LIMITING TIME VARIATIONS OF SERVOMOTOR TORQUES USING THE MODIFIED BANG-BANG CONTROLLER

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LIMITING TIME VARIATIONS OF SERVOMOTOR TORQUES USING THE MODIFIED BANG-BANG CONTROLLER

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This paper describes how to control a servomechanism when the maximum time rate variations of motor torque are specified. Limiting torque variations may be necessary to increase reliability. This approach uses a modified bang-bang controller. Introducing new controller parameters has the effect of guaranteeing enough torque during deceleration to prevent overshoot. The theory is developed and verified using simulation of a microcomputer.
TABLE OF CONTENTS

INTRODUCTION ............................................................... 1
MODIFIED BANG-BANG CONTROLLER ............................................ 1
APPROACH FOR LIMITING TORQUE GRADIENTS .............................. 3
THEORETICAL DERIVATIONS .................................................... 4
    Bang-Bang Control Phase ....................................................... 4
    PD Control Phase ............................................................ 6
SIMULATION RESULTS .......................................................... 7
REFERENCES ................................................................. 10

TABLES

Table 1 Calculated Values of r as a Function of Maximum Torque Rate u, ......................................................... 6
Table 2 Calculated Values of s and b as a Function of Maximum Torque Rate u, ......................................................... 7

LIST OF ILLUSTRATIONS

Figure 1 Block Diagram of Modified Bang-Bang Controller ................................................................. 2
Figure 2 Velocity vs Distance for Modified Bang-Bang ................................................................. 3
Figure 3 Velocity vs Distance...Variation of r ................................................................. 4
Figure 4 Study of du/dt for u,=500, r=1.0, and s=1.0 ................................................................. 8
Figure 5 Study of du/dt for u,=500, r=0.45, and s=1.5 ................................................................. 8
Figure 6 Study of du/dt for u,=500 During Loading Only ................................................................. 9
Figure 7 Study of du/dt for u,=500 and Friction=30 lbs ................................................................. 9

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INTRODUCTION

The problem considered in this paper is how to control a servomechanism when the maximum time rate variations of the motor torque are specified by the system designer. Limits on the rate of torque application may be required to prevent excessive damage to a mechanism caused by controller demands for high instantaneous torque gradients. These high demands, in turn, can result from such causes as high feedback noise effects or from component failure which inadvertently leads to a faulty, perhaps discontinuous feedback signal.

The particular application of concern to this writer has been the development of a large caliber autoloader for tank cannon. The ramming mechanism being developed for this autoloader requires a servomotor which drives a sprocket-boom assembly. High torque rates can cause localized damage to the boom by the sprocket teeth. It has been necessary to limit the rate of torque application to minimize both short term and long term fatigue damage in order to increase reliability to acceptable levels. Limiting torque rates should also prove to be beneficial in other applications where reliability and/or fatigue life are important considerations.

The general control approach used in this study is the modified bang-bang controller (called MBB in this paper) which is also called switching zone controller elsewhere [Refs 1-5]. The MBB controller is based in part on bang-bang theory in which maximum allowable torques are applied to both accelerate and decelerate a mechanism to move from one position to another in near minimum time. Surprisingly, this control approach lends itself well to specifying maximum rates of torque application, as will be shown. The main reason is that torque levels are directly controlled in MBB. Torque levels required to slow down a mechanism fast enough to prevent overshoot are readily calculated. Other control approaches such as PD (proportional-derivative) are more difficult to modify since torque levels are not directly controlled. As a consequence, using PD control does not lend itself to insuring adequate decelerating torques.

The remainder of this paper is outlined as follows:

- modified bang-bang controller
- approach for limiting torque gradients
- theoretical derivations
- simulation results.

