OPTIMAL CONTROL OF THE STARFIRE BEAM DIRECTOR

THESIS

TROY V. LANIER

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OPTIMAL CONTROL OF THE STARFIRE BEAM DIRECTOR

THESIS

Presented to the Faculty of the School of Engineering
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Mostly, I thank my wife, Francesca, for enduring these eighteen months, while giving me the support I needed and raising our one year old boy. With the completion of this thesis I now look forward to a normal job and devoting more time to my family.

Troy V. Lanier
# Table of Contents

Acknowledgments ........................................................................................................... ii  
List of Figures .................................................................................................................... vii  
List of Tables ...................................................................................................................... x  
List of Symbols .................................................................................................................. xi  
List of Acronyms ............................................................................................................... xiv  
Abstract ............................................................................................................................. xv  

I. Introduction.................................................................................................................. 1-1  
  1.1 Background ............................................................................................................. 1-1  
  1.2 Problem Statement ............................................................................................... 1-3  
  1.3 Objectives ............................................................................................................... 1-3  
  1.4 Scope ....................................................................................................................... 1-3  
  1.5 Assumptions ............................................................................................................ 1-4  
  1.6 Tools ......................................................................................................................... 1-5  
  1.7 Thesis Organization ............................................................................................... 1-5  

II. Starfire Beam Director Modeling .............................................................................. 2-1  
  2.1 Introduction ............................................................................................................. 2-1  
  2.2 Starfire Beam Director Description ...................................................................... 2-1  
  2.3 System Configuration ............................................................................................ 2-2  
  2.4 Azimuth Axis Linear Plant Model ........................................................................ 2-4  
      2.4.1 Current Loop ................................................................................................... 2-7  
      2.4.2 Gimbal Dynamics ......................................................................................... 2-11  
      2.4.3 Nominal Plant Model ................................................................................... 2-12  
  2.5 Plant Disturbances and Sensor Noise .................................................................... 2-13  
      2.5.1 Plant Disturbances ....................................................................................... 2-14
2.5.1.1 Bearing Friction .............................................. 2-14
2.5.1.2 Wind Buffeting ............................................. 2-16
2.5.1.3 Motor Ripple and Cogging ............................. 2-17
2.5.2 Sensor Noise ...................................................... 2-19
2.6 Digital Control Effects ........................................... 2-19
2.7 SBD Model Summary ............................................. 2-20

III. Optimal Control Design Techniques ......................... 3-1
3.1 Background ........................................................ 3-1
3.2 Design Techniques ................................................. 3-4
  3.2.1 Unity Feedback Setup ....................................... 3-5
    3.2.1.1 Control Weighting Rationale ...................... 3-8
    3.2.1.2 Building the P Matrix ............................... 3-10
    3.2.1.3 H₂ and H∞ Algorithms ............................ 3-10
  3.2.2 Two-Degree of Freedom Controller Setup .............. 3-10

IV. SBD Azimuth Axis Controller Design .......................... 4-1
  4.1 Introduction ...................................................... 4-1
  4.2 Scaling Problem .................................................. 4-1
  4.3 Position Command Inputs ...................................... 4-2
  4.4 Unity Feedback Design ......................................... 4-3
    4.4.1 Control Weightings ...................................... 4-4
    4.4.2 H₂ Controller Synthesis ............................... 4-8
    4.4.3 Controller Fine Tuning .................................. 4-8
      4.4.3.1 Control Use Weighting .......................... 4-9
      4.4.3.2 "Manual" Pole Placement ....................... 4-10
  4.5 Two-Degree of Freedom Controller Design ................. 4-11
V. Azimuth Axis Simulations and Results ...................................................................... 5-1

5.1 Introduction ......................................................................................................... 5-1

5.2 Controller Comparison ......................................................................................... 5-1

5.2.1 Current SBD Controller ................................................................................ 5-3
5.2.2 H₂ Optimization Controller ........................................................................... 5-3

5.3 Simulations and Results ....................................................................................... 5-4

5.3.1 Torque Disturbance and Sensor Noise ............................................................. 5-4
5.3.2 Step Response .................................................................................................. 5-6
5.3.3 Ramp Response ................................................................................................ 5-8
5.3.4 Ephemeris Response ....................................................................................... 5-10
5.3.5 Full-Up Simulation .......................................................................................... 5-12
5.3.6 Summary of Results ....................................................................................... 5-13

VI. Elevation Axis ......................................................................................................... 6-1

6.1 Introduction ......................................................................................................... 6-1

6.2 Elevation Axis Linear Plant Model ...................................................................... 6-1

6.3 Torque Disturbance and Sensor Noise .................................................................. 6-2

6.4 Nominal and Truth Models .................................................................................. 6-4

6.5 Elevation Axis Controller Design ....................................................................... 6-5

6.6 Simulation and Results ....................................................................................... 6-6

VII. Conclusions/Recommendations ........................................................................... 7-1

7.1 Conclusions ........................................................................................................... 7-1

7.1.1 Azimuth Axis Performance ........................................................................... 7-1

7.1.2 The Price to Pay ............................................................................................. 7-2

7.2 Future Research ................................................................................................... 7-3

7.3 Closing Comments ............................................................................................. 7-5
**List of Figures**

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1 Starfire Beam Director</td>
<td>2-2</td>
</tr>
<tr>
<td>2-2 Beam Director Configuration</td>
<td>2-3</td>
</tr>
<tr>
<td>2-3 Top Level System Block Diagram</td>
<td>2-3</td>
</tr>
<tr>
<td>2-4 Fine Position Servo Loop</td>
<td>2-4</td>
</tr>
<tr>
<td>2-5 Coelostat Plant</td>
<td>2-5</td>
</tr>
<tr>
<td>2-6 Position Servo Loop Signal Measurement Setup</td>
<td>2-7</td>
</tr>
<tr>
<td>2-7 Current Loop</td>
<td>2-8</td>
</tr>
<tr>
<td>2-8 Current Loop Frequency Response</td>
<td>2-9</td>
</tr>
<tr>
<td>2-9 Second-Order Model of Current Loop Frequency Response</td>
<td>2-10</td>
</tr>
<tr>
<td>2-10 Nominal Plant Model</td>
<td>2-13</td>
</tr>
<tr>
<td>2-11 SBD Plant Model with Disturbances and Sensor Noise</td>
<td>2-14</td>
</tr>
<tr>
<td>2-12 Dahl Friction Model</td>
<td>2-15</td>
</tr>
<tr>
<td>2-13 Simulink Bearing Friction Model</td>
<td>2-16</td>
</tr>
<tr>
<td>2-14 Wind Buffeting Coloring Filter</td>
<td>2-17</td>
</tr>
<tr>
<td>2-15 Torque Disturbance Coloring Filter</td>
<td>2-18</td>
</tr>
<tr>
<td>2-16 Zero-Order Hold</td>
<td>2-20</td>
</tr>
<tr>
<td>3-1 Optimal Control Setup</td>
<td>3-1</td>
</tr>
<tr>
<td>3-2 State-Space Block Diagram Form</td>
<td>3-5</td>
</tr>
<tr>
<td>3-3 Unity Feedback Setup</td>
<td>3-7</td>
</tr>
<tr>
<td>3-4 General Shape of Sensitivity</td>
<td>3-8</td>
</tr>
<tr>
<td>3-5 Two Degree of Freedom Controller Setup</td>
<td>3-12</td>
</tr>
<tr>
<td>3-6 Track Weighting</td>
<td>3-12</td>
</tr>
<tr>
<td>4-1 Position Command Input</td>
<td>4-3</td>
</tr>
<tr>
<td>4-2 Velocity Derived From Position Command</td>
<td>4-3</td>
</tr>
<tr>
<td>Figure</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
</tr>
<tr>
<td>4-3 Unity Feedback Setup</td>
<td>4-4</td>
</tr>
<tr>
<td>4-4 Output Multiplicative Uncertainty Block Diagram</td>
<td>4-6</td>
</tr>
<tr>
<td>4-5 Closed-Loop State Weighting Block Diagram</td>
<td>4-6</td>
</tr>
<tr>
<td>4-6 Sensitivity Weighting</td>
<td>4-7</td>
</tr>
<tr>
<td>4-7 Controller Magnitude Frequency Response</td>
<td>4-9</td>
</tr>
<tr>
<td>4-8 Control Usage Weighting</td>
<td>4-10</td>
</tr>
<tr>
<td>4-9 Two Degree of Freedom Controller Setup</td>
<td>4-12</td>
</tr>
<tr>
<td>4-10 2nd Order Track Weighting</td>
<td>4-12</td>
</tr>
<tr>
<td>4-11 Step Response</td>
<td>4-14</td>
</tr>
<tr>
<td>4-12 Tracking Error to Ramp Command</td>
<td>4-14</td>
</tr>
<tr>
<td>4-13 Truncated Tracking Error to Ramp Command</td>
<td>4-15</td>
</tr>
<tr>
<td>4-14 Tracking Error to Ephemeris Command</td>
<td>4-15</td>
</tr>
<tr>
<td>4-15 Truncated Tracking Error to Ephemeris Command</td>
<td>4-16</td>
</tr>
<tr>
<td>5-1 Bode Magnitude Frequency Response</td>
<td>5-2</td>
</tr>
<tr>
<td>5-2 Bode Phase Frequency Response</td>
<td>5-2</td>
</tr>
<tr>
<td>5-3 Torque Disturbance</td>
<td>5-5</td>
</tr>
<tr>
<td>5-4 Position Error to Torque Disturbance</td>
<td>5-5</td>
</tr>
<tr>
<td>5-5 Torque Disturbance Rejection Transfer Function</td>
<td>5-6</td>
</tr>
<tr>
<td>5-6 Step Response</td>
<td>5-7</td>
</tr>
<tr>
<td>5-7 Closed-Loop Transfer Function</td>
<td>5-7</td>
</tr>
<tr>
<td>5-8 Ramp Command Tracking Error</td>
<td>5-8</td>
</tr>
<tr>
<td>5-9 Truncated Ramp Command Tracking Error</td>
<td>5-9</td>
</tr>
<tr>
<td>5-10 Bearing Friction Effect With Current Controller</td>
<td>5-10</td>
</tr>
<tr>
<td>5-11 Bearing Friction Effect With $H_2$ Controller</td>
<td>5-10</td>
</tr>
<tr>
<td>5-12 Ephemeris Command Tracking Error</td>
<td>5-12</td>
</tr>
<tr>
<td>Figure</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
</tr>
<tr>
<td>5-13 Full-Up Simulation Tracking Error</td>
<td>5-13</td>
</tr>
<tr>
<td>6-1 Elevation Axis Current Loop Frequency Response</td>
<td>6-2</td>
</tr>
<tr>
<td>6-2 Torque Disturbance Coloring Filter</td>
<td>6-3</td>
</tr>
<tr>
<td>6-3 Sensitivity Weighting</td>
<td>6-6</td>
</tr>
<tr>
<td>6-4 Torque Disturbance Rejection Transfer Function</td>
<td>6-7</td>
</tr>
<tr>
<td>6-5 Ephemeris Command Tracking Error</td>
<td>6-8</td>
</tr>
<tr>
<td>7-1 Closed-Loop Transfer Function</td>
<td>7-3</td>
</tr>
<tr>
<td>C-1 Current Controller Simulation Model &quot;SimCCEvalMod&quot;</td>
<td>C-1</td>
</tr>
<tr>
<td>C-2 Current Controller Linmod Model &quot;SimCCTotEvalMod&quot;</td>
<td>C-2</td>
</tr>
<tr>
<td>C-3 Unity Feedback Simulation Model &quot;SimSenEvalMod&quot;</td>
<td>C-3</td>
</tr>
<tr>
<td>C-4 Unity Feedback Linmod Model &quot;SimSenTotEvalModl&quot;</td>
<td>C-4</td>
</tr>
<tr>
<td>C-5 2 Degree of Freedom Controller Simulation Model &quot;SimTrckEvalMod&quot;</td>
<td>C-5</td>
</tr>
<tr>
<td>C-6 2-Degree of Freedom Controller Linmod Model &quot;SimTrckTotEvalMod&quot;</td>
<td>C-6</td>
</tr>
<tr>
<td>E-1 Current Controller Tracking Error PSD</td>
<td>E-1</td>
</tr>
<tr>
<td>E-2 H₂ Controller Tracking Error PSD</td>
<td>E-1</td>
</tr>
<tr>
<td>E-3 Current Controller Voltage</td>
<td>E-2</td>
</tr>
<tr>
<td>E-4 H₂ Controller Voltage</td>
<td>E-2</td>
</tr>
<tr>
<td>E-5 Current Controller Voltage PSD</td>
<td>E-3</td>
</tr>
<tr>
<td>E-6 H₂ Controller Voltage PSD</td>
<td>E-3</td>
</tr>
<tr>
<td>E-7 Current Controller Amps</td>
<td>E-4</td>
</tr>
<tr>
<td>E-8 H₂ Controller Amps</td>
<td>E-4</td>
</tr>
<tr>
<td>E-9 Current Controller Motor Torque</td>
<td>E-5</td>
</tr>
<tr>
<td>E-10 H₂ Controller Motor Torque</td>
<td>E-5</td>
</tr>
<tr>
<td>E-11 Current Controller Motor Torque PSD</td>
<td>E-6</td>
</tr>
<tr>
<td>E-12 H₂ Controller Motor Torque PSD</td>
<td>E-6</td>
</tr>
</tbody>
</table>
List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1</td>
<td>2-5</td>
</tr>
<tr>
<td>2-2</td>
<td>2-21</td>
</tr>
<tr>
<td>4-1</td>
<td>4-9</td>
</tr>
<tr>
<td>4-2</td>
<td>4-13</td>
</tr>
<tr>
<td>5-1</td>
<td>5-14</td>
</tr>
<tr>
<td>5-2</td>
<td>5-14</td>
</tr>
<tr>
<td>6-1</td>
<td>6-5</td>
</tr>
</tbody>
</table>
List of Symbols

$Ga(s)$ Amplifier transfer function
$Gc_l(s)$ Current loop transfer function
$A_a, B_a, C_a, D_a$ Current loop state-space realization matrices
$Gm(s)$ Motor transfer function
$Gg(s)$ Gimbal dynamics transfer function
$A_g, B_g, C_g, D_g$ Gimbal dynamics state-space realization matrices
$Gp(s)$ Plant dynamics transfer function
$A_p, B_p, C_p, D_p$ Plant dynamics state-space realization matrices
$Gc(s)$ Compensator transfer function
$K_{FB}$ Current loop feedback constant
$K_v$ Coefficient of viscous friction
$K_T$ Motor torque constant
$J$ Moment of inertia
$\omega_{mb}$ Motor break frequency
$\omega_b$ Elevation axis current loop break frequency
$\omega_{an}$ Azimuth axis current loop natural frequency
$K_a$ Current loop gain constant
$\zeta_a$ Azimuth axis current loop damping ratio
$x(t)$ HP signal generator output
$\nu(t)$ Compensator voltage output
$y(t)$ Difference between $x(t)$ and $\nu(t)$
$a(t)$ Current loop ampere output
$\Theta(t)$ Gimbal position
$\dot{\Theta}(t)$ Gimbal rate
\( \dot{\theta}_m(t) \)  
Digitally measured gimbal position

\( \theta_c(t) \)  
Position command input

\( T_m(t) \)  
Motor torque

\( T_d(t) \)  
Torque disturbance

\( F_c \)  
Running friction constant

\( T_c \)  
Running friction torque disturbance constant

\( \tau_d \)  
CPU computational time delay

\( T \)  
Sample period

\( \omega_s \)  
Sample frequency

\( \omega_c \)  
Bandwidth or zero dB crossover frequency

\( G_{pad}(s) \)  
Pade' approximation transfer function

\( G_{sd}(s) \)  
Approximated sampling effects transfer function

\( \Sigma_{d}(s) \)  
Wind buffeting PSD

\( W_d \)  
Torque disturbance coloring filter

\( A_d, B_d, C_d, D_d \)  
Torque disturbance coloring filter state-space realization matrices

