INTERACTION AND IMPACT OF FLOATING BODIES

by

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Sponsored by

Ocean Engineering Division
Office of Naval Research
Under Grant N00014-89-J-1581

IIHR Report No. 357
Iowa Institute of Hydraulic Research
The University of Iowa
Iowa City, Iowa 52242

July 1992

Approved for Public Release; Distribution Unlimited

92-23203
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I. INTRODUCTION

On the basis of two 1983 OPNAV instructions, S34705A entitled "U.S. Navy Policy Regarding Arctic Polar Region" and S3470.6 entitled "U.S. Navy Warfare Program," the U.S. Navy is preparing to operate its surface ships at high latitudes on a routine basis in support of the nation's Maritime Strategy. In addressing these operating requirements, the U.S. Navy must evaluate the ability of its surface ships to fulfill their mission when operating near the ice edge in the marginal ice zone, and when entering an ice covered port with icebreaker assistance. As the Arctic has become a principal strategic location, knowledge and prediction of sea-ice conditions and the ability to cope with them have become essential to the U.S. Navy. The long-term goal of our research project is to investigate the hydrodynamic interactions, including central and oblique impact, between two floating bodies, or between a floating body and a fixed body. The floating body is usually an ice floe, and the fixed body is an offshore structure.

Our near-term objectives are to develop a rationally formulated, computer-basec' analytical model of far-field ice-floe trajectories and near-field hydrodynamic interactions between floating ice floes and offshore structures. The offshore structures considered in the present study would be assumed to have the shape of a circular cylinder. The ice floes would have the shape of a rectangular block, a rectangular cylinder, a circular cylinder, a circular disk, or a sphere.

The analysis would be based on the equations of planar motion of an ice floe under the action of external forces due to wind, fluid viscosity, fluid inertia, and the nonuniformity of the flow field due to the presence of the offshore structure. The forces due to fluid inertia and flow nonuniformity would be expressed in terms of added-mass coefficients. Trajectories would be obtained by numerical integration of the equations of motion. Solutions for central impact as well as various oblique impacts would be obtained.

Our accomplishments on this project are summarized below.

II. RESEARCH HIGHLIGHTS

1. Central impact between two bodies

A paper entitled "Interaction Between Two Bodies Translating in an Inviscid Fluid," by Landweber, Chwang, and Guo, was published in the Journal of Ship Research [1]. In this paper, the equations of motion of two bodies in translational motion in an inviscid fluid at rest at infinity are expressed in Lagrangian form. For the case of one
body stationary and the other approaching it in a uniform stream, an exact, closed-form solution in terms of added masses is obtained, yielding simple expressions for the velocity of the moving body as a function of its relative position and for the interaction forces. This solution is applied to the case of a rectangular cylinder approaching a cylindrical one, for which the added-mass coefficients had been previously obtained in a companion paper by an integral-equation procedure.

In order to compare results with those in the literature, and to evaluate the accuracy of the present procedures, results were calculated for a pair of circular cylinders by these methods as well as by successive images. Very good agreement was found. Comparison with published results showed good agreement with the added mass but very poor agreement on the forces, including disagreement as to whether the forces were repulsive or attractive. The discrepancy is believed to be due to the omission, in these papers, of terms in the Bernoulli equation which was used to obtain the pressure distribution and then the force on a body. The Lagrangian formulation is believed to be preferable to the pressure-integral approach because it yields the hydrodynamic force directly in terms of the added masses and their derivatives, thus requiring the calculation of many fewer coefficients.

2. Added masses and forces on two bodies approaching central impact

Reference [2] differs from [3] in that it contains many details that had to be omitted in [3] for publication in the Journal of Ship Research. A new result in [3], a relation between the forces on a body and a wall when the body accelerates towards the wall, was stimulated by Prof. T. Miloh who called attention to his own paper, Bentwich-Miloh (1978) when serving as a referee of [3]; see [3] for the reference.

The following is a repetition of the Summary and conclusions section of [3]:

The motivation for undertaking the present work was to evaluate the accuracy of the interaction forces computed by the integral-equation approach. For this purpose, the classical theory of successive images, for central impact of pairs of circles or spheres, is available for obtaining highly accurate numerical solutions, against which results from integral equations could be compared. In the course of applying these approaches, new and significant advances in both areas were made and have been described. These will now be summarized.

