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Accurate solids modeling plays an important role in the design and analysis of spin and fin stabilized projectiles. In particular, accurate knowledge of a projectile's mass, center of gravity, and its axial and transverse moments of inertia are important for stability and trajectory calculations. In practice, however, error sources exist with existing solids modeling techniques with respect to the determination of the mass properties of internal and external radii (ogives), which until now could not be calculated with a closed-form solution. To address this issue, the generalized closed-form solution for the mass, axial, and transverse moments of inertia, and the center of gravity for radii is developed and the definite integrals presented. The resulting solutions are surprisingly simple to incorporate into a solids modeling package, and the input parameters are few and standardized for all forms of radii, both internal and external.

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Improved Solids Modeling for Axisymmetric Projectile Design

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1. BACKGROUND

Accurate solids modeling plays an important role in the design and analysis of spin and fin stabilized projectiles. In particular, accurate knowledge of a projectile's mass, center of gravity, and its axial and transverse moments of inertia are important for stability and trajectory calculations. Historically, solids modeling has been performed in two manners: 1) constructing the projectile thickness profile with basic axisymmetric components such as cones, cylinders, frustrums, and radii (or ogives), then having their individual mass properties calculated and summed for the entire structure; and 2) constructing a two dimensional finite element model of the structure, and then summing the mass properties of the individual quadrilateral or triangular elements of revolution. In practice, however, error sources exist with these methods with respect to the determination of the mass properties of radii, which until now could not be calculated with a closed-form solution.

Both of the above techniques rely on finite difference or finite element approaches to the determination of the mass properties of radii. Finite difference solids modeling of radii or ogives using projectile profile construction have traditionally broken the curved outline into many uniform finite segments, typically 100, which are then treated as individual frustrums, for which a closed-form solution does exist. This is the technique used in solids modeling subroutines in projectile design packages such as PRODAS[1] and CPPAC.[2] Finite element modeling is effectively the same technique, since the outline of the structure is a finite number of straight lines.

In the past, no closed-form integral existed for the transverse moment of inertia for an ogive, other than for a simple hemisphere. In addition, those definite integrals which do exist for the radius element

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[1] J. Burnett, W. Hathaway, and R. Whyte, *Projectile Design and Analysis System (PRODAS)*, AFATL-TR-81-43, Armament and Electrical Systems Department, General Electric Company, Burlington, VT. April 1981.

[2] *Computerized Projectile Performance Analysis code (CPPAC)*, Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland. 1986.

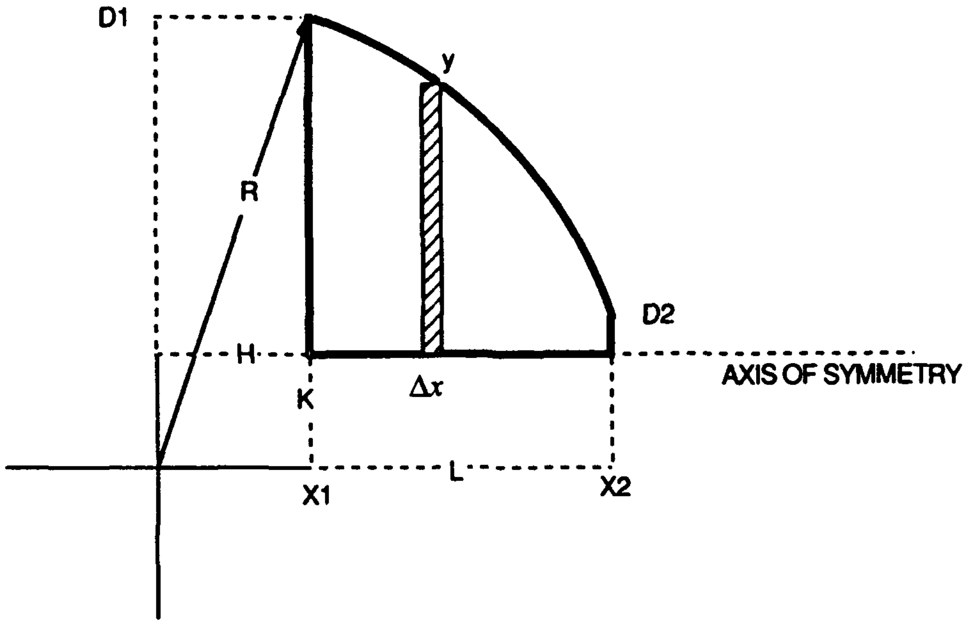
mass and axial moment of inertia are restrictive, complicated and seldom employed. Therefore, when using existing solids modeling techniques, the designer has no knowledge of the magnitude of the error with respect to the true closed-form value. In addition, no mathematical error analysis has been developed to show that by increasing the number of finite elements or discrete segments in the model construction the value converges to the closed form solution.

