The objective of this work was to study analytically, numerically, and experimentally the nonlinear dynamic behavior of metallic and composite structural elements to parametric and external excitations. The study was focused on resonance conditions that may produce large and possibly damaging motions. Special attention was given to modal coupling and consequent energy exchanges.
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ABSTRACT

The objective of this work was to study analytically, numerically, and experimentally the nonlinear dynamic behavior of metallic and composite structural elements to parametric and external excitations. The study was focused on resonance conditions that may produce large and possibly damaging motions. Special attention was given to modal coupling and consequent energy exchanges.

The following is a brief summary of the accomplishments under this grant.

A. PUBLICATIONS


In this article, we study the three-dimensional autonomous system. The method of multiple scales is used to obtain an asymptotic expansion for the amplitude and frequency of the limit cycle near the Hopf bifurcation of the system. The resulting analytical approximation is valid only in a small range near the Hopf bifurcation point. This range is ascertained using numerical simulations and stability analysis. Floquet theory is used to determine the stability of the periodic solutions. Also, Rand's analysis is discussed in light of the present study. The perturbation analysis
presented here can be easily applied to higher-dimensional systems. The discussion on the range of validity of the analytical approximation and the stability analysis are pertinent to higher-dimensional systems as well.


Newton's second law is used to develop the nonlinear equations describing the extensional-flexural-flexural-torsional vibrations of slewing or rotating metallic and composite beams. Three consecutive Euler angles are used to relate the deformed and undeformed states. Because the twisting-related Euler angle \( \phi \) is not an independent Lagrangian coordinate, twisting curvature is used to define the twist angle, and the resulting equations of motion are symmetric and independent of the rotation sequence of the Euler angles. The equations of motion are valid for extensional, inextensional, uniform and nonuniform, metallic and composite beams. The equations contain structural coupling terms and quadratic and cubic nonlinearities due to curvature and inertia. Some comparisons with other derivations are made, and the characteristics of the modeling are addressed. The second part of the paper will present a nonlinear analysis of a symmetric angle-ply graphite-epoxy beam exhibiting bending-twisting coupling and a two-to-one internal resonance.


We study the planar dynamic response of a flexible L-shaped beam-mass structure with a two-to-one internal resonance to a primary resonance. The structure is subjected to low excitation (mili g) levels and the resulting nonlinear motions are examined. The Lagrangian for weakly nonlinear motions of the undamped structure is formulated and time averaged over the period of the primary oscillation, leading to an autonomous system of equations governing the amplitudes and phases of the modes involved in the internal resonance. Later, modal damping is
assumed and modal-damping coefficients, determined from experiments. are included in the analytical model. The locations of the saddle-node and Hopf bifurcations predicted by the analysis are in good agreement, respectively, with the jumps and transitions from periodic to quasi-periodic motions observed in the experiments. The current study is relevant to the dynamics and modeling of other structural systems as well.


The behavior of single-degree-of-freedom systems possessing quadratic and cubic nonlinearities subject to parametric excitation is investigated. Both fundamental and principal parametric resonances are considered. A global bifurcation diagram in the excitation amplitude and excitation frequency domain is presented showing different possible stable steady-state solutions (attractors). Fractal basin maps for fundamental and principal parametric resonances when three attractors coexist are presented in color. An enlargement of one region of the map for principal parametric resonance reveals a Cantor-like set of fractal boundaries. For some cases, both periodic and chaotic attractors coexist.


An indirect adaptive quenching algorithm for a nonlinear single-degree-of-freedom system with unknown constant system parameters is presented. The system is subject to external or parametric sinusoidal disturbances and the resulting control signal is also sinusoidal. The quenching algorithm provides a reduction in the control effort required compared to direct disturbance cancellation. The disturbance sinusoid and the unknown parameters are incorporated into the system model and an extended Kalman filter (EKF) with modified update equations is used to estimate the system state and parameters. The estimates are then used to
form the quenching signal. The adaptive quenching algorithm is found to work well inside a quenching region defined by the separatrices and suggests the use of a hybrid control law. The algorithm was verified by implementing it on an analog computer.