MODIFIED BANG-BANG CONTROLLER

A position control approach which has proven to be valuable in applications to large caliber ammunition autoloaders and other Army systems is modified bang-bang (MBB) [ref. 5]. Essentially, this controller is comprised of bang-bang control with a boundary layer away from the desired target position and then a transition to PD control near the target position [ref. 2]. The main characteristics of this controller are:

- repositions in near minimum time
- designer can specify maximum torques
- designer can specify maximum velocities
- controller is robust and rejects disturbances
- little or no overshoot exists.
The schematic diagram for the MBB controller is shown in Figure 1. The different controller variables and parameters are defined as follows:

- \( m \) = system mass
- \( x \) = position of the mass
- \( x_d \) = desired position
- \( u \) = motor force applied to mass \( m \) = torque/lever arm
- \( u_d \) = disturbing force
- \( u_m \) = specified maximum motor force or torque
- \( a \) = nonlinear function term selected to guarantee sufficient force for deceleration
- \( a = (u_m - u_m)/u_m \) where \( u_m \) is the maximum value of the disturbance force \( u_d \)
- \( b \) = constant selected to guarantee no overshoot
- \( b = 2au_m/k_1 \)
- \( k_1,k_2 \) = positional and velocity gains
- \( \xi_a \) = \( b + mv_a^2/(2au_m) \)
  where \( \xi_a \) is essentially dependent on the specified maximum velocity \( v_a \)

\[ \begin{align*}
N_1 : & \quad u = k_1 e_f \quad (-u_m < u < u_m) \\
N_2 : & \quad \text{out} = (m/2au_m) |\dot{x}| \dot{x} \\
N_3 : & \quad \text{out} = k_2 \dot{x} \quad (-b < \text{out} < b) \\
N_4 : & \quad \text{out} = \xi_a \quad (-\xi_a < e < \xi_a) \\
\xi_a = b + (m/2au_m)v_a^2
\end{align*} \]

Figure 1   Block Diagram of Modified Bang-Bang Controller
Figure 2 is a phase diagram for a typical control problem taken from an autoloader application where \( u_s = 0.0 \) and \( a = 1.0 \). The controller in this case is designed to drive any given non-zero state toward the origin. For example, if the initial state in Figure 2 starts at point A, the full maximum force \( u = u_m \) is initially applied. The path then eventually enters the boundary layer or zone between full negative and positive forces. Once in the zone, the state is captured and is driven to the origin with little or no overshoot. See reference [5] for details and derivation of the controller parameters.

**APPRAOCH FOR LIMITING TORQUE GRADIENTS**

The objective is to fix the maximum rate of torque change \( \frac{du}{dt} \):

\[
\frac{du}{dt} = u_r \tag{1}
\]

When moving a mechanism from one position to another while limiting \( u_r \), the main concern is to begin the deceleration phase soon enough to prevent excessive speed near the end of the cycle.

In the MBB controller, starting to decelerate soon enough during the bang-bang phase can be readily accomplished by introducing a new control parameter \( r \) in the control block \( N_2 \) of Figure 1:

\[
N_2: \quad \text{output} = (m/2aru_m)\xi x\tag{2}
\]

where \( r \leq 1.0 \).

Introducing \( r \) here also necessitates adding \( r \) in the calculation of \( \xi_m \) which controls maximum velocity, \( v_m \):

\[
\xi_m = b + (m/2aru_m)v_m^2
\]

The effect of \( r \) can be seen in Figure 3 which is a phase plot for the example considered earlier for \( r = 1.0 \) and 0.5. This figure shows an earlier start of deceleration for the smaller value of \( r \). The next section discusses how to determine the required value of \( r \) as a function of the maximum torque rate \( u_m = \frac{du}{dt} \).
The control designer also needs to consider what happens when bang-bang control transitions to PD control near the target endpoint. As the next section shows, the designer may need to increase the PD zone parameter $b$ to insures no overshoot depending on the specific maximum torque rate $u_r$. No overshoot can be accomplished by introducing another control parameter $s$ to be used in redefining the PD zone parameter $b$ of Figure 1:

$$b_{new} = sb_{old}$$

where $s \geq 1.0$ and is a function of $u_r$.