To address this issue, the generalized closed-form solution for the mass, axial, and transverse moments of inertia, and center of gravity for an external and internal radius or ogive segment of a projectile is developed. In addition, these simplified equations are incorporated into a generalized computer program to determine the mass properties of complex projectile shapes. This report only presents the unique development of solids modeling solutions for internal and external radii, since solutions for cones, cylinders, and frustrums are well known and published in many standard mathematical handbooks.

2. DEVELOPMENT OF THE INTEGRAL

Figure 2.1 shows the geometric construction of the axisymmetric integral for a generic internal radius or ogive.

Figure 2.1
Geometric Construction of the Integral



In Figure 2.1 the center of the radius of curvature is located a distance *H* to the left of the left face of the radius element and a distance *K* below the axis of symmetry for this element. At the left diameter *D1*, the radius intersects at a secant point. In addition, since the radius center is below the axis of symmetry, the right diameter *D2* also intersects at a secant point. The cross-hatched area represents a finite mass of material defined by:[3]

$$R^2 = x^2 + y^2$$
$$y = \sqrt{(R^2 - x^2)}$$

$$MASS_i = \rho\pi\left(\sqrt{(R^2 - x^2)} - K\right)\Delta x \qquad (1)$$

3 R. Hudson, *The Engineers' Manual.* John Wiley and Sons, Inc., New York. Equations **(1)**, **(2)**, and **(3)** are developed from this reference.

This finite mass of material has a moment of inertia about the symmetrical axis of:

$$IZZ_i = \frac{1}{2} MASS_i \left(\sqrt{\left(R^2 - x^2 \right)} - K \right)^2 \tag{2}$$

In addition, its transverse moment of inertia about the left face of the radius element is:

$$IXX_i = \frac{1}{12} MASS_i \left[3 \left(\sqrt{\left(R^2 - x^2 \right)} - K \right)^2 + (\Delta x)^2 \right] + MASS_i \left(x_i - H \right)^2 \tag{3}$$

In order to eventually determine the center of gravity of this radius element, and to transform the transverse moment of inertia from the left face to the center of gravity, the first moment of this finite mass also needs to be determined:

$$MOM_i = x_i MASS_i \tag{4}$$

Following this, the center of gravity for the element becomes:

$$Cg = \frac{\sum MOM_i}{\sum MASS_i} \tag{5}$$

3. GENERAL MASS PROPERTY INTEGRALS

Based on the geometric construction, the following integrals may be written to define the total mass properties of the radius element in question.

$$MASS = \int_{x_1}^{x_2} \rho\pi\left(\sqrt{(R^2 - x^2)} - K\right)^2 dx \tag{6}$$

$$IZZ = \int_{x_1}^{x_2} \frac{1}{2}\rho\pi\left(\sqrt{(R^2 - x^2)} - K\right)^4 dx \tag{7}$$

$$IXX = \int_{x_1}^{x_2} \frac{1}{4}\rho\pi\left[\left(\sqrt{(R^2 - x^2)} - K\right)^4 + \left(\sqrt{(R^2 - x^2)} - K\right)^2 (x - H)^2\right] dx \tag{8}$$

$$MOM = \int_{x_1}^{x_2} \rho\pi\left(\sqrt{(R^2 - x^2)} - K\right)^2 (x - H) dx \tag{9}$$

$$CG = \frac{MOM}{MASS} \tag{10}$$

4. THE DEFINITE INTEGRALS

The above integrals were solved, given the following geometric conditions:

R = radius of curvature

H = axial offset of the radius center from the element left face

K = radial offset of the radius center from the axis of symmetry

$x_1 = H$

$x_2 = H + L$

A. The Definite Integral for the Radius Element Mass

$$MASS = \rho\pi\left(MASS(x_2) - MASS(x_1)\right)$$

where:

$$MASS(x) = R^2 x - {x^3}/{3} - 2K\left(x\frac{\sqrt{(R^2 - x^2)}}{2} + \frac{R^2}{2}\sin^{-1}\left(\frac{x}{R}\right)\right) + K^2 x \tag{11}$$

B. The Definite Integral for the Radius Element Axial Moment of Inertia

$$IZZ = \rho\frac{\pi}{2}\left(IZZ(x_2) - IZZ(x_1)\right)$$

$$IZZ(x) = R^4x - \frac{2}{3}R^2x^3 + \frac{x^5}{5} + K^4x + 6K^2\left(R^2x - \frac{x^3}{3}\right)$$
$$-4K^3\left(x\frac{\sqrt{(R^2-x^2)}}{2} + \frac{R^2}{2}\sin^{-1}\left(\frac{x}{R}\right)\right) - 4K\left(\frac{x}{4}(R^2-x^2)^{1.5} + \frac{3R^2x\sqrt{(R^2-x^2)}}{8}\right) \qquad (12)$$
$$-4K\left(\frac{3}{8}R^4\sin^{-1}\left(\frac{x}{R}\right)\right)$$

C. The Definite Integral for the Radius Element Transverse Moment of Inertia

$$IXX = \rho\pi\left(IXX(x_2) - IXX(x_1)\right)$$

$$IXX(x) = \left(-2H^2K - K^3\right)\left(\frac{x}{2}\sqrt{(R^2-x^2)} + \frac{R^2}{2}\sin^{-1}\left(\frac{x}{R}\right)\right)$$
$$-K\left[\frac{x}{4}(R^2-x^2)^{1.5} + \frac{3}{8}R^2x\sqrt{(R^2-x^2)} + \frac{3}{8}R^4\sin^{-1}\left(\frac{x}{R}\right)\right]$$
$$-2K\left[\frac{-x}{4}(R^2-x^2)^{1.5} + R^2\frac{x}{8}\sqrt{(R^2-x^2)} + \frac{R^4}{8}\sin^{-1}\left(\frac{x}{R}\right)\right] \qquad (13)$$
$$-\frac{4}{3}HK(R^2-x^2)^{1.5} - \frac{3}{20}x^5 + \frac{1}{2}Hx^4$$
$$+x^3\left(\frac{R^2}{6} - \frac{K^2}{6} - \frac{H^2}{3}\right) - x^2\left(HR^2 + K^2H\right)$$
$$+x\left(\frac{R^4}{4} + R^2\left(1.5K^2 + H^2\right) + \frac{K^4}{4} + K^2H^2\right)$$

D. The Definite Integral for the Radius Element First Moment

$$MOM(x) = 2KH\left(\frac{x}{2}\sqrt{\left(R^2 - x^2\right)} + \frac{R^2}{2}\sin^{-1}\left(\frac{x}{R}\right)\right)$$
$$+\frac{2}{3}K\left(R^2 - x^2\right)^{1.5} + \frac{x^2}{2}\left(R^2 + K^2\right) - \frac{x^4}{4}$$
$$+H\frac{x^3}{3} + xH\left(-R^2 - K^2\right) \tag{14}$$

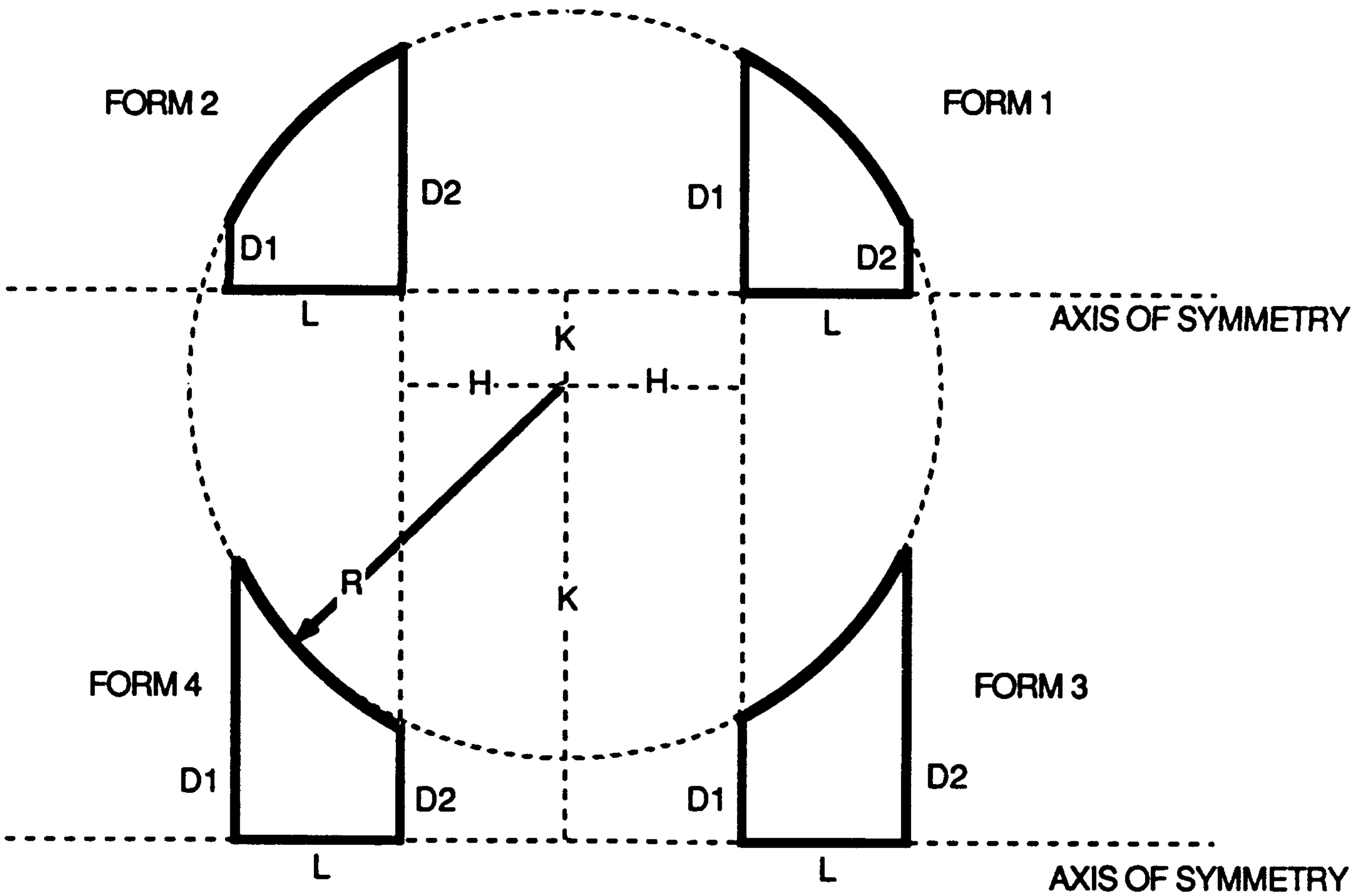
and:

