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ANNUAL MATERIALS PLAN ANALYSIS TOOL

October 1992

OPERATIONS RESEARCH AND ECONOMIC ANALYSIS OFFICE

DEPARTMENT OF DEFENSE
DEFENSE LOGISTICS AGENCY

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ANNUAL MATERIALS PLAN
ANALYSIS TOOL

October 1992

Maj Bruce Colletti, USAF

DEPARTMENT OF DEFENSE
DEFENSE LOGISTICS AGENCY
OPERATIONS RESEARCH AND ECONOMIC ANALYSIS OFFICE
CAMERON STATION
ALEXANDRIA, VA 22304-6100
FOREWORD

This report presents work done by the Defense Logistics Agency Operations Research and Economic Analysis Management Support Office (DLA-DORO). This study was made possible by the Defense National Stockpile Center, Office of Planning and Market Research, Messrs. Richard Corder and Franklin Ringquist.

The resulting product from this study will enable the Defense Logistics Agency to more quickly prepare an optimal Annual Materials Plan. Timeliness and optimality are essential since the Strategic and Critical Materials Stockpiling Act (50 U.S.C. 98 et seq.) requires the President to submit the plan to Congress each year. The Executive Branch has delegated preparation of the Annual Materials Plan to the Defense Logistics Agency, Defense National Stockpile Center.
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EXECUTIVE SUMMARY

At the request of the Defense National Stockpile Center (DNSC), the Defense Logistics Agency Operations Research and Economic Analysis Management Support Office (DLA-DORO) developed the Annual Materials Plan Analysis Tool (AMPAT). This tool quickly and accurately builds an Annual Materials Plan given macro-level factors such as minimum and maximum acquisition/disposal quantities, forecasted prices, and legislative disposal authority.

Historically, the preparation of these plans has been primarily manual, and therefore, time consuming. This becomes important when one considers the sometimes frequent changes to the plans due to Defense Department and Congressional interest and the attendant quick responses required. AMPAT has given DNSC the capability to produce the AMP calculations within hours as opposed to weeks.

Initially, it was planned to incorporate this tool in a larger computer program that would aid the DNSC industrial specialists in developing the macro-level factors. AMPAT would be incorporated in the program to lay out the optimum levels and timing of acquisitions and disposals given acquisition and disposal authority and other practical constraints. After observing an actual preparation of an AMP, it became apparent that the DNSC quickly needed a tool that solved the problem of computing an AMP that satisfied the given restrictions. Discussions with the management of DNSC led to an agreement to split the development into two parts: first, develop a tool to compute the AMP given macro-level factors; and second, a follow-on project would enhance AMPAT by using artificial intelligence concepts to develop the macro-level factors.

This report marks the completion of the first project. The initial version of AMPAT has been developed and delivered to DNSC for immediate use. The report recommends that the second phase be started immediately in order to satisfy the full requirement.
SECTION 1
INTRODUCTION

1.1 BACKGROUND

The Annual Materials Plan (AMP) states how much of what materials to add, to remove, or upgrade in the Defense National Stockpile (DNS). The AMP also states when during the next 3 years such activity should occur. Materials found in the DNS are those which would be scarce in the opening years of a global conventional war that forces full economic mobilization of the United States. Public law creates the AMP which the President submits to Congress.

The AMP is a relatively short document but is tedious and time-consuming to create. It requires coordination with many federal agencies and is affected by legislation and politics. It requires understanding the economy and political stability of foreign supply sources, international supply and demand, and being able to forecast commodity prices. Although these factors are volatile and will always be difficult to predict, it is possible to mathematically model some important interactions.

The ability to swiftly build or quickly modify an AMP is important because of the scrutiny the AMP receives at the highest levels of the federal government. It is a time-sensitive document that often undergoes rapid changes from within the Office of the Assistant Secretary of Defense/Production & Logistics (OASD/P&L), sometimes with Congressional interest.

1.2 PURPOSE

The purpose of this study is to enable the Defense National Stockpile Center (DNSC) to swiftly compute or modify an AMP.

1.3 OBJECTIVES

The general objectives of this study are to:

(1) Understand how economics and politics affect an AMP.

(2) Understand the techniques and philosophies used to build an AMP.

(3) Build a personal computer based analysis tool that implements these techniques and philosophies and which allows DNSC industrial specialists to build an AMP.

(4) Lay the foundation for a later study that uses artificial intelligence (AI) concepts to improve the process of developing the input parameters to the AMP.

1.4 SCOPE

This study uses gross macro-level factors that affect an AMP and whose interactions can be numerically modelled. Also, this study and the resultant computer based analysis tool only address the last stage of building an AMP. In this stage, the macro-level factors have already been estimated. At the
request of the DNSC, the 3 year horizon required for the AMP is extended to 10 years for the purpose of the model.

This study will not address the decision making processes or the derivation of the macro-level factors and other inputs to the model. These concerns will be addressed in a later project which uses AI and operations research concepts to model the decision making process that precedes computing the AMP.
SECTION 2
METHODOLOGY

2.1 CONDUCT BACKGROUND INTERVIEWS

In July 1991, representatives from the DNSC Office of Planning and Market Research (DNSC-R), the DLA Operations Research and Economic Analysis Office (DLA-LO), and the DLA Operations Research and Economic Analysis Management Support Office (DLA-DORO) met to discuss if AI could help ease the task of building the AMP. The result of this meeting was a concept paper that said AI and operations research concepts could be used. The resulting analysis tool would be named the Annual Materials Plan Analysis Tool (AMPAT).

