Workshop on Trajectory Optimization Methods and Applications

Presentations from the 1992 AIAA Atmospheric Flight Mechanics Conference

Compiled by:
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Preface

This report is a compilation of presentations given at the “Workshop on Trajectory Optimization Methods and Applications”, held at the 1992 AIAA Atmospheric Flight Mechanics Conference in Hilton Head, South Carolina. This workshop was co-chaired by Harry Karasopoulos and Kevin J. Langan, both of the former Flight Performance Group of the High Speed Aero Performance Branch in Wright Laboratory.

It is hoped that this document will help the attendees retain some of the ideas presented in the workshop, in addition to providing useful information to those who were unable to attend. Appreciation is expressed to the presenters and attendees. For the third year in a row, this workshop has proved to be a successful forum for highlighting current work in trajectory optimization. Thanks are also due to the American Institute of Aeronautics and Astronautics for making this workshop possible.

The following was the workshop schedule:

SCHEDULE

- 1:00 - Introduction - Harry Karasopoulos, Wright Laboratory.

- 1:00 - OMAT: An Autonomous Optimal Solution to Rendezvous Problems with Operational Constraints - Don Jezewski, McDonnell Douglas Space Systems Company, Houston, TX.

- 1:15 - MULIMP: Multi-Impulse Trajectory and Mass Optimization Program - Darla German, Science Applications International Corporation, Schaumburg, IL.

- 1:30 - Phillips Laboratory Applications of POST - Jim Eckmann, SPARTA Inc., Edwards AFB, CA.

- 1:45 - OTIS Advances at the Boeing Company - Steve Paris, The Boeing Company, Seattle, WA.

• 2:15 - Advances in Trajectory Optimization Using Collocation and Nonlinear Programming - Dr. Bruce A. Conway, University of Illinois, Urbana, IL.

• 2:45 - BREAK

• 3:00 - Flight Path Optimization of Aerospace Vehicles Using OTIS - Dr. Rajiv S. Chowdhry, Lockheed Engineering and Sciences Company, NASA LaRC, Hampton, VA.

• 3:15 - Trajectory Optimization of Launch Vehicles at LeRC: Present and Future - Dr. Koorosh Mirfakhraie, ANALEX Corp., NASA Lewis Research Center, Cleveland, OH.

• 3:30 - Collocation Methods in Regular Perturbation Analysis of Optimal Control Problems - Dr. Anthony J. Calise, Georgia Institute of Technology, Atlanta, GA.

• 3:45 - Automatic Solutions for Take-Off from Aircraft Carriers - Lloyd II. Johnson, AIR-53012D, Naval Air Systems Command, Washington, DC.

• 4:00 - Current Pratt-Whitney OTIS Applications - Russ Joyner¹, United Technologies, Pratt-Whitney, West Palm Beach, FL.


• 4:30 - Scheduled Session End

Unscheduled Speakers

• Dr. Klaus Well, University of Stuttgart.²

• Dr. Mark L. Psiaki, Cornell University.

¹Cancelled
²Presentation copy not available at the time of printing
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OMAT
An Autonomous Optimal Solution to
Rendezvous Problems with
Operational Constraints

by

D. J. Jezewski, J. P. Braxio,
B. R. Haufer, and E. E. Prust

McDonnell Douglas Space Systems Company
Houston Division
16055 Space Center Blvd.
Houston, Texas 77062-6208

AIAA Atmospheric Flight Mechanics Conference
Hilton Head, South Carolina
August 10, 1992
Sketch of Rendezvous Problem

Subscripts:
C - Chaser Vehicle
T - Target Vehicle

General Approach for Solving Optimal Rendezvous Problems

Analytical
Numerical
Definition of an Optimal Rendezvous Problem

- **Given:**
  - Chaser and Target States: \((S_C, t_C), (S_T, t_T)\)
  - Attracting Body
  - Objective Function, i.e., Delta-V, Fuel, Time, etc.

- **Subject To:**
  - Force Field
  - Perturbations
  - Terminal & Inflight Constraints and Limits

- **Define:**
  - Optimal Sequence of Maneuvers (Impulses, Finite Burns)
    - Number, Location, Magnitude or Duration, and Direction

- **Such That:**
  - Chaser & Target Vehicles Achieve a Relative Configuration at Some Time

A General Optimization Approach

- Initial Parameter Vector, \(X_0\)
- Reference Solution \(S, J, G, C\)
- Compute \(X_{i+1}\)
- Variation of Solution \(\Lambda, V, J, V, G, V, C\)
- Extremal Solution? Yes/No
- Iterate NZSOL

- **\(X\)** - Parameter Vector
- **\(S\)** - State Vector
- **\(\Lambda\)** - Costate Vector
- **\(J\)** - Objective Function
- **\(G\)** - Active Constraint Vector
- **\(C\)** - Solution Cost
Forms of Constraints

☐ Three Types of Constraints
  - Parameter Constraints
    - a < X < b
  - Linear Constraints (Constant Jacobian Matrix)
    - L(S,t) ≥ 0
  - Non-Linear Constraints
    - NL(S,t) ≥ 0

☐ Bounds
  - Constraints are Bounded by Upper and Lower Bounds (B_L, B_U)
  - Equality Defined by B_U = B_L
  - Unbounded Defined by B_L = -∞, B_U = +∞

Constraints are Defined by Five Integers

☐ I - Constraint Number
☐ J - What the the Constraint is Referenced to:
  - An Impulse
  - Another Constraint
☐ K - Reference Impulse or Constraint Number
☐ L - Condition that Triggers or Initiates Constraint
  - Time from GMT or Reference Event (Impulse or Constraint)
  - Phase Angle
  - Lighting Condition between Chaser and Target Vehicles
  - Delta-Angular Measurement in Chaser Orbit
  - Number of Revolutions
  - Chaser Vehicle’s nth Periapsis, Apoapsis Crossing

...
Constraints are Defined by Five Integers (concl'd)

- N - Type of Constraint
  - Periapsis, Apoapsis
  - Differential Height between Chaser Vehicle & Target Orbit
  - Phase Angle
  - Sleep Cycle or Quiet Time
  - Chaser Orbit Coelliptic with Target Orbit
  - Chaser Position Vector Relative to Target LVLH Frame
  - Chaser Velocity Vector Relative to Target LVLH Frame
  - Bounded Delta-V in Chaser LVLH Frame
  - Inertial Line-of-Sight Angular Rate
  - Wedge Angle

Demonstration 2:
Typical Shuttle Rendezvous

- Shuttle (Chaser)
  - 110 circular altitude
  - Inclination = 28.5°
  - Longitude of ascending node = 101°
  - Argument of perigee = 0°
  - True anomaly = 180°

- SSF (Target)
  - 190 nmi circular altitude
  - Inclination = 28.5°
  - Longitude of ascending node = 100°
  - Argument of perigee = 0°
  - True anomaly = 0°

- Limited to 2 maneuvers
Demonstration 2 (Concl'd)

Demonstration 2: Shuttle Rendezvous with SSF (Unperturbed)
OMAT Unconstrained and Constrained Solution

Results
- OMAT handles perigee constraint for unperturbed orbits
- Optimum unconstrained trajectory required 461 ft/s,
  constrained trajectory required 603 ft/s
Perturbations

- Primer Vector Theory (Presently) Requires Perturbations to have Form
  - $\mathcal{X}(R,t)$
- Largest Geopotential Perturbation for Earth & Mars is $J_2$, Has Form
  $$\mathcal{X}_{J2} = j_r R + j_k K$$
  - Where $K$ is a Unit Vector Normal to the Equator and $j_r$ and $j_k$ are Functions of the Position Vector.
- Presently Incorporating NxM Geopotential Model
- Need to Extend Theory to Functions $\mathcal{X}(R,V,t)$
- Need to Develop Theory for Third-Body Effects (Libration Point Rendezvous)

Demonstration 4:
Mars MEV Post-Ascent Rendezvous with MTV

- MEV (Chaser)
  - 117 x 135 nmi altitude
  - Inclination = 164.264662°
  - Longitude of ascending node = 194.39079°
  - Argument of perigee = 159.820571°
  - True anomaly = 0°
- MTV (Target)
  - 135 x 18,294 nmi altitude
  - Inclination = 164.2°
  - Longitude of ascending node = 195°
  - Argument of perigee = 16°
  - True anomaly = 0°

(Not to Scale)
Demonstration 4: Mars MEV Post-Ascent Rendezvous with MTV
OMAT J2 Perturbed Solution

Perturbed Lambert Problem

- A Definition of Lambert’s Problem:
- What is the Initial Velocity Vector, \( V_I \), that Generates a Trajectory that Passes between Two Radii Separated by a Given Angle in a Specified Time Interval?
Perturbed Lambert Problem (cont'd)

- Standard Approach
  - \( V_1 \) Obtained from Classical Lambert Problem
  - \( \delta V_1 \) Obtained from Solution to Variational Equations, i.e.,
    \[
    \delta V_1 = \Phi_{12}^{-1} \delta R_e
    \]
    \[\Phi(t_e, t_p) = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix}\]
  - Difficulties with this Approach
    - \( \delta R_e \) Must be "Small" (Linear Approximation)
    - \( \Phi_{12} \) Must be Well Conditioned
    - \( J_2 \) Frequently Must be Reduced (Sub-Problem)
    - Excessive Number of Iterations and Integrations