On the method of successive images, the following has been accomplished:

1. A new and more rational derivation of Herman's (1887) formulas for the doublet strengths than that given in his paper is presented.
2. A new parametric form of these formulas has been applied to derive new truncation-correction formulas, with which the infinite series for the added masses can be summed to a desired accuracy with many fewer terms.

3. A new asymptotic formula equating the mathematical parameter $\zeta$ of the aforementioned parametric form to a polynomial in the nondimensional physical parameter $\gamma$ (which is proportional to the square-root of the gap $g$ between the bodies) is derived, and shown to be highly accurate at small gaps, and for equal circular cylinders or spheres, to be usefully accurate up to $g/b = 1$.

4. Combining the truncation and asymptotic formulas yielded the results that the series for the added masses, and those for their derivatives with respect to the parameter $\zeta$, both converge at $g = 0$, and asymptotic Taylor expansions about $g = 0$, which display the added masses and their derivatives at $g = 0$, are given. These properties led to the new result that the derivatives of the added masses with respect to $g$, for small values of $\gamma$, vary as $g^{-1/2}$ for circular cylinders and, as the known result [B&M (1978), M-T (1938)], -eng for spheres. In both cases, this implies that the repulsive forces approach infinity, verifying the well-known irrotational-flow paradox that the bodies would never meet.

5. The need for considering the property of uniform convergence of the series for the added masses and their derivatives is discussed in the text. It is proved that the series for the added masses are uniformly convergent in the closed region $0 \leq g \leq \infty$, indicating that the added masses are continuous functions of $g$ at $g = 0$; and that the series for their derivatives with respect to $\zeta$ are uniformly convergent in the open region $0 < g < \infty$, i.e., although this derivative series converges at $g = 0$, the convergence is not uniform. This implies that the derivatives of the added masses may be discontinuous at $\zeta = 0$. The series of the derivatives with respect to $g$, however, diverges at $g = 0$. On the method of integral equations, three new procedures were developed to obtain more accurate solutions. The first of these was required at small gaps to eliminate the peaks of the four kernels of the two integral equations. It was shown that the transposes of these kernels eliminate not only their singularities but also their peaks, which are of the order of $1/g$ for two-dimensional and of $1/g^2$ for three-dimensional bodies of general shape. This left a residue of much smaller peaks due to the rapid variation of the source distribution in the gap region when the gap is small. These were treated by applying a quadrature formula, described in the next paragraph, which concentrates many points in the small neighborhood of the smaller peaks.

In the second procedure, the 'most accurate quadrature formula,' which requires a smooth, cyclic integrand and uniform intervals, was modified by changing the variable of integration, so that the integrand remains cyclic, uniform intervals are taken in the new
variable, and many points of the original variable are concentrated in the desired region. A sequence of such transformations, of successively increasing point concentrations in a small region, is presented. With a slight modification, this nonuniform MAQF also served to evaluate accurately the noncyclic integrals of the two-sphere problem.

The third procedure was developed in order to improve the accuracy of the added-mass derivatives obtained by numerical differentiation of the added-mass data. As is described in the text, this procedure, which is suitable for general shapes, requires accurate solutions of the integral equations, and is suggested by the asymptotic formulas for the added masses at small gaps. (For example, for the case of two circular cylinders, the added masses vary nearly linearly with \( \sqrt{g} \), where \( g \) is the gap between the cylinders. Then the error in numerical differentiation would be greatly reduced by using \( \sqrt{g} \) instead of \( g \) as the variable of differentiation. The derivative with respect to \( g \) could then be obtained by the chain rule of calculus without further error. Similarly, for other bodies, graphs of the computed added masses at small gaps, say on log-log paper, might suggest an asymptotic law to use as a differentiation variable.) Here the material between the parentheses has been inserted for clarification.

