Far-Zone Field of a Monopole Element on a Disk Ground Plane above Flat Earth

M. M. Weiner
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ABSTRACT

Richmond's moment-method analysis for the current distribution and input impedance of a monopole element on a disk ground plane above flat Earth is used to obtain the far-zone field in the free-space region. Numerical results for directivity and radiation efficiency are presented as separate entities, unlike previously reported results based on Monteath's compensation theorem or Sommerfeld's attenuation function that give only the product of the directivity and radiation efficiency.
ACKNOWLEDGMENTS

The theory is based on a report written by Dr. Jack H. Richmond (deceased) of Ohio State University when he was a member of the technical advisor group to the MITRE sponsored research project 91260 "High-Frequency Antenna Element Modeling," Melvin M. Weiner, Principal Investigator. Dr. Richmond developed the theory and computer program RICHMOND4 for the far-zone field. Christopher Sharpe and Enis Vlashi performed the computer runs and obtained the computer plots. Elinor Trottier and Sheila Lamoureux typed the manuscript.
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SECTION 1
INTRODUCTION

The modeling of monopole elements with circular ground planes in proximity to Earth has been greatly enhanced in recent years by method-of-moments programs developed by Richmond for disk ground planes [1] and by Burke, et al., for radial wire ground planes [2,3,4].

The method-of-moments models include the following advantages over models based on Monteath's compensation theorem [5,6,7,8,19] or Sommerfeld's attenuation function [9]: (1) current on the ground plane is computed rather than approximated by that for a perfect ground plane; (2) results are valid not only for moderately large ground planes, but for electrically small ground planes; (3) ground-plane edge diffraction is determined directly rather than neglected or obtained by perturbation methods; (4) analytical restrictions on evaluating Sommerfeld's integral (such as requiring that the Earth's complex relative permittivity have a modulus much greater than unity) are avoided; and (5) directivity and radiation efficiency are determined as separate entities, rather than being lumped together as a product to yield the antenna gain. Nevertheless, those other models are useful for validating method-of-moments numerical results and for treating large ground planes whose segmentation in method-of-moments models would exceed computer computational capacity and precision.

Richmond has presented a moment-method analysis for the current distribution and input impedance of a monopole element on a disk ground plane in free-space [10] and above flat Earth [1] with numerical evaluation by computer programs RICHMD1 and RICHMD3, respectively. Weiner, et al. [11,12] have used Richmond's results in reference 10 to develop a computer program, RICHMD2, for the far-zone field, directivity, and radiation efficiency for the case when the ground plane is in the free space. The present effort uses Richmond's results in reference 1 to obtain the far-zone field when the ground plane is above flat Earth. Numerical evaluation of the far zone field is achieved with Richmond's computer program RICHMOND4.
Consideration is limited to the far-zone field in the free-space region, with ionospheric effects excluded. When the observer approaches the air-Earth interface, the total far-zone field will include a small contribution from the "surface wave," but this term is not considered here. Instead of a null on the radio horizon, the surface wave will contribute a far-zone evanescent field that is small compared to the far-zone field in the direction of peak directivity. However, the significant effect of the near-field surface wave in reducing radiation efficiency is included in the present analysis.

The radiation field from the surface magnetic current density (magnetic frill) of the coaxial line feed is included in the present analysis. Although the magnetic frill is the excitation source for the current on the monopole element and the disk ground plane, its contribution to the far-zone field may usually be neglected (as was done in references 11 and 12) unless the monopole element is so short that the radiation resistance of the magnetic frill becomes comparable to that of the monopole element.

The theoretical model, numerical results, validation of numerical results, and conclusions are given in sections 2, 3, 4, and 5, respectively. A more comprehensive treatment and review of monopole elements on circular ground planes in proximity to flat Earth is given in reference 13.
SECTION 2
THEORETICAL MODEL

2.1 METHODOLOGY

The antenna geometry consists of a vertical monopole element (length \( h \) and radius \( b \)), on an infinitely thin disk ground plane of radius \( a \) at a height \( z_0 \) above flat Earth (see figure 1). The Earth, with a dielectric constant \( \varepsilon_r \), conductivity \( \sigma \) (S/m) at a radian frequency \( \omega \) (rad/s), and free-space wavelength \( \lambda \) (m), has a complex relative permittivity \( \varepsilon^*/\varepsilon_0 \)
\[
= \varepsilon_r (1 - j \tan \delta) \quad \text{where} \quad \tan \delta = \text{loss tangent} = \sigma/(\omega \varepsilon_r \varepsilon_0) = (\lambda \sigma 2\pi \varepsilon_r) (\mu_0/\varepsilon_0)^{1/2}
\]
\[
= 60 \lambda \sigma /\varepsilon_r. \]

The monopole element and disk are assumed to have infinite conductivity. The location of an arbitrary far-zone observation point \( P \) is designated by spherical coordinates \( (\rho, \theta, \phi) \) with original \( O \) at the air-Earth interface below the monopole element.

The feed for the monopole antenna is a coaxial line with its inner conductor connected through a hole of radius \( b_1 \) in the ground plane to the vertical monopole element and its outer conductor connected by means of a flange to the ground plane. The inner conductor's diameter is equal to the monopole element's diameter \( 2b \) and the outer conductor's diameter is equal to the ground-plane hole diameter \( 2b_1 \). The current on the outside of the coaxial-line feed is assumed to be zero because of the attenuation by lossy ferrite toroids along the exterior of the coaxial-line feed (see section 2.4 of reference 12). The coaxial line feed excitation may be replaced by an equivalent surface magnetic current density (magnetic frill) \( M_\phi \) given by equation (23) of section 2.4.

The magnetic frill excitation gives rise to a monopole element current distribution \( I_z(z) \) along the \( z \) axis of the monopole element and a disk current density distribution \( J_\rho(\rho) \) in the radial direction \( \rho \) in the plane of the disk. The current density \( J_\rho(\rho) \) is the net current density on the top and bottom of the disk (see equation 2.4.1 of reference 12). The method-of-moment solution for the distributions \( I_z(z) \) and \( J_\rho(\rho) \) is described in reference 1. These
Figure 1. Monopole Element on Disk Ground Plane Above Flat Earth
currents are used to determine the far-zone field at $P(r, \theta, \phi)$. The distribution $I_z(z)$ and $J_\rho(\rho)$ include the contribution of the surface wave. The theory which follows is based on a report by Richmond (see Acknowledgments).

Wherever possible the notation will agree with that of references 1 and 10, except for the following: the disk radius is denoted by $a$ (instead of $c$); the monopole element radius is noted by $b$ (instead of $a$); the disk hole radius, equal to the radius of coaxial line outer conductor, is denoted by $b_1$ (instead of $b$); the Earth complex permittivity is denoted by $e^*$ (instead of $e_2$); the free-space wave impedance is denoted by $\eta_o$ (instead of $\eta$); and the origin of the $z$ axis is on the Earth's surface rather than on the disk ground plane. These changes in notation are made to conform with the notation for the numerical results of section 3. The time dependence is $\exp(j\omega t)$. The parameters of free space are denoted by $(\mu_o, \epsilon_o)$, $\kappa = \omega \sqrt{\mu_o \epsilon_o}$, and $\eta_o = \sqrt{\mu_o / \epsilon_o}$.

The far-zone electric field intensity of the monopole/disk antenna may be regarded as the sum of the field $E^J$ radiated from the electric currents and the field $E^M$ radiated from the magnetic frill current at the antenna terminals. To calculate these fields we consider the electric current density $J$, the magnetic current density $M$ radiating in free space, and the field reflected from the air-Earth interface. In these calculations, the perfectly conducting antenna structure is removed and replaced with the equivalent currents $J$ and $M$.

One successful but tedious approach to the far-zone fields starts with the rigorous expressions in terms of Sommerfeld integrals. This formulation is interpreted as a plane-wave expansion that includes a finite spectrum of uniform "space" waves plus an infinite spectrum of evanescent plane "surface" waves. Since the evanescent waves attenuate approximately exponentially with their height above the Earth, and since their peak amplitude relative to that of the space waves approach zero with increasing distance into the far-field, they are deleted in the far-zone field derivations. Finally, the method of stationary phase is applied to evaluate the remaining integrals asymptotically as the observation point recedes to infinity.
The same far-zone field expressions can be derived more readily via Carson's reciprocity theorem [22,23] as follows. The coordinate origin is on the air-Earth interface, the disk in the plane \( z = z_0 \), and the wire monopole extends from \( z_0 \) to \( z_0 + h \) on the \( z \) axis. A point in the far-zone region is specified by the spherical coordinates \((r, \theta, \phi)\) where \( r \) denotes the radial distance from the origin and the angle \( \theta \) is measured from the \( z \) axis. The goal is to determine the far-zone field \( E(r, \theta, \phi) \) in the free-space region. To accomplish this, we let \((E', H')\) denote the total field in the vicinity of the origin produced by an infinitesimal electric test dipole located at the far-zone point \((r, \theta, \phi)\) in the plane \( \phi = 0 \). If the dipole is oriented in the \( \hat{\theta} \) direction and has a dipole moment \( p \), its field in the vicinity \((x, y, z)\) of the origin is

\[
E'_o(x, y, z) = \frac{-\hat{\theta} j k \eta_0 p \exp(-jkR')} {4\pi r}
\]

\[
R' = r - (x \sin \theta + z \cos \theta)
\]

With this field incident on the air-Earth interface, the total magnetic field above the Earth in the vicinity of the origin is

\[
H'(x, y, z) = \frac{j k p \exp(-jkr) \exp(j k x \sin \theta)} {4\pi r} \left[ \exp(j k z \cos \theta) + \Re \exp(-j k z \cos \theta) \right]
\]

where \( \Re \) denotes the plane-wave Fresnel reflection coefficient at the air-Earth interface.

The Earth conductivity, permeability, and dielectric constant are denoted by \((\sigma, \mu_2, \varepsilon_r)\). We let \( \mu_2 = \mu_0 \), in which case the reflection coefficient \( \Re \) (for parallel polarization) is given by
\[ \mathcal{R} = \mathcal{R}_\parallel = \frac{(\varepsilon^* / \varepsilon_0) \cos \theta - \sqrt{(\varepsilon^* / \varepsilon_0) - \sin^2 \theta}}{(\varepsilon^* / \varepsilon_0) \cos \theta + \sqrt{(\varepsilon^* / \varepsilon_0) - \sin^2 \theta}} \]  

(4)

The complex relative permittivity of the Earth is

\[ (\varepsilon^* / \varepsilon_0) = \varepsilon_r - j\sigma / (\omega \varepsilon_0) \]  

(5)

From equation (3) and Maxwell's curl equations, the total electric field intensity above the Earth in the vicinity of the origin is

\[ E_x = -jk\eta_0 p \exp(-jkr) \exp(jkx \sin \theta) \cos \theta \]  

\[ \frac{4\pi r}{4\pi r} \left[ \exp(jkz \cos \theta) - \mathcal{R} \exp(-jkz \cos \theta) \right] \]  

(6)

\[ E_y = 0 \]  

(7)

\[ E_z = -jk\eta_0 p \exp(-jkr) \exp(jkx \sin \theta) \cos \theta \]  

\[ \frac{4\pi r}{4\pi r} \left[ \exp(jkz \cos \theta) + \mathcal{R} \exp(-jkz \cos \theta) \right] \]  

(8)

Carson's reciprocity theorem states that

\[ \int I^t \cdot E \, dl = \iint (J \cdot E^t - M \cdot H^t) \, ds \]  

(9)

where \( E \) is the total far zone field at \((r, \theta, \phi)\).
On the right-hand side of this equation, \( J \) and \( M \) denote surface-current densities on the perfectly conducting monopole/disk antenna and the integration extends over the surface of the monopole/disk antenna. On the left side the integration extends over the infinitesimal test dipole and is readily evaluated to be \( pE_\theta(r, \theta, \phi) \), so the reciprocity theorem reduces to

\[
E_\theta(r, \theta, \phi) = \frac{1}{p} \iint (J \cdot E^t - M \cdot H^t) \, ds
\]

where \( p \) is the dipole moment.

The magnetic-frill current \( M \) is given. Upon completion of the moment-method analysis, the electric current density \( J \) is known on the vertical wire monopole and the horizontal conducting disk. The test dipole fields are given above, so equation (10) contains no unknown quantities. Thus, evaluation of the far-zone field \( E_\theta(r, \theta, \phi) \) of the monopole/disk antenna is now simply a matter of performing the integrations in equation (10). If we start with a \( \hat{\phi} \)-oriented test dipole, a similar analysis shows that the resulting far-zone \( \phi \) component of the monopole/disk antenna is \( E_\phi(r, \theta, \phi) = 0 \).

2.2. THE FIELD FROM THE MONOPOLE ELEMENT

In far-zone field calculations for the vertical wire monopole, the tubular surface current density \( J \) can be replaced with a filamentary line source \( I(z) \) on the \( z \) axis. From equations (8) and (10), the far-zone field from the vertical wire is given by

\[
E_\theta^w(r, \theta, \phi) = \frac{j k \eta_0 \exp(-j kr) \sin \theta}{4 \pi r} \cdot \left[ \int_{z=0}^{z=h} I(z) \left( \exp(j k z \cos \theta) + \Re \exp(-j k z \cos \theta) \right) \, dz \right]
\]

\[2-6\]
The vertical wire monopole is divided into $L$ segments with length $d' = L/h$. A typical segment (segment $\ell$) extends from $z_1^\ell$ to $z_2^\ell$ on the $z$-axis, with the following current distribution:

$$I_\ell(z) = \frac{I_1^\ell \sin k(z_2^\ell - z) + I_2^\ell \sin k(z - z_1^\ell)}{\sin kd'}$$ (12)

The current entering the segment at the bottom is $I_2^\ell = I(z_1^\ell)$, and the current leaving the segment at the top is $I_2^\ell = I(z_2^\ell)$.

From equations (11) and (12), the far-zone field of the wire monopole is given by

$$E^w_\theta (r, \theta, \phi) = C \sum_{\ell=1}^{L} I_1^\ell \left\{ \exp(j k z_2^\ell \cos \theta) - A \exp(j k z_1^\ell \cos \theta) \right\}$$
$$+ \Re \left[ \exp(-j k z_2^\ell \cos \theta) - B \exp(-j k z_1^\ell \cos \theta) \right]$$
$$+ C \sum_{\ell=1}^{L} I_2^\ell \left\{ \exp(j k z_1^\ell \cos \theta) - B \exp(j k z_2^\ell \cos \theta) \right\}$$
$$+ \Re \left[ \exp(-j k z_1^\ell \cos \theta) - A \exp(-j k z_2^\ell \cos \theta) \right]$$ (13)

where

$$A = \cos (kd) + j \cos \theta \sin (kd)$$ (14)

$$B = \cos(kd) - j \cos \theta \sin (kd)$$ (15)

2-7
\[ C = \frac{j\eta_0 \exp(-jkr)}{4\pi r \sin(kd) \sin \theta} \]  

(16)

On the lowest wire segment \( t = 1 \), the current at the bottom is \( I_1' = I_1 \). On the highest segment \( t = L \) the endpoint currents are \( I_1' = I_{N} \) and \( I_2' = 0 \), where \( N \) denotes the number of equations and the number of unknowns in the moment-method solution for the monopole/disk antenna.

### 2.3 THE FIELD FROM THE DISK GROUND PLANE

The electric current density \( J_p(\rho) \) on the perfectly conducting circular disk is radially directed and independent of the azimuthal angle \( \phi \). The disk lies in the plane \( z = z_0 \). If \((\zeta', \phi', z_0)\) denotes the cylindrical coordinates of a source point on the disk, the far-zone disk field is obtained from equations (6), (7), and (10) as follows:

\[
E_\theta^d(r, \theta, \phi) = \frac{-j\eta_0 \exp(-jkr) \cos \theta}{4\pi r} \left[ \exp(jkz_0 \cos \theta) - \Re \exp(-jkz_0 \cos \theta) \right]
\]

(17)

\[
\int_0^\pi J_p(\rho') \cos \phi'' \exp(jk\rho' \cos \phi'' \sin \theta) \rho' d\phi'' d\rho'
\]

\[
\phi'' = \phi' - \phi
\]

(18)

Since the disk current density \( J_p \) is independent of \( \phi' \), one integration can be evaluated as follows:

\[
\int_0^\pi \cos \phi \exp(jx \cos \phi) d\phi = 2\pi j J_1(x)
\]

(19)
where \( J_1(x) \) denotes the Bessel function. Beginning at this point, it is convenient to let \( \rho \) (instead of \( \rho' \)) denote the radial coordinate of a source point on the disk. From equations (17) and (19), the far-zone field of the circular disk is

\[
E_\phi^d = 0.5 \eta_0 \left[ \exp(-jkr)/r \right] \cos \theta
\]

\[
\left[ \exp(jkz_0 \cos \theta) - \Re \exp(-jkr \cos \theta) \right] \int_\rho^a \rho J_0(\rho) J_1(k\rho \sin \theta) \, d\rho
\]  

(20)

The perfectly conducting circular disk is divided into \( M \) concentric annular zones. A typical zone (zone \( m \)) has an inner radius \( \rho_1^m \), an outer radius \( \rho_2^m \), and a width \( d = \rho_2^m - \rho_1^m = (a - b)/M \). Let \( I_1^m \) denote the electric current entering the zone at \( \rho_1^m \), and \( I_2^m \) the current leaving at \( \rho_2^m \). Then the electric surface current density on this zone is

\[
J_\rho^m(\rho) = \frac{I_1^m \sin k(\rho_2^m - \rho) + I_2^m \sin k(\rho - \rho_1^m)}{2\pi \rho \sin kd}
\]  

(21)

From equations (20) and (21), the far-zone field of the circular disk is given by

\[
E_\phi^d(r, \theta, \phi) = \frac{\eta_0 \exp(-jkr) \cos \theta}{4\pi r \sin(kd)} \left[ \exp(jkz_0 \cos \theta) - \Re \exp(-jkr \cos \theta) \right]
\]

\[
\cdot \sum_{m=1}^M \int_{k\rho_1^m}^{k\rho_2^m} \left[ I_1^m \sin k(\rho_2^m - \rho) + I_2^m \sin k(\rho - \rho_1^m) \right] J_1(k\rho \sin \theta) \, d(k\rho)
\]  

(22)

On the first zone \( (m = 1) \), the endpoint currents are \( I_1^1 = -I_1 \) and \( I_2^1 = I_2 \). On the last zone \( (m = M) \), the endpoint currents are \( I_1^M = I_M \) and \( I_2^M = 0 \). Numerical integration techniques are required in evaluating this expression.
2.4 THE FIELD FROM THE MAGNETIC FRILL

The perfectly conducting circular disk and the coaxial-fed monopole are replaced (via Schelkunoff’s equivalence principle) with equivalent electric and magnetic surface currents radiating in free space over the flat Earth. The equivalent magnetic surface-current density, derived in section 2.4 of reference 12, is given by:

\[ M_\phi = \begin{cases} \frac{-V}{\ln(b_1/b)} \ln(b_1/b), & b \leq \rho \leq b_1 \\ 0, & \rho \text{ elsewhere} \end{cases} \]  

(23)

This "magnetic frill," located at \( z = z_0 \), is centered on the \( z \)-axis and has inner and outer radii of \( b \) and \( b_1 \), respectively. The antenna is considered to be transmitting, with a voltage generator (of \( V \) peak volts) at the terminals and the coaxial outer conductor at zero potential. The free-space field of the magnetic frill is analyzed by Tsai [14,15]. From equations (3) and (10), the far-zone field of the frill is given by

\[ E_\theta^M(r, \theta, \phi) = \frac{-jk \exp(-jkr)}{4\pi r} \left[ \exp(jkz_0 \cos \theta) + \Re \exp(-jkz_0 \cos \theta) \right] \]

\[ \cdot \int_{-\pi}^{\pi} M_\phi(\rho') \cos \phi'' \exp(jk\rho' \cos \phi'' \sin \theta) \rho' d\phi'' d\rho' \]  

(24)

Since the magnetic current density is independent of \( \phi' \), one integration can be performed with the aid of equations (19) and (23) to obtain

\[ E_\theta^M = \frac{-kV \exp(-jkr)}{2r \ln(b_1/b)} \left[ \exp(jkz_0 \cos \theta) + \Re \exp(-jkz_0 \cos \theta) \right] \cdot \int_{b}^{b_1} J_1(k\rho \sin \theta) \, d\rho \]  

(25)
The final integration is performed as follows:

\[ \int J_1(\beta x) \, dx = -J_0 \beta x / \beta \]  

(26)

Thus, the field of the magnetic frill is given by

\[
E^M_\theta = \frac{V \exp(-jkr)}{2r \ln(b_1/b)} \left[ \exp(jkz_o \cos \theta) + \Re \exp(-jkz_o \cos \theta) \right] 
\cdot \left[ J_0(kb \sin \theta) - J_0(ka \sin \theta) \right] / \sin \theta
\]  

(27)

This expression can be simplified with the following:

\[ J_0(x) = 1 - x^2/4, \quad x << 1 \]  

(28)

From equations (27) and (28), the far-zone field of the magnetic frill is given by

\[
E^M_\theta(r, \theta, \phi) = \frac{k^2 V(b^2 - b_1^2) \exp(-jkr)}{8r \ln(b_1/b)} 
\cdot \left[ \exp(jkz_o \cos \theta) + \Re \exp(-jkz_o \cos \theta) \right] \sin \theta, \quad kb << 1
\]  

(29)

2.5 THE TOTAL FAR-ZONE FIELD

The total far-zone field \( E_\theta(r, \theta, \phi) \) defined by equation (10) in the free-space region is the sum of the fields from the monopole element, the disk ground plane, and the magnetic frill. Accordingly, the total far-zone field in the free-space(air) region is given by
\[ E_\theta(r, \theta, \phi) = E_\theta(r, \theta) = E_\theta^w(r, \theta) + E_\theta^d(r, \theta) + E_\theta^M(r, \theta) \]  

(30)

where \( E_\theta^w, E_\theta^d, E_\theta^M \) are given by equations (13), (22), and (29), respectively. The fields \( E_\theta, E_\theta^w, E_\theta^d, \) and \( E_\theta^M \) are uniform with azimuthal angle \( \phi \) because of the azimuthal symmetry of the antenna geometry in figure 1.

Consider now the cases where the Earth medium either is lossy (\( \sigma > 0 \)) or is free space (\( \sigma = 0, \ \epsilon_r = 1 \)). The total far-zone radiated power \( P_r \) is given by

\[
P_r = \left\{ \begin{array}{ll}
\frac{(\pi/\eta_o)^{1/2}}{\int_0^{\pi/2} |E_\theta(r, \theta)|^2 r^2 \sin \theta \ d\theta, \ \sigma > 0} \\
\frac{(\pi/\eta_o)^{1/2}}{\int_0^{\pi} |E_\theta(r, \theta)|^2 r^2 \sin \theta \ d\theta; \ \sigma = 0}
\end{array} \right.
\]

(31)

where \( E_\theta(r, \theta) \) = the far-zone field (in the free-space region) given by equation (30).

\[
\eta_o = (\mu_o/\epsilon_o)^{1/2} = \text{free space wave impedance (ohms)}
\]

For the case of \( \sigma > 0 \), the integrand in equation (31) is integrated over only the hemisphere above the Earth because the field in lossy Earth, relative to that in free space, approaches zero at large radial distances \( r \).

The antenna directivity \( d(\theta) \) expressed as a numeric is given by

\[
d(\theta) = 2\pi r^2 |E_\theta(r, \theta)|/(\eta_o P_r)
\]

(32)
The antenna directivity $D(\theta)$, expressed in decibels, is given by

$$D(\theta) = 10 \log_{10} d(\theta) \text{ (dB)}$$  \hspace{1cm} (33)$$

The input power $P_{in}$ to the monopole element is given by

$$P_{in} = (1/2) \ Re[V(0)I^*(0)]$$  \hspace{1cm} (34)$$

where $V(0) = \text{Peak input voltage (volts)}$. The input voltage $V(0)$ is usually set equal to 1 volt in the moment-method analysis.

$I^*(0) = \text{Conjugate of the peak input current } I(0) \text{ at the base of the monopole element}$. This current is solved for by the moment-method analysis in reference 1.

The input impedance $Z_{in}$ is given by

$$Z_{in} = R_{in} + j X_{in} = V(0)/I(0)$$  \hspace{1cm} (35)$$

where $R_{in}$ and $X_{in}$ are the input resistance and reactance, respectively.