Perturbed Lambert Problem (conc'd)

- Improved Approach
  - \( V_1 \) Obtained from First-Order Correction to the Inverse-Square
    Problem Resulting from the \( J_2 \) Perturbation
    - Analytic Solution
    - Expressed in "Ideal Reference Frame"
    - Regular, No Singularities
    - Solution Expressed in Terms of Elements & their Variations
  - \( \delta V_1 \) Obtained from Solution to Variational Equations, i.e.,
    \[
    \delta V_1 = \Phi_{12}^{-1} \delta R_e
    \]
  - \( \delta R_e \approx O(10^{-3}) \) Smaller than Classical Approach
  - No Requirement for \( J_2 \) Reduction (Sub-Problems)
  - Solution to TPBVP Requires Only a Few Iterations and Integrations
Comparison of Final Position Vector Errors for the Classical Lambert and Predictor/Corrector Solutions

- Earth Centered
- Transfer Angle = 170 degrees

Outline of Optimal Solution Approach

- Unperturbed Problem
  - Force Field (Inverse-Square), i.e.,
    \[ V = -\mu \frac{R}{r^3} \]
  - State and Variation in State Obtained from Solution of
    - Kepler’s Problem (Goodyear, Analytic)
  - Boundary Value Problem Satisfied by Solution of
    - Lambert’s Problem (Gooding, Analytic)
  - Constraints and Variation of Constraints Evaluated Using
    - Keplerian Elements
  - Non-Linear Programming Algorithm, NZSOL, Solves Constrained Optimal Problem
  - Solution Obtained in seconds on Sun Sparcstation 2
Outline of Optimal Solution Approach (concl'd)

- Perturbed Problem
  - Force Field (Perturbed Inverse-Square)
    \[ V = -\mu \left( \frac{R}{r^3} \right) \cdot \dot{r} \]
  - State and Variation in State Obtained by Numerical Integration of Differential Equations
    - Variable-Step Runge-Kutta-Fehlberg 7/8 Algorithm
    - Maximum of 42 Linear & Non-Linear Differential Equations
  - Boundary Value Problem Satisfied by Solving
    - Perturbed Lambert Problem
  - Constraint Targeting Evaluated Using Non-Periodic Elements
  - Constraints and Variation of Constraints Evaluated on Perturbed Trajectory
  - NZSOL Solves Perturbed, Constrained Optimal Problem
  - Solutions Obtained (Presently) in Minutes

What Have We Done, Where Are We Going?

- Present Status of Development:
  - Proof of Concept, Using Unperturbed Solution to Solve Perturbed Problem
  - Verify Solution Approach for Handling Perturbations and Constraints and their Conditions
  - Developed Solution Approach for Perturbed Lambert Problem
  - Illustrate Initial Capability of the Algorithm, (OMAT), to Efficiently Solve Optimal Rendezvous Problems with Operational Constraints

- Planned Future Development:
  - Expand Perturbation and Constraint Models
  - Develop Multi-Rev. Capability
  - Develop Finite-Thrust Model
  - Libration Point Rendezvous
  - Solve Advanced Problems
Concluding Remarks

- Integration of Theoretical & Operational Aspects Of Optimal Rendezvous
- Development of an Autonomous Optimal Rendezvous Solution
- Primer Vector Theory Basis
- Approach Based on Solution to Lambert's Problem and it's Extension in a Perturbed Force Field
- Premise is Made that Perturbed Problem is an $\epsilon$ away from Unperturbed
- Constraints are Adjoined to Objective Function by Lagrange Multipliers
- General Mapping for Constraints from State to Parameter Space
- NZSOL Used to Solve Constrained Non-Linear Programming Problem
- Program is Fast, Reliable, Robust, Flexible, and Readily Extended
- Solution Time: Unperturbed (sec.), Perturbed (min.)
MULIMP

Multi-Impulse Trajectory and Mass Optimization Program

Darla German

Science Applications International Corporation
MULIMP GENERAL DESCRIPTION

- DESIGNED TO COMPUTE A MULTI-TARGETED TRAJECTORY AS A SEQUENCE OF "TWO-BODY" SUBARCS IN A CENTRAL GRAVITATIONAL FIELD USING KEPLER AND LAMBERT ANALYTICAL SOLUTION ALGORITHMS

- BODIES MAY BE PLANETS (ORBITAL ELEMENTS STORED INTERNALLY), ASTEROIDS OR COMETS (ORBITAL ELEMENTS CONTAINED IN ASTCOM.ELM FILE), OR FICTITIOUS (ELEMENTS INPUT BY USER)

- CENTRAL BODY MAY BE THE SUN, ANY OF THE 9 PLANETS OR AN ARBITRARY BODY (GRAVITATIONAL CONSTANT INPUT BY USER)

- UP TO 19 SUBARCS MAY BE SPECIFIED

VARIABLES IN OPTIMIZATION SEARCH

- TIMES (DATES) OF THE NODAL POINTS CONNECTING TRAJECTORY SUBARCS

- POSITION COORDINATES OF MIDCOURSE ΔV POINTS NOT MADE AT AN EPHEMERIS BODY
DEPARTURE CONDITIONS

- RENDEZVOUS DEPARTURE IN WHICH CASE THE FIRST IMPULSE $\Delta V_1$ IS EQUAL TO THE HYPERBOLIC EXCESS SPEED $V_{\infty}$.

- PARKING ORBIT DEPARTURE IN WHICH CASE THE FIRST IMPULSE IS THAT NECESSARY TO ATTAIN THE HYPERBOLIC EXCESS SPEED FROM THE PARKING ORBIT $(r_p, e)$ WITH THE MANEUVER ASSUMED TO BE COPLANAR.

- A "FREE" DEPARTURE IN WHICH CASE THE FIRST $\Delta V$ IMPULSE IS EXCLUDED FROM THE PERFORMANCE INDEX.

- A GRAVITY-ASSIST DEPARTURE IN WHICH CASE THE APPROACH HYPERBOLIC VELOCITY VECTOR MUST BE SPECIFIED BY INPUT.

INTERMEDIATE TARGET CONDITIONS

ARRIVAL

- RENDEZVOUS
- ORBIT CAPTURE (ORBIT IS USER DEFINED)
- UNCONSTRAINED FLYBY SPEED
- CONSTRAINED FLYBY SPEED (HYPERBOLIC FLYBY) SPEED IS USER INPUT

DEPARTURE

- RENDEZVOUS DEPARTURE
- ORBIT DEPARTURE (ORBIT IS USER-DEFINED)

OTHER

- GRAVITY-ASSISTED SWINGBY
GRAVITY-ASSISTED SWINGBY

- MODEL IS FORMULATED WITH POWERED MANEUVER AS THE GENERAL CASE

- $\Delta V$ WILL OFTEN ITERATE TO ZERO VALUE IF THE PROBLEM IS NOT OVERLY CONSTRAINED BY SWINGBY DATE AND DISTANCE

- USER OPTION TO SPECIFY POWERED MANEUVER LOCATION
  - INBOUND ASYMPTOTE
  - PERIAPSIS
  - OUTBOUND ASYMPTOTE
  - BEST CHOICE

TERMINAL TARGET CONDITIONS

- RENDEZVOUS

- TARGET-BODY ORBIT CAPTURE

- SATELLITE ORBIT CAPTURE

- UNCONSTRAINED FLYBY

- CONSTRAINED FLYBY

- SPECIFIED ORBIT ELEMENTS ($a,e,i$) RELATIVE TO CENTRAL BODY; FINAL TARGET MUST PROVIDE GRAVITY-ASSIST
"FREE" MIDCOURSE ΔV POINTS

MIDCOURSE VELOCITY CHANGES MAY BE MADE AT INTERIOR IMPULSE POINTS NOT OCCURRING AT AN EPHEMERIS BODY. THESE MIDCOURSE ΔV POINTS MAY BE INCLUDED IN TWO WAYS:

- TIME AND POSITION COORDINATES MAY BE ESTIMATED AND INPUT; ON USER OPTION, THE TIME AND/OR POSITION COORDINATES WILL BE OPTIMIZED

- AUTOMATIC IMPULSE ADDITION MAY BE REQUESTED

MULTIPLE REVOLUTIONS AND RETROGRADE MOTION

- MULTIPLE REVOLUTIONS AND/OR RETROGRADE MOTION ARE SPECIFIED BY INPUT

- TWO OPTIONS ARE PROVIDED FOR HANDLING MULTIPLE REVOLUTION SOLUTIONS:

  - THE NUMBER OF COMPLETE REVOLUTIONS AND ENERGY CLASS MAY BE SPECIFIED

  - THE MAXIMUM NUMBER OF COMPLETE REVOLUTIONS TO CONSIDER MAY BE ENTERED IN WHICH CASE ALL INCLUSIVE SOLUTIONS WILL BE EXAMINED AND THE "BEST" ONE SELECTED ON THE BASIS OF A VELOCITY CHANGE CRITERIA
IR&D ENHANCEMENTS

- Addition of the unpowered specification for planetary swingbys
- A new departure option of space station launches
- A new terminal arrival option of 3-impulse planet orbit capture to a final orbit determined by user input \( r_p \), \( r_e \), and inclination
- Inclusion of JPL satellite ephemerides routines for most natural satellites
- Conversion of the working coordinate system from EM050 to J2000
- Ability to constrain total transit time
Phillips Laboratory
Applications of POST

James B. Eckmann
SPARTA, Inc.
Phillips Laboratory SETA
Edwards AFB, CA

10 Aug 92
Presentation Overview

- Organization and Mission
- Simulation Work Environment
- Summary of POST Models
  - Applications and Some Results
- Future Plans

Organizational Hierarchy
Propulsion Directorate (RK) Mission

• Provide propulsion technology and expertise for U.S. space and missile systems.
• Be a center of excellence in propulsion research and development.
• Develop a broad, advanced technology base for future propulsion system designers.
• Demonstrate propulsion concepts for current systems designers.
• Assist in solving operational problems.

