QUARTZ CRYSTAL
RESONATORS AND OSCILLATORS
For Frequency Control and Timing Applications
A TUTORIAL
John R. Vig
Electronics and Power Sources Directorate
Army Research Laboratory
Fort Monmouth, NJ 07703-5601, U.S.A.
May 1993

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    Subjects covered include: applications of frequency standards; types of oscillators (quartz and atomic); quartz resonator properties; quartz growing, sweeping, and material characteristics; Q and its significance; resonator and oscillator stability, including aging, short-term instability, frequency vs. temperature characteristics, oscillator circuit caused instabilities, frequency vs. drive level effects, acceleration effects, the effect of shock, and radiation effects; emerging Technologies; atomic frequency standards; comparison of the major oscillator types; oscillator specifications and selection guidelines; time and timekeeping; clock errors; relativistic time; time transfer; time and frequency subsystem; and other applications of quartz resonators.

    The goal of this document is to assist in presenting to the nonspecialist the most frequently encountered concepts in frequency control and timing. The document originated as a set of "hard copies" of presentation visuals (i.e., vugraphs).

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Preface
Why This Tutorial?

"Everything should be made as simple as possible - but not simpler," said Einstein. The main goal of this "tutorial" is to assist with presenting the most frequently encountered concepts in frequency control and timing, as simply as possible.

In my position as Chief, Frequency Control and Timing Branch, US Army Electronics Technology and Devices Laboratory, I am often called upon to brief visitors, management, and potential users of precision oscillators. I have also been invited to present seminars and review papers before university, IEEE, and other professional groups. In the beginning, I spent a great deal of time preparing these presentations. Much of the time was spent on preparing the presentation visuals (i.e., the vu-graphs). As I accumulated more and more vu-graphs, it became easier and easier to prepare successive presentations. Since I was frequently asked for "hard-copies" of the vu-graphs, I started organizing, adding some text, and filling the gaps in the vu-graph collection. As the collection grew, I began receiving favorable comments and requests for additional copies. Apparently, others, too, found this collection to be useful. Eventually, I assembled this document, the "Tutorial."

References are listed at the end of each chapter. General references are listed just before the index at the end. Comments and suggestions for future revisions will be welcome.

John R. Vig
Applications and Requirements
# Applications of Quartz Crystals

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<td>Telecommunications</td>
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<td>Quartz Crystal Device Market</td>
<td></td>
<td></td>
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<tr>
<td>-----------------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Military</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>~1 million</td>
<td></td>
<td></td>
</tr>
<tr>
<td>~$50</td>
<td></td>
<td></td>
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<td>C³, nav., IFF, radar, fuses, sonobuoys</td>
<td></td>
<td></td>
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<tr>
<td>Higher stability, aging, noise, temperature, acceleration, radiation, lower power, smaller, less expensive, more rugged</td>
<td></td>
<td></td>
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<tr>
<td><strong>Commercial</strong></td>
<td></td>
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<tr>
<td>~1 billion</td>
<td></td>
<td></td>
</tr>
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<td>~$1</td>
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<td>Watches, clocks, color TV, autos</td>
<td></td>
<td></td>
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<tr>
<td>Less expensive, smaller</td>
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<tr>
<td>Production (no./yr.)</td>
<td></td>
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<tr>
<td>Major markets</td>
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<tr>
<td>Major R&amp;D thrusts</td>
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# Commercial - Military Comparison

<table>
<thead>
<tr>
<th>Commercial</th>
<th>Parameters</th>
<th>Military &amp; Space</th>
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<tbody>
<tr>
<td>Typical</td>
<td></td>
<td></td>
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<tr>
<td>~ 92%</td>
<td>~ 2%</td>
<td>% of Market ($)</td>
</tr>
<tr>
<td>CB radios, watches, color TVs, microcomputers</td>
<td>Instruments, Commercial Spacecraft</td>
<td>Fielded Systems</td>
</tr>
<tr>
<td>10^{-4}</td>
<td>10^{-7}</td>
<td></td>
</tr>
<tr>
<td>0°C to 60°C</td>
<td>0°C to 71°C</td>
<td>Typical Applications</td>
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<tr>
<td>No Requirement</td>
<td>No Requirement</td>
<td>PRC-77, VRC-12</td>
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<tr>
<td>100 to 1,000 g</td>
<td>100 g</td>
<td>Accuracy per year</td>
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<tr>
<td>100 cm³</td>
<td>300 cm³</td>
<td>5 x 10^{-5}</td>
</tr>
<tr>
<td>Not oven-controlled</td>
<td>4 W</td>
<td>Temperature</td>
</tr>
<tr>
<td>Not oven-controlled</td>
<td>10 min</td>
<td>-55°C to +100°C</td>
</tr>
<tr>
<td>No Requirement</td>
<td>No Requirement</td>
<td>Vibration</td>
</tr>
<tr>
<td>Radiation hardening</td>
<td></td>
<td>No Requirement</td>
</tr>
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<table>
<thead>
<tr>
<th>Military &amp; Space</th>
<th></th>
</tr>
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<tr>
<td>Evolving Systems</td>
<td></td>
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<tr>
<td>~ 1%</td>
<td>Radios, ECCM, IFF, Navigation, Surveillance</td>
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<tr>
<td>10^{-6} to 10^{-9}</td>
<td>-55°C to 85°C</td>
</tr>
<tr>
<td>10^{-12} to 10^{-10} per g</td>
<td>10^{-12} to 10^{-10} per g</td>
</tr>
<tr>
<td>up to 16,000 g</td>
<td>&lt; 20 cm³</td>
</tr>
<tr>
<td>&lt; 10 mW to &lt; 0.25 W</td>
<td>Radiation hardened</td>
</tr>
<tr>
<td>&lt; 3 min</td>
<td></td>
</tr>
</tbody>
</table>

Number of U.S. Companies ~ 60
Navigation

Precise time is essential to precise navigation. Historically, navigation has been a principal motivator in man's search for better clocks. Even in ancient times, one could measure latitude by observing the stars' position. However, to determine longitude, the problem became one of timing. Since the earth makes one revolution (360°) in 24 hours, one can determine longitude from the time difference $\Delta t$ between local time (which was determined from the sun's position) and the time at the Greenwich meridian (which was determined by a clock). Longitude in degrees = $(360°/24$ hours) $\times \Delta t$ in hours.

In 1714, the British government offered a reward of 20,000 pounds to the first person to produce a clock that allowed the determination of a ship's longitude to 30 nautical miles at the end of a six week voyage (i.e., a clock accuracy of three seconds per day). The Englishman John Harrison won the competition in 1735 for his chronometer invention.

Today's electronic navigation systems still require ever greater accuracies. Since light (radio waves) travels 300 meters per microsecond, e.g., if a vessel's timing was in error by one millisecond, a navigational error of 300 kilometers would result. In the Global Positioning System (GPS), atomic clocks in the satellites and quartz oscillators in the receivers provide nanosecond-level accuracies. The resulting (worldwide) navigational accuracies are about ten meters (see chapter 9 for further details about GPS).
Commercial Two-way Radio

Historically, as the number of users of commercial two-way radios have grown, channel spacings have been narrowed, and higher-frequency spectra have had to be allocated to accommodate the demand. Narrower channel spacings and higher operating frequencies necessitate tighter frequency tolerances for both the transmitters and the receivers. In 1940, when only a few thousand commercial broadcast transmitters were in use, a 500 ppm tolerance was adequate. Today, the millions of cellular telephones (which operate at frequency bands above 800 MHz) must maintain a frequency tolerance of 2.5 ppm. TCXOs of 2 ppm frequency accuracy are used for frequency control. The 896-901 MHz and 935-940 MHz mobile radio bands require frequency tolerances of 0.1 ppm at the base station and 1.5 ppm at the mobile station.

The need to accommodate more users will continue to require higher and higher frequency accuracies. For example, NASA is developing a personal satellite communication system, using walkie-talkie-like hand-held terminals, which employs a 30 GHz uplink, a 20 GHz downlink, and a 10 kHz channel spacing. The terminals’ frequency accuracy requirement is a few parts in $10^8$. 
Digital Network Synchronization

- Synchronization plays a critical role in digital telecommunication systems. It ensures that information transfer is performed with minimal buffer overflow or underflow events, i.e., with an acceptable level of "slips." Slips cause problems, e.g., missing lines in FAX transmission, clicks in voice transmission, loss of encryption key in secure voice transmission, and data retransmission.

- In AT&T’s network, timing is distributed down a hierarchy of nodes. A timing source-receiver relationship is established between pairs of nodes containing clocks. The clocks are of four types, in four "stratum levels."

<table>
<thead>
<tr>
<th>Stratum</th>
<th>Accuracy (Free running)</th>
<th>Clock type</th>
<th>Number used</th>
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</thead>
<tbody>
<tr>
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<td>long term</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$1 \times 10^{-11}$</td>
<td>N.A.</td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td>$1.6 \times 10^{-8}$</td>
<td>$1 \times 10^{-10}$</td>
<td>~200</td>
</tr>
<tr>
<td>3</td>
<td>$4.6 \times 10^{-6}$</td>
<td>$3.7 \times 10^{-7}$</td>
<td>1000's</td>
</tr>
<tr>
<td>4</td>
<td>$3.2 \times 10^{-5}$</td>
<td>N.A.</td>
<td>~ 1 million</td>
</tr>
</tbody>
</table>
Phase Noise in PLL and PSK Systems

The phase noise of oscillators can lead to erroneous detection of phase transitions, i.e., to bit errors, when phase shift keyed (PSK) digital modulation is used. In digital communications, for example, where 8-phase PSK is used, the maximum phase tolerance is ±22.5°, of which ±7.5° is the typical allowable carrier noise contribution. Due to the statistical nature of phase deviations, if the RMS phase deviation is 1.5°, for example, the probability of exceeding the ±7.5° phase deviation is 6 X 10^{-7}, which can result in a bit error rate that is significant in some applications.

Shock and vibration can produce large phase deviations even in "low noise" oscillators. Moreover, when the frequency of an oscillator is multiplied by N, the phase deviations are also multiplied by N. For example, a phase deviation of 10^{-3} radian at 10 MHz becomes 1 radian at 10 GHz. Such large phase excursions can be catastrophic to the performance of systems, e.g., of those which rely on phase locked loops (PLL) or phase shift keying. Low noise, acceleration insensitive oscillators are essential in such applications.
When a fault occurs, e.g., when a "sportsman" shoots out an insulator, a disturbance propagates down the line. The location of the fault can be determined from the time of arrival differences:

\[ x = \frac{1}{2} \left[ L - c(t_b - t_a) \right] = \frac{1}{2} \left[ L - c\Delta t \right] \]

where \( x \) = distance of the fault from substation A, \( L \) = A to B line length, \( c \) = speed of light, and \( t_a \) and \( t_b \) = time of arrival of disturbance at A and B, respectively.

Fault locator error = \( x_{\text{error}} = \frac{1}{2}(c\Delta t_{\text{error}}) \); if \( \Delta t_{\text{error}} = 1 \) microsecond, then \( x_{\text{error}} = 150 \text{ meters} = \frac{1}{2} \) of high voltage tower spacings.
How does NASA know where a spacecraft is in deep space? The spacecraft’s precise range, velocity and angular position are determined with the aid of highly stable frequency standards. The range is determined from the propagation time of microwave radiation between an antenna on Earth and the spacecraft. The velocity is determined from the "doppler," i.e., by comparing the phase of the incoming carrier signal with that of a reference signal generated from the ground station frequency standard. The angular position is determined by Very Long Baseline Interferometry (VLBI) in which widely separated stations (in California, Spain and Australia) simultaneously receive signals from the spacecraft. Differences between times of arrival coupled with knowledge of the baseline vectors joining the station antennas provide direct geometric determination of the angles between the baseline vectors and the direction to the spacecraft. Hydrogen masers provide the best stability (≈ 10^{-15}) for the propagation times of interest, which typically range from minutes to hours. VLBI is also used for high resolution angular measurements in radioastronomy.
Military Requirements

Military needs are a prime driver of frequency control technology. Modern military systems require oscillators/clocks that are:

- Stable over a wide range of parameters (time, temperature, acceleration, radiation, etc.)
- Low noise
- Low power
- Small size
- Fast warmup
- Low life-cycle cost
# Impacts of Oscillator Technology Improvements

- Higher jamming resistance & improved ability to hide signals
- Improved ability to deny use of systems to unauthorized users
- Longer autonomy period (radio silence interval)
- Faster signal acquisition (net entry)
- Lower power for reduced battery consumption
- Improved spectrum utilization
- Improved surveillance capability (e.g., slow-moving target detection, bistatic radar)
- Improved missile guidance (e.g., on-board radar vs. ground radar)
- Improved identification-friend-or-foe (IFF) capability
- Improved electronic warfare capability (e.g., emitter location via TOA)
- Lower error rates in digital communications
- Improved navigation capability
- Improved survivability and performance in radiation environment
- Improved survivability and performance in high shock applications
- Longer life, and smaller size, weight, and cost
- Longer recalibration interval (lower logistics costs)
Spread Spectrum Systems

- In a spread spectrum system, the transmitted signal (e.g., a voice channel of a few kHz bandwidth) is spread over a bandwidth that is much wider (e.g., many MHz) than the bandwidth required to transmit the information being sent. This is accomplished by modulating a carrier signal with the information being sent, and with a wideband pseudonoise (PN) encoding signal. A spread spectrum receiver with the appropriate PN code can demodulate and extract the information being sent. Those without the PN code may completely miss the signal, or if they detect the signal, it appears to them as noise.

- Two of the spread spectrum modulation types are: 1. direct sequence, in which the carrier is modulated by a digital code sequence, and 2. frequency hopping, in which the carrier frequency jumps from frequency to frequency, within some predetermined set, the order of frequencies being determined by a code sequence.

- Transmitter and receiver contain clocks which must be synchronized; e.g., in a frequency hopping system, the transmitter and receiver must hop to the same frequency at the same time. The faster the hopping rate, the higher the jamming resistance, and the more accurate the clocks must be.

- Advantages of spread spectrum systems include the following capabilities: 1. rejection of intentional and unintentional jamming, 2. low probability of intercept (LPI), 3. selective addressing, 4. multiple access, and 5. high accuracy navigation and ranging.
Clock for Very Fast Frequency Hopping Radio

To defeat a "perfect" follower jammer, need a hop-rate given by:

\[ t_m < (t_1 + t_2) - t_R, \]

where \( t_m \approx \text{msg. duration/hop} \approx 1/\text{hop-rate} \)

Example

Let R1 to R2 = 1 km, R1 to J = 5 km, and J to R2 = 5 km.
Then, since propagation delay = 3.3 \( \mu \)s/km, \( t_1 = t_2 = 16.5 \mu \)s, \( t_R = 3.3 \mu \)s, and \( t_m < 30 \mu \)s.

Allowed clock error \( \approx 0.2 t_m \approx 6 \mu \)s.

For a 4 hour resynch interval, clock accuracy requirement is:

\[ 4 \times 10^{-10} \]
Clocks and Frequency Hopping $C^3$ Systems

- Slow hopping $\leftrightarrow$ Good clock
- Fast hopping $\leftrightarrow$ Better clock
- Extended radio silence $\leftrightarrow$ Better clock
- Extended calibration interval $\leftrightarrow$ Better clock
- Orthogonality $\leftrightarrow$ Better clock
- Interoperability $\leftrightarrow$ Better clock
Identification-Friend-Or-Foe (IFF)

In a modern battle, when the sky is filled with friendly and enemy aircraft, and a variety of advanced weapons are ready to fire from both ground and airborne platforms, positive identification of friend and foe is critically important. For example, fratricide due to identification errors was a major problem in the 1973 Arab-Israeli war.

Current IFF systems use an interrogation/response method which employs cryptographically encoded spread spectrum signals. The interrogation signal received by a friend is supposed to result in the "correct" code being automatically sent back via a transponder on the friendly platform. The "correct" code must change frequently to prevent a foe from recording and transmitting that code ("repeat jamming"), thereby appearing as a friend. The code is changed at the end of what is called the Code Validity Interval, or CVI.

The better the clock accuracy, the shorter can be the CVI, the more resistant the system can be to repeat jamming, and the longer can be the autonomy period for users who cannot resynchronize their clocks during a mission.
Effect of Noise in Doppler Radar System

- Echo = Doppler shifted echo from moving target + large "clutter" signal
- (Echo signal) - (reference signal) → Doppler shifted signal from target
- Phase noise of the local oscillator modulates (decorrelates) the clutter signal, generates higher frequency clutter components, and thereby degrades the radar’s ability to separate the target signal from the clutter signal.
Bistatic Radar

Conventional (i.e., "monostatic") radar, in which the illuminator and receiver are on the same platform, is vulnerable to a variety of countermeasures. Bistatic radar, in which the illuminator and receiver are widely separated, can greatly reduce the vulnerability to countermeasures such as jamming and antiradiation weapons, and can increase slow moving target detection and identification capability via "clutter tuning" (receiver maneuvers so that its motion compensates for the motion of the illuminator; creates zero Doppler shift for the area being searched). The transmitter can remain far from the battle area, in a "sanctuary". The receiver can remain "quiet."

The timing and phase coherence problems can be orders of magnitude more severe in bistatic than in monostatic radar, especially when the platforms are moving. The two reference oscillators must remain synchronized and syntonized during a mission so that the receiver knows when the transmitter emits each pulse, and so that the phase variations will be small enough to allow a satisfactory image to be formed. Low noise crystal oscillators are required for short term stability; atomic frequency standards are often required for long term stability.
Doppler Shifts

Doppler radars require low-phase-noise oscillators. The velocity of the target and the radar frequency are the primary factors that determine the oscillator noise requirements. For example, to detect slow-moving targets, the noise close to the carrier must be low.

Doppler Shift for Target Moving Toward Fixed Radar (Hz)
Chapter 1 References


Oscillation

- At the frequency of oscillation, the closed loop phase shift = $2n\pi$.

- When initially energized, the only signal in the circuit is noise. That component of noise, the frequency of which satisfies the phase condition for oscillation, is propagated around the loop with increasing amplitude. The rate of increase depends on the excess; i.e., small-signal, loop gain and on the BW of the crystal network.

- The amplitude continues to increase until the amplifier gain is reduced either by nonlinearities of the active elements ("self limiting") or by some automatic level control.

- At steady state, the closed-loop gain = 1.
Oscillation and Stability

- If a phase perturbation $\Delta \phi$ occurs, the frequency must shift $\Delta f$ to maintain the $2n\pi$ phase condition, where $\Delta f/f = -\Delta \phi/2Q_L$ for a series-resonance oscillator, and $Q_L$ is the loaded $Q$ of the crystal in the network. The "phase slope" $d\phi/df$ is proportional to $Q_L$ in the vicinity of the series resonance frequency (see "Equivalent Circuit" and "Frequency vs. Reactance" in Chapt. 3).

- Most oscillators operate at "parallel resonance," where the reactance vs. frequency slope, $dX/df$, i.e., the "stiffness," is inversely proportional to $C_1$, the motional capacitance of the crystal unit.

- For maximum frequency stability with respect to phase (or reactance) perturbations in the oscillator loop, the phase slope (or reactance slope) must be maximum, i.e., $C_1$ should be minimum and $Q_L$ should be maximum. A quartz crystal unit's high $Q$ and high stiffness makes it the primary frequency (and frequency stability) determining element in oscillators.
Tunability and Stability

Making an oscillator tunable over a wide frequency range degrades its stability because making an oscillator susceptible to intentional tuning also makes it susceptible to factors that result in unintentional tuning. The wider the tuning range, the more difficult it is to maintain a high stability. For example, if an OCXO is designed to have a short term stability of $1 \times 10^{-12}$ for some averaging time and a tunability of $1 \times 10^{-7}$, then the crystal's load reactance must be stable to $1 \times 10^{-5}$ for that averaging time. Achieving such stability is difficult because the load reactance is affected by stray capacitances and inductances, by the the stability of the varactor's capacitance vs. voltage characteristic, and by the stability of the voltage on the varactor. Moreover, the $1 \times 10^{-5}$ load reactance stability must be maintained not only under benign conditions, but also under changing environmental conditions (temperature, vibration, radiation, etc.). Whereas a high stability, ovenized 10 MHz voltage controlled oscillator may have a frequency adjustment range of $5 \times 10^{-7}$ and an aging rate of $2 \times 10^{-8}$ per year, a wide tuning range 10 MHz VCXO may have a tuning range of 50 ppm and an aging rate of 2 ppm per year.
<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>XO</td>
<td>Crystal Oscillator</td>
</tr>
<tr>
<td>VCXO</td>
<td>Voltage Controlled Crystal Oscillator</td>
</tr>
<tr>
<td>OCXO</td>
<td>Oven Controlled Crystal Oscillator</td>
</tr>
<tr>
<td>TCXO</td>
<td>Temperature Compensated Crystal Oscillator</td>
</tr>
<tr>
<td>TCVCXO</td>
<td>Temperature Compensated/Voltage Controlled Crystal Oscillator</td>
</tr>
<tr>
<td>OCVCXO</td>
<td>Oven Controlled/Voltage Controlled Crystal Oscillator</td>
</tr>
<tr>
<td>MCXO</td>
<td>Microcomputer Compensated Crystal Oscillator</td>
</tr>
<tr>
<td>RbXO</td>
<td>Rubidium-Crystal Oscillator</td>
</tr>
</tbody>
</table>
provide a >1000X improvement over the crystal's f vs. T variation.

