Ionopole Elements with Disk Ground Planes on Flat Earth: Atlas of Directivity, Radiation Efficiency, Radiation Resistance, and Input Impedance

M. M. Weiner
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13. ABSTRACT (Maximum 200 words)

Richmond's moment-method programs IRCHMOND3 and RICHMOND4 for monopole elements with disk ground planes above flat Earth are used to obtain computer plots of directivity, radiation efficiency, radiation resistance, and input impedance at 15 MHz. Results, in the form of an atlas, are presented as a function of Earth classification for thin, quarter-wave monopole elements whose ground planes or radii 0 to 8 wavenumbers rest on Earth.

Results are compared with those for a perfect ground plane (of infinite extent and conductivity) and ground plane in free-space. Sea-water enhancement of radiation efficiency and low-angle directivity by comparing results for sea water with those for medium dry ground.
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Results are compared with those for a perfect ground plane (of infinite extent and conductivity) and ground plane in free-space. Sea-water enhancement of radiation efficiency and low-angle directivity is illustrated by comparing results for sea water with those for medium dry ground.
ACKNOWLEDGMENTS

Computer programs RICHMOND3 and RICHMOND4 were written by Jack H. Richmond (deceased) of Ohio State University. The latter program was written when he was a member of the technical advisor group to the MITRE-sponsored research project 91260 "High-Frequency Antenna Element Modeling," Melvin M. Weiner, Principal Investigator. Christopher Sharpe, Laurie Giandomenico, and Enis Vlashi performed the computer runs and obtained the computer plots.
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SECTION I
INTRODUCTION

The modeling of monopole elements with circular ground planes in proximity to Earth has been greatly enhanced in recent years by method-of-moments programs developed by Richmond for disk ground planes \[1,2\] and by Burke, et al. for radial-wire ground planes \[3,4,5\].

Method-of-moments models, unlike models based on Sommerfeld's attenuation function \[6\] or variational models based on Monteath's compensation theorem \[7,8,9,10\], determine the directivity and radiation efficiency as separate entities rather than lumping them together as a product to yield the antenna gain. Other advantages of the method-of-moments models include the following: more exact determination of current distributions; applicability to electrically small ground planes; direct determination of ground-plane edge diffraction; and avoidance of analytical restrictions on evaluating Sommerfeld's integral (such as requiring that the Earth's complex relative permittivity have a modulus much greater than unity).

A disadvantage of method-of-moments models is that they are restricted to relatively small ground-plane radii. This restriction is required so that ground-plane segmentation of unknown current variables does not exceed the computer's computational capacity and precision in solving for the unknown currents.

Richmond has presented a moment-method analysis for the current distributions and input impedance of a monopole element on disk ground planes in free space \[1\] and above flat Earth \[2\] with numerical evaluation by computer programs RICHMD1 and RICHMOND3, respectively. Weiner, et al. \[11,12\], have utilized the current distributions in reference 1 to develop a computer program, RICHMD2, for the far-zone field when the ground plane is in free space. Subsequently, Richmond used the current distributions in reference 2 to develop a computer program, RICHMOND4, for the far-zone field when the ground plane is above flat Earth. This latter effort has been reported by Weiner \[13\] who has also presented some numerical results and the validation of the numerical results. Listings of programs

1-1
RICHMD1 and RICHMD2 are given in reference 12. Listings of programs RICHMOND3 and RICHMOND4 are given in reference 13.

The present effort uses programs RICHMOND3 and RICHMOND4 to obtain computer plots of directivity (directive gain), radiation efficiency, radiation resistance, and input impedance for the International Radio Consultative Committee (CCIR) classifications of Earth [14]. Numerical results are presented in the form of an atlas of computer plots for thin, quarter-wave, monopole elements whose disk ground planes of radii 0 to 8 wavenumbers rest on Earth.

The antenna parameters are defined in section 2. Computer plots of numerical results are presented in section 3.

Examples of numerical results for only one type of Earth (medium dry Earth) are given in reference 13. The statement is made in reference 13 that approximately similar numerical results are obtained for other classifications of Earth with the exception of sea water. The present atlas provides numerical results to support such a statement and in particular to determine the quantitative differences in antenna performance for various classifications of Earth. The prior literature contains relatively little quantitative information on the performance of monopole elements with disk ground planes in close proximity to Earth. The intention of the present atlas is to address this deficiency for the benefit of the radar, communication, and broadcast communities.
SECTION 2
ANTENNA PARAMETERS

The antenna geometry consists of a vertical monopole element (length \( h \) and radius \( b \)) at the center of an infinitely thin disk ground plane of radius \( a \) (see figure 1). The ground plane is at a height \( z_o \) above flat Earth. The monopole element and disk are assumed to have infinite conductivity.

The Earth [with a dielectric constant \( \varepsilon_r \) and conductivity \( \sigma (S/m) \) for a waveform of time dependence \( e^{j\omega t} \) at a radian frequency \( \omega \) (rad/s) and free-space wavelength \( \lambda \) (m)] has a complex relative permittivity \( \varepsilon^{*}/\varepsilon_o = \varepsilon_r (1 - j \tan \delta) \) where \( \tan \delta = \text{loss tangent} = \sigma/(\omega \varepsilon_r \varepsilon_o) \)

\[
(\lambda\sigma/(2\pi\varepsilon_r)(\mu_o/\varepsilon_o))^{1/2} = 60\lambda \sigma/\varepsilon_r; \quad \mu_o \text{ and } \varepsilon_o \text{ are the free-space permeability and permittivity, respectively.}
\]
The location of an arbitrary far-zone observation point \( P \) is designated by spherical coordinates \((r, \theta, \phi)\).

The feed for the monopole antenna is a coaxial line with its inner conductor connected to the vertical monopole element through a hole of radius \( b \), at the center of the ground plane. The coaxial-line outer conductor of diameter \( 2b \) is connected by means of a flange to the ground plane. The coaxial-line inner-conductor diameter is equal to the monopole element diameter \( 2b \). The current on the outside of the coaxial-line outer conductor is assumed to be zero because of attenuation by lossy ferrite toroids along the exterior of the coaxial-line feed (see section 2.4 of reference 12).

The Earth constants, loss tangents, and penetration depths are summarized in table 2-1 for CCIR 527-1 classifications of Earth [14] in the 3-MHz through 30-MHz high-frequency band. Cases (2) through (9) correspond to CCIR classifications of Earth. Cases (1) and (11) correspond to perfect ground planes (of infinite conductivity and extent) and ground planes in free space, respectively. Case (10) is arbitrarily defined as Average Land and corresponds to Earth constants \( \varepsilon_r = 10.0, \sigma = 5 \times 10^{-3} \) S/m.
Figure 2-1. Monopole Element on Disk Ground Plane above Flat Earth
Table 2-1. Permittivity, Loss Tangent, and Penetration Depth of CCIR 527-1
Classifications of Earth

<table>
<thead>
<tr>
<th>Cases</th>
<th>( \varepsilon_r )</th>
<th>( \sigma ) (S/m)</th>
<th>( \sigma/(\omega \varepsilon_0) = (60\lambda) / (\sigma/c) )</th>
<th>( \delta ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Frequency (MHz)</td>
<td>Frequency (MHz)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Wavelength (m)</td>
<td>Wavelength (m)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>15</td>
<td>30</td>
</tr>
<tr>
<td>(1) Perfect Ground</td>
<td>1.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(2) Sea Water (average Salinity 20°C)</td>
<td>70.0</td>
<td>5.0</td>
<td>4.282 x 10^2</td>
<td>8.425 x 10^1</td>
</tr>
<tr>
<td>(3) Fresh Water</td>
<td>80.0</td>
<td>3.0 x 10^{-2}</td>
<td>2.251 x 10^0</td>
<td>4.497 x 10^{-1}</td>
</tr>
<tr>
<td>(4) Wet Ground</td>
<td>90.0</td>
<td>1.0 x 10^{-2}</td>
<td>1.999 x 10^0</td>
<td>3.997 x 10^{-1}</td>
</tr>
<tr>
<td>(5) Medium Dry Ground</td>
<td>15.0</td>
<td>1.0 x 10^{-3}</td>
<td>3.997 x 10^{-1}</td>
<td>7.995 x 10^{-2}</td>
</tr>
<tr>
<td>(6) Very Dry Ground</td>
<td>3.0</td>
<td>1.0 x 10^{-4}</td>
<td>1.999 x 10^{-1}</td>
<td>3.997 x 10^{-2}</td>
</tr>
<tr>
<td>(7) Pure Water, 20°C</td>
<td>80.0</td>
<td>5.0 x 10^{-4}</td>
<td>1.350 x 10^{-4}</td>
<td>-</td>
</tr>
<tr>
<td>(8) Ice (fresh water, -1°C)</td>
<td>3.0</td>
<td>6.0 x 10^{-5}</td>
<td>1.189 x 10^{-1}</td>
<td>-</td>
</tr>
<tr>
<td>(9) Ice (fresh water, -10°C)</td>
<td>3.0</td>
<td>1.0 x 10^{-5}</td>
<td>1.0 x 10^{-1}</td>
<td>-</td>
</tr>
<tr>
<td>(10) Average Land</td>
<td>10.0</td>
<td>5.0 x 10^{-3}</td>
<td>2.998 x 10^0</td>
<td>5.996 x 10^{-1}</td>
</tr>
<tr>
<td>(11) Free Space</td>
<td>1.0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Numerical results are presented in section 3 for each Earth classification of table 2-1 and antenna parameters with the following numerical values: \( h/\lambda = 0.25 \), \( b/\lambda = 10^{-6} \), \( b_1/b = 3.5 \), \( 2\pi a/\lambda = 0 \) through 8, \( z_0/\lambda = 0 \), and \( f = 15 \text{ MHz} (\lambda = 20 \text{ m}) \).

The electrical characteristics that are evaluated are antenna directivity (also called "directive gain"), radiation efficiency, radiation resistance, and input impedance.

The total far-zone electric field \( E(r, \theta, \phi) = \hat{E}_\theta E_\theta(r, \theta, \phi) \) where \( \hat{E}_\theta \) is the unit vector in the \( \theta \) direction. The far-zone electric field is zero in the azimuthal direction because of the uniformity of the antenna geometry about the \( z \) axis. The total far-zone field amplitude \( E_\theta(r, \theta, \phi) \) is the sum of the far-zone fields from the monopole element, the disk ground plane, and the equivalent magnetic current density (magnetic frill) \( E_\phi \) of the coaxial-line feed excitation [13]. The total far-zone field is independent of the azimuthal coordinate \( \phi \) because of the azimuthal symmetry of the antenna geometry in figure 2-1. Therefore, \( E_\theta(r, \theta, \phi) = E_\theta(r, \theta) \).

Consider now the cases where the Earth medium either is lossy \((\sigma > 0)\) or is free space \((\sigma = 0, \varepsilon_r = 1)\). The total far-zone radiated power \( P_r \) is given by

\[
P_r = \begin{cases} 
\frac{(\pi/\eta_o)}{2} \int_0^{\pi/2} |E_\theta(r, \theta)|^2 r^2 \sin \theta d\theta, & \sigma > 0 \\
\left(\frac{\pi/\eta_o}{2}\right) \int_0^{\pi/2} |E_\theta(r, \theta)|^2 r^2 \sin \theta d\theta; & \sigma = 0, \varepsilon_r = 1 
\end{cases}
\]

(2-1)

where \( \eta_o = (\mu_o/\varepsilon_o)^{1/2} \) is the free-space wave impedance (ohms). For the case \( \sigma > 0 \) the integrand in equation 2-1 is integrated over only the hemisphere above the Earth because the field in lossy Earth, relative to that in free space, approaches zero at large radial distances \( r \).
The antenna directivity \( d(\theta) \) expressed as a dimensionless number is given by

\[
d(\theta) = 2|E_\theta(r,\theta)|^2 / P_r.
\] (2-2)

The antenna directivity \( D(\theta) \) expressed in decibels is given by

\[
D(\theta) = 10 \log_{10} d(\theta) \text{ (dB)}.
\] (2-3)

The time-averaged input power \( P_{in} \) to the monopole antenna is given by

\[
P_{in} = (1/2) \text{Re}[V(0)I^*(0)]
\] (2-4)

where \( V(0) \) = peak input voltage (volts). The input voltage \( V(0) \) is usually set equal to 1 volt in the moment-method analysis.