\( w \)  
Exogenous inputs

\( z \)  
Controlled outputs

\( u \)  
Control input

\( y \)  
Measured output

\( w_1 \)  
Zero-mean, unit intensity, white Gaussian noise disturbance input

\( w_2 \)  
Zero-mean, unit intensity, white Gaussian sensor noise input

\( K \)  
Controller

\( P \)  
Optimal control setup plant matrix

\( A, B_w, B_u, C_z, D_{zw}, D_{zu}, C_y, D_{yw}, D_{yu} \)  
The 9 matrices that make up the \( P \) matrix

\( T_{zw} \)  
Closed-loop transfer function of \( w \) to \( z \)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_c$</td>
<td>Control usage weighting</td>
</tr>
<tr>
<td>$H$</td>
<td>State weighting</td>
</tr>
<tr>
<td>$W_s$</td>
<td>Sensitivity weighting</td>
</tr>
<tr>
<td>$A_s, B_s, C_s, D_s$</td>
<td>Sensitivity weighting state-space realization matrices</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Disturbance distribution matrix</td>
</tr>
<tr>
<td>$J$</td>
<td>LQG performance index</td>
</tr>
<tr>
<td>$Q$</td>
<td>LQG state weighting matrix</td>
</tr>
<tr>
<td>$S$</td>
<td>Sensitivity</td>
</tr>
<tr>
<td>$T$</td>
<td>Complimentary sensitivity</td>
</tr>
<tr>
<td>$W_t$</td>
<td>Tracking error minimization weighting</td>
</tr>
<tr>
<td>$W_{Rc}$</td>
<td>Dynamic control usage weighting</td>
</tr>
<tr>
<td>$err_{ss}$</td>
<td>Steady state position error.</td>
</tr>
</tbody>
</table>
### List of Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>ATP</td>
<td>Acquisition, Tracking, and Pointing</td>
</tr>
<tr>
<td>CPU</td>
<td>Central processing unit</td>
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<tr>
<td>DSP</td>
<td>Digital Signal Processor</td>
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<tr>
<td>EI</td>
<td>Experimenter's interface</td>
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<tr>
<td>EMF</td>
<td>Electro-magnetic field</td>
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<tr>
<td>FSM</td>
<td>Fast steering mirror</td>
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<tr>
<td>GBL</td>
<td>Ground based laser</td>
</tr>
<tr>
<td>LC</td>
<td>Local controller</td>
</tr>
<tr>
<td>LOS</td>
<td>Line-of-sight</td>
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<tr>
<td>LQG</td>
<td>Linear quadratic Gaussian</td>
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<tr>
<td>MIMO</td>
<td>Multi-input multi-output</td>
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<tr>
<td>PI</td>
<td>Proportional integral control</td>
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<tr>
<td>PSD</td>
<td>Power spectral density</td>
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<td>RAM</td>
<td>Random access memory</td>
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<tr>
<td>RMS</td>
<td>Root mean squared</td>
</tr>
<tr>
<td>S/N</td>
<td>Signal to noise ratio</td>
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<tr>
<td>SBD</td>
<td>Starfire Beam Director</td>
</tr>
<tr>
<td>SISO</td>
<td>Single-input single-output</td>
</tr>
</tbody>
</table>
Abstract

The Starfire Beam Director (SBD) is located at the Starfire Optical Range at Kirtland Air Force Base in Albuquerque, New Mexico. The SBD capabilities include tracking celestial objects and active or passive tracking of artificial satellites to support the Phillips Laboratory Ground Based Laser Acquisition, Tracking, and Pointing (GBL ATP) program. The pointing and tracking accuracy needed to support such experiments is microradian to sub-microradian level. To accomplish this goal requires precise pointing of the massive 6 ton 1-meter clear aperture coelostat\(^1\).

The purpose of this thesis is to use optimal control design techniques to develop a controller to meet the stringent pointing requirements. A nominal linear state-space model was built which included gimbal dynamics, plant disturbances, and sensor noise. Then optimal design techniques were used to develop unity feedback and two degree of freedom controllers. The various controllers were simulated with the coelostat "truth" model, which incorporated the higher frequency current loop and motor dynamics, non-linearities, plant disturbances, sensor noise, and discrete control effects. The best of the designs, the H\(_2\) unity feedback controller, was compared and contrasted with the performance of the controller currently being used, which was obtained by classical control design. The H\(_2\) controller exceeded tracking requirements and in most areas performed better than the current controller.

---

\(^1\)Coelostat refers to a two-mirror beam director gimbal.
I. Introduction

1.1 Background

The background discussion is intended to be a generic description of the acquisition, pointing, and tracking (ATP) functions of a beam director. Most beam directors have ATP systems and procedures that are slightly different from this description, but the general process and terminology is the same.

The term "beam director" is generally intended to describe a device which takes the output of a stationary laser device and transmits it in an arbitrary direction. To accurately illuminate an object with a laser beam the object must be tracked with minimal line-of-site (LOS) error. Typically, beam directors incorporate both passive and active tracking to minimize LOS error. Active tracking is used once an object is in the field of view and there is enough return image intensity for a coarse or fine acquisition sensor, such as a 30 Hz frame rate camera and/or a focal plane array, to close a beam steering mirror loop. The beam steering mirror is commonly called a fast steering mirror (FSM). Passive tracking, also known as predictive control, uses ephemeris computation as a command input to a beam director's gimbal position servo loop. A tracking error signal is then derived from an encoder measuring beam director gimbal position. The passive LOS stabilization is thus considered "open-loop" pointing in the sense that no precise
measurement of tracking error using a target image is fed back to close the gimbal position servo loop.

The two types of tracking can be simultaneously used to effectively reduce LOS error. The acquisition and tracking of an object starts with open-loop pointing, where assuming the ephemeris data is accurate enough to bring the object into view of a coarse acquisition camera, an update is made to the ephemeris making it more accurate. Once the return image yields enough intensity on a fine acquisition sensor, a focal plane array, the FSM loop can be closed to further reduce the LOS error effects of residual gimbal motion\(^2\), atmospheric turbulence, and ephemeris inaccuracy.

Tracking low-earth orbit satellites presents a very difficult challenge for a beam director, because the gimbal must reach velocities of 5° to 10° per second, demanding extremely large step sizes in the ephemeris position command and causing many plant disturbances that must be rejected. Assuming that the ephemeris is accurate, good open-loop pointing will have LOS error of less than 5 μrad RMS [5:60]. Of course, the measure of "good" tracking will vary with the requirements of the particular task. Typically, once the FSM loop is closed LOS error can be reduced to sub-μrad.

Although it is true that a FSM loop with high bandwidth and adequate dynamic range can effectively reduce LOS error, it is necessary to have accurate open-loop pointing for several reasons: return image intensity must initially be present on the coarse and/or fine acquisition sensor; image intensity must remain uninterrupted; if the FSM should break track and open-loop pointing is accurate, intensity will not be lost and FSM tracking can be immediately resumed; and if open-loop pointing is accurate, the more likely the FSM will not break track. For large propagating laser beam divergence, full angle 50 to 70 μrad, open-loop pointing accuracy is not as critical as it is for smaller beam divergence. As long as there is enough intensity on the acquisition sensors the

\(^2\)Residual gimbal motion refers to the LOS error not rejected or possibly even created by the closed position servo loop, or open-loop pointing.
FSM loop can be closed to reduce LOS error. However, for example, if the beam divergence is 30 μrad full angle and open-loop pointing is 5 μrad RMS the LOS error will have peaks of 15 μrad and appropriate intensity can not be maintained to close the FSM loop. Add an ephemeris bias error of 5 to 10 μrad and the problem is worse. Also, there are times when closing the FSM loop will not be possible, such as tracking dim sunlit satellites where the image does not have the required intensity, or daylight tracking of objects where the sensitive coarse acquisition camera cannot be used and signal-to-noise (S/N) ratio of the fine acquisition sensor is too low due to sunlight.

1.2 Problem Statement

The SBD linear dynamics, non-linearities, plant disturbances, and sensor noise need to be modeled, and design of a controller for the position servo loop is needed, which will effectively reject modeled plant disturbances and track actual ephemeris data for a low-earth orbiting satellite to less than 5 μrad RMS.

1.3 Objectives

The SBD currently has a controller in operation that meets the specifications described in the background section. That controller was developed by classical control design. The object of this thesis is two-fold; first, design a controller that has better performance than the current controller, and second, use optimal control design techniques to develop the controller.

1.4 Scope

While in the background section many parts of the ATP system were described, this thesis only addresses the position servo loop or open-loop pointing. From here on, closed-loop tracking will be in reference to the position servo loop as opposed to the closed FSM loop. The position servo loop is made up of two loops; the coarse position loop and the fine position loop. The coarse position loop uses "bang-bang" control to
provide a time optimal response to large position commands. A maximum voltage in the
feedforward path creates a constant acceleration or deceleration. Once the position error
falls below a predefined level, the control switches from the coarse to the fine position
loop. The controller design in this thesis is only for the fine position loop.

The plant model, for which the controller is designed, does not account for the
effects of high frequency resonant dynamics. Notch filters will have to be designed as
they have been for the current controller.

Coupling between the azimuth and elevation axes is not considered in this thesis.
Separate controllers are designed for each axis, which simplifies the design to two single-
input single-output (SISO) systems. Simplifying the design is not the motivation behind
not including coupling, but rather that the coupling is not understood well enough to
model.

1.5 Assumptions

Saturation occurs when the compensator output is greater than ±10 volts. Non-
linear saturation analysis is not performed with the fine position loop. It is assumed that
by the time hand-off from the coarse to the fine position loop occurs the system will be
approaching steady state conditions and the transients due to the hand-off will not be too
severe. Even if this is not a completely valid assumption, the results of the comparison of
the currently used controller versus the H2 controller, in Chapter 5, show that the output
voltage transients due to various commanded input types are very similar. Therefore, the
H2 controller should not perform any worse than the current controller in regard to
saturation.

A major source of disturbance in the system is back electro-magnetic field (EMF)
of the motor. The amplifier has built-in proportional plus integral (PI) control circuitry
for the purpose of rejecting the back EMF. Since measured closed-loop amplifier
dynamics are used in the plant model, back EMF is not simulated as a disturbance. It is
assumed that the built-in amplifier circuitry yields zero steady state error due to the EMF. The EMF can be simulated as a step input because it is a linear function of velocity and velocity is nearly constant throughout a satellite pass. PI control yields zero steady state error to a step input. At worst case the commanded position input has a small acceleration component, in which case the EMF is a ramp disturbance and with a near integrator in the $H_2$ controller the result should be near zero steady state error. A full description of the amplifier dynamics is presented in section 2.4.1.

1.6 Tools

All computer modeling, analysis, and word processing was done on a Macintosh IIsi with a 32K cache card, 20MHz math coprocessor, 5MB of RAM, and a 40MB hard drive. Microsoft Word 5.0 was used for word processing. Matlab/Simulink™ was used for modeling, controller design, and analysis. Linear modeling and controller design utilized the Control System, Robust-Control, Matlab, and Signal Processing Toolboxes, and the $H_2$ and $H_{\infty}$ algorithms written as Matlab script M-files by Dr. Brett Ridgely. Simulations with the controller, full linear plant, non-linearities, plant disturbances, and sensor noise were performed in Simulink.

1.7 Thesis Organization

This thesis concentrates on the controller design for the coelostat's azimuth axis for several reasons. First, it is only necessary to show the detailed design of the controller for one axis to demonstrate the optimal control design techniques. Second, the azimuth axis is chosen because more accurate data is available for the azimuth plant dynamics than for the elevation axis dynamics. Third, the comparison of the optimal design to the current azimuth axis controller is straightforward compared to the current elevation axis controller, where direct feedforward control had to be implemented for reasons not even the SBD engineers clearly understand.
Chapter 2 details the nominal and truth plant model, non-linear effects, plant disturbances, and sensor noise for the azimuth axis.

Chapter 3 gives an overview of optimal control design techniques.

Chapter 4 details the azimuth axis optimal controller design with some results of the less successful controllers.

Chapter 5 starts with a description of the controller currently used on the azimuth axis and then compares and contrasts the simulation results of the best $H_2$ controller versus the current controller.

Chapter 6 builds the model and designs the controller for the elevation axis and provides results.

Chapter 7 presents conclusions and recommendations for future research.
II. Starfire Beam Director Modeling

2.1 Introduction

The first few sections of this chapter describe the overall function and operation of the SBD. The emphasis of this chapter is contained in the last sections where the linear nominal model, plant disturbances, sensor noise, and the truth model, which incorporates the high frequency amplifier dynamics, discrete control effects, and non-linearities are developed. It is important to distinguish between the nominal and truth models because the optimal controller is designed for the nominal model, plant disturbances, and sensor noise, while simulations are performed with the truth model.

2.2 Starfire Beam Director Description

The SBD is an elevation over azimuth coelostat that maintains a 1-meter clear aperture over a full hemisphere. The coelostat weighs approximately 6 tons and has hydrostatic oil bearings in both axes to reduce bearing friction and noise. The azimuth motor can generate a maximum torque of 3000 ft-lbs and the elevation motor 300 ft-lbs. The maximum rates for the azimuth and elevation axes are set at 10' and 5' per second, respectively. It currently has the capability to track celestial objects and artificial satellites with less than 2 μrad RMS of LOS error. Figure 2-1 is a picture of the SBD.
2.3 System Configuration

Figure 2-2 represents a beam director configuration similar to the SBD. There is an optical bench on which the visiting experimenters can mount lasers and optical equipment. The Coude' path\(^3\) contains a large turning flat mirror, a mirror mounted to the gimbal's azimuth axis, and a mirror mounted to the elevation axis.

Figure 2-3 is the top level block diagram of the beam director system. The experimenter's interface (EI) allows the experimenters to select a mode to point at a selected star, planet, or satellite. The remote controller (RC) calculates the pointing vector (ephemeris) based on the mode selected. The output of the RC is the position command input to the position servo loop, or what the SBD engineers refer to as the inner loop. The local controller (LC) refers to both the coarse and fine position loops.

---

\(^3\)The Coude' path is the optical path from the edge of the optical bench to the exiting side of the elevation mirror.
From here on, this thesis only addresses the fine position loop portion of the local controller shown in Figure 2-4. The SBD utilizes high speed digital signal processors (DSP) running at 200 Hz to execute digital control. The compensator output is then
converted to an analog voltage signal to drive the plant. Gimbal position is measured by highly accurate Inductosyn™ encoders, one for each axis, and the digitally measured position is fed back to form position error. Although the controller is discrete, it is designed and simulated in the continuous domain using a continuous approximation to a zero-order hold to account for the delay effects of discrete control. The approximation is developed in section 2.6.

**Figure 2-4** Fine Position Servo Loop

### 2.4 Azimuth Axis Linear Plant Model

The plant, consisting of an amplifier, motor, and gimbal is shown in Figure 2-5. The current feedback loop is necessary to reject back EMF disturbance, which is discussed in detail in section 2.4.1. The closed current loop $G_{cl}$ can be modeled as

$$G_{cl}(s) = \frac{Gm(s)Ga(s)}{1 + Gm(s)Ga(s)K_{FB}}$$

(2.1)
The parameter values used to construct the plant dynamics are obtained from two sources. The first source is the original coelostat documentation, *Technical Description of a Coelostat System* [1]. The parameters and the values used in the model are shown in Table 2-1 [1:134]. The second and most accurate source is frequency response data taken from the actual SBD system in June 1991. The frequency responses are more accurate than the technical documentation because the documentation was written over twenty years ago and most of the parameters have changed due to system wear and small modifications. Unfortunately, complete frequency response data is not available for every component of the plant; therefore, documentation data is used for part of the model.

