A Test Fixture for Detecting the Presence of Water in Buoyant Cables

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Preface

This work was conducted under NUSC Project No. A55400; Principal Investigator C.E. Odams. The NUSC Program Manager is A.R. Susi, Code 34292. The sponsoring activity is SPAWAR PMW-153-2; Project Manager Michael Sheehan.

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A test fixture for detecting the presence of water in buoyant cables. The method makes use of test fixture in the form of a curved-plate capacitor. The plate configuration of the capacitor was designed in such a manner that its sensitivity to low levels of moisture (less than 0.1 ounce per foot) is accompanied by large percentage increases in capacitance from the nominal dry value. Typical increases in capacitance for such levels have been observed in the range of 12% to 25%, with fully saturated cables exhibiting maximum changes as large as 70%. A mathematical expression for volume of water present in a cable as a function of the percentage increase in capacitance was derived and has been shown to agree with experimental data.
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1. INTRODUCTION

This paper explores the design and implementation of a simple test fixture that indicates the presence of water or moisture inside buoyant cables. The principle employed in the development of the test fixture could be applied to any type of cable. The test fixture actually built was designed to only accommodate cables with an outer diameter of 0.65 inch. It was the leakage of water in NUSC-C-279/3-11 cable (hereafter referred to as type 3-11 cable) that motivated the development of the test fixture.

The migration of water in buoyant cables is caused in part by small cracks or holes on the outer jacket and/or by an incomplete filling with a water blocking compound (Vistanex™ and Eccogel™). These conditions, in the presence of the large hydrostatic pressure exerted by the sea, cause the migration of water in the cable. The flooding, if left unchecked, may damage the electrical connections and antenna - deployment mechanisms by causing short circuits and forming rust.

The determination of water presence, as indicated by the test fixture, is made in terms of capacitance. Since the complex relative permittivity of water (fresh or sea) has a large magnitude, changes in capacitance from the dry cable state can be readily observed. The amount of water can then be found by means of a graph relating the observed capacitance to a quantity of water, or by an algebraic expression.

The details pertaining to the year that the flooding problem arose are not precisely known. What is known, however, is that the severity (as noted by the frequency of failure reports received by NUSC), escalated to the point where serious attention to its nature and isolation was paid in the late fall of 1985. The problem was examined by repeated testing of the cable in accordance with NUSC-C-342/4141-279 specifications. The cable under consideration was the RF cable designated as type 3-11. The construction of the cable (shown in Fig. 1), caused most of the trapped water to be concentrated between the outer braid of the coaxial line and the foam fillers, although signs of flooding in the plastic sheath enveloping the twisted pair and portions of the space inside the coaxial line have been observed.
2. TEST FIXTURE DEVELOPMENT

2.1. General.

The development of an instrument to measure the moisture content in a suspect cable grew out of a 120-hour First-Article test conducted at NUSC on a sample of type 3-11 cable. In this test, 750 feet of cable was installed in a pressure vessel and subjected to 450 pounds per cubic inch gauge (PSIG). A short length of the cable (3 to 4 feet) was exposed to the atmosphere through an opening in the vessel, creating a difference in pressure. Figure 2 shows the test setup.

In order to monitor the degree of flooding, the conductors of the twisted pair (Fig. 1) were connected together, forming an extended conducting element. With the coaxial braid forming the remaining conductor, an elementary capacitor was made. An impedance analyzer was used to measure the capacitance between the combined conductors of the twisted pair and the shield of the coaxial cable. The capacitance variation with time is plotted in Fig. 3. The cable end exposed to the atmosphere began to leak after 164 hours had elapsed (the test was extended to accommodate this event), at a rate of 6.3 cubic inches per hour. Note that the leakage of water resulted in an increase in capacitance.

The procedure just mentioned provided the motivation for an alternate means of detecting water without physically cutting the cable. A noninvasive measurement of water penetration was considered important, since cutting the cable would render the cable useless for further service. Such a means would be beneficial for cables containing minute amounts of water (less than 0.1 ounce per foot). These amounts would not seriously affect the cable's buoyancy or its electrical characteristics, remaining operational.
2.2. The curved-plate capacitor.

The detection of water, as explained in § 2.1, utilized some of the conducting elements in the cable to form a capacitor. The change in capacitance between conductors within the cable could be used to identify specific regions along a sample length in which water was present. Ambiguities with regard to location and quantity of water existed.

An alternative to measuring the capacitance between conductors within the cable was realized by use of a conducting element pair in the form of thin curved strips conforming to the cable's outer jacket. The capacitance between curved strips could be used to determine the presence of water within the cable. The pair of curved metal strips will be referred to as the capacitor. The concept was tested with the model shown in Fig. 4, using the same hydrostatic test setup of Fig. 2. The variation in capacitance with time as water was forced through the cable is shown in Fig. 5.

2.3. Low-frequency electromagnetic shielding.

The use of a metal shield to house the capacitor was found to provide isolation from electrical noise. Measurements made without the shell at a frequency of 20 kHz (in earlier tests), were difficult to interpret. The instabilities in the capacitance readings due to the low frequency noise were reduced by raising the measurement frequency to 50 kHz. At this frequency, however, problems due to the handling of the device were most pronounced. Figure 6 shows the shield, made out of copper foil and plastic. A short piece of copper braid, connected to one end of the shield, allowed it to be placed at ground potential. Tests with the capacitor/shield combination were made to assess the performance of the complete assembly; the results are plotted in Fig. 7.
2.4. The location of water.

Tests described thus far to determine the device's effectiveness in detecting water were done with the device fixed at a specific location along the cable's length, using time as a frame of reference. In practice, it is desired to determine the extent of flooding as a function of position along a sample length. Figure 8 is a plot of the variation of capacitance with position, for a partially flooded cable. To make an accurate estimate of the degree of flooding, it is necessary to first establish a base-line capacitance measurement on a dry sample. This is done in order to compare both wet and dry readings. In the present example, a base-line capacitance of 19.4 ± 0.7 pF was observed throughout the entire sample length (= 36 feet).

3. TEST FIXTURE DESIGN

3.1. Test fixture design considerations.

The tests described in the previous section were performed in order to evaluate the usefulness of the proposed moisture detecting test fixture. The remaining work to be done was to improve the device. Toward this end, the areas considered were:

**Capacitor plate design.** The device's ability to detect minute traces of water is related to the size of the gaps on the plates (refer to Fig. 4). A test to determine the appropriate gap size required to increase the sensivity to low levels of moisture had to be performed.

**Shield design.** The proper design of a low-frequency shield that offered a degree of immunity to electrical noise was needed. Moreover, the need for ruggedness was of considerable importance. Lastly, a mechanism for keeping the cable sample straight during testing was desirable. This feature allowed for uniformity in testing, and was achieved by means of springs, pulleys or rollers.
Capacitance/moisture content correspondence. Calibration of the device in terms of moisture content was needed. The presence of water has been demonstrated to cause an increase in capacitance, but a relationship between the quantity of water causing an increase from a dry nominal (capacitance) value was absent. The quantity of water responsible for the change in relative permittivity is then found by the method discussed in § 4.3.

