ESTIMATING CHARACTERISTIC LIFE AND RELIABILITY OF AN AIRCRAFT ENGINE COMPONENT IMPROVEMENT IN THE EARLY STAGES OF THE IMPLEMENTATION PROCESS

BY

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DECEMBER 1992

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This thesis describes the database needed to assess improvement in the performance of a modification to the long electroless nickel (LEN) fuel tube of the TF-34-400 engine during the time the modified component is being introduced into the fleet. It also provides the methods and equations for estimating the reliability of a modified component during its implementation. The component failure times are assumed to have a Weibull distribution. When implemented, this methodology will provide engine program management teams more timely information which should enhance their decision-making process significantly.
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Estimating Characteristic Life and Reliability of an Aircraft Engine Component Improvement in the Early Stages of the Implementation Process

by

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ABSTRACT

This thesis describes the data base needed to assess improvement in the performance of a modification to the long electroless nickel (LEN) fuel tube of the TF-34-400 engine during the time the modified component is being introduced into the fleet. It also provides the methods and equations for estimating the reliability of a modified component during its implementation. The component failure times are assumed to have a Weibull distribution. When implemented, this methodology will provide engine program management teams more timely information which should enhance their decision-making process significantly.
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I. INTRODUCTION

A. BACKGROUND

The TF-34-400 turbo-fan engine is deployed on S-3 anti-submarine warfare and ES-3 electronic surveillance Naval aircraft. To ensure the S-3 and ES-3 aircraft are provided with quality engines, the TF-34-400 program management team must make critical logistic support decisions for the engine in such areas as maintenance planning, life management, and component improvements. This thesis provides information to assist the program management team in their evaluation of a specific component improvement.

It is important to first provide an overview of the objectives and functions of the Aircraft Engine Component Improvement Program (AECIP). Reference 1 states the objectives of the AECIP as

1. to maintain an engine design which allows the maximum aircraft availability at the lowest total cost to the government.

2. to correct, as rapidly as possible, any design inadequacy which adversely affects safety-of-flight; and

3. to correct any design inadequacy which causes unsatisfactory engine operation or adversely affects maintainability and logistic support in service.

Reference 1 states that the AECIP performs four functions in order to meet these objectives:
1. The problem solving function supports rapid investigation and resolution of fleet problems (safety, readiness, or reliability).

2. The problem avoidance function includes an aggressive program of mission testing, analytic sampling of fleet hardware and engineering analyses designed to forecast hardware wearout rates, life limits, and similar problems before they occur in the fleet.

3. The product improvement function is designed to develop and qualify design changes, repair procedures, and alternate sources of parts and supplies throughout the life of an engine where substantial savings in operation and support costs can be realized.

4. The product maturation function provides an infrastructure (support facilities and experienced engineers) to mature the engineering design for newly developed and fielded engines.

Currently, the program management team is interested in estimating the increase in reliability of a specific component resulting from a change thought to be an improvement as the implementation process unfolds. The component improvement of interest involves modification of the TF-34 fuel tubes which are located in the combustor section of the engine. The redesign is intended to reduce the potential for "fumers", a phenomena which can lead to hazardous fuel fumes in the cockpit or catastrophic engine fires.

Beyond safety-of-flight analysis, no readily available methods exist for analyzing the effects of an improvement to the TF-34 fuel tubes during implementation. Several theses [Refs. 2, 3] have been written at the Naval Postgraduate School which attempted to examine the effects of various component improvements by comparing a snapshot of a component’s pre-change failure rate to its failure rate after the entire fleet of engines has been modified. Since complete induction of a component
improvement into a fleet of engines may take ten years or longer [Ref. 2], there is a long lag in the feedback process utilizing this snapshot approach.

This thesis proposes a methodology to assess the effectiveness and reliability of a component improvement as the implementation process unfolds. When implemented, this methodology will allow program management teams to have more timely information and should enhance their decision-making process significantly.

B. OBJECTIVES

The primary objectives of this thesis are as follows:

1. Propose a methodology to detect improvement and estimate reliability of a specific aircraft engine component improvement during implementation so decision-makers can assess the improvement's impact on logistical support.

2. Establish a list of data elements that are required to evaluate the improvement and reliability of a specific component improvement.

C. RESEARCH QUESTIONS

The basic question of concern in this thesis is the following:

Can the improvement and reliability for the TF-34 fuel tube power plant change be assessed when only a small percentage of the entire fleet of engines has been modified to include the new fuel tubes, relatively few operating hours have accumulated on engines with the new fuel tubes, and no failures have occurred on any of the new fuel tubes?

To assist in answering this inquiry, the following questions were used to guide the research effort:

1. What data is required to detect the improvement and estimate the reliability of a component improvement during implementation?
2. How can the improvement of a power plant change be demonstrated?

3. How can the reliability of a new system be estimated for a future point in time?

D. SCOPE, LIMITATIONS, AND ASSUMPTIONS

This research effort is focused on determining the effect of a specific TF-34-400 component improvement as the implementation process unfolds. The inability to obtain "time-to-failure" data for the original fuel tube system made it impossible to generate a baseline to detect the improvement and to estimate the reliability of the new fuel tube system. Furthermore, the lack of failure data for the original system forced the author to assume a failure distribution in proposing the methodology. The methodology proposed in this thesis assumes the failures can best be described by the Weibull distribution. Therefore, the focus of this thesis concentrates on developing the following items:

1. A description of the data base required to detect improvement and estimate reliability for component improvements whose failures can be described by the Weibull distribution.

2. A methodology that can be used to determine improvement and estimate reliability of component improvements whose failures are described by the Weibull distribution.

3. A sensitivity analysis of component improvement in reliability to changes in the Weibull beta parameter.

E. THESIS PREVIEW

The remainder of this thesis focuses on the objectives discussed in section B of this chapter. Chapter II discusses the Weibull distribution. Chapter III provides a
brief overview of the TF-34-400 engine and a detailed history of the fumer problem. Chapter IV presents methodology for analyzing the effectiveness and reliability of a component improvement. Chapter V provides sensitivity analysis on the improvement and reliability of the new system. Chapter VI summarizes the thesis effort and presents the conclusions and recommendations from the effort.
II. THE WEIBULL DISTRIBUTION

A. THE WEIBULL FUNCTIONS

The Weibull distribution has been used to model the lifetime of electrical and mechanical components of systems. Additionally, "the Weibull distribution most frequently provides the best fit for the type of data experienced in the gas turbine industry" [Ref. 4]. The cumulative probability function, \( F(t; \eta, \beta) \), of the Weibull distribution is given by:

\[
F(t; \eta, \beta) = 1 - e^{-(\eta t)^\beta} : t \geq 0, \eta > 0, \beta > 0. \tag{2.1}
\]

Associated with the cumulative probability function is the hazard function. The hazard function, \( h(t) \), describes the instantaneous failure rate. Reference 5 gives the equation for \( h(t) \) as

\[
h(t) = \frac{f(t)}{1-F(t)}. \tag{2.2}
\]

Figure 2.1 from Reference 5 depicts a typical hazard function for a system or component. The hazard function for the Weibull distribution, \( h(t; \eta, \beta) \), is given by:

\[
h(t; \eta, \beta) = \left( \frac{1}{\eta} \right)^\beta t^{\beta-1} : t \geq 0, \eta > 0, \beta > 0. \tag{2.3}
\]

6
Early Random Failures

Figure 2.1 A Typical Hazard Function for a System or Component

The parameter, $\beta$, determines the slope of this hazard function at time $t$. Consequently, $\beta$ is called the slope parameter. Note: it is not the slope. The values of $\beta > 0$ are usually subdivided into three ranges corresponding to the shape of the hazard function. These are:

a. $0 < \beta < 1$; the slope of the hazard function is decreasing and is typical of early age failure or infant mortality.

b. $\beta = 1$; the slope of the hazard function is zero. This implies no wear out or early age failure.

c. $\beta > 1$; the slope of the hazard function is increasing. This implies wear out.
The probability density function $f(t; \eta, \beta)$ is the derivative of $F(t; \eta, \beta)$ with respect to $t$ and is roughly a relative measure of, but not equal to, the probability of failure at time $t$. Typical forms of this density function are shown in Figure 2.2 as $P(t)$, from Reference 4.

