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Haversian compact bone tissue is being studied theoretically and experimentally to characterize its behavior as a fiber reinforced composite material with the aim of biomimicking its salient features. Analytical and finite element based composite micromechanics techniques have been developed and evaluated for theoretical modeling of structure/property relationships. Two-dimensional finite element models of material representative volume elements have been used to predict variations in macroscopic mechanical properties with porosity, constituent properties, and fiber/matrix interface conditions. Haversian compact bone possesses a distinct 'interphase' material surrounding each osteon called the cement line. The role and properties of the cement line are of particular interest for possible biomimicking. Mechanical testing of bone specimens is accompanying the theoretical work to provide data for verification and model improvement. The experimental protocol has been established and preliminary testing is underway. Specimens are submitted to the subcontractor following testing for microstructural characterization. The microstructural parameters of interest are the density (wet & dry), percent porosity, osteon density, cement line density, and percent ash content (mineralization). Three-dimensional image reconstruction of Haversian canals, which are the main sources of porosity, has also been initiated in order to provide a more comprehensive measure of void space characteristics.
TECHNICAL PROGRESS REPORT

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Dr. Walter F. Jones, Program Manager

MICROMECHANICS MODELING AND ANALYSIS OF HAVERSIAN COMPACT BONE TISSUE AS A FIBER REINFORCED COMPOSITE MATERIAL

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INTRODUCTION

This research is part of the recent AFOSR initiative in Biomimetics, which has the broad overall goal of seeking to elucidate and understand the salient characteristics of naturally occurring biological composite materials with an aim toward mimicking these features in devising improved man-made composite materials. The biological composite of interest in this effort is bone tissue, or more specifically Haversian compact bone tissue. The main objective of the project is to develop a deterministic understanding of the relationships between microstructural details and macroscopic mechanical behavior. The unique fiber/matrix interphase material, which is distinctive of this type of bone tissue, is of particular interest because of the recognized significance of interfaces in developing new higher performance composites. The methodology combines composite micromechanics modeling techniques to analytically model structure/property relationships with companion experimental testing of tissue samples to evaluate and validate the modeling.

The specific aims of the first year of the project were:

- To develop and evaluate a microstructural model of Haversian cortical bone tissue which includes major microstructural features (e.g., Haversian porosity, a distinct cement line interphase material, interface conditions between the cement line and the osteon and interstitial bone matrix).

- To establish the appropriate experimental protocol and procedures for measuring macroscopic mechanical properties. {Preliminary testing will include both direct mechanical tensile testing of bone specimens as well as ultrasound analysis.}

- To investigate various options for quantitative characterization of microstructural details such as porosity (Haversian, lacunal and other sources), osteon density, cement line density, percent mineralization, and overall wet and dry densities.

RESEARCH DESCRIPTION

Background

The particular type of bone tissue being studied in this research is Haversian compact bone tissue because its microstructure resembles that of a fiber reinforced composite (Fig. 1). The fibers in this case, however, are hollow, layered, and surrounded by a distinct "interphase" fiber/matrix material, known as the cement line. The hollow fibers are called Haversian systems, or secondary osteons, and are not perfectly straight but in fact spiral slightly and branch. The "matrix" material is interstitial bone lying between complete osteons. Interstitial bone is actually composed of remnants of resorbed osteons and is therefore similar in composition and properties to the "fiber" component. The cement line is an amorphous substance that is typically considered to be very compliant or even viscous. Compact bone tissue is the more dense material found in the mid-shaft (diaphysis) of major load-carrying bones of humans and many mammals, as opposed to the less dense 'spongy' trabecular bone in the bulbous ends of these bones.
Figure 1. Haversian compact bone microstructure.
(a.) idealized sectional view [adapted from Martin (1984)]
(b.) photomicrograph of cross-section [after Martin and Burr (1989)]
Micromechanics Modeling

Finite element unit cell analysis. A two-dimensional finite element model assuming hexagonal packing was developed and results compared with a previous study assuming square packing (Hogan, 1992). The idealized microstructure and unit cell are presented in Fig. 2. A typical mesh generated for this case is shown in Fig. 3. The solid modeling package PATRAN was used in conjunction with the ABAQUS finite element solver to analyze this model. The mechanical properties of each constituent are summarized in Table 1. Table 2 compares the hexagonal array results with those generated using a square array. Note that $E_{11}$ is the longitudinal, or axial, modulus whereas $E_{22}$ and $E_{33}$ are moduli in the transverse directions. The Haversian porosity in this case is 3%. (The results are essentially the same for hexagonal and square packing with the exception of a slight discrepancy for $E_{22}$.) In addition, hexagonal packing offers the advantage of being able to model higher fiber volume percentages, so it has been chosen as the method for all subsequent finite element unit cell modeling.

Figure 2. Idealized microstructure of Haversian compact bone.

Figure 3. Finite element mesh analyzed.
Table 1 - Constituent properties used in micromechanics model

<table>
<thead>
<tr>
<th>Constituent</th>
<th>E (GPa)</th>
<th>v</th>
</tr>
</thead>
<tbody>
<tr>
<td>Osteon</td>
<td>12.0</td>
<td>0.3</td>
</tr>
<tr>
<td>Cement line</td>
<td>6.0</td>
<td>0.4</td>
</tr>
<tr>
<td>Interstitial bone</td>
<td>15.0</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table 2 - Macroscopic properties for hexagonal vs. square packing geometry

<table>
<thead>
<tr>
<th>Array</th>
<th>$E_{11}$</th>
<th>$E_{22}$</th>
<th>$E_{33}$</th>
<th>$v_{12}$</th>
<th>$v_{23}$</th>
<th>$v_{21}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square</td>
<td>12.3</td>
<td>11.7</td>
<td>11.7</td>
<td>0.30</td>
<td>0.30</td>
<td>0.29</td>
</tr>
<tr>
<td>Hexagonal</td>
<td>12.3</td>
<td>11.3</td>
<td>11.3</td>
<td>0.30</td>
<td>0.31</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Note: elastic moduli are in GPa

The Haversian porosity is a parameter which greatly influences the macroscopic material properties of bone and is considered an important variable in micromechanics modeling. In the finite element model, the Haversian porosity is altered by changing the diameter of the Haversian canal, i.e. the inner radius of the hollow fiber. Porosity variations from 3% to 9%, which are reasonable values found in the literature, have been studied. Table 3 details the influence of porosity on transverse and longitudinal moduli as well as Poisson's ratio. The results are as expected: the moduli generally decrease with increasing porosity, and the transverse moduli are more sensitive to porosity variations.