The antenna radiation resistance $R_{rad}$ is defined as

$$R_{rad} = 2P_r/|I(0)|^2$$  \hspace{1cm} (36)$$

The antenna radiation efficiency $\eta$ is defined as

$$\eta = P_r/P_{in} = \left[1 + (R_{rad}/R_{in})\right]^{-1}$$  \hspace{1cm} (37)$$
For the case of free-space ($\sigma = 0, \varepsilon_r = 1$), the radiation efficiency is equal to unity because the monopole element and the disk ground plane conductivities are assumed to be infinite.
SECTION 3
NUMERICAL RESULTS

Numerical evaluation of the far-zone field, directivity, radiation resistance, and radiation efficiency is executed by Richmond's computer program RICHMOND4 written in FORTRAN 77, with double precision for use on a DEC VAX computer. The program RICHMOND4 uses subroutines from Richmond's computer program RICHMOND3 that determines the current distributions on the monopole element and disk ground plane, as well as the input current $I(Z_0)$ and input impedance $Z = V/I(z_0)$. Brief descriptions, listings, and sample outputs by Richmond of programs RICHMOND3 and RICHMOND4 are given in appendices A and B, respectively. Programs RICHMOND3 and RICHMOND4 are extensions of programs RICHMD1 and RICHMD2, respectively, described in reference 12 for a monopole element on a disk ground plane in free space.

Examples of numerical results are presented here for a thin, quarter-wave monopole element on a small to moderately large disk ground plane resting on medium dry ground at 15 MHz in the high-frequency band ($b/\lambda = 10^{-6}$, $h/\lambda = 0.25$, $2\pi a/\lambda = 0$ to 8 wavenumbers, $z_0 = 0$, $\varepsilon_r = 15$, $\sigma = 0.001$ S/m, $\tan\delta = 60\lambda\sigma/\varepsilon_r = 0.08$). More extensive results, in the form of an atlas of computer plots, are presented in reference 21 as a function of Earth classification. The coaxial line feed ($b_1/b = 3.5$) has a negligible effect on the far-zone field and input current because its equivalent magnetic frill of outer diameter $2b_1/\lambda$ ($= 7 \times 10^{-6}$ wavelengths) has a radiation resistance that is small compared to that of the monopole element of length $h/\lambda$ ($= 0.25$ wavelengths). In the numerical results, the monopole element was divided into four segments. The disk was segmented into equal-width annular zones, whose numbers varied from seven for $ka = 0.025$, 0.25, 0.50; 16 for $ka = 0.75$ through 5.25; 17 for $ka = 5.5$; 18 for $ka = 5.75$ and 6.00; 19 for $ka = 6.25$; 20 for $ka = 6.5$; 21 for $ka = 6.75$, 7.0; 22 for $ka = 7.25$; 23 for $ka = 7.50$; and 24 for $ka = 7.75$ and 8.0. Results are compared with those for a perfect ground plane ($\varepsilon_r = 1.0$, $\sigma = \infty$) and for an Earth permittivity equal to that of free space ($\varepsilon_r = 1.0$, $\sigma = 0$). The results for perfect ground, medium dry ground, and free space are identified in the following figures as Case 1, Case 5, and Case 11, respectively.
The elevation numeric directivity patterns for disk radii $2\pi a/\lambda = 0.025, 3.0, 4.0, 5.0,$ and 6.5 wavenumbers are shown as polar plots on the same linear scale in figures 2 through 6, respectively. In the presence of Earth (Case 5), the directivity patterns are approximately independent of disk radius. The Earth softens the edge of the ground plane and minimizes changes in directive gain resulting from ground plane edge diffraction. The peak directivity (see figure 7) is within 0.5 dBi of that for a perfect ground plane. The direction of peak directivity (see figure 8) is approximately $30^\circ$ above the horizon with variations of less than $4^\circ$ for ground plane radii $0 \leq 2\pi a/\lambda \leq 8$ wavenumbers. The directivity at angles of incidence near the horizon (see figures 9 through 13) for $0 \leq 2\pi a/\lambda \leq 8$ wavenumbers has no improvement over that with no ground plane at all and, in fact, decreases periodically with increasing disk radius by as much as 1 dB. The directivity at angles of incidence of $82^\circ, 84^\circ, 86^\circ, 88^\circ,$ and $90^\circ$ are approximately 4 dB, 5 dB, 7 dB, 13 dB, and $\infty$ dB, respectively, below the peak directivity for these disk radii.

The directivity on the horizon (see figure 13) is $-\infty$ dB because of the space wave multipath null for Earth surface reflection at a grazing angle of $0^\circ$. In actuality, the field on the radio horizon is not zero because of the leaky evanescent surface wave that is generated in the air medium in proximity to the air-Earth interface [13]. The surface wave has an evanescent field in the air-medium only, but leaks energy into the Earth medium, not into the air medium. The amplitude of the space wave in the direction of peak directivity approaches zero with increasing distance into the far-zone.

The theoretical numeric directive gain of electrically short monopole elements on ground planes resting on lossy Earth may be approximated by an expression of the form [13]

$$d_r(\theta) = \begin{cases} A \cos^m \theta \sin^n \theta; & 0 \leq \theta \leq \pi/2 \text{ rad}, \ m > 0, n > 1 \\ 0, & -\pi/2 \leq \theta < 0 \text{ rad} \end{cases}$$

(37)
$f = 15 \text{ MHz}$

$h/\lambda = 0.25, \ b/\lambda = 1.0 \times 10^{-6}, \ Z_{0}/\lambda = 0$

Case 1, Perfect Ground $(\varepsilon_r = 1.0, \sigma = \infty)$

Case 5, Medium Dry Ground $(\varepsilon_r = 15.0, \sigma = 0.001 \text{ S/m})$

Case 11, Free Space $(\varepsilon_r = 1.0, \sigma = 0)$

Figure 2. Numeric Directive Gain Polar Plot, $2\pi a/\lambda = 0.025$
f = 15 MHz

$h/\lambda = 0.25$, $b/\lambda = 1.0 \times 10^{-6}$, $z_d/\lambda = 0$

Case 1, Perfect Ground ($\varepsilon_r = 1.0$, $\sigma = \infty$)

Case 5, Medium Dry Ground
($\varepsilon_r = 15.0$, $\sigma = 0.001 \text{ S/m}$)

Case 11, Free Space ($\varepsilon_r = 1.0$, $\sigma = 0$)

Figure 3. Numeric Directive Gain Polar Plot, $2\pi a/\lambda = 3.0$
f = 15 MHz

$h/\lambda=0.25$, $b/\lambda=1.0 \times 10^{-6}$, $z_r/\lambda=0$

Case 1, Perfect Ground ($\varepsilon_r=1.0, \sigma=\infty$)

Case 5, Medium Dry Ground
($\varepsilon_r=15.0, \sigma=0.001 \text{ S/m}$)

Case 11, Free Space ($\varepsilon_r=1.0, \sigma=0$)

Figure 4. Numeric Directive Gain Polar Plot, $2\pi a/\lambda = 4.0$
f = 15 MHz

\( h/\lambda = 0.25, \ b/\lambda = 1.0 \times 10^{-6}, \ z_0/\lambda = 0 \)

Case 1, Perfect Ground (\( \varepsilon_r = 1.0, \ \sigma = \infty \))

Case 5, Medium Dry Ground
(\( \varepsilon_r = 15.0, \ \sigma = 0.001 \text{ S/m} \))

Case 11, Free Space (\( \varepsilon_r = 1.0, \ \sigma = 0 \))

---

**Figure 5.** Numeric Directive Gain Polar Plot, \( 2\pi a/\lambda = 5.0 \)
f = 15 MHz

h/\lambda = 0.25, b/\lambda = 1.0 \times 10^{-6}, z_d/\lambda = 0

Case 1, Perfect Ground ($\varepsilon_r = 1.0, \sigma = \infty$)

Case 5, Medium Dry Ground
($\varepsilon_r = 15.0, \sigma = 0.001 \text{ S/m}$)

Case 11, Free Space ($\varepsilon_r = 1.0, \sigma = 0$)

Figure 6. Numeric Directive Gain Polar Plot, \(2\pi a/\lambda = 6.5\)
Figure 7. Peak Directivity
Figure 8. Angle of Incidence of Peak Directivity

- Case 1, Perfect ground (\( \varepsilon_r = 1.0, \sigma = \infty \))
- Case 5, Medium Dry Ground (\( \varepsilon_r = 15.0, \sigma = 0.001 \) S/m)
- Case 11, Free Space (\( \varepsilon_r = 1.0, \sigma = 0 \))
Figure 9. Directive Gain at Eight Degrees above the Horizon
Figure 10. Directive Gain at Six Degrees above the Horizon
Case 1, Perfect ground (\(\varepsilon_r = 1.0, \sigma = \infty\))
Case 5, Medium Dry Ground (\(\varepsilon_r = 15.0, \sigma = 0.001 \text{ S/m}\))
Case 11, Free Space (\(\varepsilon_r = 1.0, \sigma = 0\))

Figure 11. Directive Gain at Four Degrees above the Horizon
Figure 12. Directive Gain at Two Degrees above the Horizon
Figure 13. Directive Gain on the Horizon

f = 15 MHz

Case 1, Perfect ground (E_r = 10, σ = ∞)
Case 2, Medium Dry Ground (E_r = 15.0, σ = 0.001 S/m)
Case 3, Free Space (E_r = 10, σ = 0)

Normalized Disk Ground Plane Radius, 2πr/λ (Wavenumbers)
The exponents $m$ and $n$ are chosen to yield a peak directivity in a desired direction, and a null at $\theta = 0$ and $\pi/2$ radians. The coefficient $A$ is chosen to satisfy the condition

$$\frac{2\pi\pi/2}{(1/4\pi)} \int_0^{\pi/2} \int_0^\infty d_r(\theta) \sin \theta \, d\phi = 1$$

Accordingly,

$$A = 2\sqrt{\frac{\pi^2}{\int_0^{\pi/2} \cos^m \theta \sin^{n+1} \theta \, d\theta}}$$

The directivity of equation (37) has a null in the direction of zenith, and the horizon has a peak directivity comparable to that for a perfect ground plane.

An analytical expression that approximates the directivity obtained by numerical methods for medium dry ground and $2\pi a/\lambda = 3$ (see figure 14) is given by

$$d_r(\theta) = \begin{cases} 10 \cos \theta \sin^3 \theta; & 0 \leq \theta \leq \pi/2 \text{ rad} \\ 0; & -\pi/2 \leq \theta < 0 \text{ rad} \end{cases}$$

In the absence of Earth (Case 11), the directivity patterns (see figures 2 through 6) are strong functions of the disk radius because ground-plane edge diffraction is more pronounced. The peak directivity (see figure 7) varies from approximately 2 dBi to 5 dBi. The angle of incidence of peak directivity (see figure 8) varies from $0^\circ$ to $32^\circ$. The large changes in angle of peak directivity at $2\pi a/\lambda = 5.5$ wavenumbers do not represent significant changes in peak directivity because of the broad 3-dB beamwidth of the directivity pattern. The jump in angle of peak directivity between $2\pi a/\lambda = 5.5$ and 5.75 wavenumbers corresponds to a change in beamshape (compare figures 5 and 6). The directivity on the horizon (see figure 13) varies from 1.88 dBi for $2\pi a/\lambda = 0$ wavenumbers to the asymptotic value of -0.88 dBi for large disk ground planes of finite radius.
Figure 14. Numeric Directive Gain of a Quarter-Wave Element on a Disk Ground Plane Resting on Medium Dry Ground, $2\pi a/\lambda = 3.0$
(a) Richmond's Method-of-Moments; (b) $10 \cos \theta \sin^3 \theta$
The radiation resistance (see figure 15) increases aperiodically with increasing disk radius. The aperiodicity is more apparent in the absence of Earth because ground-plane edge diffraction is more pronounced. The radiation efficiency (see figure 16) increases monotonically with increasing disk radius in the presence of Earth: from 0.21 for $2\pi a/\lambda = 0$, to 0.69 for $2\pi a/\lambda = 8$, and to 1.0 for $2\pi a/\lambda = \infty$. In the absence of Earth, the radiation efficiency is equal to unity because the monopole element and disk are assumed to be of infinite conductivity. The reason why the radiation efficiency is so small for small disk ground planes in close proximity to Earth, regardless of whether the Earth is lossy ($\sigma > 0$) or is a pure dielectric ($\sigma = 0$), is because most of the available input energy is directed into the Earth by the leaky evanescent surface wave generated by the spherical wave source (the monopole element) in the air medium in proximity to the air-Earth interface [13]. Richmond’s method-of-moments model, in solving for the input current $I(z_0)$ indirectly, includes the surface wave and its affect on the far-zone radiation resistance and radiation efficiency. Although this paper is restricted to the calculation of the far-zone field above the Earth, Richmond’s moment method analysis for the element and disk current distributions can also be used to calculate the near-zone field including that of the surface wave. This latter effort has not yet been undertaken.
Figure 15. Radiation Resistance

- Case 1: Perfect ground ($\varepsilon_r = 1.0$, $\sigma = \infty$)
- Case 5: Medium dry ground ($\varepsilon_r = 15.0$, $\sigma = 0.001$ S/m)
- Case 11: Free space ($\varepsilon_r = 1.0$, $\sigma = 0$)

f = 15 MHz

$\tilde{h}/\lambda = 0.25$, $\tilde{h}/\lambda = 1.0 \times 10^{-6}$, $z_0/\lambda = 0$

Normalized Disk Ground Plane Radius, $2\pi a/\lambda$ (Wavenumbers)
Figure 16. Radiation Efficiency
SECTION 4
VALIDATION OF NUMERICAL RESULTS

Several approaches have been used in validating the numerical results from the programs RICHMOND3 and RICHMOND4. These approaches include validation by comparison with results from the limiting case of disk ground planes in free space; the limiting case of ground planes of zero extent in proximity to Earth; the limiting case of a perfect ground plane of infinite extent; Wait-Surtees model for input impedance; Wait-Walters model for gain; and the Numerical Electromagnetics Code (NEC) for radiation efficiency.

4.1 LIMITING CASE OF DISK GROUND PLANES IN FREE SPACE

In the limiting case of disk ground planes in free space, numerical results from programs RICHMOND3 and RICHMOND4 agree with results from programs RICHMD1 and RICHMD2. The method-of-moments programs RICHMD1 and RICHMD2, for a monopole element on a disk ground plane in free space, have received extensive validation [12]. In reference 12, numerical results for electrically thin monopole elements were compared with results from Brillouin-Stratton induced electromotive force (EMF) method for ground planes of zero extent; Bardeen's integral equation method for ground-plane radii $0 \leq ka \leq 2.75$ wavenumbers; Leitner-Spence method of oblate spheroidal wave functions for ground plane radii $3.0 \leq ka \leq 6.5$ wavenumbers; Awadalla-McClean moment method combined with the geometric theory of diffraction for ground-plane radii $8.5 \leq ka < \infty$ wavenumbers; and the method of images for $ka = \infty$. Consistent and excellent agreements of results were achieved by the RICHMD1 and RICHMD2 programs.

4.2 LIMITING CASE OF GROUND PLANES OF ZERO EXTENT

In the limiting case of ground planes of zero extent in proximity to Earth, program RICHMOND4 results for the directivity of a quarter-wave monopole element with a disk ground plane of radius $ka = 0.025$ wavenumber resting on medium dry Earth (see Case 5 of figure 2) were compared with results for $ka = 0$ from a Fresnel reflection model (MITRE 4-1).
Program MODIFIED IMAGES) and Lawrence Livermore Laboratory's method-of-moments program NEC-3 using the Sommerfeld option. Programs RICHMOND4, MODIFIED IMAGES, and NEC-3 gave identical directivity patterns with absolute values of directivity that agreed to within 0.04 dBi. The reason for the close agreement is that the directivity does not depend upon the absolute accuracy of the antenna input current.

Radiation resistance and radiation efficiency do depend upon the absolute accuracy of the antenna input current. RICHMOND4 results of radiation resistance and radiation efficiency, for the above case and various types of Earth, are compared in table 1 with results from NEC-3 (but not MODIFIED IMAGES because the omission of the surface wave in the Fresnel coefficient model affects the radiation efficiency and radiation resistance, but not directivity). The results differ by approximately 10% for radiation resistance and by more than 25% for radiation efficiency. These differences are attributable to the difference in charge density at the base of the monopole element by a factor of 4000 resulting from the different configurations of the two models [16]. In NEC-3, the current produced by the charge distribution is discharged into the Earth through an element of radius 10^-6 wavelengths, whereas in RICHMOND4 the current is discharged into the Earth through a ground plane of radius 4 x 10^-3 wavelengths. The NEC-3 results for the radiation efficiency of a quarter-wave monopole element is augmented by a 128-radial-wire ground plane of radius 0.01 wavelengths (see section 4.6). An increase in the number of monopole segments from 4 to 20 in RICHMOND4 has no significant effect in modifying the table 7 results for radiation efficiency.

4.3 LIMITING CASE OF A GROUND PLANE OF INFINITE EXTENT

In the limiting case of a perfect ground plane of infinite extent, the monopole element of length h may be modeled by the method-of-images as a free-space dipole of half-length h, but with twice the dipole input current, one-half the dipole impedance, twice the dipole directivity in the upper hemisphere, and zero times the dipole directivity in the lower hemisphere. Richmond has written a program, RICHMD6, that uses a sinusoidal-Galerkin method of moments to compute the input impedance, current distribution, and far-zone field.

4-2
Table 1. Radiation Resistance and Efficiency of a Vertical, Quarter-Wave, Monopole Element on Flat Earth; \( f = 15 \text{ MHz} \), \( b/\lambda = 1.0 \times 10^{-6} \)

<table>
<thead>
<tr>
<th>Earth Classification ((\varepsilon_r, \sigma \text{ S/m}))</th>
<th>Radiation Resistance (Ohms)</th>
<th>Radiation Efficiency (Numeric)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NEC-3</td>
<td>RICHMD4</td>
</tr>
<tr>
<td>Sea water ((70, 5))</td>
<td>34.0</td>
<td>29.5</td>
</tr>
<tr>
<td>Fresh water ((80, 3.0 \times 10^{-2}))</td>
<td>19.1</td>
<td>17.3</td>
</tr>
<tr>
<td>Wet ground ((30, 1.0 \times 10^{-2}))</td>
<td>14.5</td>
<td>13.2</td>
</tr>
<tr>
<td>Medium dry ground ((15, 1.0 \times 10^{-3}))</td>
<td>11.5</td>
<td>10.5</td>
</tr>
<tr>
<td>Very dry ground ((3, 1.0 \times 10^{-4}))</td>
<td>6.2</td>
<td>5.7</td>
</tr>
<tr>
<td>Pure water, 20°C ((80, 1.7 \times 10^{-3}))</td>
<td>19.1</td>
<td>17.3</td>
</tr>
<tr>
<td>Ice ((-1°C)) ((3, 9.0 \times 10^{-5}))</td>
<td>6.2</td>
<td>5.7</td>
</tr>
<tr>
<td>Ice ((-10°C)) ((3, 2.7 \times 10^{-5}))</td>
<td>6.2</td>
<td>5.7</td>
</tr>
<tr>
<td>Average land ((10, 5.0 \times 10^{-3}))</td>
<td>9.9</td>
<td>9.0</td>
</tr>
</tbody>
</table>

* Number of element segments, \( N = 25 \); voltage source excitation at \( N = 1 \)

** Disk ground plane radius, \( 2 \pi a / \lambda = 0.025 \) wavenumbers

*** \((\text{NEC-3} - \text{RICHMD4})/ \text{RICHMD4} \times 100\)
of the equivalent free-space dipole. A listing of program RICHMD6 is given in appendix C.

Numerical results for input impedance are in reasonable agreement with those from King-Middleton theory [17]. For example, for \( h/\lambda = 0.25 \) (corresponding to \( kh = \pi/2 \)) and \( h/b = 16.56 \) (corresponding to \( \Omega = 7 \)), RICHMD6 results for the monopole input impedance are \( Z_{in} = 46.52 + j 15.97 \) ohms which differ from the King-Middleton results of \( Z_{in} = 47.85 + j 18.50 \) ohms by 2.8 and 13.7% for input resistance and input reactance, respectively. RICHMD6 results for directivity are almost identical to the well-known results for a thin, quarter-wave monopole on a perfect ground plane [12].

4.4 COMPARISON WITH WAIT-SURTEES MODEL FOR INPUT IMPEDANCE

Program RICHMOND4 results for the input impedance of a monopole element with a disk ground plane resting on flat Earth have been compared by Richmond [1] with those obtained from a Wait-Surtees model [18]. In reference 1, the Wait-Surtees results for input reactance are inadvertently given for a disk ground plane in free space rather than for a disk ground plane on flat Earth. RICHMOND4 results for input resistance and input reactance are compared in figures 17 and 18, respectively, with those obtained from a program WAIT-SURTEES written by Richmond and based on the Wait-Surtees model. Program WAIT-SURTEES, described in Appendix D, incorporates results from program RICHMD6 for the input impedance of a monopole element on a perfect ground plane. The RICHMOND4 results are in close agreement with WAIT-SURTEES results, except at small ground-plane radii less than approximately \( ka = 1.0 \) wavenumber for which the Wait-Surtees model is not accurate. Richmond [10] has compared RICHMD1 results with WAIT-SURTEES results for the input impedance of a monopole element on a disk ground plane in free space and obtained similar agreement as above, but for ground-plane radii greater than approximately \( ka = 2.0 \) wavenumbers. The RICHMOND4 results in figure 18 for input reactance should not have a local minimum at \( ka = 0.75 \). A nonconvergent result was obtained at \( ka = 0.75 \) because of over-segmentation of the disk when the number of disk annular zones was abruptly increased from seven at \( ka = 0.5 \) to sixteen at \( ka = 0.75 \).
$f = 15 \text{ MHz}$

$h/\lambda = 0.25$, $b/\lambda = 1.0 \times 10^{-6}$, $z_0/\lambda = 0$

Case 5, Medium Dry Ground ($\varepsilon_r = 15.0$, $\sigma = 0.001 \text{ S/m}$)

Figure 17. Input Resistance
Input Reactance, $X_n$ (Ohms)

Normalized Disk Ground Plane Radius, $2\pi a/\lambda$ (Wavenumbers)

$f = 15$ MHz

$h/\lambda = 0.25$, \hspace{1em} $b/\lambda = 1.0 \times 10^{-6}$, \hspace{1em} $z_0/\lambda = 0$

Case 5, Medium Dry Ground (\hspace{1em} $\varepsilon_r = 15.0$, \hspace{1em} $\sigma = 0.001$ S/m)

Figure 18. Input Reactance
4.5 COMPARISON WITH WAIT-WALTERS MODEL FOR GAIN

Numerical results of directivity and radiation efficiency from Richmond's method-of-moments program RICHMOND4 cannot be validated against models based on Monteath's compensation theorem [5-8,19] or Sommerfeld's attenuation function [9] because those models yield only the gain (the product of directivity and radiation efficiency) rather than directivity and radiation efficiency as separate entities. Nevertheless, it is of interest to compare RICHMOND4 results for gain with those from the Wait-Walters model [6,7,8,19] based on Monteath's compensation theorem.

First consider the Wait-Walters model. The gain \( G(ka, \psi) - G(0, \psi) \) (dB) of an electrically short monopole element on a disk ground plane with radius \( ka \) wavenumbers relative to that without a disk ground plane \((ka = 0)\) is shown in figure 2 of reference 8 and in figure 23.26 of reference 19 for \( ka = 10, \epsilon_r = 9 \), and \( \sigma = 0 \). The Wait-Walters model of reference 8 computes the magnetic field intensity \( H(ka, \psi) \) with a disk ground plane as a function of the grazing angle \( \psi \) (the complement of the angle of incidence \( \theta \)) relative to that with no ground plane. At a grazing angle \( \psi = 2^\circ \), the Wait-Walters model gives a relative gain of \( G(10, 2) - G(0, 2) = 4.5 \) dB.

Now consider the Richmond model. Program RICHMOND4 results for a quarter-wave monopole element on a disk ground plane of radius \( ka = 8 \) wavenumbers on medium dry ground \((\epsilon_r = 15.0, \sigma = 0.001 \text{ S/m})\) gives a directivity at a grazing angle \( \psi = 2^\circ \), of \( D(8, 2) = -8.6 \) dBi (see figure 12) and a radiation efficiency \( \eta = 0.69 = -1.6 \) dB (see figure 16). The gain \( G(8, 2) = -8.6 \) dBi -1.6 dB = -10.2 dB; for \( ka = 0 \) and \( \psi = 2^\circ \), \( D(0,2) = -7.9 \) dBi (see figure 12) and the radiation efficiency \( \eta = 0.21 = -6.8 \) dB (see figure 16). The gain \( G(0,2) = -7.9 \) dBi - 6.8 dBi = -14.7 dBi. The relative gain \( G(8,2) - G(0,2) = -10.2 \) dBi + 14.7 dBi = 4.5 dB.