B. WEIBULL TERMINOLOGY

Since the Weibull distribution is defined by the parameters beta, $\beta$, and eta, $\eta$, a discussion of these parameters and other Weibull terminology is presented to enhance the reader's comprehension of material presented in later chapters. Reference 4 defines the following key Weibull parameters and terms:

1. Beta: Beta is also known as the slope or shape parameter of the Weibull distribution. Beta helps to determine which member of the family of Weibull failure distributions best fits or describes the data. The type of failure behavior may be any one of the types represented by the familiar reliability bathtub curve; i.e., slopes less than one which imply infant mortality, slopes of zero which imply no wear out, and slopes greater than zero which imply wear out.

2. Eta: Eta is the characteristic life of the Weibull distribution. A total of 63.2% of the lifetimes will be less than the characteristic life, regardless of the value of beta.

3. Suspensions: A test or operational unit is labeled a suspension if it has not failed at the time the life data analysis begins. Suspensions can not be ignored when establishing the Weibull plot. The argument for including them in the analysis is that if their failure had occurred in the same fashion as the other failures, the order of the other failures would have been influenced.

4. Weibayes/Weibest: Weibayes is a method for constructing a Weibull distribution based on the assumed value of beta. It is used when there are certain deficiencies in the data (i.e., when operating time has accumulated, but no failures have occurred).

5. Weibull Plot: A plot of time-to-failure data on Weibull probability paper.
1. Infant mortality
   • Inadequate burn-in, green run
   • Misassembly
   • Some quality problems

2. Random failures
   • Independent of time
   • Maintenance errors
   • Electronics
   • Mixtures of problems

3. Early wearout
   • Surprise!
   • Low cycle fatigue

4. Old age wearout (rapid)
   • Bearings
   • Corrosion

Figure 2.2 Typical Forms of the Probability Density Function
6. Ordering the data: The first step in establishing a Weibull plot is to order the data from low time to high time failures. This facilitates establishing the plotting positions on the time (horizontal) axis. Ordering is also needed to establish the corresponding ordinate or $F(t)$ values.

C. THE WEIBULL PLOT

Plotting Weibull data requires special graph paper. Reference 4 discusses how to construct Weibull paper and contains sample paper in its appendix. A reproduction of the Weibull paper from Reference 4 is included in Appendix A of this thesis.

Weibull graph paper has unique scales for its horizontal (X) and vertical (Y) axes. The horizontal axis of a Weibull plot is usually some measure of life (i.e., operating time). The vertical axis is usually the cumulative probability of failure, $F(t; \eta, \beta)$. The vertical axis value is determined by ordering the data from low-time to high-time failure and then determining the median rank for each failure. For example, the median ranks for a sample size of three would be 20.6, 50.0, and 79.3 for the low-time to high-time failures. Reference 4 provides details for ordering the data and charts for determining median ranks. Each failure can then be plotted as a data point on the Weibull paper using measure of life and median rank as entering values for the X and Y axes. The line which describes the data points is called the Weibull line.
Figure 2.3 A Typical Weibull Plot
Weibull analysis is advantageous because the Weibull plot (Figure 2.3) provides a simple graphical solution for determining Beta and Eta. Beta, the slope of the Weibull line, can be determined from a simple measurement.

The slope of the line is measured by taking the ratio of rise over run. Select a starting point and measure one inch in the horizontal direction (run). Then, measure vertically (rise), until the line is intersected." [Ref. 4]

Characteristic life is determined by locating 63.2% on the vertical axis and following the dashed line to the right until the dashed line intersects the Weibull line. If a vertical line is then drawn down from the 63.2% and Weibull line, it will intersect with the horizontal axis at the characteristic life value. Characteristic life is denoted by "*" in Figure 2.3.

D. WEIBULL PREVIEW

Weibull analysis is used in this thesis to attempt to detect improvement in characteristic life and project system reliability resulting from a power plant change to TF-34-400 fuel tube. The latter portion of Chapter III discusses the early attempts by General Electric and Naval Air Warfare Center, Trenton, to describe the fumer problem of both the long and short carbide tungsten fuel tube configurations using Weibull analysis. Chapter IV discusses a special type of analysis called "Weibayes" which could be used to detect improvement and project reliability of the new system.
III. ENGINE FUMERS

This chapter provides background information on the history of the TF-34 engine, an explanation of its fumers, a look at the initial fumer solution, and a perspective of the problem today.

A. THE TF-34 ENGINE

The TF-34 turbo-fan engine is produced by General Electric. The engine has three variants; the CF-34 for commercial use, the TF-34-100 flown by the Air Force on A-10 aircraft, and the TF-34-400 flown by the Navy on S-3 and ES-3 aircraft. As of 1 April 1991, over 2000 engines were in operation with more than 7 million engine flight hours (EFH) accumulated on those 2000 engines. [Ref. 6]

The development contract for the TF-34-400, the Naval variant, was let in March 1968. Testing of the engine began in April 1969 and full qualification occurred in June 1972. Full scale production followed the qualification and concluded in November 1977 with the completion of the 475th engine (Figure 3.1). Today there are 424 engines in the Navy inventory. Production of 50 new engines is presently underway which will increase the inventory to 474 engines. [Ref. 7]

The TF-34-400 engine has five main sections: the fan, compressor, combustor, high pressure turbine, and low pressure turbine sections (Figure 3.2). PPC-53, which sought to correct the fumer problem, modified the engine's fuel tubes which are located in the combustor section. The S-3 Naval Air Training and Operating
TF34

Historical Perspective

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Figure 3.1 Development, Production, and Activation Schedule
Procedures Standardization (NATOPS) manual's description of the combustor section is provided below. [Ref. 8]

The combustor section, located aft of the compressor section, is where the compressor discharge air and fuel are mixed and ignited. The combustor utilizes an annular configuration with a swirler type injection system. It comprises the combustion casing, combustion liner, 18 swirler plates, fuel feed tubes, and the first stage turbine nozzle. Ports are provided in the combustor section for the two igniter plugs, 18 fuel feed tubes, 14th stage bleed air, five primer fuel nozzles, and for borescope inspection. (Figure 3.3)

B. FUMERS

Prior to combustion, the 18 swirlers of the combustor section shape the fuel stream to ensure optimal ignition. Each swirler is fed by a separate fuel tube which is housed in the upper portion of the swirler, known as the preswirler. If the flow of the fuel stream is impinged upon after it leaves the fuel tube and before the fuel is ignited, deflection of the fuel can occur. Deflection results from such interferences as carbon coking on swirler walls from fuel deposits and/or swirler wall wear caused by fuel tube/swirler friction. If the deflected fuel flows back into the secondary airstream, fuel fumes can enter the bleed air via the 14th stage bleed port. The 14th stage bleed air feeds into the environmental control system of the aircraft which transmits the air to the cockpit. Therefore, safety of flight problems result when fumed air is transmitted to the cockpit. [Refs. 9, 10]

An additional hazard which results from deflected fuel has also been observed. Many field engines have accumulated carbon deposits outside the preswirler. As early as 1979 General Electric noted that if allowed to persist, this carbon backflow
Figure 3.3  TF-34 Combustion Liner
(from the deflected fuel in the secondary airstream) could progress, and in extreme cases, result in internal fire and engine damage (see arrow in Figure 3.4) [Ref. 9].