Table 3 - Effect of porosity on macroscopic properties (Square array)

<table>
<thead>
<tr>
<th>Porosity (%)</th>
<th>$E_{11}$</th>
<th>$E_{22}$</th>
<th>$v_{12}$</th>
<th>$v_{23}$</th>
<th>$v_{21}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>12.3</td>
<td>11.7</td>
<td>0.30</td>
<td>0.30</td>
<td>0.29</td>
</tr>
<tr>
<td>5</td>
<td>12.2</td>
<td>11.1</td>
<td>0.30</td>
<td>0.29</td>
<td>0.28</td>
</tr>
<tr>
<td>7</td>
<td>11.9</td>
<td>10.6</td>
<td>0.30</td>
<td>0.29</td>
<td>0.27</td>
</tr>
<tr>
<td>9</td>
<td>11.6</td>
<td>10.1</td>
<td>0.30</td>
<td>0.28</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Note: elastic moduli are in GPa
Separating the osteon from the interstitial bone is a somewhat mysterious substance known as the cement line. The exact mechanical properties (modulus and Poisson's ratio) of the cement line are not known, even in general, from experimental testing and can only be inferred from indirect testing and from the observed composition of the substance. It is generally believed that the cement line is more compliant (Burr et al., 1988) than either the osteon or the interstitial bone and that it may in fact be viscous in nature (Lakes and Saha, 1979). Moreover, the precise role of the cement line in determining the macroscopic mechanical behavior of compact is not known and much debated. Accordingly, cement line modulus values ranging widely (6.0 to 0.12 GPa, or, 50% to 1% of the osteon modulus) have been modeled and the resultant macroscopic properties documented. The thickness of the cement line was also varied from 1.4μm to 5.0μm since it is reported to be between 1μm and 5μm (Martin and Burr, 1989). The effects of these changes on macroscopic properties are shown in Table 4 (1.4μm cement line) and Table 5. As expected, the cement line affects the transverse moduli much more than the longitudinal moduli. The transverse moduli are particularly sensitive to cement line thickness. Another interesting observation is that these results suggest that the cement line modulus is most likely not as low as 0.12 GPa (i.e. 1% of the osteon modulus), because the macroscopic transverse modulus is lower (6.38 & 3.66 GPa) than generally observed experimentally.

<table>
<thead>
<tr>
<th>E_{cl} (GPa)</th>
<th>E_{11} (GPa)</th>
<th>E_{22} (GPa)</th>
<th>v_{23}</th>
<th>v_{21}</th>
</tr>
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<tbody>
<tr>
<td>6.0 (50%)*</td>
<td>12.3</td>
<td>11.5</td>
<td>0.31</td>
<td>0.28</td>
</tr>
<tr>
<td>3.6 (30%)</td>
<td>12.1</td>
<td>11.3</td>
<td>0.31</td>
<td>0.28</td>
</tr>
<tr>
<td>1.2 (10%)</td>
<td>11.9</td>
<td>10.4</td>
<td>0.33</td>
<td>0.26</td>
</tr>
<tr>
<td>0.12 (1%)</td>
<td>11.6</td>
<td>6.38</td>
<td>0.36</td>
<td>0.16</td>
</tr>
</tbody>
</table>

* percent of osteon modulus

<table>
<thead>
<tr>
<th>E_{cl} (GPa)</th>
<th>E_{11} (GPa)</th>
<th>E_{22} (GPa)</th>
<th>v_{23}</th>
<th>v_{21}</th>
</tr>
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<tr>
<td>6.0 (50%)*</td>
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</tr>
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<td>6.38</td>
<td>0.36</td>
<td>0.35</td>
</tr>
</tbody>
</table>

* percent of osteon modulus
Mori-Tanaka effective medium approach. The Mori-Tanaka method of estimating the overall mechanical properties of a fiber reinforced composite has been extended to allow for porosity by Zhao, et al. (1989). Even though experimental data is limited, reasonable predictions were found for variations in mechanical properties as a function of porosity. The simplest case to consider as a model for Haversian compact bone is that of 'needle-shaped' voids representing the Haversian canal. That is, the voids are long and slender, parallel to one another, and aligned with the overall longitudinal axis. The variation in longitudinal modulus with porosity is linear in this case, however, whereas experimental data suggests a power law relation (Schaffler and Burr, 1988).

More plausible comparisons with experimental data are nevertheless possible by using the Mori-Tanaka method for 3-D randomly oriented spheroidal voids with appropriate aspect ratios. Results are shown in Figure 4 for several aspect ratios. The ratio of the longitudinal modulus to the modulus of the solid phase \( \frac{E_{11}}{E_0} \) is plotted versus porosity. The aspect ratio (\( a \)) of 10 represents a prolate spheroidal void and the other three aspect ratios (.10, .05, .02) represent various oblate spheroidal voids. Prolate voids are more 'rod-like', while the oblate voids are more 'disk-like'. The power-law expression used to fit the experimental data is given by:

\[
E = 33.88(1-P)^{0.92} \text{ (in GPa)}
\]

The curves in Fig. 4 that best fit the data are for oblate voids. This certainly does not describe the shape of the Haversian canal, but the experimental data is for bovine bone, which contains both Haversian and plexiform types of compact bone. No experimental data is currently available for modulus versus porosity for exclusively Haversian compact. The experiments to be conducted during the second year of this research project will provide this data.

Figure 4. Longitudinal modulus vs. porosity.
A more realistic approach would be to model the solid portion of the bone microstructure as distinct components (osteon, cement line, & interstitial bone) rather than as a single phase. The work of Dasgupta and Bhandarkar (1992) considers the Mori-Tanaka technique for the case of a fiber surrounded by one or more 'interphases' embedded in a matrix material. This model has been exploited for the case of Haversian cortical bone by regarding the Haversian canal as the fiber with the osteon and cement line as two interphase materials. Results from this approach are compared with those of the finite element unit cell approach in Fig. 5. The ratios of the longitudinal to transverse moduli are plotted vs. cement line modulus for two cement line thicknesses. The methods two match well for higher cement line moduli, but the Mori-Tanaka approach seriously underestimates the transverse modulus otherwise.