The RICHMOND4 and Wait-Walters results of 4.5 dB for relative gain are identical for these similar cases.
4.6 COMPARISON WITH NEC FOR RADIATION EFFICIENCY

Numerical results of radiation efficiency obtained from programs RICHMOND4, NEC-3, and NEC-GS are compared in figure 19 for the radiation efficiency of a quarter-wave monopole element with small ground planes on or just above medium dry Earth as a function of the ground-plane radius. RICHMOND4 results are for disk ground planes (see figure 16). NEC-3 results are for a ground plane of zero extent (see table 1). NEC-GS results are for radial-wire ground planes whose wires have a radius $b_w = 10^{-5}$ wavelengths [16,20]. The results for disk ground planes are in close agreement with those for ground planes with 128 radial wires.
Figure 19. Radiation Efficiency of a Quarter-Wave Monopole Element with Different (Zero-Extent, Radial-Wire, and Disk) Ground Planes on or just above Medium Dry Earth.
SECTION 5
CONCLUSIONS

Richmond's moment-method results, for the current distribution and input impedance of a monopole element on a disk ground plane above flat Earth, are used to obtain the far-zone field, directivity pattern, radiation resistance, and radiation efficiency. This model for a disk ground plane complements the NEC method-of-moments model of Burke, et al. for a radial-wire ground plane.

Method-of-moments models, unlike models based on Sommerfeld's attenuation function or variational models based on Monteath's compensation theorem, determine the directivity and radiation efficiency as separate entities rather than lumping them together as a product to yield the antenna gain. Other advantages of the method-of-moments models are more exact determination of current distributions; applicability to electrically small ground planes; direct determination of ground-plane edge diffraction; and avoidance of analytical restrictions on evaluating Sommerfeld's integral. The segmentation of ground planes in method-of-moments models restricts the models to ground planes that are sufficiently small so that computer computational capacity and precision are not exceeded.

The far-zone field in the free-space (air) region is determined as the sum of direct and indirect (reflected from the Earth) fields from the monopole element, disk ground plane, and the magnetic frill of the coaxial-line feed excitation. The far-zone direct fields from the monopole element and disk ground plane are determined from the method-of-moments solution for their current distributions. The far-zone indirect fields are determined using the plane-wave Fresnel reflection coefficient. The significant contribution of the surface wave to the far-zone field at or near the air-Earth interface is not considered, but is small compared to the far-zone field in the direction of peak directivity. However, the significant effect of the surface-wave in determining input current and radiation efficiency are included in the present analysis.
Examples of numerical results are presented for the directivity pattern, peak directivity, radiation resistance, and radiation efficiency of a thin, quarter-wave monopole element on small to moderately large disk ground planes (of radius 0 to 8 wavenumbers) resting on medium dry Earth. Results are compared with these for a ground plane of infinite extent and for ground planes in free space. In the presence of Earth, the directivity patterns are approximately independent of disk radius for ground-plane radii at least as large as eight wavenumbers. The peak directivity is within 0.5 dBi of that for a perfect ground plane. The direction of peak directivity is approximately 30° above the horizon. The directivity at angles of incidence of 82°, 84°, 86°, 88°, and 90° are approximately 4 dB, 5 dB, 7 dB, 13 dB, and 50 dB, respectively, below the peak directivity. The numeric directivity is given approximately by the empirical expression $10 \cos \theta \sin^3 \theta$ in the hemisphere above the Earth and by zero in the hemisphere below the Earth. The radiation efficiency increases monotonically with increasing disk radius in the presence of Earth: from 0.21 for a ground plane of zero extent to 0.69 for a ground-plane radius of eight wavenumbers.

Numerical results from Richmond's method-of-moments computer programs RICHMOND3 and RICHMOND4 for a monopole element with a disk ground plane above flat Earth are in good agreement with results known from other models in the limiting cases of disk ground planes in free space, disk ground planes of zero extent in proximity to Earth, and a perfect ground plane. RICHMOND3 results for input impedance are in good agreement with results from a Wait-Surtees variational model, except for ground-plane radii less than approximately one wavenumber for which the Wait-Surtees model is not accurate. A RICHMOND4 result for antenna gain is in agreement with a result from a Wait-Walters variational model. RICHMOND4 results of radiation efficiency are in close agreement with NEC-GS method-of-moment results for ground planes with a large number of radial wires.
LIST OF REFERENCES


RE-2
APPENDIX A

COMPUTER PROGRAM RICHMOND3 FOR THE INPUT IMPEDANCE AND CURRENT DISTRIBUTION OF A MONOPOLE ELEMENT ON A DISK GROUND PLANE ABOVE FLAT EARTH
INTRODUCTION

Appendix I presents the computer program RICHMOND3 together with all the necessary subroutines. This FORTRAN program calculates the current distribution and impedance of a monopole antenna mounted at the center of a circular disk over the flat lossy earth.


Comment statements have been inserted in the main program and the subroutines to assist the user. Only a few brief comments will be required in this Introduction.

RICHMOND3 performs all calculations with double precision. The theoretical basis for this program is presented in the above published paper, and the notation in the program corresponds with the notation in the paper with one exception. The outer radius of the disk is denoted by \( b \) in the program and by \( c \) in the paper.

In Appendix I, Table I presents the antenna impedance as calculated with RICHMOND3 on a VAX computer for the following disk radii: \( BL = 0.1, 0.2, 0.3 \) and \( 0.4 \). The antenna impedance (in free space and on a lossy flat earth) agree closely with the original calculations obtained on a DATA CRAFT computer in May 1979. Table II presents the current distributions on the monopole and the disk (in free space and on a lossy flat earth).

---

1Appreciation is expressed to The MITRE Corporation for sponsoring this report.

The computer program RICHMOND3 was developed (in single precision) in 1979 with other sponsorship.
flat earth) with $BL = 0.1$. These results also agree closely with the original calculations of May 1979.

Table III presents the antenna impedance (in free space and on a flat earth) as calculated with a VAX with single precision. Comparison with Table I indicates that the need for double precision is marginal for this case.

In the original program, subroutine CROUT was employed to solve the simultaneous linear equations. In RICHMOND3, CROUT is replaced with CMINV which employs full pivoting (on rows and columns) whereas CROUT does not pivot. On the other hand CMINV is presumably slower (in solving large matrix equations) because it inverts the matrix, whereas CROUT solves the equations without inverting.

The current distribution will be printed from CMINV if $IWCJ = 1$, but the printout will be suppressed if $IWCJ = 0$.

Diagnostic data will be printed from several subroutines if $IWZ = 1$. This printout is suppressed if $IWZ = 0$.

The integer $NPH$ controls the numerical integrations in several subroutines. $NPH$ determines the number of times the integrand is to be sampled with Simpson's rule. The value $NPH = 6$ usually gives a suitable compromise between accuracy and computational expense. A larger value will increase the expense, and it may improve the accuracy in some cases.

$TL$ denotes the thickness of the circular disk, measured in free-space wavelengths. To promote convergence of the moment method (as $NEQ$ is increased), the value $TL = AL/100$ is recommended regardless of the true thickness of the metallic circular disk. This result is rather unexpected, and the interpretation is not totally understood. Of course, it is assumed that the disk thickness $TL$ (as well as the monopole wire radius $AL$) is much smaller than the wavelength.
Appendix I. RICHMOND3 Program Listing

RICHMOND3
MONOPOLE AT CENTER OF CIRCULAR DISK OVER FLAT EARTH.

CURRENT DISTRIBUTION AND IMPEDANCE.

SEE: RICHMOND, "MONOPOLE ANTENNA ON CIRCULAR DISK OVER FLAT EARTH",

IMPLICIT REAL*8 (A-H), (P-Z)
COMPLEX*16 CJ(30), VI(30), VJ(30), VIJ(30,30), E1J(30,30)
COMPLEX*16 T1, T2, E1, D12, D21, D12, D21, D21, D21, D21, D21, D21, D21
DIMENSION FB(500), LL(30), MM(30)
DATA B0, 0.008, 8.85418533678E-12, 1.25663706144E-6/
DATA ETA, PI, TP/376.730366239, 3.14159265359, 6.28318530718/
DATA IC, ITB/30, 500/
FORMAT (1X, 7F17.8)
FORMAT (1H0)

AL = RADIUS OF WIRE IN WAVELENGTHS.
BL = LENGTH OF MONOPOLE IN WAVELENGTHS.
FMC = FREQUENCY IN MEGAHERTZ.
BAR = RADIUS RATIO FOR COAXIAL FEED CABLE.
RL = RADIUS OF CIRCULAR DISK IN WAVELENGTHS = EPSLN/TP.
NSD = NUMBER OF SEGMENTS ON THE DISK.
NSW = NUMBER OF SEGMENTS ON THE WIRE.
HDL = HEIGHT OF DISK ABOVE THE EARTH IN WAVELENGTHS.
ER = RELATIVE PERMITTIVITY OF EARTH.
SIG = CONDUCTIVITY OF EARTH, A/M.

SET IVCJ=1 TO WRITE THE CURRENT DISTRIBUTION CJ(N),
OR IVCJ=0 TO SUPPRESS WRITEOUT OF CJ(N).
SET HDL = NEGATIVE, FOR MONOPOLE-DISK IN FREE SPACE,
OR HDL = POS. FOR FREE SPACE + FLAT EARTH.
SET DTHD = NEGATIVE, TO SKIP THE GAIN CALCULATIONS.

TL = 1.0-5 FOR EPSLN GREATER THAN OR EQUAL 0.25,
= AL-D-4 FOR EPSLN LESS THAN 0.25.

AL=.003
BL=.1
FMC=.300
NSD=1
NSW=4
SIG=.001
TL=AL/100.
IVCJ=1
IVS=0
MPF=6
NSD=10
NSW=4
NEQ=NSD+NSW-1
AK=TP*AL
BK=TP*BL
HK=TP*HL
TK=TP*TL
OMEG=TP/PMC*.05
EC=OMEG*(XR-.SIG/OMEG*X0))
DDX=WR-AK/WSD
DDW=KK/WSW
RHE=A+DKE
IF(VS.EQ.DOM)GO TO 400
XKDE=.0+DDK
CDDE=DCOS(DDK)
SDDE=DSIN(DDK)
CXDE=DCOS(DW)
SDDE=DSIN(DW)
MAX=NSD+1
MS=MSD+1
CALL QSM(AK, DDX, DDD, DID, DDK, DDD, DDD, DDD, DDD, TK, TWB, MPF, 411)
EIJ(1,1)=E11

A-5
IF (NSD.LT.1) GO TO 100
S1=AK
DO 60 J=2,NSD
S2=S1+DKD
S3=S1+TDKD
T1=AK
DO 50 I=2,J
T2=T1+DKD
T3=T2+TDKD
CALL QOD (CDKD,SDKD,S1,S3,T1,T3,TK,IWE,NPH,Z12)
ZIJ(I,J)=Z12
50 T1=T1+DKD
CALL QOM (AK,DKD,DKW,CDKD,SDKD,SDK,AK,SDK,TK,IWE,NPH,Z12)
ZIJ(I,J)=Z12
60 S1=S1+DKD
100 IF (NSW.LE.1) GO TO 200
CALL SFAST (AK,DKD,DKW,MAX,IWE,EJ,CJ)
L=0
DO 160 I=MA,NEQ
DO 150 J=1,NEQ
K=J-I+1
150 ZIJ(I,J)=Z12(K)
L=L+1
ZIJ(I,J)=CJ(L)
160 CONTINUE
178 IF (NSD.LE.1) GO TO 200
Z2=0
DO 190 I=2,NSD
S2=S2+DKW
S1=S1+DKW
Rh2=AK
DO 180 I=2,NSD
Rh2=Rh2+DKD
T1=Rh2+DKD
T3=Rh2+DKD
CALL XKWTK (AK, S1, S3, T1, T3, CDK, SDK, CDKD, SDKD, IWE, Z12)
180 ZIJ(I,J)=Z12
200 CONTINUE
200 CALL GRILL (AK, BAR, DKD, DKW, NEQ, NSD, NSW, TK, VJ)
DO 210 I=1,NEQ
DO 210 J=1,NEQ
WRITE (17,1)I,J,ZIJ(I,J)
210 VJ(I,J)=ZIJ(I,J)
WRITE (17,5)
CALL CNHI (CJ,VJ,ZIJ,ICC,INWCJ,1,LLL,NSM,NEQ,DET)
T1=CY(CJ(1))
Z11=I/T11
WRITE (6,2)AL,EL,Z11
WRITE (17,2)AL,EL,Z11
WRITE (17,5)
C FOR MONOPOLE ON DISK IN FREE SPACE, SKIP TO STATEMENT 320.
C
CALL D11 (AK,BAR,DKD,DKW,EC,FB,EDK,TK,DFB,D11,DV1)
E11=DF11
IF (NSD.LE.1) GO TO 265
S2=AK+DKD
DO 260 J=2,NSD
T2=AK+DKD
NI=J
DO 250 I=2,J
CALL DEDO (AK,DEBT,DKD,DKW,EC,FB,EDK,S2,T2,TEK)
2,IFB,I12,DM3,D12,D22)
IF (1.EQ.2)P12=D12
ZIJ(I,J)=D12
A-6
I12=2
250 T2=T2+DKD
  EIJ(1,J)=P12
260 S2=S2+DKD
265 IF(NSW.LE.1)GO TO 278
  DO 276 K=1,MAX
     CALL D2W, (AK, DKD, DKD, EC, MDK, K, TK, EJL, DJL)
     J=NA+K-1
     EIJ(1,J)=DZLJ
     L=1
     DO 270 I=NA,J
     EIJ(1,J)=EJ(L)
270 L=L+1
276 CONTINUE
278 IF(NSD.LE.1)GO TO 300
  E2=.0
  DO 280 J=NA,NEQ
     E2=E2+DKD
     CALL D2WD (AK, DKD, DKD, EC, MDK, NSD, TK, E2, EJ)
  DO 280 I=2,NSD
280 EIJ(I,J)=EJ(I)
290 CONTINUE
300 DO 310 I=1,NEQ
     DO 308 J=I,NEQ
     E12=V12(I,J)
     D12=E12(I,J)
     WRITE (17,1) I,J,E12,D12
     EIJ(I,J)=E12+D12
308 CONTINUE
310 CONTINUE
     WRITE (17,5)
     VJ(1)=VJ(1)+DV1
     DO 312 J=1,NEQ
     DO 312 J=1,NEQ
312 EIJ(J,J)=EIJ(J,J)
315 CONTINUE
     CALL C2MV ((CJ, VJ, EIJ, ICC, IWCJ, 1, LLL, MDK, NEQ, DET)
     Y11=CY(1)
     W11=1./Y11
     WRITE (6,2) Z11, W11
     WRITE (17,1) NSD, NSN, AL, BL, BDL
     WRITE (17,5)
     WRITE (17,2) Z11, W11
320 CONTINUE
400 CONTINUE
500 CALL EXIT
END
<table>
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<th>NSD</th>
<th>MSW</th>
<th>AL</th>
<th>BAR</th>
<th>ER</th>
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<th>ANTENNA IMPEDANCE $Z_{11}$ (on flat earth)</th>
</tr>
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Current Distribution on Monopole on Circular Disk in Free Space.

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<td>(phase)</td>
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Current Distribution on Monopole on Circular Disk on Flat Earth.

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Antenna Impedance Z11

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\[
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\text{Antenna Impedance Z11} & \\
\text{(in free space)} & \\
R & 14.84274593 \\
X & -53.87146810 \\
\text{Antenna Impedance Z11} & \\
\text{(on flat earth)} & \\
R & 29.56653347 \\
X & -32.54060135
\end{align*}
\]
<table>
<thead>
<tr>
<th>BL</th>
<th>ANTENNA IMPEDANCE $Z_{11}$ (in free space)</th>
<th>ANTENNA IMPEDANCE $Z_{11}$ (on flat earth)</th>
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<td>26.8575</td>
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</tbody>
</table>
SUBROUTINE BES10 (XX, B, B1, ID)

B = BESSEL FUNCTION J sub 0 with real argument XX.
B1 = BESSEL FUNCTION J sub 1 with real argument XX.
SET ID = (0, 1, 2) TO CALCULATE (J sub 0, J sub 1, or both).
IMPLICIT REAL*8 (A-Z), (P-Z)
B=1.
B1=0.
IF (XX.EQ.0) RETURN
X=DABS(XX)
IF (X.GT.0.01) GO TO 10
X2=X*X
X4=X2*X2
B=1.-X2/4.+X4/64.
B1=X*(1.-X2/6.)/2.
RETURN
10 DX=X
IF (X.GE.3.) GO TO 100
C=DX*DX/9.
IF (ID.EQ.1) GO TO 20
B = (((((.21D-3*C-.39444D-2)*C+.4444479D-1)*C-.3163866)*C+.126516208D+1)*C-2.2499997) *C+1.
IF (ID.EQ.0) RETURN
20 B = (((((.1109D-4*C-.31761D-3)*C+.443319D-2)*C-.3954289D-1)*C+1.21093573)*C-.56249985) *C+.5)*DX
RETURN
100 D=3./DX
C=1./DSQRT(DX)
IF (ID.EQ.1) GO TO 120
EA=C*(((1.14476D-3*D-.72805D-3)*D+.137237D-2)*D-.9512D-4)*D-4.552740D-2)*D-.77D-6)*D+.797884560803)
FB=(((1.13558D-3*D-.29333D-3)*D-.54125D-3)*D+.262573D-2)*D-5.3954D-4)*D-.4166397D-1)*D-.785398163397+DX
B = EA*DCOS(FB)
IF (ID.EQ.0) RETURN
120 EB=C*(((.20033D-3*D+.113653D-3)*D+.249511D-2)*D+.17105D-3)*D+6.1659667D-1)*D+.15D-5)*D+.797884560803)
FB=(((.29166D-3*D+.79824D-3)*D+.74348D-3)*D-.637879D-2)*D+.7560D-4)*D+.12499612)*D-2.356134480192+DX
B1 = EB*DCOS(FB)
RETURN
END
SUBROUTINE CISI(CI, CIN, SI, X)
C
CALCULATES CI = C O S I N E I N T E G R A L, AND
IMPLICIT REAL*8 (A-H), (P-Z)
DATA GAM, P2 / .57721566, 1.57079632/
A = DABS(X)
IF (A.GT.4.) GO TO 10
IF (A.GT.1) GO TO 3
IF (A.GT.0.) GO TO 2
CI = 0
CIN = 0
SI = 0
RETURN
2 X2 = A*A
SI = X*(((0.03*X2-1.)*X2/18.+1.)
CIN = .25*X2*(((X2/45.-1.)*X2/24.+1.)
GO TO 8
3 Y = (4.-A) *(4.-A)
SI = X*(((1.753119D-9*X+1.568988D-7)*X+1.374168D-5)*X+6.939888D-4)
C+Y = 1.964882D-2*Y+4.39509D-1
CIN = A*A*(((1.386985D-10*X+1.584996D-8)*X)
C+Y = 1.725752D-6*Y+1.85999D-4*Y+4.990920D-3*Y+1.315308D-1)
C = GAM+DLOG(A)-CIN
RETURN
8 SI = DSIN(A)
Y = DCOS (A)
Z = 4./A
U = ((((((4.048069D-3*X-2.279143D-2)*X+5.515070D-2)*X-7.261642D-2)
C+6.250011D-10)*Z+2.583989D-10
V = ((((((5.106896D-3*X+2.819179D-2)*X-6.537283D-2)*Z
C+7.902034D-2)*X-4.400416D-2)*X-7.945556D-3)*X+2.601293D-2)*X
C-3.764000D-4)*Z-3.122418D-2)*Z-6.646441D-7)*Z+2.5D-1
C = Z*((SINV-Y*U)+P2)
SI = Z* (SINV-Y*U) + P2
IF (X.LT.0) SI = -SI
CIN = GAM+DLOG (A) - CI
RETURN
END
SUBROUTINE CMINV(C,V,Z,IDM,INW,112,L,M,NEQ,DET)
CMINV2 INVERTS THE MATRIX Z(I,J) AND SOLVES THE
SIMULTANEOUS LINEAR EQUATIONS TO DETERMINE C(I).
V(I) = EXCITATION COLUMN.
Z(I,J) = IMPEDANCE MATRIX.
IDM = DIMENSION OF Z(IDM,IDM) IN CALLING PROGRAM.
INW = 1 IF SOLUTION IS TO BE PRINTED.
INW = 0 IF PRINTOUT IS TO BE SUPPRESSED.
112 = 1 ON FIRST CALL, WHERE CMINV MUST INVERT Z.
112 = 2 ON LATER CALLS, IF Z(I,J) HAS ALREADY BEEN INVERTED.
L(I), M(I) = WORK ARRAYS.
NEQ = NUMBER OF SIMULTANEOUS LINEAR EQUATIONS.
DET = DETERMINANT OF THE SQUARE MATRIX.
IMPLICIT REAL*8 (A-H), (P-Z)
COMPLEX*16 C(I),V(I),S
COMPLEX*16 Z(IDM,IDM),BIGZ,ROLD,DET
DIMENSION L(2),M(2)
2 FORMAT(1X,15,F0.3,15.7,F0.1)
5 FORMAT(1R0)
N-NEQ
IF(I12.NE.1)GO TO 150
C DET=CMPLX(I.DO,0.DO)
DO 80 K=1,N
L(K)=K
M(K)=K
BIGZ=Z(K,K)
DO 20 J=K,N
DO 20 I=K,N
10 IF(CDABS(BIGZ)-CDABS(Z(I,J))))15.19,19
15 BIGZ=Z(I,J)
L(K)=I
M(K)=J
19 CONTINUE
20 CONTINUE
J=L(K)
IF(J-K)35,35,25
25 CONTINUE
DO 30 I=1,N
HOLD=Z(K,I)
Z(K,I)=Z(J,I)
30 Z(J,I)=HOLD
35 I=M(K)
IF(I-K)45,45,38
38 CONTINUE
DO 40 J=1,N
HOLD=Z(J,K)
Z(J,K)=Z(J,I)
40 Z(J,I)=HOLD
45 CONTINUE
DO 55 I=1,N
IF(I-K)50,55,50
50 Z(I,K)=Z(I,K)/(-BIGZ)
55 CONTINUE
DO 65 I=1,N
IF(I-K)60,64,60
60 Z(I,J)=Z(I,J)+Z(K,J)*Z(I,K)
64 CONTINUE
65 CONTINUE
DO 75 J=1,N
IF(I-K)70,75,70
70 Z(K,J)=Z(K,J)/BIGZ
75 CONTINUE
C DET=DET*BIGZ
Z(K,K)=1./BIGZ
80  CONTINUE
    K=N
100  K=K-1
105  IF(K).150,150,105
108  CONTINUE
    DO 110 J=1,N
        HOLD=Z(J,K)
        Z(J,K)=Z(J,I)
110  CONTINUE
    DO 125 J=1,N
        IF(J-K).100,100,125
120  J=M(K)
125  CONTINUE
    DO 130 I=1,N
        Z(K,I)=Z(J,I)
    GO TO 100
150  CHK=.0
    DO 220 I=1,NEQ
        S=DCMPLX(.0D0,.0D0)
    DO 210 J=1,NEQ
        S=S+Z(I,J)*V(J)
    SA=CDABS(S)
    IF(SA.GT.ZERO)GO TO 250
220  C(I)=S
    IF(IWR.LE.0)GO TO 250
    WRITE(17,5)
    DO 240 I=1,NEQ
        S=C(I)
        SA=CDABS(S)
        SN=SA/CHK
        PH=.0
        IF(SN.LE.0)GO TO 240
        PH=57.29578*DATAN2(DIMAG(S),DREAL(S))
    240 WRITE(17,2)I, SN, SA, PH
    WRITE(17,5)
    RETURN
250  RETURN
END
SUBROUTINE DZ11 (AK, BAR, DKD, DKM, EC, FE, HX, TK, IFB, D11, DVL)

DZ11 calculates D11 = change in self-impedance of mode 1 due to reflection from flat earth.
Also DVL = one term in voltage for mode 1.