**Table 2-1** Documentation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Azimuth Axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment of Inertia (ft-lbs-sec^2)</td>
<td>J</td>
</tr>
<tr>
<td>Coefficient of Viscous Friction (ft-lbs/rad/sec)</td>
<td>K_v</td>
</tr>
<tr>
<td>Motor Torque Constant (ft-lbs/amp)</td>
<td>K_T</td>
</tr>
<tr>
<td>Motor Lag Frequency</td>
<td>( \omega_{mb} )</td>
</tr>
<tr>
<td></td>
<td>26000</td>
</tr>
<tr>
<td></td>
<td>350</td>
</tr>
<tr>
<td></td>
<td>75.6</td>
</tr>
<tr>
<td></td>
<td>60.0</td>
</tr>
</tbody>
</table>
Ideally, to accurately determine the frequency response of each component of the plant, the input and output of each component should be directly measurable. Figure 2-6 depicts the position servo loop and the signals that were measured to determine the system's frequency responses. The signal generator and the signal analyzer are shown as separate devices for clarity. Actually, they are both part of a Hewlett-Packard Dynamic Signal Analyzer. A sine signal $x(t)$ was input at the summing junction and the component transfer functions were calculated by measuring the signals $v(t)$, $y(t)$, $a(t)$, and $\theta_m(t)$ as the frequency of $x(t)$ was incremented through a range. $v(t)$ is the voltage output of the compensator, $y(t)$ is the difference between $v(t)$ and the signal generator output $x(t)$, $a(t)$ is the ampere output of the current loop, and $\theta_m(t)$ is the digitally measured gimbal position. Open and closed-loop transfer function data was acquired with this setup, where the open and closed loops were defined as

$$\frac{v(s)}{y(s)_{cl}} = G_c(s)G_g(s)K_TG_{el}(s)$$

Therefore, actual data is used to model the current loop and parameters from the original documentation are used for the torque constant and gimbal dynamics. The controller is designed with high gain and phase margins (stability robustness) to account for the uncertainty associated with using the parameters in place of actual data.
2.4.1 Current Loop. As briefly discussed in section 1.5 the amplifier was designed with PI circuitry to reject back EMF of the motor. It was also designed to include a lead to cancel the 20 dB per decade roll off at 60 rad/sec due to the motor resistance and inductance (see Eqn 2.4). Figure 2-7 is a model of the current loop used in designing the presently used controller. The current loop parameters for this model were obtained from the original coelostat document.
The transfer function equivalent of the closed current loop is

$$G_{cl}(s) = \frac{2.4 \times 10^7(s + 60.6)}{(s + 3011)(s + 1988)(s + 60.6)} \text{ amps / volt} \quad (2.4)$$

The Bode plot in Figure 2-8 shows that the closed current loop has a roll off at approximately 2000 rad/sec. or 320 Hz, and a dc (low frequency) gain of 12 dB or 4 amps/volt. As stated in Section 1.5 it is assumed that the current loop effectively rejects the back EMF; therefore, back EMF is not simulated and the measured closed current loop transfer function is used in the model.
The measured current loop data came in the form of an HP signal analyzer Bode plot and can be closely modeled as a standard second-order transfer function

\[ G_{cl}(s) = \frac{K_a \omega_{an}^2}{s^2 + 2\zeta \omega_{an} s + \omega_{an}^2} \]  

(2.5)

This plot showed another $4.5$ dB/decade attenuation at approximately 2000 rad/sec, but at such a high frequency its contribution to system dynamics would be negligible; therefore, it was neglected. The plot was reconstructed in Matlab by iteration until the shape of the simplified second order model matched the original\(^4\). The reconstructed simplified model Bode plot, Figure 2-9, shows the current loop having a natural frequency $\omega_{an}$ of approximately 560 rad/sec, or 90 Hz; a gain constant $K_a = 4.4$ amps/volt; and a damping ratio $\zeta_a = .25$. The damping ratio was determined from reference [2:253].

\(^4\)The Matlab frequency response plot has 20 dB more gain than the original HP frequency spectral analyzer plot to account for a 10 amp/volt scale factor.
It is necessary to transform all dynamics to a state-space representation to be used in optimal control design. The state-space representation of $G_{cl}(s)$ is derived as follows:

$$G_{cl}(s) = \frac{\alpha(s)}{\nu(s)} = \frac{K_a \omega_{an}^2}{s^2 + 2\zeta \omega_{an} s + \omega_{an}^2}$$

(2.6)

where $\nu(t)$ is the voltage input to the current loop and $\alpha(t)$ is the current output. After cross multiplying and taking the inverse Laplace transform of both sides the corresponding output differential equation becomes

$$\ddot{\alpha}(t) + 2\zeta \omega_{an} \dot{\alpha}(t) + \omega_{an}^2 \alpha(t) = K_a \omega_{an}^2 \nu(t)$$

(2.7)

The state variables are defined as
\[ x_a(t) = \begin{bmatrix} a(t) \\ \dot{a}(t) \end{bmatrix} \] (2.8)

The input and output are, respectively,

\[ u_a(t) = v(t), \quad y_a(t) = a(t) \] (2.9)

The phase-variable canonical form of the state-space model is

\[ \begin{aligned} \dot{x}_a(t) &= A_a x_a(t) + B_a u_a(t) \\ y_a(t) &= C_a x_a(t) + D_a u_a(t) \end{aligned} \] (2.10)

where \( A_a \) is the amplifier system matrix, \( B_a \) is the input matrix, \( C_a \) is the output matrix, and \( D_a \) is the direct feedthrough term having the values

\[ \begin{align*} A_a &= \begin{bmatrix} 0 & 1 \\ -\omega_\text{an}^2 & -2\omega_\text{an} \end{bmatrix} \\ B_a &= \begin{bmatrix} 0 \\ K_a\omega_\text{an}^2 \end{bmatrix} \\ C_a &= [1 \ 0] \\ D_a &= 0 \end{align*} \] (2.11)

2.4.2 Gimbal Dynamics. The motion of the gimbal can be described by the differential equation

\[ J\ddot{\theta}(t) + K_v\dot{\theta}(t) = T_m(t) \] (2.12)

where \( J \) is the moment of inertia, \( K_v \) is the coefficient of viscous friction due to the viscosity of the fluid in the hydrostatic bearings, and \( T_m(t) \) is the torque applied by the motor [3:133]. \( J \) and \( K_v \) are obtained from the coelostat's original documentation (see Table 2-1). The state variables are defined as
The input and output are, respectively,

\[ u_g(t) = T_m(t) \quad y_g(t) = \Theta(t). \tag{2.14} \]

The corresponding state-space representation is

\[
\begin{bmatrix}
\dot{\Theta}(t) \\
\dot{\Theta}(t)
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
0 & -K_v/J
\end{bmatrix}
\begin{bmatrix}
\Theta(t) \\
\Theta(t)
\end{bmatrix} +
\begin{bmatrix}
0 \\
1/J
\end{bmatrix} T_m(t)
\tag{2.15}
\]

or

\[
\begin{bmatrix}
\dot{\Theta}(t) \\
\dot{\Theta}(t)
\end{bmatrix} =
\begin{bmatrix}
0 & [\Theta(t)] \\
0 & [\dot{\Theta}(t)]
\end{bmatrix} + [0] T_m(t)
\]

or

\[
\begin{align*}
\dot{x}_g(t) &= A_g x_g(t) + B_g u_g(t) \\
y_g(t) &= C_g x_g(t) + D_g u_g(t)
\end{align*}
\tag{2.16}
\]

2.4.3 Nominal Plant Model. When designing a controller using optimal design techniques it is desirable to design the controller around a nominal model and then perform simulations with the truth model. Optimal control will yield controllers with at least as many states as the plant; therefore, it is desirable to keep the order of the plant small to keep the order of the controller small. The nominal plant model only includes the current loop gain constant \( K_a \), the motor torque constant \( K_T \), and the gimbal dynamics \( G_g(s) \) as shown in Figure 2-10. The high frequency current loop dynamics are included in the truth model.
The state-space representation of the nominal plant is

\[
\begin{bmatrix}
\dot{\theta}(t) \\
\ddot{\theta}(t)
\end{bmatrix} = 
\begin{bmatrix}
0 & 1 \\
0 & -K_v/J
\end{bmatrix}
\begin{bmatrix}
\theta(t) \\
\dot{\theta}(t)
\end{bmatrix} +
\begin{bmatrix}
1 \\
1/J K_T K_a
\end{bmatrix} v(t)
\]  

(2.17)

or

\[
\theta(t) = 
\begin{bmatrix}
1 & 0
\end{bmatrix}
\begin{bmatrix}
\theta(t) \\
\dot{\theta}(t)
\end{bmatrix} + [0] v(t)
\]

2.5 Plant Disturbances and Sensor Noise

There are many sources of plant disturbances and sensor noise; some are used in the nominal model to design the optimal controller, some are only included in the truth model for simulation, and others are neglected. The position servo loop is shown in Figure 2-11 with all sources of plant disturbances and sensor noise.
2.5.1 Plant Disturbances. Most plant disturbances are in the form of torque disturbances. The torque disturbance \( T_d(t) \) from Figure 2-11 is used to represent torque disturbance caused by wind buffeting, motor cogging, and motor ripple. Viscous friction is also a torque disturbance due to the viscosity of the fluid in the hydrostatic bearings and is already accounted for in the nominal plant model. Bearing noise is a bandlimited, white noise, torque disturbance that is neglected because of the smooth characteristics of hydrostatic bearings. Bearing friction is a non-linear torque disturbance that is used in the truth model and is discussed in detail in section 2.5.1.1. The last type of plant disturbance is back EMF of the motor that reduces power amp output proportional to the speed of the motor shaft and is not a simulated disturbance as discussed in section 1.5.

2.5.1.1 Bearing Friction. The Dahl friction model, shown in Figure 2-12, exhibits non-linear bearing stiffness and can be used to model bearing friction in a gimbal system, where the displacement \( x \) is equal to the change in \( \theta \) from an initially stationary position. When the bearings are rolling the running friction \( F_r \) results in a constant torque disturbance \( T_c \) opposing the direction of motion. For the azimuth axis \( T_c = 16 \) ft-
The Dahl friction model behaves like a soft spring for small deflections, yet approaches the running friction for large deflections. When force is applied to the bearings they experience plastic deformation, known as the compliance zone. Outside the bearing compliance zone, i.e. bearings are rolling, the disturbance is equal to the running friction [6:2-4].

The characteristics of the Dahl friction model can be modeled such that the friction is not a function of $\theta$ but rather a function of $\dot{\theta}$. The bearing friction is modeled using the saturation block and a constant gain in Simulink. Figure 2-13 is the Simulink bearing friction model. For $\dot{\theta}(t) \geq 0.001$ rad/sec the saturation block's output value is constant at 0.001, thus the torque disturbance is equal to a constant 16 ft-lbs, i.e. the running friction torque constant. For $\dot{\theta}(t) < 0.001$ rad/sec the torque disturbance is a linear function of $\dot{\theta}(t)$.

Figure 2-12 Dahl Friction Model
which captures the characteristic of the Dahl compliance zone, where small motions from an initially stationary position result in plastic deformation and bearing friction resistance.

\[
\text{Torque} = \left( 16000 \frac{\text{ft-lbs}}{\text{rad/sec}} \right) \dot{\theta}(t)
\]

\[\text{(2.19)}\]

---

2.5.1.2 Wind Buffeting. Wind buffeting disturbance data is not available from the SBD, so data from another beam director is used as an estimate [5:54]. The data is in the form of a power spectral density (PSD) plot with units (ft-lbs)^2/Hz/Hz. The wind disturbance PSD \( \Sigma_d(\omega) \) is characterized as low frequency colored noise and can be modeled as a coloring filter \( W_d(j\omega) \) driven by zero mean, unity intensity, white Gaussian noise \( \Sigma_n(\omega) \), where

\[
\Sigma_d(\omega) = |W_d(j\omega)|^2 \Sigma_n(\omega)
\]

\[\text{(2.20)}\]

The resulting coloring filter \( W_d(s) \) is shown in Figure 2-14.
2.5.1.3 Motor Ripple and Cogging. Ripple torque disturbance is due to a small variation in average torque during rotation of the armature. This variation is due to the fact that commutation is done in discrete steps. The ripple torque is a deterministic low frequency sinusoidal effect that is a function of the motor shaft velocity [7:13].

Cogging torque disturbance is due to the non-uniform rotation of the motor armature caused by the tendency of the armature to prefer certain discrete positions. Cogging torque is also a deterministic low frequency effect that is a function of the motor shaft velocity [7:13].

Since both disturbances are low frequency they can be effectively rejected if the position servo loop has adequate bandwidth and low frequency gain. The two disturbances can be conservatively modeled as a coloring filter driven by white noise. The only data available on the actual effects of ripple and cogging disturbance is that it is a low frequency effect that contributes, when not rejected, approximately 60 µrad of gimbal position error. The coloring filter designed for the wind buffeting disturbance creates 30 µrad of error when the position servo is simulated open loop. The final torque disturbance coloring filter used for the controller design and simulation is designed to
incorporate wind buffeting, cogging, and ripple disturbance effects and is shown in Figure 2-15.

![Figure 2-15 Torque Disturbance Coloring Filter](image)

As stated earlier, this disturbance input is used in the optimal controller design, therefore, the coloring filter must be transformed to its state-space representation. In transfer function form

\[ W_d(s) = C_d(sI - A_d)^{-1}B_d + D_d \]  \hspace{1cm} (2.21)

In state-variable form

\[ \dot{x}_d(t) = A_d x_d(t) + B_d w_1(t) \]
\[ T_d(t) = C_d x_d(t) + D_d w_1(t) \]  \hspace{1cm} (2.22)

where \( T_d(t) \) is the torque disturbance, \( x_d(t) \) is the disturbance state, and \( w_1(t) \) is zero-mean, unit intensity, white Gaussian noise. The state-space values are

\[ A_d = -1 \quad B_d = 1 \]
\[ C_d = 1000 \quad D_d = 0 \]  \hspace{1cm} (2.23)
2.5.2 Sensor Noise. The absolute angular gimbal position is measured using an Inductosyn encoder. The measurement of true angular position is corrupted by noise with an RMS of 0.2 arc seconds or 1 μrad and is modeled as zero-mean, white Gaussian noise for the controller design. For the simulation the sensor noise is rolled off at 40 dB per decade starting at 50 rad/sec to prevent the unrealistic driving of high frequency control power. A second source of encoder errors are cyclic, deterministic errors, which are a function of the rotation angle, generally comprised of modulated harmonics of the basic Inductosyn pole frequency [7:4]. Modeling of the cyclic errors is beyond the scope of this thesis.

2.6 Digital Control Effects

Digital control has several dynamic effects which can be adverse. Those effects are quantization, CPU computational time, and sample rate.

As shown in Figure 2-6 the input to the inductosyn encoder is continuous time gimbal position and the output is digital. Quantization of the gimbal position measurement is occurring due to the finite word length of the encoder. Without getting into the details of the encoder, the least significant bit is 0.75 μrad. Quantization effects will be neglected, because the noise effects of the encoder are modeled such that the noise spectrum will account for quantization uncertainty.

The controller hardware consists of a Texas Instrument 320 C-30 Digital Signal Processor (DSP) programmed in C, housed in a VME chassis, with a Macintosh PC as the host. The system runs at 200 Hz, which corresponds to a sample time $T$ of 0.005 seconds. The DSP computational time delay $\tau_D$ is assumed to be $0$, which is a valid assumption if $\tau_D$ is much less than $T$ [4:7].

The dynamic effects of sampling and holding can be neglected when

$$\omega_s \geq 30\omega_c$$  \hspace{1cm} (2.24)
where $\omega_s$ is the sample frequency and $\omega_c$ is the system cross-over or bandwidth frequency. Since $\omega_c$ is approximately 10 Hz and $\omega_s$ is 200 Hz, the ratio is 20, thus the dynamics cannot be neglected. Figure 2-16(a) shows the continuous model of the sampler and the zero-order hold (ZOH).

![Diagram showing the sampler and ZOH approximations](image)

**Figure 2-16**  
(a) Continuous Model  (b) Continuous Approximation

The sampler can be approximated by $1/T$ and a first-order Padé approximation $G_{pa}(s)$ can be used to linearize and approximate the ZOH. Thus,

$$G_{pa}(s) = \frac{1 - e^{-sT}}{s} = \frac{2}{s + \frac{2}{T}} = G_{ps}(s).$$  
(2.25)

The first order Padé approximation is valid for this system because $\omega_s \geq 10 \omega_c$. [4:213]. The combination of the sampler and ZOH approximations are defined as $G_{sd}(s)$ which represent the reduced, approximated, and linearized effects of digital control (see Figure 2-16(b)).