**Experimental Testing**

Specimens and preliminary testing. A working arrangement has been developed between the project and several laboratories in order to establish an experimental protocol for specimen acquisition, storage, machining, and testing. Fresh equine bone samples are obtained from the necropsy lab in the Department of Veterinary Physiology and Pharmacology at Texas A&M University. Sections of the middle portion of the cannon bone are cleaned and cut to lengths of approximately 2½ inches. The specimens are kept frozen until further cutting. They are then rough cut into quarters and placed in Ringer's solution and refrigerated. An Isomet diamond blade wafering saw is used next to cut rectangular slabs (approx. 2"x½"x½"). These slabs are then submitted to the Testing, Maching, and Repair Facility of the Texas Engineering Experiment
Station for machining into standard "dogbone" shaped tensile specimens (see Fig. 6). The machining is done at low speed and constant irrigation with distilled water. Samples of human bone also been acquired from the University of Texas Medical Branch at Galveston under the subcontract. Two sections of tibia and femur, and one radius and ulna have been received.

Once machined, each specimen is stored in a separate vial of Ringer's solution and refrigerated until tested. Testing must follow in less than 90 days or the specimens should be refrozen. Custom extensometers have been designed and fabricated for measuring the extension of the gage section. During preliminary testing, strain gages are also being placed on the front and back of the specimens. The gages are 'T gages' to allow determination of both axial and transverse strains. This permits calculation of longitudinal and transverse elastic moduli as well as Poisson's ratio. Several Instron and MTS testing machines are available for testing the specimens. Four specimens (2 equine and 2 human) have been prepared thus far, and the 2 equine specimens have been tested. Preliminary testing is continuing in order to further streamline the protocol and evaluate the cost/benefit of including strain gages on every specimen.

Figure 6. Specimen dimensions (all in inches).
Image analysis of microstructure. The compositional and microstructural characteristics will be determined quantitatively for each specimen tested. Following testing, a short piece of the gage section will be cut near the fracture surface and a thin section removed for microstructural analysis. The remainder will be used to determine the wet and dry density, and the percent mineralization (ash content). Facilities for computerized image analysis of the specimen microstructure have been obtained by the subcontractor. An Optimas image analysis system was purchased and procedures have been developed for automated, or semi-automated, determination of percent porosity, percent Haversian area, cement line density, and predominant collagen fiber orientation within osteons. This data will be used to correlate with mechanical properties and thereby evaluate the efficacy of the micromechanics modeling efforts.

Work has also been initiated during the first year to characterize additional features of the porosity. The main source of porosity is considered to be the void space of the Haversian canals lying at the center of each osteon. These canals are generally oriented parallel with the main longitudinal axis of the bone, but considerable misalignment and branching actually exists. Thus, a more complete characterization of porosity would include some measure of the architecture of the canals in addition to the mere volume percentage. To this end, 3-D image reconstruction of Haversian systems is being pursued. This procedure involves sectioning of a portion of the specimen near the fracture surface and tracing the paths of several osteons along the length of the specimen. The 3-D computer image can be used to calculate quantitative parameters such as misalignment angle, degree of taper of Haversian canal, and number of branches. Collaboration has been initiated with researchers in the Department of Veterinary Anatomy and Public Health at Texas A&M University to develop and implement the image reconstruction procedures. Microscope slides containing a series of sections spaced 5 μm apart have been made from specimens used in the preliminary testing. Images from the slides have been successfully captured to computer and imported into image analysis software packages. The same procedure will be repeated with human bone specimens before actual reconstruction will be attempted.

Dissemination of Results

The following presentations and publications have resulted from work done during the first year of the project:


Cement line influence - De Frese, R.J. and Hogan, H.A., "A Micromechanics Study of the Influence of the Cement Line on the Anisotropy of Compact Bone Tissue", to be
presented at the 1993 ASME/AICHE/ASCE Summer Bioengineering Conference, June 25-29, 1993, Breckenridge, CO.†


† copies of manuscripts are included in the Appendix

FUTURE WORK

The goals for the second year of the project can be summarized as follows:

1. To continue and complete mechanical testing of human, equine, and possibly bovine, bone tissue for the purpose of establishing a statistically meaningful data base of material properties which can be used to verify micromechanical models. The target number of specimens is 10 to 20 of each specimen type.

2. To quantify the compositional and microstructural characteristics of each specimen tested and correlate this data with macroscopic mechanical properties.

3. To improve the finite element micromechanics modeling by incorporating interface slip between the cement line and the surrounding bone (osteon or interstitial matrix), modeling the cement line as a viscoelastic interphase material, and radially varying the material properties of the osteon (treating it as a composite layup).

4. To continue to investigate other micromechanics approaches such as Mori-Tanaka, self-consistent schemes, and homogenization theory, the latter of which has been used by others (Crolet et al., 1993) recently for heirarchical modeling of cortical bone micromechanics.

5. To initiate ultrasonic testing of the bone specimens with the goal of obtaining transverse elastic moduli that can then also be correlated with microstructural characteristics.

6. To use the micromechanics modeling to study the mechanical role of the cement line and assess its significance and potential as a major biomimicking feature of Haversian cortical bone tissue.
REFERENCES


A COMPARISON OF METHODS FOR INCLUDING POROSITY IN MICROMECHANICS OF COMPACT BONE TISSUE

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INTRODUCTION

The next generation of advanced composite materials will have to meet increasingly severe and demanding performance requirements. Their successful development will therefore require even more rigorous and innovative development procedures. 'Tailoring' microstructural details, such as constituent composition and arrangement, in order to achieve a particular set of macroscopic mechanical properties is one such promising approach. The ultimate success of these and similar efforts, however, depends largely on an in-depth and deterministic knowledge of structure/property relations and their cause and effect mechanisms. In pursuing a greater understanding of these basic principles, a vast resource of valuable information is available through the world of naturally occurring biological composite materials. These highly complex, extremely efficient, and functionally adaptive 'natural' composites include such materials as the intricate layered shells of many marine animals, the fibrous structure of wood and plants, and the highly mineralized vertebrate skeletal system. The focus of the present work is on a particular type of bone tissue, Haversian compact bone, which comprises the dense bone in the mid-shaft region of the long bones of the legs and arms. More specifically, the emphasis is on developing composite material micromechanics modeling and analysis techniques as a mechanism for gaining insight into the fundamental relationships between bone microstructure and macroscopic mechanical behavior.

