**STRESS ANALYSIS CONSIDERATIONS FOR DEEP SUBMERGENCE CERAMIC PRESSURE HOUSINGS**

**AUTHOR(S)**

R. P. Johnson

**SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)**

Navy Engineering Logistics Office  
P. O. Box 2304, EADS Street  
Arlington, VA 22201

**ABSTRACT** (Maximum 200 words)

A brief review on the structural characteristics of ceramics is given. Three potential failure criteria that may be used to stress analyze ceramic pressure housings are discussed. Additional comments and recommendations are made concerning the task of structural analysis specific to deep submergence ceramic pressure housings.

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<tr>
<td>R. P. Johnson</td>
<td>(619) 553-1935</td>
<td>Code 944</td>
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STRESS ANALYSIS CONSIDERATIONS FOR DEEP SUBMERGENCE CERAMIC PRESSURE HOUSINGS

Richard P. Johnson
Naval Command, Control and Ocean Surveillance Center RDT&E Division
San Diego, California 92152 USA

ABSTRACT

A brief review on the structural characteristics of ceramics is given. Three potential failure criteria that may be used to stress analyze ceramic pressure housings are discussed. Additional comments and recommendations are made concerning the task of structural analysis specific to deep submergence ceramic pressure housings.

INTRODUCTION

Ceramics possess material properties that make them an attractive choice for use where low weight to displacement ratios are desirable. Their low density, high compressive strength, high elastic modulus and excellent corrosion resistance warrant consideration of ceramics over more traditional housing materials. Once a ceramic is selected, structural design criteria must be chosen that will insure integrity of the housing throughout its service life. This design process is well developed for materials such as steel, aluminum and titanium, but is not as well documented for ceramics. The intent of this paper is to familiarize the reader with the structural behavior of ceramics so that pressure housings can be analyzed. Emphasis of the paper is on designing for acceptable stress levels under static loading. This involves calculating the appropriate components of stress that can be used in failure criteria suitable for use with ceramics pressure housings. Other structural considerations such as elastic stability, cyclic loading and thermal loading will not be addressed here but should also be considered by the designer.

STRUCTURAL BEHAVIOR OF CERAMICS

Ceramics are crystalline compounds of metals and nonmetals held together by ionic and/or covalent bonds. The strength of the ionic bonds makes it difficult for slip to occur giving ceramics its characteristic of high shear strength and brittle or non-ductile nature. This in turn results in the properties of high compressive strength, high hardness, low fracture strength and low notch sensitivity. Notch sensitivity defines the reduction in properties of a material under tensile loads due to stress concentrations such as cracks or notches. The increase in nominal tensile stress in the region of a crack is directly proportional to the square root of \((c/r)\), where \(c\) is the depth of the crack and \(r\) is the radius of curvature at the crack tip. In a ductile material, as the rise in stress at a crack tip becomes excessive, local plastic deformation will occur to increase the radius of the crack tip until the local stress drops below the yield strength of the material. For nonductile materials such as ceramics, small cracks can lead to large enough stress concentrations under tensile load, to overcome the high shear strength of the material and lead to crack propagation and possible failure. Conversely, compressive loads can be supported across microcracks without degrading the material further. The compressive strength of a ceramic is typically many times greater than its tensile strength, its torsional strength and tensile strength are approximately equal.

Ceramics exhibit linear stress-strain behavior until failure occurs by fracture with a relatively low strain tolerance in comparison to most other common engineering materials. Analysis of ceramic pressure housings can be completed using the same formulas derived based on a linear elastic constitutive model as are used when analyzing metallic pressure housings. In fact this model can be considered even more appropriate for ceramics since the relative high stiffness and low strain capacity of ceramics make the chance
of geometric nonlinearities caused by large rotations and displacements less likely to occur than in an equivalent metallic structure. On the other hand since ceramics lack the ability to plastically flow to relieve the effect of stress concentrators, special care must be made to locate and eliminate local high stressed regions during design of ceramic pressure housings. Sizing of ceramic housing components using standard hand calculation techniques is appropriate but more advanced analysis or testing may be required in joint or penetration regions or any other potential local high stress areas caused by material, geometric or boundary condition discontinuities.

FAILURE CRITERIA FOR CERAMICS

Before stress analysis can be completed, selection of a failure criteria must be made to evaluate the state of stress in the ceramic housing components. Three common failure criteria used to predict failure in brittle materials such as ceramics are, the Maximum-Normal-Stress Theory, the Mohr Theory, and the Weibull Theory. All these theories share the characteristic that consideration must be given to the direction in which the stresses act within a structure (i.e. compression vs tension). This is important with materials like ceramics which exhibit radical differences in tensile and compressive strength. The Distortion-Energy Theory or Von-Mises-Hencky Theory is the most common failure theory used for designing metal parts, but would not be appropriate for use with ceramics since no consideration is given to the direction in which the stresses act.

The Maximum-Normal-Stress Theory states that failure is predicted to occur at any point subjected to a multiaxial state of stress when the maximum principal stress at that point equals the ultimate tensile strength of the material or if the minimum principal stress equals the ultimate compressive strength of the material.

Maximum-Normal-Stress Theory:
\[ \sigma_1 = S_{ut} \quad \text{or} \quad \sigma_3 = -S_{uc} \]

This theory has been shown to have good correlation with experimental data in regions where all principal stresses are of the same direction, either compressive or tensile. This situation occurs in shells of revolution where loading and geometry are such that only compressive membrane stresses are experienced such as with cylinders or hemispheres of constant wall thickness subjected to uniform hydrostatic pressure. In these cases the hoop, axial and radial stresses will be the principal stresses and the Maximum Normal Stress Theory is easily applied.