We conducted an experimental study of the influence of modal interactions on the nonlinear response of a composite-beam structure to harmonic excitations. Energy exchange between the excited and nonexcited modes of vibration leads to nonlinear periodic, periodically modulated, and chaotically modulated responses. Results obtained in the experiments with the composite structure are discussed and compared with the results obtained in earlier studies of a steel-beam structure of a similar form.


A theoretical-experimental investigation is conducted into the response of a three-degree-of-freedom structure having an autoparametric combination resonance of the additive type to a primary resonance of the higher mode. The structure consists of two light-weight beams and three concentrated masses arranged in a T-shape. The theoretically predicted natural frequencies are in good agreement with those obtained experimentally. The method of multiple scales is used to determine six first-order differential equations describing the modulation of the amplitudes and phases of the three modes with nonlinearity, damping, and resonances. Some of the predicted nonlinear phenomena include the saturation phenomenon, coexistence of stable solutions, jumps, and periodic, two-period quasiperiodic, and phase-locked or synchronized
motions. The experiment confirms these predictions and shows other phenomena that are not yet explained by the theory.


A parametrically-excited system with excitation amplitude or frequency that is a linear function of time is considered. The method of multiple scales equations and the governing equations are integrated on a digital computer. Deviations from the stationary solution, including penetration, jump, overshoot, convergence to the stationary solution, and lingering are considered. The effects of sweep rate on the maximum amplitude and unboundedness are discussed.


Newton’s second law is used to develop the nonlinear equations describing the flexural-flexural-torsional vibration of metallic and composite beams. Three consecutive Euler angles are used to relate the deformed and undeformed states. Because the twisting-related Euler angle \( \phi \) is not an independent Lagrangian coordinate, the torsional equation is used to define the real twist angle. The resulting equations contain structural coupling terms and cubic nonlinearities due to curvature and inertia. These equations are used to analyze the nonplanar responses of a cantilever composite beam to a lateral harmonic base excitation. A powerful technique based on the state space concept is used to solve for the linear mode shapes. A combination of the Galerkin procedure and the method of multiple scales is then used to construct a first-order uniform expansion for the case of a one-to-one internal resonance and a primary resonance. Quantitative results are obtained for nonplanar motions and
their dynamic behavior is investigated. For different range of parameters, the nonplanar motions can be steady whirling motions, whirling motions of the beating type, or chaotic motions.


In this paper we present the results of a theoretical-experimental investigation of the response of a three-degree-of-freedom structure with quadratic nonlinearities to a harmonic excitation. The structure consists of three light-weight steel beams and three concentrated masses, as shown in Figure 1. The positions of the masses on the vertical beams are adjusted to make \( \omega_3 \approx \omega_2 + \omega_1 \) and \( \omega_5 \approx 2\omega_1 \), where the \( \omega_n \) are the linear undamped natural frequencies. These conditions are related to combination and two-to-one internal (sometimes called autoparametric) resonances, respectively. Under these conditions the response to a simple harmonic excitation can exhibit complicated behavior, such as quasi-periodic and chaotically modulated motions. Here we call the response linear if it can be described accurately by linear equations of motion; we call the response nonlinear when nonlinear equations of motion are necessary.


We present a collection of experimental results on the influence of modal interactions (i.e., internal or autoparametric resonances) on the nonlinear response of flexible composite-beam structures to harmonic excitations. During primary resonant excitations at low excitation (mili g) levels, coupling between the directly excited and indirectly excited modes of vibration of the structure led to nonlinear planar and nonplanar periodic, nonplanar periodically modulated, and nonplanar chaotically modulated motions. Interaction between flexural and torsional modes led to the nonplanar responses. The transition to chaotically modulated motions
occurred via quasi-periodic motions. During combination resonant excitations, even at high excitation (2.0 g) levels, we could not excite the nonlinear responses of the structure. Theoretical analyses for internally-resonant systems are used to predict and explain the structure's responses. In addition, the experimental results are compared with those obtained in earlier experiments with a metallic structure of a similar form.