Ten years later, in Engineering Change Proposal AL-776, the Navy detailed the numerous instances of such fires. An example of one incident is provided below to illustrate the severity of these fires.

Engine #202404, 02/05/91: USS ROOSEVELT. IN-FLIGHT-FIRE. Fuel injector wear caused spitback, combustion frame burnthrough. Severe nacelle fire [Ref. 10].

Although the engine fire hazard is completely different than fumes in the cockpit, it is also termed a "fumer" since both hazards originate from the same fuel tube/swirler system failure.

The CF-34 and TF-34-100 (Air Force) variants do not have a fumer problem. These other variants utilize a fuel injector system vice the tube/swirler combination of the TF-34-400, Navy variant. The Navy was not able to adopt the fuel injector system because the injector system adversely affected throttle response time of the turbo-fan engine. Throttle response is a significant factor for the Navy as carrier-based engines must be able to produce high performance in short time intervals during shipboard operations. [Ref. 11]

C. THE INITIAL SOLUTION (1976 - 1986)

The first incidents of fuming occurred in the 1976 - 1977 time frame. During this period, General Electric data shows that two TF-34-400 engines produced cockpit fumers while 41 fuel tubes failed [Ref. 9]. The original fuel tube/swirler
configuration of the TF-34-400 was a long carbide tungsten (LCT) fuel tube and a Hastelloy-X preswirler.

In the late 1970's, General Electric expended CIP research and development funds to explore improvements in the fuel tube/swirler system. G.E. initially planned to shorten the fuel tube and increase the strength of the preswirler in an attempt to reduce both tube and swirler wear. The design for the strengthened preswirler was completed but manufacturing lacked the technology to produce it. [Ref. 11]

In May 1979, G.E. forwarded ECP 23KG8061 "Shortened Fuel Injectors" to Naval Air Systems Command for consideration. The ECP proposed reducing the fuel tube length by 0.2 inches (Figure 3.5). No change in the swirler was recommended. G.E. stated, "the shorter fuel tube will reduce preswirler and fuel tube wear and minimize the potential for fuel spitback, cockpit fuming and internal fires." G.E. had performed a Weibull analysis of the 41 LCT tube failures (1976 - 1977) and determined the characteristic life of the old LCT tube to be 1790 hours. G.E. conducted shake bench tests on the proposed short tube and found the tube reduced wear incidents by 90%. Based upon the shake bench test, G.E. concluded the proposed short tube would have a characteristic life of 3400 hours (90% increase from 1790 hours) and fumers would be reduced by a corresponding 90%. [Ref. 9]

The Navy approved the ECP in March 1980 and power plant change (PPC) 53, directing implementation of the short fuel tube, was developed. Implementation of PPC 53 began in 1982. In the mean time, the fumer problem had grown worse. The number of fumers occurring per 1000 EFH rose steadily as the operating hours on
the engines increased (Figure 3.6). An increase in failures per EFH should have been anticipated since a system which exhibits Weibull characteristics with beta greater than 1.0 will have an increasing hazard rate as operating hours accumulate on the system.

In 1984 and 1985 there was a dramatic increase in the number of fumers per 1000 EFH (Figure 3.6). Many of the engines which had fumed were recently modified engines incorporating PPC 53, the short tube configuration. The increase alarmed the S-3 program office in Washington D.C. and Naval Air Warfare Center Trenton, New Jersey (NAWC-T) began an engineering investigation in 1985 to determine the effects of PPC 53.
In February 1986, NAWC-T issued a report which analyzed the fumer problem and addressed the effects of PPC-53. The report reexamined General Electric's analysis of the original 41 LCT tube failures from 1976 and 1977. NAWC-T recomputed the data without suspensions and determined the characteristic life to be 852 hours vice the 1790 hours reported by G.E. NAWC-T stated G.E. had included too many suspensions and thus inflated the characteristic life of the LCT tube. [Ref. 12]. However, the report stated the 852-hour value represented "hours to fume" for the engine when it actually represented the characteristic life of the LCT fuel tube as G.E. had reported for its 1790-hour calculation. In order for mean "hours to fume" (characteristic life of the LCT/swirler system) to be calculated, NAWC-T should have utilized only the two failures which resulted in fumers as data points. The other 39 items are suspensions since those engines had not yet fumed but only experienced tube failure. The flaw in determining a proper value for the LCT/swirler system "hours to fume" means there is now an incorrect base line to determine the effectiveness of the short tube modification, and all subsequent modifications.

In the 1986 report, NAWC-T properly calculates "hours to fume" for the short tube modification, using only fumer data points. The results for the short tube system depict an extremely low characteristic life (hours-to fume) with a Weibull beta value less than 1.0 for engines having previous fumers and a beta value of 1.75 for engines having no previous fuming history. The report states that these beta values indicate previous fumer engines are experiencing infant mortality wear-out modes.
while non-fumer engines are experiencing an early wear-out failure mode. However, the report incorrectly concludes the "hours to fume" for the short tube is only 71% of the "hours to fume" for the pre-PPC-53 LCT tube. This conclusion cannot be made since the LCT/swirler system's baseline was improperly determined.

D. THE FUMER PROBLEM TODAY (1986 -1992)

By 1985 the program office realized PPC-53 was having a detrimental effect upon the fumer problem. In order to avert a major problem with fumers, the program office and G.E. sought a new solution to the problem. Both parties proposed short-term treatments to control the problem while G.E. began redesigning the system for a long-term solution. [Ref. 11]

Two short-term treatments to control the fumer problem were implemented between 1985 and 1987. First, G.E. developed a bake cleaning process for the swirler/tube system. The high temperature baking process removed carbon coking inside the swirler. The removal of the coking reduces the potential for fuel spitback into the bleed air system, a major contributing factor to fuming. The second temporary fix was the suspension of short tube modifications. Based upon conclusions from the 1986 report on PPC-53 effectiveness, the program office suspended the modifications and returned to using the LCT fuel tube. The effect of these two temporary fixes is depicted in Figure 3.6 (note years 86-88).

G.E., in the mean time, began exploring various component improvements to the system for a long-term solution to the fumer problem. In March 1988, G.E. proposed Change-In-Design (CID) 184584. The CID proposed utilizing a long fuel
tube with an electroless nickel plating. The intent of the softer, long electroless nickel (LEN) tube was to transfer the wear of the tube/swirler friction away from the swirler and onto the LEN tube. G.E. determined reducing swirler wear would decrease the probability of fuel spitback and thereby decrease the potential for fumers.

The proposed change to the LEN configuration was approved. In 1990 the first engines were modified to incorporate the change. In April 1991, a borescope was developed which allowed maintainers to visually inspect the swirler wall for wear prior to incorporation of the change. Since April 1991, all engines modified by depot level maintenance have been borescoped, and, beginning in October 1992, all engines modified at intermediate level facilities will also be borescoped. Those swirlers which have previous wear are being replaced with new swirlers [Ref. 7]. By ensuring that there are no previous defects (wear) in a swirler, the effectiveness of the LEN change can be accurately measured since there is a common reference point (baseline) from which to track the modification.

