We explored a number of issues regarding efficient updating in adaptive array signal processing. Our focus was on techniques for efficient and paralyzeable deletion of data as arises in excising obsolete data in recursive least squares, noise suppression via covariance differencing, etc. Our key result was the development of hyperbolic singular value decomposition, a new canonic matrix factorization.
COMPOSITE TASKING FOR SENSOR ARRAYS

FINAL REPORT

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DTIC QUALITY INSPECTED
(i) Hyperbolic SVD

The hyperbolic singular value decomposition (HSVD) is a new canonic matrix decomposition. We developed this matrix factorization [1], [2], [3], in order to retain the benefits of the SVD (numerical stability, detection of "fuzzy rank" etc.) in certain signal processing problems where the conventional SVD cannot be applied. The purpose of this hyperbolic decomposition is to find the eigenstructure of a matrix formed from the difference of two covariance matrices. Such a task arises in certain methods of bearing estimation in colored noise, as well as high resolution spectral estimation of nonstationary data using a sliding rectangular window. In the above papers we developed parallel algorithms for computing the HSVD for the case when the difference of two covariance matrices is full rank. The hyperbolic SVD algorithms are the first dedicated algorithms for this task. They give better numerical results than competing methods which involve explicit outer product formation. One of the algorithms can be implemented on a linear systolic array.

In applications that we initially considered the difference of two covariance matrices was full rank. It is of interest to consider the case where the subtraction of the covariance matrices yields the difference whose rank is lower than the ranks of the individual covariance matrices. We have been able to generalize the HSVD to this case proving the existence of the decomposition [4]. We present an overview, and applications, of our work on hyperbolic factorizations, and discuss the sign indefinite case in [9].

(ii) Triangular decomposition of symmetric indefinite matrices

The difference \( X = A_1A_1^\dagger - A_2A_2^\dagger \) of two covariance matrices \( A_1A_1^\dagger \) and \( A_2A_2^\dagger \) arise in regression problems, in signal processing in the context of bearing estimation and other applications. It is of practical interest to to find a triangular decomposition of \( X \). It is assumed that \( X \) is nonsingular however it can be indefinite. For numerical reasons it is desirable not to form explicitly the products \( A_1A_1^\dagger \) and \( A_2A_2^\dagger \). In considering the difference \( A_1A_1^\dagger - A_2A_2^\dagger \) it is helpful to introduce an indefinite inner product on \( \mathbb{C}^n \) induced by a weighting matrix \( \Phi, \Phi = diag(\pm 1) \). The weighting matrix \( \Phi \), often referred to as the signature matrix, defines also hyper-normal (with respect to \( \Phi \)) matrices. Hyper-normal matrices can be used in the computation of a triangular factor of the difference \( X = A_1A_1^\dagger - A_2A_2^\dagger \) (without forming the products \( A_1A_1^\dagger \) and \( A_2A_2^\dagger \)), provided \( X \) is strongly nonsingular. If a factor of \( A_1A_1^\dagger \) is known then the problem of computing a factor of \( A_1A_1^\dagger - A_2A_2^\dagger \) is referred to as downdating.

We have identified classes of matrices for which, as long as the difference of their outer products is strongly nonsingular, the implicit triangularization methods via hyper-normal
transformations are superior to the explicit triangularization methods that require formation of the difference. These results have been presented in [9].

(iii) Product SVD

The problem of computing the singular value decomposition of a product of matrices occurs in many applications (weighted least squares, canonical correlations, etc.). The singular value decomposition of a product of matrices can be computed by the Jacobi method. It is important not to form the explicit product of the matrices as this obliterates the smallest singular values. We proposed a new way of implementing the Jacobi method without explicitly forming the product of the matrices involved. We showed that the algorithm can achieve the best possible numerical accuracy in a given finite precision arithmetic [5]. Our new algorithm when used to compute the SVD of a product of two matrices provides more accurate results than existing competing schemes.

(iv) Continuous-time subspace models for sensor arrays:

The use of (discrete-time) subspace models has been popular in recent years in sensor array processing, both for enhanced convergence as well as reduction in computational complexity. The key idea behind subspace models is to identify (adaptively or via apriori information) a set of spanning vectors which describe the space of snapshots which are produced by the interference impinging on the array. One can then replace the initial sensors by "virtual" sensors by projecting into this span. The attendant reduction is dimensionality leads to faster convergence in interference-covariance estimates, as well as computational reductions. Individuals associated with this approach to interference modeling and suppression include Barry Van Veen (U. Wisconsin), K. Buckley (U. Minn.), E. Kelly (MIT LL) etc.

It is often cited in the literature that point sources exhibit a covariance matrix whose "fuzzy" rank approximately equals the time bandwidth product (TBWP) of the array. This locution arises from a rather adhoc application of the seminal work of Slepian, Widom, and others on the time/frequency uncertainty for scalar discrete time processes. Sensors arrays deal with vector valued analog data. Therefore, we have taken a more fundamental operator theoretic look at the rank of the continuous-time covariance matrix function for a sensor array, and we have developed expressions for its rank. We find that the rank formula involves the TBWP, with a correction factor involving the "edge effects" arising from the assymetries in the arrival time across the array [7].

Next, we addressed the construction of subspace models in the analog domain. The motivation being to explore the potential for subspace reduction prior to AD conversion. This has application in very high bandwidth scenarios where analog preprocessing is an attractive alternative to all-digital schemes. To mimic the conventional subspace analysis
of our predecessors we must replace eigenvector analysis with eigenfunction (i.e. integral
equations ala Karhunen Loeve) analysis. Our key result is a scheme for collapsing a matrix
valued integral equation into a scalar valued integral equation [7]. This result allows us to
find subspace models with a reasonable amount of cost and storage. (Without this collapsing
numerical solution of the integral equations becomes untenable even on large machines.)

(v) Recursive least squares

Recursive least squares problems arise in many signal processing applications when the
speed requirements can be met only by using parallel computers. Then the problem at
hand has to be mapped on a parallel architecture. Algorithms for solving recursive least
squares problems are composite tasks in the sense that they involve triangularization of the
data matrix, updating and downdating of the triangular factor, and solving the resulting
triangular systems of linear equations. We considered the problem of mapping these tasks
onto a linear array of processors. We compared block and warp mappings for a number of
different algorithms. We developed a parametric model from which the cost of the algorithms
can be predicted from a given set of parameters describing the architecture at hand. The
validity of the model was tested the on a linear array of transputers and on a hypercube-type
architecture [6].

List of papers

[1 ] “Hyperbolic Singular Value Decomposition and Applications”, R.Onn, A.Steinhardt,


[3 ] “Hyperbolic SVD decomposition and applications”, R.Onn, A.Steinhardt, A.Bojanczyk,
2nd Int. Workshop on the SVD in signal processing, Rhode Island, June 1990, pro-
cceedings (manuscript enclosed). Also presented at the 1990 SIAM Conf. on Linear
Alg. and Appl. and the 5th ASSP Workshop on spectrum estimation, Oct. 1990,
Rochester, N.Y.

to appear Linear Algebra and its Applications.

List of student support

Several students have worked on this research with us, and appear as co authors in the list of papers; Ruth Onn (Israeli), J. Choi (Korean), and S. So (Hong Kong). This support led to the following dissertations: