### Abstract (Maximum 200 words)

Scattering from the seafloor is often assumed to be controlled by two spatial scales: the larger scale associated with reflections from plane facets and the smaller one associated with diffuse scattering from height variations. Choosing the wavenumber for this partitioning has proven to be important but troublesome. For this work scattering data is simulated using Helmholtz-Kirchhoff or physical optics theory and selected input geomorphology. These data are inverted using a simpler two-scale roughness model. The work introduces a new criterion for effecting this partition based on setting a roughness parameter equal to unity. The criterion is shown to be valid for the cases analyzed based on the ability of the inversion model to cover the input geomorphology.
Estimating Geomorphology and Setting the Scale Partition with a Composite-Roughness Scattering Model

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I. INTRODUCTION

Among the approaches to the study of rough surface scattering two-scale roughness (TSR) theory, also called composite-roughness theory, is particularly attractive because it separates scattering into what appears to be its two primary mechanisms: scattering controlled by large random facets and scattering controlled by small roughness [1,2]. The reported advantage of this approach is that its applicability can span a wide range of surface roughness, since it allows for the selective application of appropriate approximations in scattering theory. While this is certainly true, it leads one to attempt to answer the troublesome question of where to set the scale partition based on the requirement for the validity of the approximations. The question should rather be posed in terms of "At which scales is the scattering like reflection from a collection of facets and at which scales is scattering like diffraction from roughness?". The goal of this work is to allow the scattering phenomenon itself to suggest what should be the scale partition.

II. APPLICABLE SCATTERING MODELS

Here we introduce the term Rough Facet Model (RFM) to refer to a part of a broader set of programs called the Bistatic Scattering Strength Model (BISSM) [3]. RFM is that part that is the mathematical description of the scattering mechanism that includes a product between a factor, F(e), describing fine-scale slope scatter and a factor, M(e), for scatter from microroughness. RFM allows inversion of the scattering problem and, thereby, the extraction of slope and height roughness for the scattering surfaces. Since no reliable experimental data are available, scattering data are simulated using a high-fidelity, 2-D scattering model and selected input seafloor morphology. This high-fidelity model is based on Monte Carlo of evaluation of the Helmholtz-Kirchhoff (H/K) integral over an ensemble of surfaces with the desired statistical properties. RFM predictions are fitted to results from the H/K simulations and the scale partition is then established when successful recovery of the correct geomorphology is attained.

The RFM algorithm gives backscatter at near normal incidence as

\[ m_{bs} = M(k, \sigma_\mu, \theta) \cdot F(\delta_f, \theta) \]  

(1)

where \( k \) is the acoustic wavenumber; \( \sigma_\mu \) is the standard deviation of the microroughness; \( \delta_f \) is the standard deviation of the fine-scale slopes; and \( \theta \) is the angle of incidence measured from normal (zenith angle). For convenient visualization we define and refer to the slope angle \( \alpha_f = \tan^{-1} \delta_f \).

The coherent reflection coefficient is given by

\[ M = \exp[-g_\mu] \]  

(2)

where \( g \) is the well known roughness parameter restricted here to the microroughness, i.e., \( g_\mu = 4\sigma_\mu^2 k^2 \cos^2 \theta \).
The function \( F(\cdot) \) is the high-frequency limit of the H/K theory in the form derived Brekhovskikh and Lysanov [41]:

\[
F = \frac{R_0^2}{8\pi \delta \cos \theta} \exp \left( \frac{-\tan^2 \theta}{26 \delta^2} \right)
\]  

(3)

where \( R_0 \) is the Rayleigh reflection coefficient. For this work \( R_0 = 1 \).

To simulate scattering data we use a model that consists of evaluating numerically the full Helmholtz-Kirchhoff integral with a minimum number of approximations. A complete description of the technique can be found in ref. 5.

### III. NUMERICAL SIMULATIONS

Although this is a purely numerical study, a practical scenario based on a multibeam, swath bathymetric sonar system is adopted here because it will allow easy testing of the results and because we envision that system as a major application of these results once they are validated. However, the scenario is also applicable to the low-frequency active sonar system used in the Acoustic Reverberation Special Research Program (ARSRP) of the Office of Naval Research (ONR). We apply it here at frequencies more relevant to ARSRP data--frequencies equal to 250, 500, 1000, and 2000 Hz. The water depth is chosen to be 2000 m and the beamwidth is two degrees. Zenith angles are set at 3 to 43 degrees in two-degree increments.

Ensembles of 2-D (i.e., surface heights are function of \( x \) and \( y \)) randomly rough surfaces were generated using a filtering technique developed by Caruthers and Novarini [6,7]. Amplitude spectra used to describe seafloor roughness is of the isotropic power-law form, \( A(K) = a K^{-b} \), where \( K \) is the wavenumber of the surface roughness (in cycles/meter).