Three nonlinear integro-differential equations of motion are used to investigate the forced nonlinear vibration of a symmetric laminated graphite-epoxy composite beam. The analysis focuses on the case of primary resonance of the first flexural mode when its frequency is approximately two times the frequency of the first out-of-plane flexural-torsional mode. A combination of the fundamental-matrix method, the Galerkin procedure, and the method of multiple scales is used to derive four first-order ordinary-differential equations describing the modulation of the amplitudes and phases of the interacting modes with damping, nonlinearity, and resonances. The eigenvalues of the Jacobian matrix of the modulation equations are used to determine the stability of constant solutions, and Floquet theory is used to determine the stability and bifurcations of limit-cycle solutions. Hopf bifurcations, symmetry-breaking bifurcations, amplitude- and phase-modulated motions, period-multiplying sequences, and chaotic motions are studied. The results show that the motion can be planar and/or nonplanar although the input force is planar. Nonplanar responses may be periodic motions, amplitude- and phase-modulated motions, or chaotically modulated motions.

The nonlinear equations of motion derived in Part I are used to investigate the response of an inextensional, symmetric angle-ply graphite-epoxy beam to a harmonic base-excitation along the flapwise direction. The equations contain bending-twisting couplings and quadratic and cubic nonlinearities due to curvature and inertia. The analysis focuses on the case of primary resonance of the first flexural-torsional (flapwise-torsional) mode when its frequency is approximately one-half the frequency of the first out-of-plane flexural (chordwise) mode. A combination of the fundamental-matrix method and the method of multiple scales is used to derive four first-order ordinary-differential equations to describe the time variation of the amplitudes and phases of the interacting modes with damping, nonlinearity, and resonances. The eigenvalues of the Jacobian matrix of the modulation equations are used to determine the stability and bifurcations of their constant solutions, and Floquet theory is used to determine the stability and bifurcations of their limit-cycle solutions. Hopf bifurcations, symmetry-breaking bifurcations, period-multiplying sequences, and chaotic solutions of the modulation equations are studied. Chaotic solutions are identified from their frequency spectra, Poincare sections, and Lyapunov's exponents. The results show that the beam motion may be nonplanar although the input force is planar. Nonplanar responses may be periodic, periodically modulated, or chaotically modulated motions.


Three nonlinear integro-differential equations of motion derived in Part I are used to investigate the forced nonlinear vibration of a symmetrically laminated graphite-epoxy composite beam. The analysis focuses on the case of primary resonance of the first in-plane flexural (chordwise) mode when its frequency is approximately twice the frequency of the first out-of-plane flexural-torsional (flapwise-torsional) mode. A combination of the fundamental-matrix method and the method of multiple scales is used...
to derive four first-order ordinary-differential equations describing the modulation of the amplitudes and phases of the interacting modes with damping, nonlinearity, and resonances. The eigenvalues of the Jacobian matrix of the modulation equations are used to determine the stability of their constant solutions, and Floquet theory is used to determine the stability and bifurcations of their limit-cycle solutions. Hopf bifurcations, symmetry-breaking bifurcations, period-multiplying sequences, and chaotic motions of the modulation equations are studied. The results show that the motion can be nonplanar although the input force is planar. Nonplanar responses may be periodic, periodically modulated, or chaotically modulated motions.


We present a collection of experimental results on the influence of modal interactions (i.e., internal or autoparametric resonances) on the nonlinear response of flexible metallic and composite structures subjected to a range of resonant excitations. The experimental results are provided in the form of frequency spectra, Poincaré sections, pseudo-phase planes, dimension calculations, and response curves. Experimental observations of transitions from periodic to chaotically modulated motions are also presented. We also discuss relevant analytical results. The current study is also relevant to other internally-resonant structural systems.


Methods for determining the response of continuous systems with quadratic and cubic nonlinearities are discussed. We show by means of a simple example that perturbation and computational methods based on first discretizing the systems may lead to erroneous results whereas
perturbation methods that attack directly the nonlinear partial-differential equations and boundary conditions avoid the pitfalls associated with the analysis of the discretized systems. We describe a perturbation technique that applies either the method of multiple scales or the method of averaging to the Lagrangian of the system rather than the partial-differential equations and boundary conditions.