Today, over seventy engines (approximately 1/6 of the inventory) have been modified to the LEN configuration (Figure 3.7). Approximately half of those engines were borescoped prior to the modification. The modified engines have accumulated over 13,000 flight hours with the high time engine having 660 flight hours (see Appendix B). There have been no documented fumer incidents on the LEN engines nor have there been any documented fuel tube failures.
Figure 3.7  Swirler/Fuel Tube Configurations (thru 01 SEP 92)
(424 Total Engines)
Currently, the S-3 program management team is interested in determining if the LEN/swirler system is as good or better than the LCT/swirler system. In other words, which system produces less fumers over time. Additionally, the management team is interested in determining the reliability of the LEN/swirler system at the 1500-hour mark since this is the first scheduled maintenance period for the engine.
IV. METHODOLOGY

This chapter explains the procedures and techniques used to formulate answers to the following questions:

1. How can the improvement in a power plant change be demonstrated?

2. How can the reliability of a new system be estimated for a future point in time?

3. What data is required to demonstrate the improvement and estimate the reliability of a component improvement during implementation?

A. DATA CONSTRAINTS

The data available for this study has three major constraints which affect the methodologies used to answer the questions posed. The three data constraints pertain to the LCT system's performance, the LEN system's operating hours, and the LEN system's failures.

1. LCT System Performance.

Chapter II briefly discussed the problem encountered when NAWC, Trenton, determined a Weibull plot of "hours to fume" for the LCT system utilizing all 41 fuel tube failures as fumer data points. The 1976 - 1977 data compiled by General Electric contained failure times for 41 LCT tubes of which only two tube/swirler systems failed (fumed). The parameters of characteristic life and slope obtained from the Weibull plots for the 41 LCT tube failures cannot be assumed to
be the same parameters which would be obtained if only the two fumer engines were
graphed on Weibull plots. The characteristic life of hours to fume and the beta
parameter for the LCT system would be dependent upon the accumulated operating
hours for both failed and unfailed (suspended) LCT systems. Establishing the
Weibull plot of the LCT system is extremely important. The plot provides both a
baseline to measure improvement (as defined in section C of this chapter) of the new
system and a beta value to use in the Weibayes analysis if no failures have occurred
on the new system.

The author attempted to retrieve data on pre-PPC-53 engines which had
fumed so this baseline Weibull could be established for the LCT system. The Navy
Aviation Logistics Data Analysis (NALDA) system was queried for engine
information from 1976 through 1981. The information retrieved did not delineate
either in the removal code or the remarks section which engines had fumed during
that time period. In fact, the removal code in the NALDA system, which specifies
why an engine has been removed from service, did not have a specific identifying
code for a "fumer" engine until the mid 1980's. Since the critical baseline for the
LCT system could not be established, Chapter V will only provide sensitivity analysis
concerning the characteristic life and the reliability of the LEN system to changes in
beta values. No conclusions will be made whether the LEN system is more or less
effective than the LCT system.
2. LEN System Operating Hours.

As mentioned in Chapter II, 72 engines (approximately 1/6 of the 424 engines in the fleet) have been modified to the LEN tube configuration as of 1 September 1992, the data cut-off date for this thesis. The 72 engines have accumulated over 13,000 engine flight hours (EFH) with the high time engine having 660 EFH (see Appendix A). Since the operating time on the modified components is relatively low, EFH projections were made for one year into the future (1 September 1993) to determine effects upon characteristic life and reliability of the LEN system.

Projections for September 1993 were determined by calculating the average monthly build (modification) rate for LEN systems and the average flight hours per month since build. The actual build rate for the past year was just over three engines per month (see Figure 4.1). The depot at NAS Alameda confirmed that they expect the total "I" and "D" level build rate to continue at three to four engines per month this next year [Ref. 12]. The projections use a three engines per month build rate which produces a conservative estimate for the number of modifications.

Figure 4.2 depicts the values used for EFH projections. As seen in Figure 4.2, the average flight hours per month since build increase rapidly and then stabilize at approximately 20 to 25 EFH per month one year from build date. The low EFH per engine in the early months after build can be attributed to logistics delay between the modifier ("I" or "D" level maintenance) and the fleet users.
Figure 4.1 Monthly Modification Rate for LEN Tubes
Figure 4.2 Average Engine Flight Hours per Engine Since Modification
3. LEN System Failures:

As of 1 Sep 1992, there have been no LEN system failures on modified engines. Predicting the success or failure of a component improvement is more difficult without failure data. Reference 4 states, "When data deficiencies such as too few or no failures exist a Weibull plot can not be generated." However, Weibayes analysis has been developed to solve problems when Weibull analysis can not be used. Weibayes is never preferred over Weibull analysis but is often required because of weaknesses in the data. Weibayes is defined as a Weibull analysis with an assumed beta value. Since the assumption requires judgement, this analysis is regarded as an informal Bayesian procedure.

B. ASSUMPTIONS ABOUT THE METHODOLOGY

This methodology makes the following assumptions:

1. The failure data of the original (LCT) system can be described by a Weibull distribution.

2. Any favorable change in reliability or characteristic life is a direct result of the component improvement and not a result of a more favorable operating environment. The operational demands upon an engine are assumed to remain constant.

3. The failure distribution of the new LEN/"clean" swirler also has a Weibull distribution with the same slope parameter, beta, as the old LCT/"clean" swirler configuration.

4. A LEN tube/"clean" swirler combination is assumed to be as good or better than a LEN tube/"unknown" swirler combination.

C. DATA REQUIREMENTS

Since deficiencies exist with the LCT data which hinder the creation of a baseline Weibull plot for the old system, the data required to perform Weibull analysis must be understood to preclude data deficiencies in future PPC's under
investigation. There are only two essential pieces of data required to create a Weibull plot and hence determine the improvement in characteristic life and reliability of a PPC whose failures can be described by the Weibull distribution. The two pieces of data are the following:

1. **Time-to-failure.**

   The individual ages of failed systems must be determined. The units of measurement for age, calculated from a common reference point, depend upon part usage and the failure mode. For example, age for a jet starter may be the number of engine starts while age for turbine parts may be the number of excursions between hot and cold [Ref. 4].

   Time-to-failure must be determined for the pre-PPC systems because this data determines the baseline Weibull plot. Time-to-failure for the PPC systems is equally important since this data determines the new component's Weibull plot. If no failures have occurred on the new system, which is the case for the LEN system, then the shape parameter (beta) from the old system may be used to establish a Weibayes plot.

2. **Suspended Component Ages.**

   The age of suspended (non-failed) units of the population is also important data. "The argument for including them (suspensions) in the analysis is that if their failure had occurred in the same fashion as other failures, the rank order of the other failures would have been influenced" [Ref. 4]. If the suspensions affect the rank order of the failed units, then the Weibull plot will shift slightly affecting
the parameters beta and eta. In fact, the ages of new systems which have zero failures provide the essential data used to estimate the Weibayes distribution.

D. METHODOLOGIES

The methods used to determine if the LEN configuration is an improvement over the LCT configuration and to estimate the reliability of the LEN configuration at time "x" are discussed in this section.