\( I^*(0) \) = conjugate of the peak input current \( I(0) \) at the base of the monopole element. This current is solved for by the moment-method analysis in reference 2.

The input impedance \( Z_{in} \) is given by

\[
Z_{in} = R_{in} + jX_{in} = V(0)/I(0)
\] (2-5)

where \( R_{in} \) and \( X_{in} \) are the input resistance and input reactance, respectively.

The antenna radiation resistance \( R_{rad} \) is defined as

\[
R_{rad} = 2P_r/|\bar{I}(0)|^2.
\] (2-6)
The antenna radiation efficiency $\eta$ is defined as

$$\eta = \frac{P_r}{P_{in}} = \left[1 + \left(\frac{R_{rad}}{R_{in}}\right)\right]^{-1}. \quad (2-7)$$

For the case of free-space ($\sigma = 0$, $\varepsilon_r = 1$), the radiation efficiency is equal to unity because the monopole element and disk ground-plane conductivities are assumed to be infinite.
SECTION 3

COMPUTER PLOTS OF NUMERICAL RESULTS

Numerical results are presented in the form of an atlas of computer plots for thin, quarter-wave monopole elements whose disk ground planes of radii 0 to 8 wavenumbers rest on Earth. Computer plots are presented in sections 3.1 through 3.9 for each Earth classification of table 1 at a frequency of 15 MHz. In each computer plot, results are compared with those for perfect ground planes (of infinite extent and conductivity) and ground planes in free space. Section 3.10 compares results for sea water with those for medium dry ground.

In the presence of Earth, the directivity patterns are approximately independent of disk radius (for ground-plane radii of 0 to 8 wavenumbers). The peak directivity is within 0.5 dBi of that for a perfect ground plane. The Earth softens the edge of the ground plane and minimizes changes in peak directivity resulting from ground-plane edge diffraction. In the absence of Earth, large changes in directivity occur because ground-plane edge diffraction is more pronounced.

The direction of peak directivity is approximately 30 degrees above the horizon, except for sea water, in which case the direction of peak directivity is approximately 10 degrees above the horizon. Relatively small changes in ground plane radius for particular radii can cause large changes in the angle of peak directivity regardless whether the ground plane is in close proximity to Earth or not and despite the broad 3 dB beamwidth of the elevation radiation pattern. The jump in angle of peak directivity between $2\pi a/\lambda = 5.5$ and 5.75 wavenumbers corresponds to a change of only 0.5 dB in directivity and is due to a change in beam shape.

The radiation efficiency increases monotonically with increasing disk radius, except for sea water, in which case the increase is not monotonic but the radiation efficiency is enhanced over that for other classifications of Earth. The radiation efficiency is small for small ground planes because most of the antenna input energy is redirected into the Earth by a surface wave that is generated at the air-Earth interface.
Each of the following subsections contains computer plots of the following antenna performance characteristics:

a. Numeric directivity patterns for disk radii $2\pi a/\lambda = 0.025, 3.0, 4.0, 5.0, 6.5$ wavenumbers. The patterns in the elevation plane at any azimuthal angle are polar plots with the same numerical scale.

b. Peak numeric directivity

c. Angle of incidence of peak directivity

d. Radiation efficiency

e. Radiation resistance

f. Input resistance

g. Input reactance

h. Directivity in decibels for angles of incidence $\theta = 82, 84, 86, 88, \text{ and } 90$ degrees.
3.1 SEA WATER (AVERAGE SALINITY, 20°C)
NUMERIC DIRECTIVE GAIN POLAR PLOT

Case 2, Sea Water (average salinity 20 deg C) at 15 MHz

\[ \frac{2\pi a}{\lambda} = 0.025 \] (Wavenumbers)

\[ h/\lambda = 0.25, b/\lambda = 1.0 \times 10^{-6}, z_0/\lambda = 0 \]
Case 1, Perfect Ground (\( \varepsilon_r = 1.0, \sigma \rightarrow \infty \))
Case 2, Sea Water (average salinity 20 deg C) (\( \varepsilon_r = 70.0, \sigma = 5.0 \text{ S/m} \))
Case 11, Free Space (\( \varepsilon_r = 1.0, \sigma = 0 \))

Figure 3.1. Directivity Pattern, \( \frac{2\pi a}{\lambda} = 0.025 \), Sea Water (Average Salinity, 20°C)
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 2, Sea Water (average salinity 20 deg C) at 15 MHz
2πa/λ = 3.0 (Wavenumbers)

h/λ=0.25, b/λ=1.0 x 10⁻⁶, z₀/λ=0
Case 1, Perfect Ground (εᵣ=1.0, σ=∞)
Case 2, Sea Water
  (average salinity 20 deg C)
  (εᵣ=70.0, σ=5.0 S/m)
Case 11, Free Space (εᵣ=1.0, σ=0)

Figure 3-2. Directivity Pattern, 2πa/λ = 3.0, Sea Water (Average Salinity, 20°C)
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 2, Sea Water (average salinity 20 deg C) at 15 MHz
$2\pi a/\lambda = 4.0$ (Wavenumbers)

$h/\lambda = 0.25, b/\lambda = 1.0 \times 10^{-6}, z_0/\lambda = 0$
Case 1, Perfect Ground ($\varepsilon_r = 1.0, \sigma = \infty$)
Case 2, Sea Water
(average salinity 20 deg C)
($\varepsilon_r = 70.0, \sigma = 5.0 \text{ S/m}$)
Case 11, Free Space ($\varepsilon_r = 1.0, \sigma = 0$)

Figure 3-3. Directivity Pattern, $2\pi a/\lambda = 4.0$, Sea Water (Average Salinity, 20°C)
NUMERIC DIRECTIVE GAIN POLAR PLOT

Case 2, Sea Water (average salinity 20 deg C) at 15 MHz

$2\pi a/\lambda = 5.0$ (Wavenumbers)

$\lambda/\lambda = 0.25$, $h/\lambda = 1.0 \times 10^{-6}$, $z_o/\lambda = 0$

Case 1, Perfect Ground ($\varepsilon_r = 1.0$, $\sigma = \infty$)
Case 2, Sea Water (average salinity 20 deg C)
($\varepsilon_r = 70.0$, $\sigma = 5.0$ S/m)
Case 11, Free Space ($\varepsilon_r = 1.0$, $\sigma = 0$)

Figure 3-4. Directivity Pattern, $2\pi a/\lambda = 5.0$, Sea Water (Average Salinity, 20°C)
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 2, Sea Water (average salinity 20 deg C) at 15 MHz

$2\pi a/\lambda = 6.5$ (Wavenumbers)

$h/\lambda=0.25, b/\lambda=1.0 \times 10^{-6}, z_d/\lambda=0$
Case 1, Perfect Ground ($\varepsilon_r=1.0, \sigma=\infty$)
Case 2, Sea Water
(average salinity 20 deg C)
($\varepsilon_r=70.0, \sigma=5.0$ S/m)
Case 11, Free Space ($\varepsilon_r=1.0, \sigma=0$)

Figure 3-5. Directivity Pattern, $2\pi a/\lambda = 6.5$, Sea Water (Average Salinity, 20°C)
**PEAK DIRECTIVITY**

Case 2, Sea Water (average salinity 20 deg C) at 15 MHz

![Graph](image)

- **Case 1**: Perfect ground ($\varepsilon_r = 1.0$, $\sigma = \infty$)
- **Case 2**: Sea Water (average salinity 20 deg C) ($\varepsilon_r = 70.0$, $\sigma = 5.0$ S/m)
- **Case 11**: Free Space ($\varepsilon_r = 1.0$, $\sigma = 0$)

Figure 3-6. Peak Directivity, Sea Water (Average Salinity, 20°C)
ANGLE OF PEAK DIRECTIVITY

Case 2, Sea Water (average salinity 20 deg C) at 15 MHz

Figure 3-7. Angle of Incidence of Peak Directivity, Sea Water (Average Salinity, 20°C)
RADIATION EFFICIENCY
Case 2, Sea Water (average salinity 20 deg C) at 15 MHz

Figure 3-8. Radiation Efficiency, Sea Water (Average Salinity, 20°C)
RADIATION RESISTANCE
Case 2, Sea Water (average salinity 20 deg C) at 15 MHz

Figure 3.9. Radiation Resistance, Sea Water (Average Salinity, 20°C)
INPUT RESISTANCE
Case 2, Sea Water (average salinity 20 deg C) at 15 MHz

$h/\lambda = 0.25, \ b/\lambda = 1.0 \times 10^{-6}, \ z_0/\lambda = 0$
Case 1, Perfect ground ( $\varepsilon_r = 1.0, \ \sigma = \infty$ )
Case 2, Sea Water
(average salinity 20 deg C) ( $\varepsilon_r = 70.0, \ \sigma = 5.0 \text{ S/m}$ )
Case 11, Free Space ( $\varepsilon_r = 1.0, \ \sigma = 0$ )

Figure 3-10. Input Resistance, Sea Water (Average Salinity, 20°C)
INPUT Reactance
Case 2, Sea Water (average salinity 20 deg C) at 15 MHz

Figure 3-11. Input Reactance, Sea Water (Average Salinity, 20°C)
DIRECTIVE GAIN AT 82 DEG ELEVATION
Case 2, Sea Water (average salinity 20 deg C) at 15 MHz

Figure 3-12. Directivity at 8 Degrees Above the Horizon, Sea Water (Average Salinity, 20°C)
DIRECTIVE GAIN AT 84 DEG ELEVATION

Case 2, Sea Water (average salinity 20 deg C) at 15 MHz

$h/\lambda = 0.25, \quad b/\lambda = 1.0 \times 10^{-6}, \quad z_0/\lambda = 0$

Case 1, Perfect ground ( $\varepsilon_r = 1.0, \quad \sigma = \infty$ )

Case 2, Sea Water (average salinity 20 deg C) ( $\varepsilon_r = 70.0, \quad \sigma = 5.0 \text{ S/m}$ )

Case 11, Free Space ( $\varepsilon_r = 1.0, \quad \sigma = 0$ )

Figure 3-13. Directivity at 6 Degrees Above the Horizon, Sea Water (Average Salinity, 20°C)
DIRECTIVE GAIN AT 86 DEG ELEVATION
Case 2, Sea Water (average salinity 20 deg C) at 15 MHz

Case 1, Perfect ground (  ε_r = 1.0, σ = ∞  )
Case 2, Sea Water (average salinity 20 deg C) (  ε_r = 70.0, σ = 5.0 S/m )
Case 11, Free Space (  ε_r = 1.0, σ = 0  )

Figure 3-14. Directivity at 4 Degrees Above the Horizon, Sea Water (Average Salinity, 20°C)
DIRECTIVE GAIN AT 88 DEG ELEVATION

Case 2, Sea Water (average salinity 20 deg C) at 15 MHz

$h/\lambda = 0.25$, $b/\lambda = 1.0 \times 10^{-3}$, $z_0/\lambda = 0$

Case 1, Perfect ground ($\varepsilon_r = 1.0$, $\sigma = \infty$)

Case 2, Sea Water
(average salinity 20 deg C) ($\varepsilon_r = 70.0$, $\sigma = 5.0$ S/m)

Case 11, Free Space ($\varepsilon_r = 1.0$, $\sigma = 0$)

Figure 3-15. Directivity at 2 Degrees Above the Horizon, Sea Water (Average Salinity, 20°C)
DIRECTIVE GAIN ON THE HORIZON
Case 2, Sea Water (average salinity 20 deg C) at 15 MHz

\[ \frac{h}{\lambda} = 0.25, \quad \frac{b}{\lambda} = 1.0 \times 10^{-6}, \quad \frac{z_0}{\lambda} = 0 \]

Case 1, Perfect ground ( \( \varepsilon_r = 1.0, \sigma = \infty \) )