We study motion near a Hopf bifurcation of a representative four-dimensional autonomous system. Special cases of the four-dimensional system represent the envelope equations that govern the amplitudes and phases of the modes of an internally resonant structure subjected to resonant excitations. Using the method of multiple scales, we derive asymptotic expansions for the frequency and amplitude of a limit cycle near a bifurcation point and determine the nature of the bifurcation (i.e., subcritical or supercritical). The range of validity of the analytical approximation is ascertained using numerical simulations. The perturbation analysis and discussions are pertinent to other autonomous systems as well.


A general nonlinear theory for the dynamics of elastic anisotropic plates undergoing moderate-rotation vibrations is presented. The theory fully accounts for geometric nonlinearities (moderate rotations and displacements) by using local stress and strain measures and an exact coordinate transformation, which result in nonlinear curvatures and strain-displacement expressions that contain the von Karman strains as a special case. The theory accounts for transverse shear deformations by using a third-order theory and for extensionality and changes in the configuration due to in-plane and transverse deformations. Five
third-order nonlinear partial-differential equations of motion describing the extension-extension-bending-shear-shear vibrations of plates are obtained by an asymptotic analysis, which reveals that laminated plates display linear elastic and nonlinear geometric couplings among all motions.


Presented here is a general theory for the three-dimensional nonlinear dynamics of elastic anisotropic initially straight beams undergoing moderate displacements and rotations. The theory fully accounts for geometric nonlinearities (large rotations and displacements) by using local stress and strain measures and an exact coordinate transformation, which result in nonlinear curvature and strain-displacement expressions that contain the von Karman strains as a special case. Extensionality is included in the formulation, and transverse shear deformations are accounted for by using a third-order theory. Six third-order nonlinear partial-differential equations are derived for describing one extension, two bending, one torsion, and two shearing vibrations of composite beams. They show that laminated beams display linear elastic and nonlinear geometric couplings among all motions. The theory contains, as special cases, the Euler-Bernoulli theory, Timoshenko's beam theory, the third-order shear theory, and the von Karman type nonlinear theory.


A general nonlinear theory for the dynamics of elastic anisotropic circular cylindrical shells undergoing small strains and moderate-rotation vibrations is presented. The theory fully accounts for extensionality and geometric nonlinearities by using local stress and strain measures and an exact coordinate transformation, which result in nonlinear curvatures and strain-displacement expressions that contain the von Karman strains as a special case. Moreover, the linear part of the theory contains, as special cases, most of the classical linear theories when appropriate stress
resultants and couples are defined. Parabolic distributions of the transverse shear strains are accounted for by using a third-order theory and hence shear correction factors are not required. Five third-order nonlinear partial differential equations describing the extension, bending, and shear vibrations of shells are obtained using the principle of virtual work and an asymptotic analysis. These equations show that laminated shells display linear elastic and nonlinear geometric couplings among all motions.


In this paper the response of a three-degree-of-freedom structure with quadratic nonlinearities subject to harmonic forcing of its third mode is investigated. The method of multiple scales is used to obtain the equations that govern its amplitudes and phases. The fixed points of these equations are obtained and their stability is determined. The fixed points are found to undergo Hopf bifurcations and hence the response undergoes amplitude- and phase-modulated motions. Regions where the amplitudes and phases undergo periodic, quasiperiodic, and chaotic motions and hence regions where the overall system response is periodically, quasiperiodically, and chaotically modulated are determined.


