Sensor Array Processing

Thomas Kailath

Information Systems Laboratory
Dept. of Electrical Engineering
Stanford University
Stanford, CA 94305-4055

Office of Naval Research
300 N. Quincy St., BCT No. 1
Arlington, VA 22217-5660

Research issues specifically related to the processing of data produced by arrays of sensors as might be encountered in various remote sensing scenarios and direction finding problems.
February 14, 1992

Dr. R. Madan  
Code 1114  
Office of Naval Research  
800 North Quincy Street  
Arlington, VA 22217-5000

re: N00014-85-K-0550

Dear Dr. Madan:

On behalf of Professor T. Kailath, I am enclosing the final report on the above referenced contract.

Sincerely yours.

Barbara McKee  
Administrative Assistant

bmck

cc: P. McCabe  
P. Biddle

encl: 1
Sensor Array Processing

Office of Naval Research Contract N00014-85-K-0550, Task NR-SDRB-002

Final Technical Report for the Period
August 1, 1985 to July 31, 1991

Thomas Kailath
Information Systems Laboratory
Department of Electrical Engineering
Stanford University
Stanford, CA 94305

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1 Introduction

This document is the final technical report for the Office of Naval Research's contract N00014-85-K-0550, Sensor Array Processing. The contract was initially granted for the period July 1, 1985 through June 30, 1988, and subsequently extended to the period August 1, 1988 through July 31, 1991. The proposals submitted for these two contract periods described research issues specifically related to the processing of data produced by arrays of sensors as might be encountered in various remote sensing scenarios and direction finding problems. We believe that our research efforts in these regards have produced significant results. Moreover, we successfully applied the understanding gained from our study of sensor arrays to other related problems including time series analysis and state-space system identification. In addition, we have developed new and computationally efficient numerical analysis procedures that render the sensor array processing algorithms developed under this contract and elsewhere practical for "real-time" implementation.

The remainder of this document is organized as follows. Section 2 provides an outline of the research results obtained during the contract period. For each related group of results, pointers to the various technical reports containing detailed descriptions of these results are provided. Section 3 lists the technical reports submitted earlier during the contract period, along with the indexing keys used in Section 2. The final section, Section 4, tabulates our publications on research related to this contract.

2 Research Results

Narrowband Sensor Array Processing Algorithms

The model for the so-called narrowband sensor array processing problem is given by equation (1). Its apparent simplicity belies its mathematical complexity and its applicability to a rich collection of engineering applications. The model applies, for instance, to the case where signal propagation times across an array are small compared to their bandwidth which is the origin of the term narrowband.

\[
x(t) = A(\Theta)s(t, \Theta) + n(t)
\]

\(\Theta\) represents the parameters to be estimated (e.g., directions of arrival) and \(s(t, \Theta)\) is the \(d\) dimensional vector of source signals. \(A(\Theta)\) is an \(m \times d\) dimensional array mapping the source signals \(s\) into \(m\) sensor outputs, and \(n(t)\) represents additive sensor noise. This same model can be used for time series analysis (the concept of sensors is replaced by delay-line taps) where the goal is to estimate the number...
and frequencies of sinusoids in noise, as well for system identification problems in which one has input and output data for a certain system and wishes to estimate a state-space model. Our group's research in this area concentrated upon the class of subspace algorithms, the progenitor of which was Ralph Schmidt's MUSIC algorithm (also developed at Stanford).

In the area of algorithm development, our goals were the development of algorithms that were more efficient than existing algorithms and the unification of existing algorithms in a common framework. A key result was the development of ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques). ESPRIT is distinguished by the fact that it does not require a priori knowledge of \( A(\Theta) \) nor does it involve a search in parameter space. These two requirements are obviated by assuming a shift invariance structure that is present in many sensor arrangements (e.g. uniform linear arrays, tapped delay lines, cross arrays). Additional invariances present in the spatial (temporal) distribution of the sensors can be exploited by the MI-ESPRIT algorithm (Multiple Invariance) to improve parameter estimates. The references for ESPRIT are I, J and L.

Another key development was the SSF (Subspace Fitting) framework and its associated algorithm WSF (Weighted Subspace Fitting). SSF provides a single unifying framework that subsumes most of the subspace-based estimation techniques, including MUSIC, MD-MUSIC, Root-MUSIC, ESPRIT, beamforming and deterministic ML (maximum likelihood). Its associated algorithm, WSF, is asymptotically efficient (i.e. it achieves the Cramér-Rao bound) for the stochastic ML formulation of the array processing problem at a fraction of the cost associated with a direct minimization of the ML cost function. The references for SSF and WSF are J, K and L.

Statistical Analysis of Narrowband Algorithms

The unifying framework of SSF has proven effective in the asymptotic statistical analysis of subspace-based techniques which it subsumes. Performance analyses have been performed for ESPRIT, stochastic ML, MUSIC and MD-MUSIC. The appropriate references are J, K and L.

Sensitivity Analysis of Narrowband Algorithms

Closely related to the statistical analysis of various subspace-based algorithms, is the analysis of these algorithms' sensitivities to modelling errors. The spacing of elements in a "uniform linear array" might not be exactly constant, for instance, or the gain and phase characteristics of the sensors might not be exactly known. The effect of these modelling errors on the algorithms' performance can be characterized using the SSF framework. The appropriate reference is L.