3.2. Capacitor plate design. Capacitance per unit length.

The capacitance per unit length of two curved cylindrical plates about a solid cylinder as shown in Fig. 9(a) can be computed from eq. (1) under the assumption that the plates are infinitesimally thin. The result is plotted in Fig. 9(b). The exact expression for this quantity, which accounts for electric field fringing, is given by [1]:

\[ C = 2 \varepsilon \frac{K(k')}{K(k)} \]  

where

C = the capacitance per unit length

\( \varepsilon \) = the permittivity of the medium surrounding the capacitor

and

\( K \) = the complete elliptic integral of the first kind, of modulus \( k \) and \( k' \) (Appendix B). The moduli are functions of the capacitor's cross-sectional dimensions.
3.2.1. Plate thickness correction.

The capacitance per unit length given by eq. (1) was derived under the assumption that the plate thickness is vanishingly small. For the practical case where a capacitor with a plate thickness "t" is given, a correction term must be added. Figure 11 is a plot of the correction term, based on work done by Bochenek [2] and Hilberg [3]. A simple approximation for a capacitor with \( t/D \geq 0.05 \) and \( s/D \leq 0.25 \) is given by [4]:

\[
\Delta C = \varepsilon \frac{\ln \left( 1 + 2 \frac{t}{D} \right)}{\sin^{-1} \left[ \frac{s}{D} \right]} \left( \frac{t}{D} \right) \sin^{-1} \left[ \frac{s}{D} \right]
\]

where \( D \) is the inner diameter of the capacitor, and is accurate to within 10 percent. The resulting expression for the total capacitance per unit length is then:

\[
C_{\text{total}} = C + \Delta C
\]

3.2.2. Plate sensitivity.

The resulting change in capacitance, due to a change in permittivity, is referred to as plate sensitivity, defined by [5]:

\[
S_{\text{plate}} = \frac{\partial C}{\partial \varepsilon}
\]

where \( \varepsilon \) is the effective relative permittivity of the medium in which the capacitor is immersed and \( C \) is \( C_{\text{total}} \), defined in eq. (3). The plate-gap distance, \( s \), is found (holding dimensions \( d \) and \( D \) constant) so as to make eq. (4) as large as possible. The gap distance is determined through a combination of calculation and experiment.
3.3. Low-frequency electromagnetic shield.

The design of a shield required to reduce electrical interference at the instrument's operating frequency, 50 kHz, was chosen to have a cylindrical shape. Unlike the earlier version (Fig. 5), the metallic wall thickness and outer diameter was increased to yield a higher degree of shielding. Shenfeld [6] gives an expression for the shielding effectiveness of a non-magnetic (\( \mu_r = 1 \)) cylinder as:

\[
\frac{H_o}{H_i} = \cosh (\gamma d) + \left(\frac{\gamma a}{2}\right) \sinh (\gamma d)
\] (5)

where

- \( H_o = \) amplitude of the incident magnetic intensity (amperes/meter)
- \( H_i = \) amplitude of the magnetic field intensity internal to the cylinder (amperes/meter)
- \( a = \) outer radius of the cylinder (meters)
- \( d = \) wall thickness (meters)
- \( \gamma = \) the wave propagation constant in the cylinder material (Nper/meter) = \( \sqrt{\frac{j \omega \mu_0 \sigma}{4\pi}} \)

The symbols \( j, \omega, \mu_0 \) and \( \sigma \) denote the complex number \( j = \sqrt{-1} \), angular frequency \( 2\pi f \), permeability of free-space \( 4\pi \times 10^{-7} \text{ H/m} \) and electrical conductivity \( \text{S/m} \), respectively. Figure 11 shows the variation in shielding effectiveness (expressed in dB) for different cylindrical sizes and materials, described by the parameter \( P \) and material skin-depth \( \delta \), as:

\[ P = \frac{a}{2d} \quad \text{and} \quad \delta = \sqrt{\frac{2}{\omega \mu_0 \sigma}} \]
3.4. Capacitance/moisture content.

An expression for determining an effective overall permittivity (\(\varepsilon^*\)) from the constituent parts of the material within the curved plate capacitor is derived in Appendix D and presented in eq. (6) below \[7,8\]:

\[
\ln(\varepsilon^*) = \sum_{i=1}^{N} \vartheta_i \ln(\varepsilon_i)
\]  

(6)

where

\(\vartheta_i\) = the \(i\)th fraction of the total volume in a substance with a relative permittivity of \(\varepsilon_i\)

\(N\) = the number of elements constituting the substance

and

\[
\sum_{i=1}^{N} \vartheta_i = 1
\]

In the analysis, the increase in capacitance due to the presence of water is found in terms of the resultant permittivity.

4. DESIGN SUMMARY

4.1. Capacitor plate design for high sensitivity.

Experiments performed to determine an appropriate plate-gap distance for high sensitivity, indicated that the use of small gaps were necessary (see Fig. 12). A suitable choice of the gap to diameter ratio (s/D) was at the designer's discretion, dictated primarily by mechanical limits (i.e., an adherence to construction tolerances). Therefore, the final choice was as follows:
4.2. Low-frequency electromagnetic shield.

For the design of the electromagnetic shield, Fig. 11 was used. The plot indicates that the shielding effectiveness varies directly with the cylinder's diameter and wall thickness. An increase in wall thickness, however, results in a heavier shield, thereby limiting the device's portability. Therefore, the diameter of the shield was considered over the wall thickness, with the following dimensions:

- i) shield outer diameter: 3 inches
- ii) shield length: 26 inches
- iii) wall thickness: 1/8 inch
- iv) material: brass

Using the dimensions given above in eq. (5), the shielding effectiveness as a function of frequency is plotted in Fig. 13, where at 50 kHz it is seen to have a value of 43 dB (the copper shield used in the earlier experiments had a shielding effectiveness of 26 dB at the same frequency).

To enhance the mechanical performance of the device, guide rollers were designed to provide tension so that the cable under test would remain straight. The degree of tension applied to the cable is adjustable by means of a lever-arm assembly. Figures 14 and 15 show the completed device.
4.3. Capacitance/moisture content correspondence.

The volume of water per unit length, \( V_f \), required to raise the dry cable capacitance \( C_m \) to a flooded value \( C_t \), as discussed in § 3 and Appendix D, is approximated by:

\[
V_f = \left\lfloor \frac{W_c}{\xi_c \rho_f g} \right\rfloor \pi a^2 \left[ \frac{\kappa \ln \left( \frac{C_t}{C_m} \right)}{1 - \kappa \ln \left( \frac{C_t}{C_m} \right)} \right], \quad \text{for} \quad 1 \leq \left( \frac{C_t}{C_m} \right) < \sqrt{\frac{\varepsilon_f}{\varepsilon_m}}
\]  

(7)

where

- \( W_c \) = the weight per unit length of the dry cable = 0.1 lb / ft
- \( \xi_c \) = the specific gravity of the dry cable = 0.74
- \( \rho_f, g \) = the density of water and acceleration due to gravity, respectively (\( \rho_f g = 62.4 \text{ lb} / \text{ft}^3 \) in English units)

\[
\kappa = \frac{1}{\ln \left( \frac{\varepsilon_f}{\varepsilon_m} \right)}
\]

and

- \( \varepsilon_f \) = the relative permittivity of water = 80 at 20° C
- \( \varepsilon_m \) = the relative permittivity of the cable material = 1.2 to 2.2
- \( a \) = radius of the center conductor in the cable = 0.092 in.