**2.7 SBD Model Summary**

The optimal controller is designed around the nominal model, which includes:

current loop gain constant $K_a$; motor torque constant $K_T$; gimbal dynamics $G_g(s)$; torque
disturbance input $T_d(t)$; and zero-mean, white Gaussian noise, of 1 μrad intensity sensor noise. The truth or simulation model includes the nominal model plus: dynamics of the current loop $G_{cl}(s)$; non-linear bearing friction; first order Padé approximation to a ZOH; and sensor noise rolled off at 40 dB per decade. The azimuth axis parameter values are summarized in Table 2-2.

<table>
<thead>
<tr>
<th>Table 2-2 Azimuth Axis Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Moment of Inertia (ft-lbs-sec²)</td>
</tr>
<tr>
<td>Coefficient of Viscous Friction (ft-lbs/rad/sec)</td>
</tr>
<tr>
<td>Motor Torque Constant (ft-lbs/amp)</td>
</tr>
<tr>
<td>Sample Period (sec)</td>
</tr>
<tr>
<td>Current Loop Gain Constant (amp/volt)</td>
</tr>
<tr>
<td>Current Loop Damping Ratio</td>
</tr>
<tr>
<td>Current Loop Natural Frequency (rad/sec)</td>
</tr>
<tr>
<td>Bearing Friction Torque Constant (ft-lbs)</td>
</tr>
</tbody>
</table>
III. Optimal Control Design Techniques

3.1 Background

$H_2$ or $H_{\infty}$ control design begins with the standard positive feedback setup shown in Figure 3-1.

\begin{equation}
\begin{bmatrix}
P_{zw} & P_{zu} \\
P_{yw} & P_{yu}
\end{bmatrix}
\end{equation}

such that

\begin{equation}
\begin{bmatrix}
z \\
y
\end{bmatrix} =
\begin{bmatrix}
P_{zw} & P_{zu} \\
P_{yw} & P_{yu}
\end{bmatrix}
\begin{bmatrix}
w \\
u
\end{bmatrix}
\end{equation}
The closed-loop transfer function $T_{zw}$, which is the transfer function from the external input $w$ to the controlled output $z$, is given by

$$T_{zw} = P_{zw} + P_{zw}K(I - P_{wu}K)^{-1}P_{yw}$$

(3.3)

The objective is to design a stabilizing controller $K$ such that the 2-norm or $\infty$-norm of the closed-loop transfer function $T_{zw}$ is minimized. Controller design by $H_\infty$ optimization for the SBD model was less than completely successful (see section 4.2). Therefore, the $H_\infty$ optimization theory and method is not presented.

The 2-norm is defined for a stable transfer function $T_{zw}$ as

$$\|T_{zw}\|_2 = \left(\frac{1}{2\pi} \int_{-\infty}^{\infty} \text{tr}\left[T_{zw}^*(j\omega)T_{zw}(j\omega)\right] d\omega\right)^{1/2}$$

(3.4)

where $\text{tr}$ is the trace and $^*$ denotes the complex conjugate transpose. Thus, the 2-norm of the transfer function $T_{zw}$ is minimizing the expected value of the energy of the output $z$ assuming the input $w$ is zero-mean, unit intensity, white Gaussian noise.

The plant $P$ is described by the state-variable differential equations

$$\dot{x} = Ax + B_w w + B_u u$$

$$z = C_z x + D_{zw} w + D_{zu} u$$

$$y = C_y x + D_{yw} w + D_{yu} u$$

(3.5)

or

$$P = \begin{bmatrix} A & B_w & B_u \\ C_z & D_{zw} & D_{zu} \\ C_y & D_{yw} & D_{yu} \end{bmatrix}$$

(3.6)
The following conditions on $P$ must be true:

1) $D_{zw} = 0$

2) $D_{yw} = 0$

3) $(A, B_u)$ stabilizable & $(C_y, A)$ detectable

4) $D_T D_{zu} & D_{yw}^T$ full rank

5) $\begin{bmatrix} A - j\omega I & B_u \\ C_z & D_{zu} \end{bmatrix}$ has full column rank for all $\omega$

6) $\begin{bmatrix} A - j\omega I & B_w \\ C_y & D_{yw} \end{bmatrix}$ has full row rank for all $\omega$

Condition i) is required or the $H_2$ problem is not well defined, as the closed-loop transfer function will then have a nonzero $D$ term for any choice of compensator, thus making the closed-loop two-norm infinite. Condition ii) makes the development easier, but can be completely removed. Condition iii) is necessary for the existence of stabilizing solutions. Condition iv) ensures that the penalty on control usage and the sensor noise intensity are nonsingular - relaxation leads to singular control problems. Conditions v) and vi) guarantee the existence of stabilizing solutions to the two Riccati equations which appear in the solution to the problem. [8:69-70]

The $H_2$ optimal controller is given by

$$K(s) = -\tilde{K}_c \left( A - K_f C_y - \tilde{B}_u K_c \right) \tilde{K}_f$$

(3.7)

where

$$K_c = \tilde{B}_u^T X + \tilde{D}_{zu}^T C_z$$

(3.8)

$$\tilde{K}_c = S_u^{-1} K_c$$

(3.9)
\[ K_f = Y\hat{C}_y^T + B_u\hat{D}_{yw} \]  
(3.10)

\[ \hat{K}_f = K_fS_y \]  
(3.11)

\(X\) and \(Y\) are the solutions to the algebraic Riccati equations

\[
(A - \tilde{B}_u\tilde{D}_{zu}^T\tilde{C}_z)X + X(A - \tilde{B}_u\tilde{D}_{zu}^T\tilde{C}_z) - X\tilde{B}_u\tilde{B}_u^TX + \hat{C}_z^T\hat{C}_z = 0
\]  
(3.12)

and

\[
(A - B_w\hat{D}_{yw}\hat{C}_y)Y + Y(A - B_w\hat{D}_{yw}\hat{C}_y)^T - Y\hat{C}_y^T\hat{C}_yX + \hat{B}_w\hat{B}_w^T = 0
\]  
(3.13)

where

\[
\hat{C}_z = (I - \hat{D}_{zu}\hat{D}_{zu}^T)\tilde{C}_z
\]  
(3.14)

\[
\hat{B}_w = B_w(I - \hat{D}_{yw}\hat{D}_{yw}^T)
\]  
(3.15)

\(S_u\) and \(S_y\) are internal scalings such that

\[
\tilde{B}_u = B_uS_u^{-1} \quad \tilde{C}_y = S_yC_y
\]  
(3.16)

\[
\hat{D}_{zu} = D_{zu}S_u^{-1} \quad \hat{D}_{yw} = S_yD_{yw}
\]

For \(H_2\) optimization the compensator \(K\) will be of the same order as the plant \(P\).

For a more thorough description of \(H_2\) and \(H_\infty\) optimization see reference [8].

### 3.2 Design Techniques

The first step in the optimal control design sequence is building the \(P\) matrix, followed by calculating the stabilizing and minimizing compensator \(K\) using \(H_2\) or \(H_\infty\) optimization, then simulating the compensator with the truth model to determine if the desired performance is achieved. Overall, this is an iterative process in which the control
weightings in $P$ are used like "dials" to "tune" system performance. It is important that
the nominal plant model is used in building $P$ to keep the order of the compensator to a
minimum.

A key tool for building the $P$ matrix is the use of a state-space block diagram shown
in Figure 3-2 representing the transfer function

$$G(s) = C(sI - A)^{-1}B + D. \quad (3.17)$$

Figure 3-2 State-Space Block Diagram Form

Once an optimal control problem is set up with the system plant, coloring filters, and
control weightings represented in state-space block diagram form, it is easy to augment
all the states into one state-space representation of the entire system. That state space
representation is Eqns. 3.5, which as discussed earlier, define the $P$ matrix.

A designer has many configurations to choose from when setting up the optimal
control problem. There are two basic optimal control design setups which are presented
in this section. Starting with a basic setup, control weightings are added depending on
the performance requirements of the system, such as weightings on control usage, states,
sensitivity, complementary sensitivity, and tracking. The two configurations presented
represent configurations of two of the final controller designs for the SBD.

3.2.1 Unity Feedback Setup. The unity feedback setup shown in Figure 3-3 has a
constant weighting $R_c$ on control usage $u(t)$, a constant weighting $H$ on the states $x_p(t)$. 
and a dynamic weighting $W_d$ on sensitivity. The blocks $A_p, B_p, \text{ and } C_p$ are the state-space representation of the nominal plant $G_p(s)$. The controller $K$ is replaced with $G_c$ and is not represented in state-space form because it is not part of the $P$ matrix. $W_d$ is the plant disturbance coloring filter and $\Gamma$ is a constant matrix used to distribute the disturbance to the proper state. The inputs $w_1$ and $w_2$ are zero-mean, white Gaussian noise of unit intensity, where $w_1$ is the disturbance input and $w_2$ represents sensor noise. The reference command $\theta_c(t)$ is not actually part of the setup for building the $P$ matrix, but is shown to make the system complete. This means that command following is not directly optimized in the unity feedback setup. Robust state regulation is the real objective of this setup. In fact, if the dynamic sensitivity weighting $W_d$ is not included, $H_2$ optimization on this setup is the same as the standard linear quadratic Gaussian (LQG) problem of minimizing the cost function

$$J = \int_0^\infty (x^TQx + u^TRu) dt$$

(3.18)

where

$$Q = H^TH \quad \text{and} \quad R = R_c^TR_c$$

(3.19)
Although good command input tracking is not a direct specification in this setup, it is indirectly achieved by use of the dynamic sensitivity weighting $W_s$. The term sensitivity refers to the output's sensitivity to plant uncertainties and/or disturbances and is defined in the SISO case as

$$S = \frac{1}{1 - GpGc} \quad (3.20)$$

It is desirable in the case of the SBD for the system to be insensitive to low frequency plant disturbances, thus $S$ would have a general shape like that shown in Figure 3-4. "General" is stressed because this discussion assumes that the plant disturbance is coming in at the plant output, where, in the case of the SBD the disturbance is actually a torque disturbance into the gimbal. The difference is elaborated when discussing the actual SBD.
design in Chapter 4. S being "small" at low frequency is achieved by making $G_c$ "large" or having high gain at low frequency. High gain at low frequency also yields good tracking as seen in the closed-loop tracking error response to the reference command and plant disturbance.

$$\text{err} = -\frac{1}{1 - G_p G_c} \theta_c + \frac{W_d}{1 - G_p G_c} w_i$$ \hfill (3.21)

![Figure 3-4 General Shape of Sensitivity](image)

3.2.1.1 Control Weighting Rationale. Choosing control weighting values is not an exact science; rather it is an iterative process. However, it is important to understand the effects of the different weightings to provide a starting point.

The control usage weighting $R_z$ must always be included and have a $D$ term in the optimal control setup or condition iv) will be violated and the problem will be singular. Theoretically, if $R_z = 0$ there is no penalty on control usage allowing an infinite amount of control power to achieve the system requirements. $R_z$ can be a dynamic weighting if, for example, it is desired to restrict high frequency control usage. The next chapter will
address that requirement. The effects of \( R_2 \) can be seen by looking at the closed-loop transfer function of \( z_2 \) to \( w \).

\[
z_2 = \frac{R_2 G_c W_d}{1 - G_p G_c} w_1 + \frac{R_2 G_c}{1 - G_p G_c} w_2
\]  

(3.22)

As \( R_2 \) is increased, the minimization of \( T_{zw} \) will yield a decreasing \( G_c \), or in other words, less control usage.

The state weighting \( H \) can be used to minimize some states more than others. For example, in the case of the SBD system, if it were necessary to limit the velocity of the gimbal a large weighting would be put on the state \( \dot{\theta}(r) \).

The sensitivity weighting \( W_s \) is chosen to achieve the desired \( S \) by picking it to generally look like \( 1/S \); therefore, \( W_s \) would have high gain at low frequency. \( W_s \) must roll off such that it does not contain a \( D \) term and violate condition i). The magnitude and bandwidth would depend on the severity of the disturbance input \( W_d \) and the command following requirements, i.e., step, ramp, speed of response, etc. The effects of sensitivity weighting can be seen by looking at the closed-loop transfer function of \( z_3 \) to \( w \).

\[
z_3 = \frac{W_s W_d}{1 - G_p G_c} w_1 + \frac{W_s}{1 - G_p G_c} w_2
\]  

(3.23)

As the low frequency magnitude of \( W_s \) is increased and/or the bandwidth is increased minimization of \( T_{zw} \) will yield an increasing \( G_c \) approximately over the same bandwidth. Note that if \( W_s \) is given a relatively small low frequency gain, minimization would still yield \( G_c \) with high gain at low frequency because the disturbance coloring filter \( W_d \) has high gain at low frequency. However, the controller would typically fall short of the desired performance, thus the "dial" \( W_s \) is needed to "tune" the performance.
3.2.1.2 Building the P Matrix. The augmented state-space representation of the optimal control setup in Figure 3-3 is described by Eqns. 3.5. $P$ is then formed by Eqn 3.6 where,

$$
A = \begin{bmatrix}
A_p & C_d & 0 \\
0 & A_d & 0 \\
B_p C_p & 0 & A_s
\end{bmatrix},
B_w = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix},
B_u = \begin{bmatrix}
B_p \\
0 \\
0
\end{bmatrix},
C_z = \begin{bmatrix}
H & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & C_s
\end{bmatrix},
D_{zw} = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix},
D_{zn} = \begin{bmatrix}
0
\end{bmatrix},
C_y = \begin{bmatrix}
C_p & 0 & 0
\end{bmatrix},
D_{yw} = \begin{bmatrix}
0 & 1
\end{bmatrix},
D_{yu} = \begin{bmatrix}
0
\end{bmatrix}
$$

(3.24)

3.2.1.3 $H_2$ and $H_{\infty}$ Algorithms. Once the $P$ matrix has been built there are several $H_2$ and $H_{\infty}$ algorithms that take the $P$ matrix as an input and then output the stabilizing and minimizing controller in state-space form. All the algorithms listed are Matlab script M-files: h2lqg and hinf are found in the Robust-Control Toolbox; hinfsyn and hinffi are $H_{\infty}$ routines found in the relatively new $\mu$-Tools that calculate output feedback sub-optimal controllers and full state feedback sub-optimal controllers, respectfully; h2opt and hinf5 are user friendly algorithms used in this thesis and were written by Dr. Brett Ridgely.

3.2.2 Two-Degree of Freedom Controller Setup. The two degree of freedom controller setup shown in Figure 3-5 is a method which allows for the direct design of command following or tracking performance. This setup fits into the standard optimal control setup of Figure 3-1, where $u = Ky$, with $K$ partitioned as
\[ K = \begin{bmatrix} Gc_1 & Gc_2 \end{bmatrix} \]  \hspace{1cm} (3.25)

and

\[ y = \begin{bmatrix} \theta_m \\ \theta_r \end{bmatrix} \]  \hspace{1cm} (3.26)

The control weightings \( R_z \) and \( H \) are used in the same manner as with the unity feedback setup. \( W_t \) is a dynamic track weighting that is used to directly minimize tracking error. The effect of \( W_t \) can be seen in the transfer function of \( z_3 \) to \( w \):

\[ z_3 = \frac{W_t W_d}{1 - Gc_1 Gp} w_i + \frac{W_t Gc_1}{1 - Gc_2 Gp} w_2 + \left( \frac{W_t Gc_2}{1 - Gc_2 Gp} - W_t \right) w_3 \]  \hspace{1cm} (3.27)

Tracking performance is normally measured in terms of the ability of a system to track step or ramp inputs; therefore, emphasis of \( W_t \) should be high gain at low frequency. An example of what \( W_t \) would generally look like is shown in Figure 3-6. An indirect result of this setup, with the disturbance coloring filter \( W_d \) having high gain at low frequency, is good sensitivity or disturbance rejection in the feedback loop. It can be seen from Eqn. 3.27 that minimization of \( T_{zw} \) will tend to make \( Gc_1 \) large at low frequency.
Figure 3-5 Two Degree of Freedom Controller Setup

Figure 3-6 Track Weighting
IV. SBD Azimuth Axis Controller Design

4.1 Introduction

This chapter is devoted to a detailed description of the two basic design setups for the azimuth axis. The simulation results of the $H_2$ optimal two degree of freedom controller are also presented in this chapter, while the results of the $H_2$ optimal unity feedback controller are presented in Chapter 5. The Matlab M-files used for building the $P$ matrix, for synthesizing the $H_2$ optimal controller, and for performance analysis are included in Appendix B. A tutorial of the $H_2$ optimization M-file h2opt and M-files specifically written for the SBD controller design is given in Appendix D.