**THEORETICAL DERIVATIONS**

In this section consideration is given to the theoretical derivation of the parameters $r$ and $s$ just introduced in the previous section as a function of the maximum torque rate $u_r$. Deriving the required parameter values will be accomplished by separately considering the two phases that comprise the MBB controller.

**Bang-Bang Control Phase**

During the bang-bang control phase, enough time must be allowed at the onset of deceleration for the motor torque to change from some given value to the maximum negative torque $-u_m$. This is to insure adequate decelerating torque prior to transition to the PD control phase. The time required is dependent on the maximum allowable rate of torque change $u_r$.

Let $t_o$ be the time at which deceleration is to start. The position $x(t_o)$ at this point is determined from the controller description given in Figure 1, along with the new definition of block $N_2$ previously given as equation (2):

$$-x(t_o) = b + (m/2ar) \times x^2(t_o)$$

where $x_r = 0.0$. At some later time $t_1$, it is assumed that the controller will have changed the torque to $u(t_1) = -u_m$. 

![Figure 3: Velocity vs Distance. Variation of $r$](image)
which is the full decelerating torque. The value of x at this time must satisfy the following equation in order to guarantee sufficient continued deceleration with no overshoot.

\[-x(t) = b + (m/2a)x(t)\]

The parameter r is absent in this equation since the motor torque is at its full negative value \(-u_m\) and, therefore, the mechanism can slow down without further restrictions.

During the deceleration phase, the maximum rate that torque can change is the designer specified value \(u_r\). The following equations are consequently satisfied:

\[\dot{u}_m = \dot{u}(t) = -u_r\]  \hspace{1cm} (6)

\[u(t) = -u_r t + C = m \dot{x}\]  \hspace{1cm} (7)

\[m \dot{x}(t) = -\frac{u_r}{2} t^2 + Ct + D\]  \hspace{1cm} (8)

\[m x(t) = -\frac{u_r}{6} t^3 + \frac{C}{2} t^2 + Dt + E\]  \hspace{1cm} (9)

Without loss of generality, let \(t_0 = 0.0\) and \(\dot{x}(t) = v_m\), the maximum velocity. Then

\[C = u(t_0)\]

\[u(t_0) = -u_m = -u_r t_1 + C\]

\[t_1 = (u(t_0) + u_m)/u_r\]  \hspace{1cm} (10)

\[D = mx(t_0) = mv_m\]

\[E = m x(t_0)\]

By combining equations (4) through (10), the control designer can derive a relation for calculating r:

\[r = f(m, u_m, v_m, u(t_0))\]  \hspace{1cm} (11)

In equation (11), it is assumed that \(x(t_0) = v_m\). The unknown quantity \(u(t_0)\) needs to be specified before specific values of r can be calculated. The worst case would occur for \(u(t_0) = +u_m\) but the more practical case is for \(u(t_0) = 0.0\). This last case happens whenever the maximum velocity \(v_m\) is reached during the acceleration portion of bang-bang in the absence of friction. For \(u(t_0) = 0.0\), equation (11) is derived to be

\[\frac{1}{r} = -\frac{\dot{u}_m}{3u_m v_m} + \frac{\dot{u}_m}{m u_m v_m} + \left(1 - \frac{u_m^2}{2m u_m v_m}\right)^2\]  \hspace{1cm} (12)

Although not shown here, a similar relation can be derived for \(u(t_0) = u_m\) or for any other point.

As an example, let \(m = 80\text{lbs/386}, u_m = 75\text{lbs},\) and \(v_m = 120\text{in/sec}\). Table 1 lists the values of r calculated for \(u(t_0) = u_m\) and 0.0 for various values of \(u_r\).
Table 1 Calculated Values of $r$ as a Function of Maximum Torque Rate $u$

<table>
<thead>
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<th>$u$, lbs/sec</th>
<th>$r$</th>
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<td>$u(\omega)=u_m$</td>
<td>$u(\omega)=0.0$</td>
</tr>
<tr>
<td>$\infty$</td>
<td>1.00</td>
</tr>
<tr>
<td>2500</td>
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</tr>
<tr>
<td>1000</td>
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</tr>
<tr>
<td>500</td>
<td>0.32</td>
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The actual values of $r$ used depend on the specific system being controlled and the deceleration requirements.