Figure 16 is a plot of the variation in water content as a function of the percentage increase in capacitance from the dry state, with experimental data.

4.4. Sensitivity of completed unit.

A check on the sensitivity of the completed device was performed in order to assess its behavior under actual conditions, using the definition given in § 3.5. The test utilized a 50 foot sample of 3-11 cable, and was set up in the manner shown in Fig. 17. With the cable under pressure (450 PSIG), the computer recorded the capacitance displayed on the impedance analyzer as a function of time (the recordings were made at one-second intervals), and was programmed to compute the derivative

\[
\Phi (\text{pF/minute}) = \frac{dC}{dt}
\]

(8)
which can be related to \( \partial C / \partial \varepsilon \). Figure 18 shows the variation of \( \Phi \) with time, where a peak value of 8.4 pF/minute is noted.

The calculation of \( \partial C / \partial \varepsilon \) is made with the aid of the relation:

\[
\frac{\partial C}{\partial \varepsilon} \approx \frac{\partial C}{\partial t} \cdot \frac{\partial t}{\partial \varepsilon} \tag{9}
\]

where \( t \) is the time taken for water to flow through the cable, and \( \varepsilon \) is the relative permittivity of the cable material. If the center conductor diameter (d) and capacitor diameter (D) are held constant, eq. (9) is simplified using the approximations below:

\[
\frac{\partial C}{\partial t} = \frac{\partial C}{\partial \varepsilon} \cdot \frac{\partial t}{\partial \varepsilon} = \frac{\partial t}{\partial \varepsilon} \cdot \Delta \varepsilon = \Delta t \tag{10}
\]

and

\[
\frac{\Delta \varepsilon \text{ (inch per minute)}}{\Delta t} = \frac{(C_{\text{wet}} - C_{\text{dry}})}{C_{\text{air}}} \cdot \frac{L}{\Delta t} \tag{11}
\]

where

- \( C_{\text{wet}} \) = the wet cable capacitance
- \( C_{\text{dry}} \) = the dry cable capacitance
- \( C_{\text{air}} \) = the capacitance with the center conductor and air as the dielectric
- \( L \) = the length of the capacitor plates (24 inches).

Substitution of eqs. (10) and (11) yields an estimate for the device's sensitivity as:

\[
S_{\text{plate}} \text{(pF/inch)} = \frac{\partial C}{\partial \varepsilon} = \frac{\Phi}{(\Delta \varepsilon / \Delta t)} = \frac{\Phi \cdot C_{\text{air}} \cdot \Delta t}{L \cdot (C_{\text{wet}} - C_{\text{dry}})} \tag{12}
\]
where $\Delta t$ is the time taken for water to flow through the cable segment enclosed by the capacitor plates (the velocity of water flow in the cable is assumed to be constant). Using the following experimental values:

$$
C_{\text{wet}} = 59.1 \text{ pF} \quad C_{\text{air}} = 21.1 \text{ pF} \\
C_{\text{dry}} = 42.8 \text{ pF} \quad \Delta t = 2 \text{ minutes}
$$

and $\Phi = 8.4 \text{ pF/inch}$ (maximum), substitution into eq. (12) yields $S_{\text{plate}} = 0.90 \text{ pF/inch}$. This value is larger than that found in the computed curve given in Fig. 12 ($s/D = 0.286$, $S_{\text{plate}} = 0.77 \text{ pF/inch}$), and can be attributed to the effects of the center conductor(s), shield assembly and the finite length of the plates.

5. TEST FIXTURE OPERATION AND USE

5.1. Instrument description.

For an external view of the instrument, refer to Fig. 14. The sample under test is inserted into port A1 or A2. The cable is held in place vertically by an adjustment of the bolts located behind the lever-pivot assemblies labelled D1 and D2, which tighten opposing rollers about the cables. The upper and lower half-shells of the RF shield assembly (C1 and C2) were designed so that both halves can be separated. This feature is necessary for cases where very long cable samples are tested, enabling the operator to survey a specific location without running the cable completely through. Ports B1 and B2 allow connection from the capacitor plates to an impedance analyzer, where the capacitance is displayed.
5.2. Moisture measurement setup and test.

A simple arrangement for checking a flooded cable is shown in Fig. 19. In this setup, the cable-under-test is fed through the instrument from its storage reel and onto the take-up spool. With the cable fastened to the take-up spool, testing can begin by rotating this spool. Experience with this setup has shown that a cable speed of approximately 5 feet per minute through the instrument is preferable, when inspecting flooded regions or "pockets" throughout a sample length.

TEST PROCEDURE

a) **Dry cable readings.** This step requires a careful measurement of dry cables for use as a base line, as mentioned in Section 2.2. A nominal value of 36 pF has been measured with the instrument, with deviation due to a combination of changes in the cable diameter, and specific gravity ($\xi_c$).

b) **Wet cable readings.** The same procedure used in (a) above is repeated with the wet cable. Readings for this case vary from 40 to 45 pF (light-to-moderate) and from 45 to 60 pF (moderate-to-extreme).

c) **Water content determination.** This procedure uses Fig. 16 or eq. (7) to determine the water content in the flooded cable. For the wet and dry readings given, this corresponds to a water-to-cable volume range of 3% to 12% (negligible) and 6% to 22% (negligible-to-light).
5.3. Flooding Chart.

The chart below was prepared to complement the plot in Fig. 16. In it, the percentage changes in capacitance, with the resulting changes in the quantity of water, are listed. This chart can be used as a guide in determining the suitability of the cable for further service.

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<th>Quantity of water</th>
<th>Flood description</th>
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<td>0 - 12</td>
<td>0 - 3</td>
<td>0 - 0.06</td>
</tr>
<tr>
<td>12 - 25</td>
<td>3 - 6</td>
<td>0.06 - 0.11</td>
</tr>
<tr>
<td>25 - 52</td>
<td>6 - 12</td>
<td>0.11 - 0.23</td>
</tr>
<tr>
<td>52 - 100</td>
<td>12 - 22</td>
<td>0.23 - 0.42</td>
</tr>
<tr>
<td>&gt; 100</td>
<td>&gt; 22</td>
<td>&gt; 0.42</td>
</tr>
</tbody>
</table>

* This quantity is computed using a cable with a volume of 3.43 in\(^3\)/ft. \(W_c = 0.1\) lb/ft, \(\xi_c = 0.74\), \(a = 0.092\) in.), and a conversion factor of 0.554 in\(^3\)/fl. oz.

** At this level, possible cable damage due to surface rupture can occur.
6. SUMMARY

This paper focused on the development, design and use of a test fixture for detecting water in buoyant cables. It was shown that the implementation of two thin strips, each bent to a radius equal to that of the cable's, allowed for this determination by means of a change in capacitance.

An examination of the properties of the capacitor plate configuration has led to a mechanical design that would yield a large tangential sensitivity, in the region where the ratio of the plate's gap-to-diameter (s/D) is less than 0.5. Such a plate shape ratio has been found to allow a fine degree of resolution in the detection of moisture. A new mathematical expression has been introduced in this paper linking the water content in a flooded sample (as a percentage of the cable's overall volume), to the percentage increase in capacitance. The expression shows that, in general, a 10% change in capacitance results in an 2.5% increase in the fractional volume of water in the cable.