Case 2, Sea Water (average salinity 20 deg C) ( \( \varepsilon_r = 70.0, \sigma = 5.0 \text{ S/m} \) )

Case 11, Free Space ( \( \varepsilon_r = 1.0, \sigma = 0 \) )

Figure 3-16. Directivity on the Horizon, Sea Water (Average Salinity, 20°C)
3.2 FRESH WATER
NUMERIC DIRECTIVE GAIN POLAR PLOT

Case 3, Fresh Water at 15 MHz

$2\pi a/\lambda = 0.025$ (Wavenumbers)

$h/\lambda = 0.25$, $b/\lambda = 1.0 \times 10^{-6}$, $z_o/\lambda = 0$

Case 1, Perfect Ground ($\varepsilon_r = 1.0$, $\sigma = \infty$)

Case 3, Fresh Water
($\varepsilon_r = 80.0$, $\sigma = 0.03$ S/m)

Case 11, Free Space ($\varepsilon_r = 1.0$, $\sigma = 0$)

Figure 3-17. Directivity Pattern, $2\pi a/\lambda = 0.025$, Fresh Water
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 3, Fresh Water at 15 MHz
\[ 2\pi a/\lambda = 3.0 \ (\text{Wavenumbers}) \]

\[ h/\lambda = 0.25, \ b/\lambda = 1.0 \times 10^{-6}, \ z_0/\lambda = 0 \]
Case 1, Perfect Ground (\( \varepsilon_r = 1.0, \sigma = \infty \))
Case 3, Fresh Water
(\( \varepsilon_r = 80.0, \sigma = 0.03 \ \text{S/m} \))
Case 11, Free Space (\( \varepsilon_r = 1.0, \sigma = 0 \))

Figure 3-18. Directivity Pattern, \( 2\pi a/\lambda = 3.0 \), Fresh Water
NUMERIC DIRECTIVE GAIN POLAR PLOT

$2\pi a/\lambda = 4.0$ (Wavenumbers)

Case 3, Fresh Water at 15 MHz

- $h/\lambda = 0.25$
- $b/\lambda = 1.0 \times 10^{-6}$
- $z/\lambda = 0$

- Case 1, Perfect Ground ($\varepsilon_r = 1.0$, $\sigma = \infty$)
- Case 3, Fresh Water ($\varepsilon_r = 80.0$, $\sigma = 0.03$ S/m)
- Case 11, Free Space ($\varepsilon_r = 1.0$, $\sigma = 0$)

Figure 3.19. Directivity Pattern, $2\pi a/\lambda = 4.0$, Fresh Water
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 3, Fresh Water at 15 MHz
$2\pi a/\lambda = 5.0$ (Wavenumbers)

Figure 3-20. Directivity Pattern, $2\pi a/\lambda = 5.0$, Fresh Water
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 3, Fresh Water at 15 MHz

$2\pi a/\lambda = 6.5$ (Wavenumbers)

$h/\lambda=0.25$, $b/\lambda=1.0 \times 10^{-6}$, $z_o/\lambda=0$

Case 1, Perfect Ground ($\varepsilon_r=1.0$, $\sigma=\infty$)

Case 3, Fresh Water
($\varepsilon_r=80.0$, $\sigma=0.03$ S/m)

Case 11, Free Space ($\varepsilon_r=1.0$, $\sigma=0$)

Figure 3-21. Directivity Pattern, $2\pi a/\lambda = 6.5$, Fresh Water
Case 3, Fresh water at 15 MHz

Figure 3-22. Peak Directivity, Fresh Water

- Case 1, Perfect ground (ɛ_r = 10, σ = 0)
- Case 3, Fresh water (ɛ_r = 80.4, σ = 0.03 S/m)
- Case 11, Free Space (ɛ_r = 10, σ = 0)

Normalized Disk Ground Plane Radius, 2πa/λ (Wavenumbers)

Peak Directivity, (dBd)
ANGLE OF PEAK DIRECTIVITY

Case 3, Fresh water at 15 MHz

\[ \frac{h}{\lambda} = 0.25, \quad \frac{d}{\lambda} = 10 \times 10^6, \quad \frac{r}{\lambda} = 0 \]

Case 1, Perfect ground (\( \varepsilon_r = 10, \sigma = \infty \))

Case 3, Fresh Water (\( \varepsilon_r = 80.0, \sigma = 0.003 \text{ S/m} \))

Case 1, Free Space (\( \varepsilon_r = 10, \sigma = 0 \))

Figure 3.23. Angle of Incidence of Peak Directivity, Fresh Water

Angle of peak directivity, \( \theta_{\text{peak}} \) (Deg)
RADIATION EFFICIENCY
Case 3, Fresh water at 15 MHz

Figure 3-24. Radiation Efficiency, Fresh Water

Normalized Disk Ground Plane Radius, $2\pi a/\lambda$ (Wavenumbers)

Radiation Efficiency, $\eta$ (Numeric)

Case 1, Perfect ground ( $\varepsilon_r = 1.0$, $\sigma = \infty$ )
Case 3, Fresh water ( $\varepsilon_r = 80.0$, $\sigma = 0.03$ S/m )
Case 11, Free Space ( $\varepsilon_r = 1.0$, $\sigma = 0$ )

$\lambda/\lambda = 0.25$, $b/\lambda = 1.0 \times 10^{-6}$, $z_0/\lambda = 0$
RADIATION RESISTANCE
Case 3, Fresh water at 15 MHz

Figure 3-25. Radiation Resistance, Fresh Water
INPUT RESISTANCE
Case 3, Fresh water at 15 MHz

$h/\lambda = 0.25, \quad b/\lambda = 1.0 \times 10^{-6}, \quad z_0/\lambda = 0$

Case 1, Perfect ground ( $\varepsilon_r = 1.0, \quad \sigma = \infty$ )
Case 3, Fresh water ( $\varepsilon_r = 80.0, \quad \sigma = 0.03 \text{ S/m}$ )
Case 11, Free Space ( $\varepsilon_r = 1.0, \quad \sigma = 0$ )

Figure 3-26. Input Resistance, Fresh Water
INPUT REACTANCE
Case 3, Fresh water at 15 MHz

Figure 3-27. Input Reactance, Fresh Water

- $h/\lambda = 0.25$, $b/\lambda = 1.0 \times 10^{-6}$, $z_0/\lambda = 0$
- Case 1, Perfect ground ( $\varepsilon_r = 10$, $\sigma = \infty$ )
- Case 3, Fresh water ( $\varepsilon_r = 80.0$, $\sigma = 0.03 \text{ S/m}$ )
- Case II, Free Space ( $\varepsilon_r = 1.0$, $\sigma = 0$ )
DIRECTIVE GAIN AT 88 DEG ELEVATION
Case 3, Fresh water at 15 MHz

Figure 3-28. Directivity at 8 Degrees Above the Horizon, Fresh Water
DIRECTIVE GAIN AT 86 DEG ELEVATION
Case 3, Fresh water at 15 MHz

$h/\lambda = 0.25$, $b/\lambda = 1.0 \times 10^{-6}$, $z_0/\lambda = 0$

Case 1, Perfect ground ( $\varepsilon_r = 10.0$, $\sigma = \infty$ )
Case 3, Fresh water ( $\varepsilon_r = 80.0$, $\sigma = 0.03$ S/m )
Case 11, Free Space ( $\varepsilon_r = 1.0$, $\sigma = 0$ )

Figure 3-29. Directivity at 6 Degrees Above the Horizon, Fresh Water
DIRECTIVE GAIN AT 84 DEG ELEVATION
Case 3, Fresh water at 15 MHz

Case 1, Perfect ground ( \( \varepsilon_r = 1.0, \sigma = \infty \) )
Case 3, Fresh water ( \( \varepsilon_r = 80.0, \sigma = 0.03 \text{ S/m} \) )
Case 11, Free Space ( \( \varepsilon_r = 1.0, \sigma = 0 \) )

Figure 3-30. Directivity at 4 Degrees Above the Horizon, Fresh Water
DIRECTIVE GAIN ON THE HORIZON
Case 3, Fresh water at 15 MHz

Figure 3-32. Directivity on the Horizon, Fresh Water
3.3 WET GROUND
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 4, Wet Ground at 15 MHz

$2\pi a/\lambda = 0.025$ (Wavenumbers)

$h/\lambda = 0.25$, $b/\lambda = 1.0 \times 10^{-6}$, $z_0/\lambda = 0$
Case 1, Perfect Ground ($\varepsilon_r=1.0$, $\sigma=\infty$)
Case 4, Wet Ground ($\varepsilon_r=30.0$, $\sigma=0.01$ S/m)
Case 11, Free Space ($\varepsilon_r=1.0$, $\sigma=0$)

Figure 3-33. Directivity Pattern, $2\pi a/\lambda = 0.025$, Wet Ground
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 4, Wet Ground at 15 MHz
$2\pi a/\lambda = 3.0$ (Wavenumbers)

$h/\lambda = 0.25$, $b/\lambda = 1.0 \times 10^{-6}$, $z_d/\lambda = 0$
Case 1, Perfect Ground ($\varepsilon_r = 1.0$, $\sigma = \infty$)
Case 4, Wet Ground
($\varepsilon_r = 30.0$, $\sigma = 0.01 \text{ S/m}$)
Case 11, Free Space ($\varepsilon_r = 1.0$, $\sigma = 0$)

Figure 3-34. Directivity Pattern, $2\pi a/\lambda = 3.0$, Wet Ground
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 4, Wet Ground at 15 MHz
2πa/λ = 4.0 (Wavenumbers)

h/λ=0.25, b/λ=1.0 x 10^{-6}, z_0/λ=0
Case 1, Perfect Ground (ε_r=1.0, σ=∞)
Case 4, Wet Ground
(ε_r=30.0, σ=0.01 S/m)
Case 11, Free Space (ε_r=1.0, σ=0)

Figure 3-35. Directivity Pattern, 2πa/λ = 4.0, Wet Ground
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 4, Wet Ground at 15 MHz

\[ 2\pi a/\lambda = 5.0 \] (Wavenumbers)

h/\lambda=0.25, b/\lambda=1.0 \times 10^{-6}, z_0/\lambda=0
Case 1, Perfect Ground (\(\varepsilon_r=1.0, \sigma=\infty\))
Case 4, Wet Ground
\(\varepsilon_r=30.0, \sigma=0.01 \text{ S/m}\)
Case 11, Free Space (\(\varepsilon_r=1.0, \sigma=0\))

Figure 3-36. Directivity Pattern, \(2\pi a/\lambda = 5.0\), Wet Ground
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 4, Wet Ground at 15 MHz

\[ 2\pi a/\lambda = 6.5 \] (Wavenumbers)

h/\lambda=0.25, b/\lambda=1.0 \times 10^{-6}, z_0/\lambda=0
Case 1, Perfect Ground (\(\varepsilon_r=1.0, \sigma=\infty\))
Case 4, Wet Ground
(\(\varepsilon_r=30.0, \sigma=0.01 \, \text{S/m}\))
Case 11, Free Space (\(\varepsilon_r=1.0, \sigma=0\))

Figure 3-37. Directivity Pattern, \(2\pi a/\lambda = 6.5\), Wet Ground
PEAK DIRECTIVITY

Case 4, Wet Ground at 15 MHz

$h_1/\lambda = 0.25, \ b/\lambda = 1.0 \times 10^{-6}, \ z_0/\lambda = 0$

Case 1, Perfect ground ( $\varepsilon_r = 1.0, \sigma = \infty$ )
Case 4, Wet Ground ( $\varepsilon_r = 30.0, \sigma = 0.01 \text{ S/m}$ )
Case 11, Free Space ( $\varepsilon_r = 1.0, \sigma = 0$ )

Figure 3-38. Peak Directivity, Wet Ground
Figure 3-39. Angle of Incidence of Peak Directivity, Wet Ground
RADIATION EFFICIENCY

Case 4, Wet Ground at 15 MHz

\[ \eta \text{ (Numeric)} \]

Normalized Disk Ground Plane Radius, \( \frac{2\pi a}{\lambda} \) (Wavenumbers)

