The flow of a wall jet embedded in an external stream or in a quiescent surrounding fluid was investigated experimentally. It was determined that the flow scales with the excess momentum injected into the stream and the viscosity of the fluid rather than the jet efflux velocity and the dimension of the nozzle. In the presence of an external stream, a velocity ratio parameter $R = (U_j - U_\infty)/(U_j + U_\infty)$ had to be added in order to obtain a universal scaling of this flow. Low-amplitude external excitation of the wall jet resulted in a reduction of skin friction and therefore a reduction of drag. The only possible cause for this behavior observed is the enhancement of the two-dimensionality of the large eddies as expressed by spanwise coherence and correlation measurements.

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THE TURBULENT WALL-JET

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by:

I. Wygnanski
Aerospace & Mechanical Engineering Department
University of Arizona
Tucson AZ 85721

ORIGINAL PURPOSE

Experimental investigations of large coherent structures in turbulent shear flows bypassed
the boundary layer in the presence of a strong, adverse-pressure-gradient and the wall-jet. Both are wall bounded flows having one characteristic in common: their mean velocity profile is inviscidly unstable to two dimensional perturbations. Thus the identification of the large coherent structures with the predominant instability modes might have been extended to this class of flows and could be quantitatively analyzed. The response of these flows to external, two dimensional excitation could promote the understanding of the interactions between the inner (wall) and the outer structures in a turbulent boundary layer. The wall jet seemed to be the ideal flow configuration for resolving these intricate interactions which dominate the conventional turbulent boundary layer, because it offers a larger degree of flexibility and controllability of flow parameters than a boundary layer does, regardless of pressure gradient. This flexibility stems from the fact that, the vorticity in the outer layer depends on the added momentum flux which for a given free stream velocity and jet efflux velocity depends also on the dimension of the nozzle. Thus the susceptibility of the wall jet to external perturbations might also depend on the ratio between the wave-length of the perturbation, the width of the nozzle and the upstream boundary layer thickness. Consequently the structure of the wall jet can be progressively altered and the influence of the outer vortical layer on the wall region, can be evaluated.

The understanding of coherent structures notwithstanding, the engineering importance of wall jets stems from their application to boundary layer control (blown flaps), enhancement of evaporation (e.g., defrosters or demisters on windshields and widows), and film cooling. Wall jets serve to cool turbine blades and other surfaces exposed to either hot or corrosive gases in order to shield them from the hostile flow environment. They are used for boundary layer and circulation control because a wall jet subjected to a streamwise curvature tends to adhere to the surface and resist separation. The Coanda effect was used recently in eliminating the tail rotor from a helicopter - the NOTAR. Even the flow over a slotted-flap, in the absence of blowing from an external source, may be considered as a particular type of a wall jet. Consequently, the wall jet was investigated extensively over the past fifty years.
MAJOR CONCLUSIONS

1. The length and velocity scales governing the wall jet were established.

It was determined that, in the absence of an external stream, the bulk of the flow is self-similar, provided it is scaled by the momentum flux at the nozzle and by the viscosity and density of the fluid. The width of the nozzle or the velocity of the jet at the nozzle, $U_J$, which commonly have been used to reduce all velocity and length scales in this flow, are not independent variables and have no part in the similarity considerations. The dependence of the rate of spread of the wall jet and its local maximum velocity on the Reynolds number was eliminated by using this scaling. Since such a dependence was not observed in a free jet at comparable Reynolds numbers, it was attributed, in the wall jet case, to the no-slip conditions at the wall and was associated with viscous dissipation. We now believe that this was an outcome of an inappropriate scaling. The new scaling parameters were used to calculate the skin friction, which could otherwise be only determined with considerable difficulty.

The most appropriate scaling parameters for the wall jet in an external stream were also investigated. It appears that the attainment of self-similarity in the mean velocity profiles is possible, provided the maximum velocity is at least twice as large as the free stream velocity. This restriction may be too severe for some film cooling applications where the injection velocity is close to the free stream velocity and may have to be relaxed in the future. However, the restricted analysis provides a good approximation for the mean motion in spite of the fact that a complete self-similarity of a turbulent wall jet in an external stream is only possible in a tailored pressure gradient (Irwin 1973).