The crystal's f vs. T variation characteristically. Analog TCXO's can provide about a 20X improvement over network. The reactance variations compensate for the crystal's f vs. T.

Voltage that is applied to a temperature-sensing variable reactance (varactor) in the crystal signal from a temperature-sensor (thermistor) is used to generate a correction output.

TCXO, temperature compensated crystal oscillator, in which the output crystal oscillator (also called PXO - packaged crystal oscillator).

XO, crystal oscillator, which does not contain means for reducing the unit's frequency vs. temperature characteristic, are:

Crystral Oscillator Categories
Crystal Oscillator Categories

- **Crystal Oscillator (XO)**
  - Voltage
  - Temperature Sensor
  - Compensation Network or Computer
  - Output

- **Temperature Compensated (TCXO)**
  - Oven
  - Oven Control
  - Xo
  - Temperature Sensor

- **Oven Controlled (OCXO)**
# Hierarchy of Oscillators

<table>
<thead>
<tr>
<th>Oscillator Type*</th>
<th>Accuracy**</th>
<th>Typical Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crystal oscillator (XO)</td>
<td>$10^{-5}$ to $10^{-4}$</td>
<td>Computer timing</td>
</tr>
<tr>
<td>Temperature compensated crystal oscillator (TCXO)</td>
<td>$10^{-6}$</td>
<td>Frequency control in tactical radios</td>
</tr>
<tr>
<td>Microcomputer compensated crystal oscillator (MCXO)</td>
<td>$10^{-8}$ to $10^{-7}$</td>
<td>Spread spectrum system clock</td>
</tr>
<tr>
<td>Oven controlled crystal oscillator (OCXO)</td>
<td>$10^{-8}$</td>
<td>Navigation system clock &amp; frequency standard, MTI radar</td>
</tr>
<tr>
<td>Small atomic frequency standard (Rb, RbXO)</td>
<td>$10^{-9}$</td>
<td>C$^3$ satellite terminals, bistatic &amp; multistatic radar</td>
</tr>
<tr>
<td>High performance atomic standard (Cs)</td>
<td>$10^{-12}$ to $10^{-11}$</td>
<td>Strategic C$^3$, EW</td>
</tr>
</tbody>
</table>

*Sizes range from $< 5 \text{ cm}^3$ for clock oscillators to $> 30 \text{ liters}$ for Cs standards. Costs range from $< $5 for clock oscillators to $> $40,000 for Cs standards.

**Including the effects of military environments and one year of aging.
Oscillator Circuit Types

Of the numerous oscillator circuit types, three of the more common ones, the Pierce, the Colpitts and the Clapp, consist of the same circuit except that the rf ground points are at different locations. The Butler and modified Butler are also similar to each other; in each, the emitter current is the crystal current. The gate oscillator is a Pierce-type that uses a logic gate plus a resistor in place of the transistor in the Pierce oscillator. (Some gate oscillators use more than one gate.)

<table>
<thead>
<tr>
<th>Pierce</th>
<th>Colpitts</th>
<th>Clapp</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Pierce Circuit" /></td>
<td><img src="image2" alt="Colpitts Circuit" /></td>
<td><img src="image3" alt="Clapp Circuit" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Butler</th>
<th>Modified Butler</th>
<th>Gate</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image4" alt="Butler Circuit" /></td>
<td><img src="image5" alt="Modified Butler Circuit" /></td>
<td><img src="image6" alt="Gate Circuit" /></td>
</tr>
</tbody>
</table>
Oscillator Circuit Types - Comments

The choice of oscillator circuit type depends on factors such as the desired frequency, stability, input voltage and power, output power and waveform, tunability, design complexity, cost, and the crystal unit's characteristics.

In the Pierce family, the ground point location has a profound effect on the performance. The Pierce configuration is generally superior to the others, e.g., with respect to the effects of stray reactances and biasing resistors, which appear mostly across the capacitors in the circuit rather than the crystal unit. It is one of the most widely used circuits for high stability oscillators. In the Colpitts configuration, a larger part of the strays appears across the crystal, and the biasing resistors are also across the crystal, which can degrade performance. The Clapp is seldom used because, since the collector is tied directly to the crystal, it is difficult to apply a dc voltage to the collector without introducing losses or spurious oscillations. (See the references for more details.)

The Pierce family usually operates at "parallel resonance" (see "Resonator Frequency vs. Reactance" in Chapt. 3), although it can be designed to operate at series resonance by connecting an inductor in series with the crystal. The Butler family usually operates at (or near) series resonance. The Pierce can be designed to operate with the crystal current above or below the emitter current.

Gate oscillators are common in digital systems when high stability is not a major consideration.
Oscillator Outputs

Most users require a sine wave, or a TTL-compatible, or a CMOS-compatible, or an ECL-compatible output. The latter three can be simply generated from a sine wave. The four output types are illustrated below, with the dashed lines representing the supply voltage inputs, and the bold solid lines, the outputs. (There is no "standard" input voltage for sine wave oscillators, and the input voltage for CMOS typically ranges from 5V to 15V.)
Chapter 2 References


Quartz Crystal Resonators
Why Quartz?

Quartz is the only material known that possesses the following combination of properties:

- Piezoelectric ("pressure-electric"; piezein = to press, in Greek)
- Zero temperature coefficient cuts exist
- Stress compensated cut exists
- Low loss (i.e., high Q)
- Easy to process; low solubility in everything, except fluoride etchants, under "normal" conditions; hard but not brittle
- Abundant in nature; easy to grow in large quantities, at low cost, and with relatively high purity and perfection. Of the man-grown single crystals, quartz, at >2,000 tons per year, is second only to silicon in quantity grown.
The Piezoelectric Effect

The direct piezoelectric effect was discovered by the Curie brothers in 1880. They showed that when a weight was placed on a quartz crystal, charges appeared on the crystal surface; the magnitude of the charge was proportional to the weight. In 1881, the converse piezoelectric effect was illustrated; when a voltage was applied to the crystal, the crystal deformed due to the lattice strains caused by the effect. The strains reversed when the voltage was reversed. The piezoelectric effect can, thereby, provide a coupling between an electrical circuit and the mechanical properties of a crystal. Under the proper conditions, a "good" piezoelectric resonator can stabilize the frequency of an oscillator circuit.

Of the 32 crystal classes, 20 exhibit the piezoelectric effect (but only a few of these are useful). Piezoelectric crystals lack a center of symmetry. When a force deforms the lattice, the centers of gravity of the positive and negative charges in the crystal can be separated so as to produce surface charges. The figures show one example (from Kelvin's qualitative model) of the effect in quartz. Each silicon atom is represented by a plus, and each oxygen atom by a minus. When a strain is applied so as to elongate the crystal along the Y-axis, there are net movements of negative charges to the left and positive charges to the right (along the X-axis).
# The Piezoelectric Effect

<table>
<thead>
<tr>
<th>STRAIN</th>
<th>FIELD along:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
</tr>
<tr>
<td>EXTENSIONAL</td>
<td>X</td>
</tr>
<tr>
<td>along:</td>
<td>Y</td>
</tr>
<tr>
<td></td>
<td>Z</td>
</tr>
<tr>
<td>SHEAR</td>
<td>X</td>
</tr>
<tr>
<td>about:</td>
<td>Y</td>
</tr>
<tr>
<td></td>
<td>Z</td>
</tr>
</tbody>
</table>

In quartz, the five strain components shown may be generated by an electric field. The modes shown on the next page may be excited by suitably placed and shaped electrodes. The shear strain about the Z-axis produced by the Y-component of the field is used in the rotated Y-cut family which includes the AT, BT, and ST-cuts.
## Modes of Motion

<table>
<thead>
<tr>
<th>Flexure Mode</th>
<th>Extensional Mode</th>
<th>Face Shear Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="flexure_mode.png" alt="" /></td>
<td><img src="extensional_mode.png" alt="" /></td>
<td><img src="face_shear_mode.png" alt="" /></td>
</tr>
<tr>
<td>Thickness Shear Mode</td>
<td>Fundamental Mode</td>
<td>Third Overtone Thickness Shear</td>
</tr>
<tr>
<td><img src="thickness_shear_mode.png" alt="" /></td>
<td><img src="fundamental_mode.png" alt="" /></td>
<td><img src="third_overtone_thickness_shear.png" alt="" /></td>
</tr>
</tbody>
</table>
Resonator Packaging

Three-and-four-point mount package

Quartz Blank Bonding Area Cover Mounting Clips Seal Base Pins

Two-point mount package

Quartz Blank Bonding Area Mounting Clips Base Pins Seal Cover
Ceramic Flatpack and Metal-Enclosed Resonators
Resonator Vibration Amplitude Distribution

In an ideal resonator, the amplitude of vibration falls off exponentially outside the electrodes. In a properly designed resonator, a negligible amount of energy is lost to the mounting and bonding structure, i.e., the edges must be inactive in order for the resonator to be able to possess a high Q. The displacement of a point on the resonator surface is proportional to the drive current. At the typical drive currents used in (e.g., 10 MHz) thickness shear resonators, the peak displacement is on the order of a few atomic spacings.
Resonant Vibrations of a Quartz Plate

X-ray topographs (21·0 plane) of various modes excited during a frequency scan of a fundamental mode, circular, AT-cut resonator. The first peak, at 3.2 MHz, is the main mode; all others are unwanted modes. Dark areas correspond to high amplitudes of displacement.
Unwanted Modes vs. Temperature

(3 Mhz rectangular AT-cut resonator, 22 x 27 x 0.552 mm)

Activity dips occur where the f vs. T curves of unwanted modes intersect the f vs. T curve of the wanted mode. Such activity dips are highly sensitive to drive level and load reactance.
Mathematical Description of a Quartz Resonator

- In piezoelectric materials, electrical current and voltage are coupled to elastic displacement and stress:

\[
\{T\} = [C] \{S\} - [\varepsilon] \{E\}
\]
\[
\{D\} = [\varepsilon] \{S\} + [\varepsilon] \{E\}
\]

where \(\{T\}\) = stress tensor, \([C]\) = elastic stiffness matrix, \(\{S\}\) = strain tensor, \([\varepsilon]\) = piezoelectric matrix, \(\{E\}\) = electric field vector, \(\{D\}\) = electric displacement vector, and \([\varepsilon]\) = is the dielectric matrix.

- For a linear piezoelectric material:

\[
\begin{bmatrix}
T_1 \\
T_2 \\
T_3 \\
T_4 \\
T_5 \\
T_6 \\
D_1 \\
D_2 \\
D_3
\end{bmatrix} =
\begin{bmatrix}
c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} & -e_{11} & -e_{12} & -e_{13} \\
c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} & -e_{12} & -e_{22} & -e_{23} \\
c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} & -e_{13} & -e_{23} & -e_{33} \\
c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} & -e_{14} & -e_{24} & -e_{34} \\
c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} & -e_{15} & -e_{25} & -e_{35} \\
c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} & -e_{16} & -e_{26} & -e_{36} \\
e_{11} & e_{12} & e_{13} & e_{14} & e_{15} & e_{16} & e_{11} & e_{12} & e_{13} \\
e_{21} & e_{22} & e_{23} & e_{24} & e_{25} & e_{26} & e_{21} & e_{22} & e_{23} \\
e_{31} & e_{32} & e_{33} & e_{34} & e_{35} & e_{36} & e_{31} & e_{32} & e_{33}
\end{bmatrix}
\begin{bmatrix}
S_1 \\
S_2 \\
S_3 \\
S_4 \\
S_5 \\
S_6 \\
E_1 \\
E_2 \\
E_3
\end{bmatrix}
\]

\[
\text{where}
\]

\[
T_1 = T_{11}, \quad S_1 = S_{11},
\]
\[
T_2 = T_{22}, \quad S_2 = S_{22},
\]
\[
T_3 = T_{33}, \quad S_3 = S_{33},
\]
\[
T_4 = T_{33}, \quad S_4 = 2S_{33},
\]
\[
T_5 = T_{13}, \quad S_5 = 2S_{13},
\]
\[
T_6 = T_{12}, \quad S_6 = 2S_{12}.
\]

- Elasto-electric matrix for quartz:

\[e\] indicates negative of \(c\)
\[\odot\] indicates twice the numerical equalities
\[\times\] indicates \(1/2\) \((c_{11}-c_{12})\)
Mathematical Description - Continued

- Number of independent non-zero constants depend on crystal symmetry. For quartz (trigonal, class 32), there are 10 independent linear constants - 6 elastic, 2 piezoelectric and 2 dielectric. "Constants" depend on temperature, stress, coordinate system, etc.

- To describe the behavior of a resonator, the differential equations for Newton's law of motion for a continuum, and for Maxwell's equation* must be solved, with the proper electrical and mechanical boundary conditions at the plate surfaces. ( \( P = ma \Rightarrow \frac{\partial T_{ij}}{\partial x_j} = \rho \ddot{u}_i; \quad \nabla \cdot D = 0 \Rightarrow \frac{\partial D_i}{\partial x_i} = 0, \)
  \( E_i = -\frac{\partial \phi}{\partial x_i}; \quad S_{ij} = \frac{1}{2} (\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}); \) etc.)

- Equations are very "messy" - have never been solved in closed form for physically realizable three-dimensional resonators. Nearly all theoretical work has used approximations.

- Some of the most important resonator phenomena (e.g., acceleration sensitivity) are due to nonlinear effects. Quartz has numerous higher order constants, e.g., 14 third-order and 23 fourth-order elastic constants, as well as 16 third-order piezoelectric coefficients are known; nonlinear equations are extremely messy.

* Magnetic field effects are generally negligible; quartz is diamagnetic.
Infinite Plate Thickness Shear Resonator

\[ f_n = \frac{n}{2h} \sqrt{\frac{c_{ij}}{\rho}} \quad n = 1, 3, 5 \]

Where

- \( f_n \) = resonant frequency of \( n \)-th harmonic
- \( h \) = plate thickness
- \( \rho \) = density
- \( c_{ij} \) = elastic modulus associated with the elastic wave being propagated

\[ T_f = \frac{d(\log f_n)}{dT} = \frac{1}{f_n} \frac{df_n}{dT} = \frac{1}{h} \frac{dh}{dT} - \frac{1}{2\rho} \frac{d\rho}{dT} + \frac{1}{2c_{ij}} \frac{dc_{ij}}{dT} \]

\( T_f \), the linear temperature coefficient of frequency, is negative for most materials (i.e., "springs" become "softer" as \( T \) increases). The coefficient for quartz can be +, - or zero (see next page).
Quartz Is Highly Anisotropic

- The properties of quartz vary greatly with crystallographic direction. For example, when a quartz sphere is etched deeply in HF, the sphere takes on a triangular shape when viewed along the Z-axis, and a lenticular shape when viewed along the Y-axis. The etching rate is more than 100 times faster along the fastest etching rate direction (the Z-direction) than along the slowest direction (the slow-X-direction).

- The thermal expansion coefficient is $7.8 \times 10^{-6}/^\circ C$ along the Z-direction, and $14.3 \times 10^{-6}/^\circ C$ perpendicular to the Z-direction; the temperature coefficient of density is, therefore, $-36.4 \times 10^{-6}/^\circ C$.

- The temperature coefficients of the elastic constants range from $-3300 \times 10^{-6}/^\circ C$ (for $C_{12}$) to $+164 \times 10^{-6}/^\circ C$ (for $C_{66}$).

- For the proper angles of cut, the sum of the first two terms in $T_f$ on the previous page is cancelled by the third term, i.e., temperature compensated cuts exist in quartz. (See next page.)
Zero Temperature Coefficient Quartz Cuts

The AT, FC, IT, SC, BT, and RT-cuts are on the loci of zero temperature coefficient cuts. The LC is a "linear coefficient" cut that is used in a thermometer.

Y-cut: $\approx +90 \text{ ppm/}^\circ\text{C}$
(thickness-shear mode)

X-cut: $\approx -20 \text{ ppm/}^\circ\text{C}$
(extensional mode)
Equivalent Circuits

The mechanically vibrating system and the circuit shown in the figure are "equivalent," because each can be described by the same differential equation. The mass, spring and damping element (i.e., the dashpot) correspond to the inductor, capacitor and resistor. The driving force corresponds to the voltage, the displacement of the mass to the charge on the capacitor, and the velocity to the current.

A crystal resonator is a mechanically vibrating system that is linked, via the piezoelectric effect, to the electrical world. In the (simplified) equivalent circuit (of one mode of vibration) of a resonator, on the next page, $C_0$ is called the "shunt" capacitance. It is the capacitance due to the electrodes on the crystal plate (plus the stray capacitances due to the crystal enclosure). The $R_1$, $L_1$, $C_1$ portion of the circuit is the "motional arm" which arises from the mechanical vibrations of the crystal. The $C_0$ to $C_1$ ratio is a measure of the interconversion between electrical and mechanical energy stored in the crystal, i.e., of the piezoelectric coupling factor, $k$, and $C_1$ is a measure of the crystal's "stiffness," i.e., its tunability - see the equation under the equivalent circuit on the next page. When a dc voltage is applied to the electrodes of a resonator, the $C_0/C_1$ is also a measure of the ratio of electrical energy stored in the capacitor formed by the electrodes to the energy stored elastically in the crystal due to the lattice strains produced by the piezoelectric effect. The $C_0/C_1$ is also a measure of the antiresonance-resonance frequency separation. (Let $r = C_0/C_1$, then $f_A - f_R = f_R/2r$, and $2r = (\pi N/2k)^2$, where $N = 1,3,5...$ is the overtone number.)

Some of the numerous advantages of a quartz crystal resonator over a tank circuit built from discrete R's, C's and L's are that the crystal is far stiffer and has a far higher Q than what could be built from normal discrete components. For example, a 5 MHz fundamental mode AT-cut crystal may have $C_1 = 0.01$ pF, $L_1 = 0.1$ H, $R_1 = 5$ $\Omega$, and $Q = 10^6$. A 0.01 pF capacitor is not available, since the leads attached to such a capacitor would alone probably contribute more than 0.01 pF. Similarly, a 0.1 H inductor would be physically large, it would need to include a large number of turns, and would need to be superconducting in order to have a $\leq 5$ $\Omega$ resistance.
Equivalent Circuit of a Resonator

Symbol for crystal unit

\[ \frac{\Delta f}{f_S} \approx \frac{C_1}{2(C_0 + C_L)} \]

\[ \begin{align*} 
1. \text{ Voltage control (VCXO)} \\
2. \text{ Temperature compensation (TCXO)} 
\end{align*} \]
Crystal Oscillator $f$ vs. $T$ Compensation

Diagram showing the relationship between frequency/voltage and temperature, with compensated and uncompensated frequency curves.
Resonator Frequency vs. Reactance

Area of Usual "Parallel Resonance"

Antiresonance

Series Resonance

\[ \frac{1}{2\pi f C_0} \]

\[ f_A \]
Equivalent Circuit Parameter Relationships

\[ C_0 \approx \varepsilon \frac{A}{t} \]

\[ f_s = \frac{1}{2\pi \sqrt{\frac{1}{C_1 L_1}}} \]

\[ Q = \frac{1}{2\pi f_s R_1 C_1} \]

\[ \tau_1 = R_1 C_1 \approx 10^{-14} \text{s} \]

\[ r = \frac{C_0}{C_1} \]

\[ f_a - f_s \approx \frac{f_s}{2r} \]

\[ \omega L_1 - \frac{1}{\omega C_1} \]

\[ \varphi = \frac{\omega L_1 - \frac{1}{\omega C_1}}{R_1} \]

\[ \frac{d\varphi}{df} \approx \frac{360}{\pi f_s} \frac{Q}{f_s} \]

\[ C_n \propto \frac{f C_1}{n^3} \quad L_n \propto \frac{n^3 L_1}{f^3} \quad R_n \propto \frac{n^3 R_1}{f} \]

\[ 2r = \left( \frac{\pi n}{2k} \right)^2 \]

- **C_0**: Static capacitance
- **C_1**: Motional capacitance
- **L_1**: Motional inductance
- **R_1**: Motional resistance
- **\varepsilon**: Dielectric permittivity of quartz
  
  \[ = 40 \times 10^{-3} \text{ pF/mm (average)} \]
- **A**: Electrode area
- **t**: Plate thickness
- **f_s**: Series resonance frequency = \( f_R \)
- **f_a**: Antiresonance frequency
- **Q**: Quality factor
- **\tau_1**: Motional time constant
- **r**: Capacitance ratio
- **\omega**: Angular frequency = \( 2\pi f \)
- **\varphi**: Phase angle of the impedance
- **n**: Overtone number
- **k**: Piezoelectric coupling factor
  
  \[ = 8.8\% \text{ for AT-cut, 4.99\% for SC} \]
What Is Q and Why Is It Important?

\[ Q = \frac{\text{Energy stored during a cycle}}{\text{Energy lost during the cycle}} \]

- Q is proportional to the decay-time, and is inversely proportional to the linewidth of resonance (see next page).