IMPLICIT REAL*8 (A-H), (P-Z)
DIMENSION FE(1)
COMPLEX*16 FST, G, GAM, RC, EC, ZAA, ZRH, FC, EG1, EG2
COMPLEX*16 ZDD, EST, ERH, ERG, CB, D11, EDM, ZMD
COMPLEX*16 DVL, VDD, VDM, VEH, VA
DATA ETA, PI, TP/376.730366239, 3.14159265359, 6.28318530718/
BAL=DLOG (BAR)
BK=BAR*AK
CDK=DCOS (DKD)
SDK=DSIN (DKD)
DK1=DKW+TK
SDK1=DSIN (DK1)
CDK1=DCOS (DK1)
EST=DCHPLX (.000, -ETA/(4. *PI*SDK1))
FST=DCHPLX (.000, -ETA/(4. *PI*SDKD))
DBET=.1
KMX=200
IF (IFX.GT.1F) KHX=IFB
C NEXT CALCULATE F (BETA) BY INTEGRATING ACROSS THE DISK.
DO 60 K=1, KMX
DRK=PI/10.
IF (BET.GT.1.) DKDRK/BET
INT=(RH2-AK)/DRK
IF (INT.LT.10) INT=10
DRK=(RH2-AK)/INT
F=0
RK=AK+DRK/2.
C NEXT INTEGRATE ACROSS THE DISK.
DO 50 I=1, INT
BR=BET*RK
CALL BE10 (BR, BJ0, BJ1, 0)
F=F+BJ0*DCOS (RH2-RK)
50 RK=RK+DRK
FB (K)=DRK*F
60 CONTINUE
C NEXT CALCULATE ZDD.
ZK=EDK+TK
Z1=EDK
Z2=EDK+TK+DKW
ERD=DCHPLX (.000, .000)
EKM=DCHPLX (.000, .000)
VDD=DCHPLX (.000, .000)
C NEXT INTEGRATE ON BETA.
DO 80 K=1, KMX
FB=BET*DBET*K-1)
CALL BE10 (BET*AK, BA0, BA1, 0)
BETS=BET*DBET
IF (BET.GT.1.) GO TO 62
ER=DSQRT (1.-BETS)
ARG=ER+EK
EG1=DCHPLX (DCOS (ARG), -DSIN (ARG))
ARG=ER*21
EG1=DCHPLX (DCOS (ARG), -DSIN (ARG))
ARG=ER*22
EG1=DCHPLX (DCOS (ARG), -DSIN (ARG))
GAM=DCHPLX (.000, ER)
GO TO 64
62 ALP=DSQRT(BETS-1.)
   EG2=DCMPLX(.0D0,.0D0)
   ARG=ALP*Z2
   IF (ARG.LT.80.) EG2=DCMPLX(DEXP(-ARG),.0D0)
   ARG=ALP*Z2
   IF (ARG.GT.80.) GO TO 81
   EG2=DCMPLX(DEXP(-ARG),.0D0)
   EG1=DCMPLX(DEXP(-ALP*Z1),.0D0)
   GAM=DCMPLX(ALP,.0D0)
64 G=DSQRT(BETS-EC)
   RC=(GAM*EC-G)/(GAM*EC+G)
   CB=EG2-CDK1*EG1
C NEXT INTEGRATE ON RHO.
   RSJ=0.
   DRS=PI/10.
   IF (BET.GT.1.) DRS=DRS/BET
   INT=0.5/DRS
   DRS=INT/10
   DRS=DRS/INT
   KK=RSJ+DRS (RH2-RK)
   DO 70 I=1,INT
      CALL BES10 (BET*RK,BJ0,BJ1,1)
      RSJ=RSJ+BJ1*DSIN(RH2-RK)
   70 RR=RSJ+DRS
   RSJ=RSJ+RR
   ERM=DDT*RC/EC
   ERM=DDT*RC/EC
   CALL BES10 (BET*RK,BJ0,BJ1,0)
   RSJ=RSJ+BJ1*DSIN(RH2-RK)
   DO 80 K=1,INT
      F=F*B(K)
      BA=BET*AK
      CALL BES10 (BA,BJ0,BJ1,0)
      BET=BET*AK
      IF (BET.GT.1.) GO TO 82
      HR=DSQRT(1.-BETS)
      ARG=HR*HDK
      EG2=DCMPLX(DCOS(ARG),-DSIN(ARG))
      ARG=HR*Z1
      EG1=DCMPLX(DCOS(ARG),-DSIN(ARG))
      ARG=HR*Z2
      EG2=DCMPLX(DCOS(ARG),-DSIN(ARG))
      GAM=DCMPLX(.0D0,HE)
      GO TO 84
   80 CONTINUE
   ERM=DDT*ERD*DCMPLX(.0D0,-ETA/(4.*PI*SDK*SDK))
   ERM=DDT*ERD*DCMPLX(.0D0,-ETA/(4.*PI*SDK*SDK))
   ERM=DDT*ERD*DCMPLX(.0D0,-ETA/(4.*PI*SDK*SDK))
   ZDD=DDT*VDD/(2.*RA*SDK)
C NEXT CALCULATE ZDM BY INTEGRATING ON BETA.
   Z1=RA+TK
   Z2=Z1+DRN
   ZDM=DCMPLX(.0D0,.0D0)
   DO 90 K=1,INT
      BET=RA-BET (K-1)
      F=F*B(K)
      BA=BET+AK
      CALL BES10 (BA,BJ0,BJ1,0)
      BET=BET+AK
      IF (BET.GT.1.) GO TO 82
      HR=DSQRT(1.-BETS)
      ARG=HR*HDK
      EG2=DCMPLX(DCOS(ARG),-DSIN(ARG))
      ARG=HR*Z1
      EG1=DCMPLX(DCOS(ARG),-DSIN(ARG))
      ARG=HR*Z2
      EG2=DCMPLX(DCOS(ARG),-DSIN(ARG))
      GAM=DCMPLX(.0D0,HE)
      GO TO 84
   82 ALP=DSQRT(BETS-1.)
   EG2=DCMPLX(.0D0,.0D0)
   ARG=ALP*Z2
   IF (ARG.LT.80.) EG2=DCMPLX(DEXP(-ARG),.0D0)
   ARG=ALP*Z2
   IF (ARG.GT.80.) GO TO 92
   EG1=DCMPLX(DEXP(-ARG),.0D0)
   EG2=DCMPLX(DEXP(-ALP*Z1),.0D0)
   GAM=DCMPLX(ALP,.0D0)
84 G=DSQRT(BETS-EC)
   RC=(GAM*EC-G)/(GAM*EC+G)
IF (K.GT.1) GO TO 86
PC = (DCMPLX (1.D0, 2.D0*DYN) *EG2 - DCMPLX (CDE, SDK) *EG1) / 4.
GO TO 90
86 PC = (GM*SDK-CDK) *EG1 + EG2 / BETS
90 EDH = EDN - BET + RC * F_B / BJ0 * EG2 * PC
92 EDN = DBET * FST * ZDM / SDK

C NEXT CALCULATE ZHR BY INTEGRATING ON H.
VHH = DCMPLX (.0D0, .0D0)
ZHH = DCMPLX (.0D0, .0D0)
DH = DBET
NS = 1./DH
DH = 1./NS
HN = DH / 2.
DO 100 I = 1, NS
BETH = 1. - HR * HR
BETB = DSQRT (BETS)
CALL BE110 (BET * AK, B0, B1, 0)
CALL BE110 (BET * BK, BB0, BB1, 0)
GAM = DCMPLX (.0D0, HR)
G = DSQRT (BETS - EC)
RC = (GAM * EC - G) / (GAM * EC + G)
EG2 = DCMPLX (DCOS (HR * SDK), - DSIN (HR * SDK))
EG1 = DCMPLX (DCOS (HR * Z1), - DSIN (HR * Z1))
EG2 = DCMPLX (DCOS (HR * Z2), - DSIN (HR * Z2))
PC = (GM * SDK - CDK) * EG1 + EG2 / BETS
HR = HR + DH
VHH = VHH + RC * (BA0 - BB0) * BA0 * EG1 * PC
100 ZHH = ZHH + RC * BA0 * BA0 * (EG2 - CDK1 * EG2) * PC
VHH = VHH + DCMPLX (.0D0, - 1.0D0 / (2.0D0 * B0 * SDK))
VHH = VHH + ETA / (4. * PI * SDK * SDK)
C NEXT CALCULATE ZAA BY INTEGRATING ON ALP.
DA = DBET
ALP = DA / 2.
VAA = DCMPLX (.0D0, .0D0)
ZAA = DCMPLX (.0D0, .0D0)
DO 110 I = 1, MX
BETS = ALP + ALP + 1.
BETB = DSQRT (BETS)
CALL BE110 (BET * AK, B0, B1, 0)
CALL BE110 (BET * BK, BB0, BB1, 0)
G = DSQRT (BETS - EC)
RC = (ALP * EC - G) / (ALP * EC + G)
EA2 = 0.
ARG = ALP * Z1
IF (ARG < 80.) EA2 = DEKF (- ARG)
ARG = ALP * Z1
IF (ARG > 80.) GO TO 112
EA1 = DEKF (- ARG)
EA2 = DEKF (- ALP * SDK)
P = (ALP * SDK - CDK) * EA1 + EA2 / BETS
VAA = VAA + RC * (BA0 - BB0) * BA0 * EA1 + P
ZAA = ZAA + RC * BA0 * BA0 * (EA2 - CDK1 * EA2) + P
110 ALP = ALP + DA
112 ZAA = DA * BET * ZAA / SDK
VAA = DA * VAA / (2. * B0 * SDK)
END = ZAA + EHR
EDM = EDM + END
DV1 = VAA + VDD + VHH
D11 = EDD + EHN
RETURN
END

A-17
SUBROUTINE D2DD(AK, DBET, DKD, DRK, EC, FB, HDK, S2, T2, TK)
D2DD CALCULATES D12 = CHANGE IN MUTUAL IMPEDANCE BETWEEN MODE 1
AND DISK DIPOLE MODE.
ALSO D22 = CHANGE IN MUTUAL IMPEDANCE BETWEEN TWO DISK DIPOLE MODES.
IMPLICIT REAL*8 (A-H), (P-Z)
DIMENSION KB(1)
COMPLEX*16 D12, D22, EC, GMT, GAM, G, RC, EGS, EDD, EDW, EZD, ES1, ES2, FC, QDN
SDKD=DSIN(DKD)
CDDK=DCOS(DKD)
S1=S2-SDKD
S3=S2+SDKD
T1=T2-SDKD
T3=T2+SDKD
QDN=DCHPLX(.000,-ETA/(4.*PI*SDKD*SDKD))
IF(ZL2.GT.1)GO TO 62
DBET=.1
KDQ=200
IF(KMX.GT.IFB)KDQ=IFB
C NEXT CALCULATE F(BETA) BY INTEGRATING ACROSS THE ANNULAR DISK.
DO 60 K=1,KMX
DRK=PI/10.
DBET=DBET+(K-1)
IF(BET.GT.1.)DBRE=DRK/BET
INT=DKD/DRK
IF(INT.LT.10)INT=10
DRK=DKD/INT
F=0
RK=S1+DRK/2.
C NEXT INTEGRATE ACROSS THE DISK.
DO 50 L=1,2
DO 40 I=1,INT
CALL BESIO (BET', BI0, 3J, 0)
IF (L.EQ.1) FF=BI0*DCOS (S3-RK)
IF (L.EQ.2) FF=BI0*DCOS (S3-RK)
40 14-K14+014
FE(K)=DRK*FI
60 CONTINUE
62 CONTINUE
C NEXT CALCULATE D22.
ZK=2.*HDK+TK
C NEXT INTEGRATE ON BETA.
D22=DCHPLX(.000,.000)
DO 100 K=1,KMX
DRK=PI/10.
DBET=DBET+(K-1)
IF(BET.GT.1.)DBRE=DRK/BET
INT=DKD/DRK
IF(INT.LT.10)INT=10
DRK=DKD/INT
F=FB(K)
BETS=BET*BET
IF (BET.GT.1.) GO TO 72
ER=DSQRT (1.-BETS)
ARG=ER*ZK
EGZ=DCHPLX(DCOS(ANG), -DSIN(ANG))
GAM=DCHPLX(.000, HR)
GO TO 74
72 ALP=DSQRT(BETS-1.)
ANG=ALP*ZK
IF (ANG.GT.80.) GO TO 102
EGZ=DCHPLX(DEEP (-ANG), .000)
GAM=DCMPLX(ALP, .OOD)
G=CDQRT(BETS-EC)
RC= (GAM*EC-G) / (GAM*EC+G)

C NEXT INTEGRATE ACROSS THE ANNULAR DISK.
R22= .0
RK=T1+DRK/2.
DO 90 I=1,2
DO 80 L=1,INT
CALL BETS10 (BET*RK, BJ0, BJ1, 1)
IF (L.EQ.1) R22=R22+BJ1*DSIN(SR-T1)
IF (L.EQ.2) R22=R22+BJ1*DSIN(T3-RK)
80 RK=RK+DRK
90 RK=T2+DRK/2.
100 D22=D22+DRK*GAM*EC*EGZ*R22
102 D22=DRK*QST*D22
IF (I12.NE.1) RETURN
C NEXT CALCULATE D12.
ZD=DCMPLX (.ODD, .ODD)
ZDN=DCMPLX (.ODD, .ODD)
R2=AK+DK
C NEXT INTEGRATE ON BETA.
E1=ED+TK
E2=ET+DK
SDK=DSIN(DKW)
CDK=DCOS(DKW)
QDN=DCMPLX (.ODD, 0.0,-ETA/(6*PI*SDK+SDK))
DO 160 K=1,IKX
DRK=PK/10.
BET=DEB+ (K-1)
IF (BET.GT.1.) DRK=DRK/BET
INT=DED/DRK
IF (INT.LE.10) INT=10
DRK=DRK/INT
F=FB(K)
BET=DE*FB
IF (BET.GT.1.) GO TO 112
ER=DSQRT (1.-BET)
ARG=ER*TK
EGZ=DCMPLX (DCOS (ARG), -DSIN (ARG))
ARG=ER*DC
E2D=DCMPLX (DCOS (ARG), -DSIN (ARG))
ARG=ER*ET
E21=DCMPLX (DCOS (ARG), -DSIN (ARG))
ARG=ER*ET
EGZ=DCMPLX (DCOS (ARG), -DSIN (ARG))
E22=DCMPLX (.ODD, .DK)
GO TO 114
112 ALP=DSQRT (BETS-1.)
E2D=DCMPLX (.ODD, .ODD)
ARG=ALP*ET
IF (ARG.LE.80.) E22=DCMPLX (DEXP (-ARG), .ODD)
EGZ=DCMPLX (.ODD, .ODD)
ARG=ALP*ET
IF (ARG.LE.80.) E21=DCMPLX (DEXP (-ARG), .ODD)
E2D=DCMPLX (DEXP (-ALP*DK), .ODD)
GAM=DCMPLX (ALP, .ODD)
114 G=CDQRT(BETS-EC)
RC= (GAM*EC-G) / (GAM*EC+G)
C NEXT INTEGRATE ACROSS THE DISK.
R22= .0
RK=AK+DRK/2.
DO 140 I=1,INT
CALL BETS10 (BET*RK, BJ0, BJ1, 1)
R22 = R22 + BJ1 * DSIN (RK-R2)

140 RX = RX + DR X

CALL RES10 (BET*AK, BJ0, BJ1, 0)

IF (K.GT.1) GO TO 150

PC = (DCMPLX (1,DO,2*D0) * EZ2 - DCMPLX (CDK, SDK) * EZ1) / 4.

GO TO 152

150 PC = ((GAM*SDK-CDK) * EZ1 + EZ2) / BETS

152 EDD = EDD + BET*RC*F*BJO*EZD*PC

160 EDD = EDD + DRX*GAM*RC*F*EZ*R22

162 EDD = DRET*QST*EDD

EDM = DBET*QDM*EDW

D12 = EDD + EDW

RETURN

END
SUBROUTINE DZWD (AK, DKE, DKN, EC, DK, NSD, TN, ZK2, ZD12)
DZWD.
C DZWD CALCULATES ZD12 = CHANGE IN MUTUAL IMPEDANCE C BETWEEN A WIRE DIPOLE MODE AND A DISK DIPOLE MODE.
IMPLICIT REAL*8 (A-H), (P-Z)
COMPLEX*16 ZD12 (1), EC, GAM, G, RC, QC, EGZ, QST
DATA ETA, PI, TP/376.730366239, 3.14159265359, 6.28318530718/
IF (NSD.LE.1) RETURN
DO 20 I=2, NSD
20 ZD12 (I)=DCMPLX (.0D0, .0D0)
TDKD=2.*DK
SDK=DCOS (DKW)
SDK=DSIN (DKW)
Z1=IDX+TK
Z2=Z1+ZK2
ZT=Z1+ZJ2
DBET=.1
KCC=200
BET=DBET/2.
C NEXT INTEGRATE ON BETA.
DO 100 K=1, KMX
BETK=BET*BT
IF (BET.GT.1.) GO TO 42
HR=DQSRT (1.-BETK)
GAM=DCMPLX (.0D0, HR)
CDG=DCOS (GAM)
CGD=DCEPLX (DCOS (ARG), -DSIN (ARG))
GO TO 44
42 ALP=DQSRT (BETK-1.)
GAM=DCMPLX (ALP, .0D0)
AAD=DEXP (ALP*DK)
CGD= (AAD+1.)/2.
ARG=ALP*BT
IF (ARG.GT.80.) GO TO 102
EAA=DEXP (-ARG)
EAG=DCMPLX (EAA, .0D0)
44 G=DQSRT (BETK-EC) (GAM*EC-G)/(GAM*EC+G)
CALL BES10 (BET, AK, BJ0, BJ1, 0)
CC=CGD-CDK
DRK=PI/10.
IF (BET.GT.1.) DRK=DRK/BET
INT=DKD/DRK
IF (INT.LT.10) INT=10
DRK=DKD/INT
AK=AK
QC= (RC*EGZ) * (DRK*BJ0*CC)
DO 80 I=2, NSD
20 DZ12 (I)=D512 (1)+QC*VK
100 BET=BET+DBET
C NEXT INTEGRATE ACROSS THE ANNULAR DISK.
FR=.0
RK=RH1+DRK/2.
DO 60 J=1, 2
DO 70 L=1, 2
CALL BES10 (BET,RK, BJ0, BJ1, 1)
IF (L.EQ.1) CI=DSIN (RK-RH1)
IF (L.EQ.2) CI=DSIN (RH3-RK)
FR=FR+BJ1*CI
60 RK=RH1+DRK
70 RK=RH2+DRK/2.
RH1=RH1+DRK
80 DZ12 (I)=DZ12 (I)+QC*FR
100 BET=BET+DBET
17

102 QST=DCMPLX(.GD6, DBET*ETA/(TP*SDK*SDK))
DO 120 I=2, NSD
120 DE12(I)=QST*DE12(I)
RETURN
END

C
C
SUBROUTINE DZW(AX, D, DX, C, UX, TX, DZIJ, DZlJ)  
DZW CALCULATES DZIJ = CHANGE IN MUTUAL IMPEDANCE OF  
TWO WIRE DIPOLE MODES,  
AND DZlJ = CHANGE IN MUTUAL IMPEDANCE BETWEEN MODE 1 AND  
A WIRE DIPOLE MODE.  
IMPLICIT REAL*8 (A-E), (F-Z)  
COMPLEX*16 DIJ, DZlJ  
COMPLEX*16 G, GAM, EC, QC, QJ, EJH, EIJ, EZl, EZ2, PC, EDW, EGI  
DATA ETA, OR/376.730366239, 3.14159265359, 6.28318530718/  
QJ=DCMPLX(.0D0,1.0D0)  
CDK=DCOS(DRW)  
SDK=DSIN(DKW)  
SDK=DSIN(DKD)  
DO 20 I=1,J  
20 DZIJ(I)=DCMPLX(.0D0,.0D0)  
DZlJ=DCMPLX(.0D0,.0D0)  
Zl=HDK+TK  
ZJ2=Z1+J*DKW  
C NEXT INTEGRATE ON H.  
C  
DH=25/EJ2  
DD=.1  
DXX=1./DH  
IF (DXX.LE.10) DXX=10  
DH=1./DXX  
HR=DD/2.  
DO 100 K=1,KXX  
BET=1.-ER*ER  
RET=DSQRT (BETS)  
CALL BES10 (BET*AK,BJO,BJ1,0)  
GAM=DCMPLX (.0D0,HR)  
G=DSQRT (BET+EC)  
RC=(GAM*EC-G) / (GAM*EC+G)  
QC=DCOS (HR*DKW)-CDK  
FAC=B30*CC  
QC=RC* (FAC*FAC*DH/BETS)  
ARG=HR*EJ2  
EJH=DCMPLX (DCOS (ARG),-DSIN (ARG))  
QC=QC*EJH  
EJ2=EJ  
DO 80 I=1,J  
80 DZIJ(I)=DCMPLX(.0D0,.0D0)  
EIJ=DCMPLX (DCOS (ARG),-DSIN (ARG))  
ARG=HR*E2  
EZl=DCMPLX (DCOS (ARG),-DSIN (ARG))  
ARG=HR*E2  
EZ2=DCMPLX (DCOS (ARG),-DSIN (ARG))  
PC=EZ2+EZ1*DCMPLX (-CDK, HR*SDK)  
DZlJ=DIJ+RC*BJ2*CC*EJH*PC*DH  
100 HR=HR+DH  
C NEXT INTEGRATE ON ALP.  
C  
AMX=6./EJ2  
DA=.1/EJ2  
SXX=AMX/DA  
DA=DH  
DXX=200  
ALP=DA/2.  
DO 200 K=1,KXX  
BET=ALP*ALP+1.  
RET=DSQRT (BETS)  
CALL BES10 (BET*AK,BJO,BJ1,0)  
G=DSQRT (BETS-EC)  

A-23
RC = (ALP*EC-G) / (ALP*EC+G)
EAD = DEXP (ALP*DKW)
CAD = (EAD+1./EAD) / 2.
CC = CAD-CDK
FAC = BJ0*CC
QC = (QC*RC) * (FAC*FAC*DA/BETS)
ARG = ALP*BJ2
IF (ARG GT 80.) GO TO 202
AJE = DEXP (-ARG)
QC = QC*AJE
I1Z = 1
DO 180 I=1, J
EIZ = EIZ+DKW
AIZ = DEXP (-ALP*EIZ)
180 DEIZ(I) = DEIZ(I) + QC*AIZ
BJ2 = BJ0*BJ0/BETS
A21 = DEXP (-ALP*E21)
A22 = DEXP (-ALP*E22)
PR = A22 + (ALP*BDK-CDK) * A21
DEIJ = DEIJ + QC*RC*BJ2*CC*AJE*PR*DA
ALP = ALP+DA
202 RIJ = ETA / (F1+SDK*SDK)
DELO = RIJ*DE13/2.
DO 220 I=1, J
220 DEIJ(I) = RIJ*DEIJ(I)
C NEXT INTEGRATE ON BETA.
RC = 200
ET = Z1+ZJ2
RH2 = AK+BDK
D2N = DCMPDLX (.00, .00)
DBET = 1
BET = DBET / 2.
DO 300 K=1, KMX
BETS = BET*BET
IF (BET GT 1.) GO TO 262
HR = DSQRT (1./BETS)
ARG = HR*ET
EGZ = DCMPLX (DCOS (ARG), -DSIN (ARG))
GAM = DCMPLX (.00, RH)
CGD = DCOS (HR*DKW)
GO TO 264
262 ALP = DSQRT (BETS-1.)
ARG = ALP*ET
IF (ARG GT 80.) GO TO 302
A21 = DEXP (-ARG)
EGZ = DCMPLX (A21, .00)
GAM = DCMPLX (ALP, .00)
BAD = DEXP (ALP*DKW)
CGD = (EAD+1./EAD) / 2.
264 QC = (GAM*EC-G) / (GAM*EC+G)
CALL RES10 (BET*AK, BJ0, BJ1, 0)
CC = CGD-CDK
QC = RC*BJ0*CC*EGZ
DRK = Z1/10.
IF (BET GT 1.) DRK = DRK/BET
INT = DKD/DRK
IF (INT LT 10) INT = 10
DRK = DKD/INT
RK = AK+DRK/2.
RDW = 0.
C NEXT INTEGRATE ON RHO.
DO 280 I=1, INT
CALL RES10 (BET*RK, BJ0, BJ1, 1)
RDW = RDW+BJ1*DSIN (RH2-RK)
280 RK = RK+DRK

A-24
EDN=EDW*DRK*QC*RDW
BET=SET+DBET
ZDW=DBET*EDW*DCMPLX(.ODD, -ETA/(TE*SDK*SDK))
D21J=D21J+ZDW
RETURN
END
SUBROUTINE EXPJ (V1,V2,W12)

EXPJ CALCULATES W12 = EXPONENTIAL INTEGRAL WITH
LOWER LIMIT V1 AND UPPER LIMIT V2.