The addition of an external stream required two length scales and two velocity scales to collapse all the velocity profiles measured. The local velocity scale in the outer region of the wall jet is the difference between the maximum velocity measured and the velocity of the free stream, $U_o = (U_m - U_\infty)$, while the no-slip condition at the wall imposes another velocity scale, $U_m$, on the inner, boundary layer-like region of the flow. The distance from the surface at which the mean velocity attains a local maximum, $Y_m$, is not affected by changing the ratio of $U_\infty/U_J$ and/or $Re_J$, it just increases linearly with $X$, while the characteristic width of the outer flow, $Y_o = (Y_m/2 - Y_m)$, depends strongly on these independent variables. Using different velocity and length scales for the inner and outer regions of the flow enabled us to collapse many measured velocity profiles onto a single universal curve. We then determined the unique dependence of all four scaling parameters ($U_m$, $U_o$, $Y_m$, and $Y_o$) on the independent variables governing this flow which are:

(i) the excess kinematic momentum flux near the nozzle exit, $J = \int_0^\infty (u - U_\infty) udY$

(ii) the kinematic viscosity of the fluid $\nu$.

(iii) a dimensionless velocity ratio $R = (U_j - U_\infty)/(U_j + U_\infty)$

(iv) the dimensionless distance from the nozzle $\xi = \left[ \frac{XJ}{\nu^2} \right]$

The various dependent parameters when scaled in this manner collapsed on universal curves. Other results, collected from papers published during the past 30 years, also collapsed on the same curves proving the universality of these scaling laws for the strong wall jets (i.e. for those flows for which $U_m/\overline{U_\infty} > 2$). Since the inner and outer velocity and length scales vary with $\xi$ at almost the same exponent, the velocity profiles self similar for the prescribed initial conditions.
2. The relevance of the "law of the wall" and the "law of the wake" (i.e. the inner and outer scaling laws) to the wall jet

The inner region of the wall-jet between the solid surface and the location at which the velocity attains its local maximum may be scaled in an analogous fashion to a turbulent boundary layer. The "defect law" which relates the difference between the local velocity and the velocity maximum \((U - U_m)\) to the skin-friction velocity \(U_f\), \(Y\) and \(Y_m\) is valid throughout most of the region with the exception of the viscous sublayer. This implies that the velocity distribution near the surface is described by a universal function \(\frac{U - U_m}{U_f} = f\left(\frac{Y}{Y_m}\right)\).

On the other hand, the derivation of the logarithmic velocity distribution depending only on \(U_f\), \(Y\) and \(r\) does not apply to the wall jet in the range of Reynolds numbers considered because whenever the viscous stress vanished \(\Re > 30\) the total stress was not a constant independent of \(Y\). Thus, one of the important assumptions used in deriving the logarithmic velocity distribution did not apply for the wall-jet. On the other hand, the derivation of the logarithmic velocity profile from the defect-law, requires that the distance of the logarithmic region from \(Y_m\) will be much larger than the width of the logarithmic region itself. This assumption does not fit the wall jet either, since the distance from \(Y_m\) to the region where the logarithmic profile occurs varies from 50 to 100 at the Reynolds numbers considered. This is in sharp contrast to channel, pipe and boundary layer flows, where the nominal thickness of the logarithmic region is an order of magnitude smaller than the thickness of the boundary layer or the radius of the pipe. The applicability of the outer scaling to the neighborhood of the wall suggests that the viscous effects are of limited significance in this flow.

3. The effects of external excitation on Reynolds-averaged quantities.

External excitation at amplitudes lower or equal to 5\% of \(U_i\) did not distort the normalized form of the mean velocity profile. We therefore concluded, that weak two dimensional excitation does not affect the shape normalized velocity distribution although it alters somewhat the rate of spread of the jet and the decay of its maximum velocity. Accounting for the changes in the "virtual origin" of the flow reduces these effects even further provided the average forcing amplitude is low.