In December 1991, DNSC industrial specialists attempted a week-long training session that would have taught DORO how to build an AMP. Unfortunately, urgent taskings from OASD/P&L regarding the AMP completely upset training. Although training could not achieve its goal, the rapid-fire taskings from OASD/P&L (arising from Congressional actions) dramatically underscored the need for the DNSC to rapidly build and modify an AMP. It was decided to split the effort into two parts, the first part addressing the most critical need of the DNSC: computing the AMP given that certain rates have already been decided. The second part would incorporate AI techniques to enhance AMPAT and to model the entire decision making process that determines the input factors used to compute an AMP.

2.2 IDENTIFY MACRO-LEVEL FACTORS AND MATHEMATICAL MODEL

Training did, however, successfully identify the macro-level factors that affect how an AMP is built. Furthermore, by viewing how the AMP was computed in response to the taskings from OASD/P&L, it became clear that the DNSC was unknowingly trying to manually solve a mathematical problem known as a linear program (LP). The LP often arises in many practical applications and a solution method is well known.

The data required to solve the LP are global and commodity-specific factors. The global factors are the operating and maintenance costs, research and development costs, the maximum spending allowed for acquisitions, and the upper/lower dollar limits imposed upon disposal sales. Commodity-specific factors give the present status of a commodity. These factors are an item's current inventory, how much extra may be currently disposed (i.e., unused disposal authority), and aggregate requirements during the planning horizon. Commodity-specific factors that vary annually are the upper/lower limits on how many units can be bought or sold, the average unit price, and additional disposals.

2.3 IDENTIFY VALUES FOR MACRO-LEVEL FACTORS

2.3.1 SOURCE DOCUMENTS

The values that macro-level factors can assume and the manner in which they can interact are partly determined by legislative and technical documents. The essential character of the AMP is found in the public law that establishes the DNS. The Strategic and Critical Materials Stockpiling Act (50 U.S.C 98 et seq.) mandates that a stock of strategic and critical materials is to be
maintained to decrease and preclude dependence upon foreign sources of supply in times of national emergency. The AMP is continually affected by legislation and political interests that affect the amount and type of materials acquired/disposed.

Another document that affects the AMP is the Report to the Congress on National Defense Stockpile Requirements. This document is submitted by the Secretary of Defense to Congress. Recommendations found in this report are based on Department of Defense assessments of national needs in conjunction with a military conflict scenario developed by the Joint Staff. This scenario determines theaters of conflict, military force structure, losses, military/industrial and essential civilian requirements, domestic production, and supplier country reliability during the first three years of a global conventional war of indefinite duration. The requirements used to build the AMP arise in the analyses considered in this Report.

2.3.2 SOURCE MODELS

There are several sources for determining the values of the macro-level factors that ultimately affect the AMP. The Joint Staff baselines military requirements by running the Joint Industrial Mobilization Planning Process Requirements Module. Requirements are simply the shortfall between supply and military demand.

Civilian and industrial requirements are also baselined to estimate what would be available for the production of defense goods. Demands for the civilian sector of the economy are computed from data in the National Income and Product Account.

The above three baselined requirements are then input into the Materials Defense Economic Impact Modeling System (MDEIMS) to refine requirements. MDEIMS consists of several economic models maintained by OASD (Program Analysis and Evaluation). Finally, the Stockpile Sizing Module (maintained by the Institute for Defense Analyses) is used to report how much of what materials should be stockpiled. These results in turn become constraints in the linear program that computes the AMP.

2.4 DEVELOP THE LINEAR PROGRAM THAT COMPUTES THE AMP

Once the input values for the macro-level AMP factors have been established, the mathematical model that computes the AMP can be solved. Appendix A shows the LP that is used in the AMP. Technical readers may obtain a summary of the exact LP approach used (Appendix B). The purpose of linear programming is to optimize some function (e.g., maximize disposal sales) subject to many constraints. The constraints that arise in building an AMP are:

1. Each commodity's annual upper/lower limits on what can be bought or sold (in units).

2. Each commodity's cumulative annual activity (in units) cannot exceed the shortfall (for acquisitions) or exceed the surplus (for disposals) in requirements.
(3) Annually, total dollars spent on acquisitions, operating/maintenance, and R&D cannot exceed the lesser of a spending cap or sales from disposals.

(4) Annual disposals (in units) of a commodity cannot exceed the cumulative annual disposal authority for that commodity.

(5) Total sales from disposals in each year have an upper and lower limit.

2.5 
IDENTIFY HOW AMPAT WOULD IMPLEMENT LINEAR PROGRAMMING

Because an LP-solver would be the heart of AMPAT, an April 1992 market survey was done to find suitable and affordable LP software. The survey sought:

(1) Source code that could be tailored for specific roles.

(2) Object code that could be integrated into other AMPAT software but not modified.

(3) Stand-alone LP solvers that were optimized for speed and efficiency but could not be modified or integrated into other AMPAT software.

The reason these different areas were considered was that choosing any one of these would fix the manner in which AMPAT would solve an LP. The source code approach would allow the most flexibility. The stand-alone LP solver would be the least flexible but would be faster. Object code stands in the middle; although it originates with source code, it has been compiled into machine language (and so can't be modified) which can be directly linked with AMPAT.

The survey paper recommended that AMPAT's LP module be created from scratch, because commercial source codes could not be found and object code/stand-alone solvers were too expensive. Furthermore, DORO had the necessary compilers and skill to write their own source code whose object code in turn could be integrated into other AMPAT software.

2.6 PROGRAMMING DESIGN OF AMPAT

Although AMPAT would center on an LP-solver, the time consuming phase of developing AMPAT was seen to be creating its user interface. Also, an eye had to be kept to the future: it was envisioned that after a later study, AMPAT would become a hybrid product that used operations research and AI to address the entire process of creating an AMP. It was thought that the future development of AMPAT would be largely in the direction of using AI to model decision making.

The three concerns, therefore, that weighed on the initial design of AMPAT were its LP-solver, its user-interface, and its future development. To address all three required software tools that would collectively allow relatively swift program development. Choosing such tools, however, can be difficult because a different analyst charged with modification and maintenance of AMPAT may prefer different tools.
In AMPAT's case, the AI language PROLOG and the procedural language C were chosen to develop AMPAT. The reason for this choice was that the creator of AMPAT had several years of experience building user interfaces and applications in these languages. Compilers in both languages were also available in DORO. The LP-solver would be created in Borland C++ 1.0, the user interfaces in PDCPROLOG 3.21, and PROLOG would control the execution of AMPAT. Of all the compilers in DORO, PDCPROLOG had the best facilities with which to build user interfaces.

It should be noted that although FORTRAN was an equally attractive language within which to develop the LP-solver, no one (to the knowledge of the PROLOG Development Center) had ever attempted an integrated PROLOG/FORTRAN product, only PROLOG/C.
SECTION 3
THE ANNUAL MATERIALS PLAN ANALYSIS TOOL (AMPAT)

3.1 GENERAL FUNCTIONALITY

AMPAT is a personal computer based analysis tool that allows industrial specialists of the DNSC to compute different AMPs and to build/maintain the associated commodity and general rates datafiles. By creating and modifying these databases, the DNSC can swiftly mix and match different rates and commodity files to build AMPs for different economic/political scenarios.

3.2 SPECIFIC FUNCTIONALITY: MAIN MENU

AMPAT's greetings screen is shown in figure 3-1. From here the user can choose to modify or create different types of datafiles, pair a rate and AMP file to compute their associated AMP, or obtain an overview of AMPAT. Scattered throughout AMPAT are different help texts, thus precluding the need for a printed users manual.

3.3 SPECIFIC FUNCTIONALITY: MASTER COMMODITY LIST

Since an AMP is built around a set of commodities, AMPAT provides a master commodity list from which items are chosen to build an AMP (figure 3-2). Over time, the master commodity list will require updating. The user may modify the list by changing the name of an item, its unit of measure, current inventory, and current disposal authority. Modifications may be done within AMPAT.

```
Annual Materials Plan Analysis Tool (AMPAT)

F1 Help
F2 Modify Master Commodity List
F3 Create/Modify an AMP File
F4 Global Rates
F5 Create Linear Program
F6 reserved
F7 reserved
F8 reserved
F9 Overview
esc Exit
```

Figure 3-1. AMPAT Greeting Screen
<table>
<thead>
<tr>
<th>Commodity</th>
<th>Standard Unit</th>
<th>Inventory</th>
<th>Current Disposal Authority</th>
</tr>
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<tr>
<td>Bauxite, refractory</td>
<td>short tons</td>
<td>15232.32</td>
<td>1900.23</td>
</tr>
<tr>
<td>Aluminum Metal</td>
<td>ST</td>
<td>57588</td>
<td>0</td>
</tr>
<tr>
<td>Aluminum Oxide, Abras Grain</td>
<td>ST</td>
<td>50786</td>
<td>0</td>
</tr>
<tr>
<td>Aluminum Oxide, Abras Grn. NSG</td>
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<td>0</td>
</tr>
<tr>
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<td>249867</td>
</tr>
<tr>
<td>Antimony</td>
<td>ST</td>
<td>36004</td>
<td>36004</td>
</tr>
<tr>
<td>Antimony, NSG</td>
<td>ST</td>
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<td>7</td>
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</tr>
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<td>9783</td>
<td>7700</td>
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<td>36</td>
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<td>Baddeleyite</td>
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<td>LDT</td>
<td>12288824</td>
<td>12457740</td>
</tr>
<tr>
<td>Bauxite, Met. Grade, Surinam</td>
<td>LDT</td>
<td>4908512</td>
<td>5299597</td>
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<td>Bauxite, Refractory</td>
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<td>276067</td>
<td>207067</td>
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<tr>
<td>Beryl Ore</td>
<td>ST</td>
<td>16074</td>
<td>17729</td>
</tr>
<tr>
<td>Beryllium Copper Master Alloy</td>
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<tr>
<td>Bismuth</td>
<td>Lb</td>
<td>1631406</td>
<td>300000</td>
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<tr>
<td>Cadmium</td>
<td>Lb</td>
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</table>

**Figure 3-2. Commodity Master List**

### 3.4 Specific Functionality: Create/Modify an AMP File

An AMP file is simply a collection of commodities (chosen from the master commodity list) whose individual data values have been established in sufficient detail to build an AMP. In this AMPAT module, users may retrieve an existing AMP file to add/delete commodities, modify their values, or build a new AMP file from scratch. Figure 3-3 shows both the menu from which an existing AMP file can be chosen and the prompt to create a new file.

**Figure 3-3. AMP File Selection Menu**

Shown is rejection of current choices in favor of building a new AMP file.
After a file has been specified, another menu appears from which the user chooses the commodity to be modified or added (figure 3-4). Commodities that already appear in the AMP file are marked with an asterisk. After the commodity is chosen, its data values screen appears (figure 3-5). The user simply enters or modifies these values which appear: inventory, requirement, annual activity levels/prices, existing disposal authority, current disposals, and the annual requested disposal authorities. The user may also attach narrative remarks to the commodity which explain the established values or which may prove helpful to a decision maker. The user may specify up to 10 years worth of data.

ESC saves/goto MAIN; D:\93A.AMP

*Aluminum Metal
*Aluminum Oxide, Abras Grain
*Aluminum Oxide, Abras Grn. NSG
Aluminum Oxide, Fused Crude
Antimony
Antimony, NSG
*Asbestos, Amosite
*Asbestos, Amosite NSG
*Asbestos, Chrysotile
*Asbestos, Chrysotile, NSG
*Asbestos, Crocidolite (DBA)
Baddeleyite
*Bauxite, Met. Grade, Jamaica
*Bauxite, Met. Grade, Surinam
Bauxite, Refractory
Beryllium Metal
Beryl Ore
Beryllium Copper Master Alloy
*Bismuth
*Cadmium
*Chromite, Chemical
*Chromite, Metallurgical
*Chromite, Metallurgical NSG

Figure 3-4. Listing of Commodities in Chosen AMP File
(* indicates items currently in file)
Commodity >> Bauxite, Met. Grade, Jamaica LDT
Inventory >> 12288824
Requirement >> 0 (A)cquisition or (D)isposal >> D

<table>
<thead>
<tr>
<th>YEAR</th>
<th>Min</th>
<th>Max</th>
<th>Price</th>
<th>Authority</th>
<th>Existing Disposal Year</th>
<th>Current Disposals</th>
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<td>600000.0</td>
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<td></td>
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</tr>
</tbody>
</table>

**Figure 3-5. Commodity Entry Screen**

3.5 **SPECIFIC FUNCTIONALITY: GLOBAL RATES SCREEN**

An AMP file containing commodity information is only the first of two components needed to compute an AMP. The second component is a datafile containing annual fiscal rates which affect the overall activity of all commodities. Figure 3-6 shows both the menu from which an existing global rates file can be chosen and the prompt to create a new file.

Figure 3-7 shows the screen which displays the rates stored in a retrieved rates file and from which the user may modify values. The annual values are the spending cap, operating and maintainence budget, research and development budget, and the minimum/maximum on disposal sales. As before, the user may attach narrative remarks to a global rates file to guide future decision making.
hit escape to abort
93A.RAT  93B.RAT  93C.RAT
93D.RAT  93E.RAT  MAC.RAT
hit escape to abort
Name of new Global Rates File >>

Figure 3-6. Global Rates Selection Menu
Shown is rejection of current choices in favor of building a new rates file.

Global Rates File: D:\MAC.RAT
(in millions)

<table>
<thead>
<tr>
<th>Year</th>
<th>$Spend Cap</th>
<th>$Spend O/M</th>
<th>$Spend R&amp;D</th>
<th>$Sales Floor</th>
<th>$Sales Cap</th>
</tr>
</thead>
<tbody>
<tr>
<td>1993</td>
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<td>32.00</td>
<td>25.00</td>
<td>150.00</td>
<td>300.00</td>
</tr>
<tr>
<td>1994</td>
<td>600.00</td>
<td>32.00</td>
<td>25.00</td>
<td>80.00</td>
<td>600.00</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1996</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1997</td>
<td></td>
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<td></td>
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<tr>
<td>1998</td>
<td></td>
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<td>2002</td>
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</tbody>
</table>

F1 Help  F2 Store  F3 Remarks  F4 DeleteFile  ESC Quit

Figure 3-7. Global Rates Entry Screen
3.6 SPECIFIC FUNCTIONALITY: COMPUTING AN AMP

After an AMP and global rates files have been chosen, the stage is set for computing an AMP. This module simply combines commodity-specific information and global rates to create a linear program which is then solved. The solution of the LP in turn becomes the levels of acquisition and disposal reported in the AMP. These levels are computed for each commodity and for each year.

Figure 3-8 shows the screen from which global rates and AMP files are chosen from. If both files begin with the same year, the user is then asked to specify the year which closes the planning horizon. If there are global rates for each year in the planning horizon and if each commodity has data for each of these years, then an AMP can be built. Otherwise, the user is told either that some years lack global rates or that some commodities lack data. The user will then return to the module in which the affected global rates or AMP file can be modified and made whole.

Otherwise, a summary is displayed for the user to review (figure 3-9). If the user accepts this data, then the underlying linear program will be built and solved. If the data is rejected, then the user may modify the appropriate global rates and/or AMP files and return again to compute an AMP. The solution to the linear program is displayed (figure 3-10) in a table that gives the annual levels of activity and associated dollar value for each commodity. AMPAT will also identify if other optimal solutions exist but will not identify them.

There is one last thing to note. Even though the user may validate the commodity and global rates files, it still may not be possible to build an AMP due to some contradiction in this input data. For example, if a disposal item's cumulative minimum activity exceeds its surplus, then the user must modify the minimum activity of the commodity. AMPAT will automatically check the accepted data and alert the user to any one of several contradictions found (Appendix A).
choose global rates file
93A.RAT  93B.RAT  93C.RAT
93D.RAT  93E.RAT  MAC.RAT*

choose AMP file
93A.AMP  93B.AMP  93C.AMP
93D.AMP  93E.AMP  NU93.AMP*

Start Year is 1993
Stop year >>1994

Figure 3-8. Building a Linear Program: Pairing Global Rates/AMP Files and Specifying Time Period
Build AMP using these global rates and (A)cquisition/(D)isposal items

<table>
<thead>
<tr>
<th></th>
<th>$Spend</th>
<th>$Spend</th>
<th>$Spend</th>
<th>$Sales</th>
<th>$Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cap</td>
<td>O/M</td>
<td>R&amp;D</td>
<td>Floor</td>
<td>Cap</td>
</tr>
<tr>
<td>1993</td>
<td>150.00</td>
<td>32.00</td>
<td>25.00</td>
<td>150.00</td>
<td>300.00</td>
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<tr>
<td>1994</td>
<td>600.00</td>
<td>32.00</td>
<td>25.00</td>
<td>80.00</td>
<td>600.00</td>
</tr>
</tbody>
</table>

C:\AMPAT\A\MAC.RAT (in millions)