IMPLICIT REAL*8 (A-H, P-Z)

DIMENSION V(21), W(21), D(16), E(16)

DATA V/ 0.22284667D 00, 20.11889321D 01, 0.29927363D 01, 0.57751436D 01, 0.98374674D 01, 20.15902874D 02, 0.93307812D-01, 0.49269174D 00, 0.12155954D 01, 20.22699495D 01, 0.36676227D 01, 0.54253366D 01, 0.75659162D 01, 20.10120228D 02, 0.13130282D-02, 0.16654408D 02, 0.20776479D 02, 20.25623894D 02, 0.31407519D 02, 0.38530683D 02, 0.48026086D 02/

DATA W/ 0.45896460D 00, 20.41700083D 00, 0.11337338D 00, 0.10399197D-01, 0.26101720D-03, 20.89854791D-06, 0.21823487D 00, 0.34221017D 00, 0.26302758D 00, 20.12642582D 00, 0.40206865D-01, 0.85638778D-02, 0.12124361D-02, 0.11167440D-03, 0.64599267D-05, 0.22263169D-06, 0.42274304D-08, 20.39218973D-10, 0.14565152D-12, 0.16059493D-14/

DATA D/ 0.22495842D 02, 20.74411568D 02, -0.41431576D 03, -0.78754339D 02, 0.11254744D 02, 20.16021761D 03, -0.23862195D 03, -0.50094687D 03, -0.68487854D 02, 20.12254778D 02, -0.10161976D 02, -0.47219591D 01, 0.79729681D 01, 20.12254778D 02, -0.23862195D 03, -0.50094687D 03, -0.68487854D 02, 20.12254778D 02, -0.10161976D 02, -0.47219591D 01, 0.79729681D 01/

DATA E/ 0.21103107D 02, -0.37959787D 02, -0.97489220D 02, 0.12900672D 00, 0.17949226D 02, 0.17949226D 02, 0.17949226D 02, 0.17949226D 02, 0.17949226D 02, 0.17949226D 02, 0.17949226D 02, 0.17949226D 02, 0.17949226D 02, 0.17949226D 02, 0.17949226D 02, 0.17949226D 02, 0.17949226D 02,

Z=V1

DO 100 J=1,2

X=DREAL(Z)

Y=DHAG(Z)

E15=DCMPLX(0.D0,0.D0)

AB=CDABS(Z)

IF(X.GE.0.) GO TO 20

IF(Y.GE.0.) GO TO 20

IF(AB.EQ.0.) GO TO 80

IF(X.GT.0.) GO TO 80

IF(Y.GT.0.) GO TO 80

IF(YA-X.GE.5.5) GO TO 20

IF(XA-X.GE.5.5) GO TO 30

IF(XA-X.GE.5.5) GO TO 30

IF(XA-X.GE.5.5) GO TO 30

IF(XA-X.GE.5.5) GO TO 30

100 N=6.43.*AB

E15=1./(N-1.2)-Z/N**2

N=N+1

E15=1./(N-1.2)-Z/E15

IF(N.GE.3) GO TO 15

15 E15=E15-DCMPLX(.577216+DLOG(AB),DATAN2(Y,X))

GO TO 90

20 J1=1

J2=6

GO TO 31

30 J1=7

J2=21

31 S=DCMPLX(0.D0,0.D0)

32 S=S+DCMPLX(.XI*CF,-YA*CF)

GO TO 54

40 T3=X*Y*Y

T4=2.*X*Y

T5=X+T3-YA*T4

54 END
22

EXPJ.2

T6=X*T4+YA*T3
UC=DCMPLX(D(11)+D(12)*X+D(13)*T3+T5-E(12)*YA-E(13)*T4,
2 E(11)+E(12)*X+E(13)*T3+T6+D(12)*YA+D(13)*T4)
VC=DCMPLX(D(14)+D(15)*X+D(16)*T3+T5-E(15)*YA-E(16)*T4,
2 E(14)+E(15)*X+E(16)*T3+T6+D(15)*YA+D(16)*T4)
GO TO 52

50 T3=X*Y
T4=Z*Y
T5=Z*X
T6=X*T3+YA*T4
T7=X*T5-YA*T6
T8=X*T6+YA*T5
T9=X*T7-YA*T8
T10=X*T8+YA*T7
UC=DCMPLX(D(1)+D(2)*X+D(3)*T3+D(4)*T5+D(5)*T7+T9-(E(2)*YA+E(3)*T4
2+E(4)*T6+D(5)*T8), E(1)+E(2)*X+E(3)*T3+E(4)*T5+D(5)*T7+T10+
3(D(2)*YA+D(3)*T4+D(4)*T6+D(5)*T8))
VC=DCMPLX(D(6)+D(7)*X+D(8)*T3+D(9)*T5+D(10)*T7+T9-(E(7)*YA+E(8)*T4
2+E(9)*T6+D(10)*T8), E(6)+E(7)*X+E(8)*T3+E(9)*T5+D(10)*T7+T10+
3(D(7)*YA+D(8)*T4+D(9)*T6+D(10)*T8))

52 EC=DC/VC
S=EC/DCMPLX(X, YA)
54 EX=DCMPLX(-X)
T=EX+DCMPLX(DCOS(YA), -DSIN(YA))
E15=S*T
56 IF(X.LT.0.)E15=DCONJG(E15)
GO TO 90

80 E15=.409319/(Z+.193044)+.421831/(Z+.181025)+.147126/(Z+.55788)+
2.2063355-1/(Z+.490355)+.1074015-2/(Z+.818215)+.1586544-4/(Z+
312.7342)+.3170313-7/(Z+19.3957)
E15=E15*DCMPLX(-Z)
90 IF(JIH.EQ.1)W12=E15
100 Z=V2
Z=V2/V1
TH=DATAN2(DIMAG(V2), DREAL(V2)) -DATAN2(DIMAG(V1), DREAL(V1))
2+DATAN2(DIMAG(V1), DREAL(V1))
AB=ABS(TH)
IF(AB.LT.1.)TH=0
IF(TH.GT.1.)TH=6.2831853
IF(TH.LT.-1.)TH=-6.2831853
W12=W12-E15*DCMPLX(.000, TH)
RETURN
END

A-27
SUBROUTINE GRILL (AK, BAR, DK, DKW, NEQ, NSD, NSW, TK, VJ)
GRILL CALCULATES THE VOLTAGE COLUMN VJ(I) FOR
A MONOPOLE ON CIRCULAR DISK IN FREE SPACE.
IMPLICIT REAL*8 (A-E), (P-Z)
COMPLEX*16 EGZ, GM, GP, GI(20), VJ(1), GII, QST, WST, VJI
DATA PI, TP/3.14159265359, 6.28318530718/
IDM=20
DO 20 I=1, NEQ
20 VJ(I)=DCMPLX(.000, .000)
DO 20 IF (BAR .LE. 1.) RETURN
VJ(I)=DCMPLX(.000, .000)
DK=DKW
SDK=DSIN(DK)
CDK=DCOS(DK)
BAL=DLOG(BAR)
QST=DCMPLX(.000, .000)
BK=AK*BAR
AKS=AK*AK
BKS=BX*BK
LIN=NSW+1
IF (LIN .GT. IDM) LIN=IDM
NPH=6
NPH=2*(NPH/2)
NPP=NPH+1
PHA=.0174533*20.
DPH=PHA/NPH
PHI=0
DO 90 LPH=1, 2
WST=DPH*QST/(3.*PI)
SQR=-1.
DO 80 IPH=1, NPP
WF=3.+SQR
IF (IPH.EQ.1) WF=1.
IF (IPH.EQ.NPP) WF=1.
CH=DCOS(PH)
IF (IPH.GT.1) GO TO 40
IF (IPh.GT.1) GO TO 40
CH=DCOS(DPH/10.)
40 RS1=2.*AKS*(1.-CPH)
RS2=AKS+BKS-2.*AK*BK*CPH
RHI=DSQRT(RS1)
RR2=DSQRT(RS2)
CALL CIS1(CA, CIN, SA, RHI)
CALL CISI2(CB, CIN, SB, RR2)
GI(1)=2.*DCMPLX(CB-CA, SA-SB)
DO 50 I=2, LIM
DS=DX*(I-1)
DZ=DZ*DS
RA=DSQRT(RS1+DS)
RB=DSQRT(RS2+DS)
CALL CISI(C1, CIN, S1, RA+DS)
CALL CISI(C2, CIN, S2, RB+DS)
GP=DCMPLX(C2-C1, S1-S2)
RAM=RS1/(RA+DS)
RBM=RS2/(RB+DS)
CALL CISI(C1, CIN, S1, RAM)
CALL CISI(C2, CIN, S2, RBM)
GH=DCMPLX(C2-C1, S1-S2)
EG=DCMPLX(DCOS(DZ), DSIN(DZ))
50 GI(I)=GP*EGZ+GH/EGZ
VJ(I)=VJ(I)+WF*WST*(GI(2)-CDK*GI(1))
IF (NSW .LE. 1) GO TO 78
K1=0

IA=NSD+1
DO 60 I=IA,NEQ
K1=K1+1
K2=K2+1
K3=K3+1
IF (K3.GT.IDM) GO TO 60
GP=GI(K1)-2.*CDR*GI(K2)+GI(K3)
VJ(I)=VJ(I)+WT*WST*GP
60 CONTINUE
78 SGN=-SGN
80 PH=PH+DPH
DPH=(PI-PHA)/NPH
90 PH=PFA
CALL CISI (CA,CIN,SA,AK)
CALL CISI (CB,CIN,SB,BK)
R2=AK+BDK
SR2=DSIN(R2)
CR2=DCOS(R2)
SDKD=DSIN(DKD)
VJ1=(SR2*(CB-CA)-CR2*(SB-SA))/(2.*BAL*SDKD)
VJ(1)=VJ1+VJ(1)
IF (NSD.LE.1) RETURN
V22=(DSIN(AK)*(CB-CA)-DCOS(AK)*(SB-SA))/(2.*BAL*SDKD)
VJO(2)=DCMPX(V22, ODO)
RETURN
END

C
C

A-29
SUBROUTINE QDD (CDKD, SDKD, S1, S3, T1, T3, TK, INW, NPH, ZZ2)  Q00.1

C QDD CALCULATES ZZ2 = MUTUAL IMPEDANCE OF TWO DISK MODES.
C
IMPLICIT REAL*8 (A-H, I-O)
COMPLEX*16 ZZ2, ZZ2, ED
DATA PI, P3/3.1415926535897932384600774/ 100.0
FORMAT (1X, 8(1X, 2F10.2))
FORFAT (5X, 'PRINTOUT FROM QDD')
FORFAT (5X, 'DISK DIPOLE TO DISK DIPOLE')
IF (INW .LE. 0) GO TO 10
WRITE (17, 4) GO TO 60
WRITE (17, 5) WRITE (17, 5)

10 PHA = .0174533*2.
PHB = .0174533*20.
NPH = 2 * (NPH/2)
DPH = PHA / NPH
PH = 0
NPH = NPH + 1
ZZ2 = (.0D0, .0D0)
DO 80 IPH = 1, 3
ED = (.0D0, .0D0)
SGI = 1.
DO 70 I = 1, NPH
WF = 3*SGI
IF (I.EQ.1) WF = 1.
IF (I.EQ.NPH) WF = 1.
CPH = DCOS (PH)
IF (I.EQ.1 .AND. IPH .GT. 1) GO TO 60

C NEXT: DISK DIPOLE TO DISK DIPOLE.
C CALL SKEWS (S1, S3, T1, T3, TK, CDKD, SDKD, SDKD, SDKD, CPB, ZZ2)

60 PHD = .57.29578*PH
IF (INW .GT. 0) WRITE (17, 2) PHD, ZZ2
SGI = -SGI
PH = PH + DPH
70 ED = ED + WF*ZZ2
ZZ2 = ZZ2 + DPH*ED/P3
PH = PHA
DPH = (PHB - PHI) / NPH
IF (IPH .EQ. 1) GO TO 80
PH = PHA
DPH = (PH - PHI) / NPH

80 CONTINUE
IF (INW .GT. 0) WRITE (17, 5)
RETURN
END
SUBROUTINE QDM (AK, DKD, DKM, SDKD, SDK, S1, S3, TK, IZW, MPH, Z12)

QDM CALCULATES Z12 = MUTUAL IMPEDANCE OF DISK DIPOLE AND MODE 1.

IMPLICIT REAL*8 (A-Z), (P-S)

COMPLEX*16 ZD, Z12, Z21, ZDH, ZM, P12, ZDD, PDM

DATA PI, P3/3.141592653589, 9.42477796077/

WRITE (17, 6)
WRITE (17, 7)
WRITE (17, 8)

READ (17, 10)

IF (S1.GT.10.*AK) IDM=0
IF (S1.GT.10.*AK) IDM=0
IF (S1.GT.10.*AK) IDM=0
PE=0
ED=(.000, .000)
T2=AK+DKD
DO 40 IPH=1, 3
ZD=(.000, .000)
SGI=1.
DO 30 I=1,NPH
WF=3.*SGI
IF (I.EQ.1) WF=1.
IF (I.EQ.1) WF=1.
CPF=DCOS (PH)
30 CONTINUE

IF (NIZ.GT.0) WRITE (17, 9)

CALL ZSDM (S1, S3, AK, T2, TK, CDK, SDKD, SDK, MPH, P12)

20 PH=57.29578*PH
IF (IZW.GT.0) WRITE (17, 10) PH, P12

WRITE (17, 11)

DO 20 I=1, 2

WRITE (17, 12)

DO 10 I=1, 2

WRITE (17, 13)

DO 10 I=1, 2

WRITE (17, 14)

A-31
WF=3.4+SGI
IF (I.EQ.1)WF=1.
IF (I.EQ.1.PF.)WF=1.
CPH=DCOS (PH)
RH=AK*DSIN (PH)
IF (I.EQ.1)RH=RGN
AC=AK*CPH
V1=S1-AC
V2=S3-AC
IF (I.EQ.1 .AND. IPH.GT.1) GO TO 70
C NEXT: DISK-DIPOLE TO WIRE-MONOPOLE.
CALL EZDM (V1,V3,TK,W2,RH,CDKD,SDFK,SDK,.ODO,F12)
70 PHD=57.29578*PH
IF (IWZ.GT.0) WRITE (17,2) PHD,P12
SGI=SGI
PH=PH+DPH
80 PDM=PDM+WF*P12
EDM=EDM+DPH*PDM/P3
DPH=(FI-PHA)/LPH
90 PHA=PHA
Z12=EDM-ZDD
IF (IWZ.GT.0) WRITE (17,5)
RETURN
C NEXT: DISK-DIPOLE TO WIRE-MONOPOLE.
100 CALL EZDM (S1,S3,TX,WZ,AK,CDKD,SDFK,SDK,.ODO,EDM)
Z12=EDM-ZDD
RETURN
END
C
C
A-32
SUBROUTINE QHM(AK, DKD, DKN, CDKD, SDKD, CDK, SDK, TK, IZW, MPH, Z11)  
QHM CALCULATES Z11 = SELF IMPEDANCE OF MODE 1,  
WHICH HAS TERMINALS AT BASE OF MONPOLE.

IMPLICIT REAL*8 (A-H, P-Z), COMPLEX*16 (A-Z)

DIMENSION Z([1:32], 1:32), Z(0), X([1:32], 1:32), X(0), Y([1:32], 1:32), Y(0)

C SUBROUTINE  
QHM (AK, DKD, DKN, CDKD, SDKD, CDK, SDK, TK, IZW, MPH, Z11)  
QHM Calculates Z11 = self impedance of mode 1, which has terminals at base of monopole.

IMPLICIT REAL*8 (A-H, (P-Z))
COMPLEX*16 FDM, FMD, FMH, FDM, PMH, FMH
COMPLEX*16 ED, EDD, EDM, EMD, EDM, EMD, Z11, P11
DATA PI, PI/3.141592653589, 9.424779960777/

1 FORMAT (1X, 8F8.0)  
2 FORMAT (1X, 8F10.2)  
5 FORMAT (1X0)  
6 FORMAT (5X, 'PRINTOUT FROM QHM')  
7 FORMAT (5X, 'DISK MONPOLE TO DISK MONPOLE')
IF (IZW.LE.0) GO TO 10
WRITE (17, 6) WRITE (17, 5)
10 AK=AK*AK
DXKM=TK+DKM
CDKM=DCOS(DKM)
SDKM=DSIN(DKM)
EMM(:,0.000,0.000)
EMM(:,0.000,0.000)
EDM(:,0.000,0.000)
ZDM(:,0.000,0.000)
LPH=0
LPH=2*(LPH/2)
LPH=LPH+1
PH=0.0174532*20.
DPM=PHA/10.
PI=0
DO 44 IP=1,2
FDM(:,0.000,0.000)
FMH(:,0.000,0.000)
SGI=1.0
DO 40 I=1,LPP
WF=3.+SGI
IF (I.EQ.1) WF=1.
IF (I.EQ.LPP) WF=1.
IF (I.EQ.1) .AND. (PH.GT.PI) GO TO 38
CPI=DCOS(PH)
SPH=DSIN(PH)
DPM=2.*(1.-CPI)
R=AK*DPM/(DPM)
IF (I.EQ.1) R=R+RNM
C NEXT: WIRE MONPOLE TO WIRE MONPOLE.
CALL ESIM (-TK, DKN, .000, DKN, R, CDKM, SDKM, SDK, 1.00, PMH)
R=AK*SPH
IF (I.EQ.1) R=R+RNM
T1=AK*(1.-CPI)
T2=T1+DKM
C NEXT: DISK MONPOLE TO DISK MONPOLE.
CALL ESIM (-TK, DKN, T1, T2, R, CDKM, SDKM, SDK, .000, PMH)
S1=AK*(1.-CPI)
S2=S1+DKM
C NEXT: DISK MONPOLE TO WIRE MONPOLE.
CALL ESIM (S1, S2, TK, DKN, R, CDKM, SDKM, SDK, .000, PMH)
38 FMH=FMH+WF*FMH
PMH=PMH+WF*FMH
FMH=FMH+WF*FMH
PHD=57.29576*PH
IF (IZW.GT.0) WRITE (17, 1) PHD, PMH, FMH, FMH
SGI=SGI

A-33
40 PH=PR+DPH
    EHM=2NM+DPH*FMD/P3
    ZMD=ZMD+DPH*FMD/P3
    EDM=ZDM+DPH*FDM/P3
    RMS=.0
    DPH=(PI-PHA)/LPH
44 PH=PHA
    IF (IWE.GT.0) WRITE (17,5)
    IF (IWE.GT.0) WRITE (17,7)
    PHA=.0174533*2.
    PHB=.0174533*20.
    NPH=2*(NPH/2)
    NPH=1
    DPH=PHA/NPH
    PH=.0
    ZD=(ZDO, ZDD)
    ZD=AK+DKD
    D=50 IPH=1.3
    ZD=(ZDO, ZDD)
    1=1.
    DO 50 I=1,NPH
      WF=3.*SGI
      IF (1.EQ.1.) WF=1.
      IF (1.EQ.NPH) WF=1.
      CPH=DCOS(PH)
      WR=25*(PI.1 .AND. IPH.GT.1) GO TO 48
C NEXT: DISK MONOPOLE TO DISK MONOPOLE.
CALL LESSH (AK, S2, AK, S2, TE, CD KD, SDD, SDKD, TPH, P11)
48 PHD=57.29578*PH
    IF (IWE.GT.0) WRITE (17,2) PHD, P11
    SGH=SGI
    PH=PH+DPH
50 ZD=ZD+WF*P11
    ZD=ZD+DPH*ED/P3
    PH=PHA
    DPH=(PHB-PHA)/NPH
    IF (IPH.EQ.1) GO TO 60
    PH=PHB
    DPH=(PI-PHB)/NPH
60 CONTINUE
    E1=E2D=E3M=E3D+ZDD
    IF (IWE.GT.0) WRITE (17,5)
    RETURN
END

A-34
SUBROUTINE SKEW(S1, S3, T1, T3, R1, CD1, SD1, CD1T, SD1T, CPSI, Z12)  
SKEW CALCULATES Z12 = MUTUAL IMPEDANCE OF CENTER-FED 
NONPLANAR-SKEW SINUSOIDAL DIPOLES WITH UNEQUAL LENGTH. 
IMPLICIT REAL*8 (A-E), (P-Z) 
COMPLEX*16 Z(2,2),F(2,2),R11,R12,E1,E2,ET1,ET2,EXPA,EXPB,EGZI 
COMPLEX*16 P11,P12,P21,P22 
DIMENSION S(3),T(3) 
DATA ETA,P3,TP/376.730366239,3.14159265359,6.28318530718/
S(1) = S1 
S(2) = S2 
S(3) = S3 
T(1) = T1 
T(2) = T2 
T(3) = T3 
Z12 = (.0D0, .0D0) 
DPSI = CPSI 
CPSI = CPSI*DPSI 
IF (CPSI.GT.1.D0) CPSI = 1.D0 
SPSI = DSQRT(1.D0-CPSI) 
IF (DABS(CPSI).LT..999999) GO TO 10 
RHS = R1X*R1K 
R12 = SPSI*(T1+T3)/2. 
RHS = RHS + R12*R12 
SGN = 1. 
IF (CPSI.GT.0) GO TO 80 
SGN = -1. 
T(1) = T3 
T(2) = T2 
T(3) = T1 
GO TO 80 
10 D = RHS 
DSQ = D*D 
CD = D/DSPI 
BD = CD*DPSI 
EB = DEXP(-BD) 
EC = DEXP(-CD) 
CST = ETA/(16.*PI*SD1*SD1T) 
TA = T1 
TB = T2 
DO 70 ITT = 1,2 
IF (ITT.EQ.1) ET1 = DCMPLX(DCOS(TA),DSIN(TA)) 
IF (ITT.EQ.2) ET2 = DCMPLX(DCOS(TB),DSIN(TB)) 
TD1 = TA 
TD2 = TB 
TS1 = TD1*TD1 
TS2 = TD2*TD2 
SA = S1 
SB = S2 
DO 60 ISS = 1,2 
IF (ISS.EQ.1) RS1 = DCMPLX(DCOS(SA),DSIN(SA)) 
IF (ISS.EQ.2) RS2 = DCMPLX(DCOS(SB),DSIN(SB)) 
DO 20 K = 1,2 
DO 20 L = 1,2 
20 K(K,L) = (.0D0,.0D0) 
SI = 1A 
DO 50 I = 1,2 
FI = (-1)**I 
SDI = SI 
SI = SDI**DI 
ST1 = 2.*SDI*TD1*DPSI
ST2 = 2.*SDI*TD2*DP5I
R1 = DSQRT(DSQ+SI5+TS1-ST1)
R2 = DSQRT(DSQ+SI5+TS2-ST2)
KK = EB
DO 40 K = 1, 2
FK = (-1)**K
SK = FK*SDI
EL = EC
DO 30 L = 1, 2
FL = (-1)**L
EKL = ER*EL
XX = FK*BD+FL*CD
TL1 = FL*TD1
TL2 = FL*TD2
RRL = R1+SK+TL1
RR2 = R2+SK+TL2
CALL EXPJ(DCMPLX(XX, RR1), DCMPLX(XX, RR2), EXPA)
CALL EXPJ(DCMPLX(-XX, RR1), DCMPLX(-XX, RR2), EXPB)
F (K, L) = (EKL + FL)*FI*(EXPA*ERL+EXPB/EKL)
30 EL = 1./EC
40 EE = 1./EB
IF (I.EQ.ISS) GO TO 50
ID = SDI*DP5I
ZC = ZD
EGZI = DCMPLX(DCOS(ZC), DSIN(ZC))
RRL = R1+ED+TD1
RR2 = R2+ED+TD2
CALL EXPJ(DCMPLX(.0D0, RR1), DCMPLX(.0D0, RR2), EXPA)
CALL EXPJ(DCMPLX(.0D0, RR1), DCMPLX(.0D0, RR2), EXPB)
F (1, 1) = (.0D0, 2.D0)*SK*EXPA/EGZI
F (1, 2) = (.0D0, 2.D0)*SDK*EXPB/EGZI
50 SI = SBB
IF (I.EQ.ISS) GO TO 54
IF (ISS.EQ.1) F22 = CST*((F(2,1)+E(2,2))*ES1-E(1,2)/ES1)*ET1
A+ = (-F(2,2)-E(2,1))*ES1-E(1,1)/ES1/ET1
B+ = (F(1,1)+E(2,1))*ES2-E(1,1)/ES2/ET1
GO TO 58
54 IF (ISS.EQ.1) F21 = CST*((-F(2,1)-E(2,2))*ES1+E(1,2)/ES1)*ET2
C+ = (F(2,2)+E(2,1))*ES1-E(1,1)/ES1/ET2
D+ = (F(1,2)+E(2,1))*ES2-E(1,2)/ES2/ET2
GO TO 58
58 SBB = S2
60 SBB = S3
70 TB = T3
Z12 = F11+P12+F21+F22
RETURN
80 DO 100 I = 1, 3
SI = S(I)
CI = 1.
IF (I.EQ.2) CI = -2.*CDK
CQX = (.0D0, .0D0)
DO 90 J = 1, 3
TJ = T(J)
CJ = 1.
IF (J.EQ.2) CJ = -2.*CDK
D = T(J)-SI
R = DSQRT(RMS**DE*DE)
X = R*DZ
IF (DE.LT...0)x = RMS/(R-DZ)
CALL CIBY (COSI, CIN, SINI, X)
EP = DCMPLX(COSI, -SINI)
X = R-DZ
IF(DZ.GT.0.)X=PFS/(P+DZ)
CALL CISI (COSI, CIN, SINI, X)
EM=DCMPLX(COSI, -SINI)
EGDZ=DCMPLX(DCOS(DZ), DSIN(DZ))
90 CQX=CQX+CJ*(EP*EGDZ+EM*EGDZ)
100 Z12=Z12+CJ*CQX
Z12= SGN*ETA*Z12/((PI*SDK*SDK)
RETURN
END
SUBROUTINE SKEWS (S1, S3, T1, T3, RHK, CDK, SDK, CDKT, SDKT, CPSI, Z12)  
SKEWS CALCULATES Z12 = MUTUAL IMPEDANCE OF CENTER-FED  
COPLANAR-SKEW SINUSOIDAL DIPOLES WITH UNEQUAL LENGTHS.  
IMPLICIT REAL*8 (A-H), (P-Z)