1. Detecting Improvement.

If the Weibayes plot for the new system lies to the right of the baseline Weibull plot for the original system there has been an improvement with the implementation of the PPC since the characteristic life of the new system is greater than the characteristic life of the old system. Two methods were used to determine if the LEN system was an improvement over the LCT system. The first method employs Weibayesian analysis to project the characteristic life of the new system. The second method uses the maximum likelihood estimator (MLE) to determine if the probability that time-to-fail (fume) of the LEN system is greater than the time-to-fail (fume) of the LCT system. Both methods are discussed below.

a. Weibayesian Analysis

Using Weibayesian analysis, a characteristic life can be determined for the LEN system. The LEN system's characteristic life can then be compared to values for the LCT system's characteristic life to determine if the new system is an improvement.
If no failures have occurred on the new system, Reference 4 gives the Weibayes equation for a lower 100(1 - \(\alpha\))% confidence bound, \(\hat{\eta}_{L(\alpha)}\), for eta, \(\eta\), as:

\[
\hat{\eta}_{L(\alpha)} = \left( \frac{\sum_{i=1}^{n} x_i^\beta}{-(\ln \alpha)} \right)^{1/\beta},
\]  

(4.1)

where

- \(\eta\) is the characteristic life;
- \(\beta\) is the assumed value of the Weibull shape parameter;
- \(x_1, x_2, \ldots, x_n\) are the operating times accumulated by engines 1, 2, \ldots, \(n\) with no failures; and
- \(n\) is the number of suspensions or non-failed units in the fleet.

Alternatively, Reference 13 shows this confidence bound may be presented as a function of the Chi-Square distribution; namely,

\[
\hat{\eta}_{L(\alpha)} = \left( \frac{2 \left[ \sum_{i=1}^{r} x_i^\beta + (n-r) x_{(r)}^\beta \right]}{x_n^2, 2(r+1)} \right)^{1/\beta},
\]  

(4.2)

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where
\[ x_{(1)}, x_{(2)}, \ldots, x_{(r)} \] are the \( r \) ordered failure times;
\( r \) denotes the number of failures; and
\[ \chi_{2(r-q)}^2 \] is the right tail 100(1 - \( \alpha \)) percentile point of the Chi-Square distribution with 2(r - q) degrees of freedom.

Equation 4.1 and 4.2 are equal when \( r = 0 \) since
\[ -\ln \alpha = \frac{1}{2} \chi_{2, 2}^2 \]

b. Estimate of \( P(X > Y) \)

Statistical inference is the process by which information from sample data is used to draw conclusions about the population from which the sample was selected. [Ref. 5]

One technique of statistical inference involves parameter estimation to obtain a point estimate. A point estimate is a single numerical value of a statistic that corresponds to that parameter. One of the best methods for obtaining a point estimator is the method of maximum likelihood. Essentially, the maximum likelihood estimator (MLE) is the value of the parameter that maximizes the probability of obtaining the observed outcome. [Ref. 5]

The maximum likelihood estimate of the probability that the time-to-failure of a new (LEN) configuration is greater than the time-to-failure of an old (LCT) configuration can be described as follows.
Suppose

\[ X = \text{time to failure of a new system and} \]

\[ Y = \text{time to failure of an old system}, \]

then the value of \( P(X > Y) \) is a measure of the improvement of the new system over the old system. Let the MLE estimate for \( P(X > Y) \) be denoted by \( \hat{P}(X > Y) \).

To obtain \( \hat{P}(X > Y) \) we first note that if \( \lambda_i = 1/\eta_i \),

\( X \) is exponential \( (\lambda_1) \), and

\( Y \) is exponential \( (\lambda_2) \),

then

\[
P(X > Y) = \int_{y=0}^{\infty} \int_{x=y}^{\infty} \lambda_1 e^{-\lambda_1 x} \lambda_2 e^{-\lambda_2 y} dx dy
\]

\[
= \int_{0}^{\infty} e^{-\lambda_2 y} \lambda_2 e^{-\lambda_2 y} dy \quad \text{(4.3a)}
\]

\[
= \frac{\lambda_2}{\lambda_1 + \lambda_2} \int_{0}^{\infty} (\lambda_1 + \lambda_2)e^{-\lambda_1 x-\lambda_2 y} dy
\]

\[
= \frac{\lambda_2}{\lambda_1 + \lambda_2}
\]

\[
= \frac{1/\eta_2}{1/\eta_1 + 1/\eta_2} \quad \text{(4.3b)}
\]

\[
= \frac{\eta_1}{\eta_1 + \eta_2}
\]
For the cases of the new (LEN) system and the old (LCT) system:

(a) If $X$ is time-to-failure of a LEN system and $X$ is Weibull ($\eta_1, \beta$), then $X^\beta$ is exponential ($\eta_1^\beta$).

(b) If $Y$ is time-to-failure of a LCT system and $Y$ is Weibull ($\eta_2, \beta$), then $Y^\beta$ is exponential ($\eta_2^\beta$). [Ref. 14]

Therefore

$$P(X > Y) = P(X^\beta > Y^\beta) = \frac{\eta_1^\beta}{\eta_1^\beta + \eta_2^\beta} .$$

(4.4)

The maximum likelihood estimate, $\hat{P}(X > Y)$, for $P(X > Y)$ is then:

$$\hat{P}(X > Y) = \frac{\hat{\eta}_1^\beta}{\hat{\eta}_1^\beta + \hat{\eta}_2^\beta} .$$

(4.5)

where $\hat{\eta}_1^\beta$ and $\hat{\eta}_2^\beta$ are the MLE estimates for $\eta_1^\beta$ and $\eta_2^\beta$, respectively, and are defined by Reference 14:

$$\hat{\eta}_1^\beta = \frac{\sum_{i=1}^{n} X_i^\beta}{r_1} ;$$

(4.6)

$$\hat{\eta}_2^\beta = \frac{\sum_{i=1}^{n_2} Y_i^\beta}{r_2} ;$$

(4.7)
where

\[ r_1 \] is the number of LEN failures in the \( n_1 \) LEN system tests;

\[ r_2 \] is the number of LCT failures in the \( n_2 \) LCT system tests; and

both \( r_1 \) and \( r_2 \) must be greater than zero for equations (4.6) and (4.7) to be valid (i.e., no division by zero occurs). [Ref. 14].

Since \( r_1 = 0 \) for the LEN data, another estimate, \( \hat{\eta}_1^* \), for \( \eta_1^\phi \) must be used. The alternate estimate chosen is formulated using equation 4.2 and is given by equation 4.8. It is not the MLE but is chosen because it is defined for the case of zero failures (\( r = 0 \)). Additionally, the 50% lower confidence limit is often used to estimate the mean of a distribution when the point estimate for the mean is undefined or has other undesirable properties. Equation 4.8 is the 50% lower confidence limit for \( \eta_1^\phi \) when zero failures have been observed.

\[
\hat{\eta}_1^* = \frac{2\sum_{i=1}^{n_1} X_i^\phi}{X_{3, 2}}. \tag{4.8}
\]

If \( r_1 = 0 \) and equation (4.8) is used to estimate \( \eta_1^\phi \), then a similar estimate, \( \hat{\eta}_2^* \), should be used to estimate \( \eta_2^\phi \); where

\[
\hat{\eta}_2^* = \frac{2\sum_{i=1}^{n_2} Y_i^\phi}{X_{3, 2(2\sigma^2 + 1)}}. \tag{4.9}
\]

Thus, from equation (4.4), the estimate for \( P(X > Y) \) becomes
\[
\hat{P}(X > Y) = \frac{\hat{\eta}_1^*}{\hat{\eta}_1^* + \hat{\eta}_2^*}.
\]  
(4.10)

Also, a lower \(100(1 - \alpha)\)% confidence bound, \(\hat{P}(X > Y)_{U(\alpha)}\), for \(P(X > Y)\) can be computed using equation (4.11) [Ref. 14]

\[
\hat{P}(X > Y)_{U(\alpha)} = \frac{1}{1 + D_{u(\alpha)}}
\]  
(4.11)

where

\(D_{u(\alpha)}\) is the \(100(1 - \alpha)\)% upper confidence bound for \(\frac{\eta_2}{\eta_1}\).