System Support Division,
Applications Branch (RKSA)

• Mr. Raymond Moszlee, Branch Chief
  • Capt. Tim Middendorf
  • 1 Lt. Paul Castro
  • 2 Lt. Naftali Dratman
  • Mr. Francis McDougall
  • Mr. Gerry Sayles
  • Ms. Pamela Tanck, SPARTA
  • Mr. James Eckmann, SPARTA
  • Maj. Leo Matuszak, AF Reservest
RKSA Simulation Environment

- Integrated Tools
- Ethernet Network (TCP/IP)
- Connectivity Software (NFS, Versaterm Pro)

Integrated Analysis

- Trajectories, Vehicle Analysis
- Sizing, Geometry
- Structures and Weights
- Presentation

* POST *

**Persuasion**
Summary of POST Models

- Atlas II
- Delta
- Titan IV *
- Space Shuttle *
- Pegasus *
- Ground Based Interceptors
- Small ICBM
- Minuteman III
- National Launch System (NLS)
- Single Stage To Orbit (SSTO)
  - Delta Clipper - VTVL Concept *
  - RASV - HTHL Concept
  - RST - VTHL Concept *

Applications of POST Models

- Atlas II
  - Monopropellants; High Energy Density Mater (HEDM) propellants; Composite shroud
- Delta
- Titan IV
  - Soviet RD-170 strap-on LRB's to replace SRB's
- Space Shuttle
  - Clean propellant SRM's
- Pegasus
  - Advanced Liquid Axial Stage (ALAS) as 4th stage; Potential booster for NASP program flight test experiment
- Ground Based Interceptors
  - Single-stage, two-stage, three-stage, and dual-pulse motor boosters; Standard Missile and SRAM 2 boosters for LEAP tests
- SICBM
  - Advanced ICBM studies baseline
Minuteman III Model

- APPLICATION:
  - The ICBM system of the future. Baseline for assessing advanced technology payoffs.
    - Clean Propellant Trade Studies
    - Impact of Reducing the Number of Warheads
    - Two-stage Missile Studies

- CONSTRUCTION:
  - Objective Function: Maximum range for fixed payload or Maximum payload for fixed range
  - Constraints (2): Maximum dynamic pressure, Minimum re-entry angle
  - Control Variables (6): Pitch rate at motor ignition for each of 3 stages; Time at which inertial attitude is held constant for each of the 3 stages
  - Phasing Events (16): 3 motor firings, 3 stage separations, initial pitch over, 3 constant attitude segments, 3 ballistic flight segments, payload shroud seperation, atmospheric re-entry, ground impact.

Sample Minuteman III Results
National Launch System (NLS) Vehicles

Trade Studies Performed:
- Castor 120's on NLS-3
- Mixture Ratio
- Tank Sizing
- Thrust Level
- Engine Out
- Throttling Effects
- Staging Algorithms
- Upper Stages

Single-Stage-To-Orbit (SSTO) Models

- McDonnell Douglas Vertical Takeoff Vertical Landing (VTVL) Delta Clipper model developed and provided by NASA/Langley
- Rockwell International Vertical Takeoff Horizontal Landing (VTHL) Reusable Space Transport (RST) model developed by Rockwell and provided by NASA/Langley
- Boeing Horizonal Takeoff Horizontal Landing (HTHL) Reusable Aerodynamic Space Vehicle (RASV) model developed in-house
Future Plans

- DEVELOPE A COMPLETE VEHICLE SIMULATION CAPABILITY
  - Apply SMART and CONSIZE to current analysis tasks
  - Complete integration of Silicon Graphics machines
  - Develop a cost analysis capability
  - Continually evaluate new analysis tools
OTIS Advances at the Boeing Company

Steve Paris
Boeing Defense & Space Group
Collocation based Optimal Control Methods

Chebytop → CTOP

Indirect Trajectory Methods

Dickmanns → TOP

Direct Explicit Trajectory Schemes

AS2530 → NTOP / SPOT → POST

OTIS Modes

Direct Trajectory Optimization using Nonlinear Programming and Collocation

Explicit Integration
- RK4
- Adams-Moulton
- Hermite
- Euler

Mode 2 Targeting
Shooting

Mode 1 Trajectory Simulation
GIGO

Mode 3 Trajectory Optimization
Parameter

Mode 4 Trajectory Optimization
Optimal Control

- State of the Art
- Optimal Control and Simulation of Aerospace Vehicles
- Flexible Operation
Explicit Integration
Collocation
Non-linear Programming
Next Wave of OTIS Advances

Problem Set-up → Data Conditioning → Program Execution → Results Interpretation

Current Resources

Goal
4 fold overall reduction
OTIS runtimes reduced by 10 fold

OTIS 3.0 Lunar Test Case

Explicit Trajectory Generation

Optimize
- Launch Date
- A parking orbit
- t0 (burn1)
- ∆V1
- t0 (burn2)
- ∆V2

burn 2
burn 1
24 hour Orbit

Low Earth orbit to high polar Earth Orbit (24hr).
OTIS Elements

OTIS 3.0 Provides Extreme Flexibility

- Global Constraints
- Analytical Arcs
- Phase Dependent
  - Equations of Motion (EQM)
  - Control Variables
  - Quadrature Variables
Future Trends

Problem Set-Up
Data Conditioning
Program Execution
Results Interpretation

Off-the-Shelf Software
- Transform by Spyglass
- Excel by Microsoft

New Code
- Object Oriented
- Software Packages
- Sparse Matrix Methods
- Defect Formulation
- Singular Arcs
- Node Placement

Off-the-Shelf Software
- AGPS - Ribbon Plots
- Agile_Vu - Animated Trajectories

Summary

- Boeing Continues OTIS Development
- Focus on Speed & Usability
- Exploit Off-the-Shelf Software
- Goal is a "Better" OTIS
AFS

OTIS ACTIVITIES
AT
MCDONNELL DOUGLAS SPACE SYSTEMS COMPANY

R. L. NELSON
10 AUGUST 1992
OTIS ACTIVITIES

- Applications
- OTIS Project Development (PD 1-301) at MDSSC-HB
- Launch Vehicle Sizing: ELVIS/OTIS
- OTIS upgrades for Wright Labs-AFB

ADVANCED APPLICATIONS
PD 1-301 OPTIMIZATION TECHNIQUES FOR ADVANCED SPACE MISSIONS

- Theater High Altitude Area Defense (THAAD)
- ENDO/EXO Atmospheric Interceptor (E^2i)
- HEDI
- DELTA
- National Aerospace Plane (NASP)
- SSRT
- Aerobrakes
- Hypersonic Advanced Weapon (HAW)
- Fighter Aircraft
  - Evasive maneuvers
  - Agility
- Military Space
- Space Transfer Vehicles
TASK 1-APPROACH (1992-1993)-NLP THEORY
PD 1-301 OPTIMIZATION TECHNIQUES FOR
ADVANCED SPACE MISSIONS

- Develop strong robust globally convergent nonlinear optimizers

  - Dense and sparse optimizers

  - NZSOL, a dense optimizer
    - Initial feasibility algorithms
    - Min - Max optimizer

  - NZSPARSE, a sparse optimizer

- Dr. Philip E. Gill, Professor, University of California, San Diego
  Dr. Michael Saunders, Research Professor, Stanford University
TASK 1 - PROGRESS-NLP THEORY
PD 1-301 OPTIMIZATION TECHNIQUES FOR ADVANCED SPACE MISSIONS

MDSSC-HB

- Developed State of the Art optimizer, NZSOL (dense)
  - NPSOL 2.1
  - NPSOL 4.02
  - Continual testing
  - Tuned NZSOL for OTIS type problems
- BREAKTHROUGH ALGORITHM: NZSPARSE (sparse)
  - Theoretical formulation
  - Development and checkout
  - MINOS
- Modified OTIS structure to accept dense / sparse optimizers