- Case 1, Perfect ground (\( \varepsilon_r = 1.0, \sigma = \infty \))
- Case 4, Wet Ground (\( \varepsilon_r = 30.0, \sigma = 0.01 \text{ S/m} \))
- Case 11, Free Space (\( \varepsilon_r = 1.0, \sigma = 0 \))

Figure 3-40. Radiation Efficiency, Wet Ground
RADIATION RESISTANCE
Case 4, Wet Ground at 15 MHz

Figure 3-41. Radiation Resistance, Wet Ground
INPUT RESISTANCE
Case 4, Wet Ground at 15 MHz

\( h/\lambda = 0.25, \ b/\lambda = 1.0 \times 10^{-6}, \ z_0/\lambda = 0 \)

Case 1, Perfect ground (\( \varepsilon_r = 1.0, \sigma = \infty \))
Case 4, Wet Ground (\( \varepsilon_r = 30.0, \sigma = 0.01 \text{ S/m} \))
Case 11, Free Space (\( \varepsilon_r = 1.0, \sigma = 0 \))

Figure 3-42. Input Resistance, Wet Ground
Figure 3-43. Input Reactance, Wet Ground

Case 1, Perfect ground (\( \varepsilon_r = 10, \sigma = 0 \))
Case 2, Perfect ground (\( \varepsilon_r = 20, \sigma = 0.001 \text{ S/m} \))
Case 3, Free Space (\( \varepsilon_r = 10, \sigma = 0 \))
Case 4, Wet Ground (\( \varepsilon_r = 20, \sigma = 0.001 \text{ S/m} \))
Figure 3-44. Directivity at 8 Degrees Above the Horizon, Wet Ground
DIRECTIVE GAIN AT 84 DEG ELEVATION
Case 4, Wet Ground at 15 MHz

Figure 3-45. Directivity at 6 Degrees Above the Horizon, Wet Ground
DIRECTIVE GAIN AT 86 DEG ELEVATION
Case 4, Wet Ground at 15 MHz

$h/\lambda = 0.25$, $b/\lambda = 1.0 \times 10^{-3}$, $z_0/\lambda = 0$
Case 1, Perfect ground ( $\varepsilon_r = 1.0$, $\sigma = \infty$)
Case 4, Wet Ground ( $\varepsilon_r = 30.0$, $\sigma = 0.01 \text{ S/m}$)
Case 11, Free Space ( $\varepsilon_r = 1.0$, $\sigma = 0$)

Figure 3-46. Directivity at 4 Degrees Above the Horizon, Wet Ground
DIRECTIVE GAIN AT 88 DEG ELEVATION

Case 4, Wet Ground at 15 MHz

Figure 3-47. Directivity at 2 Degrees Above the Horizon, Wet Ground
DIRECTIVE GAIN ON THE HORIZON
Case 4, Wet Ground at 15 MHz

$\frac{h}{\lambda} = 0.25, \quad \frac{b}{\lambda} = 1.0 \times 10^{-6}, \quad \frac{z_0}{\lambda} = 0$

Case 1, Perfect ground ( $\varepsilon_r = 1.0, \sigma = \infty$ )

Case 4, Wet Ground ( $\varepsilon_r = 30.0, \sigma = 0.01 \, \text{S/m}$ )

Case 11, Free Space ( $\varepsilon_r = 1.0, \sigma = 0$ )

Figure 3-48. Directivity on the Horizon, Wet Ground
3.4 MEDIUM DRY GROUND
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 5, Medium Dry Ground at 15 MHz
$2\pi a/\lambda = 0.025$ (Wavenumbers)

$z (\theta=0^\circ)$

$h/\lambda=0.25$, $b/\lambda=1.0 \times 10^{-6}$, $z_0/\lambda=0$
Case 1, Perfect Ground ($\varepsilon_r=1.0$, $\sigma=\infty$)
Case 5, Medium Dry Ground
($\varepsilon_r=15.0$, $\sigma=0.001$ S/m)
Case 11, Free Space ($\varepsilon_r=1.0$, $\sigma=0$)

Figure 3-49. Directivity Pattern, $2\pi a/\lambda = 0.025$, Medium Dry Ground
NUMERIC DIRECTIVE GAIN POLAR PLOT

Case 5, Medium Dry Ground at 15 MHz

$2\pi a/\lambda = 3.0$ (Wavenumbers)

$h\lambda = 0.25$, $b\lambda = 0$, $\phi\lambda = 0$

Case 1, Perfect Ground ($\varepsilon_r = 1.0, \sigma = \infty$)

Case 5, Medium Dry Ground ($\varepsilon_r = 15.0, \sigma = 0.001$ S/m)

Case 11, Free Space ($\varepsilon_r = 1.0, \sigma = 0$)

Figure 3-50. Directivity Pattern, $2\pi a/\lambda = 3.0$, Medium Dry Ground
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 5, Medium Dry Ground at 15 MHz
$2\pi a/\lambda = 4.0$ (Wavenumbers)

$\theta=0^\circ$

$\theta=90^\circ$

$\theta=180^\circ$

$\theta=270^\circ$

$h/\lambda=0.25$, $b/\lambda=1.0 \times 10^{-6}$, $z_0/\lambda=0$
Case 1, Perfect Ground ($\varepsilon_r=1.0$, $\sigma=\infty$)
Case 5, Medium Dry Ground
($\varepsilon_r=15.0$, $\sigma=0.001$ S/m)
Case 11, Free Space ($\varepsilon_r=1.0$, $\sigma=0$)

Figure 3-51. Directivity Pattern, $2\pi a/\lambda = 4.0$, Medium Dry Ground
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 5, Medium Dry Ground at 15 MHz
$2\pi a/\lambda = 5.0$ (Wavenumbers)

Figure 3-52. Directivity Pattern, $2\pi a/\lambda = 5.0$, Medium Dry Ground
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 5, Medium Dry Ground at 15 MHz
$2\pi a/\lambda = 6.5$ (Wavenumbers)

Figure 3-53. Directivity Pattern, $2\pi a/\lambda = 6.5$, Medium Dry Ground
PEAK DIRECTIVITY

Case 5, Medium Dry Ground at 15 MHz

Figure 3-54. Peak Directivity, Medium Dry Ground
ANGLE OF PEAK DIRECTIVITY
Case 5, Medium Dry Ground at 15 MHz

Figure 3-55. Angle of Incidence of Peak Directivity, Medium Dry Ground
RADIATION EFFICIENCY

Case 5, Medium Dry Ground at 15 MHz

\[ h/\lambda = 0.25, \ b/\lambda = 1.0 \times 10^{-6}, \ z_0/\lambda = 0 \]

Case 1, Perfect ground \( (\varepsilon_r = 1.0, \ \sigma = \infty) \)

Case 5, Medium Dry Ground \( (\varepsilon_r = 15.0, \ \sigma = 0.001 \text{ S/m}) \)

Case 11, Free Space \( (\varepsilon_r = 1.0, \ \sigma = 0) \)

Figure 3-56. Radiation Efficiency, Medium Dry Ground
Figure 3-57. Radiation Resistance, Medium Dry Ground

Case 5, Medium Dry Ground at 15 MHz

Normalized Disk Ground Plane Radius, $2\pi a / \lambda$ (Wavenumbers)

Radiation Resistance, $R_{\text{rad}}$ (Ohms)
INPUT RESISTANCE
Case 5, Medium Dry Ground at 15 MHz

Figure 3-58. Input Resistance, Medium Dry Ground

$h/\lambda = 0.25$, $b/\lambda = 1.0 \times 10^{-6}$, $z_0/\lambda = 0$

Case 1, Perfect ground ( $\varepsilon_r = 1.0$, $\sigma = \infty$ )

Case 5, Medium Dry Ground ( $\varepsilon_r = 15.0$, $\sigma = 0.001$ S/m )

Case 11, Free Space ( $\varepsilon_r = 1.0$, $\sigma = 0$ )
**INPUT REACTANCE**

*Case 5, Medium Dry Ground at 15 MHz*

![Graph](image1)

- $h/\lambda = 0.25, \ b/\lambda = 1.0 \times 10^{-6}, \ z_0/\lambda = 0$
- Case 1, Perfect ground ( $\varepsilon_r = 1.0, \ \sigma = \infty$ )
- Case 5, Medium Dry Ground ( $\varepsilon_r = 15.0, \ \sigma = 0.001 \text{ S/m}$ )
- Case 11, Free Space ( $\varepsilon_r = 1.0, \ \sigma = 0$ )

*Figure 3-59. Input Reactance, Medium Dry Ground*
DIRECTIVE GAIN AT 82 DEG ELEVATION

Case 5, Medium Dry Ground at 15 MHz

![Graph showing directivity gain at 82 degrees elevation for different cases.]

- **Case 1**: Perfect ground (\( \varepsilon_r = 1.0, \sigma = \infty \))
- **Case 5**: Medium Dry Ground (\( \varepsilon_r = 15.0, \sigma = 0.001 \text{ S/m} \))
- **Case 11**: Free Space (\( \varepsilon_r = 1.0, \sigma = 0 \))

Normalized Disk Ground Plane Radius, \( 2\pi a/\lambda \) (Wavenumbers)

- **Directivity Gain, \( D(\theta=82^\circ) \) (dBi)**

Figure 3-60. Directivity at 8 Degrees Above the Horizon, Medium Dry Ground
DIRECTIVE GAIN AT 84 DEG ELEVATION

Case 5, Medium Dry Ground at 15 MHz

Case 1, Perfect ground ( $\varepsilon_r = 1.0$, $\sigma = \infty$ )
Case 5, Medium Dry Ground ( $\varepsilon_r = 15.0$, $\sigma = 0.001$ S/m )
Case 11, Free Space ( $\varepsilon_r = 1.0$, $\sigma = 0$ )

Figure 3-61. Directivity at 6 Degrees Above the Horizon, Medium Dry Ground
DIRECTIVE GAIN AT 86 DEG ELEVATION

Case 5, Medium Dry Ground at 15 MHz

$h/\lambda = 0.25$, $b/\lambda = 1.0 \times 10^{-6}$, $z_0/\lambda = 0$

Case 1, Perfect ground (\( \varepsilon_r = 1.0, \sigma = \infty \))

Case 5, Medium Dry Ground (\( \varepsilon_r = 15.0, \sigma = 0.001 \text{ S/m} \))

Case 11, Free Space (\( \varepsilon_r = 1.0, \sigma = 0 \))

Figure 3-62. Directivity at 4 Degrees Above the Horizon, Medium Dry Ground
Figure 3.63. Directivity at 2 Degrees Above the Horizon, Medium Dry Ground
DIRECTIVE GAIN ON THE HORIZON
Case 5, Medium Dry Ground at 15 MHz

$\frac{h}{\lambda} = 0.25, \quad \frac{b}{\lambda} = 1.0 \times 10^{-6}, \quad \frac{z_0}{\lambda} = 0$

Case 1, Perfect ground ( $\varepsilon_r = 1.0, \sigma = \infty$ )
Case 5, Medium Dry Ground ( $\varepsilon_r = 15.0, \sigma = 0.001 \text{ S/m}$ )
Case 11, Free Space ( $\varepsilon_r = 1.0, \sigma = 0$ )

Figure 3-64. Directivity on the Horizon, Medium Dry Ground
3.5 VERY DRY GROUND
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 6, Very Dry Ground at 15 MHz
$2\pi a/\lambda = 0.025$ (Wavenumbers)

h/\lambda=0.25, b/\lambda=1.0 \times 10^{-6}, z_0/\lambda=0
Case 1, Perfect Ground ($\varepsilon_r=1.0, \sigma=\infty$)
Case 6, Very Dry Ground ($\varepsilon_r=3.0, \sigma=0.0001$ S/m)
Case 11, Free Space ($\varepsilon_r=1.0, \sigma=0$)

Figure 3-65. Directivity Pattern, $2\pi a/\lambda = 0.025$, Very Dry Ground
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 6, Very Dry Ground at 15 MHz
2πa/λ = 3.0  (Wavenumbers)