4.2 Scaling Problem

Both the $H_2$ and $H_\infty$ algorithms had numerical problems with the large controller gain required to achieve $\mu$rad tracking performance. The $H_2$ problem was solved by scaling the plant $G_p(s)$ to microradian output. Therefore, the resulting controller $G_c(s)$ had $10^6$ less gain than it would have had with an unscaled plant. $H_2$ optimization was performed with the scaled plant and simply increasing the resulting controller by $10^6$ rescaled the system. The scaling equations were

$$G_p(s)_{\text{scaled}} = G_p(s) \times 10^6 \quad \& \quad G_c(s) = G_c(s)_{\text{scaled}} \times 10^6 \quad (4.1)$$

where $G_c(s)_{\text{scaled}}$ was the controller yielded by the algorithm.

The $H_\infty$ problem was not as straightforward, and all internal system scaling attempts yielded less than completely successful results. In-depth investigation of the scaling problem $H_\infty$ had with the model was beyond the scope of this thesis; therefore, the $H_\infty$ solution was not pursued any further.
4.3 Position Command Inputs

Although in-depth performance analysis is presented in Chapter 5, some analysis is done in this chapter. Therefore, it is appropriate at this time to describe the position command inputs used to determine performance.

Step response performance was baselined against a 100 μrad step input; however, tracking commands for a beam director, whether tracking a star or a fast moving low-earth satellite, are more accurately described by a ramp input. For the SBD simulation the step sizes chosen for the ramp were 500 μrad. With a servo loop sample rate of 200 Hz this was equivalent to 0.1 rad/sec or approximately 6° per second, which approaches the maximum azimuth axis velocity of 10° per second.

A true test of performance is to simulate a system with a command input that represents real ephemeris data of a low-earth orbit satellite near culmination, for this is when the azimuth axis is at maximum velocity. A five second sample of position and velocity data extracted from ephemeris data generated at the SBD is shown in Figures 4-1 and 4-2, respectively. Like the ramp command, the gimbal velocity due to the ephemeris command had an average value of approximately 6° per second. However, as seen in Figure 4-2 there was an acceleration component in the ephemeris data, which made the tracking problem more difficult. The position data was edited such that the initial position was 0.0 and that it represented an ascending portion of a satellite pass. The original data was from a descending portion of a pass. This five second sample of data was used for all ephemeris command tracking simulations.
4.4 Unity Feedback Design

Figure 4-3 shows the design setup, where the sensitivity weighting $W_s$ was used for plant disturbance rejection and indirectly for good tracking performance. As stated in Section 3.2.1, sensitivity is defined for a disturbance added to the plant's output. The disturbance $w_1$ actually entered as a torque disturbance to the gimbal. However, setting up the optimal control problem with the disturbance modeled as torque yielded controllers of marginal performance for all weightings $W_s$ that were tried. Therefore, the optimal control $P$ matrix was built with $w_1$ entering as a position disturbance at the plant's
output and the simulation had the disturbance entering as torque. The same disturbance coloring filter $W_d$ was used for both the controller design and the simulation (see Figure 2-15). It would seem that the disturbance entering as torque would not be the same as the disturbance entering as position in $\mu$rad, because the torque disturbance goes through the gimbal dynamics, which is practically a double integrator. However, in this case, the random white noise generated by Matlab contained the appropriate low frequency spectrum such that the open loop feed-through of the torque disturbance was almost the same as the disturbance spectrum in $\mu$rad.

**Figure 4-3** Unity Feedback Setup

### 4.4.1 Control Weightings

The control weightings used in the setup were the control usage weighting $R_z$, the state weighting $H$, and the sensitivity weighting $W_s$. Larger values of $R_z$ yielded controllers of lower gain and vice versa. The final value chosen was $R_z = 1$. The nominal plant, where the only dynamics included were that of the gimbal, had two states, $\theta(t)$ and $\dot{\theta}(t)$. It was not necessary to weight the states for the
purpose of minimizing the amplitude of a particular state, which is normally the main objective of the state weighting $H$. However, including some weighting proved to yield a system with slightly better gain and phase margins than when a weighting was not included. The final value chosen was a weighting of 1 on both states such that

$$H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(4.2)

The fact that including a weighting on the states resulted in better stability robustness warrants further explanation. Consider the perturbed plant

$$\hat{G}_p(s) = (1 + \Delta)G_p(s)$$

modeled as output multiplicative uncertainty, where $G_p(s)$ is the nominal plant and $\Delta$ is the uncertainty. The closed-loop block diagram corresponding to the controller and this representation of the plant is shown in Figure 4-4. Now consider the case where $\Delta = 1$; the closed-loop transfer function of $T_{zw}$ is equal to complimentary sensitivity $T$, where

$$T = \frac{C_p(sI - A_p)^{-1} B_p G_c}{1 - G_c G_p}$$

(4.3)

$G_p(s)$ in the numerator is represented in its state-space transfer function form for demonstration. Minimization of complimentary sensitivity provides increased stability robustness; therefore, minimization of the transfer function $T_{zw}$ yields better gain and phase margins [9:53-54].
Now consider the block diagram in Figure 4-5, which represents part of the closed-loop unity feedback setup for the SBD system. $z_1$ is the state control output and $w_2$ is the sensor noise input. Indirectly, the state weighting $H$ provides for output multiplicative uncertainty such that the transfer function of $T_{z_1w_2}$ is similar to the complimentary sensitivity, thus resulting in better stability robustness. It is clear that the complimentary sensitivity $T$ in Eqn. 4.3 is similar to $T_{z_1w_2}$, where

$$T_{z_1w_2} = \frac{H(sI - A_p)^{-1}B_p G_c}{1 - GpGc}$$

(4.4)

and

$$C_p = [1 \ 0] \quad H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(4.5)

Figure 4-5 Closed-Loop State Weighting Block Diagram
The key to designing a good controller was in the sensitivity weighting. After many iterations of "dialing" $W_s$, the final had a low frequency gain of $2 \times 10^4$ and a pole at $s = -10^{-4}$ rad/sec (see Figure 4-6). The high gain of $W_s$ was necessary to achieve the system bandwidth and high gain at low frequency needed to reject the plant disturbance. The low frequency pole gave the controller a near integrator necessary for command input tracking and rejection of the non-linear bearing friction disturbance, which is discussed further in Chapter 5. The state-space representation of $W_s$ is

$$\begin{align*}
\dot{x}_s(t) &= A_s x_s(t) + B_s \theta_m(t) \\
z_3 &= C_s x_s(t) + D_s \theta_m(t)
\end{align*}$$

(4.6)

Where $z_3$ is the controlled output, $x_s(t)$ is the sensitivity weighting state, and $\theta_m(t)$ is the measured gimbal position. The state-space values are,

$$\begin{align*}
A_s &= -0.0001 & B_s &= 1 \\
C_s &= 2 & D_s &= 0
\end{align*}$$

(4.7)

Figure 4-6 Sensitivity Weighting
4.4.2 **H₂ Controller Synthesis.** With the nominal plant, exogenous inputs, and control weightings defined in state-space form, the \( P \) matrix, described by Eqns. 3.24 was built. The Matlab M-file h2opt was used to synthesize the H₂ optimization controller. As stated earlier, optimal control produces controllers of the same order as the plant \( P \); thus, with a 2\(^{nd} \) order nominal plant, one pole in \( W_d \), and one pole in \( W_s \), the resulting controller was 4\(^{th} \) order.

4.4.3 **Controller Fine Tuning.** The controller produced by the weightings presented in Section 4.4.1 exceeded the tracking and disturbance rejection requirements. However, the controller contained relatively high gain at high frequency; in other words, it did not roll off rapidly after the bandwidth of the system. The scope of this thesis did not include the consideration of the gimbal's resonant frequencies, and as stated earlier notch filtering would have to be implemented. However, knowing that the resonant frequencies exist, it was important to roll off the controller as rapidly as possible without effecting the bandwidth or decreasing the gain and phase margins to unacceptable levels. There were two ways to correct this problem: one was to put a dynamic weighting \( W_{R_c} \) on control usage to minimize high gain at high frequency, and the other was to "manually" grab the high frequency poles in the controller and place them at lower frequency, thus rolling off the controller immediately after the bandwidth. Figure 4-7 compares the magnitude frequency responses of the original controller, the controller yielded from the dynamic weighting \( W_{R_c} \) design, and the controller resulting from placing the poles manually.\(^5\) Table 4-1 shows the gain margin, phase margin, and the bandwidth. For positive feedback, the gain and phase margins are taken from the open loop system \(-G_pG_c\), while the bandwidth is defined as the 0 dB cross-over frequency \( \omega_c \) of \( 1-G_pG_c \).

Reducing high frequency control power has a negative effect of reducing robustness, but the gain and phase margins are adequate for all the controllers. The pole placement

\(^5\)The controllers have been rescaled to include the \( 10^6 \) gain.
controller had the best simulated performance of all three, and thus was chosen as the final design for the azimuth axis controller.

The next two sections elaborate on the methods used to reduce the high frequency gain of the original $H_2$ controller.

![Controller Magnitude Frequency Response](image)

**Figure 4-7** Controller Magnitude Frequency Response

**Table 4-1** Unity Feedback Controller Comparison

<table>
<thead>
<tr>
<th>Controller Design</th>
<th>Gain Margin</th>
<th>Phase Margin</th>
<th>$\omega_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original $H_2$</td>
<td>(-11.7, 16)</td>
<td>± 50</td>
<td>52</td>
</tr>
<tr>
<td>Control Usage Weighting $WR_z$</td>
<td>(-10.8, 15)</td>
<td>± 42</td>
<td>48</td>
</tr>
<tr>
<td>Final</td>
<td>(-10.5, 11)</td>
<td>± 36</td>
<td>52</td>
</tr>
</tbody>
</table>

**4.4.3.1 Control Use Weighting.** The weightings used for the original $H_2$ design did not put any restriction on high frequency control use. As such, they met all the objectives stressed by the weightings, but the controller contained high gain at high
frequency. By using a dynamic weighting on control usage $W_{Rz}$, shown in Figure 4-8, the minimization of high frequency control usage was achieved. A second order weighting would have rolled the controller off quicker without effecting bandwidth; however, the compensator would have been 6th order. Being that it was desirable to keep the order to a minimum and that this was a relatively simple SISO system, there was a more effective way of rolling off the controller.

![Figure 4-8 Control Usage Weighting](image)

**Figure 4-8 Control Usage Weighting**

**4.4.3.2 "Manual" Pole Placement.** A closer look at the compensator yielded by $H_2$ optimization, after rescaling, revealed that a complex pair of poles were placed at high frequency.

$$G_c(s) = \frac{-4.5342 \times 10^{12}(s+.013462)(s^2 + 29.4s + 441.1073)}{(s+.0001)(s+.99996)(s^2 + 44329s + 1.002624 \times 10^9)}$$  \hspace{1cm} (4.8)

After carefully decreasing the frequency of the complex pair, keeping the damping ratio of the complex pair and system bandwidth the same, and keeping the gain and phase margins acceptable, the resulting controller was

$$G_c(s) = \frac{-360 \times 10^6(s+.013462)(s^2 + 29.4s + 441.1073)}{(s+.0001)(s+.99996)(s^2 + 424s + 90000)}$$  \hspace{1cm} (4.9)
The compensator in Eqn. 4.9 was the final design used in the comparison with the current controller and it's performance analysis is presented in Chapter 5. The rescaled negative feedback state-space representation of this controller is found in Appendix A. Although the gain and phase margins differed between this compensator and the original of Eqn. 4.8, they had the same performance, i.e. disturbance rejection, speed of response, and tracking error, but the original design had substantially more control power at frequencies past the system bandwidth. For example, comparison of the two controllers tracking five seconds of real ephemeris data revealed the motor torque of the original controller system was 644 ft-lbs RMS, and the final design had 244 ft-lbs RMS. Again, since this was a SISO system, the task of grabbing the poles and dragging them to lower frequency was relatively straightforward. However, had this been a complex MIMO system, manually moving the poles could have been difficult and dangerous, because stability is not guaranteed as it is when using a control usage weighting and $H_2$ optimization.

4.5 Two-Degree of Freedom Controller Design

Figure 4-9 shows the two-degree of freedom controller design setup, where the track weighting $W_t$ was used to minimize tracking error and indirectly to yield good disturbance rejection. The disturbance coloring filter $W_d$ and the control weightings $R_z$ and $H$ had the same values as those used in the unity feedback design. $W_t$ was designed with high gain at low frequency to emphasize step and ramp tracking. Two of the more successful $W_t$ designs are presented: the first had a low frequency gain of 60 dB and a stable pole at $10^{-3}$ rad/sec; the second had a low frequency gain of 180 dB, one stable pole at $10^{-5}$ rad/sec, and another stable pole at $10^{-3}$ rad/sec. The latter is shown in Figure 4-10. The double pole $W_t$ yielded a system with higher gain at low frequency while maintaining the same bandwidth.
Gain and phase margins and bandwidth, listed in Table 4-2, for the two degree of freedom controller are dependent on the feedback loop of \(-GpGc_1\); the preprocessor \(Gc_2\).
does not effect system stability. The margins were adequate and the bandwidth was sufficient for speed of response and disturbance rejection.

Table 4-2 Two Degree of Freedom Controller Comparison

<table>
<thead>
<tr>
<th>Controller Design</th>
<th>Gain Margin (dB)</th>
<th>Phase Margin (Degree)</th>
<th>$\omega_c$ (rad/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Order $W_I$</td>
<td>(-11.8, 20)</td>
<td>$\pm 51$</td>
<td>46</td>
</tr>
<tr>
<td>2nd Order $W_I$</td>
<td>(-7.5, 20)</td>
<td>$\pm 50$</td>
<td>47</td>
</tr>
</tbody>
</table>

Figures 4-11 and 4-12 show the step and ramp responses of both controller designs, respectfully. The 1st order weighting yielded a system with a fast, well damped step response; however, the ramp tracking response had an extremely large steady state error. The 2nd order weighting yielded a system with a fast, poorly damped step response and a ramp response with acceptable steady state error of approximately 1 μrad. The steady state error is clearly seen in Figure 4-13, where the ramp response had been truncated to remove the initial transient response. Figure 4-14 shows the tracking error to ephemeris command input. The truncated data from the ephemeris command tracking error response shown in Figure 4-15 revealed that even the 2nd order track weighting system had poor steady state error. A third order track weighting design was attempted to cope with ephemeris tracking, but was unsuccessful. A gimbal velocity initial condition was given in the simulation of the ramp and ephemeris command input to prevent the initial transient from being too severe. As seen in the figures, the initial transients were still extremely large. Therefore, even if the steady state error was adequate, none of the two degree of freedom designs were acceptable for ramp or ephemeris command tracking.
**Figure 4-11** Step Response

**Figure 4-12** Tracking Error to Ramp Command
Figure 4-13 Truncated Tracking Error to Ramp Command

Figure 4-14 Tracking Error to Ephemeris Command
Figure 4-15 Truncated Tracking Error to Ephemeris Command
V. Azimuth Axis Simulations and Results

5.1 Introduction

This chapter provides detailed comparison of the final azimuth axis $H_2$ optimization unity feedback controller and the azimuth axis controller currently being used at the SBD. Comparison is based on simulations performed with the controllers and the SBD truth model developed in Chapter 2. The Simulink models used for simulations are shown in Appendix C, and a tutorial on the execution of the models is included in Appendix D.