TASK 2 - APPROACH (1992-1993)-NUMERICAL ALGORITHM:
PD 1-301 OPTIMIZATION TECHNIQUES FOR ADVANCED SPACE MISSIONS

MDSSC-HB

- Algorithms for OTIS
  - Automatic scaling
  - Automatic node placement (University of Illinois)
  - Automatic tabular data smoothing
  - Lagrange multiplier interpretation (Continuous / discrete)
  - Minimum curvature cubic control splines
TASK 2 - PROGRESS - ALGORITHMS

- AFS -

- Tabular Data Smoothing
- Enhanced Velocity Loss Model for Launch Vehicles
- Generalized Stage - Phase Concept for Sizing
- Automatic Node Placement

TASK 3 - APPROACH (1992-1993)-LOW THRUST
PD 1-301 OPTIMIZATION TECHNIQUES FOR ADVANCED SPACE MISSIONS

- MDSSC-HB -

- Develop restricted and general 3-body equations of motion
- Boundary conditions and coordinate systems
- Quantify the transition region for earth-moon low thrust / weight transfers
- SECKSPOT / NASA Code - COSMIC Library
  - Strong gravity field
  - Orbit averaging techniques
- QT2 interplanetary code
- Dr. Richard Shi, MDSSC-HB
Task 3 - Progress-Low Thrust
PD 1-301 Optimization Techniques for
Advanced Space Missions

MDSSC-HB

- Key idea to solve problem
  - The existence of the Jacobi (energy) integral for the restricted three-body problem will aid us in the general three-body problem.
  - Zero velocity or zero energy curves are the regions where the low thrust earth-moon transfers are possible
  - No integral available for the general 3-body problem

- Develop for OTIS
  - 3-body equations of motion
  - boundary conditions
  - coordinate systems

Earth-Moon Transfer
CONTOURS OF CONSTANT POTENTIAL ENERGY (REFERRED TO ROTATING SYSTEM) IN PLANE Z = 0 WITH \( \mu = 0.01213 \)

PROJECT INTERRELATIONSHIPS
PD 1-301 OPTIMIZATION TECHNIQUES FOR ADVANCED SPACE MISSIONS

<table>
<thead>
<tr>
<th>Government Agencies</th>
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<tbody>
<tr>
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<tr>
<td>Wright Labo * K230L/GRAD</td>
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<tr>
<td>NASA JSC * SD</td>
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<tr>
<td>NASA Lewis * low thrust * air breathing boosters</td>
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<tr>
<td>NABP NPO * NABP performance</td>
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<tr>
<td>NASA LaRC * vehicle analysis branch (aerobrakes) * spacecraft G&amp;C branch (OTIS / VTOS) * aircraft G&amp;C branch (Hypersonic vehicles)</td>
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<thead>
<tr>
<th>Optimizers</th>
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<tbody>
<tr>
<td>Dr. P. E. Gill * Univ. Calif. * G.D. * Dr. A. Sanders * Stanford Univ.</td>
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<table>
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<tr>
<th>Auto Nodes</th>
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<th>MDC Products</th>
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<tr>
<td>Tactical missiles</td>
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<tr>
<td>Launched vehicles</td>
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<tr>
<td>Fighter aircraft * evasive maneuvers</td>
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<tr>
<td>Branched optimization * abort * multi-vehicle</td>
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<tr>
<td>Re-entry, aerobrakes synergetic plane change</td>
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<tr>
<td>SSRT</td>
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<tr>
<td>Weapon systems * HAW</td>
</tr>
<tr>
<td>Low thrust / weight transfer</td>
</tr>
</tbody>
</table>
ELVIS / OTIS
ARCHITECTURE

Legend:
- Information flow
- Module Name
- Module function
- Platform module is executed (day) on

Exec
- Top level control
- Networking
- Data Management
- Exec Menus
- Spawn stand-alone program (APIs)
- Allow User to control APIs w/ their menus active in Exec.

LVPC
- LV parametric cost (w/o Ops Cost, June 92)
- LVPC Menus active in Exec
- Stardent (Mac)

OTIS
- Optimum constrained trajectory
- LV performance
- LV sizing
- No Menus
- Stardent

Database on Sun

Pre & Post OTIS
- Condition tabular input data for OTIS
- Write OTIS input/ output
- Menus for pre and post processor graphics active in Exec
- Stardent (PC)

LVAERO
- Tabular aero for Titan forebody shapes & aero computation for others
- Core bodies
- Strag-one
- Code to adapt aero to current design
- Automated data displays & plots
- Stardent (Vax)

GeoPlot, RealPlot
- 3D geometry display, analysis and manipulation
- Local menus
- Patran fill output
- Stardent (Vax)

LGEOEM
- Conical multi-body 3-D geometry file generation
- LGEOEM menus active in Exec
- Stardent (Vax)

3D LV Diagrams
- Laser Printer

2-D LV Diagrams from LVS 1.5

LVS 1.5
- Optimum / assumed impulsive velocity per stage
- LV sizing based on assumed velocity losses and stage mass fractions
- Propulsion system dimensions included in sizing
- 2-D diagram display and file generation
- LVS menus active in Exec
- Stardent (Mac)

data link only for LGEOEM 3-D geometry file

3D LV Diagrams
- Color Printer
OTIS UPGRADES FOR WRIGHT LABS

- TASK 1: NZSOL
- TASK 2: Automatic Variable Scaling
- TASK 3: Variable Names for NZSOL Output
- TASK 4: Minny Heating Model

McDonnell Douglas Space Systems Company

R. Nelson/2
Advances in Trajectory Optimization Using Collocation and Nonlinear Programming

Bruce A. Conway
Dept. of Aeronautical & Astronautical Engineering
University of Illinois
Urbana, IL

August 1992
Outline

Introduction
Progress to Date - Theory
Progress to Date - Solved Problems
Continuing and Proposed Research - Problems

Progress - Theory

1. Use of costates to improve an optimal trajectory.

Lagrange multipliers for the discrete (NPSOL) solution are a representation of the Lagrange multipliers of the continuous case. (Enright & Conway, JGC&D 15, No. 4, 1992)

Knowledge of the Lagrange multipliers allows a posteriori determination of the optimality of the solution, e.g., can examine the switching function.

2. Generalized defects

Can be used when the differential equation for a state variable is integrable, e.g., on a coast arc.

May significantly reduce the number of NLP parameters and hence execution time.

3. Coordinate transformation within the H-P structure

Necessary for orbit transfer when changing sphere of influence

Keeps state variables near one order of magnitude, as NPSOL prefers
4. Method of parallel shooting

Replaces single Hermite-Simpson "integration step" with multiple Runge-Kutta steps allowing use of larger intervals.

Results in smaller NLP problems for a given accuracy.

5. Automatic node placement

Computer solves a succession of NLP problems in which additional nodes are inserted as needed to achieve a given accuracy.

More efficient than using a uniform distribution of nodes

6. Neighboring optimal feedback control

Determines gains for linear feedback controller to yield neighboring optimal controller.

Unnecessary to solve NLP problem for small change in initial or terminal conditions.

Feedback gain history easily loaded into small memory

Illustration of Generalized Defects
Illustration of Parallel Shooting

Progress - Solved Problems

1. Optimal low-thrust escape trajectory (Enright Ph. D. thesis)
2. Optimal 2 and 3 burn circle-circle low-thrust rendezvous (Enright Ph. D. thesis)
4. Optimal spacecraft detumbling (A. Herman M.S. thesis)
6. Optimal 2D and 3D direct ascent time-bounded interception (J. Downey Ph. D. thesis)
7. Neighboring optimal feedback control for continuous-thrust ascent maximizing horizontal velocity (F. Chen Ph. D. research)
Low-thrust Minimum Fuel Escape

\[ \mu = 1 \]
\[ r_i = 1 \]
\[ a_t = 0.0125 \]
\[ t_{\text{final}} = 16\pi \]
All in canonical units

<table>
<thead>
<tr>
<th>Method</th>
<th>Variables</th>
<th>CPU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hermite/Simpson (60)</td>
<td>427</td>
<td>190 sec</td>
</tr>
<tr>
<td>Parallel shooting (34 x 3)</td>
<td>385</td>
<td>95 sec</td>
</tr>
<tr>
<td>Parallel shooting (5 x 20)</td>
<td>270</td>
<td>72 sec</td>
</tr>
</tbody>
</table>

Optimal 2 and 3 Burn Circle-Circle Rendezvous

Fig. 3 Two-burn rendezvous trajectory.

Fig. 4 Two-burn rendezvous distance vs time for burn 1.

Fig. 5 Three-burn rendezvous trajectory.

Fig. 6 Three-burn rendezvous distance vs time.

COPY AVAILABLE TO DTIC DOES NOT PERMIT FULLY LEGIBLE REPRODUCTION
Optimal Low-Thrust Earth-Moon Transfer

Optimal Low-Thrust Earth-Moon Transfer, cont'd

![Diagram](image)

Fig. 1 Earth-moon transfer trajectory.

Optimal Low-Thrust Earth-Moon Transfer, cont'd

![Graph](image)

thrust angle (degrees) vs time (days)
Optimal Spacecraft Passivation (Detumbling)

View of OMV / Disabled Satellite System

Spacecraft Passivation, cont'd.