\(D_{u(\alpha)}\) can be written as a function of the data as follows [Ref. 14]:

\[
D_{u(\alpha)} = \frac{2(r_1 + 1) \sum_{i=1}^{n_1} Y_i^\beta}{2(r_2 + 1) \sum_{i=1}^{n_1} X_i^\beta} \left[ F_{a, 2(r_1 + 1), 2(r_2 + 1)} \right].
\]  
(4.12)

Here \(F_{a, 2(r_1 + 1), 2(r_2 + 1)}\) is the \(100(1 - \alpha)\) percentile point of the F distribution with \(2(r_1 + 1)\) and \(2(r_2 + 1)\) degrees of freedom. [Ref. 14]. See Appendix C for the derivation of equation (4.12) for \(D_{u(\alpha)}\).

2. Reliability of the LEN System.

The reliability of a system is the probability that, when operating in the manner intended, a system will perform its intended function satisfactorily for a
specified interval of time [Ref. 4]. Reference 4 shows the reliability at time \( x \) for a Weibull distribution, with a known beta, is expressed as:

\[
R(x; \eta, \beta) = e^{-\left(\frac{x}{\eta}\right)^\beta} .
\] (4.13)

If \( X \) is the time to failure of a LEN system and if \( X \) has a Weibull distribution with known \( \beta \), then \( X_i^\beta \) has an exponential distribution with mean \( 1/\eta^\beta \). It follows from the monotonicity of \( R(x; \eta, \beta) \) as a function of \( \eta \) that the lower 100(1 - \( \alpha \))% confidence limit for the reliability at time \( x \) for a LEN system can be expressed as:

\[
R(x)_{L(\alpha)} = \exp\left[-\left(\frac{1}{\eta_{L(\alpha)}}\right) (x^\beta)\right];
\] (4.14)

where \( \eta_{L(\alpha)}^\beta \) is a lower 100(1 - \( \alpha \))% confidence limit for \( \eta^\beta \). [Ref. 14]

From Reference 14,

\[
\frac{2}{\eta^\beta} \sum_{i=1}^{n} X_i^\beta
\] (4.15)

is a Chi-Square variable with 2(\( r + 1 \)) degrees of freedom where \( r \) is the observed number of failures and, since \( r = 0 \), the Chi-Square variable has 2 degrees of freedom. Then, from the definition of \( \chi^2_{x, 2} \) we have,
\[ 1 - \alpha = P \left( \chi^2 \leq \chi^2_{\alpha, \nu} \right) \]

\[ = P \left( \frac{2 \sum_{i=1}^{n} X_i^\beta}{\eta^\beta} \leq \chi^2_{\alpha, \nu} \right) \]

\[ = P \left( \frac{2 \sum_{i=1}^{n} X_i^\beta}{\chi^2_{\alpha, \nu}} \leq \eta^\beta \right) \]  \hspace{2cm} (4.16)

It follows from the definition of a lower confidence limit for \( \eta^\beta \) that the 100(1 - \( \alpha \))% lower confidence limit, \( \eta^\beta_{L(\alpha)} \), is

\[ \eta^\beta_{L(\alpha)} = \frac{2 \sum_{i=1}^{n} X_i^\beta}{\chi^2_{\alpha, \nu}}. \]  \hspace{2cm} (4.17)

Therefore

\[ \hat{R}(x)_{L(\alpha)} = \exp \left[ -\left( \chi^2_{\alpha, \nu} / 2 \sum_{i=1}^{n} X_i^\beta \right) x^\beta \right]. \]  \hspace{2cm} (4.18)

3. Expected number of failures.

Next we derive the formulas for the expected number of engines that will fail (fume) in the next year given the current time accumulated on each non-fumed system. The determination of the expected number of failures will assist decision-makers with the projection of the engine's logistical support requirements for the time period under analysis.
Let

\[ x_i = \text{current total EFH for engine } i; \ i = 1, 2, \ldots, n; \]

\[ X_i = \text{time to failure for engine } i; \ i = 1, 2, \ldots, n; \]

\[ Y = \text{projected EFH per engine during the next year; and} \]

\[ N = \text{total number of LEN modifications by the end of a year from now.} \]

Then the conditional probability, \( P_i \), that the \( i \)th engine will not fail before the end of a year from now is given by\(^1\)

\[
P_i = P(X_i > (x_i + Y) | X_i > x_i) \\
= \frac{P(X_i > x_i + Y \text{ and } X_i > x_i)}{P(X_i > x_i)} \\
= \frac{P(X_i > x_i + Y)}{P(X_i > x_i)} \\
= \frac{R(x_i + Y)}{R(x_i)} \\
= \exp \left\{ -\frac{1}{\eta} ((x_i + Y)^{\beta} - x_i^{\beta}) \right\}.
\]

Using the estimate, \( \hat{\eta}^{\beta} \), for \( \eta^{\beta} \) given by equation 4.8 the estimate, \( \hat{P}_i \), for \( P_i \) is

\[
\hat{P}_i = \exp \left\{ -\left( x_{5.2}^2 / 2 \sum_{i=1}^n x_i^{\beta} \right) (x_i + Y)^{\beta} - x_i^{\beta} \right\}.
\]

\[ (4.20) \]
The expected number of engines which will not fail prior to next year is
\[
\sum_{i=1}^{N} P_i = \sum_{i=1}^{N} P(X_i > x_i + Y | X_i > x_i).
\] (4.21)

Therefore, the expected number of engines to fail during the next year, \(E(N)\), is
\[
E(N) = N - \sum_{i=1}^{N} P(X_i > x_i + Y | X_i > x_i)
\] (4.22)
\[
= N - \sum_{i=1}^{N} P_i.
\]

The estimate, \(\hat{E}(N)\), for \(E(N)\) is
\[
\hat{E}(N) = N - \sum_{i=1}^{N} \hat{P}_i.
\] (4.23)

E. **SUMMARY**

The methodology presented provides a sound framework for analyzing the characteristic life and reliability of a PPC whose failures can be described by the Weibull distribution. The inability to generate a baseline Weibull plot due to data constraints of the LCT system will preclude the author from both determining if the
characteristic life of the LEN system has improved and projecting the reliability of the LEN system. However, in the following chapter, the author will analyze the LEN system’s data to determine the sensitivity of characteristic life and reliability projections to various beta values.
V. ANALYSIS

This chapter analyzes data using the methodologies and assumptions presented in Chapter IV. The data was obtained from NAWC, Trenton, and the NALDA database. It is presented in Appendix B.

A. OBJECTIVES

The objectives of the analysis are as follows:

1. To determine the characteristic life of the LEN system for various failure modes (beta values).

2. To estimate the reliability of the LEN system at the 1000 and 1500 hour marks for various beta values.

3. To estimate the expected number of failures (failures) of the LEN system during the next year for various beta values.

B. PRESENTATION OF DATA

Beta values greater than one were analyzed since the LEN system is beyond the initial "test and fix" stage of its life cycle where beta values are often less than one. The system should experience wear out from day one of operation which is consistent with beta values greater than one.