The use of a metallic shield for electromagnetic isolation has been found to enhance the performance of the capacitor, by providing a shielding effectiveness of 43 dB at the operating frequency, 50 kHz. Finally, a simple procedure for testing cables having water damage has been described. In this procedure, the cable (wound on a reel), is fed through the detector by means of a drive mechanism located near a take-up (empty) reel.
THE "HOSING" PROBLEM

- COPPER BRAID IS SEA WATER GROUND. THEREFORE, WATER PENETRATION NOT A PROBLEM

- WATER "HOSING" PATHS ARE:
  1. COPPER BRAID WITH INSUFFICIENT VISTANEX COATING
  2. KEVLAR STRENGTH MEMBERS, IF INSUFFICIENT ELASTOMER TO FILL VOIDS

Fig. 1. Potential leak paths.
Pressure gauge reading: 450 PSIG

Type 3-11 cable

Pressure tank

Twisted pair

Coax outer conductor

Alligator clips (from impedance analyzer)

Fig. 2. Initial two-element capacitance test.
TEST CONDITIONS

1. Cable manufacturer: Blake wire and cable
2. Sample length: 750 ft.
3. Pressure: 450 PSIG
4. Cable type: NUSC-C-279/3-11
5. Three to four feet of cable exposed to atmospheric pressure.

Fig. 3. Capacitance between the combined conductors of the twisted pair and the braid of the coax.
Fig. 4. Curved plate capacitor. Dimensions given are in inches.
TEST CONDITIONS
1. Cable manufacturer: Consolidated Products
2. Cable type: NUSC-C-279/3-11
3. Pressure: 450 PSIG
4. Length of strips: 18 inches
5. Gap width: 3/8 inch
6. Measurement frequency: 50 kHz.

Fig. 5. Capacitance between curved strips exterior to the cable with water migration.
Fig. 6. Low frequency shield.
TEST CONDITIONS

1. Cable manufacturer: Consolidated Products
2. Cable type: NUSC-C-279/3-11
3. Sample length: 100 ft.
4. Cable length exposed to atmospheric pressure: approx. 20 ft.
5. Location of capacitor plates on cable: approx. 15 ft. from tank
6. Measurement frequency: 50 kHz
7. Low frequency shield in place.

Fig. 7. Capacitance between curved strips with water migration; shield in place.
Fig. 8. Capacitance between curved strips; variation with distance

TEST CONDITIONS
1. Cable Manufacturer: Consolidated Products
2. Cable type: NUSC-C-279/3-11
3. Sample length: approx. 36 ft.
4. Low frequency shield in place
5. Measurement frequency: 50 kHz
Fig. 9(a). Curved plate capacitor with center conductor.
Fig. 9(b). Capacitance per unit length of a curved plate capacitor for various $s/D$ and $d/D$ ratios.
Fig. 10. Change in capacitance due to plate thickening for various $s/D$ and $t/D$ ratios.
Fig. 11. Shielding effectiveness of non-magnetic (μ_r = 1) cylinders.
Fig. 12. Plate sensitivity (\(\partial C/\partial \varepsilon\)) for various s/D ratios.
NOTES
1. Infinitely long tube assumed
2. Shell material: Brass
3. Material resistivity ($\rho$) = 6.63 micro-ohm cm
4. Tube radius ($a$): 1-1/2 in.
5. Wall thickness ($d$): 1/8 in.

Fig. 13. Low frequency shielding effectiveness of cylinder used in the prototypes.
Moisture sensing assembly
Fig. 18. Variation of $\Phi$ with time.
Fig. 19. Hydrostatic pressure / Capacitance test setup.
Appendix A

EXPLODED VIEW OF TEST FIXTURE
Fig. A.1. Exploded view of test fixture.
# MOISTURE DETECTOR - PARTS LIST

<table>
<thead>
<tr>
<th>Item Number</th>
<th>Quantity</th>
<th>Description</th>
<th>Overall Dimensions (Inch)</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>Washer / Spacer</td>
<td>5/8 O.D. X 21/64 I.D. X 0.062 Thick</td>
<td>Teflon</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>Cable Guide Roller</td>
<td>1.5 O.D. X 0.314 I.D. X 0.975 Thick</td>
<td>Teflon</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>Cable Tension Arm</td>
<td>Arm Segment: 5.937 L X 0.5 W</td>
<td>Naval Brass</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Roller Anchor: 1.43 H X 1.5 W X 5/8 D</td>
<td>Pivot: 0.562 H X 0.5 W X 0.5 D</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>Washer / Spacer</td>
<td>0.5 O.D. X 17/64 I.D. X 0.062 Thick</td>
<td>Teflon</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>Pivot Shaft</td>
<td>0.25 Diam. X 1.50 L</td>
<td>Stainless Steel</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>Capacitor RF Shield / Housing (TOP)</td>
<td>Body: 3.00 O.D. X 2.75 I.D. X 26 L</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Pivot: 0.75 H X 1.0 W X 0.25 Thick</td>
<td>Brass</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Alignment Post: 0.5 H X 0.37 W X 0.37 D</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>BNC Connector Stand:</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.75 L X 0.75 W X 0.25 D</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>Pan Head Machine Screw</td>
<td># 4 - 40 X 3/8 L</td>
<td>Stainless Steel</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>BNC Female RF Connector</td>
<td></td>
<td>Nickel Plated Brass</td>
</tr>
</tbody>
</table>
# MOISTURE DETECTOR - PARTS LIST (CON'T)

<table>
<thead>
<tr>
<th>Item Number</th>
<th>Quantity</th>
<th>Description</th>
<th>Overall Dimensions (inch)</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>20</td>
<td>Flat Head Machine Screw</td>
<td># 4 - 40 X 3/8 L</td>
<td>Stainless Steel</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>Tension Adjust Bolt</td>
<td>1/4 - 20 X 1.00 L</td>
<td>Stainless Steel</td>
</tr>
<tr>
<td>11</td>
<td>2</td>
<td>Tension Adjust Nut</td>
<td>1/4 - 20 X 0.25 H</td>
<td>Stainless Steel</td>
</tr>
<tr>
<td>12</td>
<td>6</td>
<td>Roller Support Shaft</td>
<td>0.25 Diam. X 2.00 L</td>
<td>Stainless Steel</td>
</tr>
<tr>
<td>13</td>
<td>2</td>
<td>Capacitor Plate Support Bed</td>
<td>2.75 O.D. X 0.75 I.D. X 26 L</td>
<td>Lexan (Polycarbonate)</td>
</tr>
<tr>
<td>14</td>
<td>4</td>
<td>RF Shield End - Caps</td>
<td>3.00 O.D. X 0.75 I.D. X 0.125 Thick</td>
<td>Brass</td>
</tr>
<tr>
<td>15</td>
<td>2</td>
<td>Capacitor Plates</td>
<td>0.75 O.D. X 0.70 I.D. X 24 L</td>
<td>Brass</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Plate Angle of Arc : 146.8 Degrees</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>4</td>
<td>Housing Alignment Pins</td>
<td>0.1875 Diam. X 1.00 L</td>
<td>Stainless Steel</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>1</td>
<td>Capacitor RF Shield / Housing (BOTTOM)</td>
<td>5.25 L X 1.50 W X 1.56 H</td>
<td>Brass</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cable Guide Roller Support;</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Alignment Post :0.5 H X 0.37 W X 0.37 D</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>BNC Connector Stand :</td>
<td>0.75 L X 0.75 W X 0.25 D</td>
<td></td>
</tr>
</tbody>
</table>
Appendix B