Figure 3-66. Directivity Pattern, 2πa/λ = 3.0, Very Dry Ground
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 6, Very Dry Ground at 15 MHz

\[ 2\pi a/\lambda = 4.0 \] (Wavenumbers)

\( h/\lambda = 0.25, \ b/\lambda = 1.0 \times 10^{-6}, \ z_0/\lambda = 0 \)
Case 1, Perfect Ground (\( \varepsilon_r = 1.0, \sigma = \infty \))
Case 6, Very Dry Ground
(\( \varepsilon_r = 3.0, \sigma = 0.0001 \, \text{S/m} \))
Case 11, Free Space (\( \varepsilon_r = 1.0, \sigma = 0 \))

Figure 3-67. Directivity Pattern, \( 2\pi a/\lambda = 4.0 \), Very Dry Ground
NUMERICAL DIRECTIVE GAIN PATTERN PLT

2πa/λ = 5.0 (Wavenumbers)

Case 1, Perfect Ground (εr = 1.0, σ = 0)
Case 2, Very Dry Ground (εr = 3.0, σ = 0.001 S/m)
Case 6, Very Dry Ground (εr = 3.0, σ = 0.001 S/m)
Case 11, Free Space (εr = 1.0, σ = 0)

Figure 3-8. Directivity Pattern, 2πa/λ = 5.0, Very Dry Ground
PEAK DIRECTIVITY
Case 6, Very Dry Ground at 15 MHz

Figure 3-70. Directivity Pattern, Very Dry Ground
ANGLE OF PEAK DIRECTIVITY

Case 6, Very Dry Ground at 15 MHz

\[ \frac{h}{\lambda} = 0.25, \quad \frac{b}{\lambda} = 1.0 \times 10^{-6}, \quad \frac{z_0}{\lambda} = 0 \]

Case 1, Perfect ground ( \( \varepsilon_r = 1.0, \sigma = \infty \) )
Case 6, Very Dry Ground ( \( \varepsilon_r = 3.0, \sigma = 0.0001 \text{ S/m} \) )
Case 11, Free Space ( \( \varepsilon_r = 1.0, \sigma = 0 \) )

Figure 3-71. Angle of Incidence of Peak Directivity, Very Dry Ground
RADIATION EFFICIENCY
Case 6, Very Dry Ground at 15 MHz

- Case 1, Perfect ground (εᵣ = 1.0, σ = ∞)
- Case 6, Very Dry Ground (εᵣ = 3.0, σ = 0.0001 S/m)
- Case 11, Free Space (εᵣ = 1.0, σ = 0)

Figure 3-72. Radiation Efficiency, Very Dry Ground
RADIATION RESISTANCE
Case 6, Very Dry Ground at 15 MHz

Figure 3-73. Radiation Resistance, Very Dry Ground
INPUT RESISTANCE
Case 6, Very Dry Ground at 15 MHz

$h/h = 0.25$, $b/\lambda = 1.0 \times 10^{-6}$, $c_0/\lambda = 0$

Case 1, Perfect ground ( $\varepsilon_r = 1.0$, $\sigma = \infty$ )
Case 6, Very Dry Ground ( $\varepsilon_r = 3.0$, $\sigma = 0.0001$ S/m )
Case 11, Free Space ( $\varepsilon_r = 1.0$, $\sigma = 0$ )

Figure 3-74. Input Resistance, Very Dry Ground
INPUT REACTANCE
Case 6, Very Dry Ground at 15 MHz

\[ h/\lambda = 0.25, \quad b/\lambda = 1.0 \times 10^{-6}, \quad z_0/\lambda = 0 \]

Case 1, Perfect ground ( \( \varepsilon_r = 10, \quad \sigma = \infty \) )

Case 6, Very Dry Ground ( \( \varepsilon_r = 3.0, \quad \sigma = 0.0001 \text{ S/m} \) )

Case 11, Free Space ( \( \varepsilon_r = 1.0, \quad \sigma = 0 \) )

Figure 3-75. Input Reactance, Very Dry Ground
DIRECTIVE GAIN AT 82 DEG ELEVATION
Case 6, Very Dry Ground at 15 MHz

$h/\lambda = 0.25, \quad b/\lambda = 1.0 \times 10^{-6}, \quad z_0/\lambda = 0$

Case 1, Perfect ground ( $\varepsilon_r = 1.0, \quad \sigma = \infty$ )
Case 6, Very Dry Ground ( $\varepsilon_r = 3.0, \quad \sigma = 0.0001 \text{ S/m}$ )
Case 11, Free Space ( $\varepsilon_r = 1.0, \quad \sigma = 0$ )

Figure 3-76. Directivity at 8 Degrees Above the Horizon, Very Dry Ground
DIRECTIVE GAIN AT 84 DEG ELEVATION
Case 6, Very Dry Ground at 15 MHz

$h/\lambda = 0.25$, $b/\lambda = 1.0 \times 10^{-6}$, $z_0/\lambda = 0$

Case 1, Perfect ground ($\varepsilon_r = 1.0$, $\sigma = \infty$)

Case 6, Very Dry Ground ($\varepsilon_r = 3.0$, $\sigma = 0.0001 \text{ S/m}$)

Case 11, Free Space ($\varepsilon_r = 1.0$, $\sigma = 0$)

Figure 3-77. Directivity at 6 Degrees Above the Horizon, Very Dry Ground
DIRECTIVE GAIN AT 86 DEG ELEVATION
Case 6, Very Dry Ground at 15 MHz

\[ \frac{h\lambda}{\lambda} = 0.25, \quad \frac{b\lambda}{\lambda} = 1.0 \times 10^{-6}, \quad \frac{z_0\lambda}{\lambda} = 0 \]

Case 1, Perfect ground ( \( \varepsilon_r = 1.0, \sigma = \infty \) )
Case 6, Very Dry Ground ( \( \varepsilon_r = 3.0, \sigma = 0.0001 \text{ S/m} \) )
Case 11, Free Space ( \( \varepsilon_r = 1.0, \sigma = 0 \) )

Figure 3-78. Directivity at 4 Degrees Above the Horizon, Very Dry Ground
DIRECTIVE GAIN AT 88 DEG ELEVATION
Case 6, Very Dry Ground at 15 MHz

![Graph showing directive gain at 88 deg elevation](image)

- Case 1: Perfect ground ($\varepsilon_r = 1.0$, $\sigma = \infty$)
- Case 6: Very Dry Ground ($\varepsilon_r = 3.0$, $\sigma = 0.0001$ S/m)
- Case 11: Free Space ($\varepsilon_r = 1.0$, $\sigma = 0$)

Normalized Disk Ground Plane Radius, $2\pi a/\lambda$ (Wavenumbers)

Figure 3-79. Directivity at 2 Degrees Above the Horizon, Very Dry Ground
DIRECTIVE GAIN ON THE HORIZON
Case 6, Very Dry Ground at 15 MHz

Figure 3-80. Directivity on the Horizon, Very Dry Ground
3.6 PURE WATER (20°C)
NUMERIC DIRECTIVE GAIN POLAR PLOT

Case 7, Pure Water (20 deg C) at 15 MHz

$2\pi a/\lambda = 0.025$ (Wavenumbers)

$h/\lambda = 0.25$, $b/\lambda = 1.0 \times 10^{-6}$, $z_0/\lambda = 0$

Case 1, Perfect Ground ($\varepsilon_r = 1.0$, $\sigma = \infty$)

Case 7, Pure Water (20 deg C)

($\varepsilon_r = 80.0$, $\sigma = 0.0005$ S/m)

Case 11, Free Space ($\varepsilon_r = 1.0$, $\sigma = 0$)

Figure 3-81. Directivity Pattern, $2\pi a/\lambda = 0.025$, Pure Water (20°C)
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 7, Pure Water (20 deg C) at 15 MHz

$2\pi a/\lambda = 3.0$ (Wavenumbers)

Figure 3-82. Directivity Pattern, $2\pi a/\lambda = 3.0$, Pure Water (20°C)
NUMERICAL DIRECTIVE GAIN POLAR PLOT
Case 7, Pure Water (20 deg C) at 15 MHz
$2\pi a/\lambda = 4.0$ (Wavenumbers)

$h/\lambda=0.25$, $b/\lambda=1.0 \times 10^{-6}$, $z_0/\lambda=0$
Case 1, Perfect Ground ($\varepsilon_r=1.0$, $\sigma=\infty$)
Case 7, Pure Water (20 deg C)
($\varepsilon_r=80.0$, $\sigma=0.0005$ S/m)
Case 11, Free Space ($\varepsilon_r=1.0$, $\sigma=0$)

Figure 3-83. Directivity Pattern, $2\pi a/\lambda = 4.0$, Pure Water (20°C)
NUMERIC DIRECTIVE GAIN POLAR PLOT

Case 7, Pure Water (20 deg C) at 15 MHz

$2\pi a/\lambda = 5.0$ (Wavenumbers)

$h/\lambda=0.25$, $b/\lambda=1.0 \times 10^{-6}$, $z_f/\lambda=0$

Case 1, Perfect Ground ($\varepsilon_r=1.0$, $\sigma=\infty$)

Case 7, Pure Water (20 deg C)

($\varepsilon_r=80.0$, $\sigma=0.0005$ S/m)

Case 11, Free Space ($\varepsilon_r=1.0$, $\sigma=0$)

Figure 3-84. Directivity Pattern, $2\pi a/\lambda = 5.0$, Pure Water (20°C)
NUMERIC DIRECTIVE GAIN POLAR PLOT

Case 7, Pure Water (20 deg C) at 15 MHz

$2\pi a/\lambda = 6.5$ (Wavenumbers)

$\frac{h}{\lambda} = 0.25$, $\frac{b}{\lambda} = 1.0 \times 10^{-6}$, $z_o/\lambda = 0$

Case 1, Perfect Ground ($\varepsilon_r = 1.0$, $\sigma = \infty$)

Case 7, Pure Water (20 deg C)

($\varepsilon_r = 80.0$, $\sigma = 0.0005 \text{ S/m}$)

Case 11, Free Space ($\varepsilon_r = 1.0$, $\sigma = 0$)

Figure 3-85. Directivity Pattern, $2\pi a/\lambda = 6.5$, Pure Water (20°C)
PEAK DIRECTIVITY
Case 7, Pure Water (20 deg C) at 15 MHz

Figure 3-86. Peak Directivity, Pure Water (20°C)
ANGLE OF PEAK DIRECTIVITY
Case 7, Pure Water (20 deg C) at 15 MHz

Figure 3-87. Angle of Incidence of Peak Directivity, Pure Water (20°C)
RADIATION EFFICIENCY
Case 7, Pure Water (20 deg C) at 15 MHz

Figure 3-88. Radiation Efficiency, Pure Water (20°C)
RADIATION RESISTANCE

Case 7, Pure Water (20 deg C) at 15 MHz

$\frac{h_d}{\lambda}$ = 0.25, $\frac{b}{\lambda}$ = $10 \times 10^{-6}$, $\sigma_d/\lambda$ = 0

Case 1, Perfect ground ($\varepsilon_r = 10$, $\sigma = \infty$)

Case 7, Pure Water (20 deg C) ($\varepsilon_r = f_{\omega_d}$, $\sigma = 0.0005$ S/m)

Case 11, Free Space ($\varepsilon_r = 10$, $\sigma = 0$)

Figure 3-80. Radiation Resistance, Pure Water (20°C)

Normalized Disk Ground Plane Radius, $2\pi a/\lambda$ (Wavenumbers)

Radiation Resistance, $R_m$ (Ohms)

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INPUT RESISTANCE
Case 7, Pure Water (20 deg C) at 15 MHz

\[ h/\lambda = 0.25, \quad b/\lambda = 1.0 \times 10^{-6}, \quad z_0/\lambda = 0 \]

Case 1, Perfect ground \( (\varepsilon_r = 1.0, \quad \sigma = \infty) \)

Case 7, Pure Water
\( (20 \text{ deg C}) \quad (\varepsilon_r = 80.0, \quad \sigma = 0.0005 \text{ S/m}) \)

Case 11, Free Space \( (\varepsilon_r = 1.0, \quad \sigma = 0) \)

