A knowledge state is represented by two sets of statements, rather than one. One set of statements represents evidence; it corresponds to recorded data, together with general knowledge that is not open to question in the context at hand. We refer to this as the evidential corpus of knowledge. The other set of statements represents a body of practical certainties, based on the statements constituting the evidential corpus. It consists of statements whose probabilities, relative to the evidential corpus, exceed some explicit level determined by the context. Probabilities are assigned to statements, relative to a body of evidence - called evidential corpus. We require statistical knowledge (not just statistical evidence) as a basis for every probability statement. Two facts render this constraint acceptable: It doesn’t take much statistical data to yield an approximate statistical hypothesis. And if we adopt the principle that statements known to have the same truth value are to be assigned the same probability, we may link many statements to the same statistical foundation.
### General Instructions for Completing SF 298

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Almost without exception, the interesting things we know might not be so. They might not be so in the very strong sense that the statements expressing them might, to our shock and surprise, turn out to be false. The exceptions are statements, like mathematical theorems, that are interesting precisely because they cannot be false.

This would suggest that a fundamental concern of knowledge representation should be the treatment of uncertainty. There are a number of approaches to uncertainty that might be considered: There is the purely Bayesian approach, in which one assigns probabilities [Cheeseman, 1985], [Pearl, 1988]; there are various alternative numerical measures that have been proposed [Shafer, 1976, 1987], [Zadeh, 1975], [Shortliffe, 1976]; Higher order probabilities have been suggested [Domotor, 1980], [Skyrms, 1980]; there is a wide variety of non-monotonic formalisms that might be used to capture the uncertainty of inference, if not the uncertainty of knowledge [McCarthy, 1980, 1987], [Reiter, 1980], [McDermott, 1980], etc.

The relations among these approaches have been discussed in a number of places [Kyburg, 1987, 1988a, 1988b, 1988c]. We do not propose to discuss these relations further here, but simply to adopt an interval-valued epistemic notion of probability (which we shall briefly characterize in the next section) and to show how this approach can be used for inference, decision-making, evidential and inductive reasoning, and commonsense reasoning, as well as nonmonotonic reasoning.
2. Since the membership of statements in these bodies of knowledge depends on probability, we had best begin with a brief characterization of the sense of probability we are employing. We construe probability as objective, and not subjective. But we specifically think of probability as epistemic: that is, it concerns individual cases, and not merely classes of cases.

Probabilities are assigned to statements, relative to a body of evidence -- what I have called the evidential corpus. We require statistical knowledge (not just statistical evidence) as a basis for every probability statement. Two facts render this constraint acceptable: It doesn't take much statistical data to yield an approximate statistical hypothesis. And if we adopt the principle that statements known to have the same truth value are to be assigned the same probability, we may link many statements to the same statistical foundation.

Many people lament the fact that we do not have the statistical knowledge to use probabilities (McCarthy and Hayes, 1969). In fact the opposite is the case. Once you admit the linkage among statements known to have the same truth value, and once you admit approximate probabilities, the difficulty is to choose the appropriate reference class among a possibly large number of potential candidates.

Two principles suffice to perform this selection. They include as a special case the various principles of maximum specificity that have been proposed both in non-monotonic logic and in the philosophy of scientific explanation [Etherington, 1987], [Horty, 1987], [Poole, 1985]. The principles include two other cases that have not been noted in the AI literature.

We assume, as usual, a formal language, and a fixed body of knowledge. A sentence $S$ of our language determines a class of inference structures. An inference structure is a 5-tuple of the form <ind, prop, ref.class, low, high>, where in the body of knowledge we know "$S$ <- ind has prop," we know "ind is in ref.class," and the most
accurate statistical knowledge we have about the frequency of the property in the reference class is that it lies between low and high.

Two inference structures differ, if neither mentioned interval is included in the other.

Principle I: If two inference structures $IS_1$ and $IS_2$ differ from each other, delete both from the original set, unless

(a) One ref.class is known to be included in the other, or

(b) [A dual condition concerning sampling] or

(c) [A condition concerning sequential experiments -- the classical "Bayesian" case]

These last two conditions are slightly complicated to state, but versions have been offered in [Kyburg, 1961, 1974, and 1983]. The output of the application of principle I is a reduced class of inference structures, no two of which differ. We then apply principle II.