DIMENSION (3), T(3)  
DATA ETA, PI, TP/376.730366239, 3.14159265359, 6.28318530718/
S(1)=S1  
S(2)=(S1+S3)/2.  
S(3)=S3  
T(2)=(T1+T3)/2.  
T(3)=T3  
Z12=(.0D0, .0D0)  
DPSI=CPSI  
IF (DABS (CPSI).LT..999999)GO TO 10  
RHS=RHK*RHK  
CPSS=DPSI*DPSI  
IF (CPSS.GT.1.D0) CPSS=1.D0  
SPSI=SQRT((1.D0-CPSS))  
RH2=SPSI*(T1+T3)/2.  
RHS=RHS+RH2*RH2  
SGN=1.  
IF (CPSS.GT..0D0) GO TO 60  
SGN=-1.  
T(1)=-T3  
T(2)=T(2)  
T(3)=-T1  
GO TO 60  
10 DO 50 I=1,3  
S1=S(I)  
SIS=S*I*S1  
CI=1.  
IF (I.EQ.2).CI=-2.*CDK  
DO 50 J=1,3  
TJS=TJ*TS  
R=DSQRT((SJS+TJS-2.*S*TJS)*DPSI)  
CJ=1.  
IF (J.EQ.2).CJ=-2.*CDKT  
CQX=(.0D0, .0D0)  
DO 40 K=1,2  
FK=1.-1.)**K  
DO 40 L=1,2  
FL=(-1.)**L  
XX=FK*SI+FL*TS  
XX=XXD  
XXK=DCMLX(DCOS (XX), DSIN (XX))  
XXK=RXD  
X=DABS (XXD)  
CALL CISI (COSI, CIN, SINI, X)  
IF (XXX.LT.0) SINI=-SINI  
CQX=CQX+DCMLX(COSI, -SINI)*XXK*FK*FL  
40 CONTINUE  
S12=S12+CQX*CI*CJ  
50 CONTINUE  
S12=-ETA*S12/(8.*PI*SDK*SDKT)  
RETURN  
60 DO 80 I=1,3  
S1=S(I)  
CI=1.  
IF (I.EQ.2).CI=-2.*CDK  
CQX=(.0D0, .0D0)
DO 70 J=1,3
   TJ=T(J)
   CJ=1.
   IF(J.EQ.2) CJ=-2.*CDK
   DZ=TJ-SI
   R=DSQRT(RHS+DZ*DZ)
   X=R+DZ
   IF(DZ.LT.0) X=RHS/(R-DZ)
   CALL CISI (COSI,CIN,SINI,X)
   EP=DCHPLX(COSI,-SINI)
   X=R-DZ
   IF(DZ.GT.0) X=RHS/(R+DZ)
   CALL CISI (COSI,CIN,SINI,X)
   EM=DCHPLX(COSI,-SINI)
   EGZ=DCHPLX(DCOS(DZ),DSIN(DZ))
  70  CQX=CQX+CJ*(EP*EGZ+EM/EGZ)
  80  Z12=Z12+CI*IQX
RETURN
END

C
C

A-39
C
SUBROUTINE SKEW(AK,S1,S3,T1,T3,CDX,SDK,CDKD,SDKD,IMZ,Z12) 
SKEW CALCULATES Z12 = MUTUAL IMPEDANCE OF WIRE DIPOLE
AND DISK DIPOLE.
IMPLICIT REAL*8 (A-H), (P-Z)
COMPLEX*16 P12,Z12,Q12
DATA P1,P3/3.14159265359,9.4247796077/
2 FORMAT (1X,8F10.2)
5 FORMAT (1H0)
6 FORMAT (5X, 'PRINTOUT FROM SKEW')
7 FORMAT (5X, 'WIRE DIPOLE TO DISK DIPOLE')
IF(IMZ.LE.0)GO TO 20
WRITE(17,6)
WRITE(17,7)
WRITE(17,5)
WHAT=AK/100.
NPH=6
NPF=NPH+1
PHA=0.0174533*20.
DPH=PHA/NPH
Z12=(.000,.000)
PH=0
C WIRE DIPOLE TO DISK DIPOLE.
DO 80 IPH=1,2
Q12=(.000,.000)
SGI=-1.
DO 60 I=1,NPF
WF=3.+SGI
IF(I.EQ.1)WF=1.
IF(I.EQ.NPF)WF=1.
RH=AK*DIN(PH)
IF(I.EQ.1)RH=RH+RMN
AC=AK*DCOS(PH)
V1=T1-AC
V3=T3-AC
IF(I.EQ.1)AND.IPH.EQ.2)GO TO 50
CALL SKEW (S1,S3,V1,V3,AK,CDX,SDK,CDKD,SDKD,.000,P12)
50 Q12=Q12+WF*P12
PFD=57.29578*PH
IF(IMZ.GT.0)WRITE(17,2)PFD,P12
SGI=-SGI
60 PH=PH-DPH
Z12=Z12+DPH*Q12/P3
BM=-.0
DPH=(P1-PHA)/NPH
80 PH=PHA
IF(IMZ.GT.0)WRITE(17,5)
RETURN
END
C
SUBROUTINE SPART(AK, DKD, DKW, MAX, IWZ, Z1)
SPAR calculates Z = mutual impedance of two wire dipole modes, and Z1 = mutual impedance between a wire dipole mode and mode 1.
IMPLICIT REAL*8 (A-H), (P-Z)
COMPLEX*16 EID(20), EM(20), EP(20), EPD(20), Z(1), Z1(1)
COMPLEX*16 CEM, CEP, END, EPD, END2, EPD2, S11, S22, G11, G11
DIMENSION CID(20), S1D(20), C1D(20), P(20), SM(20), SP(20)
DATA GAM,P2/.577215664,1.57079632/
DATA ETA,PI/376.727,3.14159/
DIMX=20
1 FORMAT(3X,'MUST INCREASE DIMENSIONS IN SUBROUTINE SPAR')
2 FORMAT(3X,'ACTUAL DIMENSION IDM = ',I5,6X, 'REQUIRED DIMENSION MAX2 = ',I5)
3 FORMAT(1X,6F10.2)
5 FORMAT(1X0)
6 FORMAT(5X,'PRINTOUT FROM SPAR')
7 FORMAT(5X,'FIRST: WIRE DIPOLE TO MODE ONE')
8 FORMAT(5X,'THEN: WIRE DIPOLE TO WIRE DIPOLE')
IF (MAX.LE.0) RETURN
MAX2=MAX+2
DO 14 I=1,MAX
Z1(I)=.0DO, .0D0
14 Z(I)=.0DO, .0D0
IF (MAX2.LE.IDM) GO TO 16
WRITE (17,1)
WRITE (17,2) IDM, MAX2
RETURN
16 DK=DKW
IF (IWZ.LE.0) GO TO 18
WRITE (17,6)
WRITE (17,7)
WRITE (17,8)
WRITE (17,5)
18 TDK=2.*DK
SDK=DSIN(DKD)
S11=0
S13=TDK
S21=DK
S23=3.*DK
DO 20 N=1,MAX2
I=N-1
DI=I*DK
CID(N)=DCOS(DIK)
SID(N)=DSIN(DIK)
20 EID(N)=DCMPLX(CID(N),SID(N))
CDK=DCOS(DK)
SDK=DSIN(DK)
EPD=DCMPLX(CDK,SDK)
END=DCMPLX(CDK,-SDK)
EPD2=EPD*EPD
END2=END*END
CEM=2.*CDK*END
CEP=2.*CDK*EPD
AKX=AK*AK
CSB=ETA/(8.*PI*SDK*SDK)
MPH=6
NPH=2*(NPH/2)
NPH=MPH+1
PHA=0.0174533*20.
DPH=PHA/NPH
PNA=.0
DO 100 JPN=1,2
CST=DPH*ETA/(24.*PI*SDK*SDK)
C22=DPH/ (3. *PI) 
SGN=-1. 
DO 80 IPh=1, NPP 
CPH=DCOS (PH) 
SPH=DSIN (PH) 
IF (IPh. GT. 1) GO TO 30 
IF (IPh. GT. 1) GO TO 30 
PHO=DPH/10. 
CPH=DCOS (PH) 
SPH=DSIN (PH) 
T1=AK* (1. -CPH) 
T2=T1+DKD 
Rh=AK*SPH 
DRG=2. *AK2* (1. -CPH) 
R=R-SQRT (DRG) 
R3=DRG 
WF=1. *SGN 
IF (IPH. EQ.1) WF=1. 
IF (IPH. EQ. NPP) WF=1. 
WST=WF*CST 
W22=WF*C22 
DO 40 N=1, MAX2 
I=N-1 
DZ=1. *DK 
DZS=DZ*DS 
R=R-SQRT (R3+DZS) 
ARG=R+DS 
IF (N. EQ. 1) ARG=RK 
CALL CISI (CP (N), CIN, SP (N), ARG) 
EN (N)=DCMPLX (CP (N), -SP (N)) 
IF (N. GT. 1) GO TO 30 
CH (1)=CP (1) 
SN (1)=SP (1) 
EN (1)=EP (1) 
GO TO 40 
ARG=RS/ARG 
CALL CISI (CH (N), CIN, SH (N), ARG) 
EN (N)=DCMPLX (CH (N), -SH (N)) 
CONTINUE 
R=4. *(CH (1)-2. *CP (1)-CP (2)) 
A=2. *CID (3) *{+CH (3)-2. *CH (2)+2. *CP (1)-2. *CP (2)+CP (3)} 
B=2. *SID (3) *{+SH (3)-2. *SH (2)-2. *SP (2)+SP (3)} 
C=4. *{SH (2)-2. *SP (1)+SP (2)} 
C=2. *(CID (3)-{SH (3)-2. *SH (2)-2. *SP (1)+SP (2)-2. *SP (2)+SP (3)}) 
D=2. *SID (3) *{+CH (2)-2. *CP (2)+CP (3)} 
G11=C8*DCMPLX (R, X) 
E (1)=E (1)+WST*DCMPLX (R, X) 
22. *EPD*CDK*EP (2)-EPD*EM (3) 
CALL ZSDM (E11, S11, T1, T2, Rh, CdK, SDK, SDKD, DDDO, E22) 
G11=C8*E11*E22 
S11=E11*E22 
V=57.29578*PH 
IF (IWE. GT. 0) WRITB (17, 3, FED, G11, Q11 
IF (MAX. EQ. 1) GO TO 70 
R=2. *CID (2) *{+CH (3)+3. *CH (2)-4. *CP (1)+3. *CP (2)+CP (3)} 
H=2. *SID (2) *{+SH (3)-2. *SH (2)+2. *SP (2)+SP (3)} 
F+CID (4) *{CH (4)-2. *CH (3)+CP (2)-2. *CP (3)+CP (4)} 
G+SID (4) *{SH (4)-2. *SH (3)-SP (2)-2. *SP (3)+SP (4)} 
X=2. *(CID (2) *{EM (5)-3. *EM (4)+4. *SP (4)-3. *SP (5)+SP (6)} 
H+2. *SID (2) *{CH (5)-2. *CH (4)+2. *CP (2)-CP (3)} 
I+CID (4) *{SH (4)+2. *SH (3)-SP (2)+2. *SP (3)+SP (4)} 
J+SID (4) *{CH (4)-2. *CP (3)+CP (2)-2. *CP (3)+CP (4)} 
E (2)=E (2)+WST*DCMPLX (R, X) 
2CEP* (EPD*EP (3)-EM (EM (2)))-EPD2*EP (4)-EMD2*EM (4)
CALL ZSDM(S1, S2, T1, T2, RH, CDK, SDK, SDKD, .0D0, .Z22)
E1 = E1 + M * E11 - W22 * Z22
IF (MAX.EQ.2) GO TO 70
S1 = DK
DO 60 N = 3, MAX
M1 = N - 1
M2 = N - 2
M3 = 1
N2 = N + 2
E11 = EXP(N1) * EID(N1) + EM(N1) / EID(N1)
+ EXP(N1) * EID(N1) - EM(N1) / EID(N1)
2 * EXP(N1) * EID(N1) / EID(N1)
- EXP(N2) * EID(N1) / EID(N1)
S1S1 = DK
S3 = S1 + TDK
CALL ZSDM(S1, S3, T1, T2, RH, CDK, SDK, SDKD, .0D0, .Z22)
E1(N) = E1(N) + M * E11 - W22 * Z22
CPA = CP(N2) - 2 * CP(N1) + CP(N)
CPF = 2 * CF(N) - CP(N1) - CF(N1)
CFC = CP(N2) - 2 * CF(N1) + CP(N)
CDH = CH(N2) - 2 * CH(N1) + CH(N)
CHH = 2 * CH(N) - CH(N1) - CH(N1)
CIF = CH(N2) - 2 * CH(N1) + CH(N)
SPF = SP(N2) - 2 * SP(N1) + SP(N)
SFB = 2 * SP(N) - SP(N1) - SP(N1)
SFC = SP(N2) - 2 * SP(N1) + SP(N)
SMF = SM(N2) - 2 * SM(N1) + SM(N)
SMF = 2 * SM(N) - SM(N1) - SM(N1)
SMF = SM(N2) - 2 * SM(N1) + SM(N)
1 + CID(N2) * (CPA+CIA) + 2 * CIC(N) + (CBF+CHC) * 2 * SID(N) * (SFB-SMB)
X + CID(N2) * (CFC+CHC) + SID(N2) * (SFB-SMB)
IF (N.GT.3) X = X + SID(N2) * (SFB-SMB)
X + SID(N2) * (SFB-SMB) + 2 * SID(N) * (SFB-SMB)
X + SID(N2) * (SFC+CIC) + SID(N2) * (SFB-SMB)
IF (N.GT.3) X = X + SID(N2) * (SFB-SMB)
S1S1 = Z(N) + WST * DCMPLX(R, X)
70 PH = PH + DFH
80 SGN = -SGN
DPH = PI - PHA
100 PH = PHA
IF (NW = GT.0) WRITE(17, 5)
RETURN
END
A-43
SUBROUTINE ZSDM(S1, S3, T1, T2, RHK, CDK, SDK, SDKT, CPSI, Z12)
          ZSDM.1
CALLS Z12 = MUTUAL IMPEDANCE BETWEEN SINUSOIDAL DIPOLE
AND SINUSOIDAL MONPOLE WITH SKewed ORIENTATION.
          IMPlicit REAL*8 (A-H), (P-Z)
COMPLEX*16 E(2,2), F(2,2), ES2, ET2, EXPA, EXPB, EZ1, ES1
COMPLEX*16 CI, CIXX, SI2, EPI, EM1, PI1, P21, EGDZ, EM, EP
DIMENSION S(3)
DATA ETA, PI, TP/376.730366239, 3.14159265359, 6.28318530718/
S(1)=S1
S2=(S1+S3)/2.
S(2)=S2
S(3)=S3
SI2=(.0D0, .0D0)
DPSI=CPSI
CPSS=DPBI*DPSI
IF(CPSS.GT.1.DO)CPSS=1.DO
SPSI=DSQRT(1.DO-CPSS)
IF(DABS(CPSI).LT.999999)GO TO 10
RHS=RHS*RHK
RHS=CPSS*(T1+T2)/2.
RHS=RHS+RHS*RH2
SGN=1.
IF(CPSI.GT.0)GO TO 80
S(1)=S3
S(2)=S2
S(3)=S1
SGN=-1.
GO TO 80
10 D=RHK
DSQ=D*D
CD=D/DSPI
RD=CD*DPSI
EB=DEXP(-BD)
EC=DEXP(-CD)
TD1=T1
TD2=T2
TS1=TD1*TD1
TS2=TD2*TD2
CST=ETA/(16. *PI*SDK*SDKT)
SA=S1
SB=S2
ST2=DCMPLX(DCOS(T2), DSIN(T2))
DO 60 ISS=1,2
IF(18.EQ.1)ES1=DCMPLX(DCOS(SA), DSIN(SA))
IF(18.EQ.2)ES2=DCMPLX(DCOS(SB), DSIN(SB))
DO 20 K=1,2
DO 20 L=1,2
DO 40 K=1,2
DO 40 L=1,2
20 K(K,L)=(.0D0, .0D0)
SI=SA
DO 50 I=1,2
FI=(-1)**I
SI0=SI
SI0=SI0*SDI
ST1=2.*SDI*TD1*DPSI
ST2=2.*SDI*TD2*DPSI
R1=DSQRT(DSQ+SI0+TS1-ST1)
R2=DSQRT(DSQ+SI0+TS2-ST2)
ER=EB
DO 40 K=1,2
FK=(-1)**K
SK=FK*SDI
EL=EC
DO 30 L=1,2
30 CONTINUE
40 CONTINUE
50 CONTINUE
60 CONTINUE
FL= (-1)**L
EKL=EK*EL
XX=FK*BD+FL*CD
TL1=FL*TD1
TL2=FL*TD2
RR1=RR1+SD+TL1
RR2=RR2+SD+TL2
CALL EXPJ (DCMPLX (XX, RR1), DCMPLX (XX, RR2), EXPA)
CALL EXPJ (DCMPLX (-XX, RR1), DCMPLX (-XX, RR2), EXPB)
E(K, L)=E(K, L)+FI*(EXPAL*EKL+EXPB/EKL)

30 EL=1./EC
40 EE=1./EB
IF (I.EQ.ISS) GO TO 50
SD=SDI*DSF1
EC=EC
EG21=DCMPLX (DCOS (EC), DSIN (EC))
RR1=RR1+SD-TD1
RR2=RR2-SD-+TD2
CALL EXPJ (DCMPLX (.OD0, RR1), DCMPLX (.OD0, RR2), EXPB)
RR1=RR1-SD+TD1
RR2=RR2-SD+TD2
CALL EXPJ (DCMPLX (.OD0, RR1), DCMPLX (.OD0, RR2), EXPB)
F (1, 1)= (.OD0, 2. DOO)*SDK*EXPA/EG21
F (1, 2)= (.OD0, 2. DOO)*SDK*EXPA*EG21
50 SI=SB
IF (ISS.EQ.1) A21=CST* ((-F (2, 1)+E (2, 2)*ES1+E (1, 2)/ES1)*ET2
B= (F (2, 2)+E (2, 1)*ES1-E (1, 1)/ES1)/ET2)
IF (ISS.EQ.2) CP1=CST* (( F (1, 1)+E (2, 2)*ES2-E (1, 2)/ES2)*ET2
D= (-F (1, 2)+E (2, 1)+ES2+E (1, 1)/ES2)/ET2)
SA=SB
60 SB=SS
E12=F11+P21
RETURN
80 DO 100 I=1, 3
CI=I.
IF (I.EQ.2) CI=-2.*CDK
SI=SI
TJ=T1
DO 90 J=1, 2
DZ=TJ-SI
R=DSQRT (RHS+DZ*DZ)
X=R+DZ
IF (DZ.LT.0) X=RHS/(R-DZ)
CALL CIS1 (CO3I, CIN, SIN1, X)
EP=DCMPLX (CO3I, -SIN1)
X=R-DZ
IF (DZ.GT.0) X=RHS/(R+DZ)
CALL CIS1 (CO3I, CIN, SIN1, X)
EP=DCMPLX (CO3I, -SIN1)
IF (J.EQ.2) GO TO 90
EP1=EP
EM1=EM
90 TO=T2
X=T2-SI
EGDI=DCMPLX (DCOS (X), DSIN (X))
Z12=Z12+CI* ((EP-EP1)*EGDI+(EM-EM1)/EGDI)
100 CONTINUE
Z12= 8G1*ETAَ*Z12/(8.*FI*SDK*SDK)
RETURN
END

A-45
SUBROUTINE ZSMH(S1, S2, T1, T2, D, CD3, SDS, SDT, CPSI, P11)

C CALCULATES MUTUAL IMPEDANCE OF COPLANAR-SKew

IMPLICIT REAL*8 (A-H), (P-Z)

COMPLEX*16 E(2,2), F(2,2), GAM, P11, P12, F21, F22
COMPLEX*16 ES1, ES2, ET1, ET2, EXPA, EXPB
COMPLEX*16 EG2, EM, EP, EM1, EP1
DATA ETA, GAM, PI/376.730366239, (.DO,.1.DO), 3.14159265359/

DD=D
DQ=DQ*DD
DPSI=CFPSI
CPSS=CPSS*DPSS
IF (CPSS.GT.1.D0) CPSS=1.D0
SPSI=DSQRT(1.D0-CPSS)
SGDS=SDS
IF (S2.LT.S1) SGDS=-SDS
SGDT=SDT
IF (T2.LT.T1) SGDT=-SDT
IF (3ABS(CPSI).LT.999999) GO TO 6
DO = CPSI*(T1+T2)/2.
DQ = DQ*DD*DD
GO TO 110

6 ES1=DCMPLX(DCOS(S1),DSIN(S1))
ES2=DCMPLX(DCOS(S2),DSIN(S2))
ET1=DCMPLX(DCOS(T1),DSIN(T1))
ET2=DCMPLX(DCOS(T2),DSIN(T2))

ID=1
IF (ID.EQ.0) ID=0
TD1=T1
TD2=T2
CD=DD/SPSI
C=CD
BD=BD*DPSSI
B=BD
EB=0
EC=0
IF (ID.EQ.0) GO TO 8
EB=DEXP(-B)
EC=DEXP(-C)

8 DO 10 K=1,2
DO 10 L=1,2

10 E(K,L) = (.DO,.1.DO)
TS1=TD1*TD1
TS2=TD2*TD2
S1=S1
DO 100 I=1,2
FI=(-1)**I
SI=SI
SI=SI*SI
ST1=2.*SI*TD1*DPSSI
ST2=2.*SI*TD2*DPSSI
R1=DSQRT(DPQ+SI*TS1-ST1)
R2=DSQRT(DPQ+SI*TS2-ST2)
XK=EB
DO 50 K=1,2
FK=(-1)**K
SK=FK*SI
KL=EC
DO 50 L=1,2
FL=(-1)**L
KL=KL*KL
XX=FK*BD*FL*CD
Y=FL*TD1
TL2=FI^2TD2  
RR1=RI+SK+TL1  
RR2=R2+SK+TL2  
AXX=DABS(XX)  
IF(AXX.GT.DABS(XX))/100.) GO TO 28  
IF(AXX.GT.DABS(XX))/100.) GO TO 28  
IF(AXX.GT.001) GO TO 28  
IF(XX/RR1.0) GO TO 28  
CALL CISI (C0S1,CIN,SIN1,RR1)  
CALL CISI (C0S2,CIN,SIN2,RR2)  
EXPB=DCMPLX(COS2-COS1,SIN1-SIN2)  
E(K,L)=E(K,L)+PF1*EXPB(1.0)  
GO TO 40  
28  
CALL EXPJ(DCMPLX(XX,RR1),DCMPLX(XX,RR2),EXPB)  
CALL EXPJ(DCMPLX(-XX,RR1),DCMPLX(-XX,RR2),EXPB)  
E(K,L)=E(K,L)+PF1*EXPB  
GO TO 100  
40  
EX=1.0  
IF(XX.EQ.2) GO TO 100  
ZD=SDI*DPSI  
EG=DCMPLX(DCOS(XX),DSIN(XX))  
RR1=RI+ZD-TD1  
RR2=R2+ZD-TD2  
CALL CISI (C0S1,CIN,SIN1,RR1)  
CALL CISI (C0S2,CIN,SIN2,RR2)  
EXPB=DCMPLX(COS2-COS1,SIN1-SIN2)  
RR1=RI-ZD+TD1  
RR2=R2-ZD+TD2  
CALL CISI (C0S1,CIN,SIN1,RR1)  
CALL CISI (C0S2,CIN,SIN2,RR2)  
EXPB=DCMPLX(COS2-COS1,SIN1-SIN2)  
F(I,1)=2.*SSGS*(.000,.000)  
F(I,2)=2.*SSGS*(.000,.000)  
100  
S1=62  
CST=E*ETA/(16.*PI*SSGS*SGDT)  
P11=CST*((F(1,1)+E(2,2)*SS2-E(1,2)/ES2)*ET2  
A+(F(1,2)-E(2,1)*SS2-E(1,2)/ES2)/ET2  
RETURN  
110  
IF(CPS.0.) GO TO 120  
TA=TA  
TB=TB  
GO TO 130  
120  
TA=-TA  
TB=-TB  
SGDT=-SGDT  
130  
SI=SI  
C0S=DCS  
P11=(.000,.000)  
DO 150  I=1,2  
TA=TJ  
DO 140  J=1,2  
TA=TJ  
R=SQR(DSQ+DS*DS)  
X=R+DS  
IF(DS.LT.0) X=DSQ/(R-DS)  
CALL CISI (COSI,CIN,SINI,X)  
EP=DCMPLX(COSI,-SINI)  
X=R-DS  
IF(DS.LT.0) X=DSQ/(R+DS)  
CALL CISI (COSI,CIN,SINI,X)  
EM=DCMPLX(COSI,-SINI)  
IF(J.EQ.2) GO TO 140  
EP1=EP  
EM1=EM  
140  
TO-TB  
A-47
APPENDIX B

COMPUTER PROGRAM RICHMOND4 FOR THE FAR-ZONE FIELD OF A MONOPOLE ELEMENT ON A DISK GROUND PLANE ABOVE FLAT EARTH
INTRODUCTION ¹

Appendix I presents the computer program RICHMOND4. This FORTRAN program calculates the current distribution, terminal impedance, and directive gain \( G(\theta) \) of a base-fed monopole antenna mounted at the center of a circular disk over the flat lossy earth. The detailed theory behind this moment-method solution is presented in the following paper: (J. H. Richmond, "Monopole Antenna on Circular Disk over Flat Earth," IEEE Transactions, Vol. AP-33, pp. 633-637, June 1985.)