Results from the TPBVP solver

Results from the NLP method

External Torque Histories
Optimal 2D & 3D Direct -Ascent Interception

- Target is assumed to be in a general Keplerian orbit with orbital elements

\[ \mathbf{E^T} = [a, e, i, \Omega, \omega, f] \]

- Geometry of the problem

Continuing Research - Problems

1. Automatic node placement. (A. Herman)
2. Optimal very-low-thrust trajectories (W. Scheel)
3. Optimal Earth-Mars low-thrust transfer including escape and arrival spirals and coordinate transformations at sphere of influence of each planet. (S. Tang)
4. Neighboring optimal feedback control for complex problems
   Automation of NOFC using symbolic programming (F. Chen)
5. Optimal trajectories for interception of Earth-crossing asteroids (B. Conway)
FLIGHT PATH OPTIMIZATION OF AEROSPACE VEHICLES USING OTIS

Rajiv S. Chowdhry

Lockheed Engineering & Sciences Company
MS 489
Aircraft Guidance & Control Branch
NASA, LaRC, Hampton VA.
Outline

- Accuracy of OTIS solutions.
- Overview of OTIS applications at AGCB

Accuracy of OTIS Solutions

OTIS: Optimal control solutions via direct transcription
Combination of collocation and nonlinear programming

Question:
How do OTIS solutions compare to the "exact" or TPBVP solutions?

- Analytical Approach
  estimate adjoint variables, examine discretized necessary conditions

- Engineering Approach
  numerical comparison of collocation solution to exact solution for a representative problem
FLIGHT PATH OPTIMIZATION OF
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- Analytical Approach
  estimate adjoint variables, examine discretized necessary conditions
- Engineering Approach
  numerical comparison of collocation solution to exact solution for a representative problem
Example : ALS Ascent to Orbit

Example Problem :

Steer a two stage launch vehicle from a given initial condition to a specified target orbit in minimum fuel.

- Exact or TPBVP solution available in literature (Ref. Hans Seywald and E. M. Cliff)
- Care was taken to keep the vehicle/atmosphere/planet models same in OTIS
- Only solution methodologies were different

Comparison of Optimal ALS Ascent with OTIS solutions

<table>
<thead>
<tr>
<th></th>
<th>OTIS</th>
<th>Solutions</th>
<th>TBPVP solution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12 nodes</td>
<td>22 nodes</td>
<td>32 nodes</td>
</tr>
<tr>
<td>$t_f$ sec</td>
<td>477.2</td>
<td>477.2</td>
<td>477.2</td>
</tr>
<tr>
<td>Mass ($m_f$) Kgs</td>
<td>149.881</td>
<td>149.877</td>
<td>149.895</td>
</tr>
<tr>
<td>Velocity ($v_f$) m/sec</td>
<td>7855</td>
<td>7855</td>
<td>7857</td>
</tr>
<tr>
<td>Altitude ($h_f$) km</td>
<td>148.68</td>
<td>148.74</td>
<td>147.80</td>
</tr>
<tr>
<td>Apogee (km)</td>
<td>274.50</td>
<td>274.11</td>
<td>279.52</td>
</tr>
<tr>
<td>Perigee (km)</td>
<td>148.53</td>
<td>148.68</td>
<td>147.80</td>
</tr>
<tr>
<td>CPU Time (secs)</td>
<td>74</td>
<td>377</td>
<td>935</td>
</tr>
</tbody>
</table>
Figure [1]. Comparison of optimal ALS ascent with OTIS solution, mass (kg) vs. time.

Figure [2]. Comparison of optimal ALS ascent with OTIS solution, altitude above spherical Earth vs. time.
Figure [3]. Comparison of optimal ALS ascent with OTIS solution, Earth relative velocity (m/sec) vs. time.

Figure [4]. Comparison of optimal ALS ascent with OTIS solution, local horizontal flight path angle (deg) vs. time.
OTIS Applications at AGCB

- Fuel efficient ascent for SSTO airbreathing hypersonic vehicle.
  - fuel optimal path definition for G&C studies

- HL-20 abort maneuvers : ELSA (Efficient Launch Site Abort)
  - Parameter sensitivity studies to support design activities.
  - Guidance algorithm development & real time validation.

- ALS ascent for OTIS calibration.

- Optimal maneuvers for a high performance fighter aircraft (HARV) in air combat situation.
Conclusions

For the ALS Ascent Problem:

- Excellent match of the collocation solution to the TPBVP solution
- Relatively quick turnaround time for OTIS solutions
- Very robust to initial guesses
Trajectory Optimization of Launch Vehicles at LeRC: Present and Future

Presented by
Koorosh Mirfakhraie

at
Workshop on Trajectory Optimization Methods and Applications
Hilton Head, SC
August 10, 1992
Outline

- Introduction
- Present method of solution and code
- Capabilities of the present code
- Motivation for replacing the code (and method)
- Examination of methods using collocation

Introduction

Trajectory optimization* of ELV's at the Advanced Space Analysis Office at LeRC is performed for:

- Mission design for approved programs
- Feasibility and planning studies
- Corroboration of contractors' data for NASA missions flown on Atlas and Titan

* Trajectory optimization: Maximizing the final payload subject to a set of intermediate and final constraints.
Introduction (Cont’d)

Mission profiles for launch vehicle systems with booster and upper stages include:

- Launches from ER and WR
- LEO, GTO, and GSO insertion
- Interplanetary escape trajectories
- Orbit transfers

Present Method of Solution and Code

- Calculus of Variations is used to formulate the problem. The resulting two point boundary value problem is solved using a Newton-Raphson algorithm.
- The computer program (DUKSUP) was written entirely at LeRC during 1960’s and early 70’s.
- DUKSUP is a 3-D.O.F. code written for performance analysis of multi-stage high-thrust launch vehicles.
DUKSUP Features

- Detailed modeling (e.g., propulsion and aerodynamic) of a launch vehicle is possible.

- A variety of constraints can be imposed on the model. They include:
  - Instantaneous and total aerodynamic heating
  - Maximum dynamic pressure
  - Parking orbit parameters (e.g., radius of perigee, energy, velocity, etc.)
  - G-limit staging

DUKSUP Features (Cont’d)

- Several in-plane and out-of-plane final target conditions can be specified (e.g., energy, radius, true anomaly, inclination, declination of outgoing asymptote, etc.).

- Variables free for optimization include:
  - Upper stage burn and coast times
  - ‘Kick angle’
  - Payload fairing jettison time
  - Thrust angle in the non-atmospheric flight
Motivation for Replacing DUKSUP

- Sensitivity to initial guesses
- Difficulty in reformulating the C.O.V. problem when adding new features and constraints to the code
- Difficulty in modifying and expanding the code due to lack of documentation and outdated programming practices

Examination of Methods Using Collocation

- Two main features of collocation making it attractive are
  - Lack of sensitivity to initial guesses
  - Relative ease of formulation
- Concerns about using collocation for ELV optimization are
  - Ability to handle complex modeling requirements and constraints typical of ELV flight
  - Computer run time
  - Fidelity of the solution vis a vis C.O.V.
- Evaluation of collocation uses DUKSUP as the benchmark for comparison.
Using Collocation (Cont'd)

- Available collocation codes are used as testbeds with necessary modifications.
- A simple LV model is used first and moved progressively to a full DUKSUP model.
- Enright's orbit transfer program was used for the first simple model comparison. Results matched those of DUKSUP.
- OTIS is used for the more sophisticated comparisons.
- OTIS is currently used to model an Atlas II/Centaur to LEO.
Collocation Methods in Regular Perturbation Analysis of Optimal Control Problems*

August 10, 1992

Prepared for

Workshop on Trajectory Optimization Methods and Applications
AIAA Guidance, Navigation, and Control Conference
Hilton Head, SC

Anthony J. Calise** & Martin S.K. Leung
Georgia Institute of Technology
School of Aerospace Engineering
Atlanta, GA 30332

* See conference paper no. 92-4304. Research supported by NASA Langley under grant No. NAG-1-939
**Phone: (404) 894-7145, Fax: (404) 894-2760, E-Mail: AE231TC@GIVE11.GATECH.EDU
Overview

Motivation

Regular Perturbation Analysis

The Method of Collocation

Hybrid Collocation / Regular Perturbation Analysis Approach

Examples

Duffing Equation

Launch Vehicle Guidance Application (presented at 1-GNC-1)
Advantages / Disadvantages

Analytical Methods

Approximates solution by expansion in an asymptotic series in a small parameter

Zero order problem is simpler to solve ⇒ Insight
Higher order problems are linear

Zero order problem must reasonably approximate the full order problem

For practical applications, zero order problem must be analytically tractable or reducible to a simple algebraic problem

Significant amount of analysis is required for each problem formulation of interest

Advantages / Disadvantages (continued)

Numerical (Collocation) Methods

Finite element method that enforces interpolatory constraints at specific points within each element

Simple to use for a wide variety of optimization problems

Large dimensional nonlinear programming problem

No general guarantee of convergence

Note: Advantages of analytical and numerical methods are in many respects complimentary in the sense that if the advantages can be combined in some way, then most of the important disadvantages for real-time applications can be removed.
Regular Perturbations in Optimal Control