Time Series Analysis

Many researchers have noted that the problem of identifying directions of arrival
with a uniformly linear array is nearly identical to that of identifying the frequencies of sinusoids in a uniformly sampled time series. The key difference is that in the former case many data records, or "snapshots", are obtained while in the latter a single data record is obtained. In the time series case, one typically forms a matrix of ersatz snapshots from the single data record before applying a subspace technique for the purpose of frequency estimation. We obtained results establishing the conditions under which this procedure will result in the desired estimates, the appropriate reference is L.

Signal Estimation

It is often desirable to obtain more information than just the directions of arrival in a sensor array processing scenario or the sinusoid frequencies in the case of time series analysis. One might want estimates of the individual signals in the array processing case (so-called signal copy). This difficulty of this problem is exacerbated when the source signals are correlated. Methods for estimating the source coherency structure and for estimating signals in the case of correlated sources are described in reference L. Similar problems arise in time series analysis when the sinusoids are phase-coupled — our results in this area are described in references J and K.

State-Space System Identification

Although linear state space models have become increasingly important in estimation and control since the pioneering work of Kalman, they have not been widely used in identification. One of the goals of our research has been to demonstrate that there are some important advantages in explicitly considering general state space forms in identification. The methods we studied exploit a certain shift structure in the input/output model for the data. The motivation for these new algorithms comes from some interesting connections linking the state space identification problem with sensor array signal processing. In particular, we have shown how the identification problem can be cast in the SSF framework. Of special interest is the fact that the dominant subspace can be weighted to emphasize certain directions in the so-called "signal" subspace where the signal-to-noise ratio is high. It has been demonstrated that for the identification problem, the ability to properly weight this subspace provides our algorithm with superior performance in cases involving nearly unobservable systems or an insufficiently exciting input sequence. The appropriate reference is M.

Numerical Analysis Results

A step common to all subspace algorithms is the estimation of the so-called "signal subspace". Classically, this is obtained via an eigendecomposition of the estimated measurement covariance matrix, requiring $O(m^3)$ operations. This op-
eration often dominates the computational requirements of a subspace algorithm and often renders real-time implementations infeasible. The FSD (Fast Subspace Decomposition), developed under this contract, obtains the signal subspace in $O(m^2d)$ steps. In time series analysis problems, for example, $m$ can be on the order of 100 (typically $d \ll m$) and the computational savings afforded by FSD are manifest. The appropriate reference is M.

Wideband Sensor Array Processing Algorithms

When signal propagation times across an array are not small compared to their bandwidths, the model of equation (1) no longer applies. A number of approaches are possible in this case; we investigated two in particular. Using a wideband subspace model developed by Su and Morf, we investigated a wideband extension of the ESPRIT algorithm. Sensor outputs are preprocessed in the time domain to identify the poles of the emitter signals and ESPRIT is subsequently applied at the identified poles. ESPRIT is particularly well suited to this problem as knowledge of the array response (as a function of frequency as well as, say, angle) is not required. This work is reported in I, J and K.

Alternatively, one can exploit structural features of the emitter signals in the wideband case. Many digital communication signals (e.g. BPSK, QPSK, FSK) exhibit cyclostationarity. This property can be used to advantage in the wideband scenario and also for the purpose of reducing co-channel interference. The appropriate references are L and M.

The issue of adaptive beamforming for wideband systems was also addressed. The typical structure employed for this purpose is a bank of parallel tapped delay lines with adaptive weights at the output of each sensor. We developed a fast and numerically stable multichannel RLS (Recursive Least Squares) algorithm based on an underlying transversal filter architecture. These results are significant not only because they provides a fast algorithm for RLS, but because they debunked the widely held notion that stable transversal filter RLS implementations were impossible. This work is discussed in reference I.

Nearfield Sensor Array Processing

In both the narrow and wideband situations described above, it is assumed that the emitters are in the array's far-field so that the emitter wavefronts may be modelled as plane waves at the array. When the emitters are in the array's nearfield, the curvature of the emitter wavefronts must be taken into account. Using a Fresnel approximation for the wavefronts, an algorithm employing the Wigner-Ville Distribution of the sensor outputs and ESPRIT was developed that generated
range and bearing estimates for the emitters. A discussion of this algorithm and Cramér-Rao bound comparisons is provided in reference 1.

3 Technical Reports

The following table lists the various technical reports submitted in accordance with the contract. The column labeled “Key” is the indexing key used in Section 2 to refer a particular report. The interim technical report of July, 1988, I, subsumes the reports predating it and is therefore referred to in Section 2 to their exclusion.

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<th>Type</th>
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<td>A</td>
<td>Jan, 1986</td>
<td>Semi-Annual</td>
<td>Sensor Array Processing</td>
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<td>B</td>
<td>Jun, 1986</td>
<td>Semi-Annual</td>
<td>Sensor Array Processing</td>
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<td>C</td>
<td>Jul, 1986</td>
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<td>Oct, 1987</td>
<td>Progress</td>
<td>Wideband Direction of Arrival Techniques</td>
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<td>Interim</td>
<td>Recent Advances in Sensor Array Processing</td>
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<td>Annual</td>
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<td>M</td>
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4 Publications

Papers Published in Refereed Journals.


Papers Accepted by Refereed Journals.


Papers Submitted to Refereed Journals.


Contributed Presentations at Technical Conferences


Books (and Sections Thereof) Published.