1. Determining characteristic life of the LEN system.

The characteristic life of the LEN system was determined using equation 4.2 from Chapter IV. Figure 5.1 illustrates a 90% lower confidence bound for the
Figure 5.1 LEN Characteristic Life: 90% Lower Confidence Bound

Characteristic Life - Hrs (Thousands)
characteristic life of the LEN system for 1 September 1992 EFH's and 1 September 1993 EFH projections. A 90% lower confidence bound for the characteristic life means that we can be 90% confident, using the data obtained, that the true characteristic life is greater than lies above the plotted value. Weibayes plots, discussed in Chapter II, are presented in Appendix D for selected beta values of the data depicted in Figure 5.1.

2. Estimating the reliability of the LEN system.

The reliability of the LEN system was estimated using equation 4.18 from Chapter IV. Figures 5.2 and 5.3 present 1 September 1992 and 1 September 1993 estimates for the LEN system's reliability at the 1000-hour mark for various lower confidence bounds assuming different beta values. Figures 5.4 and 5.5 present 1 September 1992 and 1 September 1993 reliability estimates for the 1500-hour mark. The 1500-hour mark represents the age of the LEN system at the first scheduled maintenance period. The 1000-hour mark was arbitrarily chosen to contrast the estimated system reliability for near-term (1000-hour) and long-term (1500-hour) decisions points.
Figure 5.2 LEN System Reliability at 1000 hours (01SEP92 Data)
Lower Confidence Limit for the Probability of No Failure Before 1000 hours

CONFIDENCE LEVEL (%)

- □ BETA = 2  + BETA = 2.5  ○ BETA = 3  △ BETA = 3.5  × BETA = 4
- ▽ BETA = 4.5

Figure 5.3 LEN System Reliability at 1000 hours (01 SEP 93 Projections)
Figure 5.4 LEN System Reliability at 1500 hours (01 SEP 92 Data)

Lower Confidence Limit for the Probability of No Failure before 1500 hours
Figure 5.5 LEN System Reliability at 1500 hours (01 SEP 93 Projections)
3. Projected number of fumers.

The expected number of LEN fumers from 1 September 1992 through 1 September 1993 was estimated using equation 4.23 from Chapter IV. The 1 September 1993 flight hours for each engine were projected using the data presented in Figure 4.2. Each engine's projected 1 September 1993 EFH are presented in Appendix B. Figure 5.6 depicts the expected number of fumers for the LEN system between 1 September 1992 and 1 September 1993 for various beta values.

C. CONCLUSIONS ABOUT THE RESULTS

Based upon the data presented, the following conclusions concerning the LEN system can be made:

1. The characteristic life of the LEN system increases as operating time accumulates on individual, modified systems.

2. The estimated reliability of the LEN system is greater for periods in the near future (1000 hours) than for periods in the more distant future (1500 hours).

3. As beta increases, the reliability of the system decreases while the projected number of fumers (failures) for the period increases. A larger beta signifies a more rapid wearout or failure mode.

4. The projections for a system's characteristic life, reliability, and expected number of failures are all highly sensitive to the assumed beta value.

The analysis demonstrates there is viable method for determining characteristic life and projecting reliability of a component improvement which has not experienced a failure. Furthermore, the analysis demonstrates the importance of determining the
baseline Weibull plot for the original system. The baseline Weibull plot provides both an assumed beta value to determine characteristic life and project reliability of the new system and a yard stick for detecting improvement in characteristic life. Without the baseline Weibull plot, the decision maker will only have sensitivity analysis (as provided in this chapter) to provide feedback on the characteristic life and reliability of the component improvement.
VI. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

A. SUMMARY

The original objective of this thesis was to provide the TF-34-400 program management team with feedback which would assist them in evaluating the improvement and reliability of a specific component improvement, the incorporation of the long electroless nickel (LEN) fuel tube. As the research effort progressed, a significant lack of data was discovered for the original long carbide tungsten (LCT) fuel tube/swirler system. The lack of LCT system data precluded the author from establishing the critical baseline to detect improvement and project reliability of the new LEN system.

The data limitations forced the author to modify the original objective to the following two objectives:

1. Establish a list of data elements that are required to detect the improvement and estimate the reliability of a specific component improvement as the implementation process unfolds.

2. Propose methodologies to detect the improvement and estimate the reliability of a specific component improvement as the implementation process unfolds.

The author made the following two assumptions while determining the required data elements and proposing the methodologies:
1. The failure data of the old LCT system could be described by a Weibull distribution.

2. The failure data of the new LEN system would also be describable by a Weibull distribution and have the same slope parameter, beta, as the old LCT system.

The Weibull distribution and its parameters were discussed in Chapter II. Chapter III presented an overview of the TF-34-400 engine and an in-depth discussion of the fumer problem. Chapter IV discussed data constraints of the LCT and LEN systems, data requirements for PPC evaluation, and methodologies to detect the improvement and project the reliability of a PPC when the failure data can be described by a Weibull distribution. Chapter V presented sensitivity analyses of the characteristic life and reliability for the new LEN system to changes in the beta parameter.

B. CONCLUSIONS

The establishment of data requirements and development of methodologies necessary to detect improvement and project reliability provides a framework for engine program management teams to assess the impact of PPC’s during implementation. This thesis demonstrated the importance of determining the old LCT system’s baseline (point of origin), gathering the proper data elements to measure, and converting that data into useful feedback via the proposed methodologies for determining improvement and reliability.
This thesis concluded that when a system's failures can be described by a Weibull distribution there are two critical pieces of data to measure for both the old and new systems. First, the time-to-failure (age) of each failed system must be determined using a common reference point (i.e., engine flight hours). Second, the ages of suspended (non-failed) units must also be determined.

This thesis proposed two methodologies for detecting improvement and one methodology for projecting reliability. Improvement in characteristic life can be detected when the characteristic life of the new system is greater than the characteristic life of the old system (Equation 4.8). Additionally, improvement is detected when the probability of the time-to-failure of the new system is greater than the time-to-failure of the old system (Equation 4.10). Reliability for the new system can be projected using Equation 4.18.

Valuable feedback, such as system improvement and reliability, can be gained from tracking a PPC as implementation unfolds. However, before the methodologies proposed in this thesis can be fully implemented, the program management team must ensure the required data is being tracked at operational levels.

As more testing is accumulated on the engines that have the new design and "fumer" failures begin to occur, alternate statistical methods for assessing the improvement of the new design should be explored. When the number of failures is between one and five, the methods used in this thesis could be used but with the degrees of freedom equal to 2(r+1), where r is the number of failures; that is, use Equation (4.2) with an assumed Beta. It would be wise, however, to compute the
MLE point estimate, $\hat{\beta}$, of $\beta$ using existing statistical methods found in Reference 15. The MLE point estimate, $\hat{\beta}$, should be compared to the assumed beta value in the Weibayes method. If these values differ by more than roughly 20%, then alternative methods to the Weibayes procedure should be considered. The variance of this estimate, $\hat{\beta}$, will be quite large when the ratio of the number of failures to the number of items tested is less than 1/3. Bain in reference 15 has developed constants which when multiplied by the MLE estimate, $\hat{\beta}$, yields an unbiased estimate. However, the variance of the unbiased estimate still remains large.

After a significant number of failures ($\geq 5$) become available, the alternative statistical procedures that have been developed in Reference 15 can be used to develop improved versions of the measures of improvement developed in this thesis. In addition, other measures such as exact confidence limits on the reliability and on the parameters $\eta$ and $\beta$ could be introduced that would provide added meaning and credibility to the assessment on improvement.

C. RECOMMENDATIONS

The author makes four recommendations.