Given:
\[
dx/dt = f(x,u,t) + \varepsilon g(x,u,t); \quad x(t_0) = x_0
\]
\[J = \Phi(x, t) |_{t_f}\]

Find the control that minimizes \(J\) subject to the terminal time constraints:
\[\psi(x, t) |_{t_f} = 0\]

Optimality condition:
\[H_u = 0 \quad \text{assuming } H_{uu} > 0 \quad \Rightarrow \quad u = U(x, \lambda, t)\]

where:
\[H = \lambda^T (f + \varepsilon g) + g; \quad H(t_f) = - (D_t |_{t_f}); \quad \Phi = \phi + v^T \psi\]
\[d\lambda/dt = - H_x; \quad \lambda(t_f) = \Phi_x |_{t_f}\]

Regular Perturbation Analysis

Based on a simplified model (when \(\varepsilon\) is set to zero)
- Treat neglected dynamics as perturbation
- Define a normalized independent variable, \(\tau = (t - t_0) / T\)
  where \(T = t_f - t_0\)
- Compute zero order solution

Consider an asymptotic series in \(x, \lambda,\) and \(T\)

Evaluate high order corrections from sets of nonhomogeneous, time-varying linear O. D. E's.
\[
\frac{d}{dt} \begin{bmatrix} x_k \\ \lambda_k \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_k \\ \lambda_k \end{bmatrix} + \begin{bmatrix} T_k \cdot C_1 \\ T_0 \cdot C_2 \end{bmatrix} + \begin{bmatrix} P_{1k} \\ P_{2k} \end{bmatrix}
\]

enforcing all boundary conditions to \(k^{th}\) order

Compute feedback control at current time \((t_0)\) using \(x(t_0)\) and \(k^{th}\) order approximation for \(\lambda(t_0)\)
Regular Perturbation Analysis (continued)

- A's and C's depend only on the zero order \((k = 0)\) values.
- C's are the explicit correction term for free final time, \(T\).
- P's are the forcing functions involving lower order \((k-1, \ldots, 1, 0)\) terms.

Solution:

\[
\begin{bmatrix}
x_k(i) \\
\lambda_k(i)
\end{bmatrix} = \Omega_A(i, t_o) \begin{bmatrix} x_k(t_o) \\
\lambda_k(t_o)
\end{bmatrix} + T_k \begin{bmatrix} \dot{x}_0(i) \\
\dot{\lambda}_0(i)
\end{bmatrix} + \int_{t_o}^{i} \Omega_A(i, \tau) \begin{bmatrix} P_{1k}(\tau) \\
P_{2k}(\tau)
\end{bmatrix} d\tau
\]

Higher order correction involves simple operations of quadrature and solution of linear algebraic equations

Can be easily modified to account for discontinuous dynamics

Solution of Optimal Control Problems by Collocation

Methodology

- a finite element approach
- approximates the solution with interpolating functions
- consider first order polynomials

\[x(i) = x_j + p_j(i - i_{j-1}); \lambda(i) = \lambda_{j-1} + q_j(i - i_{j-1}); j = 1, 2, \ldots, N\]

- enforce the derivative constraints at the mid point of each element

\[p_j = \frac{x_j - x_{j-1}}{i_j - i_{j-1}} = \frac{\partial H}{\partial x}|_{i=(i_j + i_{j-1})/2}; x=(x_j + x_{j-1})/2; \lambda=(\lambda_j + \lambda_{j-1})/2\]

\[q_j = \frac{\lambda_j - \lambda_{j-1}}{i_j - i_{j-1}} = -\frac{\partial H}{\partial x}|_{i=(i_j + i_{j-1})/2}; x=(x_j + x_{j-1})/2; \lambda=(\lambda_j + \lambda_{j-1})/2\]

- \(N\) is the number of elements, \(x_j\) and \(\lambda_j\) are nodal values
- control assumed to be eliminated using optimality condition
Hybrid Collocation / Regular Perturbation

A Regular Perturbation Formulation

- rewrite the actual dynamics as
  \[ \dot{x} = p_j + \varepsilon \left( \frac{\partial H}{\partial x} - p_j \right) \quad ; \quad \dot{\lambda} = q_j + \varepsilon \left( \frac{\partial H}{\partial x} - q_j \right) \]

  - perturbation terms are zero at mid point of each element.
  - for cases that control cannot be eliminated explicitly, use an
    analytic portion \( \Pi(x, \lambda, u) \)
    \[ 0 = \Pi + \varepsilon \left( \frac{\partial H}{\partial u} - \Pi \right) \]

Carry out a Regular Perturbation Analysis

- expand about the zero order solution (derived from collocation)
- provides higher order corrections to collocation solution
- further exploitation of the analytically tractable portion of the dynamics
  will result in more intelligent interpolating functions (see simple example)

A Simple Example

Duffing's equation in first order form:

\[ \begin{align*}
\dot{x} &= v \\
\dot{v} &= -x - ax^3 + u
\end{align*} \quad ; \quad x(0) = x_0 \quad ; \quad v(0) = v_0 \\
J &= S_x x^2(t_f) + S_v v^2(t_f) + \int_0^{t_f} (1 + u^2/2) \, dt
\]

Notes:

- hardening effect is given by the nonlinear term, \( ax^3 \)
- the optimal control problem is a fourth order example
- will demonstrate different levels of intelligent interpolating functions
  that enhance the approximation with fewer number of elements
Simple Example (continued)

Level 0 Formulation:
- degenerate case, uses only regular perturbation with a completely analytic zero order solution
- let $\varepsilon = a = 0.4$, and treat the nonlinear terms as perturbations
- $S_x = S_y = 100$
  
  \[ \dot{x} = v \qquad \text{; } x(0) = x_0 \]
  \[ \dot{v} = -x + u - \varepsilon x^3 \qquad \text{; } v(0) = v_0 \]
  \[ \dot{\lambda}_x = \lambda_y + \varepsilon 3 \lambda_y x^2 \qquad \text{; } \lambda_x(t_f) = 2S_x x(t_f) \]
  \[ \dot{\lambda}_y = -\lambda_x \qquad \text{; } \lambda_y(t_f) = 2S_y v(t_f) \]
  \[ H_u = u + \lambda_y = 0 \quad \text{; } \{ H = \lambda_y v + \lambda_y (-x + u - \varepsilon x^3) + 1 + u^2 / 2 \} | t_f = 0 \]

- zero order problem is linear and time-invariant
- compute up to second order corrected solutions (Figs. 4.1 and 4.2)
- series not convergent, most accurate approximation is first order
- nonlinear term $ax^3$ is too large to be neglected in the zero order problem

![Figure 4.1. Level 0 Result in $x$.](image1)

![Figure 4.3. Level 0 Result in $\lambda_x$.](image3)

![Figure 4.2. Level 0 Result in $v$.](image2)

![Figure 4.4. Level 0 Result in $\lambda_y$.](image4)
Simple Example (continued)

**Level 1 Formulation:**
- Use hybrid approach, approximate all state and costates as piecewise linear functions

\[
\begin{align*}
\dot{x}_0(i) &= x_{0j-1} + p_{xj}(\hat{i} - \hat{i}_{j-1}) \\
n_0(i) &= n_{0j-1} + p_{nj}(\hat{i} - \hat{i}_{j-1}) \\
\lambda_{x0}(i) &= \lambda_{x0j-1} + q_{xj}(\hat{i} - \hat{i}_{j-1}) \\
\lambda_{n0}(i) &= \lambda_{n0j-1} + q_{nj}(\hat{i} - \hat{i}_{j-1})
\end{align*}
\]

- Number of unknowns is \(4N + 5\)
- 1st and 2nd order corrections are computed for \(N = 3\) (Fig's. 4.9 - 4.12)
- Discontinuity in slope is smoothed as order of correction increases
- Correction by regular perturbation analysis allows use of crude number of element representation in the zero order collocation solution
Simple Example (continued)

**Level 2 Formulation:**
- enhanced level 1 formulation by interpolating only those variables that have nonlinear coupling
- decompose the dynamics as:

\[
\frac{dx}{dt} = v
\]

\[
\frac{dv}{dt} = p_{vj} + \varepsilon(-x - \lambda_v - ax^3 - p_{vj})
\]

\[
\frac{d\lambda_x}{dt} = q_{xj} + \varepsilon(\lambda_v(1 + 3ax^2) - q_{xj})
\]

\[
\frac{d\lambda_v}{dt} = -\lambda_x
\]
- number of unknowns is \(2N + 5\)
- both zero and first order results for \(N = 2\) are superior than the \(N = 3\) results for the Level 1 formulation (Fig's. 4.13 - 4.16)
Simple Example (continued)

**Level 3 Formulation:**
- enhanced level 2 formulation by fully utilizing analytically tractable portion of the necessary conditions
- decompose the dynamics as:

\[
\begin{align*}
\dot{x} &= v \\
\dot{v} &= -x - \lambda_v + p_{ij} + \varepsilon(-ax^3 - p_{ij}) \quad ; \ j = 1, 2, \ldots, N \\
\dot{\lambda}_x &= \lambda_v + q_{xj} + \varepsilon(3a\lambda_v x^2 - q_{xj}) \\
\dot{\lambda}_v &= -\lambda_x
\end{align*}
\]