Because engineering cables are usually lengthy and flexible, their vibrations are essentially nonlinear and dominated by geometric nonlinearities. Large-amplitude vibrations and nonlinear phenomena of cables have been extensively studied analytically and experimentally. There are many nonlinear cable models due to the many ad hoc assumptions adopted by different researchers during different steps in the derivation. Because extensional forces are the only elastic loads acting on cables, compressibility and Poisson's effect can be significant. For
example, for rubber-like materials the Poisson's effect results in a nonlinear relationship between the tension force and the axial strain. In the literature, most of cable models do not account for the change in the cross-section area due to Poisson's effect or assume that the material is incompressible, and hence the Poisson's ratio $\nu$ is absent from the equations of motion: such theories are valid only for materials with $\nu \approx 0$ or $\nu \approx 0.5$. But, the Poisson's ratios of most engineering materials are between 0.25 and 0.35 except those of rubber and paraffin, for which $\nu \approx 0.5$. To account for geometric nonlinearities, some researchers account only for cubic nonlinearities, and some researchers account only for quadratic nonlinearities. However, cubic nonlinearities can greatly change the dynamic responses due to quadratic nonlinearities. Moreover, some theories do not account for initial sags, static loads, and/or extensionality, but these effects always exist in real cable structures.


A higher-order shear-deformation theory is used to analyze the interaction of two modes in the response of thick laminated rectangular plates to transverse harmonic loads. The case of a two-to-one autoparametric resonance is considered. Four first-order ordinary differential equations describing the modulation of the amplitudes and phases of the internally resonant modes are derived using the averaged Lagrangian when the higher mode is excited by a primary resonance. The fixed-point solutions are determined and their stability is analyzed. It is shown that besides the single-mode solution, two-mode solutions exist for a certain range of parameters. It is further shown that, in the multi-mode case, the lower mode, which is indirectly excited through the internal resonance may dominate the response. For a certain range of parameters, the fixed points lose stability via a Hopf bifurcation, thereby giving rise to limit-cycle solutions. It is shown that these limit cycles undergo a series of period-doubling bifurcations, culminating in chaos.

The exact natural frequencies and mode shapes of a slewing isotropic beam are determined, and the effects of the distributed rotary inertia and the hub inertia are investigated. The results show that the influence of the rotary inertia is not negligible for a beam vibrating in the principal direction of strong flexural rigidity and it decreases the natural frequencies and increases the modal rotation angles at the hub. Because the modal rotation angles at the hub and the natural frequencies are large when the hub inertia is small, a small hub inertia is desirable for controlling a slewing beam. For a composite beam made of three prismatic segments and exhibiting bending-twisting coupling, a powerful technique based on the state-space concept and the fundamental-matrix solution is used to obtain the exact frequencies and mode shapes. The influence of the hub inertia on the modal frequencies and mode shapes is similar to that for an isotropic beam. But, the bending-twisting coupling results in a serious control problem because it creates a large twisting angle at the tip of the beam, especially for higher modes. The results obtained by the fundamental-matrix method are compared with those obtained by a finite-element method.


Generalized Levy-type solutions are obtained for the stability of cross-ply laminated plates. The governing equations, which are derived by using the principle of virtual displacement and a third-order shear-deformation plate theory, are transformed into a set of first-order linear ordinary-differential equations with constant coefficients. The general solution of these equations can be obtained by using the state-space concept. Then, application of the boundary conditions yields equations for the buckling loads. Fortunately, a straightforward
application of the state-space concept yields numerically ill-conditioned problems as the plate thickness is reduced. A combination of an initial-value method and the modified Gram-Schmidt orthonormalization procedure is used to overcome this problem. It is shown that this method not only yields results that are in excellent agreement with the results in the literature, but also it converges fast and gives all the buckling loads regardless of the plate thickness.


Structural identification is of paramount importance for the prediction of the response of a given structure to various loading conditions. Although the responses of many structures used in practice can be determined by modeling them with linear equations, there are many physical phenomena that cannot be modeled by linear equations. These phenomena include subharmonic, superharmonic, and combination resonances, jumps, flutter limit cycles, saturation, multi-mode responses, and aperiodic responses. In several structures, some of the natural frequencies are commensurate or nearly commensurate. In the presence of appropriate nonlinearities, these frequency relationships can lead to modal interactions. In this event, these frequency relationships are called internal resonances. Modal interactions can occur during both free and forced oscillations of a structure and can lead to the aforementioned nonlinear phenomena. With the advent of complex space and aircraft structures, there is a growing need for the identification of nonlinear systems. Conditions for modal interactions do exist in most such structures. However, the identification of systems which exhibit modal interactions is just beginning to receive attention.