To assist the user, comment statements have been inserted in the main program and the subroutines. Only a few additional brief comments will be required in this Introduction.

RICHMOND4 performs all calculations with double precision. In this program the notation corresponds closely with the notation in the above paper, with one exception: In the paper \( z_o \) denotes the height of the circular disk above the surface of the earth, whereas in the program HDL denotes \( z_o/\lambda \). (The wavelength in free space is denoted by \( \lambda \) or WAVM.)

¹Appreciation is expressed to The MITRE Corporation for sponsoring this report.
RICHMOND4 requires all the subroutines used by RICHMOND3. In addition, RICHMOND4 requires the following additional subroutines which are listed after the main program in Appendix I: EDISK1, EDISK2, GAIN1, and GAIN2.

Subroutines EDISK1 and GAIN1 calculate the far-zone electric field intensity $E_\theta(\theta)$ for the monopole on the circular disk in free space, with $0 \leq \theta \leq \pi$. In these calculations, the factor $\exp(-jkr)/(kr)$ is suppressed. For the monopole on a circular disk over the flat earth, EDISK2 and GAIN2 calculate $E_\theta(\theta)$ with $0 \leq \theta \leq \pi/2$. In GAIN1 and GAIN2, $ET$ denotes the quantity $kr \exp(jkr) E_\theta(r, \theta)$, which may be called “the normalized far-zone electric field intensity” corresponding to the space wave.

GAIN1 and GAIN2 each makes two passes through the range of angles $\theta$. On the first pass ($M = 1$), the time-average radiated power $P$, is calculated via numerical integration using appropriately small increments DTH in the angle $\theta$. On the second pass ($M = 2$), the directive gain $D(\theta)$ is calculated and stored using the angular increments DTHD specified in the input data.

Tables I and II show numerical results (with RICHMOND4) for circular disks with radii $ka = 1.5$ and $3.0$ respectively, where $k$ denotes the wavenumber in free space. For the monopole on a disk in free space, the radiation resistance $R(RAD)$ and the directive gain $GAIN$ agree closely with Tables A2-6 and A2-12 in the following:


In addition, Tables I and II list the radiation resistance and gain for the monopole on a disk on the flat lossy earth, as well as the antenna terminal impedance in free space and on the flat earth. Let us define the radiation efficiency to be the ratio between the power radiated (via the “space wave” into the free-space region) and the power input at the antenna terminals. This radiation efficiency, then, is equal to the ratio of the radiation resistance and the real part of the antenna terminal impedance. In Tables I and II the radiation efficiency is 100% for the antenna in free space. For
the antenna on flat earth the radiation efficiency is 25.1% with the smaller
disk, and 46.6% with the larger disk.

RICHMOND4 has been tested with several of the examples in the pub-
lished paper cited earlier (Richmond, IEEE, 1985), with excellent agree-
ment on the antenna terminal impedance. In addition, the RICHMOND4
calculations converge properly as the number of segments (NSD and NSW)
increases.

In the calculation of the normalized far-zone electric field intensity ET,
GAIN1 and GAIN2 include only the "space wave" component of the field.
The effects of the round earth and the ionosphere are not included. Even
with flat earth, as the observer approaches the air-earth interface, the
"ground wave" or "surface wave" field may become significant, but is not
included in ET.
TABLE I. Numerical Results with $ka = 1.5$

DOUBLE PRECISION
MONOPOLE ON CIRCULAR DISK

<table>
<thead>
<tr>
<th>NSD</th>
<th>NSW</th>
<th>AL</th>
<th>CK</th>
<th>CL</th>
<th>HL</th>
<th>HDL</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>2</td>
<td>1.E-6</td>
<td>1.5</td>
<td>0.2387</td>
<td>0.25</td>
<td>0.0</td>
</tr>
<tr>
<td>R(RAD)</td>
<td>R</td>
<td>X</td>
<td>(IN FREE SPACE)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22.0733</td>
<td>22.0710</td>
<td>13.7758</td>
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<td></td>
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<table>
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<tr>
<th>THETA</th>
<th>GAIN (IN FREE SPACE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>10.0000</td>
<td>0.0582</td>
</tr>
<tr>
<td>20.0000</td>
<td>0.2202</td>
</tr>
<tr>
<td>30.0000</td>
<td>0.4521</td>
</tr>
<tr>
<td>40.0000</td>
<td>0.7103</td>
</tr>
<tr>
<td>50.0000</td>
<td>0.9540</td>
</tr>
<tr>
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<tr>
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<td>80.0000</td>
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<td>90.0000</td>
<td>1.4008</td>
</tr>
<tr>
<td>100.0000</td>
<td>1.3674</td>
</tr>
<tr>
<td>110.0000</td>
<td>1.2775</td>
</tr>
<tr>
<td>120.0000</td>
<td>1.1306</td>
</tr>
<tr>
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<td>0.6906</td>
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<tr>
<td>150.0000</td>
<td>0.4387</td>
</tr>
<tr>
<td>160.0000</td>
<td>0.2135</td>
</tr>
<tr>
<td>170.0000</td>
<td>0.0564</td>
</tr>
<tr>
<td>180.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

THETA GAIN (ON FLAT EARTH)

| 0.0000 | 0.0000 |
| 10.0000 | 0.1550 |
| 20.0000 | 0.6084 |
| 30.0000 | 1.3146 |
| 40.0000 | 2.1628 |
| 50.0000 | 2.9360 |
| 60.0000 | 3.2924 |
| 70.0000 | 2.8331 |
| 80.0000 | 1.3814 |
| 90.0000 | 0.0000 |

<table>
<thead>
<tr>
<th>NSD</th>
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<th>AL</th>
<th>BAR</th>
<th>CL</th>
<th>ER</th>
<th>FMC</th>
<th>HDL</th>
<th>HL</th>
<th>SIG</th>
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<tbody>
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<td>3.0</td>
<td>0.2387</td>
<td>4.0</td>
<td>10.0</td>
<td>0.0</td>
<td>0.25</td>
<td>0.001</td>
</tr>
<tr>
<td>R(RAD)</td>
<td>R</td>
<td>X</td>
<td>(ON FLAT EARTH)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.4736</td>
<td>29.7531</td>
<td>12.3685</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
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</tbody>
</table>
TABLE II. Numerical Results with $ka = 3$

**DOUBLE PRECISION**

**MONOPOLE ON CIRCULAR DISK**

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<th>CL</th>
<th>CL</th>
<th>HL</th>
<th>HDL</th>
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</thead>
<tbody>
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<td>3.</td>
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<td>0.2500</td>
<td>0.0</td>
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<tr>
<td>R(RAD)</td>
<td>R</td>
<td>X</td>
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<td>(IN FREE SPACE)</td>
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</tr>
<tr>
<td>40.0556</td>
<td>40.0120</td>
<td>33.0500</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>THETA</th>
<th>GAIN</th>
<th>(IN FREE SPACE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0000</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>10.0000</td>
<td>0.2238</td>
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</tr>
<tr>
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<td>0.7637</td>
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</tr>
<tr>
<td>30.0000</td>
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</tr>
<tr>
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<td>1.6823</td>
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</tr>
<tr>
<td>50.0000</td>
<td>1.7417</td>
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</tr>
<tr>
<td>60.0000</td>
<td>1.5725</td>
<td></td>
</tr>
<tr>
<td>70.0000</td>
<td>1.2923</td>
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<table>
<thead>
<tr>
<th>THETA</th>
<th>GAIN</th>
<th>(ON FLAT EARTH)</th>
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<td>0.0000</td>
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<td></td>
</tr>
<tr>
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<td>0.2630</td>
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<tr>
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<td>2.6187</td>
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<table>
<thead>
<tr>
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<th>AL</th>
<th>BAR</th>
<th>CL</th>
<th>ER</th>
<th>FMC</th>
<th>HDL</th>
<th>HL</th>
<th>SIG</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>2</td>
<td>1.E-6</td>
<td>3.</td>
<td>0.4775</td>
<td>4.</td>
<td>10.</td>
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<tr>
<td>R(RAD)</td>
<td>R</td>
<td>X</td>
<td></td>
<td>(ON FLAT EARTH)</td>
<td></td>
<td></td>
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<tr>
<td>18.2413</td>
<td>39.1551</td>
<td>27.5684</td>
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</table>

B-7
APPENDIX I. RICHMOND4 and the Subroutines

RICHMOND4
MONOPOLE AT CENTER OF CIRCULAR DISK OVER FLAT EARTH.

DOUBLE PRECISION.
CURRENT DISTRIBUTION, IMPEDANCE, AND FAR-FIELD PATTERN.
LINK: BES10, CIS1, CHINV, DZ11, DZDD, DZDM, EDISK1, EDISK2, EXPJ.
GAIN1, GAIN2, GRILL, QDD, QD0, QOH, QMM, QEW, QEWS, QEW, SFARAT, ZSDN, ZSMM.

IMPLICIT REAL*8 (A-H), (P-Z)
COMPLEX*16 CJ(30), VJ(30), ZJ(30), Z1J(30, 30), Z2J(30, 30)
COMPLEX*16 PI1, PI2, P21, P22, ZDD, ZDM, ZSM, ZDD, Z22, Z12, Z21

DIMENSION FB(500), G(182), LLL(-A0), MM(30), MMR0(30)

DATA EO, UO /8.854l8!336770-12, 1.25663706144D-6/
DATA ETA, PI, T? /376.730366239, 3.14159265359, 6.28318530718/
DATA ICC, IFB /50, 30, 500/

1 FORMAT(1X, 2F10.4)
2 FORMAT(1X, 8F10.4)
3 FORMAT(1H0)
4 FORMAT(1H0)
5 FORMAT(1H0)

AL = RADIUS OF WIRE IN WAVELENGTHS.
BAR = RADIUS RATIO FOR COAXIAL FEED CABLE.
CL = RADIUS OF CIRCULAR DISK IN WAVELENGTHS = EPSLN/TP.
DTHD = INCREMENT IN FAR-FIELD ANGLE THETA (DEGREES)
ER = RELATIVE PERMITTIVITY OF EARTH.
FMC = FREQUENCY IN MEGAHERTZ.
HL = LENGTH OF MONOPOLE IN WAVELENGTHS.
HD = HEIGHT OF DISK ABOVE THE EARTH IN WAVELENGTHS.
NSD = NUMBER OF SEGMENTS ON THE DISK.
NSW = NUMBER OF SEGMENTS ON THE WIRE.
SIG = CONDUCTIVITY OF EARTH, MHO/M.
SET DTHD = NEGATIVE TO SKIP THE GAIN CALCULATIONS.
SET HDL = NEGATIVE FOR MONOPOLE-DISK IN FREE SPACE.
SET HDL = POS. FOR FREE SPACE + FLAT EARTH.
SET IWCJ = 1 TO WRITE OUT THE CURRENTS CJ(N),
OR IWCJ = 0 TO SUPPRESS WRITEOUT.

TL = 1.0D-5 FOR EPSLN GREATER THAN OR EQUAL 0.25,
OR 1.0D-4 FOR EPSLN LESS THAN 0.25.

AL=1.0D-6
BAR=3.
CL=3./TP
DTHD=10.
ER=4.
FMC=10.
HL=25
HD=1.0D-5
IWCJ=0
NSD=8
NSW=2
SIG=0.001
TL=1.0D-5
WAVM=300.0/FMC
IM=0
NPD=6
NEQ=NSD+NSW-1
IF(NEQ.GT.ICC) GO TO 400
AX=TP*AL
CK=TP*CL
HF=TP*HL
HD=TP*HD
TK=TP*TL
OMEG=TP*FMC*1.D6
EC=DCHIPLX(ER, -SIG/(OMEG*EO))
DKD=(CK-AK)/NSD
DKR=KK/NSW
RK=AK+DK
IF(RK.LT.BAR*AR)GO TO 400
TDKD=2.*DKD

B-8
CDDO(DID)
MAIK.2
CDK-DCOS (DKW)
SDK-DSIN (Dlii)
NA-NSD+l
C&LL QiMi(A1,DXD,DKW, CDXW,SDXD, CDX, SDX, TX,INZ,NPB, El)
IF(NSD.LE.1)GO TO 100
S3=S1+TDKD
DO 50 1-2,J
T2=T1+TDKD
T3=T1+TDKD
CALL QDM(CDKD,SDKD,S1,S3,T1,T3,TK,INZ,NPH,322)
ZIJ(I,J)=222
DO 50 T1=T1+TDKD
CALL QDM(AK,DKD,DKW,CDKD,SDKD,SDK,S1,S3,TK,INZ,NPH,312)
ZIJ(I,J)=212
60 S1=S1+TDKD
100 IF(NSW.LE.1)GO TO 200
CALL SPART(AK,DKD,DKW,CDKD,SDKD,SDK,S1,S3,TK,INZ,CJ)
L=0
DO 160 I=NA,NEQ
DO 150 J=I,NEQ
K=J-I+1
150 ZIJ(I,J)=ZJ(K)
L=L+1
ZIJ(I,J)=CJ(L)
160 CONTINUE
178 IF(NSW.LE.1)GO TO 200
ZL=0
DO 190 J=NA,NEQ
Z2=Z2+DKW
S1=S1-DKW
S3=S3-DKW
RH2=AK
DO 180 I=2,NSD
RH2=RH2+DKD
T1=RH2+DKD
T3=RH2+DKD
CALL SKW(MT,AK,S1,S3,T3,CDK,SDK,CDKD,SDKD,INZ,312)
180 ZIJ(I,J)=Z12
190 CONTINUE
200 CALL GRILL(AK,AR,DID,DKN,NEQ,NSD,NSW,TK,VJ)
DO 210 I=1,NEQ
DO 210 J=I,NEQ
WRITE(17,1)I,J,ZIJ(I,J)
ZIJ(I,J)=ZIJ(I-1,J)
WRITE(6,1)NSD,NSW,AL,CL,HL,EDL
WRITE(17,1)NSD,NSW,AL,CL,HL,EDL
WRITE(6,5)
WRITE(17,5)
CALL GINV(CJ,VJ,ZIJ,ICC,INW1,1,LLL,NSW,NSW,DET)
Y11=CJ(1)
Z11=1./Y11

CALCULATE DIRECTIVE GAIN G(N) IN FREE SPACE.
RR=0
IF(DTHD.LE.0)GO TO 212
. GAIN = DIRECTIVITY IN FREE SPACE.
C

THETA = ANGLE OF MAXIMUM GAIN IN FREE SPACE.
CALL GAIN (AK,CK,CJ,DTHD,G,GAINA,HK,NSD,  
2 NSW,NTH,PR,THA,WAVM)
AJ=CDABS(CJ(1))
PR1=PR/(AJ*AJ)

CONTINUE
C
WRITE(6,2)HDL,GAINA,RRI,Z11
WRITE(15,2)HDL,GAINA,RRI,Z11
WRITE (6,5)
WRITE (17,5)
IF(DTHD.LE.0.0)GO TO 222
DO 220 N=1,NTH
TH=DTHD*(N-1)
WRITE(6,2)TH,G(N)
WRITE(17,2)TH,G(N)
CONTINUE
WRITE (6,5)
WRITE (17,5)
CONTINUE

C
C CALCULATE DIRECTIVE GAIN G(N) OVER FLAT EARTH.

C
IF(HDL.LT.0.0)GO TO 350
DO 400 NHD1,4
HDL=NHD
HDL=T*HDL
C DELETE STATEMENT 230 UNLESS THE CURRENT DISTRIBUTION IS TO BE
C APPROXIMATED BY THE CUR. DIST. FOR ANTENNA IN FREE SPACE.
C 230 IF(NHD.GT.0)GO TO 316
CALL D11(AK,BAR,DKD,DKW,EC,FB,HDK,TK,IFB,D11,DV1)
Z1J(1,1)=D11
IF(NSD.LE.1)GO TO 265
S2=AK+DKD
DO 260 J=2,NSD
T2=AK+DKD
I12=1
DO 250 I=2,J
CALL D2DD(AK,DD,DKD,DKW,EC,FB,DKW,S2,T2,TK)
2,IF(I.EQ.2)P12=D12
Z1J(1,J)=D12
Z12=2
T2=T2+DKD
Z1J(1,J)=P12
260 S2=S2+DKD
265 DO 280 K=1,MAX
CALL DPDW(AK,DKD,DKW,EC,HDK,K,TK,2J,D21J)
2,J=NA+K-1
Z1J(1,J)=D21J
L=1
DO 270 I=NA,J
Z1J(I,J)=ZJ(L)
270 L=L+1
CONTINUE
276 IF(NSD.LE.1)GO TO 300
Z2=0
DO 290 J=NA,NEQ
Z2=Z2+DKW
CALL DPDW(AK,DKD,DKW,EC,HDK,NSD,TK,Z2,ZJ)
DO 280 I=2,NSD
280 Z1J(I,J)=ZJ(I)
CONTINUE
290 DO 310 I=1,NEQ
300 DO 308 J=1,NEQ
Z12=V1J(I,J)
B-10
D12 = ZIJ(I,J)
WRITE(17,1) I, J, Z12, D12
ZIJ(I,J) = Z12 + D12
CONTINUE

WRITE(17,5)
CALL GRILL(AK, BAR, DKD, DKW, NEQ, NSD, NSW, TK, VJ)
VJ(1) = VJ(1) + DV1
DO 315 I = 1, NEQ
DO 312 J = 1, NEQ
ZIJ(J, I) = ZIJ(I, J)
CONTINUE
CALL CMINV(CJ, VJ, ZIJ, ICC, IWCJ, 1, LLL, NSD, NEQ, DET)

RR2 = 0.
IF (DTHD .LE. 0.) GO TO 322
GAINB = DIRECTIVITY OVER FLAT EARTH.
THE = ANGLE OF MAXIMUM GAIN OVER FLAT EARTH.
CALL GAINB(AK, CK, CJ, DTHD, ECC, G, GAINB, EDK, EK,
2 NSD, NSW, NT, PR, TEB, WAVH)
AJ = CDABS(CJ(1))
RR2 = PR/(AJ*AJ)
IF (DTHD .LE. 0.) GO TO 322
DO 320 N = 1, NT
TH = DTHD*(N-1)
WRITE(6,2) TH, G(N)
WRITE(17,2) TH, G(N)
CONTINUE
WRITE(6,5)
WRITE(17,5)
Y11 = CJ(1)
W11 = 1./Y11
DBB = 10.*ALOG10(GAINB)
WRITE(6,2) HDL, DBB, TEB, RR2, W11
WRITE(16,2) HDL, DBB, TEB, RR2, W11
WRITE(17,5)
CONTINUE
WRITE(6,5)
WRITE(17,5)

DBA = 10.*ALOG10(GAINA)
WRITE(6,2) DBA, TBA, RR1, Z11
WRITE(16,2) DBA, TBA, RR1, Z11
CALL EXIT
END
SUBROUTINE GAIN1(AK, CK, CJ, DTHD, G, GAINA, HK, MSD, NSW, GAM1.1)

SEPTMBER 21, 1990.

CALCULATE DIRECTIVE GAIN G(N) FOR MONPOLE ON DISK IN FREE SPACE.

GAINa = DIRECTIVITY IN FREE SPACE.

THA = ANGLE OF MAXIMUM GAIN IN FREE SPACE.

IMPLICIT REAL*8 (A-H), (P-Z)

COMPLEX*16 CJ(1), CJ1, CJ2, CQ, CQS, CK1, CK2, CK3, ET, ETH, ETHD

DIMENSION G(1)

DATA ETA, PI, TP/376.730366239, 3.14159265359, 6.28318530718/

BET=TP/WAVM

DKD=(CK-AK)/NSD

SDK=DSIN(DKD)

CDK=DCOS(DKD)

DN=HK/NSD

SDK=DSIN(DKD)

MDG=NS+NSW-1

DK=HK

IF(CK.GT.HK)DE=CR

NS=10.*DK/PI

NS=2*(NS/2)

IF(NS.LT.20)NS=20

NTH=NS+1

DTH=PI/NS

CQ=DCMPLX(.OD0, BET*ETA/(4.*PI*SDK))

20=DK

20=0

LS=10.*DK/PI

IF(NS.LT.4)LS=4

LS=2*(LS/2)

PR=.0

GAINA=.0

DO 250 M=1, 2

IF M=1, USE SIMPSON'S RULE INTEGRATION TO

CALCULATE PR = POWER RADIATED.

IF M=2, CALCULATE THE DIRECTIVE GAIN G(N).

SGN=-1.

DO 200 NT=1,NTH

IF (NT.EQ.1) GO TO 200

IF (NT.EQ.NTH) GO TO 200

ET=(.ODD0, .0DD)

WF=3.*SGN

SELECT THE FAR-FIELD ANGLE TH = THETA.

TH=DTH*(NT-1)

ETH=DCOS(TH)

ETHD=DSIN(TH)

CQS=CQ/ETH

CALCULATE FAR-ZONE FIELD OF MODE #1 IN FREE SPACE.