A section of compact bone in the tubular mid-shaft portion of a femur is shown in Fig. 1(a.). The major microstructural features are included, although somewhat idealized. This bone tissue can be considered to resemble a fiber reinforced composite in many respects but with some important exceptions. In adult human bone, secondary osteons (also called Haversian systems) predominate and are considered to represent the 'fiber' component. The fibers in this case however are hollow, less regularly spaced, and actually composed of several concentric layers of bone material (called lamellar bone). Osteons tend to be roughly aligned with the main axis of the long bone but also spiral somewhat at an angle with this axis. Haversian canals form the hollow centers of osteons, whereas Volkmann's canals tend to traverse radially (or transversely) through the osteons connecting the Haversian canals of adjacent osteons. The 'matrix' is also made up of lamellar bone and...
is denoted interstitial bone. Most interstitial bone consists of remnants from osteons that have been resorbed as the tissue continually replenishes itself with new osteons. Lacunae are small cavities located between layers of lamellar bone which contain bone cells (osteocytes). Lacunae are connected with Haversian and Volkmann's canals by very fine passageways called canaliculae. Mature compact bone is also characterized by a distinct 'interphase' material located at the fiber/matrix interface. This material is an amorphous substance containing minerals, polysaccharides, and collagen and is called the cement line. The precise mechanical role of the cement line is largely unknown. The photomicrograph of Fig. 1(b.) shows a cross-section of Haversian compact bone stained to highlight the cement lines.

Figure 1. The microstructure of compact bone.
(a.) idealized sectional view (after [1])
(b.) photomicrograph of cross-section (after [2])
Several features of Haversian compact bone make it a particularly attractive candidate material for ‘biomimicking’, i.e. trying to identify fundamental characteristics of ‘biological composites’ that can be exploited in developing new man-made composite materials. The mechanical properties of the fiber and matrix components in compact bone are of the same order of magnitude since the materials are quite similar (osteons and interstitial bone). A similar situation exists for many ceramic and metal-matrix composites. Furthermore, a common toughening mechanism for such composites is the creation of weak fiber/matrix interface, whereas this already exists to a certain extent in compact bone. The mechanical significance of the cement line material of compact bone is not fully understood but is believed to play a similar role [3]. Many consider the cement line to be much more compliant than the surrounding bone [4,5] and perhaps even viscous-like [6]. Thus, it acts as a crack arrestor and probably plays a greater role in determining the dynamic behavior of compact bone. The osteons in compact bone are anisotropic due to the orientation of collagen fibers in the concentric layers of lamellar bone, which can be considered to be a functionally graded, or tailored, fiber component. Composite micromechanics modeling is a promising approach for quantifying these effects, but a confounding factor that must be included is porosity, or void space. The mechanical effects of porosity must be adequately accounted for before the advantages of micromechanics modeling can be fully realized for studying further structure/property relations. The potential medical benefits of a valid micromechanics model for compact bone should not be overlooked as well. Such a model would permit prediction of changes in mechanical properties due to mechanical adaptation or pathological conditions such as osteoporosis.

ANALYSIS

Finite Element Unit Cell Approach

A composite material micromechanics model for compact bone has recently been developed and studied [7]. A quarter-fiber finite element micromechanics model for fiber reinforced composites was adapted and modified for this purpose. Alternate configurations have also been investigated [8]. Predictions of macroscopic mechanical properties are reasonable but somewhat difficult to judge since the mechanical properties of the constituents are not well established. The components of the quarter-fiber unit cell are shown in Fig. 2. Porosity due to the Haversian canal is represented explicitly by modeling the fiber component as hollow. The interstitial bone and cement line were treated as isotropic materials with elastic moduli of 15GPa and 6GPa, respectively. The osteon was assigned a longitudinal modulus ($E_{11}$) of 24GPa and a transverse modulus ($E_{22} = E_{33}$) of 12GPa. Poisson’s ratios were taken to be 0.4 for the cement line and 0.3 for the osteon and interstitial bone components.

The unit cell depicted in Fig. 2 was modeled using 2-D continuum finite elements. A generalized plane strain formulation was employed to permit out-of-plane loading and deformation. The displacements of nodes along each of the boundaries of the unit cell were coupled together to require the sides to remain straight and parallel to their original positions. This prevents the material being modeled from forming gaps along these sides. By applying a homogeneous normal stress in the longitudinal direction ($\sigma_{11}$), the overall macroscopic, or effective, longitudinal elastic modulus ($E_{11}$) and Poisson’s ratio ($\nu_{12}$) can be determined by:

$$E_{11} = \frac{\sigma_{11}}{\epsilon_{11}}$$

(1)
The transverse modulus ($E_{22}$) and minor Poisson's ratios ($v_{23}$ and $v_{33}$) can likewise be determined by applying a homogeneous normal stress in the transverse direction ($\sigma_{22}$):

$$E_{22} = \frac{\sigma_{22}}{\varepsilon_{22}}$$  \hspace{1cm} (3)

$$v_{23} = -\frac{\varepsilon_{33}}{\varepsilon_{22}}$$  \hspace{1cm} (4)

The requisite strain components are calculated from the appropriate displacement components of the unit cell boundaries.

Figure 2. Quarter-fiber unit cell for Haversian compact bone.
Mori-Tanaka Method

Zhao, Tandon and Weng [9] have recently applied the Mori-Tanaka method to the analysis of porous materials. The work represents an extension of previous developments for composite materials but the inclusions have been replaced by voids. The voids are considered to be spheroidal in shape, ranging from prolate to spherical to oblate. Expressions are presented for the effective constitutive constants of porous materials in terms of porosity, the aspect ratio of the voids, the properties of the solid phase, and the Eshelby transformation tensor. Several cases of assumed distributions and orientations have been analyzed.

Perhaps the simplest case to consider as a model for Haversian compact bone is that of 'needle-shaped' voids, which is basically the same as a continuous fiber reinforced composite with voids instead of fibers. That is, the voids are long and slender, parallel to one another, and aligned with the overall longitudinal axis. The aspect ratio of the spheroidal voids is taken to be very high creating long extremely prolate voids. The macroscopic behavior is then transversely isotropic and the five independent constants are:

\[
\frac{E_{11}}{E_0} = c_0 \quad (6a)
\]

\[
\frac{v_{12}}{v_0} = 1 \quad (6b)
\]

\[
\frac{G_{12}}{G_0} = \frac{c_0}{1 + c_1} \quad (6c)
\]

\[
\frac{G_{23}}{G_0} = \frac{c_0}{1 + c_1(3 - 4v_0)} \quad (6d)
\]

\[
\frac{\kappa_{23}}{\kappa_0} = \frac{c_0}{1 + c_1(1 - 2v_0)} \quad (6e)
\]