The slope of the mean velocity profile in the immediate vicinity of the wall was significantly reduced indicating a reduction of the wall shear stress. Reductions in the skin friction of approximately 10\% were prevalent at most frequencies corresponding to initial excitation amplitudes which were lower or equal to 5\%. However, reductions in \(\tau_w\) of approximately 40\% were also recorded by forcing at much higher amplitudes corresponding to 10\% or 20\% of the efflux velocity at the nozzle. Alternately, forcing at a preselected frequency which is amplified by the flow achieves similar drag reduction at a much lower input amplitude. Local coherent motion having an average amplitude smaller than 0.5\% may be responsible for a reduction in \(\tau_w\) of approximately 15\% while an additional increase in local amplitude by an order of magnitude resulted in a small incremental reduction in \(\tau_w\).

A clear definition of the most effective frequency responsible for a reduction in drag was not possible because a typical wave length associated with the forcing might be comparable with the slot-width, or with a characteristic length of the apparatus. Therefore the use of
the viscosity of the fluid and the jet momentum in the definition of a Strouhal number might not be unique. Inviscid amplification of two-dimensional disturbances in free shear flows occurs wherever the local disturbance wave-length is commensurate with the local width of the flow. In the case of the wall-jet, the maximum amplitude attainable at a prescribed streamwise location for a given excitation level depends on: \( \beta = \frac{2\pi f \frac{Y_m}{U_m}}{L} \). Consequently the dimensionless \( \tau_m \) is also sensitive to this parameter which depends on \( X \) as well as on the frequency of the excitation.

It seems that external excitation modifies the velocity distribution in what appears to be the logarithmic region, although the existence of such a region in the range of \( Re \) considered is doubtful. The turbulent energy production in the immediate neighborhood of the surface is reduced by the excitation and so is the intensity of the longitudinal component of the velocity fluctuations. The intensity of \( u'^2 \) is somewhat increased due to forcing in the vicinity of \( Y = Y_m \).

4 Large Coherent Structures in the wall jet.

The identification of the large coherent structures with the predominant instability modes of the mean motion was accomplished in part by resorting to spectral methods. Power spectral densities of the streamwise component of the velocity fluctuations were measured at several transverse and streamwise locations with and without external flow. The predominant frequency measured at a given \( Y/Y_{m/2} \) decreased in the direction of streaming so that the product \( (fY_{m/2})/U_m \) was approximately constant at all \( X \) locations. When this predominant frequency was scaled with the independent variables to form: \( fR^2 v^2 J^2 \) its dependence on \( \xi \) was universal for all velocity ratios and Reynolds numbers.

The measurements taken near the surface (at \( Y/Y_m = 0.5 \)) have consistently a higher frequency than those taken further away. For example the ratio of the predominant frequency measured at \( Y/Y_m = 0.5 \) and at \( Y/Y_{m/2} = 1 \) is 1.7 for all \( X \) locations and velocity ratios \( R \). This might be associated with the possible existence of two instability modes one dominating the outer flow while the other the wall region. The existence of two instability modes was inferred from flow visualization in laminar flow, they were also calculated for the temporal evolution of disturbances in the laminar wall jet by Tsuji & Morikawa 1977. There is reason to believe that the primary large coherent structures existing in the turbulent wall jet might also originate from two unstable linear modes.

5 The Evolution of Harmonic Perturbations in the Wall-Jet

Harmonic excitation of the wall-jet enables one to digitize the measured velocities at any streamwise location together with the forcing signal. One may Fourier decompose the phase-locked velocities and plot the transverse distributions of amplitude and phase of the signals at the frequency of forcing. The measured amplitude and phase distributions thus obtained, may be compared with same quantities calculated from the linear stability model. When such a comparison was done the lateral distribution of amplitudes across the flow was found to be reasonably represented by the model but the agreement between "theory" and experiment was inferior to the agreement achieved in free shear flows (e.g. the mixing layer - Weisbrot et al. 1988; the plane wake Marasli et al. 1989). This may stem from the fact that in the free shear flows mentioned, the shape of the normalized mean velocity distribution is insensitive to the forcing and it does not even change when the flow undergoes transition from a laminar to a turbulent state. The velocity profile in the inner portion of the wall-jet is distorted.
by strong perturbations as best witnessed by the difference between laminar and turbulent profiles (e.g. Tsuji et al. 1977). Mean flow distortion by the primary instability makes the velocity profile susceptible to a secondary instability which might have a very different eigen function associated with it. This susceptibility was not yet investigated. The possible existence of another unstable mode in turbulent flow was sought for a long time but only recently it was found to be feasible. The investigation started by examining carefully the conditions prevailing in a laminar wall jet.