$$X_{cg} = \frac{\rho\pi\left(MOM(x_2) - MOM(x_1)\right)}{MASS} \tag{15}$$

5. GENERAL SOLUTIONS FOR ALL FORMS OF INTERNAL AND EXTERNAL RADII

Figure 5.1 shows that four distinct forms of internal and external radii exist, including the one used to develop the closed-form solution (Form 1). As it turns out, the above definite integrals are valid for all four cases, provided the geometric inputs are defined accordingly, as shown in Figure 5.1. The rules for determining the mass properties of all of these forms follows.

Figure 5.1
Generalized Input Parameters for all Radii



A. Form 1

1. The value of H is always positive.

2. The value of K is positive only when the center of the radius of curvature is below the axis of symmetry. Enter K as negative if the radius center is above the axis of symmetry.

B. Form 2

1. The value of H is always positive, and is now the distance to the right face of the element.

2. The value of K is positive only when the center of the radius of curvature is below the axis of symmetry. Enter K as negative if the radius center is above the axis of symmetry.

3. Since this form is the mirror image of Form 1, the center of gravity for this element from its left face is determined by subtracting the calculated center of gravity using the above integral from the element length L.

C. Form 3

1. H is always positive.

2. K is always positive and is always above the axis of symmetry.

D. Form 4

1. The value of H is always positive, and is now the distance to the right face of the element.

2. K is always positive and is above the axis of symmetry.

3. This form is the mirror image of Form 3, subtract the calculated center of gravity from the element length.

6. SOME EXAMPLE CALCULATIONS

Figure 6.1 shows the cross-section of eight axisymmetric radius or ogive elements drawn with the software developed under this project. Table 6.1 shows the geometric inputs for each element according to the rules defined above. At the bottom of this table are the calculated closed-form mass properties for these eight elements, as well as the cumulative properties if all of these elements were connected.