The material properties of the isotropic solid phase are \(E_0, v_0, G_0\), and \(\kappa_0\) which represent the elastic modulus, Poisson's ratio, shear modulus, and plane strain bulk modulus, respectively. The porosity is \(c_1\) (volume fraction of voids) and the volume fraction of solid is \(c_0\). The effective properties are the longitudinal elastic modulus \(E_{11}\), Poisson's ratio \(v_{12}\), shear moduli \(G_{12}\) and \(G_{23}\), and plane strain bulk modulus \(\kappa_{23}\). The transverse elastic modulus \(E_{22}\) is readily calculated from these by:

\[
E_{22} = E_{11} \left[ v_{12}^2 + E_{11} \left( \frac{1}{\mu_{23}} + \frac{1}{\kappa_{23}} \right) \right]^{-1} \quad (7)
\]
Since osteons, and therefore Haversian canals, are not perfectly aligned with the longitudinal axis of the bones, it is also useful to transform the resulting effective moduli through a mild angle (θ) to determine the longitudinal modulus when the axis of the voids is rotated off the main longitudinal axis. The transformed longitudinal modulus aligned with the global x-axis is then given by the familiar transformation equation:

$$\frac{1}{E_{xx}} = \cos^2 \theta \cdot \frac{1}{E_{11}} + \frac{\sin^2 \theta}{E_{22}} - \frac{\sin^2 \theta}{E_{11}} \cdot 4 \left( \frac{1}{G_{12}} \right) \sin^2 \theta$$  \hspace{1cm} (8)

Another case examined as a model for compact bone is that of three-dimensional randomly oriented voids. The voids are again spheroidal but are considered to be randomly oriented in three-dimensional space creating a macroscopically isotropic material. This not only accounts for the non-alignment of Haversian canals but also for other sources of porosity such as Volkmann's canals and lacunae. Expressions for the effective shear modulus (G) and bulk modulus (K) are given in detail by Zhao. Tandon and Weng [9], but the complexity prohibits repeating them here. The effective elastic modulus can then be calculated by:

$$E = \frac{9 \pi G}{3 \pi + G}$$  \hspace{1cm} (9)

RESULTS AND DISCUSSION

Predictions of the effective longitudinal modulus ($E_{xx}$) using the finite element unit cell analysis are summarized in Fig. 3. The open squares are model results and the filled squares are experimental data from Schaffler and Burr [10]. The heavy solid line is a power-law expression fit to the data by a least squares regression in log-log coordinates [10]. The correlation coefficient is 0.84 and the equation is:

$$E = 33.88 (1-P)^{10.92} \text{ (in GPa)}$$  \hspace{1cm} (10)

The most striking feature of these results is that the experimental data exhibit much greater sensitivity to variations in porosity than do the model predictions. The finite element results decrease essentially linearly with porosity as would be expected for the unit cell configuration employed. The experimental results, however, decrease much more rapidly with increasing porosity, especially for the lower porosity specimens.

A similar trend is also apparent in Fig. 4 for predictions from the Mori-Tanaka approach for needle-shaped voids. The modulus results are all normalized in this case, using a value of 33.88GPa from equation (10) for $E_0$. The solid curve labeled '0 deg' is for the long cylindrical voids aligned with the global x-axis, in which case $E_{11}/E_0$ is given simply by equation (6a). This linear variation with porosity is basically the same as the finite element unit cell results. Rotating the axis of the voids off the global x-axis increases the sensitivity of the longitudinal modulus to variations in porosity but not dramatically. The
Figure 3. Comparison of finite element unit cell results with experimental data [10].
($E_{11}$ is in units of GPa)

Figure 4. Comparison of results from Mori-Tanaka method for needle-shaped voids with experimental data [10].
curves in Fig. 4 labeled '15 deg' and '30 deg' are actually \( E_{\alpha} \) as given by equation (8) for \( \theta \) of 15° and 30°, respectively. The longitudinal modulus decreases more because the transformation to the global \( x \)-axis incorporates the effect of the modulus transverse to the axis of the voids, i.e. \( E_{\gamma \gamma} \) in equation (7). \( E_{\alpha \alpha} \) is shown for reference in Fig. 4 as the '90 deg' curve. This also serves to highlight the fact that even the most extreme case for the needle-shaped model (90°) does not produce as severe a variation in longitudinal modulus with porosity as reported experimentally.

Putting too much emphasis on the results of Schaffler and Burr [10] as a basis for judging the usefulness of the analysis may, however, constitute an unfair comparison in several respects. First, Schaffler and Burr point out themselves that since their test specimens came from bovine bone (steers), the lower porosity specimens likely represent mostly plexiform bone whereas the higher porosity specimens should contain more secondary osteonal (i.e. Haversian) bone. Plexiform bone has a microstructure quite different from Haversian bone. The lamellar bone in plexiform bone is arranged in layers circumferentially around the medullary canal in the center of long bones in somewhat of a brick and mortar configuration (with elongated bricks). Porosity in plexiform bone comes primarily from vascular channels but they traverse in fairly irregular directions. Thus, it is likely that a high percentage of these vascular channels deviate considerably from the main longitudinal axis of the bone. Another important consideration is that the ash content of the test specimens ranged from 0.664 to 0.723. Ash content is a measure of the degree of mineralization of the tissue. No statistically significant correlation was found between ash content and modulus for the 20 specimens, but Currey [11] has documented a strong positive correlation between modulus and calcium content for compact bone from a wide variety of species. The variation in ash content therefore almost certainly contributes to the variation in modulus for the experimental data plotted in Figs. 3 and 4, but the models do not account for this variable in their present form. It should be pointed out that the results of Currey [11] have not been used for comparison with the analysis herein because the wide variety of species makes it highly unlikely that much Haversian bone is included. Haversian bone is most prominent in adult humans but is mixed with plexiform and other bone types in many vertebrate animals.

The Mori-Tanaka method for three-dimensionally oriented voids would be expected to capture the more general spatial orientation of the vascular channels of plexiform bone as well as the inherent misalignment of Haversian canals in Haversian bone. The results shown in Fig. 5 seem to confirm this. The data and curve fit are the same as before, but the other curves represent the results of Zhao, Tandon and Weng [9] for several aspect ratios (\( a \)). The aspect ratio of 10 represents a prolate spheroidal void and the other three aspect ratios (10, 05, 02) represent various oblate spheroidal voids. Prolate voids are more 'rod-like' while the oblate voids are more 'disk-like'. The lower the aspect ratio for the oblate voids the flatter, or thinner, the disk. For oblate voids in general, and particularly so for very 'thin disks', proportionally more of the total volume of the void space is taken out of the plane perpendicular to the axis of loading. That is, oblate voids reduce the 'net section' more than do prolate voids for the same level of porosity. This gives rise to the more dramatic decrease in modulus with porosity for oblate voids. The dashed line curves in Fig. 5 for aspect ratios of .05 and .02 show the greatest sensitivity to variation with porosity and bound the experimental data.