**Figure 3-90. Input Resistance, Pure Water (20°C)**
INPUT REACTANCE
Case 7, Pure Water (20 deg C) at 15 MHz

Figure 3-91. Input Reactance, Pure Water (20°C)
DIRECTIVE GAIN AT 82 DEG ELEVATION

Case 7, Pure Water (20 deg C) at 15 MHz

Case 1, Perfect ground \( (\varepsilon_r = 1.0, \sigma = \infty) \)
Case 7, Pure Water (20 deg C) \( (\varepsilon_r = 80.0, \sigma = 0.0005 \text{ S/m}) \)
Case 11, Free Space \( (\varepsilon_r = 1.0, \sigma = 0) \)

Figure 3-92. Directivity at 8 Degrees Above the Horizon, Pure Water (20°C)
DIRECTIVE GAIN AT 84 DEG ELEVATION

Case 7, Pure Water (20 deg C) at 15 MHz

\[
\begin{align*}
    h/\lambda &= 0.25, \quad b/\lambda = 1.0 \times 10^{-3}, \quad z_0/\lambda = 0 \\
    \text{Case 1, Perfect ground ( } \varepsilon_r = 1.0, \sigma = \infty \text{ )}
\end{align*}
\]

\[
\begin{align*}
    \text{Case 7, Pure Water} \\
    (20 \text{ deg C}) \quad ( \varepsilon_r = 80.0, \sigma = 0.0005 \text{ S/m }) \\
    \text{Case 11, Free Space ( } \varepsilon_r = 1.0, \sigma = 0 \text{ )}
\end{align*}
\]

**Figure 3-93.** Directivity at 6 Degrees Above the Horizon, Pure Water (20°C)
DIRECTIVE GAIN AT 86 DEG ELEVATION

Case 7, Pure Water (20 deg C) at 15 MHz

\[ \frac{h}{\lambda} = 0.25, \quad \frac{b}{\lambda} = 1.0 \times 10^{-3}, \quad \frac{z_0}{\lambda} = 0 \]

Case 1, Perfect ground ( \( \varepsilon_r = 1.0, \quad \sigma = \infty \) )

Case 7, Pure Water (20 deg C) ( \( \varepsilon_r = 80.0, \quad \sigma = 0.0005 \text{ S/m} \) )

Case 11, Free Space ( \( \varepsilon_r = 1.0, \quad \sigma = 0 \) )

Figure 3-94. Directivity at 4 Degrees Above the Horizon, Pure Water (20°C)
DIRECTIVE GAIN AT 88 DEG ELEVATION

Case 7, Pure Water (20 deg C) at 15 MHz

$D(\theta=88^\circ)$ (dB)

Normalized Disk Ground Plane Radius, $2\pi a/\lambda$ (Wavenumbers)

$h/\lambda = 0.25$, $b/\lambda = 1.0 \times 10^{-6}$, $z_0/\lambda = 0$

Case 1, Perfect ground ($\varepsilon_r = 1.0$, $\sigma = \infty$)

Case 7, Pure Water (20 deg C) ($\varepsilon_r = 80.0$, $\sigma = 0.0005$ S/m)

Case 11, Free Space ($\varepsilon_r = 1.0$, $\sigma = 0$)

Figure 3-95. Directivity at 2 Degrees Above the Horizon, Pure Water (20°C)
DIRECTIVE GAIN ON THE HORIZON
Case 7, Pure Water (20 deg C) at 15 MHz

$h/\lambda = 0.25, \ b/\lambda = 1.0 \times 10^{-6}, \ z_0/\lambda = 0$

Case 1, Perfect ground \( (\varepsilon_r = 10, \sigma = \infty) \)

Case 7, Pure Water (20 deg C) \( (\varepsilon_r = 80.0, \sigma = 0.0005 \text{ S/m}) \)

Case i, Free Space \( (\varepsilon_r = 1.0, \sigma = 0) \)

Figure 3-96. Directivity on the Horizon, Pure Water (20°C)
3.7 ICE (FRESH WATER, -1°C)
NUMERIC DIRECTIVE GAIN POLAR PLOT

Case 8, Ice (fresh water, -1 deg C) at 15 MHz

\(2\pi a/\lambda = 0.025\) (Wavenumbers)

h/\lambda=0.25, b/\lambda=1.0 \times 10^{-6}, z/\lambda=0
Case 1, Perfect Ground (\(\varepsilon_r=1.0, \sigma=\infty\))
Case 8, Ice (fresh water, -1 deg C)  
(\(\varepsilon_r=3.0, \sigma=0.00009\) S/m)
Case 11, Free Space (\(\varepsilon_r=1.0, \sigma=0\))

Figure 3-97. Directivity Pattern, \(2\pi a/\lambda = 0.025\) Ice (Fresh Water, -1°C)
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 8, Ice (fresh water, -1 deg C) at 15 MHz
$2\pi a/\lambda = 3.0$ (Wavenumbers)

Figure 3-98. Directivity Pattern, $2\pi a/\lambda = 3.0$, Ice (Fresh Water, -1°C)
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 8, Ice (fresh water, -1 deg C) at 15 MHz
$2\pi a/\lambda = 4.0$ (Wavenumbers)

h/\lambda=0.25, b/\lambda=1.0 \times 10^{-6}, z_0/\lambda=0
Case 1, Perfect Ground ($\varepsilon_r=1.0, \sigma=\infty$)
Case 8, Ice (fresh water, -1 deg C)
($\varepsilon_r=3.0, \sigma=0.00009 \text{ S/m}$)
Case 11, Free Space ($\varepsilon_r=1.0, \sigma=0$)

Figure 3-99. Directivity Pattern, $2\pi a/\lambda = 4.0$, Ice (Fresh Water, -1°C)
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 8, Ice (fresh water, -1 deg C) at 15 MHz
2\pi a/\lambda = 5.0 \ (\text{Wavenumbers})

\[ h/\lambda = 0.25, \ b/\lambda = 1.0 \times 10^{-6}, \ z_0/\lambda = 0 \]
Case 1, Perfect Ground (\(\varepsilon_r = 1.0, \sigma = \infty\))
Case 8, Ice (fresh water, -1 deg C)
(\(\varepsilon_r = 3.0, \sigma = 0.00009 \text{ S/m}\))
Case 11, Free Space (\(\varepsilon_r = 1.0, \sigma = 0\))

Figure 3-100. Directivity Pattern, 2\pi a/\lambda = 5.0, Ice (Fresh Water, -1°C)
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 8, Ice (fresh water, -1 deg C) at 15 MHz
$2\pi a/\lambda = 6.5$ (Wavenumbers)

Figure 3-101. Directivity Pattern, $2\pi a/\lambda = 6.5$, Ice (Fresh Water, -1°C)
PEAK DIRECTIVITY
Case 8, Ice (fresh water, \(-1\) deg C) at 15 MHz

![Graph showing peak directivity vs. normalized disk ground plane radius.]

- Case 1, Perfect ground \((\varepsilon_r = 10, \sigma = \infty)\)
- Case 8, Ice (fresh water, \(-1\) deg C) \((\varepsilon_r = 3.0, \sigma = 0.0009 \text{ S/m})\)
- Case 11, Free Space \((\varepsilon_r = 1.0, \sigma = 0)\)

Figure 3-102. Peak Directivity, Ice (Fresh Water, \(-1^\circ\text{C}\))
ANGLE OF PEAK DIRECTIVITY

Case 8, Ice (fresh water, -1 deg C) at 15 MHz

Case 1, Perfect ground ( $\varepsilon_r = 1.0$, $\sigma = \infty$ )

Case 8, Ice
(fresh water, -1 deg C) ( $\varepsilon_r = 3.0$, $\sigma = 0.00009$ S/m )

Case 11, Free Space ( $\varepsilon_r = 1.0$, $\sigma = 0$ )

Figure 3-103. Angle of Incidence of Peak Directivity, Ice (Fresh Water, -1°C)
RADIATION EFFICIENCY
Case 8, Ice (fresh water, -1 deg C) at 15 MHz

Figure 3-104. Radiation Efficiency, Ice (Fresh Water, -1°C)
RADIATION RESISTANCE
Case 8, Ice (fresh water, -1 deg C) at 15 MHz

Figure 3-105. Radiation Resistance, Ice (Fresh Water, -1°C)
INPUT RESISTANCE
Case 8, Ice (fresh water, -1 deg C) at 15 MHz

$h/\lambda = 0.25$, $b/\lambda = 1.0 \times 10^{-6}$, $z_{o}/\lambda = 0$

Case 1, Perfect ground ( $\varepsilon_{r} = 10$, $\sigma = \infty$ )

Case 8, Ice
(fresh water, -1 deg C) ( $\varepsilon_{r} = 3.0$, $\sigma = 0.00009$ S/m )

Case 11, Free Space ( $\varepsilon_{r} = 1.0$, $\sigma = 0$ )

Figure 3-106. Input Resistance, Ice (Fresh Water, -1°C)
INPUT REACTANCE
Case 8, Ice (fresh water, -1 deg C) at 15 MHz

\[ h/\lambda = 0.25, \quad b/\lambda = 1.0 \times 10^{-6}, \quad z_0/\lambda = 0 \]

Case 1, Perfect ground \( (\varepsilon_r = 1.0, \quad \sigma = \infty) \)

Case 8, Ice
(fresh water, -1 deg C) \( (\varepsilon_r = 3.0, \quad \sigma = 0.00009 \text{ S/m}) \)

Case 11, Free Space \( (\varepsilon_r = 1.0, \quad \sigma = 0) \)

Figure 3-107. Input Reactance, Ice (Fresh Water, -1°C)
DIRECTIVE GAIN AT 82 DEG ELEVATION

Case 8, Ice (fresh water, -1 deg C) at 15 MHz

Figure 3-108. Directivity at 8 Degrees Above the Horizon, Ice (Fresh Water, -1°C)
DIRECTIVE GAIN AT 84 DEG ELEVATION

Case 8, Ice (fresh water, -1 deg C) at 15 MHz

Case 1, Perfect ground ( $\varepsilon_r = 1.0$, $\sigma = \infty$ )
Case 2, Ice (fresh water, -1 deg C) ( $\varepsilon_r = 3.0$, $\sigma = 0.00009$ S/m )
Case 11, Free Space ( $\varepsilon_r = 1.0$, $\sigma = 0$ )

Figure 3-109. Directivity at 6 Degrees Above the Horizon, Ice (Fresh Water, -1°C)
DIRECTIVE GAIN AT 86 DEG ELEVATION

Case 8, Ice (fresh water, -1 deg C) at 15 MHz

Figure 3-110. Directivity at 4 Degrees Above the Horizon, Ice (Fresh Water, -1°C)
DIRECTIVE GAIN AT 88 DEG ELEVATION

Case 8, Ice (fresh water, -1 deg C) at 15 MHz

Figure 3-111. Directivity at 2 Degrees Above the Horizon, Ice (Fresh Water, -1°C)
3.8 ICE (FRESH WATER, -10 °C)
NUMERIC DIRECTIVE GAIN POLAR PLOT

Case 9, Ice (fresh water, -10 deg C) at 15 MHz

$2\pi a/\lambda = 0.025$ (Wavenumbers)

$h/\lambda = 0.25$, $b/\lambda = 1.0 \times 10^{-6}$, $z_0/\lambda = 0$

Case 1, Perfect Ground ($\varepsilon_r = 1.0$, $\sigma = \infty$)

Case 9, Ice (fresh water, -10 deg C)

$\varepsilon_r = 3.0$, $\sigma = 0.000027 \, S/m$

Case 11, Free Space ($\varepsilon_r = 1.0$, $\sigma = 0$)

Figure 3-113. Directivity Pattern, $2\pi a/\lambda = 0.025$, Ice (Fresh Water, -10°C)
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 9, Ice (fresh water, -10 deg C) at 15 MHz
2πa/λ = 3.0 (Wavenumbers)

h/λ=0.25, b/λ=1.0 x 10^{-6}, z_0/λ=0
Case 1, Perfect Ground (ε_r=1.0, σ=∞)
Case 9, Ice (fresh water, -10 deg C)
(ε_r=3.0, σ=0.000027 S/m)
Case 11, Free Space (ε_r=1.0, σ=0)