By means of these new procedures, the minimum gap at which accurate results could be obtained was reduced to about one-tenth of those reported in the original work on this problem for the Mobil Research and Development Corporation. Results for circle pairs of diameter ratios 1, 4, 16 and \( \infty \), two ellipse-circle pairs and a pair of equal spheres are presented. The treatment of the infinite-diameter ratio (a circle approaching a wall), which is also applied to a sphere and a wall, yields results for the added masses and forces on bodies of arbitrary shape. The Bentwich-Miloh (1978) result, that the interaction force on a solid wall is three times as large as that on a sphere at a very great distance from the wall moving in the direction normal to it, has been generalized to apply to any rigid body at any distance from the wall, moving in the direction normal to it. The new result is as follows: The algebraic sum of the interaction force \( F_1 \) on the wall and \( F_2 \) on the moving body is equal to the force required to accelerate the mass of fluid displaced by the body, that is

\[
F_1 + F_2 = \rho \Delta U \frac{dU}{dt}
\]

All the other applications were to two-dimensional and axisymmetric forms, for which the integral-equation approach yielded highly accurate solutions. An IBM RISC/6000 was used to obtain the numerical results.
3. Oblique impact between two circular cylinders

The oblique motion of a circular cylinder through an inviscid and incompressible fluid, conveyed by a uniform flow at infinity, in the vicinity of another cylinder fixed in space is considered. In a relative polar coordinate system moving with the stream, the kinetic energy of the fluid is expressed as a function of six added masses due to motions parallel and perpendicular to the line joining the centers of the cylinder pair. The Lagrange equations of motion are then integrated for the trajectories of the moving cylinder. In order to evaluate the added masses and their derivatives with respect to the separation distance between the cylinders in terms of the hydrodynamic singularities, the method of successive images, initiated by Hicks, and the Taylor added-mass formula are applied, and analytic solutions due to Herman are presented. The dynamic behavior of a drifting body in close proximity of a fixed one is investigated by considering the limiting values of the fluid kinetic energy and the interaction forces on each body. The reliability of the numerical approximation of added masses and their derivatives is also discussed in the present study. The integral equations, in terms of surface source distributions and their derivatives on both circles, are carefully modified for obtaining accurate numerical solutions. The results of these studies are reported in publications [41, 42, and 43].

4. Near-field interaction between two bodies moving in a fluid

The planar translational motion of a pair of bodies and the hydrodynamic interaction forces acting on them in an inviscid fluid are studied based on Lagrange's equations of motion. In a relative coordinate system moving with the fluid at infinity, the kinetic energy of the fluid is expressed as a function of six added masses due to motions parallel and perpendicular to the line joining the centers of two bodies. The velocity components and the moving trajectories of each body are obtained by integrating the equations of motion in terms of the added masses, which are evaluated in terms of source distributions on the surfaces of two bodies by solving a set of Fredholm integral equations of the second kind.

Numerical results for several practical engineering problems involving central and oblique motions between two bodies are presented. It is found that the hydrodynamic interaction force depends on the separation distance between two bodies and on the direction of the flow with respect to the centerline joining the centers of two bodies. The velocity component along the centerline produces a repulsive force, which prevents the collision of two bodies, while the component perpendicular to it produces an attractive force. These results are reported in publications [7] and [8].
5. **Planar translation of two three-dimensional bodies**

The general planar translation of two bodies of revolution through an inviscid and incompressible fluid is considered in [9] and [10]. The moving trajectories and the hydrodynamic interactions are computed based on the generalized Lagrange equation of motion, including the effects of solid constraints, external forces in the plane of motion, and a uniform stream in any direction parallel to the plane of motion. In a relative coordinate system moving with the stream, the kinetic energy of the fluid is expressed as a function of six added masses due to motions parallel and perpendicular to the line joining the centers of two bodies. Analytical solutions of added masses in series form are obtained for the motion of two spheres. A new iterative formula based on Basset's (1887) analysis of velocity potentials around each body is developed for added masses and their derivatives with respect to the separation distance due to the transverse motion. The method of successive images and Taylor's added-mass formula are applied to determine the added masses and their derivatives due to the centroidal motion. These results are compared with the numerical solution of added masses computed by the boundary-integral method and the generalized Taylor added-mass formula. The integral equations, in terms of surface-source distributions on both surfaces, are modified for obtaining accurate numerical solutions. Numerical results are given for several practical engineering problems.