Improved predictions for strictly Haversian bone can be achieved through a somewhat \textit{ad hoc} modification to the Mori-Tanaka method for needle-shaped voids. The properties given by equations (6a)-(6e) are retained to represent the basic configuration of long cylindrical voids aligned in the longitudinal direction. Equation (7) for \( E_{\alpha \alpha} \), however is replaced by an alternate expression for the modulus for uni-directionally aligned oblate spheroidal voids. The expression chosen is actually for the longitudinal modulus, or \( E_{\alpha \alpha} \), in
Figure 5. Comparison of results from Mori-Tanaka method for 3-D randomly oriented voids with experimental data [10].

In this case and is given in [9]. The aligned oblate spheroids represent the effect of the Haversian canals, which run perpendicular to the axis of loading for the transverse modulus $(E_{tt})$. Thus, the modulus $E_{11}$ for the uni-directionally aligned oblate spheroidal solution is used for $E_{tt}$ in equation (8). Results are plotted in Fig. 6 for two aspect ratios (0.02 & 0.01). The '0 deg' curves are included for reference. The other two curves from the analysis are for off-axis angles of 20° and 30°. The only experimental data shown is for porosities of 4% or greater. The assumption is that the results for higher porosities more likely represent exclusively Haversian bone and not plexiform bone. Linear and power-law regressions of the 7 data points yield the following expressions:

\[
E = 26.23 - 150.98P \quad (\text{in GPa}; \ r^2 = 80) \quad (11a)
\]

\[
E = 28.18(1-P)^{11} \quad (\text{in GPa}; \ r^2 = 83) \quad (11b)
\]

which are plotted in Fig. 6 as well. The slopes of the '20 deg' and '30 deg' curves for $a = 0.02$ (Fig. 6(a)) are at least similar to those of the curves fit to the data. Further decreasing the aspect ratio to 0.01 (Fig. 6(b)) provides the best match between predictions and experiment, especially for an off-axis angle of 30°. It should again be emphasized, however, that no data currently exists in the literature on mechanical properties of exclusively Haversian compact bone as a function of porosity. While the present results are encouraging and increase the
Figure 6. Comparison of results from Mori-Tanaka method for modified needle-shaped voids with experimental data [10] for porosity greater than 4%.
prospects that theoretical models can eventually be devised to predict the mechanical properties of compact bone, more definitive development must be accompanied by additional experimental data.

CLOSURE

This comparison of micromechanics methods for modeling Haversian compact bone suggests that the Mori-Tanaka approach holds reasonable promise for predicting the variation in mechanical properties with porosity. The experimental data is scant, however, so the findings are quite tentative. The 2-D finite element unit cell analysis is limited in its present form to predicting an essentially linear decrease in modulus with porosity. The finite element method nevertheless possesses significant advantages: (1) several solid phases (including the cement line interphase material) can be readily incorporated, (2) the anisotropy of constituents can also be included easily, and (3) imperfect bonding (or even slip) between phases can be modeled as well. Several options are therefore being investigated for improving the unit cell approach. These include extending it to three dimensions and/or devising alternate 2-D unit cell configurations. Current efforts are also considering possible ways to further improve and adapt the Mori-Tanaka method for this problem. The solid phase, or matrix material, in the current approach has isotropic material properties only, yet osteons and lamellar bone are anisotropic, at least to a certain extent. Another approach being examined as this line of research continues is to combine the unit cell and Mori-Tanaka methods in some fashion to make use of the relative strengths of each.

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A MICROMECHANICS STUDY OF THE INFLUENCE OF THE CEMENT LINE ON THE ANISOTROPY OF COMPACT BONE TISSUE

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ABSTRACT

A finite element micromechanics model of Haversian compact bone has been used to investigate the influence of the cement line on the anisotropy of the macroscopic mechanical properties. The micromechanics approach utilizes a 2-dimensional unit cell of material in a plane transverse to the main axis of the bone. The microstructure is idealized as consisting of three components: secondary osteons, cement line, and interstitial bone. Osteons are assumed to be arranged in a regular hexagonal array parallel to one another. All constituents are assumed to be isotropic. The macroscopic mechanical properties examined are the longitudinal elastic modulus and the transverse elastic modulus. The ratio of the two is taken as an indicator of the degree of anisotropy. The thickness of the cement line has been varied from 1μm to 5μm and its elastic modulus varied from 50% of that of the osteon to 1%. The effects on the longitudinal modulus are minimal, but the transverse modulus changes more significantly. For most of the range of parameters varied the modulus ratio is lower than experimentally observed, but the ratio approaches the experimental range at the extremes of higher cement line thickness and lower cement line modulus.

INTRODUCTION

Micromechanics modeling and analysis provides explicit and deterministic insight into the relationships between the microstructure and macroscopic mechanical behavior of a variety of engineering materials. This approach typically entails idealizing the geometry and architecture of the heterogeneous constituents of the material and identifying a repeating unit cell of material that can then be analyzed in detail. A valid micromechanics model for compact bone tissue could be a powerful tool in helping elucidate the specific mechanical roles of the various microstructural elements. This would not only contribute directly to the basic science of bone tissue mechanics but it also represents an essential tool in the newly emerging field of biomimetics. Biomimetics refers to efforts to examine naturally occurring composite materials (e.g. wood, bone, insect exoskeletons, etc.) and identify salient 'design features' that can then be mimicked in producing improved man-made-composite materials.

Initial efforts to model bone as a composite material treated it as a two-phase material of collagen and apatite crystals (Currey, 1964, 1969; Pickarski, 1973). Katz (1976, 1981) proposed a hierarchical model for compact bone with the 'microstructural' level being that for which secondary osteons, or Haversian systems, could be considered to be the main microstructural element. On this level, the osteons are considered to be analogous to the fibers of a fiber-reinforced composite material (although hollow) and the cement line and interstitial bone are taken together as a binder material forming the matrix of the composite. The same basic approach was followed in developing the current micromechanics model (Hogan, 1988, 1990, 1992), which utilizes a finite element analysis of the material unit cell to permit more general and explicit treatment of microstructural details. Others have also used a similar approach in studying Haversian compact bone (Crolet, 1990; Pidorapu & Burr, 1991).