- The higher the Q, the higher the frequency stability and accuracy **capability** of a resonator. If, e.g., Q=10^6, then 10^{-10} accuracy requires determining center of resonance curve to 0.01% of the linewidth, and stability (for some averaging time) of 10^{-12} requires ability to stay at same point on resonance curve to 10^{-6} of linewidth.

- Phase noise close to the carrier has an especially strong dependence on Q \( (S_\phi \propto 1/Q^4) \).
Decay Time, Linewidth, and Q

\[ W = \frac{1}{t_d} \]

\[ Q = \frac{v_0}{W} = v_0 t_d \]
Factors That Determine Resonator Q

The maximum Q of a quartz crystal resonator is given by:

$$Q_{\text{max}} = \frac{1}{2\pi f \tau},$$

where \( f \) is the frequency in Hz, and \( \tau \) is an empirically determined time constant in seconds, which varies with the angles of cut and the mode of vibration. For example, \( \tau = 1 \times 10^{-14} \text{s} \) for the AT-cut’s c-mode (\( Q_{\text{max}} = 3.2 \text{ million at 5 MHz} \)), \( \tau = 9.9 \times 10^{-15} \text{s} \) for the SC-cut’s c-mode, and \( \tau = 4.9 \times 10^{-15} \text{s} \) for the BT-cut’s b-mode.

Other factors which affect the Q of a resonator include:

- Overtone
- Surface finish
- Material impurities and defects
- Mounting stresses
- Bonding stresses
- Temperature
- Electrode geometry and type
- Blank geometry (contour, dimensional ratios)
- Drive level
- Gases inside the enclosure (pressure, type of gas)
- Interfering modes
- Ionizing radiation
Resonator Fabrication Steps

1. Design Resonators
2. Grow Quartz
3. Sweep
4. Etch (Chemical Polish)
5. Clean
6. Orient in Mask
7. Deposit Contacts
8. Prepare Enclosure
9. Mount
10. Bond
11. Inspect
12. Contour
13. Angle Correct
14. X-Ray Orient
15. Clean
16. Lap
17. Round
18. Final Clean
19. Plate
20. Frequency Adjust
21. Seal
22. Test
23. Oscillator
## Milestones in Quartz Technology

<table>
<thead>
<tr>
<th>Year</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>1880</td>
<td>Piezoelectric effect discovered by Jacques and Pierre Curie</td>
</tr>
<tr>
<td>1905</td>
<td>First hydrothermal growth of quartz in a laboratory - by G. Spezia</td>
</tr>
<tr>
<td>1917</td>
<td>First application of piezoelectric effect, in sonar</td>
</tr>
<tr>
<td>1918</td>
<td>First use of piezoelectric crystal in an oscillator</td>
</tr>
<tr>
<td>1926</td>
<td>First quartz crystal controlled broadcast station</td>
</tr>
<tr>
<td>1927</td>
<td>First temperature compensated quartz cut discovered</td>
</tr>
<tr>
<td>1927</td>
<td>First quartz crystal clock built</td>
</tr>
<tr>
<td>1934</td>
<td>First practical temp. compensated cut, the AT-cut, developed</td>
</tr>
<tr>
<td>1949</td>
<td>Contoured, high-Q, high stability AT-cuts developed</td>
</tr>
<tr>
<td>1956</td>
<td>First commercially grown cultured quartz available</td>
</tr>
<tr>
<td>1956</td>
<td>First TCXO described</td>
</tr>
<tr>
<td>1972</td>
<td>Miniature quartz tuning fork developed; quartz watches available</td>
</tr>
<tr>
<td>1974</td>
<td>The SC-cut (and TS/TTC-cut) predicted; verified in 1976</td>
</tr>
<tr>
<td>1982</td>
<td>First MCXO with dual c-mode self-temperature sensing</td>
</tr>
</tbody>
</table>
Chapter 3 References


Oscillator Stability
The Units of Stability in Perspective

- What is one part in $10^{10}$? (As in $1 \times 10^{-10}$ per day aging.)
  - $\sim 1/2$ cm out of the circumference of the earth.
  - $\sim 1/4$ second per human lifetime ($\sim 80$ years).

- What is -170 dB? (As in -170 dBc/Hz phase noise.)
  - $-170$ dB = 1 part in $10^{17}$ = thickness of a sheet of paper out of the total distance traveled by all the cars in the USA in a day.
Accuracy, Precision and Stability

<table>
<thead>
<tr>
<th>Condition</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accurate and precise</td>
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</tr>
<tr>
<td>Accurate but not precise</td>
<td></td>
</tr>
<tr>
<td>Not accurate and not precise</td>
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</tr>
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<td>Precise but not accurate</td>
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</tr>
<tr>
<td>Stable but not accurate</td>
<td></td>
</tr>
<tr>
<td>Not stable and not accurate</td>
<td></td>
</tr>
</tbody>
</table>
Influences on Oscillator Frequency

- **Time**
  - Short term (noise)
  - Intermediate term (e.g., due to oven fluctuations)
  - Long term (aging)

- **Temperature**
  - Static frequency vs. temperature
  - Dynamic frequency vs. temperature (warmup, thermal shock)
  - Thermal history ("hysteresis," "retrace")

- **Acceleration**
  - Gravity (2g tipover)
  - Vibration
  - Acoustic noise
  - Shock

- **Ionizing radiation**
  - Steady state
  - Pulsed
  - Photons (X-rays, γ-rays)
  - Particles (neutrons, protons, electrons)

- **Other**
  - Power supply voltage
  - Humidity
  - Magnetic field
  - Atmospheric pressure (altitude)
  - Load impedance
Idealized Frequency-Time-Influence Behavior

- Temperature Step
- Vibration
- Shock
- Oscillator Turn Off & Turn On
- 2-g Tipover
- Radiation
- Off
- Aging
- On
- Short-Term Instability

\[ \frac{\Delta f}{f} \times 10^8 \]
Aging and Short-Term Stability

Short-term instability
(Noise)
Aging Mechanisms

- Mass transfer due to contamination
  Since $f \propto 1/t$, $\Delta f/f = -\Delta t/t$; e.g., $f_{5\text{MHz}} \approx 10^6$ molecular layers, therefore, 1 quartz-equivalent monolayer $\Rightarrow \Delta f/f \approx 1$ ppm

- Stress relief in the resonator's: mounting and bonding structure, electrodes, and in the quartz (?)

- Other effects
  - Quartz outgassing
  - Diffusion effects
  - Chemical reaction effects
  - Pressure changes in resonator enclosure (leaks and outgassing)
  - Oscillator circuit aging (load reactance and drive level changes)
  - Electric field changes (doubly rotated crystals only)
  - Oven-control circuitry aging
Typical Aging Behaviors

A(t) = 5 Ln(0.5t+1)

A(t) + B(t)

B(t) = -35 Ln(0.006t+1)

Aging can be positive or negative. Occasionally, a reversal of aging direction is observed. The above (computer generated) curves illustrate the three types of aging behaviors. The curve showing the reversal is the sum of the other two curves. Reversal indicates the presence of at least two aging mechanisms.
Thermal Expansion Coefficient of Quartz

The graph shows the thermal expansion coefficient, $\alpha$, of AT-cut quartz, $10^{-6} ^\circ K^{-1}$, as a function of orientation, $\psi$, with respect to $xx^\dagger$. The graph includes two curves: one for radial expansion and one for tangential expansion. The thermal expansion coefficient for thickness is $11.64 \times 10^{-6} ^\circ K^{-1}$. The orientation range is from $0^\circ$ to $90^\circ$.
Force-Frequency Coefficient

\[ \Delta f = K_F \frac{(\text{Force})(\text{Frequency-constant})}{(\text{Diameter})(\text{Thickness})} \]
Photograph of a 1 cm diameter AT-cut resonator and its X-ray topograph. The topograph shows the lattice distortion due to the mounting stresses.
Strains Due To Bonding Cements

X-ray topographs showing lattice distortions caused by bonding cements; (a) Bakelite cement - expanded upon curing, (b) DuPont 5504 cement - shrank upon curing.
Mounting Force Induced Frequency Changes

The force-frequency coefficient, $K_F(\Psi)$, is defined by

$$\frac{\Delta f}{f} = K_F \frac{\text{(Force)} \ (\text{Frequency-constant})}{\text{(Diameter)} \ (\text{Thickness})}$$

Maximum $K_F$ (AT-cut) = $24.5 \times 10^{-15}$ m-s/N at $\Psi = 0^\circ$

Maximum $K_F$ (SC-cut) = $14.7 \times 10^{-15}$ m-s/N at $\Psi = 44^\circ$.

As an example, consider 5 MHz 3rd overtone, 14 mm diameter resonators. Then, since 1 gram = $9.81 \times 10^{-3}$ newtons, and assuming the presence of diametrical forces only,

$$\left(\frac{\Delta f}{f}\right)_{\text{Max}} = \begin{cases} 2.9 \times 10^{-8} \text{ per gram for an AT-cut resonator} \\ 1.7 \times 10^{-8} \text{ per gram for an SC-cut resonator} \end{cases}$$

$$\left(\frac{\Delta f}{f}\right)_{\text{Min}} = 0 \text{ at } \Psi = 61^\circ \text{ for an AT-cut resonator, and at } \Psi = 82^\circ \text{ for an SC-cut.}$$
Bonding Strains Induced Frequency Changes

When 22 MHz fundamental mode AT-cut resonators were reprocessed so as to vary the bonding orientations, the $f$ vs. $T$ characteristics of the resonators changed as if the angles of cut had been changed. The resonator blanks were 6.4 mm in diameter, plano-plano, and were bonded to low-stress mounting clips by nickel electrobonding.
Short Term Instability (Noise)

Stable Frequency (Ideal Oscillator)

\[ V(t) = V_0 \sin(2\pi v_0 t) \]

\[ \Phi(t) = 2\pi v_0 t \]

Unstable Frequency (Real Oscillator)

\[ V(t) = [V_0 + \varepsilon(t)] \sin[2\pi v_0 t + \phi(t)] \]

\[ \Phi(t) = 2\pi v_0 t + \phi(t) \]

Instantaneous frequency, \( v(t) = \frac{1}{2\pi} \frac{d\Phi(t)}{dt} = v_0 + \frac{1}{2\pi} \frac{d\phi(t)}{dt} \)

\( V(t) = \) Oscillator output voltage, \( V_0 = \) Nominal peak voltage amplitude

\( \varepsilon(t) = \) Amplitude noise,

\( v_0 = \) Nominal (or "carrier") frequency

\( \Phi(t) = \) Instantaneous phase, and \( \phi(t) = \) Deviation of phase from nominal (i.e., the ideal)
Impacts of Oscillator Noise

- Limits the ability to determine the current state and the predictability of precision oscillators
- Limits synthonization and synchronization accuracy
- Limits receivers' useful dynamic range, channel spacing, and selectivity; can limit jamming resistance
- Limits radar performance (especially Doppler radar's)
- Causes timing errors $[\sim \tau \sigma_y(\tau)]$
- Causes bit errors in digital communication systems
- Limits navigation accuracy
- Limits ability to lock to narrow-linewidth (atomic) resonances
- Can cause loss of lock; can limit acquisition/reacquisition capability in phase-locked-loop systems
Causes of Short Term Instabilities

- Temperature fluctuations - thermal transient effects
  - activity dips at oven set-point
- Johnson noise (thermally induced charge fluctuations, i.e., "thermal emf" in resistive elements)
- Acoustic losses (i.e., Q)
- Random vibration
- Fluctuations in the number of adsorbed molecules
- Stress relief, fluctuations at interfaces (quartz, electrode, mount, bond)
- Noise due to oscillator circuitry (active and passive components)
- Shot noise in atomic frequency standards
- ???
Time Domain - Frequency Domain

Example (a) shows a sine wave and its second harmonic. A signal consisting of the sum of the two waves is shown in the time domain (b), and in the frequency domain (c). In the time domain, all frequency components are summed together. In the frequency domain, signals are separated into their frequency components and the power level at each frequency is displayed.
# Short-Term Stability Measures

<table>
<thead>
<tr>
<th>Measure</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two-sample deviation (square-root of Allan variance)</td>
<td>$\sigma_y(\tau)$*</td>
</tr>
<tr>
<td>Spectral density of phase deviations</td>
<td>$S_\phi(f)$</td>
</tr>
<tr>
<td>Spectral density of fractional frequency deviations</td>
<td>$S_y(f)$</td>
</tr>
<tr>
<td>Phase noise</td>
<td>$\mathcal{L}(f)$*</td>
</tr>
</tbody>
</table>

* Most frequently found on oscillator specification sheets

\[
f^2 S_\phi(f) = \nu^2 S_y(f); \quad \mathcal{L}(f) \equiv 1/2[S_\phi(f)] \text{ (per IEEE Std. 1139-1988)},
\]

and

\[
\sigma_y^2(\tau) = \frac{2}{(\pi \nu \tau)^2} \int_0^\infty S_\phi(f) \sin^4(\pi f \tau) df
\]

where $\tau = \text{averaging time}$, $f = \text{Fourier frequency}$, or "frequency from the carrier", and $\nu = \text{carrier frequency}$. 
Allan Variance

The two-sample deviation, or square-root of the "Allan variance," is the standard method of describing the short-term stability of oscillators in the time domain. It is usually denoted by $\sigma_y(\tau)$,

where

$$\sigma_y^2(\tau) = \frac{1}{2} <(y_{k+1} - y_k)^2> .$$

The fractional frequencies, $y = \frac{\Delta f}{f}$, are measured over a time interval, $\tau$; $(y_{k+1} - y_k)$ are the differences between pairs of successive measurements of $y$, and, ideally, $<>$ denotes a time average of an infinite number of $(y_{k+1} - y_k)^2$. A good estimate can be obtained by a limited number, $m$, of measurements ($m \geq 100$). $\sigma_y(\tau)$ generally denotes $\sqrt{\sigma_y^2(\tau, m)}$, i.e.,

$$\sigma_y^2(\tau) = \sigma_y^2(\tau, m) = \frac{1}{m} \sum_{j=1}^{m} \frac{1}{2} (y_{k+1} - y_k)^2_j$$
Why Allan Variance?

- Classical variance: \( \sigma^2 = \frac{1}{m-1} \sum (y_i - \bar{y})^2 \),

diverges for commonly observed noise processes, such as random walk, i.e., the variance increases with increasing number of data points.

- Allan variance:
  o Converges for all noise processes observed in precision oscillators.
  o Has straightforward relationship to power law spectral density types.
  o Is easy to compute.
  o Is faster and more accurate in estimating noise processes than the Fast Fourier Transform.
Frequency Noise and $\sigma_y(\tau)$

- $0.1$ s averaging time

- $1.0$ s averaging time

- $\sigma_y(\tau)$

Averaging time, $\tau$, s
Time Domain Stability

For $\sigma_y(\tau)$ to be a proper measure of random frequency fluctuations, aging must be properly subtracted from the data at long $\tau$'s.
Power Law Dependence of $\sigma_y(\tau)$

Below the flicker of frequency noise (i.e., the "flicker floor") region, crystal oscillators typically show $\tau^{-1}$ (white phase noise) dependence. Atomic standards show $\tau^{-1/2}$ (white frequency noise) dependence down to about the servo-loop time constant, and $\tau^{-1}$ dependence at less than that time constant. Typical $\tau$'s at the start of flicker floors are: 1 second for a crystal oscillator, $10^3$ s for a Rb standard and $10^5$ s for a Cs standard.
Spectral Densities

\[ V(t) = [V_0 + \varepsilon(t)] \sin [2\pi v_0 t + \phi(t)] \]

In the frequency domain, due to the "phase noise", \(\phi(t)\), some of the power is at frequencies other than \(v_0\). The stabilities are characterized by "spectral densities." The spectral density \(S_v(f)\), the mean-square voltage \(<V^2(t)\>\) in a unit bandwidth centered at \(f\), is not a good measure of frequency stability because both \(\varepsilon(t)\) and \(\phi(t)\) contribute to it, and because it is not uniquely related to frequency fluctuations (although \(\varepsilon(t)\) is usually negligible in precision frequency sources.)

The spectral densities of phase and fractional-frequency fluctuations, \(S_\phi(f)\) and \(S_y(f)\), respectively, are used to characterize stabilities in the frequency domain. The spectral density \(S_g(f)\) of a quantity \(g(t)\) is the mean square value of \(g(t)\) in a unit bandwidth centered at \(f\). Moreover, the RMS value of \(g^2\) in bandwidth \(BW\) is given by \(g_{\text{RMS}}^2 \equiv \int_{-\text{BW}}^{\text{BW}} S_g(f) \, df\).
## Pictures of Noise

<table>
<thead>
<tr>
<th>Plot of $z(t)$ vs. $t$</th>
<th>$S_z(f) = h_\alpha f^\alpha$</th>
<th>Noise name</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Plot" /></td>
<td>$\alpha = 0$</td>
<td>White</td>
</tr>
<tr>
<td><img src="image" alt="Plot" /></td>
<td>$\alpha = -1$</td>
<td>Flicker</td>
</tr>
<tr>
<td><img src="image" alt="Plot" /></td>
<td>$\alpha = -2$</td>
<td>Random walk</td>
</tr>
<tr>
<td><img src="image" alt="Plot" /></td>
<td>$\alpha = -3$</td>
<td></td>
</tr>
</tbody>
</table>

Plots show fluctuations of a quantity $z(t)$, which can be, e.g., the output of a counter ($\Delta f$ vs. $t$) or of a phase detector ($\phi[t]$ vs. $t$). The plots show simulated time-domain behaviors corresponding to the most common (power-law) spectral densities; $h_\alpha$ is an amplitude coefficient.