The Mohr Theory may be more applicable in cases where the pressure housing experiences bending and shear stresses in addition to membrane stresses as may be caused by geometric, material, loading or boundary condition discontinuities. In this state of stress the direction of the principal stresses may no longer be the same and the Maximum Normal Stress Theory could predict an acceptable level of stress when in actuality the part could fail. The Mohr Theory states that failure is predicted to occur at any point subjected to a multiaxial state of stress when the largest Mohr's circle associated with the state of stress at that point exceeds a failure region defined by two lines constructed tangent to two Mohr's circles, one corresponding to failure under uniaxial compression, the other to failure under uniaxial tension.

Mohr Theory:
\[ \sigma_1 - \sigma_2 = 1 \quad \sigma_1 \geq 0 \]
\[ S_{ut} \quad S_{uc} \quad \sigma_1 \leq 0 \]

The Mohr Theory has been shown to provide conservative estimates of material failure when compared to experimental data and is therefore adequate for use in design of ceramic pressure housings. Variations on this theory such as the modified Mohr Theory give slightly more accurate prediction of failure for brittle materials in regions where the principal stresses are both compressive and tensile, lending itself to consideration when pursuing an optimal design.

In ceramic pressure housings subjected to hydrostatic loads, principal stresses are typically all compressive except in localized regions such as the bearing surfaces of joint interfaces. For this load case the Maximum-Normal-Stress Theory is adequate for gauging the effect of membrane stresses away from the joint but the Mohr Theory should be considered in any localized regions where tensile principal stresses
occur. For load cases associated with the handling of ceramic pressure housings this same philosophy should be used. Consideration should be given to preloading the ceramic housing via design features like tie rods to minimize the areas of the housing subjected to tensile stresses under inertial loading. For each load case, the state of stress should be calculated and converted to principal stresses and then evaluated with the appropriate failure criteria.

The last failure criteria discussed here is the Weibull Theory which has gained popularity of recent as a tool for use in the design of structural ceramics, particularly for ceramic components used in engines. This theory assumes that failure is caused by tensile stresses and ignores the possibility of compressive failure. The Weibull Theory assumes the ceramic will have statistical variations in strength properties and defines the reliability of a part based on a probabilistic system of design given the tensile stress distribution of a part. This theory may warrant further consideration in the regions described earlier where tensile stresses have been shown to develop since local crack propagation and the ensuing degraded capability of the part to bear load has been shown to be a potential failure mode for ceramic pressure housings.

Application of these theories with appropriate factors of safety will insure that the ceramic housing is adequately designed from a stress analysis point of view. For depth loading, ceramic housings currently being designed at NCCOSC utilize a minimum factor of safety of two for compressive membrane stress (minimum principal stress less than zero) and a minimum factor of safety of five for tensile membrane stresses (maximum principle stresses greater than zero). Factor of safety is defined here to be the ratio of the compressive or tensile strength of the material over the calculated operational principal stress.

**ADDITIONAL CONSIDERATIONS**

As previously stated, the design of ceramic pressure housings is more challenging than their metal counterparts in the sense that special care must be taken to avoid features that may induce locally high stressed areas in the ceramic. This especially true of the ceramic metal interfaces that occur at joints and penetrations. These regions present the most significant challenges to the designer of ceramic pressure housings and warrant further discussion here. Figure 1 depicts the cross section of a potential joint design for the interface of a ceramic cylinder and a ceramic hemisphere.

There are several factors that can contribute to high stresses at the ceramic metal interface and should be considered during the design and analysis process. A mismatch in radial expansion will occur at the ceramic metal juncture as the housing is subjected to axial compression under depth loading. The radial expansion of a material is dependent upon the ratio of its Poisson’s ration to its elastic modulus. For a titanium/alumina joint, the titanium would expand almost four times more than the alumina. The net effect is that if no relative radial motion is allowed between the two materials, the alumina will develop tensile stresses and crack propagation could occur. Precision grinding of the ceramic bearing surfaces and the use of a compliant interface layer may help control this effect. Matching the radial deflection between mating parts subjected to depth pressure will eliminate stiffness induced bending and shear stress that would otherwise be present at their juncture. Local geometry at the interface, as for example in the titanium endcaps shown in Figure 1, and the method used for joining the ceramic and metal components together will effect the interface stresses. Bending and shear stresses induced by a mismatch in thermal expansion of the ceramic and metal may also be a concern in applications where significant thermal loading may occur.

The finite element method is very useful in lending insight into design and analysis of joint regions. The effect of various design parameters on the stress distribution at the joint interface can be studied and computed stress values can be incorporated into one of the failure criteria mentioned previously. Figure 2 depicts maximum principal stress contours obtained from finite element analysis for the joint shown in Figure 1 under depth loading. Regions are indicated where the maximum principal stresses were positive. The other isostress contours that diverge from the tensile stress areas represent regions where the maximum principal stresses...
were increasingly compressive. The effect of tensile stresses should be considered when analyzing ceramic pressure housings since their presence may well define the capability of the housing to perform its mission. Ceramic housings designed at NCCOSC subjected to 9000 psi pressure cycles eventually failed because of cracks that propagated from bearing surfaces where tensile stresses occurred.

BIBLIOGRAPHY