Figure 6.1
Cross-Sections of Example Radii

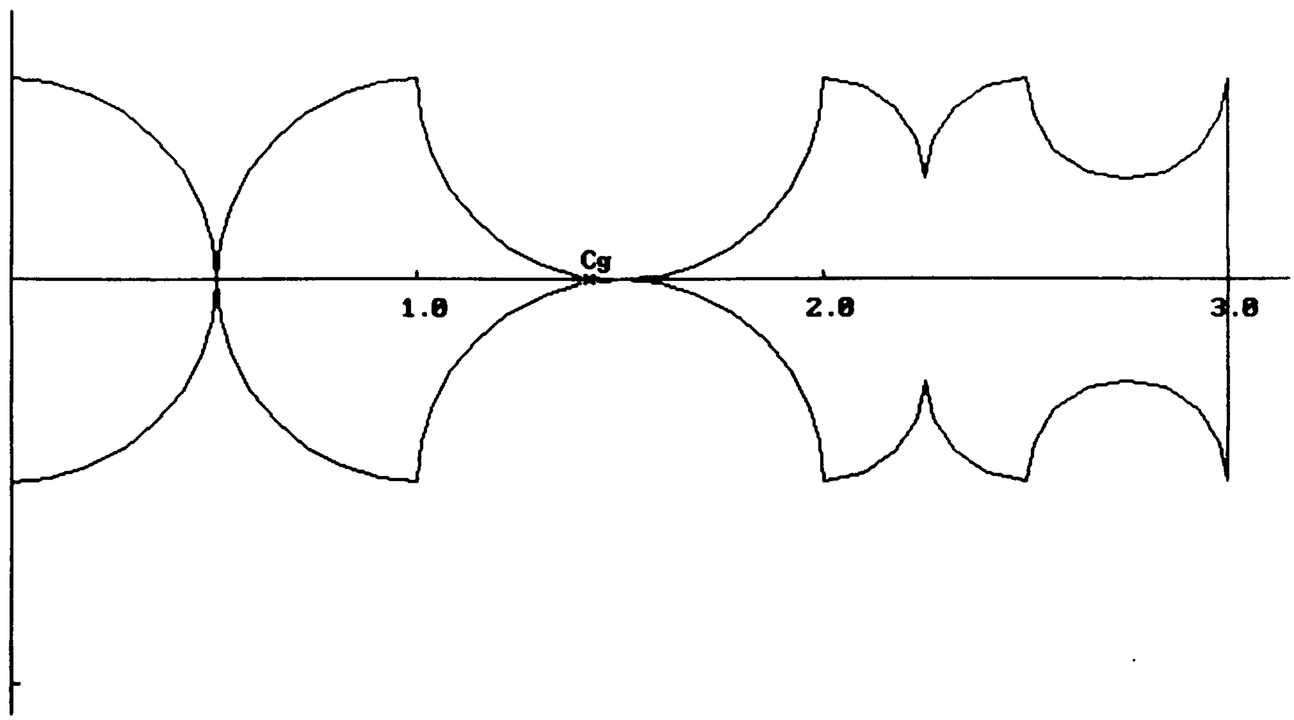


Table 6.1
Geometric Inputs and Mass Properties of Example Radii

ELEM	D1	D2	L	Rho	FORM	R	H	K
1	1.0000	.0000	.5000	10.0000	41	.5000	.0000	.0000
2	.0000	1.0000	.5000	10.0000	42	.5000	.0000	.0000
3	1.0000	.0000	.5000	10.0000	44	.5000	.0000	.5000
4	.0000	1.0000	.5000	10.0000	43	.5000	.0000	.5000
5	1.0000	.5000	.2500	10.0000	41	.2500	.0000	-.2500
6	.5000	1.0000	.2500	10.0000	42	.2500	.0000	-.2500
7	1.0000	.5000	.2500	10.0000	44	.2500	.0000	.5000
8	.5000	1.0000	.2500	10.0000	43	.2500	.0000	.5000

ELEM	MASS (lb)	Izz (axi) (lb-in^2)	Ixx (trans)	Cg
1	2.6180	.2618	.1698	.1875
2	2.6180	.2618	.1698	.3125
3	.3765	.0174	.0102	.0654
4	.3765	.0174	.0102	.4346
5	1.5892	.1692	.0920	.1094
6	1.5892	.1692	.0920	.1406
7	.7486	.0412	.0246	.0997
8	.7486	.0412	.0246	.1503

TOTAL MASS	Izz,g (axi)	Ixx,g (trans)	Cg (from left)
10.6646	.9794	10.6164	1.4080

Figure 6.2 shows the cross-section of a complex axisymmetric projectile component, developed with this software. This component contains nine discrete elements, one of which is an internal radius. The mass properties of each of these elements and those of the combined structure are presented in Table 6.2