- similar to Level 0 except for additional unknown constants \( p_{ij}, q_{xj} \)
- use piecewise constant terms to approximate the nonlinear parts
- both zero and first order results for \( N = 1 \) are superior than the Level 0 case (Fig's. 4.17 - 4.20)
- Level 2 and 3 cases demonstrate the use of intelligent interpolating functions
Alternative Implementations

Repeat zero order solution and perform quadratures at each control update interval

Or

Compute zero order solution and quadratures off line, and store for in-flight use

Implements reliability and computational efficiency with some loss in accuracy
Summary

Benefits of Hybrid Approach:

Significantly improves a collocation solution

First and higher order corrections are obtained by quadrature

Intelligent interpolation functions obtained by retaining as much of the analytically tractable portion of the solution as possible

Possible to implement the control solution so that the zero order solution and quadratures are performed once off-line and stored

Significantly improve a regular perturbation solution

Retain more of the nonlinearities in the zero order problem by using finite elements and collocation to construct an improved zero order solution

Important implications in real-time guidance applications

Computational efficiency and reliability
AUTOMATIC SOLUTIONS FOR TAKE-OFF FROM AIRCRAFT CARRIERS

Lloyd H Johnson
AIR-53012D
CATAPULT IN BATTERY POSITION

AIRCRAFT ATTACHED TO SHUTTLE AND HOLDBACK UNIT
GRAB EXERTS FORWARD PRESSURE ON SHUTTLE FOR BRODE TENSIONING.

CATAPULT FIRES
1. HOLDBACK UNIT RELEASES
2. GRAB RELEASES SHUTTLE
3. SHUTTLE TOWS AIRCRAFT FORWARD

SHUTTLE HALTED BY BRAKE MECHANISM

GRAB ADVANCES AND LATCHES TO SHUTTLE

GRAB RETRACTS SHUTTLE TO BATTERY POSITION
4.02 BRIDLE/PENDANT LAUNCH METHOD.

This method of launching aircraft is no longer a design option since the nose gear launch method became standard (see paragraph 4.0.1). With this method, the aircraft is coupled to the catapult tow fitting by means of a wire, rope, bridle or pendant. The bridle is "V" shaped and requires two new fittings on the aircraft, whereas the pendant needs only one new fitting on the aircraft.

The holdback device used with bridle/pendant launch is a wire rope, chain, or metal link. The release element is either a ring or a tension bar. The holdback assembly attaches one point on the aircraft to the well and the other to the holdback dock cleat. Section VIII describes typical holdback and release assembly.

This method of launching requires manual backup of the bridle or pendant and the holdback by the catapult deck crew after the aircraft has been moved into position on the catapult. When the aircraft reaches the end of the catapult power run and the tow force decays, the bridle or pendant drops from the aircraft tow fittings and is brought to a stop on the flight deck by the bridle arrestor system.

4.1 LAUNCHING EQUIPMENT.

4.0 GENERAL.

4.0.0 CATAPULT. A direct-drive, steam-type catapult is used on all carriers. Steam, piped from the ship's boilers to a series of large main receivers, is released suddenly to the launching engine to drive two pistons. The pistons are directly connected to the catapult tow fitting through the closed cylinder walls. Reversion is accomplished by a separate hydropneumatic reverser.

4.0.1 DECK EQUIPMENT. The catapult is equipped with a shuttle which moves along the catapult track during the launching operation and transmits the catapulting force from the catapult engine to the aircraft through the launch bar, bridle or pendant. A ramp, secured to the catapult tow fitting, enables the aircraft to roll over the tow fitting. For bridle-launched aircraft, the holdback bar is provided and connects the aircraft to the catapult track. The bridle release arm and release the bridle or pendant after it is shed from the aircraft. Finally, the deck edge control panel provides the primary control power for operating the catapult.

Figure 6-1. Nose Gear Launch Configuration
THE CATAPULT LAUNCH SIMULATION CONSISTS OF FIVE PHASES

- STATIC BALANCE
- HOLDBACK
- CATAPULT STROKE
- DECK RUN
- FLYAWAY

THE CATAPULT LAUNCH SIMULATION INCLUDES:

- CATAPULT FORCES
- HOLDBACK FORCES
- HIGH FIDELITY LANDING GEAR MODEL
- AERODYNAMIC DATA AS A FUNCTION OF
  - ANGLE OF ATTACK OR LIFT COEFFICIENT
  - NOZZLE DEFOCTION
  - THRUST COEFFICIENT OR NOZZLE PRESSURE RATIO (NPR)
  - FLAP DEFOCTION
  - PITCH TRIM SURFACE DEFOCTION
- GENERIC FLIGHT CONTROL AND STABILITY AUGMENTATION SYSTEM
- LONGITUDINAL THRUST VECTORING
PROGRAM OPTIONS

- AUTOMATIC WIND OVER DECK SOLUTION
- SOLUTION TERMINATION
- POWERSSETTING
- FLAP DEFLECTION
- FLIGHT CONTROL AND STABILITY AUGMENTATION SYSTEM
- THRUST VECTORING CONTROL SYSTEM
- LANDING GEAR
- ENGINE FAILURE
- STORE JETTISON
- LANDING GEAR RETRACTION

AUTOMATIC WIND OVER DECK SOLUTION

TWO CONSTRAINTS:
- MAXIMUM SINK
- MAXIMUM ANGLE OF ATTACK
  OR
  MAXIMUM PITCH RATE
  OR
  MINIMUM LONGITUDINAL ACCELERATION

TWO VALUES DETERMINED:
- WIND OVER DECK
- STICK DISPLACEMENT (OR TAIL DEFLECTION)
SOLUTION TERMINATION

- POSITIVE Tmax
  - TIME HISTORIES STOP AT THE SPECIFIED Tmax

- NEGATIVE Tmax
  - TIME HISTORIES STOP WHEN A POSITIVE RATE OF CLIMB HAS BEEN ACHIEVED AND ANGLE OF ATTACK HAS PEAKED.
  - IF A POSITIVE RATE OF CLIMB IS NOT ACHIEVED OR ANGLE OF ATTACK IS CONTINUOUSLY INCREASING, THE TIME HISTORY WILL STOP AT THE ABSOLUTE VALUE OF Tmax.

AERODYNAMIC DATA INCLUDES PROPULSION–INDUCED EFFECTS

COEFFICIENTS ARE FUNCTIONS OF:

- ANGLE OF ATTACK OR LIFT COEFFICIENT
- NOZZLE DEFLECTION
- THRUST COEFFICIENT OR NPR
- FLAP DEFLECTION
- TRIM SURFACE DEFLECTION
TYPICAL AERODYNAMIC DATA
FOR A THRUST VECTORING CONFIGURATION

AERODYNAMIC COEFFICIENTS HAVE THE DIRECT PROPULSION EFFECTS REMOVED AND ARE FUNCTIONS OF:

- ANGLE OF ATTACK OR LIFT COEFFICIENT
- THRUST COEFFICIENT OR NPR
- NOZZLE DEFLECTION
- FLAP DEFLECTION
- TRIM SURFACE DEFLECTION

FORCES ACTING ON THE AIRCRAFT
INERTIAL AXES
BODY CENTERLINE AXES
WIND AXES
NOSE GEAR AXES
MAIN GEAR AXES
AIRCRAFT REF. SYSTEM

\[ \vec{x}, \vec{z} \]
\[ \vec{x}, \vec{z} \]
\[ \vec{x}_w, \vec{z}_w \]
\[ \vec{x}_1, \vec{z}_1 \]
\[ \vec{x}_2, \vec{z}_2 \]
\[ \vec{x}_{FB}, \vec{z}_{FB} \]

\[ \text{FIGURE 2-1: BODY AXES, GEAR AXES AND WIND AXES ORIENTATION WITH RESPECT TO THE INERTIAL FRAME OF REFERENCE} \]

\[ \text{FIGURE 9a: CONTROL SURFACE DEPLOYMENT AS A FUNCTION OF TIME} \]
FIGURE 9b - PITCH RATE AS A FUNCTION OF TIME

FIGURE 9c - ANGLE OF ATTACK AS A FUNCTION OF TIME
FIGURE 3d - TRUE AIRSPEED AS A FUNCTION OF TIME
AIRBREATHING BOOSTER PERFORMANCE OPTIMIZATION USING MICROCOMPUTERS

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AIAA Astrodynamics Conference
Workshop on Trajectory Optimization

10 August, 1992
OVERVIEW

• MICROCOMPUTER-BASED OPTIMIZING SIMULATION OF TRAJECTORIES (MOST)\(^1\)
  - MOTIVATION - Fill void in preliminary design tools
  - Easy to use fast running modes
  - TPBV solution for truth model
• OTIS PROGRAM OPERATION ON PC
• LOW-THRUST TRAJECTORY OPTIMIZATION PROGRAM (MICROTOP)

\(^1\) Work performed under Air Force Contract F33615-91-C-2100.