6. **Hydrodynamic interaction between a 3-D body and a 2-D cylinder**

A joint paper [11] by Guo and Chwang, entitled "Planar motion of a three-dimensional body near a circular cylinder in a uniform flow," has been published in the Proceedings of the Second (1992) International Offshore and Polar Engineering Conference (ISOPE-92). In this paper, hydrodynamic interactions between a three-dimensional body of revolution and an infinitely-long cylinder moving relatively in an inviscid fluid at rest at infinity are studied by means of the Lagrangian equations of motion and the boundary-integral method. A set of four integral equations of the second kind are solved numerically, and a numerical technique is developed to evaluate integrations over steep peaks accurately and efficiently. Least-squares approximations are constructed on the basis of Legendre polynomials to fit the discrete values of added masses. As a practical example, the moving trajectories of a sphere, conveyed by a uniform flow, around a fixed circular cylinder are computed and presented.
7. Wind effect on oblique motion of two bodies

A joint paper [12] by Chwang and Huang, entitled "Wind effect on oblique motion of two bodies in a uniform flow," has been published in the Proceedings of the Ninth ASCE Engineering Mechanics Conference, College Station, Texas, 1992. In this paper, the wind effect on the general translational motion of a circular cylinder through an inviscid and incompressible fluid, conveyed by a uniform flow at infinity, in the vicinity of another cylinder fixed in space is considered. It is found that the trajectories of the moving cylinder do not change much, if the magnitude of the wind velocity is the same as that of the current velocity, except that they shift slightly in the direction of the wind. However, when the wind velocity is five times the magnitude of the current velocity, the trajectories of the moving cylinder change greatly, particularly when the direction of the wind is perpendicular to the current velocity.

8. Nonlinear viscous waves produced by a moving solid body

A paper [13] by Yang and Chwang, entitled "Numerical Simulation of Nonlinear Viscous Waves," has been published in the Proceedings of the Seventh Congress of the Asian and Pacific Regional Division of IAHR, Vol. 3, 1990. In this paper, the nonlinear viscous waves produced by a moving solid boundary is studied numerically. The unsteady, two-dimensional Navier-Stokes equations are discretized using the finite-analytic scheme, a branch of the finite-difference method, with a built-in automatic upwind ability. The SIMPLER algorithm of Patankar (1980) is adopted to solve the velocity and pressure fields in the flow domain. The continuity equation and the dynamic boundary conditions on normal and tangential stresses at the free surface are applied to determine the pressure and two velocity components at the free surface. Numerical results obtained from the present computer code are also discussed in publication [14].

In order to validate the present computer code, numerical results on the free-surface profile and the pressure distribution due to an impulsive, horizontal motion of a vertical boundary are compared with experimental measurements carried out by Yang and Chwang (15, 16). The agreement is fairly satisfactory. For this impulsive motion of a vertical boundary, both the numerical result and the physical experiment indicate that the water surface in front of the vertical boundary simply rises up during the initial stage of its acceleration. The potential-flow solution of Chwang (Physics of Fluids, Vol. 26, 1983, pp. 383-387) on the corresponding problem is also evaluated.
9. About the moving contact line

In publications [17], [18], and [19], the flow field around a moving contact line, generated by an impulsive, horizontal motion of a vertical plate, is studied numerically. The two-dimensional, unsteady Navier-Stokes equations, coupled with the kinematic and dynamic boundary conditions on the free surface, are solved. The contact angle between the free surface and the vertical plate is found to be very close to that formed by the free surface of a fluid contained in a rectangular tank which moves with the same constant acceleration. The velocity of the moving contact line is thereby derived as a function of the acceleration of the plate based on the principle of mass conservation.

III. PUBLICATIONS RESULTING FROM THIS PROJECT


IV. CONCLUSIONS

Analytical, numerical, and experimental investigations on the Interaction and Impact of Floating Bodies have been conducted under the sponsorship of the Ocean Engineering Division, the Office of Naval Research, from February 1, 1989 to March 31, 1992. This paper consists of a summary of the accomplishments on this project.

ACKNOWLEDGMENTS

The research described in this report was sponsored by the Ocean Engineering Division, the Office of Naval Research, under Grant N00014-89-J-1581. The participants in this research project are grateful to Dr. Steven Eric Ramberg, Director of the Ocean Engineering Division, for his continuous encouragement and support.