C:\AMPAT\A\NU93.AMP

<table>
<thead>
<tr>
<th>Inventory Requirements</th>
<th>Remaining CDA</th>
<th>1993</th>
<th>1994</th>
</tr>
</thead>
<tbody>
<tr>
<td>(ST) Aluminum Metal D</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>57588.0</td>
<td>10000.0</td>
<td>10000.0</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>20000.0</td>
<td>20000.0</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>850.00</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>60000.0</td>
<td>0.0</td>
</tr>
<tr>
<td>(ST) Aluminum Oxide, Abras Grain D</td>
<td>50786.0</td>
<td>15000.0</td>
<td>15000.0</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
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<td>20000.0</td>
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<td></td>
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<td>120.00</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>20000.0</td>
<td>20000.0</td>
</tr>
<tr>
<td>(ST) Aluminum Oxide, Abras Grain NSG D</td>
<td>118.0</td>
<td>118.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>118.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
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<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>120.0</td>
<td>0.0</td>
</tr>
<tr>
<td>(ST) Asbestos, Amosite D</td>
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<td>10000.0</td>
<td>10000.0</td>
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</tr>
<tr>
<td></td>
<td>34000.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

HOME beginning of line   END end of line   F10 Accept   ESC Abort

*Figure 3-9. Review of Factors Before Linear Program is Computed (Partial Listing)*
**SOLUTION:** multiple optima  **OBJECTIVE:** 4590907.00  **REPORT:** 09041253.45

**Annual Materials Plan 1993-1994**

**Goal:** maximize total raw unit activity over planning horizon

**Acquisitions:**

<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
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<td>5300000.00</td>
<td>250000.00</td>
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<td>50000.00</td>
<td>400000.00</td>
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<td><strong>43000071.0</strong></td>
<td><strong>105774492.8</strong></td>
<td><strong>151870358.7</strong></td>
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</tbody>
</table>

**Disposals:**

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</tr>
</thead>
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<tr>
<td>Aluminum Metal (ST)</td>
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<td>1800000.00</td>
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<tr>
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<td>0.0</td>
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<td>0.00</td>
</tr>
<tr>
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</tr>
<tr>
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<td>600000.00</td>
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</tr>
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<td>2000000.00</td>
<td>200000.00</td>
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<td>41920000.0</td>
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<td>0.0</td>
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<tr>
<td><strong>TOTAL</strong></td>
<td><strong>105774492.8</strong></td>
<td></td>
<td><strong>151870358.7</strong></td>
<td></td>
</tr>
</tbody>
</table>

---

**INPUT DATA**

C:\AMPAT\T1.RAT (920822 20:01)
C:\AMPAT\T1.AMP (920822 19:59)

**Figure 3-10.** Results of Solving Linear Program: Levels of Commodity Activity and Associated Costs to Report in AMP (partial listing)
3.7  BENEFIT OF USING AMPAT

AMPAT empowers the DNSC to swiftly answer the frequent and urgent taskings from OASD/P&L for a new AMP. By retrieving data from a user-created library, the user of AMPAT may quickly establish the characteristics of an economic/political scenario for which an AMP will be computed. Once the scenario has been set, its underlying mathematical model is quickly built and solved. AMPAT takes the solution and builds the AMP which (after review) may lead to other scenarios and AMPS. The feedback loop between OASD/P&L and the DNSC is thus greatly accelerated and may lead to a better "final" AMP being submitted to the Executive Branch.
SECTION 4
CONCLUSION

This study reached the following conclusions:

(1) A mathematically based approach to building an AMP does exist.

(2) Such an approach would use well-known solution methods that would quickly determine the optimal levels to report in an AMP.

(3) Errors that may otherwise arise from using multiple methods to compute an AMP are reduced by using a single analysis tool such as AMPAT.
SECTION 5
RECOMMENDATION

This study recommends that AMPAT be:

1. Used to compute the Annual Materials Plan. This will allow the Defense National Stockpile Center to submit more timely AMPs as mandated in public law.

2. Incorporated in a follow-on system that uses AI and operations research concepts to model the decision making process that leads to computing an AMP.
APPENDIX A
LINEAR PROGRAMMING FORMULATION FOR THE ANNUAL MATERIALS PLAN
### TABLE OF CONTENTS

**APPENDIX A**

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<th>Title</th>
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</thead>
<tbody>
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<td>A-5</td>
</tr>
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<td>A-1.2</td>
<td>Statement Of The Original Linear Program</td>
<td>A-5</td>
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<tr>
<td>A-1.3</td>
<td>Statement Of The Revised Linear Program</td>
<td>A-6</td>
</tr>
<tr>
<td>A-1.4</td>
<td>Simple Inspection For Infeasibility</td>
<td>A-7</td>
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</table>
APPENDIX A
LINEAR PROGRAMMING FORMULATION FOR THE ANNUAL MATERIALS PLAN

A-1.1 DEFINITIONS

This appendix establishes the mathematical model—a linear program—whose solution is equivalent to computing an Annual Materials Plan. The model's commodity-specific factors for commodity i in Year j are:

- \( a_{ij} \) requested disposal authority
  -- \( a_{i0} \) is current remaining disposal authority
  -- equals current disposal authority less disposals to-date
- \( L_{ij} \) lower activity bound
- \( P_{ij} \) per unit price
- \( U_{ij} \) upper activity bound
- \( x_{ij} \) activity level
- \( \delta_i \) shortfall or surplus, i.e., requirement - inventory

The model's global rates for Year j are:

- \( C_j \) sales cap
- \( F_j \) sales floor
- \( S_j \) spending cap
- \( O_j \) combined O&M/R&D budget

Lastly, define these index sets:

- \( A \) acquisition commodities
- \( D \) disposal commodities
- \( Y \) planning horizon \([\text{Year1}, \text{YearN}]\)