1. Determine data requirements.

The data reference points (ie., engine flight hours, hot/cold cycles, number of starts) must be determined and tracked for systems of interest. This data is not only essential for assessing component improvements but can also be valuable for enhancing other logistical support decisions such as maintenance planning and life management.
2. Establish a baseline.

Prior to implementation of a PPC, establish a baseline for the system being replaced so the effect of the change can be accurately assessed.

3. Employ a standard methodology for evaluating component improvements.

For example, if the failure data of various component improvements can each be described by the Weibull distribution, use the methodologies presented in Chapter IV to evaluate improvement and project reliability for each component improvement. Standardization generally reduces variability and therefore should improve the quality of the feedback.

4. Establish a lead engine program for component improvements.

A lead engine program accelerates operating time (age) on a percentage of the engine fleet so advanced feedback on improvement and reliability will be available to enhance decision-making. The Air Force has implemented the lead engine concept with their Pacer Century Program for the F-15 and F-16 engine programs. "The purpose of the Pacer Century Program is to gain, from actual operation, early intelligence on engine integrity, reliability, and maintainability" [Ref. 16]. Reference 16 states the objectives of the Pacer Century Program are

1. "to identify potential premature engine component failures by an early analysis of trends/failure rates and assist in the identification of corrective actions."

2. "to assess scheduled maintenance requirements."

3. "to rapidly advance engine/module/accessory maximum Life Limits consistent with the capability of the hardware."
4. "to identify potential impact on future spares support."

5. "to identify hardware life impacts on system support costs."

The Pacer Century Program allows the Air Force to pursue a proactive vice reactive strategy in their F-15 and F-16 engine programs.

D. RECOMMENDATIONS FOR FURTHER RESEARCH

The following two items are recommendations for further thesis research.

1. Perform an in-depth analysis of the Air Force Pacer Century Program to determine the viability of incorporating its concepts and techniques into the Navy's Component Improvement Program.

2. Conduct an analysis of the various data bases (ie., Naval Aviation Logistics Data, Aircraft Engine Maintenance System, and Engine Component Improvement Feedback Report) to determine an efficient, "user-friendly" format for reporting the required data elements for evaluating component improvements.
APPENDIX A WEIBULL PAPER
## APPENDIX B  LEN DATA

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APPENDIX C: FORMULA DERIVATION FOR \( D_{\alpha(e)} \)

Recall that equation 4.12 gave a formula for \( D_{\alpha(e)} \), the upper 100(1-\( \alpha \))% confidence limit for \( \left( \frac{\eta_2}{\eta_1} \right)^{\beta} \). That formula is derived in this Appendix.

From equation 4.4 recall that

\[
P = P(X^\beta > Y^\beta) = \frac{\eta_1^\beta}{\eta_1^\beta + \eta_2^\beta} \]

\[
= \frac{1}{1 + \left( \frac{\eta_2}{\eta_1} \right)^\beta}
\]

(C1)

Clearly the last fraction in (C1) decreases as \( \left( \frac{\eta_2}{\eta_1} \right)^{\beta} \) increases. Consequently, an upper bound for \( \left( \frac{\eta_2}{\eta_1} \right)^{\beta} \) is a lower bound for \( \frac{1}{1 + \left( \frac{\eta_2}{\eta_1} \right)^\beta} \); i.e., a lower bound for

\[
P(X^\beta > Y^\beta)
\]

Specifically, suppose \( D_{\alpha(e)} \) is an upper 100 (1-\( \alpha \))% lower confidence limit for \( \left( \frac{\eta_2}{\eta_1} \right)^{\beta} \), then
\[ 1 - \alpha = P\left( \frac{\eta_2}{\eta_1}^\beta < D_{u(a)} \right) \]
\[ = P\left( 1 + \left( \frac{\eta_2}{\eta_1} \right)^\beta < 1 + D_{u(a)} \right) \]
\[ = P\left( \frac{1}{1 + \left( \frac{\eta_2}{\eta_1} \right)^\beta} > \frac{1}{1 + D_{u(a)}} \right) \]
\[ = P\left( P(X^\beta > Y^\beta) > \frac{1}{1 + D_{u(a)}} \right) \]  
\hfill (C2)

From the definition of a 100 (1-\(\alpha\))% lower confidence limit, \(\frac{1}{1 + D_{u(a)}}\) is a 100 (1-\(\alpha\))% lower confidence limit for \(P(X^\beta > Y^\beta)\).

To find the (100-\(\alpha\))% upper confidence bound for \(P(X^\beta > Y^\beta)\), recall that the F distribution random variable \(F_{r_1, r_2}\), with \(r_1\) and \(r_2\) degrees of freedom is defined by \(\frac{\chi^2_{r_1}}{\chi^2_{r_2}}\) [Ref. 14].

Additionally,
\[ \left( \frac{2\sum_{i=1}^{n_1} X_i^\beta}{\eta_1^\beta} \right) \frac{1}{\chi^2_{2(r_1 + 1)}} \]  
\hfill (C3)
and

\[ \left( \frac{2 \sum_{i=1}^{n_2} y_i^\beta}{\eta_2^\beta} \right) \] is \( \chi^2_{2(r_2+1)} \). \hfill (C4)

were \( r_1, n_1, r_2 \) and \( n_2 \) are defined in equations 4.6 and 4.7.

Therefore

\[
\frac{\left( \sum_{i=1}^{n_1} x_i^\beta \right)}{2(r_1 + 1)} / \frac{\left( \sum_{i=1}^{n_2} y_i^\beta \right)}{2(r_2 + 1)} = \left( \frac{\eta_1}{\eta_2} \right)^\beta \left( \frac{\sum_{i=1}^{n_1} x_i^\beta}{\sum_{i=1}^{n_2} y_i^\beta} \right) \left( \frac{2(r_2+1)}{2(r_1+1)} \right)
\]

\[ = \frac{\chi^2_{2(r_2+1)} / 2(r_1+1)}{\chi^2_{2(r_1+1)} / 2(r_2+1)} \]
\[
= F_{2(r_1+1), 2(r_2+1)} \quad \hfill (C5)
\]

If we let \( F_{x, 2(r_1+1), 2(r_2+1)} \) be the 100 \((1-\alpha)\)th percentile point of the \( F_{2(r_1+1), 2(r_2+1)} \) probability distribution, then
(1 - \alpha) = P \left[ F_{2(n_1-1), 2(r_2+1)} \leq F_{\alpha, 2(n_1 - 1), 2(r_2 + 1)} \right] \\
= P \left( \frac{(\eta_2)^\beta}{(\eta_1)^\beta} \right) \left( \frac{\sum_{i=1}^{n_1} X_i^\beta}{\sum_{i=1}^{n_2} Y_i^\beta} \right) \frac{2(r_2+1)}{2(r_1+1)} \leq F_{\alpha, 2(r_1+1), 2(r_2+1)} \\
= P \left( \frac{(\eta_2)^\beta}{(\eta_1)^\beta} \right) < \frac{2(r_1+1)}{2(r_2+1)} \frac{\sum_{i=1}^{r_1} Y_i^\beta}{\sum_{i=1}^{r_2} X_i^\beta} \leq F_{\alpha, 2(r_1+1), 2(r_2+1)} \\
= P \left( \frac{(\eta_2)^\beta}{(\eta_1)^\beta} < D_{u(\alpha)} \right)

This, by definition of an upper 100 (1 - \alpha) % confidence limit, says that \( D_{u(\alpha)} \) is the expression in Equation 4.12. [Ref. 14]
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