**PD Control Phase**

Bang-bang control essentially hands off to PD control at

$$x = b/k_2 : -x \geq b - au_m/k_1$$

At this point, the decelerating torque is as much as $-u_m$. The parameter $b$ in equation (13) depicts a horizontal shift of the bang-bang portion of MBB. The value of $b$ for the no overshoot condition is derived in reference [3]:

$$b = 2au_m/k_1$$

This value of $b$ assumes no limitation on torque rate $du/dt$. Introducing the parameter $s$ as in equation (3) and then deriving the resulting torque rate from the PD differential equations of motion [ref. 6] yields the following expression:

$$\frac{du}{dt} = au_m\omega [\sqrt{(2-s) + (s-1)\omega}]$$

where $\omega = 2k_2 = \sqrt{k_1/m}$

The initial conditions at $t = 0$ are those given by equation (13).

From equation (15), the maximum value of $du/dt$ can be derived for different values of $s$:

$$\left(\frac{du}{dt}\right)_{\text{max}} = au_m\omega (2 - s) ; \quad 1 < s \leq 1.5$$

$$= au_m\omega (s - 1) ; \quad 1.5 < s \leq 2.0$$
Table 2 lists the values of $u_i=(du/dt)_{max}$, $s$, and the corresponding $b$ calculated for the example given earlier.

<table>
<thead>
<tr>
<th>$u_i$</th>
<th>$s$</th>
<th>$b$</th>
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<tr>
<td>1.0</td>
<td>1.0</td>
<td>4.0</td>
</tr>
<tr>
<td>1008</td>
<td>1.0</td>
<td>4.0</td>
</tr>
<tr>
<td>504</td>
<td>1.5</td>
<td>6.0</td>
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<tr>
<td>371</td>
<td>2.0</td>
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SIMULATION RESULTS

A nonlinear simulation program called SIMION [ref. 7] was used to study the problem of limiting motor torque rate. The dynamic models for these studies were derived and tested on a microcomputer.

Consider the autoloader ramming problem presented earlier where $m = 80\text{lbs}/386$, $u_m = 75\text{lbs}$, and $v_m = 120\text{in/sec}$. Let $u_i = (du/dt)_{max}$ be specified as 500 lbs/sec. This value of $u_i$ is lower than what was required to prevent excessive damage in actual applications. Figures 4 and 5 show the results comparing the original MBB control to control using the additional parameters $r$ and $s$ as presented earlier. As can be seen, satisfactory results are obtained using the parameter values $r = 0.45$ and $s = 1.5$.

Figure 6 shows a different case where the maximum rate of 500 lbs/sec was applied only during loading, either in the positive or negative direction. On unloading, it might be assumed that no structural damage will occur. This eases the control requirements as can be seen where a higher value of $r = 0.8$ was sufficient to yield a satisfactory response. Finally, Figure 7 shows the beneficial effects of the presence of friction on smoothing out response fluctuations, even for a value of $r = 1.0$ which represents the original MBB. The slight overshoot shown in Figure 7 can be eliminated by fixing $r$ to some value less than 1.0.
Figure 4  Study of $du/dt$ for $u=500$, $r=1.0$, and $s=1.0$

Figure 5  Study of $du/dt$ for $u=500$, $r=0.45$, and $s=1.5$
Figure 6  Study of \( \frac{du}{dt} \) for \( u_i = 500 \) During Loading Only

Figure 7  Study of \( \frac{du}{dt} \) for \( u_i = 500 \) and Friction=30 lbs
REFERENCES


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