It should be noted that, in the RFM scattering model, the two surface parameters (the rms height of the micro-scale surface and the rms slope of the fine-scale surface) are band limited quantities. The variance of the microroughness for a 2-D isotropic surfaces is given by

\[
\sigma^2_{\mu} = 2\pi \int_{K_c}^{K_h} A^2(K) K dK
\]  

(4)

The variance of the slopes of the fine-scale surface by

\[
\delta_i = 2\pi \int_{K_c}^{K_i} A^2(K) K^3 dK
\]  

(5)

Table I shows the spectral parameters \( a \) and \( b \) for the surfaces used in this work. Given in the table are \( \sigma_T \), the total rms height (i.e., presuming that all the deformation of the surface is considered to be microroughness) and \( \delta_T \), the total rms slope (i.e., presuming that all the deformation of the surface in attributed to slope variance). Also given in the table is the slope angle \( \alpha_T \).

The dimensions for each of the surfaces is 256 by 256 meters (1024 x 1024 grid points, with a grid spacing of 0.25 m). Each surface realization was split into a number of ensonified regions for ensemble averaging purposes. The number of subregions depends on geometry of the simulation and the zenith angle of incidence.

### IV. RESULTS

Here we hypothesize that the scattering phenomenon itself will establish a facet size such that the microroughness that exists upon it leads to \( g_{\rho} \approx 1 \). Larger facets contain too much roughness to produce a coherent reflection. Smaller facets are not the full extent of the region that can cause a coherent reflection. Therefore, the partition wavenumber can be determined from Eqs. (4) and \( \delta_i \) determined from Eq. (5). For
simplicity we will approximate $g_\mu$ for all angles (0 to 45 deg) by its value at $\Theta = 0^\circ$, i.e., $g_{\mu_0} = 4\mu_0^2k^2 = 1$. Scattering described by RFM is strongly dependent on the spectrum slope. And second, the best fit occurs at about $b = 2.14$. This departure from the expected $b = 2.27$ is not particularly troublesome because spectral analysis of the actual surface realizations resulted in spectral slope values between 2.10 and 2.35. Examination in greater detail bears out this best fit value for $b$ and also confirms that $g_{\mu_0}$ is precisely 1.0.

To test the robustness of the criterion the RFM was run for surface 0D at three other frequencies (250 Hz, 1 kHz and 2 kHz). In each case there is a narrow range of $b$ values that fit the H/K curve within 3 dB. The value

Fig. 1. Comparison of the RKM and H/K results at 500 Hz for surface 0D for different values of spectral slope $b$ assuming that $g_{\mu_0} = 1.0$.

Fig. 2. Comparison of the RKM and H/K results at 250 Hz for surface 0D assuming that $g_{\mu_0} = 1.0$ and $b = 2.14$.

Fig. 3. Comparison of the RKM and H/K results at 1 kHz for surface 0D assuming that $g_{\mu_0} = 1.0$ and $b = 2.14$.

Fig. 4. Comparison of the RKM and H/K results at 2000 Hz for surface 0D assuming that $g_{\mu_0} = 1.0$ and $b = 2.14$. 
TABLE I: BAND-LIMITED RMS ROUGHNESS FOR SURFACE 0D FOR $g_{0d} = 1.0$ CORRESPONDING TO THE BEST FITTING PARAMETERS $a = 6.10^3$ AND $b = 2.14$.

<table>
<thead>
<tr>
<th>$g_{0d}$</th>
<th>$a$</th>
<th>$b$</th>
<th>$a_1$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 Hz</td>
<td>0.17</td>
<td>0.23</td>
<td>0.12</td>
</tr>
<tr>
<td>500 Hz</td>
<td>0.27</td>
<td>0.33</td>
<td>0.14</td>
</tr>
<tr>
<td>1 kHz</td>
<td>0.37</td>
<td>0.43</td>
<td>0.15</td>
</tr>
<tr>
<td>2 kHz</td>
<td>0.47</td>
<td>0.53</td>
<td>0.16</td>
</tr>
</tbody>
</table>

V. CONCLUSIONS

The partition wavenumber between the two-scales of roughness in composite-surface scattering theory for surfaces having power-law spectra, typical of the seafloor, has been investigated through numerical simulation. Results indicate that a criterion based on the smoothness of the micro-scale component may be adequate to establish the partition. We find that the partition wavenumber appears to be set by the physics scattering to be fairly precisely $g_{0d} = 1.0$. We introduced the hypothesis that a facet will be formed such that its roughness is approaching a value that begins to destroy the coherence of scatter from the facet. Larger facets are not allowed because coherent reflection is not allowed. Smaller facets are not allowed because there still could be a coherent reflection from a large one. For the cases analyzed here the hypothesis appears the be valid.

REFERENCES