MOST TECHNICAL OBJECTIVES BEING MET

• SUITABLE FOR RAPID PRELIMINARY DESIGN
• MICROCOMPUTER OPERATION - Run time less than 5 mins. on PC AT
• AIRBREATHING & ROCKET PROPULSION VIA TABLES AND EQUATIONS - Including realistic flight constraints
• EARTH-TO-ORBIT (ETO) FLIGHT
• PLANAR FLIGHT - Simple rotating earth model facilitates 3-D type results with minimal complexity
• EASE-OF-USE - Easy input, good graphics, & robust convergence
HYBRID APPROACH OFFERS SPEED AND PRECISION

MICROCOMPUTER-BASED OPTIMIZING SIMULATION OF TRAJECTORIES (MOST)
A HYBRID APPROACH

PHASE 1
RAPID OPTIMIZATION MODEL (ROM)
FEATURES:
- 2-DIMENSIONAL FLIGHT
- ACCELERATING AND ROCKET PROFILES
- FLIGHT CONSTRAINTS
- MULTIPLE STAGES
- PRINTED AND GRAPHIC OUTPUT
- MULTIPLE STAGES
- FAST RUNNING APPROPRIATE METHODS
APPLICATIONS:
- CONCEPT FEASIBILITY ASSESSMENT
- PRELIMINARY DESIGN TRADEOFFS
- PERFORMANCE ESTIMATION

PHASE 2
PRECISION OPTIMIZATION MODEL (POM)
FEATURES:
- SAME AS ROM PLUS
- IMPLICIT INTEGRATION OF STATE EQUATIONS
- FLIGHT AND VEHICLE DESIGN PARAMETER OPTIMIZATION
- IF REQUIRED, 3-DIMENSIONAL FLIGHT SIMULATION
- DYNAMIC OPTIMIZATION
APPLICATIONS:
- ACCURATE TRAJECTORY TIMELINE GENERATION AND PERFORMANCE ANALYSIS
- SUPPORT DETAIL DESIGN ANALYSIS

SINGLE-STAGE-TO-ORBIT OPTIMIZATION GOALS ACHIEVED ON PC

MOST versus OTIS H-V Flight Profile Comparison
Test Case No. 2, Single-Stage-to-Orbit

Altitude, h (feet)

Relative Velocity, Vr (fps)
2 STAGES TO ORBIT
OPTIMIZATION GOALS ACHIEVED ON PC

MOST vs OTIS Test Case 1 H-V Profile Comparison
Liftoff-to-Rocket Burnout

GRAPHICS ENHANCE UNDERSTANDING
OF PERFORMANCE OPTIMIZATION

CONTOURS OF CONSTANT MANEUVERING EFFICIENCY
CONCLUSIONS -
ETO PERFORMANCE OPTIMIZATION
ACHIEVED ON PERSONAL COMPUTER

- **MOST** - LOW COST, RAPID RESPONSE TOOL FOR PRELIM. DESIGN SUCCESSFULLY ACHIEVED

- ADVANTAGES OF PC DEMONSTRATED - Low Cost, portability, and good graphics and support software (LOTUS, Harvard Graphics, Freelance, etc.)

- USER-FRIENDLY ENVIRONMENT FOR PREPARATION OF INPUT FILES & OUTPUT DATA - Facilitates OTIS input file preparation

- **MOST** FAST RUNNING MODES DEMONSTRATED - Good agreement with OTIS results. Early engineering model delivered

- PRELIMINARY RESULTS FROM 2-D NLP/COLLOCATION ALGORITHMS (MINI-OTIS) ARE ENCOURAGING

- FAST RUNNING MODES FACILITATE NEW APPLICATION - Trajectory optimizer simple and fast enough to imbed in vehicle design optimization code

- OTIS HOSTED ON PC - ETO flight achievable with large RAM (≈ 40KBYTES)
An Algorithm for Trajectory Optimization on a Distributed-Memory Parallel Processor

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Acknowledgement:

Work supported by NASA/LaRC
Continuous-Time Problem to be Solved

find: \( u(t) \) and \( x(t) \) for \( t_0 \leq t \leq t_f \)

to minimize: \( J = \int_{t_0}^{t_f} L[x(t),u(t),t] \, dt + V[x(t_f)] \)

subject to: \( x(t_0) \) given
\[
\dot{x} = f[x(t),u(t),t] \\
a_e[x(t),u(t),t] = 0 \\
a_i[x(t),u(t),t] \leq 0 \\
a_{ef}[x(t_f)] = 0 \\
a_{if}[x(t_f)] \leq 0
\]

Approach

- Use zero-order-hold control parameterization
- Model as a multi-stage parameter optimization problem
- Retain state variables and dynamic constraints explicitly
- Solve using a nonlinear programming algorithm that ...
  ... has fast local and robust global convergence
  ... allows infeasible intermediate results
  ... parallelizes function, gradient, etc. evaluations at different time steps
  ... exploits dynamic structure and parallelism to get search directions

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Multi-Stage Nonlinear Programming Problem

find: \( \mathbf{x} = \left[ u_0^T, x_1^T, u_1^T, x_2^T, \ldots, u_{N-1}^T, x_N^T \right]^T \)

to minimize: \( J = \sum_{k=0}^{N-1} L_k(x_k, u_k) + V[x_N] \)

subject to:
- \( x_0 \) given
- \( x_{k+1} = f_k(x_k, u_k) \) for \( k = 0 \ldots N-1 \)
- \( a_{ek}(x_k, u_k) = 0 \) for \( k = 0 \ldots N-1 \)
- \( a_{ik}(x_k, u_k) \leq 0 \) for \( k = 0 \ldots N-1 \)
- \( a_{eN}(x_N) = 0 \)
- \( a_{iN}(x_N) \leq 0 \)

A Static/Dense Nonlinear Programming Problem

find: \( \mathbf{x} \)

to minimize: \( J(\mathbf{x}) \)

subject to:
- \( c_{e}(\mathbf{x}) = 0 \)
- \( c_{i}(\mathbf{x}) \leq 0 \)

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Status of Project

- Parallel search direction algorithm:
  FORTRAN version tested on 32-node INTEL iPSC/2

- General static NP algorithm:
  FORTRAN version tested on 1 node of INTEL iPSC/2
  Compared to NPSOL version 4.02 on static problems

- Full parallel trajectory optimization algorithm:
  A "next generation" of the NP algorithm that exploits parallelism and dynamic problem structure
  FORTRAN components currently being tested on 32-node INTEL iPSC/860
Plans
(the LORD willing)

- Finish component and full algorithm testing (Present-Oct. '92?)
- Model and solve guidance problems for NASP and generic hypersonic vehicles (Oct. '92 - Dec. '93)
- Compare to existing codes (199?)
- Evaluate suitability for real-time guidance updates (199?)
- Make code user-friendly and disseminate (199?)

Distribution of Problem Stages on Parallel Processors

24 Stage Problem on 8 Processors
Divide-and-Conquer Trajectory Optimization

Iteration 1: x is fixed at these times

Iteration 2: x is fixed at these times

Iteration 3: x is fixed at these times

Iteration 4: x is fixed at this time

Test Problem 2 FOR STATIC NP ALGORITHM

find: $x_1, x_2$

to minimize: $J = -x_2$

subject to: $(x_1 - 1)^2 + x_2^2 + 10000 (x_1^2 + x_2^2 - 1)^2 - .0625 \leq 0$

Test Results

- Augmented Lag.: 77 iterations
  (52 feasibility and 25 optimality)

- NPSOL 4.02: failed

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Test Problem 5 for Static MP Algorithm

find: \( m_0, \Delta V_1, \Delta v, \Delta V_2 \)

to minimize: \( J = m_0 \)

subject to: Newton's laws for a spherical Earth

Fixed fuel specific impulse

\[
 r_{\text{LEO}} - e_r \leq r_t \leq r_{\text{LEO}} + e_r \\
 V_{\text{circ}} - e_v \leq V_t \leq V_{\text{circ}} + e_v \\
 -e_v \leq \gamma_t \leq e_v \\
 28^\circ - e_i \leq i_t \leq 28^\circ + e_i \\
 m_{\text{empty}} \leq m_f
\]

Test Results

- Augmented Lag.: 14/22 iterations
  \( \bar{\lambda}_{\text{run}} \approx 1000 \)
  \( \bar{\lambda}_{\text{final}} \approx 100 \)
  3/3 feas. 11/19 opt. 
- NPSOL 4.02: 9 iterations

Aero-Assisted Orbital Maneuvering Example
(taken from Miele, 1989 ACC)

Problem: Minimize Fuel for GEO to LEO transfer with +28° inclination change

\[
x = [V, \gamma, \psi, r, \phi, \theta]^T
\]

\[
u = [V_c, \gamma_c, \psi_c]^T \text{ or } [C_L, \sigma, \tau]^T
\]

Constraint: Heating rate \( \leq 100 \text{ watts/cm}^2 \)

LOR-like problem derivation: Linear-quadraticize about guessed solution