5.2 Controller Comparison

The open-loop Bode frequency responses of $G_p(s)G_c(s)$ for the current SBD system and $-G_p(s)G_c(s)$ for the $H_2$ system are shown in Figures 5-1 and 5-2, where $G_p(s)$ is the truth model. Recalling that the $H_2$ controller is based on positive feedback, the open-loop Bode frequency response is defined as $-G_p(s)G_c(s)$. The current SBD system is very robust with gain margins of (-35 dB, 16 dB) and phase margin $\pm 60^\circ$. The system with the $H_2$ controller is not as robust, but has adequate gain margins of (-10.5 dB, 11 dB) and phase margin $\pm 36^\circ$. The bandwidths for the current and $H_2$ systems are 49.7 rad/sec or 7.9 Hz and 51.6 rad/sec or 8.2 Hz, respectively.
Figure 5-1 Bode Magnitude Frequency Response

Figure 5-2 Bode Phase Frequency Response
5.2.1 Current SBD Controller. The current SBD azimuth axis controller is described by the transfer function

\[ G_c(s) = \frac{10^6(s + 0.3)(s + 8.1)}{s(s + 256)} \]  (5.1)

The controller is designed in a classical sense in that it consists of PI control and a lead, where high gain and the lead are required to achieve the necessary bandwidth for speed of response and plant disturbance rejection, and the integrator is necessary to provide a near type 3 system to track the parabolic ephemeris command with zero steady state error. "Near" type 3 is used to describe the system because the gimbal dynamics have one pure integrator and one "near" integrator, a low frequency pole due to the damping effect of viscous friction as described in Chapter 2.

An integrator in the compensator is also needed to reject the non-linear bearing friction. When the bearings are rolling the constant running friction torque \( T_c \) of 16 ft-lbs is like a step torque disturbance input. The definition of system type for unity feedback is based on the reference command input to the controller, not as an input disturbance to the plant; therefore, even though the gimbal is practically a double integrator it does not necessarily yield zero steady state error to a step torque disturbance. It can be seen from Eqn. 5.2 that \( G_c(s) \) must contain an integrator to achieve zero steady state error to a step torque disturbance input.

\[
err_{ss} = \lim_{s \to 0} s \left( \frac{G_g(s)}{1 - G_g(s)K_TG_{cl}(s)G_c(s)} \right) \frac{1}{s} \]  (5.2)

5.2.2 H₂ Optimization Controller. Again, it is pointed out that the final controller used for comparison and contrast with the current controller was obtained by placing the high frequency poles of the original H₂ design at lower frequency, as described in Section 5-3.
4.4.3.2. The controller, repeated as Eqn. 5.3, is similar to the current controller in that it has high gain and leads to achieve the bandwidth necessary for speed of response and disturbance rejection. Although it does not contain a free integrator, it does have a "near" integrator that proved to yield more than adequate steady state error to a pure ramp and constant torque disturbance $T_c$ input. Simulation indicated that the steady state error due to a pure ramp input was less than .05 \mu rad and was also less than .05 \mu rad due to $T_c$.

$$G_c(s) = \frac{-360 \times 10^6(s+.013462)(s^2 + 29.4s + 441.1073)}{(s+.0001)(s+.99996)(s^2 + 424s + 90000)}$$ (5.3)

5.3 Simulations and Results

Simulations were performed with the torque disturbance input $w_1$, the sensor noise input $w_2$, step, ramp, and ephemeris command inputs as described in Section 4.3, and the non-linear bearing friction torque disturbance $T_c$. The inputs were simulated separately to determine the individual effects, and combinations of the inputs were simulated to determine the total system performance.

5.3.1 Torque Disturbance and Sensor Noise. The simulated torque disturbance driven by zero mean, unit intensity, white Gaussian noise, and shaped by $W_d$, had the time response shown in Figure 5-3. The position error due to the torque disturbance is shown in Figure 5-4. The current and $H_2$ systems had 2.12 and 0.47 \mu rad RMS of position error, respectively. The disturbance rejection transfer function with units of \mu rad per ft-lb is shown in Figure 5-5. It is clear that the $H_2$ controller had better disturbance rejection. However, as stated earlier the torque disturbance was modeled conservatively; therefore, the error due to torque disturbance may not be as severe in the actual SBD system.
Figure 5-3 Torque Disturbance

Figure 5-4 Position Error to Torque Disturbance
Sensor noise was simulated with zero mean, unit intensity, white Gaussian noise, and filtered to prevent the unrealistic driving of high frequency control power. The position error due to sensor noise was approximately 0.3 µrad RMS for both systems.

5.3.2 Step Response. Figure 5-6 shows the responses to a 100 µrad step input, and Figure 5-7 is the closed-loop transfer function \( \theta(s)/\theta_c(s) \) of the two systems. The \( \text{H}_2 \) system had a 2% settling time of approximately 300 msec. The current controller had a 2% settling time of 250 msec and was better damped with a damping ratio of 0.6 compared with 0.3 for the \( \text{H}_2 \) system. Although analysis of voltage saturation (± 10 volts) was not performed, both systems would have saturated with a 100 µrad step. The current system required approximately 50% more control power than the \( \text{H}_2 \) system to achieve the well damped performance; therefore, had saturation been modeled, saturation for the current system would have been more severe. Greater saturation results in more overshoot and longer settling time [2:217-218]. Therefore, the well damped response does not necessarily mean that the current controller is a better design for step inputs.
Figure 5-6  Step Response

Figure 5-7  Closed-Loop Transfer Function
5.3.3 **Ramp Response.** As described in Section 1.4, bang-bang control is used to move the gimbal large distances. Thus, by the time control is switched to the fine position loop, the gimbal will have some initial velocity. To make the simulation more realistic an initial velocity condition is given. Actually, all the states, including the controller state, will have initial conditions, but experimenting with the initial conditions indicated that only the velocity initial condition was important. The first simulation presented was done without an initial condition for the purpose of contrasting a key difference between the two controllers. Figure 5-8 revealed that the amplitudes of the initial transients were about the same for both systems. In Figure 5-9 the initial transient response was removed by truncating the data to get a closer look at the steady state error. The $H_2$ controller produced near zero steady state error within 300 msec, while the current system had a settling time of greater than 10 seconds. This was the most significant difference between the two controllers.

![Figure 5-8 Ramp Command Tracking Error](image)

**Figure 5-8** Ramp Command Tracking Error
The ramp command input has a slope of 0.1 rad/sec; therefore, the next simulation included an initial velocity of 0.1 rad/sec. The simulation also included the effects of the constant torque disturbance $T_c$. It would have been more realistic to pick an initial velocity less than 0.1 rad/sec, but to clearly illustrate the effects of bearing friction it was necessary to keep the initial transient to a minimum. Figure 5-10 and Figure 5-11 compare the effects of bearing friction of the current and $H_2$ controllers, respectively. Again, the large settling time with the current controller is seen. Both controllers did an adequate job rejecting the bearing friction torque disturbance. The small steady state error due to bearing friction with the $H_2$ controller can be seen in Figure 5-11 as the difference between the two curves.

Figure 5-9 Truncated Ramp Command Tracking Error
Figure 5-10 Bearing Friction Effect With Current Controller

Figure 5-11 Bearing Friction Effect With H2 Controller
5.3.4 Ephemeris Response. It is necessary at this time to make the distinction between RMS tracking error and steady state tracking error. Pointing and tracking requirements are usually split into two specifications: the first being pointing error defined as the bias or mean error, referred to in this thesis as the steady state error $err_{ss}$; the second is the tracking jitter, which is the RMS error about the mean.

The simulation presented in this section only included the response to ephemeris command input. The velocity derived from the ephemeris command data had an initial value of 0.0875 rad/sec; therefore, an initial condition of .085 rad/sec was given to simulate the transient due to the hand-off from coarse to fine control. Figure 5-12 shows the resulting tracking error for both systems. The H2 system had a settling time of 300 msec, $err_{ss}$ of 0.36 μrad, and an RMS tracking error of 1.0 μrad. For the current system, it was not possible to determine the exact settling time or steady state error from the five second response, but as was seen with the ramp response in Section 5.3.3 the settling time was at least 10 seconds and the steady state error was asymptotically approaching zero. The RMS tracking error for the current system was 0.95 μrad.
Figure 5-12 Ephemeris Command Tracking Error

5.3.5 Full-Up Simulation. Total system performance was analyzed in this section by simulating ephemeris command input with an initial velocity condition of 0.085 rad/sec, torque disturbance $w_1$, sensor noise $w_2$, and bearing friction torque disturbance $T_c$. Figure 5-13 shows the resulting tracking error for both systems. The conservatively modeled torque disturbance was apparent in the current system, whereas the torque disturbance was effectively rejected in the $H_2$ system. Time histories and PSD plots of all system signals for the full-up simulation are included in Appendix E.

The accuracy of the RMS tracking error values for the full-up simulation presented in the summary section in Table 5-2 is questionable. All of the RMS energy associated with the non-rejected low frequency torque disturbance was not captured in the 5 second sample of data. Therefore, the RMS values given were smaller than they would have been had the simulation been run longer. A five second sample of ephemeris data was the longest available. However, since the torque disturbance was modeled conservatively...
this is not an important issue. Also, the point of the simulation was to compare the two controllers, and clearly the $H_2$ controller had the better disturbance rejection and ephemeris tracking performance.

![Graph showing tracking errors for Current Controller and $H_2$ Controller over time.]

**Figure 5-13** Full-Up Simulation Tracking Error

### 5.3.6 Summary of Results

Tables 5-1 and 5-2 are the compiled results from all simulations. Although the current system had a better step response, the $H_2$ controller proved to yield better overall performance, especially in the "true test of performance" of ephemeris tracking. One might think that the high price to pay for the better tracking performance would have been more control usage; however, as seen in the motor torque RMS values, the $H_2$ controller required only 10% more control power than the current controller.
### Table 5-1 Simulation Results

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Settling Time</th>
<th>Over-shoot</th>
<th>RMS ( \text{err} ) (( \mu \text{rad} ))</th>
<th>( \text{err}_{ss} ) (( \mu \text{rad} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Torque Disturbance</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current Controller</td>
<td></td>
<td></td>
<td>2.12</td>
<td></td>
</tr>
<tr>
<td>( H_2 ) Controller</td>
<td></td>
<td></td>
<td>0.47</td>
<td></td>
</tr>
<tr>
<td><strong>Sensor Noise</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current Controller</td>
<td></td>
<td></td>
<td>0.30</td>
<td>0</td>
</tr>
<tr>
<td>( H_2 ) Controller</td>
<td></td>
<td></td>
<td>0.30</td>
<td>&lt;0.05</td>
</tr>
<tr>
<td><strong>Bearing Friction</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current Controller</td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>( H_2 ) Controller</td>
<td></td>
<td></td>
<td></td>
<td>&lt;0.05</td>
</tr>
<tr>
<td><strong>Step Response</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current Controller</td>
<td>250 msec</td>
<td>10%</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>( H_2 ) Controller</td>
<td>300 msec</td>
<td>40%</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td><strong>Ramp Response</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current Controller</td>
<td>=10 sec</td>
<td></td>
<td>approaches 0</td>
<td></td>
</tr>
<tr>
<td>( H_2 ) Controller</td>
<td>300 msec</td>
<td></td>
<td>&lt;0.05</td>
<td></td>
</tr>
<tr>
<td><strong>Ephemeris Response</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current Controller</td>
<td>=10 sec</td>
<td></td>
<td>0.95</td>
<td>approaches 0</td>
</tr>
<tr>
<td>( H_2 ) Controller</td>
<td>300 msec</td>
<td></td>
<td>1.0</td>
<td>0.36</td>
</tr>
</tbody>
</table>

### Table 5-2 Full-Up Simulation Results

<table>
<thead>
<tr>
<th>Full-Up Simulation</th>
<th>RMS ( \text{err} )</th>
<th>( \text{err}_{ss} )</th>
<th>RMS</th>
<th>Mean Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current Controller Voltage</td>
<td>2.6 ( \mu \text{rad} )</td>
<td></td>
<td>0.66 volts</td>
<td>0.57 volts</td>
</tr>
<tr>
<td>Motor Torque</td>
<td>221 ft-lbs</td>
<td>195 ft-lbs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( H_2 ) Controller Voltage</td>
<td>1.14 ( \mu \text{rad} )</td>
<td>0.47 ( \mu \text{rad} )</td>
<td>0.60 volts</td>
<td>0.56 volts</td>
</tr>
<tr>
<td>Motor Torque</td>
<td>244 ft-lbs</td>
<td>190 ft-lbs</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
VI. Elevation Axis

6.1 Introduction

As stated earlier, the emphasis in this thesis is on controller design for the azimuth axis, and the elevation axis controller design is merely included for completeness. The data available for the elevation model is not complete; therefore, many assumptions are made in developing the model. Direct feedforward had to be implemented in the actual SBD elevation axis control to achieve adequate tracking for reasons not even the SBD engineers clearly understand. This indicates that there is something missing from the model developed in this thesis, because it is not necessary to implement feedforward to achieve the tracking requirements. Therefore, since the model does not accurately represent the actual system, a comparison between the currently used elevation axis controller and the controller developed in this chapter is not made. However, it is important to present the elevation axis model and controller design to provide a starting point for possible follow-on research.

6.2 Elevation Axis Linear Plant Model

Like the azimuth axis, the elevation axis consists of the current loop $G_{el}(s)$, motor torque $K_T$, and the gimbal $G_g(s)$. Most of the model plant dynamics are obtained from the original coelostat documentation and from data taken from the actual SBD system. However, the data is not complete; therefore, some of the values for the model are estimated.

The motor torque constant and the gimbal's moment of inertia are given in the documentation as $K_T = 19.5$ ft-lbs/amp and $J = 1700$ ft-lbs-sec$^2$. No value for the
coefficient of viscous friction $K_v$ is given, therefore, $K_v$ is estimated such that the viscous damping is the same as it is in the azimuth axis. $K_v = 23 \text{ ft-lbs/rad/sec}$.

The current loop dynamics are obtained from frequency response plots taken from the actual system. The plot was reconstructed in Matlab by iteration until the shape matched the original. Figure 6-1 shows that the current loop is a first order lag system with a break frequency $\omega_b$ of approximately 300 rad/sec and a gain constant $K_a$ of 9 dB or 2.8 amps/volt.

![Figure 6-1 Elevation Axis Current Loop Frequency Response](image)

6.3 Torque Disturbance and Sensor Noise

The motor cogging and ripple torque disturbances are estimated to be 10% of the magnitude of the disturbance used for the azimuth axis. Justification for the disturbance estimate came from data for a similar beam director that indicated that the magnitude of the elevation axis disturbance was 10% of that in the azimuth axis [7:14]. Wind buffeting
in the elevation axis will also not be as severe as in the azimuth axis. The torque disturbance is modeled as a coloring filter $W_d$, shown in Figure 6-2, driven by zero-mean, unit intensity, white Gaussian noise. $W_d$ is described in state-space form as

\[
\dot{x}_d(t) = A_d x_d(t) + B_d w_1(t) \\
T_d(t) = C_d x_d(t) + D_d w_1(t)
\]

(6.1)

where $T_d(t)$ is the torque disturbance, $x_d(t)$ is the disturbance state, and $w_1(t)$ is zero-mean, unit intensity, white Gaussian noise. The state-space values are

\[
A_d = -1 \quad B_d = 1 \\
C_d = 100 \quad D_d = 0
\]

(6.2)

![Figure 6-2 Torque Disturbance Coloring Filter](image)

No data is available on the bearing friction; therefore, knowing that it will be less than the bearing friction in the azimuth axis, it is modeled as $T_r = 8$ ft-lbs.