Figure 3-114. Directivity Pattern, 2πa/λ = 3.0, Ice (Fresh Water, -10°C)
NUMERIC DIRECTIVE GAIN POLAR PLOT

Case 9, Ice (fresh water, -10 deg C) at 15 MHz

$2\pi a/\lambda = 4.0$ (Wavenumbers)

$\theta=0^\circ$

$z(\theta=0^\circ)$

$\theta=90^\circ$

Figure 3-115. Directivity Pattern, $2\pi a/\lambda = 4.0$, Ice (Fresh Water, -10°C)
NUMERIC DIRECTIVE GAIN POLAR PLOT

2πa/λ = 5.0 (Wavenumbers)

Case 9, Ice (fresh water, -10 deg C) at 15 MHz

Figure 3-116. Directivity Pattern, 2πa/λ = 5.0, Ice (Fresh Water, -10 deg C)
NUMERIC DIRECTIVE GAIN POLAR PLOT

Case 9, Ice (fresh water, -10 deg C) at 15 MHz

$2\pi a/\lambda = 6.5$ (Wavenumbers)

$z (\theta=0^\circ)$

$h/\lambda=0.25$, $b/\lambda=1.0 \times 10^{-6}$, $z/\lambda=0$

Case 1, Perfect Ground ($\varepsilon_r=1.0$, $\sigma=\infty$)

Case 9, Ice (fresh water, -10 deg C)

($\varepsilon_r=3.0$, $\sigma=0.000027 S/m$)

Case 11, Free Space ($\varepsilon_r=1.0$, $\sigma=0$)

Figure 3-117. Directivity Pattern, $2\pi a/\lambda = 6.5$, Ice (Fresh Water, -10°C)
PEAK DIRECTIVITY
Case 9, Ice (fresh water, -10 deg C) at 15 MHz

Figure 3-118. Peak Directivity, Ice (Fresh Water, -10°C)
ANGLE OF PEAK DIRECTIVITY

Case 9, Ice (fresh water, -10 deg C) at 15 MHz

Case 1, Perfect ground ( $\varepsilon_r = 10, \sigma = \infty$ )

Case 9, Ice (fresh water, -10 deg C) ( $\varepsilon_r = 3.0, \sigma = 0.000027$ S/m )

Case 11, Free Space ( $\varepsilon_r = 10, \sigma = 0$ )

Figure 3-119. Angle of Incidence of Peak Directivity, Ice (Fresh Water, -10°C)
RADIATION EFFICIENCY
Case 9, Ice (fresh water, -10 deg C) at 15 MHz

Figure 3-120. Radiation Efficiency, Ice (Fresh Water, -10°C)
RADIATION RESISTANCE
Case 9, Ice (fresh water, -10 deg C) at 15 MHz

Figure 3-121. Radiation Resistance, Ice (Fresh Water, -10°C)
INPUT RESISTANCE
Case 9, Ice (fresh water, -10 deg C) at 15 MHz

$h/\lambda = 0.25, \ b/\lambda = 1.0 \times 10^{-6}, \ \varepsilon_0/\lambda = 0$

Case 1, Perfect ground \( (\varepsilon_r = 10, \ \sigma = \infty) \)

Case 9, Ice
(fresh water, -10 deg C) \( (\varepsilon_r = 3.0, \ \sigma = 0.000027 \ \text{S/m}) \)

Case 11, Free Space \( (\varepsilon_r = 10, \ \sigma = 0) \)

---

Figure 3-122. Input Resistance, Ice (Fresh Water, -10°C)
INPUT REACTANCE
Case 9, Ice (fresh water, -10 deg C) at 15 MHz

Figure 3-123. Input Reactance, Ice (Fresh Water, -10°C)
DIRECTIVE GAIN AT 82 DEG ELEVATION

Case 9, Ice (fresh water, -10 deg C) at 15 MHz

$h/\lambda = 0.25, \quad b/\lambda = 1.0 \times 10^{-6}, \quad z_\alpha/\lambda = 0$
Case 1, Perfect ground \quad $\varepsilon_r = 1.0, \quad \sigma = \infty$
Case 9, Ice
(fresh water, -10 deg C) \quad $\varepsilon_r = 3.0, \quad \sigma = 0.000027 \text{ S/m}$
Case 11, Free Space \quad $\varepsilon_r = 1.0, \quad \sigma = 0$

Figure 3-124. Directivity at 8 Degrees Above the Horizon, Ice (Fresh Water, -10°C)
DIRECTIVE GAIN AT 84 DEG ELEVATION
Case 9, Ice (fresh water, -10 deg C) at 15 MHz

\[
\frac{h}{\lambda} = 0.25, \quad \frac{b}{\lambda} = 1.0 \times 10^{-6}, \quad \frac{z_0}{\lambda} = 0
\]
Case 1, Perfect ground ( \( \varepsilon_r = 1.0, \sigma = \infty \) )
Case 9, Ice
(fresh water, -10 deg C) ( \( \varepsilon_r = 3.0, \sigma = 0.000027 \text{ S/m} \) )
Case 11, Free Space ( \( \varepsilon_r = 1.0, \sigma = 0 \) )

Figure 3-125. Directivity at 6 Degrees Above the Horizon, Ice (Fresh Water, -10°C)
DIRECTIVE GAIN AT 86 DEG ELEVATION

Case 9, Ice (fresh water, -10 deg C) at 15 MHz

$h/\lambda = 0.25$, $b/\lambda = 1.0 \times 10^{-6}$, $z_0/\lambda = 0$

Case 1, Perfect ground ($\varepsilon_r = 10$, $\sigma = \infty$)
Case 9, Ice
(fresh water, -10 deg C) ($\varepsilon_r = 3.0$, $\sigma = 0.000027$ S/m)
Case 11, Free Space ($\varepsilon_r = 10$, $\sigma = 0$)

Figure 3-126. Directivity at 4 Degrees Above the Horizon, Ice (Fresh Water, -10°C)
DIRECTIVE GAIN AT 88 DEG ELEVATION

Case 9, Ice (fresh water, -10 deg C) at 15 MHz

$h/\lambda = 0.25$, $b/\lambda = 1.0 \times 10^{-6}$, $z_0/\lambda = 0$

Case 1, Perfect ground \( E_r = 1.0, \sigma = \infty \)

Case 9, Ice
(fresh water, -10 deg C) \( E_r = 3.0, \sigma = 0.000027 \text{ S/m} \)

Case 11, Free Space \( E_r = 1.0, \sigma = 0 \)

Figure 3-127. Directivity at 2 Degrees Above the Horizon, Ice (Fresh Water, -10°C)
DIRECTIVE GAIN ON THE HORIZON

Case 9, Ice (fresh water, -10 deg C) at 15 MHz

\[ \frac{h}{\lambda} = 0.25, \quad \frac{b}{\lambda} = 10 \times 10^6, \quad \frac{z_o}{\lambda} = 0 \]

Case 1, Perfect ground ( \( \varepsilon_r = 1.0, \quad \sigma = \infty \) )

Case 9, Ice
(fresh water, -10 deg C) ( \( \varepsilon_r = 3.0, \quad \sigma = 0.000027 \text{ S/m} \) )

Case 11, Free Space ( \( \varepsilon_r = 1.0, \quad \sigma = 0 \) )

Figure 3-128. Directivity on the Horizon, Ice (Fresh Water, -10°C)
3.9 AVERAGE LAND
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 10, Average Land at 15 MHz

\[ 2\pi a/\lambda = 0.025 \] (Wavenumbers)

\[ h/\lambda=0.25, b/\lambda=1.0 \times 10^{-6}, z_0/\lambda=0 \]
Case 1, Perfect Ground (\(\varepsilon_r=1.0, \sigma=\infty\))
Case 10, Average Land
(\(\varepsilon_r=10.0, \sigma=0.005 \text{ S/m}\))
Case 11, Free Space (\(\varepsilon_r=1.0, \sigma=0\))

Figure 3-129. Directivity Pattern, \(2\pi a/\lambda = 0.025\), Average Land
NUMERIC DIRECTIVE GAIN POLAR PLOT

Case 10, Average Land at 15 MHz

$2\pi a/\lambda = 3.0$ (Wavenumbers)

h/\lambda = 0.25, b/\lambda = 1.0 \times 10^{-6}, z_0/\lambda = 0
Case 1, Perfect Ground ($\varepsilon_r = 1.0, \sigma = \infty$)
Case 10, Average Land
($\varepsilon_r = 10.0, \sigma = 0.005 \text{ S/m}$)
Case 11, Free Space ($\varepsilon_r = 1.0, \sigma = 0$)

Figure 3-130. Directivity Pattern, $2\pi a/\lambda = 3.0$ Average Land
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 10, Average Land at 15 MHz
$2\pi a/\lambda = 4.0$ (Wavenumbers)

$h/\lambda=0.25$, $b/\lambda=1.0 \times 10^{-6}$, $z_0/\lambda=0$
Case 1, Perfect Ground ($\varepsilon_r=1.0$, $\sigma=\infty$)
Case 10, Average Land
($\varepsilon_r=10.0$, $\sigma=0.005 \text{ S/m}$)
Case 11, Free Space ($\varepsilon_r=1.0$, $\sigma=0$)

Figure 3-131. Directivity Pattern, $2\pi a/\lambda = 4.0$, Average Land
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 10, Average Land at 15 MHz
$2\pi a/\lambda = 5.0$ (Wavenumbers)

$h/\lambda=0.25$, $b/\lambda=1.0 \times 10^{-6}$, $z_0/\lambda=0$
Case 1, Perfect Ground ($\varepsilon_r=1.0$, $\sigma=\infty$)
Case 10, Average Land
($\varepsilon_r=10.0$, $\sigma=0.005$ S/m)
Case 11, Free Space ($\varepsilon_r=1.0$, $\sigma=0$)

Figure 3-132. Directivity Pattern, $2\pi a/\lambda = 5.0$, Average Land
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 10, Average Land at 15 MHz
$2\pi a/\lambda = 6.5$ (Wavenumbers)

$h/\lambda = 0.25$, $b/\lambda = 1.0 \times 10^{-6}$, $z/\lambda = 0$
Case 1, Perfect Ground ($\varepsilon_r = 1.0, \sigma = \infty$)
Case 10, Average Land
($\varepsilon_r = 10.0, \sigma = 0.005 \text{ S/m}$)
Case 11, Free Space ($\varepsilon_r = 1.0, \sigma = 0$)

Figure 3-133. Directivity Pattern, $2\pi a/\lambda = 6.5$, Average Land
PEAK DIRECTIVITY

Case 10, Average Land  at 15 MHz

Figure 3-134. Peak Directivity, Average Land
ANGLE OF PEAK DIRECTIVITY

Case 10, Average Land at 15 MHz

\( h/\lambda = 0.25, \ b/\lambda = 1.0 \times 10^{-6}, \ \rho/\lambda = 0 \)

Case 1, Perfect ground (\( \varepsilon_r = 10, \sigma = \infty \))

Case 10, Average land (\( \varepsilon_r = 10.0, \sigma = 0.005 \text{ S/m} \))

Case 11, Free Space (\( \varepsilon_r = 1.0, \sigma = 0 \))

Figure 3-135. Angle of Incidence of Peak Directivity, Average Land
RADIATION EFFICIENCY

Case 10, Average Land at 15 MHz

Figure 3-136. Radiation Efficiency, Average Land

$h/\lambda = 0.25, \quad b/\lambda = 10 \times 10^{-6}, \quad z_0/\lambda = 0$
Case 1, Perfect ground ( $\varepsilon_r = 10, \sigma = \infty$ )
Case 10, Average land ( $\varepsilon_r = 10.0, \sigma = 0.005 \text{ S/m} $ )
Case 11, Free Space ( $\varepsilon_r = 10, \sigma = 0$ )
RADIATION RESISTANCE