ELLiptic Integrals

1. Variable names.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>Modulus</td>
<td>$0 \leq k \leq 1$</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>Amplitude</td>
<td>$0 \leq \Psi \leq \pi/2$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Characteristic</td>
<td>$-\infty \leq \rho \leq \infty$</td>
</tr>
</tbody>
</table>

2. Integrals.

2.1. Elliptic integral of the first kind.

a) Incomplete

$$F(\Psi \setminus k) = \int_{0}^{\Psi} \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}}$$  \hspace{1cm} (1a)

b) Complete

$$K(k) = F\left(\frac{\pi}{2} \setminus k\right) = \int_{0}^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}}$$  \hspace{1cm} (1b)
2.2. Elliptic integral of the second kind.

a) Incomplete

\[ E(\Psi \setminus k) = \int_0^\Psi \frac{1}{\sqrt{1 - k^2 \sin^2 \theta}} \, d\theta \]  \hspace{1cm} (2a)

b) Complete

\[ E(k) = E\left(\frac{\pi}{2} \setminus k\right) = \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{1 - k^2 \sin^2 \theta}} \, d\theta \]  \hspace{1cm} (2b)

2.3. Elliptic integral of the third kind.

a) Incomplete

\[ \Pi(\rho; \Psi \setminus k) = \int_0^\Psi \frac{d\theta}{(1 - \rho \sin^2 \theta) \sqrt{1 - k^2 \sin^2 \theta}} \]  \hspace{1cm} (3a)

b) Complete

\[ \Pi(\rho \setminus k) = \Pi(\rho; \frac{\pi}{2} \setminus k) = \int_0^{\frac{\pi}{2}} \frac{d\theta}{(1 - \rho \sin^2 \theta) \sqrt{1 - k^2 \sin^2 \theta}} \]  \hspace{1cm} (3b)
Appendix C

COMPUTATION OF THE CAPACITANCE PER UNIT LENGTH OF A CURVED PLATE CAPACITOR WITH A CENTER CONDUCTOR

1. General.

An exact method and an approximate method of calculating the capacitance of a curved plate capacitor with a center conductor are given in this appendix. These methods were not used directly in the development of the test fixture, but are presented here for the sake of completeness. They show the relationship between the capacitance and the physical characteristics of the curved plate capacitor. A method of accounting for the thickness of the plates is also given.

A cross-sectional view of the capacitor is Fig. C.1. In this figure, the center conductor of diameter "d" is held at ground potential, while the upper and lower plates are set at potentials of +V₀ and -V₀, respectively. The capacitance computed from such an arrangement is called the odd-mode capacitance, and refers to the odd symmetry of the potential distribution on the plates.

1.1. Exact formula for infinitesimally-thin plates.

The exact expression for the odd-mode capacitance per unit length, found using conformal mapping techniques is given by [9]:

\[
C \text{ (Farads per meter)} = 2 \varepsilon \frac{K(k')}{K(k)}
\]  (1a)

where

\[
k = \frac{1}{\sqrt{1 + \beta}} \quad \text{and} \quad k' = \sqrt{1 - k^2}
\]  (1b)
The parameter $\beta$ is a function of the angle subtended by the plates in the gap region ($\varphi$) and the ratio $D/d$, and is determined from the set of simultaneous parametric equations given below:

\[
\ln\left(\frac{d}{D}\right) = \frac{[(\alpha - 1) \Pi(\rho_1/\kappa_1) - (\alpha + \gamma) K(\kappa_1)]}{\sqrt{\alpha(\beta + 1)}}
\]  

(2a)

\[
\cos^{-1}\left(\frac{s}{D}\right) = \frac{\pi}{2} \cdot \varphi = \frac{[(\gamma + 1) F(\Psi/\kappa_1) - \Pi(\rho_2/\Psi/\kappa_1)]}{\sqrt{\alpha(\beta + 1)}}
\]  

(2b)

and

\[
\frac{\pi}{2} = \frac{[(\gamma - \beta) K(\kappa_1) + (\beta + \alpha) \Pi(\rho_3/\kappa_1)]}{\sqrt{\alpha(\beta + 1)}}
\]  

(2c)

where

\[
k_1 = \sqrt{\frac{\beta(\alpha + 1)}{\alpha(\beta + 1)}} \quad \text{and} \quad k_1' = \sqrt{1 - k^2}
\]  

(3a)

also

\[
\rho_1 = -\frac{1}{\alpha} \quad \text{and} \quad \rho_2 = -\left(\frac{\beta}{\beta + 1}\right)
\]  

(3b)

\[
\rho_3 = -\left(\frac{\alpha - 1}{\beta + 1}\right) \quad \text{and} \quad \Psi = \sin^{-1}\left(\sqrt{\frac{\gamma(\beta + 1)}{\beta(\gamma + 1)}}\right)
\]  

(3c)
1.2. Approximate computation: variational formulas.

An alternate method of determining the capacitance for the configuration shown in Fig. C.1 is through the use of the Rayleigh-Ritz technique, used in the Calculus of Variations. In the present application, a mathematical expression describing the electrostatic charge distribution on the capacitor plates is chosen in a manner such that electric field energy stored in the system is a minimum. Due to the nature of this technique, an upper and lower bound for the capacitance is sought. Collin [10] has derived these bounds (converted here in terms of capacitance, Farads/meter) in the form:

a) Lower-bound capacitance:

\[
C_{\text{lower bound}} = \frac{\pi \varepsilon}{16} \frac{(\pi - 2\phi)^2}{\sum_{n = 1,3,5,...}^{\infty} \frac{\cos^2 n\phi}{1 + \zeta \frac{n^2 (\pi - 2\phi)^2}{n^2 (\pi - 2\phi)^2 - 4\pi^2}}}^{2}
\]

where

\[
\zeta = \frac{\sum_{n = 1,3,5,...}^{\infty} \frac{\cos^2 n\phi}{n [1 + \coth (n \ln \frac{D}{d})] [n^2 (\pi - 2\phi)^2 - 4\pi^2]}}{\sum_{n = 1,3,5,...}^{\infty} \frac{n \cos^2 n\phi [\pi - 2\phi]^2}{[1 + \coth (n \ln \frac{D}{d})] [n^2 (\pi - 2\phi)^2 - 4\pi^2]^2}}
\]
b) Upper-bound capacitance:

\[
C_{\text{upper bound}} = \frac{4}{\pi} \varepsilon \frac{\varphi^2}{\sum_{n=1,3,5,...}^{\infty} \left[1 + \coth \left( n \ln \frac{D}{d} \right) \right] \frac{\sin^2 n\varphi}{n^3}}
\]  

(4c)

where \( \varphi \) and \( D/d \) were defined in Fig. C.1. The plot in Fig. C.2 was made by computing the arithmetic average of the upper and lower bounds.

