get as a function of the number of days tested and confidence level required. From Figure 17, if a test lasts for 9 days, we can be 90 percent confident that \( \frac{\theta}{\bar{x}} \) will be between 0.624 and 1.92, a factor of 3.1, or 95 percent confident that \( \frac{\theta}{\bar{x}} \) is within a factor of 3.9. Note that these results are independent of the sample mean.

The plot in Figure 17 shows that a system should be tested for 9 to 12 days to be within a factor of 4 of the true value at 90 percent and 95 percent confidence levels respectively. The nature of the curves suggests that very long term testing is required to bound the uncertainty to narrow intervals. For 60 days tested at the mid latitude link, the true mean is estimated to be between 7.6 percent and 12.6 percent, a factor of 1.66 at a 95 percent confidence.

The expected percent variation (expected mean) \( N \) days later is calculated as:

\[ \theta (N) = \theta \left( 1 + \frac{\theta}{100} \right)^{N - 1} \]  

(3)
Where

\[ \text{Where} \]

\[ 0 = \text{Percent variation, day} + 1 \]

\[ N \geq 1 \]

The results for \( N = 1 \) through 8 are calculated and compared to the data taken on the mid latitude link in Table 3. These results show that for a short time period, about 5 to 6 days, the exponential distribution is a reasonable fit. The data in Table 3 suggests that the average arrival rate variation, or expected mean, for \( N \) greater than 6 days is more conservative than the exponential distribution predicts. For example, a test conducted over 9 days would yield a mean arrival variation with certain bounds. One could use the exponential model to predict the mean \( N + 7 \) days later, but because the exponential hypothesis yields a greater mean than the data suggests, the predicted exponential mean would be a conservative estimate. However, one should be careful using this to extrapolate this much more than 8 days, as the exponential curve will yield a mean too large to be of use.

Table 3. Expected and Calculated Mean for Days 1 Through 8 Apart.

<table>
<thead>
<tr>
<th></th>
<th>Day+1</th>
<th>Day+2</th>
<th>Day+3</th>
<th>Day+4</th>
<th>Day+5</th>
<th>Day+6</th>
<th>Day+7</th>
<th>Day+8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std Dev</td>
<td>8.50</td>
<td>9.92</td>
<td>9.81</td>
<td>12.16</td>
<td>10.58</td>
<td>9.96</td>
<td>10.31</td>
<td>9.91</td>
</tr>
<tr>
<td>No Samples</td>
<td>60</td>
<td>59</td>
<td>58</td>
<td>57</td>
<td>56</td>
<td>55</td>
<td>54</td>
<td>53</td>
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<tr>
<td>Expected mean</td>
<td>10.56</td>
<td>11.57</td>
<td>12.69</td>
<td>13.91</td>
<td>15.25</td>
<td>16.72</td>
<td>18.33</td>
<td></td>
</tr>
</tbody>
</table>

5. CONCLUSION

The arrival rate data from two different sites shows that the day-to-day percent change in the arrival rate, at least in the short term, can be modeled as an exponential distribution with mean of near 10 percent. This suggests that meteor scatter communication testing to evaluate performance should be conducted for 9 to 12 days at a minimum to approximate medium term, (30 to 60 days) day-to-day arrival rate variation.
Figure 2. Receiver Noise Power in 100 Hz Bandwidth, High Latitude Link during April 1992

Figure 3. Receiver Noise Power in 100 Hz Bandwidth, Mid Latitude Link during April 1992
Figure 4. Receiver Noise Power in 100 Hz Bandwidth, High Latitude Link during May 1992

Figure 5. Receiver Noise Power in 100 Hz Bandwidth, Mid Latitude Link during May 1992
Figure 6. Sporadic E Duty Cycle above -126 dBm RSL, High Latitude Link during April 1992

Figure 7. Sporadic E Duty Cycle above -126 dBm RSL, Mid Latitude Link during April 1992
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Figure 15. K-S Test, High Latitude Data Set, 95 Percent Confidence Level
Figure 16. K-S Test, Mid Latitude Data Set, 95 Percent Confidence Level

Figure 17. High and Low Ratio of True Mean to Sample Mean vs the Number of Days Tested for 90 Percent and 95 Percent Confidence Levels
References