A-1.2 STATEMENT OF THE ORIGINAL LINEAR PROGRAM

The initial objective function of this linear program is:

\[
\text{maximize } \sum_{i \in A \cup D} \sum_{j \in Y} c_{ij} x_{ij}
\]

where the cost coefficients \( c_{ij} \) are specific to the type of LP solved. The initial constraints of the LP are:

a. \( x_{ij} \in [L_{ij}, U_{ij}] \), for each \( i \in A \cup D \) and \( j \in Y \) (activity bounds)

b. \( \sum_{j \in Y} x_{ij} \leq |\delta_i| \), for each \( i \in A \cup D \) (shortfall/surplus bounds)
c. \( o_j + \sum_{i \in A} p_{ij}x_{ij} \leq \sum_{i \in D} p_{ij}x_{ij} \), for each \( j \in Y \) (spending \leq sales)

d. \( \sum_{j \in k} x_{ij} \leq \sum_{j \in k} a_{ij} \), for each \( i \in D \) and \( k \in Y \) (disposal authority limits)

e. \( o_j + \sum_{i \in A} p_{ij}x_{ij} \leq S_j \), for each \( j \in Y \) (acquisition spending cap)

f. \( \sum_{i \in D} p_{ij}x_{ij} \in [F_j, C_j] \), for each \( j \in Y \) (sales floor and cap)

A-1.3 STATEMENT OF THE REVISED LINEAR PROGRAM

To obtain the LP used in AMPAT's LP-solver, transform the above LP so that all variables have a lower bound of zero. This transformation will yield the initial basic feasible solution needed to begin a Phase I optimization (Appendix B, Section 1.5) and is accomplished by the substitution:

\[ x_{ij} = L_{ij} + y_{ij} \]

where \( y_{ij} \in [0, U_{ij} - L_{ij}] \). The resulting LP is thus:

\[
\text{maximize } \sum_{i \in A U D} c_{ij}y_{ij} \\
\text{subject to:}
\]

a. \( y_{ij} \in [0, U_{ij} - L_{ij}] \), for each \( i \in A U D \) and \( j \in Y \)

b. \( \sum_{j \in Y} y_{ij} \leq |d_i| - \sum_{j \in Y} L_{ij} \), for each \( i \in A U D \)

c. \( \sum_{i \in A} p_{ij}y_{ij} - \sum_{i \in D} p_{ij}y_{ij} \leq o_j + \sum_{i \in D} p_{ij}L_{ij} - \sum_{i \in A} p_{ij}L_{ij} \), for each \( j \in Y \)

d. \( \sum_{j \in k} y_{ij} \leq \sum_{j \in k} a_{ij} - \sum_{j \in k} L_{ij} \), for each \( i \in D \) and \( k \in Y \)

e. \( \sum_{i \in A} p_{ij}y_{ij} \leq S_j - o_j - \sum_{i \in D} p_{ij}L_{ij} \), for each \( j \in Y \)

f. \( \sum_{i \in D} p_{ij}y_{ij} \leq C_j - \sum_{i \in D} p_{ij}L_{ij} \), for each \( j \in Y \)

g. \( -\sum_{i \in D} p_{ij}y_{ij} \leq -F_j + \sum_{i \in D} p_{ij}L_{ij} \), for each \( j \in Y \)
SIMPLE INSPECTION FOR INFEASIBILITY

It is possible that a linear program built to compute the AMP may not have a solution, i.e., it is infeasible. Infeasibility may arise for many different reasons, e.g., specifying a minimum level of acquisition that exceeds the spending cap. Although it is generally difficult to identify an infeasible linear program a priori, AMPAT does perform several rudimentary checks for infeasibility and alerts the user when any of these are encountered.

To begin with, AMPAT only accepts positive real inputs. Thus all the variables in (except for $\delta_i$) will be positive. Further inspection of item data and global rates are made to ensure that:

- $a_{i0} \geq 0$, $i \in D$ (current disposals valid)
- $L_{ij} \leq U_{ij}$, $i \in A \cup D$, $j \in Y$ (activity levels are valid)
- $S_j, F_j \leq C_j$, $j \in Y$ (sales cap exceeds spending cap and sales floor)
- $O_j \leq \min(S_j, F_j)$, $j \in Y$ (O&M and R&D costs exceed neither the spending cap nor sales floor)
- $L_{i1} \leq a_{i1} + a_{i0}$, $i \in D$ (first year minimum disposal level doesn't exceed first year cumulative disposal authority)

After these item-level inspections, AMPAT next combines factors across the planning horizon to search for infeasibility. These checks ensure that:

- $\sum_{i \in D} U_{i j} P_{ij} \geq F_j$, $j \in Y$ (maximum disposal sales exceed sales floor)
- $\sum_j L_{ij} \leq |\delta_i|$, $i \in A \cup D$ (an item's minimum acquisition/disposal across the planning horizon doesn't exceed its shortfall/surplus)
- $\sum_{i \in D} L_{ij} P_{ij} \leq C_j$, $j \in Y$ (minimum disposal sales in a year does not exceed the sales cap)
- $\sum_{i \in A} L_{ij} P_{ij} \leq \min\{S_j, \sum_{i \in D} U_{i j} P_{ij} - O_j\}$, $j \in Y$ (total expense for minimum acquisitions in a given year does not exceed what remains after subtracting O&M/R&D costs from the smaller of the annual spending cap and maximum possible disposal sales)
Finally, each disposal item is checked to ensure that its requested disposal authorities will support its minimum activity levels:

- let RemDA_{ij} be the remaining maximum disposal authority for each disposal item \( i \in D \) at the end of Year \( j \in Y \)

  \[
  \text{RemDA}_{ij} = \text{RemDA}_{i,j-1} + a_{ij} - L_{ij}
  \]

  \[
  \text{RemDA}_{i0} = a_{i0}
  \]

- if any RemDA_{ij} < 0, then the LP is infeasible
APPENDIX B

SIMPLEX ALGORITHM USED IN AMPAT
<table>
<thead>
<tr>
<th>Paragraph</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-1.1</td>
<td>Introduction</td>
<td>B-5</td>
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<tr>
<td>B-1.2</td>
<td>Canonical Form</td>
<td>B-5</td>
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<td>B-1.3</td>
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B-1.1 INTRODUCTION

This appendix gives the mathematical background of the simplex algorithm used in the AMPAT LP-solver. In a nutshell, this version of the simplex uses known variable upper bounds to reduce the number of constraints considered, and uses a single artificial variable in a Phase I optimization to find an initial basic feasible solution to the given LP. It is assumed that the reader is familiar with elementary linear algebra/matrix theory and has been exposed to the vocabulary surrounding the simplex algorithm.