Principle II: If the interval mentioned by one inference structure is properly included in the interval mentioned by a second inference structure, delete the second.

The outcome of the application of these two principles is a class of inference structures that agree precisely. The common interval mentioned by these inference structures is the probability of $S$, and also, in virtue of the use we have made of the biconditional, of any statement we know to have the same truth value as $S$. This procedure is deterministic, and in fact has been implemented in limited domains [Loui, 1986].

3. A knowledge state is represented by two sets of statements, rather than one. One set of statements represents evidence; it corresponds to recorded data, together with general knowledge that is not open to question in the context at hand. We refer to this as the evidential corpus of knowledge. We will say more about it shortly.
The other set of statements represents a body of practical certainties, based on the statements constituting the evidential corpus. It consists of statements whose probabilities, relative to the evidential corpus, exceed some explicit level determined by the context. (This is to be contrasted with the idea, to be found in [Pearl, 1988], for example, that probabilistic acceptance requires an arbitrarily high probability.) This set of statements we will call the practical corpus.

A statement is in the practical corpus just in case its probability exceeds a level we take to correspond to "practical certainty" in a given context. (For a suggestion as to how that level might be determined, see [Kyburg, 1988d].) This has the important and useful consequence that the practical corpus is not deductively closed, since in general the probability of a conjunction, even in the epistemic sense, is less than (has a lower bound less than) the probability of either of its conjuncts. We do have limited closure:

If $S$ is in the practical corpus, and $T$ is deductively implied by $S$, then $T$ will also be in it.

A further consequence that is of considerable significance is that the practical corpus, since it is not deductively closed, may be "inconsistent." We draw the fangs of the lottery paradox [Kyburg, 1961], by refusing to countenance deductive or conjunctive closure. This not only allow us to have "ticket $i$ will not win" in our corpus (for large lotteries) but, more important, allows us to have statements of the form, "the error of measurement $i$ is less than $d$" in our corpus, even when there are so many that we can be practically certain that at least one of those measurements is in error by more than $d$.

If statements get in the practical corpus by being probable enough relative to the evidential corpus, how do they get in the evidential corpus? Presumably the evidential corpus is even more demanding than the practical corpus. And is the evidential corpus deductively closed? In a given context, we take the contents of the evidential corpus for granted: to ask the provenance of statements in the evidential corpus is to shift context -- to
regard it as "practical" relative to a new "evidential" corpus. This suggests that we take the structure of the evidential corpus to be the same as that of the practical corpus.

4. Probabilities are defined relative to the practical corpus in the same way that they can be defined relative to the evidential corpus. This yields a natural decision theory. (It is weak, due to the fact that probabilities are intervals.)

       It is clear that as evidence is added to the evidential corpus, statements will come and go in the practical corpus, reflecting the nonmonotonicity of ordinary reasoning. (The practical corpus will be incomplete.) The conventional examples are easy to handle.

       In planning, we do not in general want to have to consider outlandish possibilities -- the potato in the tailpipe. Outlandish possibilities are not represented in the practical corpus: they do not represent possibilities that we should take seriously. But they can be represented as possibilities in the evidential corpus, and an addition to that corpus can change their probabilities, and thus lead to their significant probability relative to the practical corpus.

       In some planning situations, we wish to take advantage of external inputs to modify our plans. In general, this will be helpful only if we can deal quantitatively with the possibility of error in the input. The suggested approach allows this: the evidential corpus can contain general error distributions, from which we can infer in the practical corpus statements about errors in particular cases.

       There are many cases in which we want our system to take as fact, ceteris paribus, a certain statement; and at the same time, be sensitive to the fact that circumstances can arise when ceteris is no longer paribus.

       The cost of being able to do this is that any addition to the evidential corpus may make a crucial difference to what is contained in the practical corpus. But that difference can only make itself felt in a change of probabilities, relative to the evidential corpus, of
statements that are relevant to the decision or goal we are concerned with. We may think of this as vertical modularity.

We must also consider the possibilities of horizontal modularity: there are some domains that are quite independent of other domains, ordinarily, and we should be able to take advantage of those independencies. But we would want to allow the boundaries of these domains to shift as our evidential corpus changes: it is always possible that there is a link, after all, between the number of missionaries in Papua and the rainfall in South Bend, and that we could discover it and incorporate it in our evidential corpus.

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