The objective of the present study has been to use the micromechanics model to examine the potential role of the cement line in determining the anisotropy of the mechanical properties of Haversian compact bone. A prominent and consistent cement line surrounding secondary osteons is distinctive of Haversian compact bone, and its mechanical role has been the subject of much study and speculation. The macroscopic mechanical properties of compact bone are rather well documented to be anisotropic, either transversely isotropic or orthotropic. In either case, the longitudinal elastic modulus is typically much greater than the elastic modulus transverse to the longitudinal axis. The potential sources of this anisotropy include porosity and anisotropy of the osteons or interstitial bone, but the
purpose of the present study is to examine the extent to which the anisotropy can be attributed to the cement line and its characteristics.

**MICROMECHANICS MODELING**

The idealized microstructure of Haversian compact bone tissue is shown in Fig. 1. The material is assumed to extend continuously perpendicular to the transverse plane shown allowing a 2-dimensional micromechanics model. The only void space included explicitly in this model is the Haversian canal. Three components make up the solid phase: (1) secondary osteons representing a hollow fiber component, (2) interstitial bone representing the matrix material, and (3) the cement line representing a distinct fiber/matrix 'interphase' material. The cement line is assumed to be perfectly bonded to the osteon and interstitial bone. The osteon fibers are spaced in a regular hexagonal array. The rectangular unit cell has been identified based upon symmetry of the material microstructure as well as symmetry of loading conditions.

A typical finite element mesh for the model is shown in Fig. 2. The model consisted of 322 8-noded isoparametric elements and 1053 nodes. The elements include a generalized plane strain formulation to allow out-of-plane loading perpendicular to the plane of the model. All nodes on the straight boundaries of the unit cell are constrained to deflect such that the boundaries remain straight and parallel to their original positions. This prevents the material from forming new internal 'voids' during deformation. The overall dimensions of the unit cell are 100μm × 173.2μm. The outer radius of the osteon is 87.85μm creating a fiber volume percent (or percent Haversian area) of 70% for all cases studied. The radius of the Haversian canal for the mesh shown is 18.2μm yielding a porosity of 3%. A porosity of 5% was also studied by increasing this radius to 23.5μm. The thickness of the cement line was either 1μm, 3μm, or 5μm. All three solid components were modeled as linearly elastic and isotropic. The elastic modulus was taken to be 12GPa for the osteon and 15GPa for the interstitial bone. A Poisson's ratio of 0.3 was used for both of these as well. The cement line was considered to be much more compliant than the bone components. Its precise mechanical properties are unknown, however, so the following values were used for the elastic modulus of the cement line: 6, 3.6, 1.2, 0.6, and 0.12 (all in GPa). As a percent of the osteon modulus, these correspond to 50%, 30%, 10%, 5%, and 1%, respectively. A Poisson's ratio of 0.4 was used for the cement line. The model was loaded with a uniform applied stress in the horizontal direction to determine the transverse elastic modulus. The in-plane Poisson's ratio can also be determined from this loading case using the lateral contraction displacements of the upper and lower boundaries of the unit cell. The model can also be loaded in the longitudinal direction (perpendicular to the plane of the model) to determine the longitudinal elastic modulus and additional Poisson's ratios.
The macroscopic elastic modulus predicted by the model is essentially the same as that given by a rule-of-mixtures (uniform strain) approach for the longitudinal direction. The cement line is a very small percentage of the total solid phase components (ranging from 1.7% to 8.6%), so the variation in the longitudinal modulus with cement line properties and thickness is minimal as shown in Table 1. The cement line modulus is given as a percent of the osteon modulus. Results are presented for cement line thicknesses of 1μm, 3μm, and 5μm, and for porosities of 3% and 5%. As would be expected, the modulus decreases for lower cement line moduli and for higher cement line thicknesses. Note as well that the moduli are generally slightly lower for 5% porosity versus 3%. Anisotropy in the macroscopic moduli is clearly evident in comparing the macroscopic transverse moduli and for porosities of 3% and 5%. The transverse moduli are all lower than the corresponding longitudinal moduli in Table 1. The transverse moduli are sensitive to changes in cement line modulus and thickness. The transverse modulus drops off much more significantly as the cement line modulus decreases to 10% or less of the osteon modulus. At the extreme case of 1% the transverse modulus ranges from 64% to 34% of the corresponding value for a cement line modulus of 50% of the osteon modulus. The effect is again slightly greater for the 5% porosity results.

As a quantitative indication of the degree of anisotropy in the macroscopic modulus predictions, the ratio of the longitudinal modulus to the transverse modulus has also been calculated. These results are presented in detail in Table 3 and summarized here. As would be expected, the modulus decreases for lower cement line moduli and for higher cement line thicknesses. The transverse modulus drops off much more significantly as the cement line modulus decreases to 10% or less of the osteon modulus. At the extreme case of 1% the transverse modulus ranges from 64% to 34% of the corresponding value for a cement line modulus of 50% of the osteon modulus. The effect is again slightly greater for the 5% porosity results.

As a quantitative indication of the degree of anisotropy in the macroscopic modulus predictions, the ratio of the longitudinal modulus to the transverse modulus has also been calculated. These results are presented in detail in Table 3 and summarized in Fig. 3 for 5% porosity and the two extreme cement line thicknesses only (1μm and 5μm). The longitudinal modulus is only 10 to 20 percent higher than the transverse modulus for cement line moduli of 50% and 30% of the osteon modulus. As the cement line modulus further decreases, however, the ratio increases rather dramatically, particularly for the thickest (5μm) cement line. Overall, these ratios are lower than those typically reported in the literature. Cowin (1989) summarized results for human compact bone from several sources, and these yield the following ratios: 1.48, 1.46, 2.66, 2.16, 1.67, and 1.49.