B-1.2 CANONICAL FORM

Before an LP can be solved by AMPAT, it must first be pre-processed into a required form before solution is attempted. The LP first seen by AMPAT must have the following canonical form:

\[
\begin{align*}
\text{maximize} & \quad c'X \\
\text{subject to} & \quad AX \leq b \\
& \quad 0 \leq X \leq U
\end{align*}
\]

where \( c, b, X, \) and \( U \) are columnar vectors, \( ' \) is the transpose operator, and \( A \) is an \( m \times n \) matrix of real numbers.

B-1.3 STANDARD FORM

Next, AMPAT converts the above canonical-LP into the standard form:

\[
\begin{align*}
\text{maximize} & \quad c'X \\
\text{subject to} & \quad AX = b \\
& \quad 0 \leq X \leq U
\end{align*}
\]

through the introduction of positive slack variables into the previous inequalities. Note that although the same variable names are used, the matrices in the standard and canonical LPs are not the same. At this point, only the form of the LPs are being established. Also note that the slack variables have no upper bound, unlike the variables in [1]. In this case, the appropriate entry in \( U \) for a slack variable is \( \infty \).

B-1.4 USE ONE ARTIFICIAL VARIABLE

The next step is to transform the above equalities so that all of the right
hand sides are positive, the coefficients of the slack variables remain equal to 1, and that only one artificial variable has been introduced. This is accomplished by the following:

- let \( AX = (V \cdot I)X = b \), where \( I \) is an identity matrix
- let \( (V'' \cdot I'')X = b'' \) be the cross-section of \( AX = b \) all of whose right hand side values are negative
- let \( S \) be the index set of rows in this cross-section
- let row \( j \) from this cross-section be represented by the equation:
  \[
  E_j + s_j = b_j
  \]
  where \( s_j \) is the slack variable and \( b_j \) is the right hand side
- let \( b_k = \min_{j \in S} \{b_j\} \)
- for all \( j \in S - \{k\} \), subtract the equation represented by row \( k \) from the equation represented by row \( j \):
  \[
  \begin{align*}
  E_j + s_j &= b_j \\
  (E_k + s_k) &= b_k \\
  E_j - E_k + s_j - s_k &= b_j - b_k
  \end{align*}
  \]
  and substitute equation \( j \) by the result, i.e.,
  \[
  E_j + s_j = b_j \quad \text{is replaced by} \quad E_j - E_k + s_j - s_k = b_j - b_k
  \]
  Notice that \( b_j - b_k \) is positive because \( b_k \leq b_j \) by design. Also note that the coefficient of the slack variable \( s_j \) remains equal to 1.
- finally, replace \( E_k + s_k = b_k \) by \( -E_k - s_k + \alpha = -b_k \), where \( \alpha \) is the sole artificial variable.

**B-1.5 FINAL FORM OF LINEAR PROGRAM**

After the above steps have been taken, the standard-LP of equation (2) has been replaced by an LP of the form:

\[
\begin{align*}
\text{maximize} \quad & c'X \\
\text{subject to} \quad & AX = b \geq 0 \\
& 0 \leq X \leq U
\end{align*}
\]

where \( A \) is an \( m \times n \) matrix and by construction, has \( m \) columns that comprise a standard bases for Euclidean \( m \)-space, and where a member of this bases corresponds to the artificial variable. Note that an initial basic feasible solution exists to the above LP: values of basic variables correspond to entries in \( b \) and non-basic variables equal zero. It is to obtain this
initial basic feasible solution that the transformation found in Appendix A, Section 1.3, was done. The pre-processing phase is now complete and the above LP is ready to be solved by AMPAT.

**B-1.6 PHASE I OPTIMIZATION**

If indeed an artificial variable exists in the above LP (as will usually be the case), then it is necessary to drive its value to zero to obtain an initial basic feasible solution to the original problem given to AMPAT. To do this using Phase I optimization, simply change the above objective function to:

$$\text{maximize } -\alpha$$

and solve the new LP using the procedure outlined below. If the optimal objective value is nonzero, then $\alpha$ cannot be driven to zero and the LP given to AMPAT doesn't have a solution. Otherwise, if the artificial variable is non-basic, then proceed with Phase II optimization. However, if the artificial variable is basic, try to swap it with a non-basic variable and then proceed with Phase II. If this isn't possible, then simply recognize that redundancy has been discovered and proceed with Phase II. In all cases, before going to Phase II optimization, drop the artificial variable from the LP and do not ever consider it again.