Note: since $S_{\Delta f} = f^2 S_{\phi}$, e.g. white frequency and random walk of phase are equivalent.
Mixer Functions

\[ V_1 = A_1 \sin(\omega_1 t + \phi_1) \]
\[ V_2 = A_2 \sin(\omega_2 t + \phi_2) \]

Trigonometric identities: \( \sin(x)\sin(y) = 1/2 \cos(x-y) - 1/2 \cos(x+y) \)
\( \cos(x \pm \pi/2) = \sin(x) \)

Let \( \omega_1 = \omega_2; \ \Phi_1 = \omega_1 t + \phi_1, \) and \( \Phi_2 = \omega_2 t + \phi_2. \) Then the mixer can become:

- **Phase detector**: When \( \Phi_1 = \Phi_2 + \pi/2 \) and \( A_1 = A_2 = 1, \) then
  \[ V_0 = \frac{1}{2} \sin(\phi_1 - \phi_2) = \frac{1}{2}(\phi_1 - \phi_2) \] for small \( \phi \)'s

- **AM detector**: When \( A_2 = 1 \) and the filter is a low-pass filter, then
  \[ V_0 = \frac{1}{2} A_1 \cos(\phi_1 - \phi_2); \] if \( \phi_1 \approx \phi_2, \) then \( V_0 \approx \frac{1}{2} A_1 \)

- **Frequency multiplier**: When \( V_1 = V_2 \) and the filter is band-pass at \( 2\omega_1, \) then
  \[ V_0 = \frac{1}{2} A_1^2 \cos(2\omega_1 t + 2\phi_1) \] \( \Rightarrow \) Doubles the frequency and phase error.
Phase Detector

The device under test (DUT) and a reference source, at the same frequency and in phase quadrature (i.e., 90° out of phase), are input to a double-balanced mixer. Then,

\[ V_O(t) = V(t) \quad V_R(t) = K \cos[\phi(t) - \phi_R(t) + \pi/2] + K \cos[2\pi(v + \nu_R)t + \ldots] \]

The low-pass filter (LPF) eliminates the second cosine term. For

\[ \phi_R(t) \ll \phi(t) \ll \pi/2, \quad V_\phi(t) = K \phi(t), \]

i.e., the phase detector converts phase fluctuations to voltage fluctuations.
Phase Noise Measurements

\[ V(t) = V_o \sin [2\pi v_o t + \Phi(t)] \]

\[ \Phi(t) \]

\[ \Phi_{\text{RMS}}(t) \text{ in BW of meter} \]

\[ S_\phi(f) \text{ vs. } f \]
Frequency - Phase - Time Relationships

\[ v(t) = v_0 + \frac{1}{2\pi} \frac{d\phi(t)}{dt} \]

"instantaneous" frequency,

\[ y(t) = \frac{v(t) - v_0}{2\pi} \]

normalized frequency,

\[ S_\phi(f) = \frac{\phi_{\text{RMS}}}{\text{BW}} = \frac{V_0}{\sqrt{2}} \int_s^t S_y(f) \sin^2(\pi f_c dt) \]

\[ \sigma_y^2(t) = \frac{1}{2} \left< (y_{k+1} - y_k)^2 \right> = \frac{2}{\pi v_0^2 (\pi f_c)^2} \int_s^t S_\phi(f) \sin^4(\pi f_c dt) \]

The five common power-law noise processes in precision oscillators are:

\[ S_y(f) = h_2^2 f^2 + h_1 f^1 + h_0 + h_1 f^1 + h_2 f^2 \]

(White PM) (Flicker PM) (White FM) (Flicker FM) (Random-walk FM)

Time deviation = \[ x(t) = \int_0^t y(t') dt' = \frac{2\pi v_0}{\phi(t)} \]

* MIL-O-55310B's definition of phase noise is \( L(f) = 10 \log \left[ S_\phi(f) / 2 \right] \), where

the unit of \( L(f) \) is dBc.
$S_\phi(f)$ to SSB Power Ratio Relationship

Consider the "simple" case of sinusoidal phase modulation at frequency $f_m$. Then, $\phi(t) = \phi_0(t)\sin(2\pi f_m t)$, and $V(t) = V_0\cos[2\pi f_c t + \phi(t)] = V_0\cos[2\pi f_c t + \phi_0(t)\sin(2\pi f_m t)]$, where $\phi_0(t) = $ peak phase excursion, and $f_c = $ carrier frequency. Cosine of a sine function suggests a Bessel function expansion of $V(t)$ into its components at various frequencies via the identities:

\[
\begin{align*}
\cos(X + Y) &= \cos X \cos Y - \sin X \sin Y \\
\cos X \cos Y &= \frac{1}{2}[\cos(X + Y) + \cos(X - Y)] \\
-\sin X \sin Y &= [\cos(X + Y) - \cos(X - Y)] \\
\cos(B\sin X) &= J_0(B) + 2\sum_{n=1}^\infty J_{2n}(B) \cos(2nX) \\
\sin(B\sin X) &= 2\sum_{n=0}^\infty J_{2n+1}(B) \sin((2n + 1)X)
\end{align*}
\]

After some messy algebra, $S_v(f)$ and $S_\phi(f)$ are as shown on the next page. Then,

SSB Power Ratio at $f_m = \frac{V_0^2 J_1^2[\Phi(f_m)]}{V_0^2 J_0^2[\Phi(f_m)] + 2\sum i=1^\infty J_i^2[\Phi(f_m)]}

if $\Phi(f_m) << 1$, then $J_0 = 1, J_1 = 1/2\Phi(f_m), J_n = 0$ for $n > 1$, and

SSB Power Ratio = $L(f_m) = \frac{\Phi^2(f_m)}{4} = \frac{S_\phi(f_m)}{2}$
$S_{\phi}(f)$, $S_{v}(f)$ and $\mathcal{L}(f)$

\[ \Phi(t) = \Phi(f_m) \cos(2\pi f_m t) \]

\[ S_{\phi}(f) = \frac{\Phi^2}{2} \delta(f - f_m) \]

\[ V(t) = V_0 \cos[2\pi f_c t + \Phi(f_m)] \]

\[ S_{v}(f) = \frac{V_0^2 J_0^2}{2} \delta(f - f_m) + \frac{V_0^2 J_1^2}{2} \delta(f - f_c) + \frac{V_0^2 J_2^2}{2} \delta(f - f_{c+f_m}) + \frac{V_0^2 J_3^2}{2} \delta(f - f_{c+3f_m}) \]

SSB Power Ratio $= \frac{V_0^2 J_1^2[\Phi(f_m)]}{V_0^2 J_0^2[\Phi(f_m)] + 2 \sum_{i=1}^{n} J_i^2[\Phi(f_m)]}$

$\equiv \mathcal{L}(f_m) \equiv \frac{S_{\phi}(f_m)}{2}$
Types of Phase Noise

\[ L(f_f) \]

- 40 dB/Decade \((f_f^{-4})\)
  - Random Walk of Frequency
- 30 dB/Decade \((f_f^{-3})\)
  - Flicker of Frequency
- 20 dB/Decade \((f_f^{-2})\)
  - White Frequency; Random Walk of Phase
- 10 dB/Decade \((f_f^{-1})\)
  - Flicker of Phase
- 0 dB/Decade \((f_f^0)\)
  - White Phase

\(~\text{BW of Resonator}\)

\(~\text{Fourier Frequency}\)
\((\text{Sideband Frequency})\)
\((\text{Offset Frequency})\)
\((\text{Modulation Frequency})\)
Noise in Crystal Oscillators

- The resonator is the primary noise source close to the carrier; the oscillator circuitry is the primary source far from the carrier.

- Frequency multiplication by N increases the phase noise as N^2 (i.e., by 20 log N, in dB's).

- Vibration-induced "noise" can dominate all other sources of noise in many applications (see acceleration effects section, later).

- Close to the carrier (within BW of resonator), S_y(f) varies as 1/f, S_\phi (f) as 1/f^3, where f = offset from carrier frequency, v; S_\phi(f) also varies as 1/Q^4, where Q = unloaded Q. Since Q_{max}v = const., S_\phi(f) \propto v^4. (Q_{max}v)_{BAW} = 1.6 \times 10^{13} \text{ Hz}; (Q_{max}v)_{SAW} = 1.05 \times 10^{13} \text{ Hz}.

- In the time domain, noise floor is \sigma_y(\tau) \geq (2.0 \times 10^{-7})Q^{-1} \approx 1.2 \times 10^{-20} v, v in Hz. In the regions where \sigma_y(\tau) varies as \tau^{-1} and \tau^{-1/2} (\tau^{-1/2} occurs in atomic frequency standards), \sigma_y(\tau) \propto (Q S_R)^{-1}, where S_R is the signal-to-noise ratio; i.e., the higher the Q and the signal-to-noise ratio, the better the short term stability (and the phase noise far from the carrier, in the frequency domain).

- Loaded Q of oscillator affects noise when the oscillator circuitry is a significant noise source.

- Noise floor is limited by Johnson noise; noise power, kT = -174 dBm/Hz at 290^0K.

- Higher signal level will improve the noise floor but not the close-in noise. (In fact, high drive levels generally degrade the close-in noise.)

- Low noise SAW vs. low noise BAW multiplied up: BAW is lower noise at f < ~1 kHz, SAW is lower noise at f > ~1 kHz; can phase lock the two to get the best of both.
Low-Noise SAW and BAW Multiplied to 10 GHz
(in a nonvibrating environment)

BAW = bulk-acoustic wave oscillator
SAW = surface acoustic wave oscillator

 Offset frequency in Hz

 $L(f)$ in dBc/Hz
Low-Noise SAW and BAW Multiplied to 10 GHz
(in a vibrating environment)

Vibration induced phase noise dominates the phase noise of both (whichever has lower acceleration sensitivity will have lower phase noise; currently, BAW can provide lower sensitivity than SAW.) Illustration assumes $1 \times 10^{-9}/g$ acceleration sensitivity for both BAW and SAW, and 0.01 $g^2/Hz$ random vibration power spectral density at all vibration frequencies.
Effects of Frequency Multiplication

\[ f_i \equiv f_{in} \]
\[ \Delta f_i \]
\[ \frac{\Delta f_i}{f_i} \equiv y \]
\[ \Delta \phi_i \]
\[ \mathcal{L}(f)_i \]
\[ S_\phi(f)_i \]
\[ S_y(f)_i \]
\[ \sigma_y(\tau)_i \]

\[ f_i \times M = f_o \]

Noiseless Multiplier

\[ f_o \equiv f_{out} = Mf_i \]
\[ \Delta f_o = M\Delta f_i \]
\[ \frac{\Delta f_o}{f_o} = \frac{\Delta f_i}{f_i} \]
\[ \Delta \phi_o = M\Delta \phi_i \]
\[ \mathcal{L}(f)_o = \mathcal{L}(f)_i + 20 \log M \]
\[ S_\phi(f)_o = M^2 S_\phi(f)_i \]
\[ S_y(f)_o = S_y(f)_i \]
\[ \sigma_y(\tau)_o = \sigma_y(\tau)_i \]

Note that \( y = \frac{\Delta f}{f} \), \( S_y(f) \), and \( \sigma_y(\tau) \) are unaffected by frequency multiplication.
Quartz Wristwatch Accuracy vs. Temperature

Temperature coefficient of frequency = -0.035 ppm/°C²
Frequency vs. Temperature Characteristic

This frequency vs. temperature characteristic is typical of AT-cut and SC-cut resonators. The upper and lower turnover points (UTP and LTP) are the points where \( \frac{df(T)}{dT} = 0 \). The inflection point is where \( \frac{d^2f(T)}{dT^2} = 0 \).

The inflection temperatures are \( \approx 26^\circ C \) for AT-cuts, and \( \approx 96^\circ \) to \( 105^\circ C \) for SC-cuts.
Resonator $f$ vs. $T$ Determining Factors

- **Primary:** Angles of cut

- **Secondary:**
  - Overtone
  - Blank geometry (contour, dimensional ratios)
  - Material impurities and strains
  - Mounting & bonding stresses (magnitude and direction)
  - Electrodes (size, shape, thickness, density, stress)
  - Drive level
  - Interfering modes
  - Load reactance (value & temperature coefficient)
  - Temperature rate of change
  - Thermal history
  - Ionizing radiation
Frequency-Temperature vs. Angle-of-Cut, AT-cut

\[ \theta = 35^\circ 20' + \Delta \theta, \phi = 0 \]

for 5th overtone AT-cut

\[ \theta = 35^\circ 12.5' + \Delta \theta, \phi = 0 \] for fundamental mode plano-plano AT-cut
Desired $f$ vs. $T$ for SC-cut Resonator

frequency remains within $\pm$ 1 ppm over a $\pm$ 25°C range about $T_i$
OCXO Oven’s Effect on Stability

Typical $f$ vs. $T$ characteristic for AT and SC-cut resonators

**Oven Parameters vs. Stability for SC-cut Oscillator**
assuming $T_i - T_{LTP} = 10^\circ C$

<table>
<thead>
<tr>
<th>$T_i - T_{LTP} = 10^\circ C$</th>
<th>Oven Cycling Range (millidegrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Oven Offset (millidegrees)</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>$4 \times 10^{-12}$</td>
<td>$4 \times 10^{-13}$</td>
<td>$4 \times 10^{-14}$</td>
<td>$4 \times 10^{-15}$</td>
</tr>
<tr>
<td>10</td>
<td>$6 \times 10^{-13}$</td>
<td>$4 \times 10^{-14}$</td>
<td>$4 \times 10^{-15}$</td>
<td>$4 \times 10^{-16}$</td>
</tr>
<tr>
<td>1</td>
<td>$2 \times 10^{-13}$</td>
<td>$6 \times 10^{-15}$</td>
<td>$4 \times 10^{-16}$</td>
<td>$4 \times 10^{-17}$</td>
</tr>
<tr>
<td>0.1</td>
<td>$2 \times 10^{-13}$</td>
<td>$2 \times 10^{-15}$</td>
<td>$6 \times 10^{-17}$</td>
<td>$4 \times 10^{-18}$</td>
</tr>
<tr>
<td>0</td>
<td>$2 \times 10^{-13}$</td>
<td>$2 \times 10^{-15}$</td>
<td>$2 \times 10^{-17}$</td>
<td>$2 \times 10^{-19}$</td>
</tr>
</tbody>
</table>

A comparable table for AT and other non-thermal-transient compensated cuts of oscillators would not be meaningful because the dynamic $f$ vs. $T$ effects would generally dominate the static $f$ vs. $T$ effects.
Effect of Load Capacitance on $f$ vs. $T$

The $f$ vs. $T$ characteristics with and without a load capacitor: 1. $C_L$ raises the frequency at all $T$'s (curve with $f_L$ has been vertically displaced for clarity), 2. $C_L$ rotates the $f$ vs. $T$ to lower apparent angle of cut, i.e., it reduces peak-to-peak $f$ and turning-point-to-turning-point $T$, and 3. temperature coefficient of $C_L$ can greatly amplify $f$ vs. $T$ rotation.
Effects of Harmonics on $f$ vs. $T$

AT-cut

Reference angle-of-cut ($\theta$) is about 8 minutes higher for the overtone modes. (For the overtone modes of the SC-cut, the reference $\theta$-angle-of-cut is about 30 minutes higher.)
Warmup of AT- and SC-cut Resonators

\[ T = \tilde{\alpha} \frac{dT}{dt}, \]
where, for example, \( \tilde{\alpha} \approx -2 \times 10^{-7} \text{ s/K}^2 \)
for a typical AT-cut resonator.
TCXO Thermal Hysteresis

TCXO = Temperature Compensated Crystal Oscillator
In (a), the oscillator was kept on continuously while the oven was cycled off and on. In (b), the oven was kept on continuously while the oscillator was cycled off and on.
In TCXO's, temperature sensitive reactances are used to compensate for f vs. T variations. A variable reactance is also used to compensate for TCXO aging. The effect of the adjustment for aging on f vs. T stability is the "trim effect." Curves show f vs. T stability of a "0.5 ppm TCXO," at zero trim and at ±6 ppm trim. (Curves have been vertically displaced for clarity.)
Why the Trim Effect?

\[ \frac{\Delta f}{f_s} \approx \frac{C_1}{2(C_0 + C_L)} \]

Compensating \( f \) vs. \( T \)

Compensating \( C_L \) vs. \( T \)
Activity dips in the f vs. T when operated with an without load capacitors. (Curves have been vertically displaced for clarity.) Dip temperatures are a function of $C_L$, which indicates that the dip is caused by a mode (probably flexure) with a large negative temperature coefficient. See also "Unwanted Modes vs. Temperature" in Chapter 3.
Oscillator Circuit Caused Instabilities

- **Effect of load reactance change:**

  Let \( \delta f \equiv \frac{\Delta f}{f} = \frac{C_1}{2(C_0 + C_L)} \),

  then, \( \frac{\Delta(\delta f)}{\Delta C_L} = -\frac{C_1}{2(C_0 + C_L)^2} \).

**Example:** If \( C_0 = 5 \text{ pF}, C_1 = 14 \text{ fF} \) and \( C_L = 20 \text{ pF} \), then \( \Delta C_L = 10 \text{ fF (}=5 \times 10^{-4}) \) causes \( \approx 1 \times 10^{-7} \) frequency change, and \( C_L \) aging of 10 ppm per day causes \( 2 \times 10^{-9} \) per day of oscillator aging.

- **Drive level changes:** Typically \( 10^{-8} \) per \( \text{ma}^2 \) for a 10 MHz 3rd SC-cut.

- **DC bias** on the crystal also contributes to oscillator aging.
Frequency vs. Drive Level

Frequency Change (parts in $10^6$)

Crystal Current (μ amps)

- 5 MHz AT
- 3 diopter 10 MHz SC
- 2 diopter 10 MHz SC
- 1 diopter 10 MHz SC
- 10 MHz BT
At high drive levels, resonance curves become asymmetric due to the nonlinearities of quartz.
Drive Level vs. Resistance

Anomalous starting resistance

Normal operating range

Drive level effects

Resistance $R_x$

$I_x$ (mA)

$10^{-3}$ $10^{-2}$ $10^{-1}$ $1$ $10$ $100$
A 'good' crystal will follow the path OABCBAO without hysteresis. A 'bad' crystal will follow a path OADBCBAO: hence the term 'second level of drive'. On again increasing the drive, there is a tendency for the magnitude of the effect to decrease, but in a very irregular and irreproducible manner. The effect is usually due to particulate contamination, loose electrodes, or other surface defects.
Frequency shift is a function of the magnitude and direction of the acceleration, and is linear with magnitude up to at least 50 g’s.
2-g Tipover Test

(Δf vs. attitude about three axes)

- 10,000 MHz tipover test - ppb
  (F(max) - F(min))/2 = 1.889 - 09 (ccw)
  (F(max) - F(min))/2 = 1.863 - 09 (cw)
  delta THETA = 106.0 deg.

(F(max) - F(min))/2 = 6.841 - 10 (ccw)
(F(max) - F(min))/2 = 6.896 - 10 (cw)
delta THETA = 150.0 deg.

(F(max) - F(min))/2 = 1.882 - 09 (ccw)
(F(max) - F(min))/2 = 1.859 - 09 (cw)
delta THETA = 16.0 deg.
Sinusoidal Vibration Modulated Frequency

$$f_0 - \Delta f \quad f_0 + \Delta f$$

$$t = 0$$

$$t = \frac{\pi}{2f_v}$$

$$t = \frac{\pi}{f_v}$$

$$t = \frac{3\pi}{2f_v}$$

$$t = \frac{2\pi}{f_v}$$
## Acceleration Levels and Effects

<table>
<thead>
<tr>
<th>Environment</th>
<th>Acceleration typical levels*, in g's</th>
<th>Frequency change x10^-11, for 1x10^-9/g oscillator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buildings**, quiescent</td>
<td>0.02 rms</td>
<td>2</td>
</tr>
<tr>
<td>Tractor-trailer (3-80 Hz)</td>
<td>0.2 peak</td>
<td>20</td>
</tr>
<tr>
<td>Armored personnel carrier</td>
<td>0.5 to 3 rms</td>
<td>50 to 300</td>
</tr>
<tr>
<td>Ship - calm seas</td>
<td>0.02 to 0.1 peak</td>
<td>2 to 10</td>
</tr>
<tr>
<td>Ship - rough seas</td>
<td>0.8 peak</td>
<td>80</td>
</tr>
<tr>
<td>Propeller aircraft</td>
<td>0.3 to 5 rms</td>
<td>30 to 500</td>
</tr>
<tr>
<td>Helicopter</td>
<td>0.1 to 7 rms</td>
<td>10 to 700</td>
</tr>
<tr>
<td>Jet aircraft</td>
<td>0.02 to 2 rms</td>
<td>2 to 200</td>
</tr>
<tr>
<td>Missiles - boost phase</td>
<td>15 peak</td>
<td>1,500</td>
</tr>
<tr>
<td>Railroads</td>
<td>0.1 to 1 peak</td>
<td>10 to 100</td>
</tr>
</tbody>
</table>

* Levels at the oscillator depend on how and where the oscillator is mounted. Platform resonances can greatly amplify the acceleration levels.

** Building vibrations can have significant effects on noise measurements.
Acceleration Sensitivity Vector

\[ \vec{\Gamma} = \gamma_1 \hat{i} + \gamma_2 \hat{j} + \gamma_3 \hat{k} \]

\[ |\vec{\Gamma}| = \sqrt{\gamma_1^2 + \gamma_2^2 + \gamma_3^2} \]

Acceleration-sensitivity is a vector, i.e., the acceleration-induced frequency shift is maximum when the acceleration is along the acceleration-sensitivity vector: \( \Delta f = \vec{\Gamma} \cdot \vec{A} \).
Vibration-Induced Allan Variance Degradation

Vibration modulates the frequency and, thereby, degrades the short-term stability. The typical degradation due to sinusoidal vibration varies with averaging time, as shown. Since a full sine wave averages to zero, the degradation is zero for averaging times that are integer multiples of the period of vibration. The peaks occur at averaging times that are odd multiples of half the period of vibration. The $\sigma_y(\tau)$ due to a single-frequency vibration is:

$$\sigma_y(\tau) = \frac{\Gamma \cdot a \cdot \tau_v}{\pi} \sin^2\left(\frac{\pi \tau}{\tau_v}\right),$$

where $\tau_v$ is the period of vibration, $\tau$ is the measurement averaging time, $\Gamma$ is the acceleration sensitivity vector, and $a$ is the acceleration.