A fully nonlinear theory for the dynamics and active control of elastic laminated plates with integrated piezoelectric actuators and sensors undergoing large-rotation and small-strain vibrations is presented. The theory fully accounts for geometric nonlinearities (large rotations and displacements) by using local stress and strain measures and an exact coordinate transformation. Moreover, the model accounts for continuity of interlaminar shear stresses, extensionality, orthotropic properties of piezoelectric actuators, dependence of piezoelectric strain constants on induced strains, and arbitrary orientations of the integrated actuators and sensors. Extension and shearing forces and bending and twisting moments are introduced onto the plate along the boundaries of the piezoelectric actuators. Five nonlinear partial-differential equations describing the extension-extension-bending-shear-shear vibrations of laminated plates are obtained, which display linear elastic and nonlinear geometric couplings among all motions. Also, the piezoelectric actuator-induced warping is addressed, and comparisons with other simplified models and nonlinear theories are made.


A geometrically exact nonlinear beam model is developed for naturally curved and twisted solid composite rotor blades undergoing large vibrations in three-dimensional space. The theory accounts for in-plane warpings due to bending and extensional loadings, out-of-plane warpings due to shearing and torsional loadings, elastic couplings among warpings, and three-dimensional stress effects by using the results of a two-dimensional, static, sectional, finite-element analysis. Also, the theory fully accounts for extensionality, initial curvatures, and geometric nonlinearities by using local stress and strain measures and an exact coordinate transformation, which result in fully nonlinear curvature and strain-displacement expressions. Six fully nonlinear equations of motion
describing one extension, two bending, one torsion, and two shearing vibrations of composite beams are obtained by using a combination of the extended Hamilton principle and the concept of virtual local rotations. The equations display linear elastic couplings due to structural anisotropy and initial curvatures and nonlinear geometric couplings. The theory contains most of beam theories as special cases. Moreover, the formulation is based on an energy approach, but the derivation is fully correlated with the Newtonian approach and the final equations of motion are put in compact matrix form.


A nonlinear theory of curved and twisted beams can be used to model helicopter rotor blades, aviation propeller blades, turbine blades, arm-type positioning mechanisms of magnetic disk drives, robot manipulators, helical springs, etc. Moreover, beam-type elements are usually used in material characterization as well as basic structural elements. Recently, the rapid developments in aerospace exploration stimulated extensive research into the dynamics and control of flexible structures because space structures must be extremely light weight and hence very flexible in order to be cost effective. In this paper, we develop a general nonlinear theory for curved and twisted beams.


The response of a structure to a simple-harmonic excitation is investigated theoretically and experimentally. The structure consists of two light-weight beams arranged in a T-shape turned on its side. Relatively heavy and concentrated weights are placed at the upper and lower free ends and at the point where the two beams are joined. The base of the "T" is clamped to the head of a shaker. Because the masses of the
concentrated weights are much larger than the masses of the beams, the first three natural frequencies are far below the fourth; consequently, for relatively low frequencies of the excitation, the structure has, for all practical purposes, only three degrees of freedom. The lengths and weights are chosen so that the third natural frequency is approximately equal to the sum of the two lower natural frequencies, an arrangement that produces an autoparametric (also called an internal) resonance. A linear analysis is performed to predict the natural frequencies and to aid in the design of the experiment; the predictions and observations are in close agreement. Then a nonlinear analysis of the response to a prescribed transverse motion at the base of the "T" is performed. The method of multiple scales is used to obtain six first-order differential equations describing the modulations of the amplitudes and phases of the three interacting modes when the frequency of the excitation is near the third natural frequency. Some of the predicted phenomena include periodic, two-period quasiperiodic, and phase-locked (also called synchronized) motions; coexistence of multiple stable motions and the attendant jumps; and saturation. All the predictions are confirmed in the experiments, and some phenomena that are not yet explained by theory are observed.

B. PRESENTATIONS


C. SEMINARS


D. STUDENTS