The elevation gimbal position is measured using an Inductosyn encoder. The measurement of true angular position is corrupted by zero mean, white Gaussian noise with an RMS of .2 arc seconds or 1 μrad and is modeled as such for the controller design.
6.4 Nominal and Truth Models

The nominal model includes the current loop gain constant $K_a$, motor torque constant $K_T$, gimbal dynamics $G_g(s)$, torque disturbance input $T_d(t)$, and zero-mean, white Gaussian noise, of 1 μrad intensity sensor noise. The state-space representation of the nominal plant is

$$\begin{bmatrix}
\dot{\theta}(t) \\
\dot{\dot{\theta}}(t)
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
0 & -\frac{K_v}{J}
\end{bmatrix}
\begin{bmatrix}
\theta(t) \\
\dot{\theta}(t)
\end{bmatrix} + \begin{bmatrix}
0 \\
\frac{1}{J}K_T K_a
\end{bmatrix} v(t)$$

or

$$\theta(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \theta(t) \\
\dot{\theta}(t) \end{bmatrix} + [0] v(t)$$

(6.3)

The truth or simulation model includes the nominal model plus the dynamics of the current loop $G_c(t)$, non-linear bearing friction, first order Padé approximation to a ZOH, and the sensor noise is rolled off at 40 dB per decade. The elevation axis parameter values are summarized in Table 6-1.
### Table 6-1 Elevation Axis Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment of Inertia (ft-lbs-sec(^2))</td>
<td>(J) 17000</td>
</tr>
<tr>
<td>Coefficient of Viscous Friction (ft-lbs/rad/sec)</td>
<td>(K_v) 23</td>
</tr>
<tr>
<td>Motor Torque Constant (ft-lbs/amp)</td>
<td>(K_T) 19.5</td>
</tr>
<tr>
<td>Sample Period (sec)</td>
<td>(T) 0.005</td>
</tr>
<tr>
<td>Current Loop Gain Constant (amp/volt)</td>
<td>(K_a) 2.8</td>
</tr>
<tr>
<td>Current Break Frequency (rad/sec)</td>
<td>(\omega_b) 300</td>
</tr>
<tr>
<td>Bearing Friction Torque Constant (ft-lbs)</td>
<td>(T_c) 8</td>
</tr>
</tbody>
</table>

### 6.5 Elevation Axis Controller Design

The same unity feedback setup used for the azimuth axis controller design is used for the elevation design. As in the azimuth axis design, \(R_z\) is chosen to be 1 and \(H\) is a 2 by 2 identity matrix. The final sensitivity weighting \(W_s\), chosen to achieve the performance requirements of disturbance rejection and command tracking, is shown in Figure 6-3 and its state-space representation is

\[
\dot{x_s}(t) = A_x x_s(t) + B_s \theta_m(t) \\
z_3 = C_x x_s(t) + D_s \theta_m(t)
\]

(6.5)

where \(z_3\) is the controller output, \(\zeta_s(t)\) is the sensitivity weighting state, and \(\theta_m(t)\) is the measured gimbal position. The state-space values are

\[
A_x = -0.0001 \quad B_x = 1 \\
C_x = 0.5 \quad D_x = 0
\]

(6.6)
The state-space description of the resulting controller is contained in Appendix A. The system had gain and phase margins of (-11.1 dB, 20 dB) and ±43°, and a bandwidth of 49 rad/sec. Like the azimuth axis H2 optimization put a pair of complex poles at high frequency resulting in more high frequency control power than was necessary for the required system performance. The high frequency poles could be moved to a lower frequency without decreasing system performance or decreasing the gain and phase margins below acceptable values.

6.6 Simulation and Results

Simulations were performed with the truth model. The disturbance rejection transfer function, shown in Figure 6-4, was not as good as it was in the azimuth axis. This is to be expected, because the moment of inertia of the azimuth axis is 15 times greater than that of the elevation axis, thus making it naturally more resistant to torque disturbances. Even though the input torque disturbance was modeled as 10% of that in the azimuth axis, the position error due to torque disturbance was greater in the elevation axis: \( \text{err} = 0.78 \text{μrad RMS} \). However, the disturbance rejection was still more than adequate.
In general, the elevation rate of a satellite ephemeris vector is considerably less than the azimuth rate for an elevation over azimuth beam director. The elevation position ephemeris had an average rate of 0.007 rad/sec compared to 0.1 rad/sec for the azimuth position ephemeris. Therefore, the tracking environment for the elevation axis is not as severe and is evident in the time response shown in Figure 6-5. Even with no initial velocity condition given, the initial transient was not too severe, the RMS jitter and $err_{ss}$ were only 0.06 and 0.05 μrad, respectively. The full-up simulation of ephemeris command input, torque disturbance, bearing friction disturbance, and sensor noise yielded RMS jitter and $err_{ss}$ of 1.0 and 0.62 μrad, respectively.
Figure 6-5 Ephemeris Command Tracking Error
VII. Conclusions/Recommendations

7.1 Conclusions

The two objectives of this thesis were met: a controller for the azimuth axis was designed that had better performance than the currently used controller, and optimal control was used to develop the controller.

The modeling (including the plant, disturbances, and sensor noise) and defining system requirements proved to be the most difficult tasks in designing the controller. Since the model was a SISO system, classical design techniques could have been used to design a controller with better ramp tracking performance than the currently used controller, because it appears that the original design was based on step response performance and not ramp tracking performance. The optimal control design techniques proved to be no more difficult than a classical approach, because once the optimal control problem was set up and it was understood how the control weightings affect system performance, it was relatively straightforward to "dial" the weightings to "tune" system performance.

7.1.1 Azimuth Axis Performance. The currently used controller yielded a system with better robustness than the $H_2$ controller. However, the gain and phase margins for the $H_2$ system were more than adequate.

The $H_2$ controller produced better disturbance rejection than the currently used controller. However, as stated in Section 5.3.1 the disturbance in the actual SBD system may not be as severe as the modeled disturbance; therefore, a controller with better disturbance rejection may not be necessary.
The currently used controller yielded a better damped, slightly faster, step response than the $H_2$ controller. However, this is not necessarily an advantage for three reasons: first, as described in section 5.3.2 the current controller requires more control power for step command following, and thus would have a larger saturation problem; second, the true test of performance is tracking ephemeris commands which are more akin to ramp commands than step commands; and third step commands are more realistically a job for the coarse position loop or "bang-bang" controller.

The most important difference between the two controllers was in their ability to track ramp or ephemeris commands. The settling time to achieve near zero steady state error for the $H_2$ system was 300 msec, while the settling time for the current system was greater than 10 seconds.

### 7.1.2 The Price to Pay.

The most obvious difference between the two controllers is that the currently used controller is 2nd order and the $H_2$ controller is 4th order. Generally, optimal control produces high order controllers. However, if classical control design had been used to develop a controller with the same tracking performance as the $H_2$ controller, it is likely that it would have been higher than 2nd order.

The gimbal has resonant frequencies at 14, 30, and 45 Hz, or 88, 188, and 282 rad/sec, respectively. Notch filters were implemented on the current SBD system to account for the gimbal's natural modes. If the $H_2$ controller is implemented on the SBD, notch filters will also have to be used. As described in Section 4.4.2.2, the high frequency poles of the $H_2$ controller were moved to roll off the controller to minimize high frequency control use that would drive the resonant frequencies. As seen in the closed-loop transfer function $\Theta(t)/\Theta_c(t)$, Figure 7-1, the final $H_2$ controller still has more gain than the current controller out to 300 rad/sec. Placing the poles at an even lower frequency to roll off the compensator will have many adverse effects: lower bandwidth, lower gain and phase margins, and less damping. The notch filtering will be a more...
difficult task with the H\textsubscript{2} controller for the resonant frequencies of 88 and 188 rad/sec, because the gain is higher than the current system at those frequencies.

![Graph showing H2 Controller and Current Controller](image)

**Figure 7-1 Closed-Loop Transfer Function**

### 7.2 Future Research

Better understanding of the elevation axis dynamics and the coupling between the azimuth and elevation axes would enable one MIMO system model to be built. The two inputs would be azimuth and elevation position commands and the two outputs would be the measured azimuth and elevation positions. The resulting controller would be a coupled 2 by 2 matrix controller. The challenge would be to utilize the true advantage of optimal control design for a MIMO system and provide the SBD with a controller that addresses the coupling problem.

One of the original objectives of this thesis was to use both H\textsubscript{2} and H\textsubscript{\infty} optimization for the controller design and compare and contrast the two. However, the scaling
problem described in Section 4.2 prevented completely successful $H_\infty$ optimization results. An $H_\infty$ synthesized controller was designed with marginal performance, and more effort would have eventually yielded an adequate controller. However, even if a successful $H_\infty$ design was achieved, the comparison would have been difficult at best, because the scaled plant used for $H_\infty$ optimization was drastically different than the scaled plant used for $H_2$ optimization. Further investigation is needed to determine the cause of the numerical problems $H_\infty$ has with this model, and to determine the proper scaling necessary to develop a controller with good performance. It may have been possible to develop an adequate controller of the uncoupled SISO system, but a much better understanding of the $H_\infty$ numerical problems will be necessary to develop a controller for a coupled MIMO system.

The most interesting suggested follow-on research is to design a two degree of freedom controller for the SBD model that has better performance than the unity feedback controller for ramp or ephemeris command tracking. A two degree of freedom controller has more flexibility than a unity feedback controller and theoretically, should provide at least as good if not better tracking performance. One of the two degree of freedom controllers designed in this thesis did yield a faster, better damped step response than the unity feedback controller. However, ramp tracking was non-existent and efforts to better the performance by changing the track weighting were less than completely successful. "Dialing" of the track weighting $W_t$ was exhausted to achieve the desired performance of fast, well damped, zero steady state error ramp tracking, while maintaining the well damped step response. Other weightings in conjunction with the track weighting, such as a sensitivity weighting and a weighting on the command input, were also attempted to no avail. In order to get the two degree of freedom controller to yield adequate performance alternate approaches need to be investigated.
7.3 Closing Comments

This thesis demonstrates that optimal control design is a powerful tool that can be used not only to achieve robustness but also microradian tracking performance. The ultimate test of performance would be to integrate the discretized azimuth axis $H_2$ controller into the SBD. Obviously, that decision will be made by the SBD engineers who have to determine if the improved performance demonstrated in this thesis is worth the effort.
Appendix A: Final Compensator Design

A.1 Azimuth Axis Compensator

Transfer function form of negative feedback controller:

\[ G_c(s) = \frac{360 \times 10^6(s + 0.013462)(s^2 + 29.4s + 441.1073)}{(s + 0.0001)(s + 0.99996)(s^2 + 424s + 90000)} \]

State-space form:

\[ G_c(s) = C_c(sI - A_c)^{-1}B_c + D_c \]

\[
A_c = \begin{bmatrix}
-425.0 & -90424.0 & -90006.0 & -8.9996 \\
1.0 & 0 & 0 & 0 \\
0 & 1.0 & 0 & 0 \\
0 & 0 & 1.0 & 0
\end{bmatrix}, \quad
B_c = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad
C_c = \begin{bmatrix} 3.6 \times 10^8 & 9.6019 \times 10^9 & 1.2561 \times 10^{11} & 1.6892 \times 10^9 \end{bmatrix}, \quad
D_c = 0
\]

A.2 Elevation Axis Compensator

State-space form:

\[
A_c = \begin{bmatrix}
-14141.0 & 1.0 & 100.0 & 0 \\
-1273.0 & -50.458 & -4945.9 & -0.016059 \\
-999860.0 & 0 & -1.0 & 0 \\
0 & 0 & 0 & -0.0001
\end{bmatrix}, \quad
B_c = \begin{bmatrix} 0.014141 \\ 0 \end{bmatrix}, \quad
C_c = \begin{bmatrix} 39636.0 & 1570.6 & 153990.0 & 0.5 \end{bmatrix}, \quad
D_c = 0
\]
Appendix B: Matlab M-Files

B.1 Introduction

M-files were used extensively. Data, such as model parameter values were stored in M-files as opposed to data or MAT-files. It was easier to edit an M-file where a parameter and its value were clearly displayed. Simply typing the M-file name loaded its parameter contents into the Matlab work space. The comment sign "%" was also used extensively not only to add descriptive comments, but to comment out certain commands that were not wished to be executed for a particular analysis. Contained in this appendix is a list of all M-files and a short description used in the execution of this thesis, and a print out of some of the key M-files.

B.2 List of M-files

M-Files
ave - Calculates the average or mean value of a time history.
azplant - Contains all the azimuth axis plant parameter values, and builds state-space form of the nominal and truth models.
cctoteval - Creates linear state-space model of Simulink model "SimCCTotEvalMod" for the currently used controller to calculate the closed-loop transfer function and the disturbance rejection transfer function. Also calculates Bode frequency response and gain and phase margins of $G_p(s)G_c(s)$, where $G_p(s)$ is the truth model.
cesca - executes the sequence of M-files; pbazswl5, H2, sentoteval, which, builds the $P$ matrix, calculates the $H_2$ optimal controller, and plots the Bode frequency plot of $-G_p(s)G_c(s)$.
classcntr - builds the currently used controller state-space, plots Bode frequency response, and calculates gain and phase margin from $G_p(s)G_c(s)$, where $G_p(s)$ is the truth model.
comproll - builds the final controller state-space.
elplant - Contains all the elevation axis plant parameter values, and builds the nominal and truth models.
H2 - Calls the h2opt and split M-files to calculate the optimal controller and split it into its state-space form $A_c$, $B_c$, $C_c$, $D_c$.
h2opt - $H_2$ optimization algorithm
Hin - Calls the hinf5 M-file to calculate the $H_{\infty}$ optimized controller.
**hinf5-** $H_\infty$ optimization algorithm

**param** - Builds state-space of torque disturbance coloring filter, sensor noise shaping filter, and all weightings.

**pbazswl1-14** - Builds azimuth axis $P$ matrix for various unity feedback setups with 2nd and 3rd order sensitivity and complementary sensitivity weightings

**pbazswl15** - Builds azimuth axis $P$ matrix for the final unity feedback design with a 1st order $W_s$

**pbaztwl1-12** - Builds azimuth axis $P$ matrix for various 2 degree of freedom controller setups with weightings $W_t$, $W_s$, and input weighting $W_r$.

**pbaztwl13** - Builds azimuth axis $P$ matrix for 2 degree of freedom controller setup with 1st order $W_s$.

**pbaztwl14** - Builds azimuth axis $P$ matrix for 2 degree of freedom controller setup with 2nd order $W_t$.

**pbelsw2** - Builds elevation axis $P$ matrix for the final elevation axis unity feedback design with a 1st order $W_s$

**psd** - Calculates RMS and Plots PSD of a vector.

**scale3** - Performs internal scaling of the optimal control setup

**sentoteval** - Creates linear state-space model of Simulink model "SimSenTotEvalMod" for the unity feedback design to calculate the closed-loop transfer function and the disturbance rejection transfer function. Also calculates Bode frequency response and gain and phase margins of $-G_p(s)G_c(s)$, where $G_p(s)$ is the truth model.

**split** - Slits the $H_2$ optimal controller $P_c$ into $A_c$, $B_c$, $C_c$, $D_c$.

**trcktoteval** - Creates linear state-space model of Simulink model "SimTrckTotEvalMod" for the 2 degree of freedom controller design to calculate the closed-loop transfer function and the disturbance rejection transfer function. Also calculates Bode frequency response and gain and phase margins of $-G_p(s)G_c(s)$, where $G_p(s)$ is the truth model.

**truncate** - truncates all of the time histories generated from a Simulink simulation to eliminate the initial transient response from a 5 second simulation.

**truncatetensec** - truncates all of the time histories generated from a Simulink simulation to eliminate the initial transient response from a 10 second simulation.

### B.3 Azimuth Axis Plant "azplant"

This M-file contains all the azimuth axis parameter values, and builds the state-space nominal and truth models used in $H_2$ optimization and Simulink simulation.