Example:

$f_v = 20$ Hz
$a = 1.0$ g along $\Gamma$
$|\Gamma| = 1 \times 10^{-9}/g$
Vibration-Induced Phase Excursion

The phase of a vibration modulated signal is

\[ \phi(t) = 2\pi f_o t + \left( \frac{\Delta f}{f_v} \right) \sin(2\pi f_v t). \]

When the oscillator is subjected to a simple sinusoidal vibration, the peak phase excursion is

\[ \Delta \phi_{peak} = \frac{\Delta f}{f_v} = \frac{(\overline{\Gamma} \cdot \overline{a}) f_o}{f_v} \]

**Example:** if a 10 MHz, 1 x 10^{-9}/g oscillator is subjected to a 10 Hz sinusoidal vibration of amplitude 1 g, the peak vibration-induced phase excursion is 1 x 10^{-3} radians. If this oscillator is used as the reference oscillator in a 10 GHz radar system, the peak phase excursion at 10 GHz will be 1 radian. Such a large phase excursion can be catastrophic to the performance of many systems, such as those which employ phase locked loops (PLL) or phase shift keying (PSK).
Vibration-Induced Sidebands

\[ L(f) \]

- LEVEL = 10g
- VIBRATION SENSITIVITY = 1.4 \times 10^{-9}/g
Vibration-Induced Sidebands

Each frequency multiplication by 10 increases the sidebands by 20 dB.
Sine Vibration-Induced Phase Noise

Sinusoidal vibration produces spectral lines at \( \pm f_v \) from the carrier, where \( f_v \) is the vibration frequency.

\[
\mathcal{L}'(f_v) = 20 \log \left( \frac{\bar{\Gamma} \cdot A f_0}{2f_v} \right)
\]

e.g., if \( |\bar{\Gamma}| = 1 \times 10^{-9}/g \) and \( f_0 = 10 \) MHz, then even if the oscillator is completely noise free at rest, the phase "noise," i.e., the spectral lines, due solely to a sine vibration level of 1g will be:

<table>
<thead>
<tr>
<th>Vibr. freq., ( f_v ), in Hz</th>
<th>( \mathcal{L}'(f_v) ), in dBC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-46</td>
</tr>
<tr>
<td>10</td>
<td>-66</td>
</tr>
<tr>
<td>100</td>
<td>-86</td>
</tr>
<tr>
<td>1,000</td>
<td>-106</td>
</tr>
<tr>
<td>10,000</td>
<td>-126</td>
</tr>
</tbody>
</table>
Random Vibration-Induced Phase Noise

Random vibration’s contribution to phase noise is given by:

\[ \mathcal{L}(f) = 20 \log \left( \frac{\overline{\Gamma} \cdot A f_0}{2f} \right), \quad \text{where } |A| = [(2)(PSD)]^{1/2} \]

e.g., if \( |\overline{\Gamma}| = 1 \times 10^{-9}/g \) and \( f_0 = 10 \) MHz, then even if the oscillator is completely noise free at rest, the phase noise due solely to a vibration PSD = 0.1 \( g^2/\text{Hz} \) will be:

<table>
<thead>
<tr>
<th>Offset freq., f, in Hz</th>
<th>( \mathcal{L}(f) ), in dBc/Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-53</td>
</tr>
<tr>
<td>10</td>
<td>-73</td>
</tr>
<tr>
<td>100</td>
<td>-93</td>
</tr>
<tr>
<td>1,000</td>
<td>-113</td>
</tr>
<tr>
<td>10,000</td>
<td>-133</td>
</tr>
</tbody>
</table>
Random-Vibration-Induced Phase Noise

Phase noise under vibration is for $\Gamma = 1 \times 10^{-9}$ per g and $f = 10$ MHz

Typical Aircraft Random Vibration Envelope
Acceleration Sensitivity vs. Vibration Frequency

The acceleration sensitivity can be calculated from the vibration induced sidebands. The preferred method is to measure the sensitivity at a number of vibration frequencies in order to reveal resonances. The example above shows the results for an OCXO. The resonance at 424 Hz amplified the sensitivity 17-fold.
Acceleration Sensitivity of Quartz Resonators

Resonator acceleration sensitivities range from the low parts in $10^{10}$ per g for the best commercially available SC-cuts, to parts in $10^7$ per g for tuning-fork-type watch crystals. When a wide range of resonators was examined: AT, BT, FC, IT, SC, AK, and GT-cuts; 5 MHz 5th overtones to 500 MHz fundamental mode inverted mesa resonators; resonators made of natural quartz, cultured quartz, and swept cultured quartz; numerous geometries and mounting configurations (including rectangular AT-cuts); nearly all of the results were within a factor of three of $1 \times 10^{-9}$ per g. On the other hand, the fact that a few resonators have been found to have sensitivities of less than $1 \times 10^{-10}$ per g (for unknown reasons) indicates that the observed acceleration sensitivities are not due to any inherent natural limitations. Recent theoretical and experimental evidence indicates that the major variables yet to be controlled properly are the mode shape and location (i.e., the amplitude of vibration distribution), and the strain distribution associated with the mode of vibration. Until the acceleration sensitivity problem is solved, acceleration compensation and vibration isolation can provide lower than $1 \times 10^{-10}$ per g, for a limited range of vibration frequencies, and at a cost.
Phase Noise Degradation Due to Vibration

- Data shown is for a 10 MHz, 2 x 10^{-9} per g oscillator
- Radar spec. shown is for a coherent radar (e.g., SOTAS)

Impacts on Radar Performance
- Lower probability of detection
- Lower probability of identification
- Shorter range
- False targets

'GOOD' OSCILLATOR ON VIBRATING PLATFORM (1g)

Radar Oscillator Specification

Required to 'see' 4Km/hr target: 53 dB

Offset from Carrier (Hz):

~150 dBc
Coherent Radar Probability of Detection

To "see" 4 km/h targets, low phase noise 70 Hz from the carrier is required. Shown is the probability of detection of 4 km/h targets vs. the phase noise 70 Hz from the carrier of a 10 MHz reference oscillator. (After multiplication to 10 GHz, the phase noise will be at least 60 dB higher.) The phase noise due to platform vibration, e.g., on an aircraft, reduces the probability of detection of slow-moving targets to zero.

![Graph showing the probability of detection vs. phase noise](image)

- Probability of Detection (%)
- Phase Noise (dBc/Hz)

(at 70 Hz from carrier, for 4 km/h targets)
Vibration Isolation

Limitations

- Poor at low frequencies
- Adds size, weight and cost
- Ineffective for acoustic noise
# Vibration Compensation

<table>
<thead>
<tr>
<th>Stimulus</th>
<th>Response</th>
<th>Compensated Oscillator</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="DC Voltage on Crystal" /></td>
<td><img src="image" alt="Response" /></td>
<td><img src="image" alt="Compensated Oscillator" /></td>
</tr>
<tr>
<td>DC Voltage on Crystal</td>
<td>$f$</td>
<td>Vibration Compensated Oscillator</td>
</tr>
<tr>
<td><img src="image" alt="AC Voltage on Crystal" /></td>
<td><img src="image" alt="Response" /></td>
<td>ACC = accelerometer</td>
</tr>
<tr>
<td>AC Voltage on Crystal</td>
<td>$f_0 - f_v$, $f_0$, $f_0 + f_v$</td>
<td>Response to Vibration</td>
</tr>
<tr>
<td><img src="image" alt="Crystal Being Vibrated" /></td>
<td><img src="image" alt="Response" /></td>
<td></td>
</tr>
<tr>
<td>Crystal Being Vibrated</td>
<td>$f_0 - f_v$, $f_0$, $f_0 + f_v$</td>
<td></td>
</tr>
</tbody>
</table>
Vibration Sensitivity Measurement System

CONTROLLER

SPECTRUM ANALYZER

FREQUENCY MULTIPLIER (x10)

SYNTHESIZER (LOCAL OSCILLATOR)

SIGNAL GENERATOR $F_V$

ACCELEROMETER

TEST OSCILLATOR

POWER AMPLIFIER

VIBRATION LEVEL CONTROLLER

PLOTTER OR PRINTER
Shock

The frequency excursion during a shock is due to the resonator's stress sensitivity. The magnitude of the excursion is a function of resonator design, and of the shock induced stresses on the resonator. (Resonances in the mounting structure will amplify the stresses.) The permanent frequency offset can be due to: shock induced stress changes, the removal of (particulate) contamination from the resonator surfaces, and changes in the oscillator circuitry. Survival under shock is primarily a function of resonator surface imperfections. Chemical-polishing-produced scratch-free resonators have survived shocks of up to 36,000 g in air gun tests, and have survived the shocks due to being fired from a 155 mm howitzer (16,000 g, 12 ms duration).
Radiation-Induced Frequency Shifts

\[ f_0 = \text{original, preirradiation frequency} \]

\[ \Delta f_{ss} = \text{steady-state frequency (0.2 to 24 hours after exposure)} \]

\[ f_t = \text{instantaneous frequency at any time (t)} \]

Idealized frequency vs. time behavior for a quartz resonator following a pulse of ionizing radiation.

\[ \Delta f_{ss}/\text{rad}^* = \begin{cases} 
10^{-11} & \text{for natural quartz (R increase can stop oscillation)} \\
10^{-12} & \text{for cultured quartz} \\
10^{-13} & \text{for swept cultured quartz} 
\end{cases} \]

* for 1 Mrad dose
Effects of Repeated Irradiations

Five irradiations; responses during 4th and 5th repeated 3rd; at least 2 days elapsed between successive irradiations

Initial slopes:
1st: $-1 \times 10^{-9}$/rad
2nd: $+1 \times 10^{-11}$/rad
3rd: $+3 \times 10^{-12}$/rad
4th: $+3 \times 10^{-12}$/rad
5th: $+5 \times 10^{-12}$/rad
Radiation Induced $\Delta f$ vs. Dose and Quartz-Type

- 10 MeV Electrons,
- 5 MHz 5th overtone
- AT-cut resonators

- Z-growth Cultured
- Swept Z-growth Cultured
- Natural

Diagram showing frequency change (Hz) vs. rads (Si).
Annealing of Radiation Induced $f$ Changes

- For 4 MHz AT-cut resonator, X-ray dose of $6 \times 10^6$ rads produced $\Delta f = 41$ Hz.

- Activation energies were calculated from the temperature dependence of the annealing curves. The experimental results can be reproduced by two processes, with activation energies $E_1 = 0.3 \pm 0.1$ eV and $E_2 = 1.3 \pm 0.3$ eV.

- Annealing was complete in less than 3 hours at $> 240^\circ$C.
Transient $\Delta f$ After a Pulse of $\gamma$ Radiation

- Experimental data, dose = $1.3 \times 10^4$ rads, SC-cut
- Experimental data, dose = $2.3 \times 10^4$ rads, AT-cut
- Model calculation: AT-cut

The graph shows the transient frequency shift $\Delta f/f$ over time, with different symbols representing experimental data at varying doses and a model calculation line.
Effects of Flash X-rays on $R_s$

The curves show the series resonance resistance, $R_s$, vs. time following a $4 \times 10^4$ rad pulse. Resonators made of swept quartz show no change in $R_s$ from the earliest measurement time (1 ms) after exposure, at room temperature. Large increase in $R_s$ (i.e., large decrease in the $Q$) will stop the oscillation.
Frequency Change due to Neutrons

Curve shows the nearly linear increase in resonant frequency of a crystal unit as a function of reactor irradiation. At other fluences, the slopes are, for example, $8 \times 10^{-21}/n/cm^2$ at $10^{10}$ to $10^{12}n/cm^2$, and $5 \times 10^{-21}/n/cm^2$ at $10^{12}$ to $10^{13}n/cm^2$. 
Neutron Damage

A fast neutron can displace about 50 to 100 atoms before it comes to rest. Most of the damage is done by the recoiling atoms. Net result is that each neutron can cause numerous vacancies and interstitials.
Summary - Steady-State Radiation Results

- Dose vs. frequency change is nonlinear; f change per rad is larger at low doses.
- At doses > 1 KRad, f change is quartz impurity dependent. The ionizing radiation produces electron-hole pairs; the holes are trapped by the impurity Al sites while the compensating cation (e.g., Li or Na) is released. The freed cations are loosely trapped along the optic axis. The lattice near the Al is altered, the elastic constant is changed; therefore, the f shifts. Ge impurity is also troublesome.
- At a 1 MRad dose, f change ranges from pp $10^{11}$ per rad for natural quartz to pp $10^{14}$ per rad for high quality swept quartz.
- Frequency change is negative for natural quartz; it can be positive or negative for cultured and swept cultured quartz.
- Frequency change saturates at doses > $10^6$ rads.
- Q degrades if quartz contains high concentration of alkali impurities; Q of resonators made of properly swept cultured quartz is unaffected.
- High dose radiation can also rotate f vs. T characteristic.
- Frequency change anneals at $T > 240^\circ$C in less than 3 hours.
- Preconditioning (e.g., with doses > $10^5$ rads) reduces the high dose radiation sensitivities upon subsequent irradiations.
- At doses < 100 rad, f change is not well understood. Radiation induced stress relief and surface effects (adsorption, desorption, dissociation, polymerization and charging) may be significant.
Summary - Pulse Irradiation Results

- For applications requiring circuits hardened to pulse irradiation, quartz resonators are the least tolerant element in properly designed oscillator circuits.

- Resonators made of unswept quartz or natural quartz can experience a large increase in $R_s$ following a pulse of radiation; the radiation pulse can stop the oscillation.

- Natural, cultured, and swept cultured AT-cut quartz resonators experience an initial negative frequency shift immediately after exposure to a pulse of X-rays (e.g., $10^4$ to $10^5$ Rad of flash X-rays), $\Delta f/f$ is as large as -3 ppm at 0.02 sec after burst of $10^{12}$ Rad/sec.

- Transient $f$ offset anneals as $t^{-1/2}$; the nonthermal-transient part of the $f$ offset is probably due to the diffusion and retrapping of hydrogen at the Al$^{3+}$ trap.

- Resonators made of properly swept quartz experience a negligibly small change in $R_s$ when subjected to pulsed ionizing radiation (the oscillator circuit does not require a large reserve of gain margin).

- SC-cut quartz resonators made of properly swept high Q quartz do not exhibit transient frequency offsets following a pulse of ionizing radiation.

- Crystal oscillators will stop oscillating during an intense pulse of ionizing radiation because of the large prompt photoconductivity in quartz and in the transistors comprising the oscillator circuit. Oscillation will start up within 15 $\mu$sec after burst if swept quartz is used in the resonator and the oscillator circuit is properly designed for the radiation environment.
Summary - Neutron Irradiation Results

- When a fast neutron (~MeV energy) hurtles into a crystal lattice and collides with an atom, it is scattered like a billiard ball. The recoiling atom, having an energy (~$10^4$ to $10^6$ eV) that is much greater than its binding energy in the lattice, leaves behind a vacancy and, as it travels through the lattice, it displaces and ionizes other atoms. A single fast neutron can thereby produce numerous vacancies, interstitials, and broken interatomic bonds. Neutron damage thus changes both the elastic constants and the density of quartz. Of the fast neutrons that impinge on a resonator, most pass through without any collisions, i.e., without any effects on the resonator. The small fraction of neutrons that collide with atoms in the lattice cause the damage.

- Frequency increases approximately linearly with fluence. For AT- and SC-cut resonators, the slopes range from $+0.7 \times 10^{-21}/n/cm^2$ at very high fluences ($10^{17}$ to $10^{18} n/cm^2$) to $5 \times 10^{-21}/n/cm^2$ at $10^{12}$ to $10^{13} n/cm^2$, and $8 \times 10^{-21}/n/cm^2$ at $10^{10}$ to $10^{12} n/cm^2$. Sensitivity probably depends somewhat on the quartz defect density and on the neutron energy distribution. (Thermonuclear neutrons cause more damage than reactor neutrons.)

- Neutron irradiation also rotates the frequency vs. temperature characteristic.

- When a heavily neutron irradiated sample was baked at 500° C for six days, 90% of the neutron-induced frequency shift was removed (but the 10% remaining was still 93 ppm).
Other Effects on Stability

- **Electric field** - affects doubly-rotated resonators; e.g., a voltage on the electrodes of a 5 MHz fundamental mode SC-cut resonator results in a $\Delta f/f = 7 \times 10^{-9}$ per volt. The voltage can also cause sweeping, which can affect the frequency (of all cuts).

- **Magnetic field** - quartz is diamagnetic, however, magnetic fields can induce Eddy currents, and will affect magnetic materials in the resonator package and the oscillator circuitry. Induced ac voltages can affect varactors, AGC circuits and power supplies. Typical frequency change of a "good" quartz oscillator is $<<10^{-10}$ per gauss.

- **Ambient pressure (altitude)** - deformation of resonator and oscillator packages, and change in heat transfer conditions affect the frequency.

- **Humidity** - can affect the oscillator circuitry, and the oscillator's thermal properties.

- **Power supply voltage, and load impedance** - affect the oscillator circuitry, and indirectly, the resonator's drive level and load reactance. A change in load impedance changes the amplitude or phase of the signal reflected into the oscillator loop, which changes the phase (and frequency) of the oscillation. The effects can be minimized through voltage regulation and buffering.

- **Gas permeation** - stability can be affected by excessive levels of atmospheric hydrogen and helium diffusing into "hermetically sealed" metal and glass enclosures (e.g., hydrogen diffusion through nickel resonator enclosures, and helium diffusion through glass Rb standard bulbs).
## Interactions Among Influences

In attempting to measure the effect of a single influence, one often encounters interfering influences, the presence of which may or may not be obvious.

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Interfering Influence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resonator aging</td>
<td>ΔT due to oven T (i.e., thermistor) aging</td>
</tr>
<tr>
<td></td>
<td>Δ drive level due to osc. circuit aging</td>
</tr>
<tr>
<td>Short term stability</td>
<td>Vibration</td>
</tr>
<tr>
<td>Vibration sensitivity</td>
<td>Induced voltages due to magnetic fields</td>
</tr>
<tr>
<td>2-g tipover sensitivity</td>
<td>ΔT due to convection inside oven</td>
</tr>
<tr>
<td>Resonator f vs. T (static)</td>
<td>Thermal transient effect, humidity</td>
</tr>
<tr>
<td></td>
<td>T-coefficient of load reactances</td>
</tr>
<tr>
<td>Radiation sensitivity</td>
<td>ΔT, thermal transient effect, aging</td>
</tr>
</tbody>
</table>
Chapter 4 References


4-8 Provided by Arthur Ballato, U.S. Army LABCOM, private communication, circa 1978.


4-10 R. A. Young, R. B. Belser, A. L. Bennett, W. H. Hicklin, J. C. Meaders, and C. E. Wagner, "Special X-ray Studies of Quartz


4-30 The analysis and graphs on pp. 4-30 and 4-31 were provided by Raymond L. Filler, U.S. Army LABCOM, 1989.


Hydrothermal Growth of Quartz

- The autoclave is filled to some predetermined factor with water plus mineralizer (NaOH or Na$_2$CO$_3$).
- The baffle localizes the temperature gradient so that each zone is nearly isothermal.
- The seeds are thin slices of (usually) Z-cut single-crystals.
- The nutrient consists of small (~1/2") pieces of single-crystal quartz ("lascas").
- The temperatures and pressures are typically about 350ºC and 800 to 2,000 atmospheres; $T_2 - T_1$ is typically 4ºC to 10ºC.
- The nutrient dissolves slowly (30 to 60 days per run), diffuses to the growth zone, and deposits onto the seeds.
The Quartz Lattice
## Quartz Properties' Effects on Device Properties

<table>
<thead>
<tr>
<th>Quartz Property</th>
<th>Device and Device-Fabrication Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>Oscillator short-term stability, phase noise close to carrier, long-term stability, filter loss</td>
</tr>
<tr>
<td>Purity (Al, Fe, Li, Na, K, -OH, H₂O)</td>
<td>Radiation hardness, susceptibility to twinning, optical characteristics</td>
</tr>
<tr>
<td>Crystalline Perfection, Strains</td>
<td>Sweepability, etchability for chem. polishing and photolithographic processing, optical properties, strength, aging(?), hysteresis (?)</td>
</tr>
<tr>
<td>Inclusions</td>
<td>High-temperature processing and applications, optical characteristics, etchability</td>
</tr>
</tbody>
</table>
Models show the positions of $\text{H}^+$ and alkali ions in the channels of the quartz lattice and the corresponding trends of the potential energy curves.
Aluminum Associated Defects

Al-OH\(^-\) center

\(\text{OH}^-\) molecule

Al-M\(^+\) center

Interstitial Alkali

\([\text{Al}_{E^+}]^0\) center
(aluminum-hole center)

Hole trapped in nonbonding oxygen p orbital
Sweeping

Sweeping is a purification process which removes certain impurities from the quartz and thereby improves the radiation hardness and etching properties of quartz crystals. It is an electric-field driven, solid-state diffusion process that is performed at an elevated temperature. The major steps of a typical sweeping process consist of applying electrodes to the Z-surfaces of a lumbered quartz bar, heating the bar slowly to 500°C, applying a voltage to the electrodes such that the electric field along the Z-direction is about 1 kV/cm, monitoring the current through the bar (as the sweeping progresses, the current decreases), and after the current decays to some constant value, cooling the bar slowly to room temperature, then removing the voltage.

Under the influences of the high electric field and the high temperature, the positive impurity ions, such as Li⁺ and Na⁺, diffuse to the cathode and are removed when the electrodes are removed in subsequent processing. In addition to improving radiation hardness, sweeping also greatly reduces the number of etch channels that are produced when quartz is etched.
Typical Sweeping Method

Oven
$T = 500^\circ C$

Cr-Au
Quartz Bar

$E = 1000 \text{ V/cm}$

Thermometer
Ammeter
High V Power Supply

$I$ vs Time
$0.5 \mu \text{a/cm}^2$
Quartz Quality Indicators

- Infrared absorption coefficient *
- Etch-channel density *
- Etch-pit density
- Inclusion density *
- Impurity analysis
- X-ray topography
- UV absorption
- Birefringence along the optic axis
- Thermal shock induced fracture
- Electron spin resonance
- ???