Figure 6.2
Cross-Section of Example Projectile Component

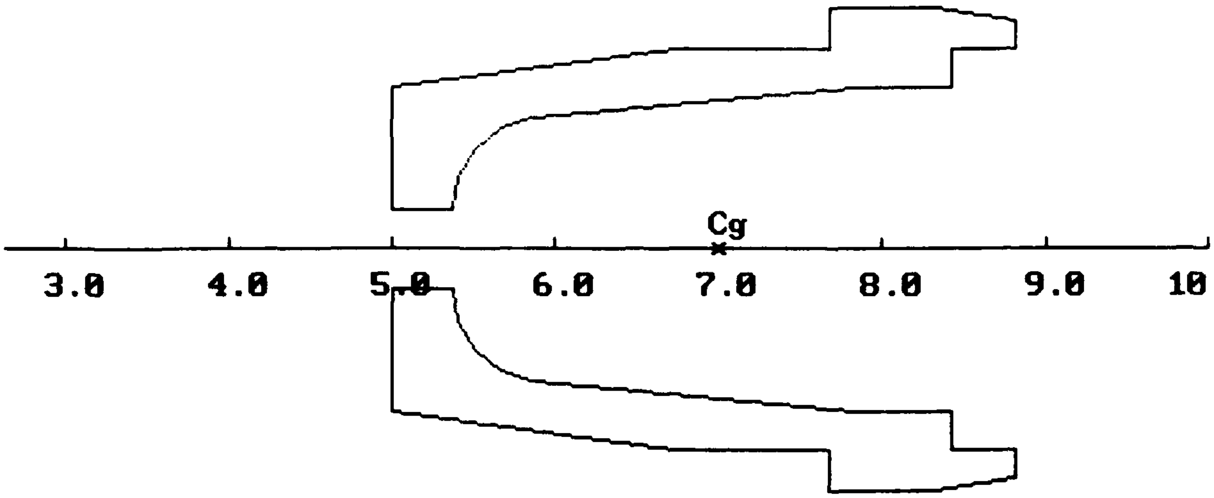


Table 6.2
Geometric Inputs and Mass Properties of Example Component

AFT CAVITY
5.0000
4 5
.2810

ELEM	D1	D2	L	Rho	FORM	R	H	K
1	2.0000	2.5000	1.7500	.2810	3	.0000	.0000	.0000
2	2.5000	2.5000	.9375	.2810	2	.0000	.0000	.0000
3	3.0000	3.0000	.6250	.2810	2	.0000	.0000	.0000
4	3.0000	2.8438	.5000	.2810	3	.0000	.0000	.0000
5	.5000	.5000	.3750	-.2810	2	.0000	.0000	.0000
6	.5000	1.6163	.4927	-.2810	42	.5625	.0698	-.2500
7	1.6163	2.0000	2.0000	-.2810	3	.0000	.0000	.0000
8	2.0000	2.0000	.5625	-.2810	2	.0000	.0000	.0000
9	2.5000	2.5000	.3823	-.2810	2	.0000	.0000	.0000

ELEM	MASS (lb)	Izz (axi) (lb-in^2)	Ixx (trans)	Cg
1	1.9633	1.2679	1.1285	.9395
2	1.2931	1.0103	.5998	.4688
3	1.2414	1.3966	.7387	.3125
4	.9423	1.0068	.5230	.2455
5	-.0207	-.0006	-.0006	.1875
6	-.2049	-.0534	-.0301	.2912
7	-1.4485	-.6031	-.7786	1.0705
8	-.4966	-.2483	-.1372	.2813
9	-.5273	-.4120	-.2124	.1911

TOTAL MASS	Izz,g (axi)	Ixx,g (trans)	Cg (from left)
2.7421	3.3642	5.1451	1.9582

7. SUMMARY AND CONCLUSIONS

Although solving the definite integrals for the closed-form mass properties of radii is a laborious process, the results appear to be useful. The geometric construction for each internal and external radius is standardized and simple, requiring only the two extra inputs, *H* and *K*, to locate the radius center. One may also have observed that in these calculations, the left diameter *D1* and the right diameter *D2* were never used. As it turns out, these values are largely redundant given knowledge of *H* and *K* and the element length *L*, since only two possible radii elements can exists for each combination of these three numbers. To distinguish between the differences between the external radii, Forms 3 and 4, from the internal radii, Forms 1 and 2, simply adopt the convention that a negative *R* is an external radius, and proceed to calculate *H* and *K* internal to the software routine. Therefore, if one wishes to avoid entering *H* and *K*, they may be calculated in a straight-forward manner using the standard inputs of *D1*, *D2*, *L* and +-*R*. In addition, one set of equations has been shown to be valid for all four radii geometries. This further simplifies incorporation of these equations into a generic solids modeling routine. The mathematical approach developed here is also applicable to more complex axisymmetric shapes such as parabolic and hyperbolic curves. One only needs to substitute these equations for the circular cross-section used in these integrals to develop those closed-form solutions. One would expect the resulting equations to be equally standardized and simple to incorporate into a solids modeling package.