**M-File azplant:**

% Azimuth axis symbol definitions and values

% $K_a$ - current loop gain constant (AMP/VOLT)
% $W_a$ - amplifier natural frequency (rad/sec)
% $z$ - amplifier damping coefficient
% $K_T$ - motor Torque Constant (ft-lbs/AMP)
% $J$ - gimbal moment of inertia (ft-lbs-sec^2)
% $K_v$ - Coefficient of viscous friction (ft-lbs/rad/sec)
% T - Sample rate of digital controller

Ka=4.4;
Wan=560;
z=.25;
KT=75.6;
J=26000;
Kv=350;
T=.005;

%Aa - amplifier A matrix
%Ba - amplifier B matrix
%Ca - amplifier C matrix
%Da - amplifier D matrix
%
Aa=[0 1;-Wan^2 -2*z*Wan];
Ba=[0;Ka*Wan^2];
Ca=[1 0];
Da=[0];

%Ag - gimbal A matrix
%Bg - gimbal B matrix
%Cg - gimbal C matrix
%Dg - gimbal D matrix
%
Ag=[0 1;0 -Kv/J];
Bg=[0;1/J];
Cg=[1e6 0];
Dg=0;

%Aasd - Sampling Delay A matrix
%Bsd - Sampling Delay B matrix
%Csd - Sampling Delay C matrix
%Dsd - Sampling Delay D matrix
%
Asd=-2/T;
Bsd=2/T;
Csd=1;
Dsd=0;

% The following augmented plant state-space is for the
% optimization routines and therefore does not include the current
% loop dynamics. The current loop is modeled by the gain constant Ka.
%Ap - plant A matrix
%Bp - plant B matrix
%Cp - plant C matrix
%Dp - plant D matrix
%
Ap=Ag;
Bp=KT*Ka*Bg;
Cp=Cg;
Dp=0;

% The following state-space is the full plant "truth model" including current loop
% and sampling delay dynamics
Afp=[Asd zeros(1,4);Ba*Csd Aa zeros(2,2);zeros(2, 1) Bg*KT*Ca Ag];
Bfp=[Bsd;0;0;0;0]
Cfp=[0 0 0 Cg];
Dfp=0;

\textbf{B.4 Elevation Axis Plant "elplant"}

This M-file contains all the elevation axis parameter values, and builds the state-

space nominal and truth models used in H\textsubscript{2} optimization and simulation.

\textbf{M-File elplant:}
% Elevation axis symbol definitions values and model
% Ka - amplifier gain constant (AMP/VOLT)
% Wab - amplifier break frequency (rad/sec)
% KT - motor Torque Constant (ft-lbs/AMP)
% J - gimbal moment of inertia (ft-lbs-sec\textsuperscript{2})
% Kv - Coefficient of viscous friction (ft-lbs/rad/sec)
% T - Sample rate of digital controller
Ka=2.8;
Wab=300;
KT=19.5;
J=1700;
Kv=23;
T=.005;

%Aa - amplifier A matrix
%Ba - amplifier B matrix
%Ca - amplifier C matrix
%Da - amplifier D matrix
%
Aa= [-Wab]; 
Ba=[Ka*Wab]; 
Ca=[1];
Da= 0;

%Ag - gimbal A matrix
%Bg - gimbal B matrix
%Cg - gimbal C matrix
%Dg - gimbal D matrix
%
\[ Ag = [0; 1; -Kv/J]; \]
\[ Bg = [0; 1/J]; \]
\[ Cg = [1e6; 0]; \]
\[ Dg = 0; \]

\% Asd - Sampling Delay A matrix
\% Bsd - Sampling Delay B matrix
\% Csd - Sampling Delay C matrix
\% Dsd - Sampling Delay D matrix

\%
Asd = -2/T;
Bsd = 2/T;
Csd = 1;
Dsd = 0;

\%
The following augmented plant state-space is for the
\%
optimization routines and therefore does not include the amplifier
\%
dynamics. The amplifier is modeled by the gain constant \( K_a \).
\%
Ap - plant A matrix
\%
Bp - plant B matrix
\%
Cp - plant C matrix
\%
Dp - plant D matrix

\%
Ap = Ag;
Bp = KT*Ka*Bg;
Cp = Cg;
Dp = 0;

\%
The following state-space is the full plant including amplifier
\%
and sampling delay dynamics

\%
Ap = [Asd zeros(1,3); Ba*Csd Aa zeros(1,2); zeros(2,1) Bg*KT*Ca Ag];
Bp = [Bsd; 0; 0; 0];
Cp = [0 0 Cg];
Dp = 0;

\%
B.5 Weighting, Disturbance, and Noise Parameters "param"

This M-file builds the state-space representation of the torque disturbance coloring
filter, the sensor noise shaping filter, and the dynamic weightings used in the various
optimal control designs. This print out has been edited to remove all but the 1st order
weighting \( W_i \) used in the final azimuth axis unity feedback design.

M-File param:
\%
param - Parameters for disturbance, noise, and weightings.
% Torque Disturbance distribution vector
gam=[1;0];

% Disturbance state-space where:
% -1
% Wd(s) = Cd(sI-Ad) Bd + Dd
% Disturbance parameters used in optimal control for performance robustness
Gain=1000;
wb=1;
numd=Gain*wb;
dend=[1 wb];
[Ad,Bd,Cd,Dd]=tf2ss(numd,dend);

% Disturbance parameters used in simulation
Gain=1000;
wb=1;
numd=Gain*wb;
dend=[1 wb];
[Adsim,Bdsim,Cdsim,Ddsim]=tf2ss(numd,dend);

% Sensor Noise parameters used in simulation
Gain=1;
wb1=50;
wb2=50;
numn=Gain*wb1*wb2;
denn=conv([1 wb1],[1 wb2]);
[Ansim,Bnsim,Cnsim,Dnsim]=tf2ss(numn,denn);

% Sensitivity weighting state-space where Ws is First Order
Gain=5000;
wb1=.0001;
numws=Gain*wb1;
denws=[1 wb1];
[As,Bs,Cs,Ds]=tf2ss(numws,denws);

% LQG weightings
Rz=1;
H=[1 0; 0 1];

\textbf{B.6 Building the P matrix "pbazsw15"}

This M-file calls the M-files \textit{azplant} and \textit{param} to build the state-space form of the azimuth axis model, the torque disturbance and sensor noise coloring filters, and the
weightings. Then it builds the $P$ matrix and defines the dimension of $P$ necessary for the $H_2$ optimization algorithm.

**M-File pbazsw15:**

% pbazsw15- Azimuth Axis P Matrix Build
% Unity feedback setup

% Build state-space azimuth axis plant
azplant

% Disturbance, noise, and weighting parameters
param

% Nine matrices that make up the state-space of $P(s)$
A=[A p gam*Cd zeros(2,1);zeros(1,2) Ad 0;Bs*Cp 0 As];
Bw=[zeros(2,2);Bd 0 ;0 Bs];
Bu=[Bp;0; 0];
Cz=[H zeros(2,2) ;zeros(1,4);zeros(1,3) Cs];
Dzw=[zeros(4,2)];
Dzu=[zeros(2,1); Rz;0];
Cy=[Cp 0 0];
Dyw=[0 1 ];
Dyu=[0];

% P=[A,Bw,Bu;Cz,Dzw,Dzu;Cy,Dyw,Dyu];
%
% Define the dimensions of $P$ where
% dims=[ns,nw,nu,nz,ny]
% ns - number of states
% nw - number of inputs w
% nu - number of inputs u
% nz - number of outputs z
% ny - number of outputs y
[m,n]=size(A);
s=

[m,n]=size(Bw);
$nw=n;
[m,n]=size(Bu);
$nu=n;
[m,n]=size(Cz);
$nz=m;
[m,n]=size(Cy);
$ny=m;
dims=[ns,nw,nu,nz,ny];
B.7 H₂ Optimization "H2"

This M-file calls the M-files h2opt and split to calculate the H₂ optimal controller and split the controller into its state-space form.

M-File H2
% H2- H2 Optimization
[Pc,su,sy,Kc,Kf]=h2opt(P,dims);
ns=dims(:,1);
[Ac,Bc,Cc,Dc]=split(Pc,ns);

B.8 H₂ Optimization "h2opt"

This M-file calculates the H₂ optimal controller.

M-File h2opt
function [pk,su,sy,kc,kf] = h2opt(p,dims)
%[pk,su,sy,kc,kf]=h2opt(p,dims)
ns = dims(1);nw = dims(2);nu = dims(3);nz = dims(4);ny = dims(5);
[a,b,c,d] = split(p,ns);
[d11,d12,d21,d22] = split(d,nz,nw);
if any(any(d11 ~= 0)), error('d11 not a zero matrix');end;
if any(any(d22 ~= 0)), error('d22 not a zero matrix');end;
[pscl,su,syl] = scale3(p,dims);
[a,b,c,d] = split(pscl,ns);
[d11,d12,d21,d22] = split(d,nz,nw);
b1 = b(:,1:nw);
b2 = b(:,nw+1:nw+nu);
c1 = c(1:nz,:);
c2 = c(nz+1:nz+ny,:);
f = a - b2*d12*c1;
g = b2*b2';
c1hat = (eye(nz) - d12*d12')*c1;
h = c1hat*c1hat;
x2 = are(f,g,h);
f = (a - b1*d21'*c2);
g = c2'*c2;
b1hat = b1*(eye(nw) - d21'*d21);
h = b1hat*b1hat';
y2 = are(f,g,h);
kc = b2*x2 + d12'*c1;
kf = y2*c2' + b1'*d21';
ak = a - kf*c2 - b2*kc;
bk = kf*sy;
ck = -inv(su)*kc;
dk = 0*ones(nu,ny);
pk = [ak, bk; ck, dk];

**B.9 Frequency Responses "sentoteval"**

This M-file calculates the open loop Bode frequency response and gain and phase margins of \(-G_P(s)G_C(s)\). It also calculates the closed-loop transfer function \(\theta(s)/\theta_c(s)\) and the torque disturbance rejection transfer function by calculating the closed-loop linear model of the Simulink model "SimSenTotEvalMod".

**M-File sentoteval**

```
M-File sentoteval
"% sentotaleval - Total Evaluation Model
"% Gain and Phase Margins
"% Closed-Loop and Torque Disturbance Rejection Transfer Functions

"% Frequency range for all analysis
w=logspace(-2,3,300);

"% Calculate the SISO gain and phase margins
"% where Aol, Bol, Col, & Dol is the open loop state-space of -G_P(s)G_C(s)
[Aol,Bol,Col,Dol]=series(Ac,Bc,Cc,Dc,Afp,Bfp,Cfp,Dfp);
"% Since feedback is positive the margins are obtained from -GcGp
[magol,phaol]=bode(Aol,Bol,-Col,Dol,1,w);
[GM,PM,wg,wp]=margin(magol,phaol,w);
margin(magol,phaol,w)

"% Set up for transfer function calculations, null out filters
"% and calculate linear model
Ansim=[ ];
Bnsim=[ ];
Cnsim=[ ];
Dnsim=1;
Adsim=[ ];
Bdsim=[ ];
Cdsim=[ ];
Ddsim=1;
[Acl,Bcl,Ccl,Dcl]=linmod('SimSenTotEvalMod');

"% Torque disturbance rejection transfer function
[magds,phads] = bode(Acl,Bcl,Ccl,Dcl,1,w);
magds=20*log10(magds);

"% Closed-loop transfer function
"% Complimentary Sensitivity
[magns,phans]=bode(Acl,Bcl,Ccl,Dcl,2,w);
magns=20*log10(magns);
```
Appendix C: Simulab Block Diagrams

Figure C.1 Current Controller Simulation Model "SimCCEvalMod"
Figure C-2 Current Controller Linmod Model "SimCCTotEvalMod"
Figure C-3 Unity Feedback Simulation Model "SimSenEvalMod"
Figure C-4 Unity Feedback Linmod Model "SimSenTotEvalMod1"
Figure C-5. 2 Degree of Freedom Controller Simulation Model "SimTrkEvalMod"
Figure C-6 2-Degree of Freedom Controller Linmod Model "SimTrckTotEvalMod"
Appendix D: SBD Controller Design and Simulation Tutorial

The following is the sequence of steps taken to design the azimuth axis $H_2$ optimal unity feedback controller and perform Simulink simulations.

1. Build the azimuth axis nominal and truth state-space models by typing `azplant` in the Matlab workspace.

2. Build the torque disturbance and sensor noise coloring filters, and weightings in state-space form by typing `param`.

3. Build the $P$ matrix by typing `pbazsw15`.

4. Calculate the $H_2$ optimal controller in state-space format by typing `H2`.

5. Calculate the Bode frequency response and gain and phase margins of $-G_p(s)G_c(s)$, the closed-loop transfer function $\theta(s)/\theta_c(s)$, and the torque disturbance rejection transfer function by typing `sentoteval`.

6. Perform simulations with the Simulink model "SimSenEvalMod". The Simulink model was built mostly with state-space blocks such that the parameters for those blocks are automatically read in from the Matlab workspace.

   a. Simulations were performed with the maximum and minimum step sizes set at 0.005 sec to correspond to the 200 Hz sample rate of the SBD. The integration algorithm used was Linsim.

   b. Ramp and Ephemeris command tracking simulations were performed by using the "From Workspace" block to load in the command vector. The data for all the command vectors are stored in the MAT-file `poscmd`. The 5 and 10 second ramp command vectors are called `poscmdfivecramp` and `poscmdtensecramp`, respectively. The 5 second ephemeris command vector is called `poscmdascend`. 
Appendix E: Full-Up Simulation Plots

Pxx - X Power Spectral Density

Figure E-1 Current Controller Tracking Error PSD
Pxx - X Power Spectral Density

Figure E-2 H₂ Controller Tracking Error PSD

Figure E-3 Current Controller Voltage
Figure E-4  H\textsubscript{2} Controller Voltage

Figure E-5  Current Controller Voltage PSD
Figure E-6 H₂ Controller Voltage PSD

Figure E-7 Current Controller Amps
Figure E-8 $H_2$ Controller Amps

Figure E-9 Current Controller Motor Torque
Figure E-10  H$_2$ Controller Motor Torque

Figure E-11  Current Controller Motor Torque PSD
Figure E-12  H$_2$ Controller Motor Torque PSD
References


Vita

Captain Troy V. Lanier was born 25 November 1963 in Germany. He graduated from Iolani School in Honolulu, Hawaii in 1982 and attended the Virginia Military Institute, graduating in 1986 with Distinction with a Bachelor of Science in Mechanical Engineering. Upon graduation he received a regular commission in the USAF and served his first tour of duty at Kirtland AFB, New Mexico. He began as a structural engineer performing modal analysis of a large flexible space structure via computer simulation. He was a qualified project officer, and was program manager for the $10 million space-based Strategic Defense Initiative's (SDI) Wideband Angular Vibration Experiment that was part of the Relay Mirror Experiment (RME). When the RME was fielded in 1989 he worked as a member of the Science Team analyzing system performance data for the duration of the one year experiment. He was also heavily involved in the very successful RME additional science experiment Adaptive Noise Cancellation for Closed-Loop Systems. He entered the School of Engineering, Air Force Institute of Technology, in May of 1991.
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The Starfire Beam Director (SBD) is located at the Starfire Optical Range at Kirtland Air Force Base in Albuquerque, New Mexico. The SBD capabilities include tracking celestial objects and active or passive tracking of artificial satellites to support the Phillips Laboratory Ground Based Laser Acquisition, Tracking, and Pointing (GBL ATP) program. The pointing and tracking accuracy needed to support such experiments is μrad to sub-μrad level. To accomplish this goal requires precise pointing of the massive 6 ton 1-meter clear aperture coelostat. The purpose of this thesis is to use optimal control design techniques to develop a controller to meet the stringent pointing requirements. A nominal linear state-space model was built which included gimbal dynamics, plant disturbances, and sensor noise. Then optimal control design techniques were used to develop a controller to meet the stringent pointing requirements.  
A nominal linear state-space model was built which included gimbal dynamics, plant disturbances, and sensor noise. Then optimal control design techniques were used to develop a controller to meet the stringent pointing requirements.  
The best of the designs, the $H_2$ unity feedback controller, was compared and contrasted with the performance of the controller currently being used, which was obtained by classical control design. The $H_2$ controller exceeded tracking requirements and in most areas performed better than the current controller.  

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