* EIA Standard 477-1 contains standard test method for this quantity
Infrared Absorption

![Graph showing infrared absorption spectrum with peaks at 3585, 3410, 3200, and 3300 cm\(^{-1}\).]
Infrared Absorption

One of the factors that determine the maximum achievable resonator Q is the OH content of the quartz. Infrared absorption measurements are routinely used to measure the intensities of the temperature-broadened OH defect bands. The infrared absorption coefficient $\alpha$ is defined by EIA Standard 477-1 as

$$\alpha = \frac{A(3500 \text{ cm}^{-1}) - A(3800 \text{ cm}^{-1})}{\text{Y-cut thickness in cm}}$$

where the A’s are the logarithm (base 10) of the fraction of the incident beam absorbed at the wave numbers in the parentheses.

<table>
<thead>
<tr>
<th>Grade</th>
<th>$\alpha$, in cm$^{-1}$</th>
<th>Approx. max. Q*</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.03</td>
<td>3.0</td>
</tr>
<tr>
<td>B</td>
<td>0.045</td>
<td>2.2</td>
</tr>
<tr>
<td>C</td>
<td>0.060</td>
<td>1.8</td>
</tr>
<tr>
<td>D</td>
<td>0.12</td>
<td>1.0</td>
</tr>
<tr>
<td>E</td>
<td>0.25</td>
<td>0.5</td>
</tr>
</tbody>
</table>

* In millions, at 5 MHz ($\alpha$ is a quality indicator for unswept quartz only).
Quartz Twinning

- The X-axes of quartz, the electrical axes, are parallel to the line bisecting adjacent prism faces; the +X-direction is positive upon extension due to tension.

- Electric twinning (also called Dauphiné twinning) consists of localized reversal of the X-axes. It usually consists of irregular patches, with irregular boundaries. It can be produced artificially by inversion from high-quartz, thermal shock, high local pressure (even at room temperature), and by an intense electric field.

- In right-handed quartz, the plane of polarization is rotated clockwise as seen by looking toward the light source; in left handed, it is CCW. Optically twinned (also called Brazil twinned) quartz contains both left and right-handed quartz. Boundaries between optical twins are usually straight.

- Etching can reveal both kinds of twinning.
The diagrams illustrate the relationship between the axial system and hand of twinned crystals. The arrows indicate the hand.
Quartz Lattice and Twinning

Z-axis projection showing electric (Dauphiné) twins separated by a twin wall of one unit cell thickness. The numbers in the atoms are atom heights expressed in units of percent of a unit cell height. The atom shifts during twinning involve motions of < 0.03 nm.

- Silicon
- Oxygen
Quartz Inversion

- Quartz undergoes a high-low inversion (α – β transformation) at 573°C. (It is 573°C at 1 atm on rising temperature; it can be 1° to 2°C lower on falling temperature.)

- Bond angles between adjoining (SiO₄) tetrahedra change at the inversion. Whereas low-quartz (α - quartz) is trigonal, high quartz (β - quartz) is hexagonal. Both forms are piezoelectric.

- An abrupt change in nearly all physical properties takes place at the inversion point; volume increases by 0.86% during inversion from low to high quartz. The changes are reversible, although Dauphiné twinning is usually acquired upon cooling through the inversion point.

- Inversion temperature decreases with increasing Al and alkali content, increases with Ge content, and increases 1°C for each 40 atm increase in hydrostatic pressure.
Phase Diagram of Silica (SiO₂)
Chapter 5 References


5-7a J. J. Martin, "Electrodiffusion (Sweeping) of Ions in Quartz," IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency


Emerging Oscillator Technologies
Emerging/Improving Technologies

- SC-cut resonators
- Resonator theory and finite element modeling
- Advanced fabrication techniques
  - Surface cleaning (UV-ozone, plasma; ice scrubber for particle removal)
  - Chemical polishing & chemical milling
  - Plate, mount and electrode geometries (lateral field, BVA, polygonal)
  - Bonding (parallel gap, thermocompression)
  - Packaging (ceramic flatpack, ceramic-metal, all-quartz)
  - Ultrahigh vacuum, high temperature & automated processing
- High purity, low defect density quartz
- UHF and miniature (photolithography/etching produced) resonators
- Acceleration sensitivity reduction and compensation techniques
- Microcomputer compensation (temperature, acceleration, radiation)
- Miniature fast warmup OCXO & directly heated crystal plate
- Rubidium-crystal oscillator (RbXO)
- Optically pumped atomic frequency standards
Comparison of SC and AT-cuts

- Advantages of the SC-cut
  - Thermal transient compensated (allows faster warmup OCXO)
  - Static and dynamic f vs. T allow higher stability OCXO and MCXO
  - Better f vs. T repeatability allows higher stability OCXO and MCXO
  - Far fewer activity dips
  - Lower drive level sensitivity
  - Planar stress compensated; lower Δf due to edge forces and bending
  - Lower sensitivity to radiation
  - Higher capacitance ratio (less Δf for oscillator reactance changes)
  - Higher Q for fundamental mode resonators of similar geometry
  - Less sensitive to plate geometry - can use wide range of contours

- Disadvantage of the SC-cut: More difficult to manufacture for OCXO (but is easier to manufacture for MCXO than is an AT-cut for precision TCXO)

- Other Significant Differences
  - B-mode is excited in the SC-cut, although not necessarily in LFR's
  - The SC-cut is sensitive to electric fields (can be used for compensation)
Lateral Field Resonator

In lateral field resonators (LFR): 1. the electrodes are absent from the regions of greatest motion, and 2. varying the orientation of the gap between the electrodes varies certain important resonator properties. Advantages of LFR are:

- Ability to eliminate undesired modes, e.g., the b-mode in SC-cuts
- Potentially higher Q (less damping due to electrodes and mode traps)
- Potentially higher stability (less electrode and mode trap effects, smaller $C_1$)
Microcomputer Compensated Crystal Oscillator (MCXO)

- Accuracy: 5 msec per day ($5 \times 10^{-8}$), with < 50 mW power

- Major barriers: thermal hysteresis, thermometry, circuit instabilities

- Solutions: high stability overtone SC-cut and lateral field resonators, dual-mode oscillator and digital compensation techniques

- Advantages over analog TCXO: much higher accuracy possible, rapid and easy compensation and recalibration
MCXO - Description of Operation

The following analogy illustrates the difference between an MCXO-based clock and a conventional TCXO-based clock. Suppose one has a clock that gains 24 seconds per day. The conventional way to maintain accurate time with such a clock is to adjust the frequency of the internal oscillator to the proper frequency, and then to maintain that frequency, e.g., with a TCXO. Another way to maintain accurate time is to set the clock, and then to stop the clock for 1 second every hour, or for 1/60 sec every minute, or for 1/3600 sec every second, etc. For a conventional clock, the second method would be very inconvenient and difficult to use for accurate timekeeping. The MCXO, however, operates somewhat as the second method, i.e., the MCXO provides accurate time from an inaccurate (but highly reproducible) frequency source, by correcting for the known frequency inaccuracies.

Simplified block diagrams of two implementations of the MCXO are shown on the two pages following this description. In the pulse deletion method, the dual-mode oscillator provides output signals at two frequencies, one of which, fβ, is the resonator temperature indicator. The signals are processed by the microcomputer which, from fβ, determines the necessary correction to fc and then subtracts the required number of pulses from fc to obtain the corrected output fo. Fractions of pulses that cannot be subtracted within the update interval (~ 1 s) are used as a carry, so that the long-term average is within the ±2 x 10⁻⁸ design accuracy. Correction data in the PROM are unique to each crystal and are obtained from a precise thermal characterization of the fc and fβ output signals. The corrected output signal fo can be divided down to produce a 1 pps time reference or can be used directly to drive a clock. Due to the objectionable noise characteristics
MCXO - Description of Operation, Cont’d

created by the pulse deletion process, additional signal processing is necessary to provide a useful RF output for frequency control applications. This can be accomplished by, for example, imparting the MCXO frequency accuracy to a second, low-noise, low-cost, voltage controlled crystal oscillator.

A sinewave RF output may be obtained directly by using an alternate MCXO approach that is based on phase-locked-loop frequency summing instead of pulse deletion. As in the pulse deletion method, a dual-mode oscillator generates the two output frequencies, \( f_c \) and \( f_\beta \). The microcomputer computes a number \( N \) which is used to control a direct digital synthesizer (DDS). The DDS generates a correction frequency \( df_c \) which, when added to \( f_c \), results in the compensated output frequency \( f_0 \). The phase-locked-loop frequency summer incorporates a VCXO that is adjusted, in frequency and phase, to the desired sum frequency \( f_0 \).

The MCXO has major advantages over conventional temperature compensated crystal oscillators (TCXO’s) for the following reasons: 1. the MCXO circumvents the need for pulling the crystal frequency and, therefore, permits the use of "stiff", high stability SC-cut crystal units, 2. the MCXO allows resonator self-temperature sensing, using a dual-mode oscillator; thermometry-caused errors are thus eliminated, 3. the trim effect is eliminated, 4. automatic recalibration features can be designed into the MCXO algorithm; an offset can be stored in memory following simple injection of an external, higher-accuracy reference signal, and 5. an accurate but very low-power clock is possible through duty-cycling the MCXO to periodically update a low-power, wristwatch-type clock (e.g., six seconds on, one minute off).
MCXO - Pulse Deletion Method

Microcomputer compensated crystal oscillator (MCXO) block diagram - pulse deletion method.
MCXO - PLL Frequency Summing Method

Microcomputer compensated crystal oscillator (MCXO) block diagram - phase-locked-loop frequency summing method.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>TCXO</th>
<th>MCXO</th>
</tr>
</thead>
<tbody>
<tr>
<td>AT-cut, fund.</td>
<td>Tight</td>
<td>Tight</td>
</tr>
<tr>
<td>SC-cut, 3rd</td>
<td>Loose</td>
<td>Loose</td>
</tr>
<tr>
<td>Angle-of-cut tolerance</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Blank f and plating tolerance</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Activity dip incidence</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hysteresis (-55°C to +85°C)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aging per year</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Rubidium - Crystal Oscillator (RbXO)

| Rubidium Frequency Standard ($\approx 25W @ -55^\circ C$) | RbXO Interface | Low-power Crystal Oscillator |

The RbXO provides the best of both worlds - the long term stability of a Rb standard and the low power requirement of a crystal oscillator. Occasionally, power is applied to the Rb standard for a few minutes. Upon warmup of the Rb standard, the RbXO interface syntonizes the crystal oscillator and cuts off power to the Rb standard. When the crystal oscillator is an MCXO, the MCXO digital circuit can include the RbXO interface, and the average RbXO power consumption can be less than 100 mW.
The Rb reference is a miniature Rb frequency standard (RFS) that has been modified to control an external crystal oscillator. The OCXO includes a digital tuning memory to hold the frequency control voltage while the Rb reference is off. The OCXO is ON continually. Periodically, the system applies power to the RFS. After the warmup of the RFS (a few minutes), the interface circuits adjust the frequency of the OCXO to the RFS reference, then shut off the RFS. For manpack applications, the OCXO will be separable from the rest of the RbXO so that the manpack can operate with minimum size, weight, and power, and with nearly the accuracy of the RFS for the duration of a mission. An MCXO can replace the OCXO for even lower power consumption.
Rubidium Crystal Oscillator
Accuracy vs. Power-Requirement*

(Goal is to move the technologies toward the upper left)

* Accuracy vs. size, and accuracy vs. cost have similar relationships.
<table>
<thead>
<tr>
<th>Problem</th>
<th>Major Causes</th>
<th>Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aging</td>
<td>Contamination transfer, stress relief, material defects (?)</td>
<td>Ultraclean processing, SC-cut, &quot;good&quot; quartz, mounting, bonding, electrodes, and package</td>
</tr>
<tr>
<td></td>
<td>Thermal Hysteresis</td>
<td>&quot;good&quot; quartz, mounting, bonding, electrodes, and package</td>
</tr>
<tr>
<td></td>
<td>Frequency vs. Temperature</td>
<td>Incorrect angles-of-cut, interfering modes</td>
</tr>
<tr>
<td></td>
<td>Thermal Shock (Warmup)</td>
<td>Stress sensitivity of quartz, thermal time constant of package</td>
</tr>
<tr>
<td></td>
<td>Vibration</td>
<td>Stress sensitivity of quartz (non-lineairties), mode shape</td>
</tr>
<tr>
<td></td>
<td>Mechanical Shock</td>
<td>Surface and bulk imperfections, deformation in mount/bond</td>
</tr>
<tr>
<td>Radiation</td>
<td></td>
<td>High purity quartz (sweeping), compensation</td>
</tr>
</tbody>
</table>
Chapter 6 References


Atomic Frequency Standards*

* There are two important reasons for including this section: 1. atomic frequency standards are one of the most important applications of precision quartz oscillators, and 2. those who study or use crystal oscillators ought to be aware of what is available in case they need an oscillator with better long-term stability than what crystal oscillators can provide.
Precision Frequency Standards

- Quartz crystal resonator-based ($f \sim 5$ MHz, $Q \sim 10^6$)

- Atomic resonator-based
  - Rubidium$^{87}$ ($f_0 = 6.8$ GHz, $Q \sim 10^7$)
  - Cesium$^{133}$ ($f_0 = 9.2$ GHz, $Q \sim 10^8$)
  - Hydrogen ($f_0 = 1.4$ GHz, $Q \sim 10^9$)
  - Trapped ions ($f_0 > 10$ GHz, $Q > 10^{11}$)
Atomic Frequency Standard Basic Concepts

When an atomic system changes energy from an excited state to a lower energy state, a photon is emitted. The photon frequency $\nu$ is given by Planck's law

$$\nu = \frac{E_2 - E_1}{h}$$

where $E_2$ and $E_1$ are the energies of the upper and lower states, respectively, and $h$ is Planck's constant. An atomic frequency standard produces an output signal the frequency of which is determined by this intrinsic frequency rather than some property of a bulk material (as it is in quartz oscillators).

The properties of isolated atoms at rest, and in free space, would not change with space and time. Therefore, the frequency of an ideal atomic frequency standard would not change with time or with changes in the environment. Unfortunately, in real atomic frequency standards: 1) the atoms are moving at thermal velocities, 2) the atoms are not isolated but experience collisions and electric and magnetic fields, and 3) some of the components needed for producing and observing the atomic transitions contribute to instabilities.
Hydrogen-Like Atoms

Hydrogen-Like (or Alkali) Atoms

Hyperfine structure of $^{87}$Rb, with nuclear spin $I = 3/2$, $v_0 = \Delta W/h = 6,834,682,605$ Hz and $X = [(-\mu J/J) + (\mu I/I)]H_0/\Delta W$ calibrated in units of $2.44 \times 10^3$ Oe.
A voltage controlled crystal oscillator (VCXO) is locked to the atomic resonator, which is a highly stable frequency reference generated from an atomic transition. Of the many atomic transitions available, the ones selected are from those which are least sensitive to environmental effects and which can be conveniently locked to the VCXO. The long term stability is determined by the atomic resonator, the short term stability, by the crystal oscillator.
Generalized Atomic Resonator

- Let A and B be two possible energy states of an atom, separated by energy $\hbar \nu_0$; then $\nu_0$ is the frequency of the electromagnetic radiation required to convert the atoms from A to B, or from B to A; $\nu_0$ is in the microwave range for all currently manufactured atomic standards.

- Population difference between energy states, when $\hbar \nu_0 << kT$, is near zero. Therefore, in a natural ensemble of atoms, when $\nu_0$ is applied, about half the atoms absorb $\hbar \nu_0$ and half emit $\hbar \nu_0$; the net effect is zero.

- A nonthermal distribution is prepared, i.e., one of the states is "selected," by optical excitation from one of the levels to a third level or by magnetic deflection of an atomic beam.

- Microwave energy is absorbed in the process of converting the selected atoms to the other energy state, e.g., from A to B. Thus, the applied microwave frequency can be "locked" to the frequency corresponding to the atomic transition.
Atomic Resonator Concepts

- The energy levels used are due to the spin-spin interaction between the atomic nucleus and the outer electron in the ground state \( ^2S_{1/2} \) of the atom; i.e., the ground state hyperfine transitions.

- Nearly all atomic standards use Rb or Cs atoms; nuclear spins \( I = 3/2 \) and \( 7/2 \), respectively.

- Energy levels split into \( 2(I + 1/2) + 1 \) sublevels in a magnetic field; the "clock transition" is the transition between the least magnetic-field-sensitive sublevels. A constant magnetic field, the "C-field," is applied to minimize the probability of the more magnetic-field-sensitive transitions.

- Magnetic shielding is used to reduce external magnetic fields at least 100-fold (e.g., the earth’s).

- The Heisenberg uncertainty principle limits the achievable accuracy: \( \Delta E \Delta t \geq \hbar/2\pi \), \( E = h\nu \), therefore, \( \Delta v \Delta t \geq 1 \), therefore, long observation time \( \rightarrow \) small frequency uncertainty.

- Resonance linewidth (i.e., \( 1/Q \)) is inversely proportional to coherent observation time \( \Delta t \); \( \Delta t \) is limited by: 1.) when atom enters and leaves the apparatus, and 2.) when the atom stops oscillating due to collisions with other atoms or with container walls (collisions disturb atom’s electronic structure).

- Since atoms move with respect to the microwave source, resonance frequency is shifted due to the Doppler effect \( (\mathbf{k} \cdot \mathbf{v}) \); velocity distribution results in "Doppler broadening"; the second-order Doppler shift \( (1/2 \nu^2/c^2) \) is due to relativistic time dilation.
Cesium-Beam Frequency Standard

Cs atomic resonator schematic diagram

Atomic state selection

Cs atom detection
Cesium-Beam Frequency Standard

- The atomic resonance used is at 9,192,631,770 Hz - by definition (of the second).

- Oven is at ~100°C, Cs pressure in the oven ~ 10^-3 torr, cavity is at ~ 10^-9 torr; typical atom speed is 100 m/s; typical cavity length in commercial standards is 10 to 20 cm; interaction time ~ 1 to 2 x 10^-3 s; linewidth ~ 0.5 to 1 kHz; Q ~ 10^7; in standard lab’s, length ~ 4 meters, Q ~ 10^8.

- It would be desirable to operate at zero magnetic field - all transitions would behave as a single transition, signal would be 7X larger, but that would require < 10^-8 gauss for errors < 1 x 10^-12; not feasible; C-field must be applied; a 0.06 gauss C-field separates the sublevels by 40 kHz.

- The (3,0) to (4,0) clock transition has a small quadratic dependence on magnetic field; C-field must be stable and uniform; high degree of shielding is required for ±1x10^-13/ gauss (e.g., the HP 004 uses a triple shield).

- State selecting magnet A "selects" one of the two atomic levels; the applied microwave causes a state change; the second magnet deflects to the detector the atoms which have undergone the state change; A and B magnets’ peak field ~ 10 kgauss.

- Atom detector is a ribbon or wire (e.g., W or Pt) at ~ 900°C; Cs atoms are ionized, ions are collected, current is amplified and fed back into feedback network; microwave frequency is locked to the frequency of maximum ion current, thus the atomic transition frequency controls the microwave frequency, i.e., the frequency of the crystal oscillator. Much less than 1% of the Cs atoms reach the detector in conventional Cs standards (hence optical pumping’s advantage.)
Cs Hyperfine Energy Levels

Magnetic field dependence of the hyperfine energy levels in the ground state of the cesium atom (nine in the upper state, seven in the lower). The magnetic field is plotted up to the value $H_0$. The solid arrow represents the "clock" transition; the dashed arrows depict the magnetic-field-sensitive (Zeeman) transitions. $F$ is the hyperfine quantum number, and $m_F$ is the magnetic quantum number of the atom.
Rubidium Frequency Standard

Energy level diagram for a rubidium atomic standard

Rubidium atomic resonator schematic diagram
Rubidium Gas Cell Resonator

- The atomic resonance used is at 6,834,682,608 Hz.

- Cell contains Rb gas at $\sim 10^{-6}$ torr and an inert buffer gas at $\sim 1$ torr; Rb atom oscillation lifetime is limited by collisions to $\sim 10^{-2}$ s; linewidth $\sim 100$ Hz; $Q \sim 5 \times 10^7$. Buffer gas, a mixture of positive (e.g., N$_2$) and negative (e.g., Ar) pressure-shift gases, provides zero temperature coefficient at some T, confines Rb atoms to small region to reduce wall-collision and 1st order Doppler effects.

- Optical pumping relies on the natural coincidence of optical resonance frequencies between $^{85}$Rb and $^{87}$Rb, both at 795 nm.