Case 10, Average Land at 15 MHz

Figure 3-137. Radiation Resistance, Average Land
INPUT RESISTANCE
Case 10, Average Land at 15 MHz

\[ \frac{h}{\lambda} = 0.25, \quad \frac{b}{\lambda} = 1.0 \times 10^{-6}, \quad \frac{z_0}{\lambda} = 0 \]
Case 1, Perfect ground ( \( \varepsilon_r = 1.0, \quad \sigma = \infty \) )
Case 10, Average land ( \( \varepsilon_r = 10.0, \quad \sigma = 0.005 \text{ S/m} \) )
Case 11, Free Space ( \( \varepsilon_r = 1.0, \quad \sigma = 0 \) )

Figure 3-138. Input Resistance, Average Land
INPUT REACTANCE
Case 10, Average Land at 15 MHz

\[ h/\lambda = 0.25, \quad b/\lambda = 1.0 \times 10^{-6}, \quad z_0/\lambda = 0 \]

Case 1, Perfect ground \( E_r = 1.0, \quad \sigma = \infty \)
Case 10, Average land \( E_r = 10.0, \quad \sigma = 0.005 \text{ S/m} \)
Case 11, Free Space \( E_r = 1.0, \quad \sigma = 0 \)

Figure 3-139. Input Reactance, Average Land
DIRECTIVE GAIN AT 82 DEG ELEVATION

Case 10, Average Land at 15 MHz

Figure 3-140. Directivity at 8 Degrees Above the Horizon, Average Land
DIRECTIVE GAIN AT 84 DEG ELEVATION

Case 10, Average Land at 15 MHz

Figure 3-141. Directivity at 6 Degrees Above the Horizon, Average Land
DIRECTIVE GAIN AT 86 DEG ELEVATION
Case 10, Average Land at 15 MHz

Case 1, Perfect ground \( (\varepsilon_r = 10, \sigma = \infty) \)
Case 10, Average land \( (\varepsilon_r = 10.0, \sigma = 0.005 \text{ S/m}) \)
Case 11, Free Space \( (\varepsilon_r = 1.0, \sigma = 0) \)

Figure 3-142. Directivity at 4 Degrees Above the Horizon, Average Land
DIRECTIVE GAIN AT 88 DEG ELEVATION
Case 10, Average Land at 15 MHz

Figure 3-143. Directivity at 2 Degrees Above the Horizon, Average Land
DIRECTIVE GAIN ON THE HORIZON

Case 10, Average Land at 15 MHz

- Case 1, Perfect ground ( \( \varepsilon_r = 1.0, \sigma = \infty \) )
- Case 10, Average land ( \( \varepsilon_r = 10.0, \sigma = 0.005 \text{ S/m} \) )
- Case 11, Free Space ( \( \varepsilon_r = 1.0, \sigma = 0 \) )

Figure 3-144. Directivity on the Horizon, Average Land
3.10 SEA WATER COMPARED WITH MEDIUM DRY GROUND
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 2, Sea Water (average salinity 20 deg C) at 15 MHz
Case 5, Medium Dry Ground at 15 MHz

$2\pi a/\lambda = 0.025$ (Wavenumbers)

$h/\lambda = 0.25, b/\lambda = 1.0 \times 10^{-6}, z/\lambda = 0$
Case 1, Perfect Ground ($\varepsilon_r = 1.0, \sigma = \infty$)
Case 2, Sea Water
  (average salinity 20 deg C)
    ($\varepsilon_r = 70.0, \sigma = 5.0 \text{ S/m}$)
Case 5, Medium Dry Ground
  ($\varepsilon_r = 15.0, \sigma = 0.001$)
Case 11, Free Space ($\varepsilon_r = 1.0, \sigma = 0$)

Figure 3-145. Directivity Pattern, $2\pi a/\lambda = 0.025$, Sea Water Compared with Medium Dry Ground
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 2, Sea Water (average salinity 20 deg C) at 15 MHz
Case 5, Medium Dry Ground at 15 MHz

\[ 2\pi a/\lambda = 3.0 \] (Wavenumbers)

- Case 1, Perfect Ground \((\varepsilon_r=1.0, \sigma=\infty)\)
- Case 2, Sea Water
  - (average salinity 20 deg C)
  - \((\varepsilon_r=70.0, \sigma=5.0 \text{ S/m})\)
- Case 5, Medium Dry Ground
  - \((\varepsilon_r=15.0, \sigma=0.001)\)
- Case 11, Free Space \((\varepsilon_r=1.0, \sigma=0)\)

Figure 3-146. Directivity Pattern, \(2\pi a/\lambda = 3.0\), Sea Water Compared with Medium Dry Ground
NUMERIC DIRECTIVE GAIN POLAR PLOT

Case 2, Sea Water (average salinity 20 deg C) at 15 MHz
Case 5, Medium Dry Ground at 15 MHz

\[ 2\pi a/\lambda = 4.0 \] (Wavenumbers)

- **Case 1**, Perfect Ground \((\varepsilon_r=1.0, \sigma=\infty)\)
- **Case 2**, Sea Water
  - (average salinity 20 deg C)
  - \((\varepsilon_r=70.0, \sigma=5.0 \text{ S/m})\)
- **Case 5**, Medium Dry Ground
  - \((\varepsilon_r=15.0, \sigma=0.001)\)
- **Case 11**, Free Space \((\varepsilon_r=1.0, \sigma=0)\)

Figure 3-147. Directivity Pattern, \(2\pi a/\lambda = 4.0\), Sea Water Compared with Medium Dry Ground
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 2, Sea Water (average salinity 20 deg C) at 15 MHz
Case 5, Medium Dry Ground at 15 MHz

$2\pi a/\lambda = 5.0$ (Wavenumbers)

$h/\lambda=0.25$, $b/\lambda=1.0 \times 10^{-6}$, $z_0/\lambda=0$
Case 1, Perfect Ground ($\varepsilon_r=1.0$, $\sigma=\infty$)
Case 2, Sea Water
  (average salinity 20 deg C)
  ($\varepsilon_r=70.0$, $\sigma=5.0$ S/m)
Case 5, Medium Dry Ground
  ($\varepsilon_r=15.0$, $\sigma=0.001$)
Case 11, Free Space ($\varepsilon_r=1.0$, $\sigma=0$)

Figure 3-148. Directivity Pattern, $2\pi a/\lambda = 5.0$, Sea Water Compared with Medium Dry Ground
NUMERIC DIRECTIVE GAIN POLAR PLOT
Case 2, Sea Water (average salinity 20 deg C) at 15 MHz
Case 5, Medium Dry Ground at 15 MHz

\[ 2\pi a/\lambda = 6.5 \] (Wavenumbers)

\[ h/\lambda = 0.25, b/\lambda = 1.0 \times 10^{-6}, z/\lambda = 0 \]
Case 1, Perfect Ground \((\varepsilon_r = 1.0, \sigma = \infty)\)
Case 2, Sea Water
\((\text{average salinity 20 deg C})\)
\((\varepsilon_r = 70.0, \sigma = 5.0 \text{ S/m})\)
Case 5, Medium Dry Ground
\((\varepsilon_r = 15.0, \sigma = 0.001)\)
Case 11, Free Space \((\varepsilon_r = 1.0, \sigma = 0)\)

Figure 3-149. Directivity Pattern, \(2\pi a/\lambda = 6.5\), Sea Water Compared with Medium Dry Ground
**PEAK DIRECTIVITY**

Case 2, Sea Water (average salinity 20 deg C) at 15 MHz
Case 5, Medium Dry Ground at 15 MHz

![Graph showing peak directivity for different cases.]

- Case 1: Perfect Ground ($\varepsilon_r = 1.0, \sigma = \infty$)
- Case 2: Sea Water (Average Salinity 20 deg C) ($\varepsilon_r = 70.0, \sigma = 5.0 \text{ S/m}$)
- Case 5: Medium Dry Ground ($\varepsilon_r = 15.0, \sigma = 0.001 \text{ S/m}$)
- Case 11: Free Space ($\varepsilon_r = 1.0, \sigma = 0$)

Figure 3-150. Peak Directivity, Sea Water Compared with Medium Dry Ground
ANGLE OF PEAK DIRECTIVITY

Case 2, Sea Water (average salinity 20 deg C) at 15 MHz
Case 5, Medium Dry Ground at 15 MHz

Figure 3-151. Angle of Incidence of Peak Directivity, Sea Water Compared with Medium Dry Ground
RADIATION RESISTANCE

Case 2, Sea Water (average salinity 20 deg C) at 15 MHz
Case 5, Medium Dry Ground at 15 MHz

Figure 3-152. Radiation Efficiency, Sea Water Compared with Medium Dry Ground
RADIATION EFFICIENCY
Case 2, Sea Water (average salinity 20 deg C) at 15 MHz
Case 5, Medium Dry Ground at 15 MHz

Figure 3-153. Radiation Resistance, Sea Water Compared with Medium Dry Ground
INPUT RESISTANCE

Case 2, Sea Water (average salinity 20 deg C) at 15 MHz
Case 5, Medium Dry Ground at 15 MHz

Case 1, Perfect Ground (\(\varepsilon_r = 1.0, \sigma = \infty\))
Case 2, Sea Water (Average Salinity 20 deg C) (\(\varepsilon_r = 70.0, \sigma = 5.0 \text{ S/m}\))
Case 5, Medium Dry Ground (\(\varepsilon_r = 15.0, \sigma = 0.001 \text{ S/m}\))
Case 11, Free Space (\(\varepsilon_r = 1.0, \sigma = 0\))

Figure 3-154. Input Resistance, Sea Water Compared with Medium Dry Ground
INPUT REACTANCE

Case 2, Sea Water (average salinity 20 deg C) at 15 MHz
Case 5, Medium Dry Ground at 15 MHz

Figure 3-155. Input Reactance, Sea Water Compared with Medium Dry Ground
DIRECTIVE GAIN AT 82 DEG ELEVATION

Case 2, Sea Water (average salinity 20 deg C) at 15 MHz

Case 5, Medium Dry Ground at 15 MHz

Figure 3-156. Directivity at 8 Degrees Above the Horizon, Sea Water Compared with Medium Dry Ground
DIRECTIVE GAIN AT 84 DEG ELEVATION

Case 2, Sea Water (average salinity 20 deg C) at 15 MHz
Case 5, Medium Dry Ground at 15 MHz

Case 1
Case 2
Case 11
Case 5

\[ \phi_\lambda = 0.25, \, \phi_\lambda = 1.0 \times 10^{-6}, \, \eta_\lambda/\lambda = 0 \]

Case 1, Perfect Ground \( (\varepsilon_r = 1.0, \sigma = \infty) \)
Case 2, Sea Water (Average Salinity 20 deg C) \( (\varepsilon_r = 70.0, \sigma = 5.0 \text{ S/m}) \)
Case 5, Medium Dry Ground \( (\varepsilon_r = 15.0, \sigma = 0.001 \text{ S/m}) \)
Case 11, Free Space \( (\varepsilon_r = 1.0, \sigma = 0) \)

Figure 3-157. Directivity at 6 Degrees Above the Horizon, Sea Water Compared with Medium Dry Ground
Case 2, Sea Water (average salinity 20 deg C) at 15 MHz
Case 5, Medium Dry Ground at 15 MHz

\[ \text{Normalized Disk Ground Plane Radius, } \frac{2\pi a}{\lambda} (\text{Wavenumbers}) \]

Figure 3-158. Directivity at 4 Degrees Above the Horizon, Sea Water Compared with Medium Dry Ground

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DIRECTIVE GAIN AT 88 DEG ELEVATION

Case 2, Sea Water (average salinity 20 deg C) at 15 MHz
Case 5, Medium Dry Ground at 15 MHz

Figure 3-159. Directivity at 2 Degrees Above the Horizon, Sea Water Compared with Medium Dry Ground
DIRECTIVE GAIN ON THE HORIZON

Case 2, Sea Water (average salinity 20 deg C) at 15 MHz
Case 5, Medium Dry Ground at 15 MHz

Figure 3-160. Directivity on the Horizon, Sea Water Compared with Medium Dry Ground
LIST OF REFERENCES


LIST OF REFERENCES (CONCLUDED)