**B-1.7 THE SIMPLEX ALGORITHM**

**B-1.7.1 INITIAL LINEAR PROGRAM**

Because both Phase I and Phase II optimizations follow the same procedure, we drop the distinction between the two and discuss how to find the optimal solution of a general LP using the simplex algorithm. Let the general LP be:

$$\begin{align*}
\text{maximize } & \; c'_B x_B + c'_R x_R \\
\text{subject to } & \; A_R x_B + A_R x_R = b \\
& \; 0 \leq x_{B,R} \leq U
\end{align*}$$

where $B, R$ are index sets to the basic and nonbasic variables respectively. Since $A_B$ is invertible by definition of $B$, multiply equation [4] by the inverse of $A_B$ to obtain:

$$x_B + A_B^{-1} A_R x_R = A_B^{-1} b$$

[5]
B-1.7.2 TRANSFORM LINEAR PROGRAM/CHOOSE ENTERING NONBASIC

Solve for $X_B$ in the above equation and substitute into the above LP to obtain the equivalent LP:

$$\text{maximize } [c'R - c'R A_B^{-1} A_R] * X_R \quad [5.1]$$

subject to

$$X_B + A_B^{-1} A_R X_R = A_B^{-1} b$$

$$0 \leq X_{B,R} \leq U$$

Now choose the nonbasic whose per unit change increases the objective most (cost is synonymous with objective coefficient):

- of all nonbasics at lower bound, let $\Phi$ have the largest positive cost
- of all nonbasics at upper bound, let $\Omega$ have the least negative cost
- let ENTER be the nonbasic from $\{\Phi, \Omega\}$ whose cost has the largest absolute value
- if ENTER is null, then optimality has been reached

Now ask the question "by how much can ENTER change such that $X_{B,R}$ remains feasible?" Let this max allowable change be $\delta$. From [5]:

$$X_B = A_B^{-1} b - A_B^{-1} A_R X_R$$

$$= A_B^{-1} b - Y*X_R$$

Let

$$Xl_R = X_R + D \quad [6.1]$$

where $\delta$ is the only nonzero in $D$ and appears in the position that corresponds to ENTER. From [6], the new $X_B$ is:

$$Xl_B = A_B^{-1} b - Y*Xl_R$$

$$= A_B^{-1} b - Y*(X_R + D)$$

$$= A_B^{-1} b - Y*X_R - YD$$

$$= \delta - \delta*Y_{ENTER} \quad [7]$$

where $Y_{ENTER}$ is the column in the matrix $Y$ that corresponds to the variable ENTER. Note that $YD = \delta*Y_{ENTER}$ only because $\delta$ is the sole nonzero in $D$ and appears in the row that corresponds to the variable ENTER.

B-1.7.3 CHOOSE EXITING BASIC

It remains to compute $\delta$ and to determine which basic variable gets replaced by ENTER. Per the original LP, $0 \leq X_{B,R} \leq U$ and so:

$$0 \leq Xl_B \leq U_B$$

$$0 \leq Xl_R \leq U_R \quad [7.1]$$
Thus from (7):

\[ 0 \leq x_B - \delta y_{ENTER} \leq u_B \]

which implies:

\[ x_B - u_B \leq \delta y_{ENTER} \leq x_B \]  \hspace{1cm} (8)

Let \( y_{ENTER} = [y_i] \), \( x_B = [x_i] \), and \( u_B = [u_i] \). From (8):

\[ \delta \leq \min \{ \min \{ \frac{x_i}{y_i} : y_i > 0 \}, \min \{ \frac{x_i - u_i}{y_i} : y_i < 0 \} \} = \alpha_1 \quad \text{(pushes a basic to its extreme)} \]

\[ \delta \geq \max \{ \max \{ \frac{x_i}{y_i} : y_i < 0 \}, \max \{ \frac{x_i - u_i}{y_i} : y_i > 0 \} \} = \alpha_2 \quad \text{(pushes a basic to its extreme)} \]

But since only one nonbasic variable changes (namely, \( x_{ENTER} \)) and that this variable has an upper bound:

\[ 0 \leq x_{ENTER} + \delta \leq u_{ENTER} \]

from which it follows:

\[ -x_{ENTER} \leq \delta \leq u_{ENTER} - x_{ENTER} \]

But since nonbasics are always at their extreme values:

\[ 0 \leq \delta \leq u_{ENTER} \quad \text{(ENTER at lower bound)} \]

\[ -u_{ENTER} \leq \delta \leq 0 \quad \text{(ENTER at upper bound)} \]

Thus, C9-C12 imply:

\[ \delta = \min \{ \alpha_1, u_{ENTER} \} \quad \text{ENTER at lower bound} \]

\[ \delta = \max \{ \alpha_2, -u_{ENTER} \} \quad \text{ENTER at upper bound} \]

Although we have an entering nonbasic ENTER, it remains to determine if it will indeed replace an existing basic. If \( \delta \) arises from [9]-[10], then the leaving basic variable is the \( x_i \) whose value determines \( \delta \). The optimization process resumes anew starting at equation (4) (revised in [5.1]) with new sets \( B \) and \( R \), or if \( B \) and \( R \) remain unchanged, then at equation (6) with the new values for the nonbasic variables (given in [6.1]).
**REPORT DOCUMENTATION PAGE**

**4. TITLE AND SUBTITLE**
Annual Materials Plan Analysis Tool

**6. AUTHOR(S)**
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**13. ABSTRACT (Maximum 200 words)**
AMPAT is a personal computer based analysis tool that the Defense National Stockpile Center (DNSC) will use to build the Annual Materials Plan (AMP). Per public law, the President submits the AMP to Congress and in it states how much of what materials to modify in the Defense National Stockpile. AMPAT allows the user to create and maintain many commodity and financial information databases. These databases can be freely paired to construct an economic/political scenario for which an AMP is computed and solved using linear programming. AMPAT allows the DNSC to swiftly respond to the frequent and urgent taskings from the Assistant Secretary of Defense (Production and Logistics) for new AMPs, taskings often generated by Congressional actions. AMPAT reduces the time used to compute the AMP from one week to one hour and thus accelerates feedback between OASD/P&L and the DNSC. AMPAT was created using the artificial intelligence language PROLOG and the procedural language TURBO C++.

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