1.3. Plate thickness correction.

The problem of accounting for the finite plate thicknesses found in actual practice can be made by the addition of a correction, \( \Delta C \), to the vanishingly-thin plate capacitance computed from the expressions given earlier, with no center conductor. This assumption is made for cases where the plate thickness-to-diameter ratio \( (t/D) \) is very small \( (<< 1) \) and the plate shape ratio \( (s/D) \) is small \( (< 0.5) \) so that the electric field is not seriously distorted. Bochenek [11] has analyzed this problem, but the expressions, like those derived by Smolarska [12], are difficult to evaluate in their exact form. Figure C.3 illustrates the problem under consideration.

The capacitance per unit length of Fig. C.3a is found from:

\[
C_1 = 2 \varepsilon \frac{K(k_1)}{K(k_1)}
\]

(5a)
where

\[ k_1 = \frac{1}{\sqrt{1 + \left[ \frac{E(k)}{k \sin^{-1} \left( \frac{s}{D} \right)} \right]^2}} \quad \text{and} \quad k_1^* = \sqrt{1 - k_1^2} \quad (5b) \]

and \( k \) is the zero of eq. (5c):

\[ \sin^{-1} \left( \frac{s}{D} \right) \left[ K(k') - E(k') \right] - \ln \left( 1 + 2 \frac{1}{D} \right) E(k) = 0 \quad (5c) \]

where

\[ k' = \sqrt{1 - k^2} \quad (5d) \]

The capacitance per unit length of Fig. C.3b is given by Hilberg [13] as:

\[ C_2 = 2 \varepsilon \frac{K(k_2)}{K(k_2)} \quad (6a) \]

where

\[ k_2 = \frac{s}{D} \quad \text{and} \quad k_2^* = \sqrt{1 - k_2^2} \quad (6b) \]

Finally, the value of \( \Delta C \) to be added to the thin-plate capacitance per unit length of a capacitor with a center conductor is found from:

\[ \Delta C = C_1 - C_2 \quad (6c) \]
The plot shown in Fig. C.4 has been constructed using the expressions given above. By knowing the value of \( t/D \) and \( s/D \), the value of \( \Delta C \) is found and added to the infinitesimally-thin plate capacitance per unit length, given by Fig. C.2.
Fig. C.1. Capacitor with center conductor.
Fig. C.2. Calculated capacitance of a curved plate capacitor with a center conductor.

NOTES

1. Curves are computed from the arithmetic mean of the upper and lower bound variational formulas given by Collin [10].

2. Plates are considered to be vanishingly thin.
Small gaps are assumed: \( s/D < 0.5 \)

Curved plate capacitor with finite plate thickness "t" with capacitance \( C_1 \)

Curved plate capacitor with infinitesimally thin plates, with capacitance \( C_2 \)

Fig. C.3. Plate thickness correction. The correction, \( \Delta C \), is the difference of the two capacitances \( C_1 \) and \( C_2 \): \( \Delta C = C_1 - C_2 \).
Fig. C.4. Plate thickness correction for curved plate capacitors.
Appendix D

THE DETERMINATION OF THE WATER CONTENT IN A FLOODED BUOYANT CABLE IN TERMS OF CAPACITANCE

1. Volume of water.

To arrive at an expression for the amount of water trapped in a cable in terms of capacitance, some assumptions are made to simplify the analysis. For a given length:

i) The cable is of constant volume made up of a material having uniform density and permittivity. There are voids within the cable that become filled with material

ii) The cable cross-section is uniform, i.e., no bulges or other defects are present on the cable's outer jacket

iii) The flooded cable weight is the sum of the cable's dry weight and that due to the volume of water (to be determined)

iv) The distribution of water is random within the air voids inside the cable.

Statement (iii) can be written as:

\[ W_t = W_m + W_f \]  \hspace{1cm} (1)

where

\[ W_t = \text{the flooded cable weight (excluding the center conductor)} \]

\[ W_m = \text{the dry cable weight (excluding the center conductor)} \]

\[ W_f = \text{the weight imparted by a volume of water in the cable.} \]
Let:

\[ \rho = \text{density} = \text{mass} / \text{volume} \]

\[ g = \text{acceleration due to gravity} \]

\[ V = \text{volume} \]

\[ \xi = \text{specific gravity: material density relative to water} \]

then

\[ \rho_t \, g \, V_t = \rho_m \, g \, V_m + \rho_f \, g \, V_f \]

or

\[ \frac{\rho_t}{\rho_f} \, g \, V_t = \frac{\rho_m}{\rho_f} \, g \, V_m + \frac{\rho_f}{\rho_f} \, g \, V_f \]

and finally

\[ \xi_t \, V_t = \xi_m \, V_m + V_f \quad (2) \]

Since the volume of the cable remains constant, it must be composed of unequal amounts of cable material and water such that:

\[ V_t = V_m + V_f \quad (3) \]

this relation establishes bounds on the maximum possible volume that water can occupy, since the cable volume is finite.
Combining eqs. (2) and (3):

$$
\xi (V_m + V_f) = \xi_m V_m + V_f
$$

(4)

2. The effective permittivity of a wet cable.

The effective permittivity resulting from a mixture of substances having permittivities \(\varepsilon_{1,2,3,...}\) while occupying a fraction \(\vartheta_{1,2,3,...}\) of the total substance volume can be computed from [14]

$$
\ln (\varepsilon^*) = \sum_{i=1}^{N} \vartheta_i \ln (\varepsilon_i)
$$

(5)

where

\(\varepsilon^* = \text{effective permittivity of the substance (the wet cable)}\)

\(\vartheta_i = \text{the fraction of the total volume containing an element with a permittivity of } \varepsilon_i\)

\(N = \text{number of constituent permittivities}\)

and

$$
\sum_{i=1}^{N} \vartheta_i = 1
$$
3. Quantity of water inside a cable.

The cable under consideration is composed of the insulator and a volume of water.

Let:

\[ \vartheta_1 = \frac{\text{volume of cable material}}{\text{total volume}} = \frac{1 - \xi_t}{1 - \xi_m} \]  \hspace{1cm} (6a)

\[ \vartheta_2 = \frac{\text{volume of water}}{\text{total volume}} = \frac{\xi_t - \xi_m}{1 - \xi_m} \]  \hspace{1cm} (6b)

using eq. (5):

\[ \ln (\varepsilon^*) = \vartheta_1 \ln (\varepsilon_1) + \vartheta_2 \ln (\varepsilon_2) \]  \hspace{1cm} (6c)

\[ = \frac{1 - \xi_t}{1 - \xi_m} \ln (\varepsilon_1) + \frac{\xi_t - \xi_m}{1 - \xi_m} \ln (\varepsilon_2) \]  \hspace{1cm} (6d)

solving for \( \xi_t \),

\[ \xi_t = \xi_m \left[ \ln \left( \frac{\varepsilon_2}{\varepsilon_1} \right) \right] + \left[ \ln \left( \frac{\varepsilon^*}{\varepsilon_1} \right) \right] \]  \hspace{1cm} (6e)

(here, \( \xi_m \) is the specific gravity of the cable material = 0.72).
Eq. (6e) can be rewritten in terms of capacitance. Let:
\[
\varepsilon_1 = \frac{C_m}{C_{\text{air}}} \quad \varepsilon^* = \frac{C_{\text{air}}}{C_{\text{air}}}
\]

where
\[
C_{\text{air}} = \text{the capacitance of the curved plate capacitor with a center conductor and an air dielectric}
\]
\[
C_m = \text{the dry cable capacitance}
\]
\[
C_t = \text{the wet cable capacitance}
\]

and \(\varepsilon_2\) is the relative permittivity of water (≈ 80 at 20° C).