- Rf excited $^{87}$Rb lamp emits wavelengths corresponding to both the F=1 and F=2 transitions; $^{85}$Rb filter cell absorbs more of the F=2 transition light; light which passes through filter is absorbed by the $^{87}$Rb F=1 state; excited atoms relax to both the F=1 and F=2 states, but the F=1 states are excited again; F=2 state is overpopulated: 6.8 GHz converts F=2 back to F=1, which provides more atoms to absorb light. Microwave resonance causes increased light absorption, i.e., a ($< 1\%$) dip in the light detected by the photocell; microwave frequency is locked to photocell detection dip, thus the atomic transition frequency controls the microwave frequency, i.e., the frequency of the crystal oscillator.
Atomic Resonator Instabilities

- **Noise** - due to the circuitry, crystal resonator, and atomic resonator. (See next page.)

- **Cavity pulling** - microwave cavity is also a resonator; atoms and cavity behave as two coupled oscillators; effect can be minimized by tuning the cavity to the atomic resonance frequency, and by maximizing the atomic resonance Q to cavity Q ratio.

- **Collisions** - cause frequency shifts and shortening of oscillation duration.

- **Doppler effects** - 1st order is classical, can be minimized by design; 2nd order is relativistic.

- **Magnetic field** - this is the only influence that directly affects the atomic resonance frequency.

- **Microwave spectrum** - asymmetric frequency distribution causes frequency pulling; can be made negligible through proper design.

- **Environmental effects** - magnetic field changes, temperature changes, vibration, shock, radiation, atmospheric pressure changes, and He permeation into Rb bulbs.
Noise in Atomic Frequency Standards

If the time constant for the atomic-to-crystal servo-loop is $t_0$, then at $\tau < t_0$, the crystal oscillator determines $\sigma_y(\tau)$, i.e., $\sigma_y(\tau) \sim \tau^{-1}$. From $\tau > t_0$ to the $\tau$ where the "flicker floor" begins, variations in the atomic beam intensity (shot-noise) determine $\sigma_y(\tau)$, and $\sigma_y(\tau) \sim (i\tau)^{-1/2}$, where $i =$ number of signal events per second. Shot noise within the feedback loop shows up as white frequency noise (random walk of phase). Shot noise is generally present in any electronic device (vacuum tube, transistor, photodetector, etc.) where discrete particles (electrons, atoms) move across a potential barrier in a random way.

In commercial standards, $t_0$ ranges from 0.01 s for a small Rb standard to 60 s for a high-performance Cs standard. In the regions where $\sigma_y(\tau)$ varies as $\tau^{-1}$ and $\tau^{-1/2}$, $\sigma_y(\tau) \propto (QS_R)^{-1}$, where $S_R$ is the signal-to-noise ratio, i.e., the higher the Q and the signal-to-noise ratio, the better the short term stability (and the phase noise far from the carrier, in the frequency domain).
Short-Term Stability of a Cs Standard

*The 60 s time constant provides better short-term stability, but it is usable only in benign environments.*
Short-Term Stability of a Rb Standard

\[ f_L (\text{LOOP BW}) = \frac{1}{2\pi \tau} \]

\[ f_L = 100\text{Hz} \text{ (STANDARD)} \]

OTHERS OPTIONAL

- **RUBIDIUM-WORST CASE**
- **VCXO**

\[ \sigma_y(\tau) \]

\[ 10^{-9} \]

\[ 10^{-10} \]

\[ 10^{-11} \]

\[ 10^{-12} \]

\[ 0.001 \quad 0.01 \quad 0.1 \quad 1 \quad 10 \quad 100 \quad \tau \text{ (seconds)} \]
Acceleration Sensitivity of Atomic Standards

Let the servo loop time constant = $t_0$, let the atomic standard's $\Gamma = \Gamma_A$, and the VCXO's $\Gamma = \Gamma_O$. Then,

- For fast acceleration changes ($f_{vib} \gg 1/2\pi t_0$), $\Gamma_A = \Gamma_O$
- For slow acceleration changes, ($f_{vib} \gg 1/2\pi t_0$), $\Gamma_A \ll \Gamma_O$
- For $f_{vib} = f_{mod}$, $2f_{mod}$, servo confused, $\Gamma_A \approx \Gamma_O$, plus f offset
- For small $f_{vib}$, (at Bessel function null), loss of lock, $\Gamma_A \approx \Gamma_O$
Atomic Standard Acceleration Effects

In Rb cell standards, high acceleration can cause $\Delta f$ due to light shift, power shift, and servo effects:

- Location of molten Rb in the Rb lamp can shift
- Mechanical changes can deflect light beam
- Mechanical changes can cause rf power changes

In Cs beam standards, high acceleration can cause $\Delta f$ due to changes in the atomic trajectory with respect to the tube & microwave cavity structures:

- Vibration modulates the amplitude of the detected signal. Worst when $f_{\text{vib}} = f_{\text{mod}}$.
- Beam to cavity position change causes cavity phase shift effects
- Velocity distribution of Cs atoms can change
- Rocking effect can cause $\Delta f$ even when $f_{\text{vib}} < f_{\text{mod}}$

In H-masers, cavity deformation causes $\Delta f$ due to cavity pulling effect
Magnetic Field Sensitivities of Atomic Clocks

Clock transition frequency $v = v_o + C_H H_o^2$, where $C_H$ is the quadratic Zeeman effect coefficient (which varies as $1/v_o$).

<table>
<thead>
<tr>
<th>Atom</th>
<th>Transition Frequency</th>
<th>C-field* (milligauss)**</th>
<th>Shielding Factor*</th>
<th>Sensitivity* per gauss**</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rb</td>
<td>$v = 6.8 \text{ GHz} + (574 \text{ Hz/G}^2) B_o^2$</td>
<td>250</td>
<td>5,000</td>
<td>$10^{-11}$</td>
</tr>
<tr>
<td>Cs</td>
<td>$v = 9.2 \text{ GHz} + (427 \text{ Hz/G}^2) B_o^2$</td>
<td>60</td>
<td>50,000</td>
<td>$10^{-13}$</td>
</tr>
<tr>
<td>H</td>
<td>$v = 1.4 \text{ GHz} + (2750 \text{ Hz/G}^2) B_o^2$</td>
<td>0.5</td>
<td>50,000</td>
<td>$10^{-13}$</td>
</tr>
</tbody>
</table>

* Typical values.
** 1 gauss = $10^{-4}$ Tesla; Tesla is the SI unit of magnetic flux density.
Crystal's Influences on Atomic Standard

- **Short term stability** - for averaging times less than the atomic-to-crystal servo loop time constant, $\tau_L$, the crystal oscillator determines $\sigma_y(\tau)$.

- **Loss of lock** - caused by large phase excursions in $t < \tau_L$ (due to shock, attitude change, vibration, thermal transient, radiation pulse). At a Rb standard's 6.8 GHz, for a $\Delta f = 1 \times 10^{-9}$ in 1s, as in a 2g tipover in 1s, $\Delta \phi \sim 7\pi$. Control voltage sweeping during reacquisition attempt can cause the phase and frequency to change wildly.

- **Maintenance or end of life** - when crystal oscillator frequency offset due to aging approaches EFC range (typically $\sim 1$ to $2 \times 10^{-7}$).

- **Long term stability** - noise at second harmonic of modulation $f$ causes time varying $\Delta f$'s; this effect is significant only in the highest stability (e.g., H and Hg) standards.
Miniature Optically Pumped Cs Standard

The proper atomic energy levels are populated by optical pumping with a laser diode, which provides superior utilization of Cs atoms. The potential advantages include: higher S/N, longer life, lower weight, and the possibility of trading off size for accuracy. The main goals of the miniature Cs standard development program are $1 \times 10^{-11}$ accuracy, and a 1 liter volume, i.e., about 100x higher accuracy than a Rb standard, in about the same volume (but not necessarily the same shape factor).
Chapter 7 References


- Several review papers, including three on the environmental sensitivities of atomic frequency standards, are contained in the Proc. 22nd Ann. Precise Time and Time Interval (PTTI) Applications and Planning Meeting, NASA Conference Publ. 3116, 1990; AD-A239372.


7-14 Hewlett-Packard 5061B Data Sheet (Pub. 5952-7912D), Hewlett-Packard, Attn: Inquiry Manager, 1820 Embarcadero Road, Palo Alto, CA 94303.


Oscillator Comparison and Specification
# Oscillator Comparison

<table>
<thead>
<tr>
<th></th>
<th>Quartz Oscillators</th>
<th>Atomic Oscillators</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TCXO</td>
<td>MCXO</td>
</tr>
<tr>
<td><strong>Accuracy</strong> (per year)</td>
<td>2 x 10^-6</td>
<td>5 x 10^-8</td>
</tr>
<tr>
<td><strong>Aging/Year</strong></td>
<td>5 x 10^-7</td>
<td>2 x 10^-8</td>
</tr>
<tr>
<td><strong>Temp. Stab.</strong> (range, °C)</td>
<td>5 x 10^-7 (-55 to +85)</td>
<td>2 x 10^-8 (-55 to +85)</td>
</tr>
<tr>
<td><strong>Stability, σ_y(τ) (τ = 1 s)</strong></td>
<td>1 x 10^-9</td>
<td>1 x 10^-10</td>
</tr>
<tr>
<td><strong>Size</strong> (cm³)</td>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td><strong>Warmup Time</strong> (min)</td>
<td>0.1 (to 1 x 10^-6)</td>
<td>0.1 (to 2 x 10^-8)</td>
</tr>
<tr>
<td><strong>Power (W)</strong> (at lowest temp.)</td>
<td>0.05</td>
<td>0.04</td>
</tr>
<tr>
<td><strong>Price (~$)</strong></td>
<td>100</td>
<td>1,000</td>
</tr>
</tbody>
</table>

* Including environmental effects (note that the temperature ranges for Rb and Cs are narrower than for quartz).
Accuracy vs. Power-Requirement*

* Accuracy vs. size, and accuracy vs. cost have similar relationships.
Stability Ranges of Various Frequency Standards

- Quartz
- Rubidium
- Cesium
- Hydrogen Maser

Log \( \sigma_y (\tau) \) vs. Log \( \tau \), seconds

1 day, 1 month
Phase Instabilities of Various Frequency Standards

Typical One-sided Spectral Density of Phase Deviation vs. Offset Frequency, for Various Standards, Calculated at 5 MHz.
<table>
<thead>
<tr>
<th>Weaknesses and Wearout Mechanisms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wearout Mechanisms</td>
</tr>
<tr>
<td>None</td>
</tr>
<tr>
<td>Rubidium depletion</td>
</tr>
<tr>
<td>Buffer gas depletion</td>
</tr>
<tr>
<td>Glass contaminants</td>
</tr>
<tr>
<td>Cesium supply depletion</td>
</tr>
<tr>
<td>Spent cesium gettering</td>
</tr>
<tr>
<td>Ion pump capacity</td>
</tr>
<tr>
<td>Electron multiplier</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Weaknesses</th>
<th>Aging</th>
<th>Rad hardness</th>
<th>Life</th>
<th>Power</th>
<th>Weight</th>
<th>Life</th>
<th>Power</th>
<th>Weight</th>
<th>Cost</th>
<th>Temp. range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartz</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rubidium</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cesium</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Oscillator Selection Considerations

- Frequency accuracy or reproducibility requirement
- Recalibration interval
- Environmental extremes
- Power availability - must it operate from batteries?
- Allowable warmup time
- Short term stability (phase noise) requirements
- Size and weight constraints
- Cost to be minimized - acquisition or life cycle cost
Crystal Oscillator Specification MIL-O-55310

MILITARY SPECIFICATION

OSCILLATORS, CRYSTAL,

GENERAL SPECIFICATION FOR

This specification is approved for use by all Departments and Agencies of the Department of Defense.

1. SCOPE

1.1 Statement of scope. This specification covers the general requirements and quality and reliability assurance requirements for bulkwave quartz crystal oscillators designed for frequency control or timekeeping in Armed Services electronic equipment.
Chapter 8 References


8-3 The graphs on pp.8-3 and 8-4 were prepared and provided by Richard Sydnor, Jet Propulsion Laboratory, 1989.

8-7 Copies of MIL-O-55310 are available by mail from: Military Specifications and Standards, Bldg. 4D, 700 Robbins Avenue, Philadelphia, PA 19111-5094. Customer Service telephone: (215) 697-2667/2179; Telephone Order Entry System (requires a touch tone telephone and a customer number): (215) 697-1187 thru 1195.
9

Time and Timekeeping
What Is Time?

- "What, then, is time? If no one asks me, I know; if I wish to explain to him who asks, I know not." --- Saint Augustine, circa 400 A.D.

- The question, both a philosophical and a scientific one, has no entirely satisfactory answer. "Time is what a clock measures." "It defines the temporal order of events." "It is an element in the four-dimensional geometry of space-time." "It is nature’s way of making sure that everything doesn’t happen at once."

- Why are there "arrows" of time? The arrows are: entropy, electromagnetic waves, expansion of the universe, k-meson decay, and psychological. Does time have a beginning and an end? (Big bang; no more "events", eventually.) See, e.g., Time’s Arrows, by Richard Morris, Simon & Schuster, NY, 1985.

- The unit of time, the second, is one of the seven base units in the International System of Units (SI units). Since time is the quantity that can be measured with the highest accuracy, it plays a central role in metrology.
Dictionary Definition of "Time"

(From The Random House Dictionary of the English Language, © 1987)

time (tim), n., adj., v., timed, timing. —n. 1. the system of those sequential relations that any event has to any other, as past, present, or future; indefinite and continuous duration regarded as that in which events succeed one another; duration regarded as belonging to the present life as distinct from the life to come or from eternity; finite duration. 2. (sometimes cap.) a system or method of measuring or reckoning the passage of time: mean time; apparent time; Greenwich Time. 4. a limited period or interval, as between two successive events; a long time. 5. a particular period considered as distinct from other periods: Youth is the best time of life. 7. times, a. in the history of the world, or contemporary with the life or activities of a notable person: prehistoric times; in those times. b. the period or era now or previously present; a sign of the times; how times have changed! c. a period considered with reference to its events or prevailing conditions, tendencies, ideas, etc.: holy time and stately order. 7. a prescribed or allotted period, as of one's life, for payment of a debt, etc. 8. the end of a prescribed or allotted period, as of one's life or a pregnancy: His time had come, but there was no one left to mourn over him. When her time came, her husband accompanied her to the delivery room. 9. a period with reference to personal experiences of a specified kind: to have a good time; a hot time in the old town tonight. 10. a period of work of an employee, or the pay for its working hours or days or an hourly or daily pay rate. 11. informal: a term of enforced duty or imprisonment: to serve time in the army; do time in prison. 12. the period necessary for or occupied by something: The time of the baseball game is two hours and five minutes. 13. the time taken or required: The bus takes too much time, so I'll take a plane. 14. the time sufficient or spare time: to have time for a vacation; I have no time to stop now. 15. a particular or definite point in time, as indicated by clock: What time is it? 15. a particular part of a day, year, etc.; season or period: It's time for lunch. 16. an appointed, fixed, or scheduled period of time: On time; once, at the usual time; on time; never, at all times; all the time. 17. the time when the sun crosses the meridian: Chicago time is the time when the sun crosses the meridian. 18. an infinitely, indefinitely prolonged period or duration in the future: Time will tell if we have done the right thing today. 19. the right occasion or opportunity: to watch one's time. 20. on occasion of a recurring action or event: to do a thing five times; it's the pitcher's time at bat. 21. times, used as a multiplicative word in phrase combinations expressing how many instances of a quantity or factor are taken together: Two goes into six three times; five times faster. 22. Drama. one of the three units. Cf. unity (def. 8). 23. Proc. a unit or a group of units in the measurement of time, meter. 24. Music. a. tempo; relative rapidity of movement. b. the metrical duration of a note or rest. c. proper or characteristic tempo. d. the general movement of a particular kind of musical composition with reference to its rhythm, metric structure, and tempo. 25. the movement of a dance or the like to music so arranged: a waltz time. 26. Mil. rate of marching, calculated on the number of paces taken per minute; double time; quick time. 27. Menge. each completed action or movement of the horse. 27. against time, in an effort to finish the game in its limited period; we worked against time to get out the newspaper. 28. behind schedule, before the time due; early. The building was completed ahead of schedule. 29. at one time, a. once; in a former time. b. at one time, at the same time; at once. They all tried to talk at once. 30. at the same time, nevertheless; yet. I'd like to try it, but at the same time I'm a little afraid. 31. at times, at intervals; occasionally. At times the city becomes intolerable. 32. beat someone's time, Slang. to compete for or win a person being dated or courting by another; prevail over a rival; He was trying to beat his time. 33. behind the times, old-fashioned; dated. These attitudes are behind the times. 34. for the time being, temporarily. For the present. 35. from time to time, on occasion; occasionally; at intervals: She comes to see us from time to time. 36. gain time, to postpone in order to make preparations or gain an advantage; delay the outcome of: He hoped to gain time by putting off signing the papers for a few days more. 37. good time, a. at the right time; on time; punctually. b. in advance of the right time; early. We arrived at the appointed spot in good time. 38. in no time, in a very short time; at first; before long. 39. in time, early enough: to come in time for dinner. 40. keep time, a. to record time, as a watch or clock does; to mark or observe the tempo. c. to perform rhythmic movements in unison. 41. kill time, to occupy oneself with some activity to make time pass quickly: While I was waiting, I killed time counting the cars on the freight train. 42. make time, a. to move quickly, esp. in an attempt to recover lost time. b. to travel at a particular speed. 43. make time with, Slang. to pursue or take as a sexual partner. 44. many a time, again and again; frequently. Many a time they didn’t have enough to eat and went to bed hungry. 45. mark time, a. to suspend progress temporarily, to make a stand; to stop developments; fail to advance. b. Mil. to move the feet alternately as in marching, but without advancing. 46. on one's own time, during one's free time; without payment. He worked out more efficient production methods on his own time. 47. on time, a. at the specified time; punctually. b. to be paid for within a designated period of time. Many people are never out of debt because they buy everything on time. 48. out of time, not in the proper rhythm: His singing was out of time with the music. 49. pass the time of day, to converse briefly with or about. 50. stop time, to stop in the market to pass the time of day. 50. take one's time, to be slow or leisurely, dawdle. Speed was important here, but he just took his time. 51. time after time, again and again; repeatedly; often. I've told him time after time not to slam the door. 52. time and time again, repeatedly; often. Time and time again I urged her to stop smoking. 53. time bomb, a. time bomb, b. explosive device containing a clock so that it will detonate at the desired moment. c. time bomb, d. Com. payable at a stated period of time after performance: time drafts or notes. 54. time of one's life, Informal. 55. of pertaining to, or showing the passage of time; past or present. 56. of an explosive device containing a clock so that it will detonate at the desired moment: a time bomb. 57. Com. payable at a stated period of time after performance: time drafts or notes. 58. of or pertaining to purchases on the installment plan, or with payment postponed. 59. at or by. 60. to measure or record the speed, duration, or rate of: to time a race. 60. to time a race. 61. to time a race. 62. to execute, to perform. 63. to appoint or choose the moment or occasion for: schedule. 64. to time a race. 65. to time a race. 66. to time a race. 67. to time a race.
The Second

- The SI unit of time is the second (symbol s).

- The second was defined, by international agreement, in October, 1967, at the XIII General Conference of Weights and Measures.

- The second is "the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium atom 133."

- Prior to 1967, the unit of time was based on astronomical observations; the second was defined in terms of ephemeris time, i.e., as "1/31,556,925.9747 of the tropical year..."