At least two views are possible when comparing the modeling results with experimental data. One is that the model simply lacks sufficient realism in its present form because it predicts a...
much lower modulus ratio for most of the range of parameters studied. In this case, other sources of anisotropy may need to be included in the model. Prominent among these would be to consider the osteon to be anisotropic due to the preferential orientation of collagen and apatite in the concentric layers of osteonic lamellar bone. This can be modeled explicitly within the framework of the present approach but the precise degree of anisotropy of the osteon is unknown. This becomes to a large extent then another adjustable parameter that can be varied over a wide range as needed. Osteon anisotropy has been investigated theoretically (Katz, 1981; Pidaparti et al., 1992), but no direct experimental evidence is currently available to firmly establish anisotropic osteon properties. Another view is that the lower cement line modulus cases in the present analysis more accurately reflect the true nature of the cement line and this is why the predictions match the experimental data more closely for these cases. The precise mechanical properties of the cement line are also unknown at this point, but most consider it to be very compliant (Burr et al., 1988; Frasca, 1981; Schaffler et al., 1987) and perhaps even viscous (Lakes and Saha, 1979). The current model treats the cement line as perfectly bonded to the surrounding bone, which may therefore overestimate the model relative to the actual physical situation. The lowest cement line modulus values of 5% and 1% could be considered somewhat unrealistic since they are 20 and 100 times, respectively, lower than the osteon modulus, but the net effect taken together with the perfect bonding may in fact combine to produce a response quite close to the actual case. In either case, the overriding need is for more definitive experimental data on the mechanical properties of the osteon and the cement line. Such information will not be easy to come by, however, and the present study demonstrates the utility of micromechanics modeling in exploring the possible effects of microconstituent properties on macroscopic properties. Specifically, the results herein suggest that the cement line is likely a significant contributor to Haversian compact bone anisotropy. The macroscopic anisotropy in Haversian bone mechanical properties is not necessarily due solely to osteon anisotropy and can conceivably be explained based upon cement line properties alone. These possibilities should therefore remain open to inquiry until direct experimental evidence indicates otherwise.

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THEORETICAL MODELING OF THE EFFECTS OF POROSITY ON
THE MECHANICAL PROPERTIES OF COMPACT BONE TISSUE

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INTRODUCTION
At the microstructural level compact bone contains a network of canals and cavities that house
blood vessels, nerves, and cells. The porosity of these void spaces can range from 2% to 20%. A
basic first step in establishing theoretical structure/property relationships for compact bone tissue is
to account for the separate effects of void space and the solid phase. Compact bone tissue can
actually be classified into several different types depending on the details of the microstructure. The
specific type of interest in the present work is Haversian compact bone tissue, which has secondary
osteons as the predominant microstructural element. These osteons are commonly considered to be
approximately cylindrical in shape and aligned roughly with the long axis of the bone. The main
void space in Haversian compact bone is the Haversian canal, which is a hollow cylindrical canal
that runs down the center of each osteon. The objective of the present study has been to evaluate
and assess various theoretical approaches for modeling these types of void spaces and their effects
on mechanical properties. As a beginning point, the mechanical property of interest is simply the
elastic modulus of the material.

POROSITY MODELING
Most attempts to develop analytical expressions relating porosity and mechanical properties
of various engineering materials have resulted in empirical correlations based upon extensive
experimental data. These expressions typically take the form of power law relations with particular
coefficients and exponents for particular materials. Theoretical (Zhao et al., 1989) and semi-
empirical (Bert, 1985) treatments are also available and have been examined as candidates for
modeling the cylindrical voids of Haversian compact bone tissue. Zhao et al. (1989) use an effective
medium approach to develop expressions for the elastic constants of materials with spheroidal voids.
The voids can be either aligned or randomly oriented and can have aspect ratios less than one
(oblative) or greater than one (prolate). Cylindrical voids are a special case in which the voids are
aligned and have an aspect ratio approaching infinity. Bert (1985) developed a combined theoretical
and empirical treatment for cylindrical and spheroidal voids. The cylindrical and spheroidal voids
can be taken to be arranged in a square array or a hexagonal array. Finite element micromechanics
modeling has also been used for studying compact bone tissue (Hogan, 1992). Porosity due to the
Haversian canal is modeled explicitly, and distinct osteon, cement line, and interstitial bone
components are included separately for the solid phase.

RESULTS AND DISCUSSION
Two fundamental factors to be considered in a theoretical model for porosity effects in
compact bone are: (1) the variation in mechanical properties with porosity; and (2) the degree of
anisotropy in the mechanical properties due to the presence of porosity. Elastic modulus predictions
are compared in Fig. 1 for the analytical models of Bert (1985) and Zhao et al. (1989) and for the
current finite element micromechanics model. The moduli values are normalized by dividing by the
elastic modulus of the solid phase. All three approaches yield the same results for the elastic
modulus in the axial direction, i.e. along the main longitudinal axis of the long bone. The cylindrical
voids are also aligned with this axis so the decrease in modulus is linear and proportional to the
porosity. The elastic modulus transverse to the longitudinal axis is more sensitive to changes in
porosity. The results for Bert's model are for a hexagonal array of cylindrical voids and show a slightly greater decrease in modulus with porosity than the other two approaches. The rate of decrease, however, is greatest for the finite element results. The degree of anisotropy predicted by the models is depicted by plotting the ratio of the axial modulus to the transverse modulus. Bert's model predicts greater anisotropy due to the lower transverse modulus values. The ratio increases from approximately 1.0 to 1.35 for Zhao et al. and the finite element model, but reaches a maximum of 1.55 for Bert. These are generally lower than typical ratios reported for human compact bone. Cowin (1989) summarized data from four sources which yield the following ratios: 1.48, 1.46, 2.66, 2.16, 1.67, 1.49. Six values are included because two of the studies found the bone to be orthotropic so two different transverse moduli were reported. None of the studies included any microstructural information about the specimens tested so the porosities are not known. An important point to consider as well is that the two analytical approaches assume the solid phase to be homogeneous and isotropic. Each component of the solid phase in the finite element model is likewise treated as isotropic. The observed degree of anisotropy is likely due in part to the microstructural architecture of the solid phase as well as the presence of voids. The theoretical models nevertheless clearly demonstrate that a significant portion of the anisotropy of Haversian compact bone can be attributable to porosity. Only limited experimental data is available in the literature on the variation in elastic modulus of compact bone with porosity. Currey (1988) examined the influence of porosity and mineralization on the elastic modulus of bone 18 different species of animals but none were from human sources. Volume fraction (1-p) was correlated using a power law expression with the exponent ranging from 3.1 to 3.5. Schaffler and Burr (1988) report a power law exponent of 10.92 for modulus as a function of volume fraction 10.92. These are not linear relations as the models herein predict, but neither study included human bone, which is predominantly Haversian bone. Other species are typically a mix of plexiform and Haversian bone, so the models would not be expected to predict well for these.

Fig. 1. Predictions of modulus vs. porosity.

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