Substitution of the capacitance ratios given above into eq. (6e), along with some algebraic manipulation, yields an expression for the total specific gravity of the flooded cable material, as:
\[
\xi_t = \xi_m + \kappa \left(1 - \xi_m\right) \ln \left(\frac{C_t}{C_m}\right)
\]

(7)

where
\[
\kappa = \frac{1}{\ln \left(\frac{\varepsilon_2}{\varepsilon_1}\right)} = 0.26, \text{ using } \varepsilon_1 = 1.7.
\]

The amount of water in the cable can be found using eq. (7):
\[
\frac{\text{volume of water}}{\text{volume of cable}} = \frac{V_t}{V_m} = \frac{\xi_t - \xi_m}{1 - \xi_t} = \frac{\kappa \ln \left(\frac{C_t}{C_m}\right)}{1 - \kappa \ln \left(\frac{C_t}{C_m}\right)}
\]

(8)
In eq. (8), $V_m$ can be expressed as the difference between the total volume of the
cable, $V_c$, and the volume of the center conductor, $V_a$:

$$V_m = V_c - V_a \quad (9)$$

The cable volume ($V_c$) can be computed from:

$$V_c = L \left( \frac{W_c}{\xi_c \rho_f \rho_g} \right) \quad (10a)$$

where

- $W_c$ = the weight per unit length of the cable = 0.1 lb / ft
- $\xi_c$ = the specific gravity of the cable = 0.74
- $\rho_f$ = the mass density of fresh water = 1.94 slug / ft$^3$ at 20°C
- $g$ = the acceleration due to gravity = 32.2 ft / sec$^2$
- $L$ = the capacitor plate length = 2 ft.

The cable volume computed from eq. (10a) is useful because it accounts for air voids
in the cable. The volume of the center conductor, $V_a$, is simply

$$V_a = \pi a^2 L \quad (10b)$$

where $a$ is the radius of the conductor (≈ 0.092 inch). The material volume, $V_m$, is then

$$V_m = L \left[ \frac{W_c}{\xi_c \rho_f \rho_g} - \pi a^2 \right] \quad (10c)$$

Substitution of eq. (10c) into eq. (8) gives the total volume of water in the cable as:

$$V_f = L \left[ \frac{W_c}{\xi_c \rho_f \rho_g} - \pi a^2 \right] \left[ \frac{\kappa \ln \left( \frac{C_L}{C_m} \right)}{1 - \kappa \ln \left( \frac{C_L}{C_m} \right)} \right] \quad (11)$$

where $\kappa$ is defined in eq. (7). The expression given as eq. (7) in the main text gives $V_f$ in
per unit length terms, by dropping the plate length $L$ in eq. (11).
4. Upper limit determination.

An upper limit for eq. (8) can be determined by assigning an extreme case when $V_f = V_m$ in eq. (4) (This condition is hypothetical. Experiments carried out on type 3-11 cables have indicated that saturation occurs when $V_f = 0.25 V_m$, thus preventing further increases in capacitance from the nominal dry value). When this condition is set, eq. (4) has acquires the form

$$\xi_t = \frac{1 + \xi_m}{2}.$$  \hspace{1cm} (12)

Inserting eq. (12) into the expression for the specific gravity, eq. (7), and solving for the capacitance ratio, yields

$$\left(\frac{C_t}{C_m}\right) = \sqrt{\frac{\xi_f}{\xi_m}}.$$  \hspace{1cm} (13)
Appendix E

SENSITIVITY

1. General.

There are generally three ways to characterize a device's sensitivity [15]:

**Tangent.** For a given input variable \( x \), an output deflection \( y \), the sensitivity of the device, holding all other system variables constant is

\[
S_t = \frac{\partial y}{\partial x} \quad \text{other system variables remain constant}
\]  

(1)

**Chord.** This measure, as the tangent, utilizes a \( y \) versus \( x \) profile to define \( S \). It, however, differs from the tangential case since it uses the end points of the profile, through which a straight line is drawn. The slope of this line, or chord, is the sensitivity:

\[
S_c = \frac{y_2 - y_0}{x_2 - x_0}
\]  

(2)

**Secant.** This means of defining output deflection to input change, is used to specify a nominal sensitivity for a given range. In mathematical terms, it is defined in a manner identical to that of the chord, except that the end points are confined to a smaller region of interest within the profile:

\[
S_s = \frac{y_2 - y_1}{x_2 - x_1}
\]  

(3)

Because of this definition, several secant sensitivities are used to specify an instrument's responsiveness. Fig. E.1 shows a graphical definition of the terms discussed above.
From the foregoing discussion, it is seen that there are three distinct ways in which to specify instrument sensitivity. Since the capacitor under consideration varies non-linearly with the plate gap-to-diameter ratio \(s/D\), the tangential sensitivity \(S_t\) was chosen as the measure of effectiveness for the device. The plate tangential sensitivity, borrowing the earlier definition, becomes:

\[
S_t = \frac{\partial C}{\partial \varepsilon} = \psi \frac{\partial C}{\partial x}
\]  

The independent variable \(x\) is chosen to be the length of the capacitor containing a dielectric. An increase in the length of the dielectric will cause a capacitance rise (deflection) that will depend on the ratio, \(s/D\). The constant of proportionality, \(\psi\), relates \(\partial C/\partial x\) to \(\partial C/\partial \varepsilon\) and is discussed in § 2.1.

2. Sensitivity experiment.

In order to experimentally determine the tangential sensitivity, a test fixture, shown in Fig. E.2, was made. It was constructed from thin-walled plastic tubing. Along its length, notches were made at half-inch intervals for a total of one foot and thin capacitor plates (4 mils) were affixed on the tube's outer surface.

With the plates cut to a given width (fixing the value of \(s/D\)), the sensitivity was determined by filling the cylinder with a fluid having a known relative permittivity at the interval markings along the side of the tube. Figures E.3, E.4 and E.5 show that the capacitance varies linearly with fluid level for different values of \(s/D\), enabling the sensitivity to be determined. Figure E.6 is a plot of the slopes computed for each line (for the three fluids used), as a function of \(s/D\), where the slopes are seen to vary inversely with the plate gap separation. Figure E.7 is a plot of \(\partial C/\partial \varepsilon\) computed from Figs. E.3 through E.6.
2.1. Derivation of $\frac{\partial C}{\partial \varepsilon}$.

a) Theoretical result.