6 The Stability of the Laminar Wall-Jet

The plane laminar wall-jet flowing over a flat surface in the absence of an external stream possesses two modes of temporal instability provided the Reynolds number is sufficiently large (Chun & Schwarz 1967, Tsuji et al. 1977 and Mele et al. 1986). The large scale (low frequency) disturbances are associated with the outer inflection point in the mean velocity profile while the small scale (high frequency) unstable disturbances evolve mostly near the surface. A limited number of spatial stability calculations done at Re = 800 and 900 (Cohen et al. 1992) also predict the existence of two spatially amplified modes. The power spectra measured near the surface and in the outer flow suggest that both the laminar and the turbulent wall jets contain two dominant scales (Cohen et al. 1992, Katz et al. 1992) in contrast to most free shear flows which are naturally (i.e. without external forcing) dominated by a single scale. Tsuji et al. (1977), who excited the laminar wall jet acoustically failed to confirm the existence of a second instability mode in their experiment, while Katz et al. did not manage to predict its possible existence in a turbulent wall jet at any of the Reynolds numbers they considered. The main purpose of the laminar flow investigation was to determine the range of parameters (both frequencies and Reynolds numbers) giving rise to spatially amplifying waves and to confirm the calculations experimentally for both modes of instability. Based on these findings, the interaction between the modes can be quantitatively specified.

Laminar wall jet was attained when the experiment was moved to a closed-loop wind tunnel thus eliminating the adverse effects of room drafts which hinder the uniformity of the low speed laminar flow. The velocity in the wall-jet was well represented by the Glauert solution even when the flow was externally excited at low amplitudes (mostly below 1%). Forcing the jet at the “inviscid mode” (at low frequency) had no effect on its width of the flow while the rate of spread quintupled when the frequency of forcing at identical initial amplitude increased to the region where the viscous mode became highly amplified. The effect of excitation on the decay of the maximum jet velocity is much less spectacular. This may have many implications on the spurious results obtained in turbulent flow.

It was also observed that laminar wall jet is susceptible to two independent and linearly amplifying modes of instability provided Re > 460. The phase velocity (and thus the wave length) of each mode is different although the dimensionless frequency $\beta$ and the Re are identical. The same holds true for the form of the eigen functions and hence the phase and amplitude distribution of the fluctuations across the flow varies depending on the particular mode considered. Only by a careful selection of parameters in an experiment one may follow the evolution of a pure mode over a limited spatial distance. In general both modes coexist and they can not be easily uncoupled.

7 Some Implications of the Stability of the Laminar Wall-Jet on the Study of Coherent Structures in Turbulent Flow

One may solve the Orr-Sommerfeld equation for a variety basic state velocities $U(y)$, of which Glauert's solution is a particular case. Such profiles may represent laminar wall jets...
I, flowing over concave or convex surfaces or distorted flows due to finite amplitude perturbations, the mean velocity distribution measured in a turbulent wall jet may be a special case. This prompted us to develop mathematically a universal family of wall-jet-like velocity profiles and study their stability characteristics. By increasing progressively the slope of the velocity profile near the surface we examined the sensitivity of the various instabilities to the change in the form of the profile. We finally proved that the turbulent wall jet is also susceptible to two independent modes of instability at high Reynolds numbers and this might be the source of the disagreement between theory and experiment observed by Katz et al (1992). Therefore, as long as we do not know how to separate these modes in a clear way and how to prevent the non linear interaction between them we will have difficulties in modeling coherent structures in wall bounded flows.

**APPENDICES**


**PUBLICATIONS NOT ENCLOSED**


Wygnanski I., "Active Control of Skin friction and Separation in certain class of Wall Bounded Flows" Proceedings of Monte Verita Colloquium on Turbulence Th. Dracos Editor to be published by Springer verlag 1991.

Rothstein, J. "Stability of Viscous Incompressible Wall-Jets Flowing on Curved Surfaces" Dipl. Ing. thesis at the Technical University Berlin 1992 for theoretical research done at the U of A.