- The unit of frequency is defined as the hertz (symbol Hz). One hertz equals the repetitive occurrence of one "event" per second.
Frequency and Time

\[ f = \frac{1}{\tau} \]

where \( f \) = frequency (= number of "events" per unit time), and \( \tau \) = period (= time between "events")

Accumulated clock time = \( \frac{\text{Total number of events}}{\text{Number of events per unit of time}} \)

Example: \( \frac{3 \text{ rotations of the earth}}{1 \text{ rotation/day}} = 3 \text{ days.} \)

Frequency source + counting mechanism \( \rightarrow \) clock

Examples of frequency sources: the rotating earth, pendulum, quartz crystal oscillator, and atomic frequency standard.
Typical Clock System

\[ t = t_0 + \Sigma \Delta \tau \]

where \( t \) is the time output, \( t_0 \) is the initial setting, and \( \Delta \tau \) is the time interval being counted.
Evolution of Clock Technologies

- Sundials, and continuous flow of:
  - Water (clepsydra)
  - Sand (hour glass)
- Falling weights, with frictional control of rate
- Vibrating, but non-resonant motion - escapement mechanisms:
  - Falling weight applies torque through train of wheels; rate control depends on moments of inertia, friction and torque; period is the time it takes to move from one angular position to another.
- Resonant control
  - Mechanical: pendulum, hairspring and balance wheel
  - Mechanical, electrically driven: tuning fork, quartz resonator
- Atomic and molecular
# Progress in Timekeeping

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Clock / Milestone</th>
<th>Accuracy Per Day</th>
</tr>
</thead>
<tbody>
<tr>
<td>4th millennium B.C.</td>
<td>Day &amp; night divided into 12 equal hours</td>
<td>~ 1 h</td>
</tr>
<tr>
<td>Up to 1280 A.D.</td>
<td>Sundials, water clocks (clepsydrae)</td>
<td>~ 30 to 60 min</td>
</tr>
<tr>
<td>~ 1280 A.D.</td>
<td>Mechanical clock invented - assembly time for prayer was first regular use.</td>
<td>~ 15 to 30 min</td>
</tr>
<tr>
<td>14th century</td>
<td>Invention of the escapement; clockmaking becomes a major industry</td>
<td></td>
</tr>
<tr>
<td>~ 1345</td>
<td>Hour divided into minutes and seconds</td>
<td>~ 2 min</td>
</tr>
<tr>
<td>15th century</td>
<td>Clock time used to regulate people’s lives (work hours)</td>
<td></td>
</tr>
<tr>
<td>16th century</td>
<td>Time’s impact on science becomes significant (Galileo times physical events, e.g., free-fall)</td>
<td>~ 1 min</td>
</tr>
<tr>
<td>~ 1657</td>
<td>First pendulum clock (Huygens)</td>
<td>~ 100 s</td>
</tr>
<tr>
<td>18th century</td>
<td>Temperature-compensated pendulum clocks</td>
<td>1 to 10 s</td>
</tr>
<tr>
<td>19th century</td>
<td>Electrically driven free-pendulum clocks</td>
<td>$10^{-2}$ to $10^{-1}$ s</td>
</tr>
<tr>
<td>~1910 to 1920</td>
<td>Wrist watches become widely available</td>
<td></td>
</tr>
<tr>
<td>1920 to 1934</td>
<td>Electrically driven tuning forks</td>
<td>$10^{-3}$ to $10^{-2}$ s</td>
</tr>
<tr>
<td>1921 to present</td>
<td>Quartz crystal clocks (and watches, since ~1971)</td>
<td>$10^{-5}$ to $10^{-1}$ s</td>
</tr>
<tr>
<td>1949 to present</td>
<td>Atomic clocks</td>
<td>$10^{-9}$ to $10^{-7}$ s</td>
</tr>
</tbody>
</table>
Clock Errors

\[
T(t) = T_0 + \int_0^t R(t)dt + \varepsilon(t) = T_0 + (R_0 t + 1/2 At^2 + ...) + \int_0^t E_i(t)dt + \varepsilon(t)
\]

Where,

- \(T(t)\) = time difference between two clocks at time \(t\) after synchronization
- \(T_0\) = synchronization error at \(t = 0\)
- \(R(t)\) = the rate (i.e., fractional frequency) difference between the two clocks under comparison; \(R(t) = R_0 + At + ... E_i(t)\)
- \(\varepsilon(t)\) = error due to random fluctuations \(\approx \tau \sigma_y(\tau)\)
- \(R_0 = R(t)\) at \(t = 0\)
- \(A\) = linear aging term (higher order terms are included if the aging is not linear)
- \(E_i(t)\) = rate difference due to environmental effects (temperature, etc.)

Example: If a watch is set to within 0.5 seconds of a time tone \((T_0 = 0.5\) s\), and the watch initially gains 2 s/week \((R_0 = 2\) s/week\), and the watch rate ages -0.1 s per week\(^2\), \((A = -0.1\) s/week\(^2\)), then after 10 weeks (and assuming \(E_i(t) = 0\)):

\[
T (10 \text{ weeks}) = 0.5 + (2 \times 10) + 1/2 (-0.1 \times (10)^2) = 15.5 \text{ seconds.}
\]
Frequency Error vs. Time Error

\[ f_r = \text{reference (i.e., the "correct") frequency} \]
Clock Error vs. Resynchronization Interval

Aging/Day = 5 x 10^{-10}
Temp Stability = 2 x 10^{-8}
Resync Interval = 4 days

<table>
<thead>
<tr>
<th></th>
<th>TCXO</th>
<th>OCXO</th>
<th>MCXO</th>
<th>RbXO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. Temp. Stab.</td>
<td>1 x 10^{-6}</td>
<td>2 x 10^{-8}</td>
<td>2 x 10^{-8}</td>
<td>2 x 10^{-8}</td>
</tr>
<tr>
<td>Aging/Day</td>
<td>1 x 10^{-8}</td>
<td>1 x 10^{-10}</td>
<td>5 x 10^{-11}</td>
<td>5 x 10^{-13}</td>
</tr>
<tr>
<td>Resynch Interval* (A/J &amp; Security)</td>
<td>10 min</td>
<td>6 hr</td>
<td>6 hr</td>
<td>4 da</td>
</tr>
<tr>
<td>Recal Interval* (Maint Cost)</td>
<td>10 yr</td>
<td>80 da</td>
<td>50 yr</td>
<td>1.5 yr</td>
</tr>
</tbody>
</table>

* Calculated for an accuracy requirement of 25 milliseconds. Many modern systems need much better.
To Estimate the Accumulated Time Error

1. Estimate the initial frequency offset plus the average expected offsets due to temperature and other environmental effects.
2. Find the time error caused by the sum of the offsets.
3. Find the time error caused by the oscillator’s specified aging rate.
4. Add the results of 2 and 3 to estimate the total time error.
On Using Time for Clock Rate Calibration

It takes time to measure the clock rate (i.e., frequency) difference between two clocks. The smaller the rate difference between a clock to be calibrated and a reference clock, the longer it takes to measure the difference ($\Delta t/t \approx \Delta f/f$).

For example, assume that a reference timing source (e.g., Loran or GPS) with a time uncertainty of 100 ns is used to calibrate the rate of a clock to $1 \times 10^{-11}$ accuracy. A frequency offset of $1 \times 10^{-11}$ will produce $1 \times 10^{-11} \times 3600$ s/hour $= 36$ ns time error per hour. Then, to have a high certainty that the measured time difference is due to the frequency offset rather than the reference clock uncertainty, one must accumulate a sufficient amount ($\geq 100$ ns) of time error. It can take hours to perform the calibration. (See the next page for a different example.) If one wishes to know the frequency offset to a $\pm 1 \times 10^{-12}$ precision, then the calibration will take more than a day.

Of course, if one has a cesium standard for frequency reference, then, for example, with a high resolution frequency counter, one can make frequency comparisons of the same precision much faster.
Calibration With a 1 pps Reference.

Let $A$ = desired clock rate accuracy after calibration
$A'$ = actual clock rate accuracy
$\Delta \tau$ = jitter in the 1 pps of the reference clock, rms
$\Delta \tau'$ = jitter in the 1 pps of the clock being calibrated, rms
$t$ = calibration duration
$\Delta t$ = accumulated time error during calibration

Then, what should be the $t$ for a given set of $A$, $\Delta \tau$, and $\Delta \tau'$?

Example: The crystal oscillator in a clock is to be calibrated by comparing the 1 pps output from the clock with the 1 pps output from a standard. If $A = 1 \times 10^{-9}$; $\Delta \tau = 0.1 \mu s$, and $\Delta \tau' = 1.2 \mu s$, then, $[(\Delta \tau)^2 + (\Delta \tau')^2]^{1/2} \approx 1.2 \mu s$, and when $A = A'$, $\Delta t = (1 \times 10^{-9})t \equiv (1.2 \mu s)N$, and $t = (1200N)$ s. The value of $N$ to be chosen depends on the statistics of the noise processes, on the confidence level desired for $A'$ to be $\leq A$, and on whether one makes measurements every second or only at the end points. If one measures at the end points only, and the noise is white phase noise, and the measurement errors are normally distributed, then, with $N = 1$, 68% of the calibrations will be within $A$; with $N = 2$, and 3, 95% and 99.7%, respectively, will be within $A$. One can reduce $t$ by about a factor $2/N^{3/2}$ by making measurements every second; e.g., from 1200 s to $2 \times (1200)^{2/3} = 225$ s.
# Time Transfer Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Accuracy</th>
<th>Cost ('92)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portable Cs clock</td>
<td>10 - 100 ns</td>
<td>$40K</td>
</tr>
<tr>
<td>GPS time dissemination</td>
<td>50 - 100 ns</td>
<td>$3K - 10K*</td>
</tr>
<tr>
<td>GPS common view</td>
<td>2 - 20 ns</td>
<td></td>
</tr>
<tr>
<td>Two-way via satellite</td>
<td>~ 1 ns</td>
<td>~ $100K</td>
</tr>
<tr>
<td>Loran-C</td>
<td>100 ns</td>
<td>$10K</td>
</tr>
<tr>
<td>HF (WWV)</td>
<td>200 μsec</td>
<td>$3K</td>
</tr>
<tr>
<td>Portable quartz &amp; Rb clocks</td>
<td>Calibration interval dependent</td>
<td>$1K - 10K</td>
</tr>
</tbody>
</table>

* Projected to be < $1K by 1994
The Global Positioning System

The Global Positioning System (GPS) is the most precise worldwide navigation system available. As it is capable of providing nanosecond-level timing accuracies, it is also one of the most accurate sources of time.

GPS is a satellite-based radio navigation and positioning system that is designed to provide global, all-weather, 24-hour, accurate navigation to an unlimited number of users. Each of the satellites contains four atomic clocks. The satellites transmit a navigation message that provides satellite position, time, and atmospheric propagation correction data. The GPS receiver, which contains a quartz crystal clock, measures the transit time of the satellite signal and multiplies that time by the speed of light to compute range to the satellite. The satellite clocks are more accurate than the receiver clocks. Therefore, although three satellites can provide latitude, longitude and altitude, the signal from a fourth satellite is used to correct for the navigational error caused by the receiver clock’s inaccuracy, i.e., the receivers calculate their x, y, z, and t from receiving each of four satellite’s x, y, z, and t. Velocity is determined from the doppler shifts of the transmitted carrier frequencies.
Global Positioning System

- GPS can provide global, all-weather, 24-hour, real-time, accurate navigation and time reference to an unlimited number of users.

- **GPS Accuracies (2\sigma)**
  
  Position: 120 m (Standard Positioning Service, SPS), 40 m (Precise Positioning Service, PPS), 1 cm + 1ppm (differential, static land survey); Velocity: 0.3 m/s (SPS), 0.1 m/s (PPS).

  **Time:** 350 ns to < 10 ns

- 24 satellites in 6 orbital planes; 6 to 10 visible at all times; partially deployed now, fully operational by 1993; \(~12\ h\) period 20,200 km orbits; types of military sets: 1 channel (manpack, vehicular), 2 channel (helicopter), 5 channel (high dynamic platforms); 1 and 2 ch. sets acquire sequentially.

- PRN navigation signals broadcast at \(L_1 = 1.575\ GHz\) (19 cm) and \(L_2 = 1.228\ GHz\) (24 cm); two codes, C/A and P are sent; messages provide satellite position, time, and atmospheric propagation data; users select the optimum 4 satellites to track. PPS (for DOD users) uses L1 and L2, SPS uses L1 only.
Oscillators' Impact on GPS

- Satellite oscillator (clock) inaccuracy is a major source of navigational inaccuracy. In sequencing receivers, the oscillator's stability during the time it takes to acquire the satellites sequentially (minutes) affects nav. accuracy.

- Receiver oscillator affects GPS performance, as follows:

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Time Scales

- A "time scale" is a system of assigning dates, i.e., a "time," to events; e.g., 6 January 1989, 13 h, 32 m, 46.382912 s, UTC, is a date.

- A "time interval" is a "length" of time between two events; e.g., five seconds.

- Universal time scales, UT0, UT1, and UT2, are based on the earth's spin on its axis, with corrections.

- Celestial navigation: clock (UT1) + sextant \rightarrow \text{position}.

- International Atomic Time (TAI) is maintained by the International Bureau of Weights and Measures (BIPM), and is derived from an ensemble of more than 160 atomic clocks, from more than 25 nations.

- Coordinated Universal Time (UTC) is the time scale today, by international agreement. The rate of UTC is determined by TAI, but, in order to not let the time vs. the earth's position change indefinitely, UTC is adjusted by means of leap seconds so as to keep UTC within 0.9 s of UT1.
Relativistic Time

- Time is not absolute. The "time" at which a distant event takes place depends on the observer. For example, if two events, A and B, are so close in time or so widely separated in space that no signal traveling at the speed of light can get from one to the other before the latter takes place, then, even after correcting for propagation delays, it is possible for one observer to find that A took place before B, for a second to find that B took place before A, and for a third to find that A and B occurred simultaneously. Although it seems bizarre, all three can be right.

- Rapidly moving objects exhibit a "time dilation" effect. (Twin on a spaceship moving at 0.87c will age 6 months while twin on earth ages 1 year. There is no "paradox" because spaceship twin must accelerate; i.e., there is no symmetry to the problem.)

- A clock's rate also depends on its position in a gravitational field. A high clock runs faster than a low clock.
Relativistic Time Effects

- Transporting "perfect" clocks slowly around the surface of the earth along the equator yields $\Delta t = -207$ ns eastward and $\Delta t = +207$ ns westward (portable clock is late eastward). The effect is due to the earth’s rotation.

- At latitude 40°, for example, the rate of a clock will change by $1.091 \times 10^{-13}$ for each kilometer above sea level. Moving a clock from sea level to 1km elevation makes it gain 9.4 nsec/day at that latitude.

- In 1971, atomic clocks flown eastward then westward around the world in airlines demonstrated relativistic time effects; eastward $\Delta t = -59$ ns, westward $\Delta t = +273$ ns; both values agreed with prediction to within the experimental uncertainties.

- Spacecraft Examples:
  - For a space shuttle in a 325 km orbit, $\Delta t = t_{\text{space}} - t_{\text{gnd}} = -25 \mu\text{sec/day}$
  - For GPS satellites (12 hr period circular orbits), $\Delta t = +44 \mu\text{sec/day}$

- In precise time and frequency comparisons, relativistic effects must be included in the comparison procedures.
Relativistic Time Corrections

The following expression accounts for relativistic effects, provides for clock rate accuracies of better than 1 part in $10^{14}$, and allows for global-scale clock comparisons of nanosecond accuracy, via satellites:

$$
\Delta t = \frac{-1}{c^2} \int_0^T \left[ \frac{1}{2} (v_s^2 - v_g^2) - (\phi_s - \phi_g) \right] dt + \frac{2\omega}{c^2} A_E
$$

Where $\Delta t =$ time difference between spacecraft clock and ground clock, $t_s - t_g$
- $v_s =$ spacecraft velocity ($< c$), $v_g =$ velocity of ground station
- $\Phi_s =$ gravitational potential at the spacecraft
- $\Phi_g =$ gravitational potential at the ground station
- $\omega =$ angular velocity of rotation of the earth
- $A_E =$ the projected area on the earth’s equatorial plane swept out by the vector whose tail is at the center of the earth and whose head is at the position of the portable clock or the electromagnetic signal pulse. The $A_E$ is taken positive if the head of the vector moves in the eastward direction.

Within 24 km of sea level, $\Phi = gh$ is accurate to $1 \times 10^{-14}$ where $g = (9.780 + 0.052 \sin^2 \Psi )m/s^2$, $\Psi =$ the latitude, $h =$ the distance above sea level, and where the $\sin^2 \Psi$ term accounts for the centrifugal potential due to the earth’s rotation. The "Sagnac effect," $(2\omega/c^2)A_E = (1.6227 \times 10^{-21}s/m^2)A_E$, accounts for the earth-fixed coordinate system being a rotating, noninertial reference frame.
Some Useful Relationships

- Propagation delay = 1 ns/30 cm = 1 ns/ft = 3.3 μs/km ≈ 5 μs/mile

- 1 day = 86,400 s; 1 year = 3.1536 x 10^7 s

- Clock accuracy: 1 ms/day ≈ 1 x 10^-8

- At 10 MHz: period = 100 ns; phase deviation of 1° = 0.3 ns of time deviation

- Doppler shift* = Δf/f = 2v/c

* Doppler shift example: if v = 4 km/h and f = 10 GHz (e.g., a slow-moving vehicle approaching an X-band radar), then Δf = 74 Hz, i.e., low phase noise 74 Hz from the carrier is necessary in order to "see" the vehicle.
"The leading edge of the BCD code (negative going transitions after extended high level) shall coincide with the on-time (positive going transition) edge of the one pulse-per-second signal to within ±1 millisecond." See next page for the MIL-STD BCD code.
BCD Time Code
(MIL-STD-188-115)

Example: Selected Time is 12:34:56
Rate: 50 Bits per Second
Bit Pulse Width: 20 msec
H = +6V dc ± 1V
L = -6V dc ± 1V

24 Bit BCD Time Code*

* May be followed by 12 bits for day-of-year and/or 4 bits for figure-of-merit (FOM). The FOM ranges from better than 1 ns (BCD character 1) to greater than 10 ms (BCD character 9).
Time and Frequency Subsystem

- Oscillator and Clock Driver
  - Power Source
  - Frequency Distribution
    - $f_1$
    - $f_2$
    - $f_3$
  - Time Code Generator
    - TOD
    - 1 pps
The MIFTTI Subsystem

MIFTTI = Modular Intelligent Frequency, Time and Time Interval

* The microcomputer compensates for systematic effects (after filtering random effects), and performs: automatic synchronization and calibration when an external reference is available, and built-in-testing.
Chapter 9 References


Related Devices and Applications
Discrete-Resonator Crystal Filter

A Typical Six-pole Narrow-band Filter

Layout

Circuit
Monolithic Crystal Filter

Two-pole filter and its response

Four-pole filter
electrode arrangement

Frequency

Attenuation (dB)
Surface Acoustic Wave (SAW) Devices

BAW

SAW, One-port

SAW, Two-port

Simplified Equivalent Circuits

BAW and One-port SAW

Two-port SAW
SAW Devices

- The primary application of SAW devices is in filters. Applications in precision frequency control and timing are limited because the long term stability and temperature stability of the best bulk-acoustic-wave (BAW) devices are significantly better than those of the best SAW devices.

- For BAW resonators, the plate thickness determines the fundamental-mode frequency. For SAW resonators (SAWR), the interdigital transducers' (IDT) spacings determine the frequency. For quartz, a 300 MHz BAWR plate is 6 \( \mu \text{m} \) thick. A 2.6 GHz SAWR has 0.3 \( \mu \text{m} \) IDT spacings, and can be produced by e-beam lithography.

- In SAWRs, wave motion is concentrated at the surface of the crystal; motion decays exponentially with distance from surface; 90 to 95% of the energy is within one acoustic wavelength of the surface.

- In one-port SAWRs and BAWRs, the static capacitance, \( C_0 \), provides a low-impedance path that can mask out the desired resonance at high f's. An external inductor is usually placed in parallel with \( C_0 \) to "resonate out" \( C_0 \). In two-port SAWRs \( C_0 \) does not shunt the motional arm of the equivalent circuit, therefore, two-port SAWRs are preferred in many applications.
Quartz Bulk-Wave Resonator Sensors

In frequency control and timekeeping applications, resonators are designed to have minimum sensitivity to environmental parameters. In sensor applications, the resonator is designed to have a high sensitivity to an environmental parameter, such as temperature, force, pressure and acceleration.

Quartz resonators’ advantages over other sensor technologies are:

- High resolution and wide dynamic range (due to excellent short-term stability); e.g., one part in $10^7$ ($10^{-6}$ g out of 20 g) accelerometers are available, and quartz sorption detectors are capable of sensing $10^{-12}$ grams.

- High long-term accuracy and stability, and

- Frequency counting is inherently digital.
Tuning Fork Resonator Sensors

Photolithographically produced tuning forks, single- and double-ended (flexural-mode or torsional-mode), can provide low-cost, high-resolution sensors for measuring temperature, pressure, force, and acceleration. Shown are flexural-mode tuning forks.
Chapter 10 References


General References

- Proceedings of the Annual Symposium on Frequency Control - see the next page for information on obtaining copies of these Proceedings.

- Proceedings of the Annual Precise Time and Time Interval (PTTI) Applications and Planning Meeting - information on obtaining copies of these Proceedings is available from the U.S. Naval Observatory, Time Services Department, 34th and Massachusetts Avenue, N.W., Washington, D.C. 20392-5100.

- Proceedings of the European Frequency and Time Forum - copies available from the Swiss Foundation for Research in Microtechnology (FSRM), Rue de l’Orangerie 8, CH-2000 Neuchâtel, Switzerland.


- B. Parzen, Design of Crystal and Other Harmonic Oscillators, John Wiley and Sons, 1983.
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*NTIS - National Technical Information Service
5285 Port Royal Road, Sills Building
Springfield, VA 22161, U. S. A.
Tel: 703-487-4650

*IEEE - Inst. of Electrical & Electronics Engineers
445 Hoes Lane
Piscataway, NJ 08854, U.S.A.
Tel: 800-678-4333 or 908-981-0030

* Prior to 1992, the name of the Symposium was the "Annual Symposium on Frequency Control".
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