The total capacitance of the configuration shown in Fig. E.8 is:

$$C_{\text{total}} = \left[ \varepsilon_{r, \text{eff}}^{(1)} (1 - h) + \varepsilon_{r, \text{eff}}^{(2)} h \right] C_0$$  \hspace{1cm} (5)

where

- $C_0 =$ capacitance in air (no dielectric)
- $\varepsilon_{r, \text{eff}}^{(1)} =$ effective relative permittivity of empty tube
- $\varepsilon_{r, \text{eff}}^{(2)} =$ effective relative permittivity of tube filled with fluid
- $h =$ the fluid height in the tube
- $l =$ the tube length

Let:

$$\frac{\partial C}{\partial \varepsilon} = \frac{\partial C}{\partial h} \cdot \frac{\partial h}{\partial \varepsilon}$$  \hspace{1cm} (6)

and

$$\frac{\partial C}{\partial h} = [\varepsilon_{r, \text{eff}}^{(2)} - \varepsilon_{r, \text{eff}}^{(1)}] C_0$$  \hspace{1cm} (7a)

$$\frac{\partial \varepsilon}{\partial h} = [\varepsilon_{r, \text{eff}}^{(2)} - \varepsilon_{r, \text{eff}}^{(1)}]$$  \hspace{1cm} (7b)
multiplication of eqs. (7a) and (7b) yields the tangential sensitivity as:

\[
\frac{\partial C}{\partial \varepsilon} = \left[ \varepsilon_{r, \text{eff}}^{(2)} - \varepsilon_{r, \text{eff}}^{(1)} \right] C_0 \times \frac{1}{\left[ \varepsilon_{r, \text{eff}}^{(2)} - \varepsilon_{r, \text{eff}}^{(1)} \right]} = C_0. \tag{8}
\]

**b) Comparison of theoretical result with measurement.**

In the present measurements, we have \( \partial C / \partial h \). To obtain \( \partial C / \partial \varepsilon \), the expression,

\[
\frac{\partial C}{\partial \varepsilon} = \psi \frac{\partial C}{\partial h}
\]

is used. In the expression, \( \psi \) is the reciprocal of the expression for \( \partial \varepsilon / \partial h \) given in part (a). We must, therefore, find an expression for \( \psi \) in terms readily measured parameters. The derivation for \( \psi \) is as follows:

1. We note the following:

\[
\varepsilon_{r, \text{eff}}^{(2)} = \frac{C_{\text{full tube}}}{C_0} \tag{9a}
\]

\[
\varepsilon_{r, \text{eff}}^{(1)} = \frac{C_{\text{empty tube}}}{C_0} \tag{9b}
\]

2. Casey [16] has derived an expression that gives the effective dielectric constant of a curved plate capacitor (no center conductor) with an interior filled with a dielectric, and an air exterior as:

\[
\varepsilon_{r, \text{eff}} = \frac{1 + \varepsilon_r}{2} \tag{10}
\]
where $\varepsilon_r$ is the relative permittivity of the interior medium.

3. The determination of $\varepsilon_r$ is complicated by the fact that it is composed of a tube filled with a fluid. King [17] gives an approximate expression for the case where the surface of a dielectric mixture is perpendicular to the direction of the electric field (present case), as:

$$\frac{1}{\varepsilon_r} = \frac{p}{\varepsilon_r(1)} + \frac{1-p}{\varepsilon_r(2)}$$  \hspace{1cm} (11)

where

$\varepsilon_r(1)$ = the relative permittivity of the plastic tube ($= 3.2$)

$\varepsilon_r(2)$ = the relative permittivity of the fluid in the tube

$p$ = the fraction of the total volume occupied by medium 1 (the tube)

$$V_{\text{empty tube}} = \frac{\pi (d_o^2 - d_i^2) l}{4} = 1 - \left(\frac{d_i}{d_o}\right)^2$$

4. From the foregoing discussion, we now write:

$$\varepsilon_r^{(2)}_{\text{eff}} = \frac{1 + \varepsilon_r\text{, full tube}}{2}$$  \hspace{1cm} (12a)

$$\varepsilon_r^{(1)}_{\text{eff}} = \frac{1 + \varepsilon_r\text{, empty tube}}{2}$$  \hspace{1cm} (12b)
then:

\[
\varepsilon_{r, \text{eff}}^{(2)} - \varepsilon_{r, \text{eff}}^{(1)} = \left(\frac{1 + \varepsilon_r, \text{empty tube}}{2}\right) \cdot \left(\frac{1 + \varepsilon_r, \text{full tube}}{1 + \varepsilon_r, \text{empty tube}} - 1\right)
\]

\[
= \left(\frac{1 + \varepsilon_r, \text{empty tube}}{2}\right) \cdot \left(\frac{C_{\text{full tube}}}{C_{\text{empty tube}}} - 1\right)
\]

therefore

\[
\frac{\partial C}{\partial \varepsilon} = \left(\frac{\partial C}{\partial h}\right) \cdot \left(\frac{1 + \varepsilon_r, \text{empty tube}}{2}\right) \cdot \left(\frac{C_{\text{full tube}}}{C_{\text{empty tube}}} - 1\right)
\]

Given:

\[
d_i = 1 \text{ inch}
\]
\[
d_o = 1 - 1/8 \text{ inch}
\]
\[
\varepsilon_r (1) = 3.2
\]
\[
\varepsilon_r (2) = 1 \text{ (air)}
\]

then:

\[
\varepsilon_r, \text{empty tube} = 1.17 \quad \text{and} \quad \varepsilon_{r, \text{eff}}^{(1)} = \frac{1 + \varepsilon_r, \text{empty tube}}{2} = 1.08.
\]

The quantities \(C_{\text{full tube}}, C_{\text{empty tube}}\) and \(\partial C/\partial h\) are taken from Figs. E.3 through E.6 and are substituted in the expression(s) given earlier, resulting in the plot of Fig. E.7. The solid curve is the capacitance per unit length computed using the equations derived by Bochenek and Hilberg [2,3].
Fig. E.1. Graphical definition of sensitivity.
Fig. E.2. Test fixture used for sensitivity measurements.
Fig. E.3. Sensitivity test; capacitance vs. fluid (water, \(\varepsilon_r = 80\)) level in the graduated cylinder.
Fig. E.4: Sensitivity test, capacitance vs. fluid height in the graduated cylinder.

Sensitivity test: capacitance vs. fluid level (2-Propyl Alcohol, \( \varepsilon_r = 22.2 \) level)
Fig. E.5. Sensitivity test; capacitance vs. fluid (Light Machine Oil, $\varepsilon_r = 2.2$) level in the graduated cylinder.

Measurement frequency: 50 kHz

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Fig. E.6. Variation of $\partial C/\partial h$ with $s/D$.

Legend

- Water, $\varepsilon_r = 80$
- 2-Propyl Alcohol, $\varepsilon_r = 22.2$
- Light Machin. Oil, $\varepsilon_r = 2.2$

* All three curves are interpolated between measured points.
TEST CONDITIONS

1. No center conductor: d/D=0
2. Capacitor plate thickness(t): 0.004 in.
4. Wall thickness of graduated cylinder: 1/8 in.
5. Cylinder material: clear polyvinyl chloride (PVC)
6. Relative permittivity of cylinder material ($\varepsilon_r$): 3.2 at 1 kHz
7. Measurement frequency: 50 kHz

Fig. E.7. Variation of Plate Sensitivity ($\partial C/\partial \varepsilon$) with s/D.
Fig. E.8. Computation of $\partial C/\partial e$. (a) Side view of tube with fluid; (b) Top view. The plates affixed to tube are not shown for clarity.
Fig. E.8. Computation of $\frac{\partial C}{\partial z}$. (a) Side view of tube with fluid; (b) Top view. The plates affixed to tube are not shown for clarity.
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