Z1=20

Z2=21*DK

ARG=Z2*ETH

EK2=DCMPLX(DCOS(ARG), DSIN(ARG))

ARG=Z2*ETH

EK1=DCMPLX(DK, ETH*SDK) *DCMPLX(DCOS(ARG), DSIN(ARG))

ETH=CQS*CJ(1) * (EK2-EK1)

CJ1=CJ(1)

CJ2=(.OD0, .0DD)

NX=AK

NK2=MK+DKD

CALL EDISK1(CJ1, CJ2, CTH, DKD, ETHD, LS, RK1, RK2, SDKD, ETH, BET, 20)

ET=ET+ETH+ETHD

B-12
IF (NSD .LE. 1) GO TO 100

C CALCULATE FAR FIELD FROM DISK CURRENT IN FREE SPACE.
DO 60 J=1,NSD
RK1=AK+(J-1)*DKD
RK2=RK1+DKD
CJ1=CJ(J)
IF (J.EQ.1) CJ1=(.0D0,.0D0)
CJ2=CJ(J+1)
IF (J.EQ.NSD) CJ2=(.0D0,.0D0)
C EDISK1 CALCULATES FIELD FROM ANNULAR ZONE J OF THE DISK.
C RK1 AND RK2 ARE INNER AND OUTER RADII OF ZONE J.
C CJ1 AND CJ2 DENOTE RADIAL CURRENTS AT INNER AND OUTER RADII.
CALL EDISK1 (CJ1,CJ2,CTH,DKD,EZH,LS,RK1,RK2,SDKD,STH,BET,ETO)
ET=ET+ETH
60 CONTINUE

100 IF (NSD .LE. 1) GO TO 162
C CALCULATE FAR FIELD FROM THE WIRE DIPOLE MODES IN FREE SPACE.
JB=NSW-1
DO 160 J=1,JB
Z2=Z0+J*DKW
Z1=Z2-DKW
Z3=Z2+DKW
L=NSD+J
ARG=Z1*CTH
EK1=DCMPLX (DCOS (ARG), DSIN (ARG))
ARG=Z2*CTH
EK2=DCMPLX (DCOS (ARG), DSIN (ARG))
ARG=Z3*CTH
EK3=DCMPLX (DCOS (ARG), DSIN (ARG))
ETH=CJ (L)*COS*(EK1+EK3-2.*CDK*EK2)
ET=ET+ETH
160 CONTINUE

162 AT=CDABS (ET)
ATS=AT**2
IF (M.EQ.1) PR=PR+WF*ATS*STH
IF (M.EQ.2) G (NT)=ATS
IF (ATS.LT.GAINA) GO TO 200
GAINA=ATS
THA=57.29578*TH
200 SGN=-SGN
IF (M.EQ.1) PR=TP*PR*DTH/(3.*ETA*BET*BET)
DTH=.01745329*DTHD
IF (DTHD.GT.0.) GO TO 248
NS=1
NTH=1
GO TO 250
248 NS=180./DTHD
NTH=NS+1
250 CONTINUE
G(1)=0
G(NTH)=0
C NORMALIZE THE DIRECTIVE GAIN G(N).
CST=4.*PI/(ETA*BET*BET*PR)
GAINA=CST*GAINA
DO 300 N=1,NTH
GN=CST*G(N)
G(N)=GN
300 CONTINUE
RETURN
END
SUBROUTINE GAIN2(AX,CK,CJ,DTHD,EC,G,GAINB,HDK,HK, GAIN2.L
2 NSD,NSW,NTH,PR,THB,NAVM)
3 SEPT. 21, 1990.
4 GAINB = DIRECTIVITY OVER FLAT EARTH.
5 THB = ANGLE OF MAXIMUM GAIN.
6 CALCULATE DIRECTIVE GAIN G(N) FOR MONOPOLE ON DISK OVER FLAT EARTH.
7 ALSO PR = TIME-AVERAGE POWER RADIATED INTO UPPER HALF-SPACE.
8 DOUBLE PRECISION
9 IMPLICIT REAL*8 (A-H), (P-Z)
10 COMPLEX*16 CJ (1), CJ1, CJ2, CO, CQS
11 COMPLEX*16 EC, EK1, EK2, EK3, ET, ETH, ETMD, ETI, QST, RC
12 DIMENSION G(1)
13 DATA ETA, PI, TP/376.730366239, 3.14159265359, 6.28318530718/
14 BET=TP/NAVM
15 DKD=(CK-AK)/NSD
16 SDK=DSIN (DKD)
17 CDK=DCOS (DKD)
18 DK=HK/NSW
19 SDK=DCOS (DKW)
20 NEQ=NSD+NSW-1
21 IF (NS.GT.90) NEQ=90
22 NS=NS+1
23 NT=NS+1
24 DTH=PI/(2.*NS)
25 CO=DCMPLX (.ODD, BET* ETA/(4.*PI*SDK))
26 LS=10.*SDK/PI
27 LS=2.*LS/2
28 FR=.0
29 GAINB=.0
30 DO 250 M=1,2
31 IF (M.GT.1), USE SIMPSON'S RULE INTEGRATION TO
32 CALCULATE PR = POWER RADIATED.
33 IF (M.GT.2), CALCULATE DIRECTIVE GAIN G(N).
34 SGN=1.
35 DO 200 NT=1,NTH
36 IF (NT.EQ.1) GO TO 200
37 ET=(.ODD, .ODD)
38 WT=3.4*SGN
39 IF (NT.EQ.NTH) WT=1.
40 SELECT THE FAR-FIELD ANGLE TH = THETA.
41 TH=DTH*(NT-1)
42 CTH=DCOS (TH)
43 STH=DSIN (TH)
44 CQS=CQ/STH
45 QST=DCSQR (EC-SSTH*STH)
46 RC = REFLECTION COEFFICIENT AT AIR-EARTH INTERFACE.
47 RC=(EC*CSTH-QST)/(EC*CSTH+QST)
48 CALCULATE FAR-ZONE FIELD OF MODE #1.
49 Z1=20
50 Z2=21+DKW
51 ARG1=21*CTH
52 EK1=DCMPLX (DCOS (ARG1), DSIN (ARG1))
53 ET1=DCMPLX (CDK, CTH*SDK) *EK1
54 ARG2=22*CTH
55 EK2=DCMPLX (DCOS (ARG2), DSIN (ARG2))

B-14
ETH=EK2-ET1
EK1=DCONJG(EK1)
ET1=DCMPLX(DK, CTH*SDK)*EK1
EK2=DCONJG(EK2)
ETH=CQS*(ETH-RC*(EK2-ET1))
CJ1=-CJ1
CJ2=(.0D0,.0D0)
RK1=AK
RK2=RK1+DKD
CALL EDISK2(CJ1, CJ2, CTH, DKD, ETHD, LS, RC, RK1, RK2, SDKD, STH, BET, Z0)
ET=ET+ETHD
IF(NSD.LE.1)GO TO 100
CALL FAR FIELD FROM DISK CURRENT IN FREE SPACE.

DO 60 J=1,NSD
RK1=AK+(J-1)*DKD
RK2=RK1+DKD
CJ1=CJ(J)
IF(J.EQ.1)CJ1=(.0D0,.0D0)
CJ2=CJ(J+1)
IF(J.EQ.NSD)CJ2=(.0D0,.0D0)
EDISK2 CALCULATES FIELD FROM ANNULAR ZONE J OF THE DISK.
RK1 AND RK2 ARE INNER AND OUTER RADII OF ZONE J.
CJ1 AND CJJ DENOTE RADIAL CURRENTS AT INNER AND OUTER RADII.
CALL EDISK2(CJ1, CJ2, CTH, DKD, ETHD, LS, RC, RK1, RK2, SDKD, STH, BET, Z0)
ET=ET+ETHD
60 CONTINUE
100 IF(NSW.LE.1)GO TO 162
CALL FAR FIELD FROM THE WIRE DIPOLE MODES IN FREE SPACE.

JB=NSW-1
DO 160 J=1,JB
Z2=Z0+(J-1)*DKW
Z1=Z2-DKW
Z3=Z2+DKW
L=NSD+J
ARG^Z1*CTH
EK1=DCMPLX(DCOS(ARG),DSIN(ARG))
ARG=Z2*CTH
EK2=DCMPLX(DCOS(ARG),DSIN(ARG))
ARG=Z3*CTH
EK3=DCMPLX(DCOS(ARG),DSIN(ARG))
ETH=EK1+EK3-2.*CDK*EK2
EK1=DCONJG(EK1)
EK2=DCONJG(EK2)
EK3=DCONJG(EK3)
ETH=CJ(L)*CQS*(ETH+RC*(EK1+EK3-2.*CDK*EK2))
ET=ET+ETH
160 CONTINUE
162 AT=CDASS(ET)
ATS=AT+Z
IF(M.EQ.1)PR=PR+MF*ATS*STH
IF(M.EQ.2)G(NT)=ATS
IF(ATS.LT.GAINB)GO TO 200
GAINB=ATS
THE=57.29578*ETR
200 SGN=-SGN
IF(M.EQ.1)PR=TP*PR+DTD/(3.*ETA*BET*BET)
IF(DTHD.GT.0.)GO TO 248
MS=1
NTH=1
GO TO 250
248 DTH=.01745329*DTD
MS=.0/.DTD
NTH=MS+1

B-15
NORMALIZE THE DIRECTIVE GAIN $G(N)$.  
\[ \text{CST} - 4. \cdot \text{FI} / (\text{ETA} \cdot \text{BET} \cdot \text{BET} \cdot \text{PR}) \]
\[ \text{GAINC} = \text{CST} \cdot \text{GAINB} \]
\[ \text{DO 300 } \text{N} = 1, \text{NTH} \]
\[ \text{GN} = \text{CST} \cdot G(N) \]
\[ G(N) = \text{GN} \]

CONTINUE  
RETURN  
END
APPENDIX C

COMPUTER PROGRAM RICHMD6 FOR THE INPUT IMPEDANCE, CURRENT DISTRIBUTION, AND FAR-ZONE FIELD OF A MONOPOLE ELEMENT ON A PERFECT GROUND PLANE
PROGRAM "RICHMD6.FOR"


THIS COMPUTER PROGRAM, IN FORTRAN LANGUAGE, WAS WRITTEN BY DR. JACK RICHMOND OF OHIO STATE UNIVERSITY. IT USES A SINUSOIDAL-GALERKIN METHOD OF MOMENTS TO COMPUTE THE INPUT IMPEDANCE, CURRENT DISTRIBUTIONS, AND ANTENNA PATTERN OF A MONOPOLE ELEMENT OF LENGTH $h$ AND RADIUS $b$ ON A PERFECT GROUND PLANE OF INFINITE EXTENT. A DETAILED DERIVATION IS PUBLISHED IN REFERENCE 1.

REFERENCE:


THIS COMPUTER PROGRAM REQUIRES FIVE INPUTS WHICH ARE ENTERED FROM AN INPUT FILE NAMED "RICH6_IN.DAT".

- $b/\lambda = AL = MONOPOLE ELEMENT RADIUS IN WAVELENGTHS$
- $h/\lambda = BL = MONOPOLE ELEMENT LENGTH IN WAVELENGTHS$
- $b_1/b_2 = BAR = RATIO OF OUTER TO INNER CONDUCTOR RADII OF THE COAXIAL LINE FEED.$
- $\sigma = CMM = CONDUCTIVITY OF MONOPOLE ELEMENT (MEGAMOHMS/METER)$
- $\gamma = -1$ FOR PERFECTLY CONDUCTING MONOPOLE ELEMENT
- $IFLAG = FLAG FOR MONOPOLE ELEMENT CURRENT DISTRIBUTIONS$
- $0 = MONOPOLE ELEMENT CURRENTS COMPUTED BY METHOD OF MOMENTS$
- $-1 = MONOPOLE ELEMENT CURRENTS WITH IDEALIZED SINUSOIDAL DISTRIBUTION AND FOR INFINITE CONDUCTIVITY OF THE MONOPOLE ELEMENT$

$F = FMC = FREQUENCY IN MEGAHERTZ$

NOTE: IF $CMM = -1$, IT IS NOT NECESSARY TO SPECIFY AN INPUT VALUE FOR FREQUENCY.

C**************************
C LINK CISI;FRILLS;TPLZ;TSPAR;ZSURF
REAL G(100),ANG(100),DIRECD(100),DIRECR(100),DIRMAX
INTEGER HALFNTH,NTH
COMPLEX COO,CII,CJA,CJB,DELZ
COMPLEX EC,EGZ,EJS,ETA2,ETD,ETH
COMPLEX GM,GP,01,02,Q3,RC
COMPLEX Y11,Zll,ZFIN,ZINF,ZS,ZSG
COMPLEX CJ(99),GI(99),U(99),VJ(99),W(99),ZJ(99)
OPEN(UNIT=4,FILE='RICH6_IN.DAT',STATUS='OLD',READONLY)
OPEN(UNIT=10,FILE='RICH6_OUT.DAT',STATUS='NEW')
DATA ETA,PI,TF/376.73036239,3.14159265359,6.28318530718/
DATA BO,0/1.57079632679,2.718281828/
DATA P2,8/1.57079632679,2.718281828/
DATA IMD/99/
C**************************
C WAWM = WAVELENGTH IN FREE-SPACE (METERS).
C NOTE: NOT USED IF $CMM = -1$. 
C

C-3
C******************************************************************************************
2 FORMAT(7X,8F11.5)
3 FORMAT(8X,8F16.10)
4 FORMAT(4X,8F11.5)
5 FORMAT(5X)
521 FORMAT(1X,A)
C******************************************************************************************
C INPUTS
C******************************************************************************************
READ(4,*),AL,HL,BAR,CMN,IFLAG
IF(CHM.EQ.0.GT-1)GOTO 8
READ(4,*),PMC
VAVM=300./PMC
8 CONTINUE
IVR=1
C******************************************************************************************
C INPUT WRITE STATEMENTS
C******************************************************************************************
CALL HEADING2(AL,HL,BAR,CMN,IFLAG)
AK=TP*AL
HK=TP*HL
SK2=2.*HJ
EJS=CMPLX(COS(SK),SIN(SK))
CJ=CMJX(HK)
SHK=SIN(HK)
EJH=CMPLX(CHK,SHK)
NS=15.*BL
IF(NS.LT.6)NS=6
IF(NS.GT.IDM)NS=IDM
IG=NS/2
IF(IFLAG.EQ.0)GOTO 43
IG=1
CMN=1.
43 CONTINUE
NS=IG
N=NS-I
ZS=(0.0)
IF(CHM.GT.0.)CALL ZSURF(AK,CMN,PMC,ZS)
C******************************************************************************************
C ZS = SURFACE IMPEDANCE OF MONOPOLE WIRE.
C******************************************************************************************
DK=HR/IG
CDK=COS(DK)
SDK=SIN(DK)
C******************************************************************************************
C ZJ(N) = FIRST ROW OF IMPEDANCE MATRIX FOR DIPOLE IN FREE SPACE, *
C USING GALEKKIN'S METHOD WITH OVERLAPPING SINUSOIDAL BASIS FUNCTIONS. *
C (DIPOLE LENGTH = TWICE THE MONOPOLE LENGTH, USING IMAGE THEORY.) *
C******************************************************************************************
CALL TSPAR(AK,DK,N,ZJ)
IF(CHM.LT.0.)GO TO 52
GK=2.*(DK-CDK*SDK)
GL=SDK-DK*CDK
FB=4.*PI*AK*SDK*SDK
ZJ(1)=ZJ(1)+ZS*GK/FB
ZJ(2)=ZJ(2)+ZS*GL/FB
52 **
C******************************************************************************************
C SET UP THE MOMENT-METHOD VOLTAGE COLUMN VJ(N) FOR DIPOLE IN FREE SPACE.*
CALL FRILLS(AK,BAR,DK,N,CJ)
DO 60 I=1,N
K=1+IABS(IG-I)
60 VJ(I)=CJ(K)

C*****************************************************************************
C SOLVE THE SIMULTANEOUS LINEAR EQUATIONS TO DETERMINE THE CURRENTS CJ(N)*
C ON THE WIRE DIPOLE IN FREE SPACE. (THE IMPEDANCE MATRIX IS TOEPOLITZ.) *
C*****************************************************************************

CALL PRCURR
CALL TPLZ(CJ,U,VJ,IG,IER,IWR,112,N)
WRITE(10,521)CHAR(12)

C*****************************************************************************
C ZINF = IMPEDANCE OF MONOPOLE ANTENNA OVER INFINITE GROUND PLANE. *
C*****************************************************************************

CALL HEADING2(AL,EL,BAR,CMH,IFLAG)
ZINF=.5/CJ(IG)
RIN=REAL(ZINF)
XIN=AIMAG(ZINF)
CALL FRES(RRAD,RIN,XIN)
Y11=CJ(IG)
PD=0
PIN=REAL(Y11)
PRAD=PIN-PD
EFF=100.*PRAD/PIN
DTH=2.
NTH=1.*90./DTH

DO I=1,NTH
ANG(I)=(I-1)*DTH
TB=ANG(I)
CALL GAIN(CDK,CJ,DGAIN,DK,U,HK,N,PRAD,SDK,TH,VJ,WAVM)
G(I)=DGAIN*2
END DO

DO I=NTH+1,91
G(I)=0
ANG(I)=(I-1)*DTH
END DO

C DO I=1,NTH
C WRITE(10,2)ANG(I),G(I),ANG(I+NTH),G(I+NTH)
C END DO

C*****************************************************************************
C OUTPUT STATEMENTS FOR PAR-FIELD PATTERN HEADINGS (WITH EARTH) *
C*****************************************************************************

WRITE(10,850)
850 FORMAT(//1X,
   & 'ELEVATION',5X,'DIRECTIVE',6X,'DIRECTIVE',5X,'RELATIVE',12X,
   & 'ELEVATION',5X,'DIRECTIVE',6X,'DIRECTIVE',5X,'RELATIVE',)
   & WRITE(10,852)
852 FORMAT(3X,'ANGLE',9X,'GAIN',11X,'GAIN',9X,'POWER',
   & 16X,'ANGLE',9X,'GAIN',11X,'GAIN',9X,'POWER' )
   & WRITE(10,854)
854 FORMAT(3X,'(DEG)',7X,'(NUMERIC)',8X,'(DBI)',9X,'(DB)',
   & 16X,'(DEG)',7X,'(NUMERIC)',8X,'(DBI)',9X,'(DB)/5X )

C*****************************************************************************
C*****************************************************************************

DIRMAX=0.0
DO N=1,NTH
IF(G(N).NE.0)DIRECD(N) = 10.0*LOG10(G(N))
IF(DIRMAX.LT.DIRECD(N))DIRMAX = DIRECD(N)
END DO
DO N=1,NTH
DIRECRP(N) = DIRECD(N) - DIRMAX
END DO
C*****************************************************************************
C OUTPUT STATEMENTS FOR FAR FIELD-PATTERN (WITH EARTH)
C*****************************************************************************
HALFNTH = NTH/2
DO N=1,HALFNTH
IF( G(N).NE.0) THEN
WRITE(10,750)ANG(N),G(N),DIRECD(N),DIRECRP(N),
ANG(N+HALFNTH),G(N+HALFNTH),DIRECD(N+HALFNTH),
DIRECRP(N+HALFNTH)
750 FORMAT(3X,F4.0,2F16.5,F13.5,14X,F4.0,2F16.5,F13.5)
ELSE
WRITE(10,604)ANG(N),ANG(N+HALFNTH),G(N+HALFNTH),
DIRECD(N+HALFNTH),DIRECRP(N+HALFNTH)
604 FORMAT(3X,F4.0,9X,'0.00000',7X,'-INFINITY',4X,'-INFINITY',
14X,F4.0,2F16.5,F13.5)
END IF
END DO
320 CONTINUE
C*****************************************************************************
C*****************************************************************************
CLOSE(UNIT=4,STATUS='KEEP')
CLOSE(UNIT=10,STATUS='KEEP')
400 CALL EXIT
END
C
C*****************************************************************************
C SUBROUTINE GAIN
C*****************************************************************************
C SUBROUTINE GAIN(CDK,CJ,DGAIN,DK,ETT,HK,N,PRAD,SDK,TB,VJ,
& VAVM)
C
COMPLEX CJ(1),ETT(1),VJ(1),CQQ,ETH,ETD
DATA ETA/376.727/
CQQ=CMPLX(-.0,2./ETA)
CTH=COS(-.0174533*TB)
STH=SIN(-.0174533*TB)
ETH=(.0,.0)
IF(TB.GT.1.)
& ETD=-60.*(.0,1.)*(CDK-COS(DK*CTH))/(SDK*STH)
Z=E-EK+DK
ETH=(.0,.0)
DO 125 I=1,N
ANG=2*CTH
ETT(I)=ETD*CMPLX(COS(ANG),SIN(ANG))
VJ(I)=CQQ*ETT(I)*VAVM
IF(PRAD.GT.0)ETH=ETH+CJ(I)*ETT(I)
IF(PRAD.LT.0)CJ(I)=VJ(I)
125 Z=Z+DK

C-6
IF(PRAD.GT..0)DGAIN=-(CABS(ETB)**2)/(30.*PRAD)
RETURN
END

SUBROUTINE PRCURR
C******************************************************************************
C OUTPUT STATEMENTS FOR CURRENT DISTRIBUTIONS
C******************************************************************************
WRITE(10,502)
502 FORMAT('//17X,'CURRENT DISTRIBUTIONS ON MONOPOLE')
WRITE(10,506)
506 FORMAT('/5X,'I',12X,'CJ(I)',15X,'CJ(1)',14X,'CJ(I)')
WRITE(10,508)
508 FORMAT(17X,'(NORM)',15X,'(MAG)',BX,'(PHASE IN DEGREES)1,5X)
C******************************************************************************
RETURN
END

SUBROUTINE PRRES(RRAD,RI,XI)
C******************************************************************************
C OUTPUT STATEMENTS FOR RADIATION RESISTANCE, INPUT IMPEDANCE, AND
C RADIATION EFFICIENCY
C******************************************************************************
REAL*8 ETA
WRITE(10,954)RRAD
954 FORMAT(IX,'RADIATION RESISTANCE IN OHMS, Rrad', &
5X,'Rrad = ',F10.4)
WRITE(10,952)RI,XI
952 FORMAT(IX,'INPUT IMPEDANCE IN OHMS, Rin + jXin', &
4X,'Rin = ',F11.4, &
7X,'Xin = ',F11.4)
ETA=RRAD/RI
WRITE(10,956)ETA
956 FORMAT(IX,'RADIATION EFFICIENCY, ETA = Rrad/Rin ', &
4X,'ETA = ',F10.3)
C******************************************************************************
RETURN
END

SUBROUTINE EARTHTYPE
C******************************************************************************
C PRINTS OUT THE PROGRAM NAME AND EARTH TYPE
C******************************************************************************
INTEGER CASE
CHARACTER*40 TYPEEARTH

TYPEEARTH = 'PERFECT GROUND'
CASE = 1

C-7
**IRITE(10, 100)CASE,TYPEARTH**

**FORMAT(/21X,'PROGRAM RICHMD6',5X,'CASE NO. ',I2,',',A))**

C****************************************************************************************************
C****************************************************************************************************
RETURN
END

**SUBROUTINE HEADING2(AL,HL,BAR,CMN,IFLAG)**

PRINTS THE HEADINGS FOR OVER FLAT LOSSY EARTH

C****************************************************************************************************
C****************************************************************************************************
C PROGRAM DESCRIPTION AND ECHOED INPUT (WITH EARTH) *
C****************************************************************************************************
C****************************************************************************************************
CALL EARTHTYPE
WRITE(10,224)AL
224 FORMAT(/1X,'MONOPOLE ELEMENT RADIUS IN WAVELENGTHS, AL',35X, 
A 'AL = ',F18.10)
WRITE(10,226)HL
226 FORMAT(/1X,'MONOPOLE ELEMENT LENGTH IN WAVELENGTHS, HL',35X, 
A 'HL = ',F18.10)
WRITE(10,228)
228 FORMAT(/1X,'RATIO OF OUTER TO INNER CONDUCTOR RADI energies OP 
A COAXIAL')
WRITE(10,36)BAR
230 FORMAT(/1X, 'LINE FEED, BAR',62X,'BAR = ',F10.3)
WRITE(10,83)CMN
83 FORMAT(/1X,'CONDUCTIVITY OF MONOPOLE ELEMENT (MCGHRO/METER)', 
A ',',CMN',')
WRITE(10,84)CMM
84 FORMAT(/1X,'CONDUCTIVITY OF MONOPOLE ELEMENT (MCGHRO/METER)', 
A ',CMM = ',F10.3)
WRITE(10,36)
36 FORMAT(/1X,'FLAG FOR MONOPOLE ELEMENT CURRENT DISTRIBUTIONS', 
A ', IFLAG')
WRITE(10,37)
37 FORMAT(/1X,'0, MONOPOLE ELEMENT CURRENTS COMPUTED BY METHOD', 
A 'OF')
WRITE(10,11)
11 FORMAT(/7X,'MOMENTS')
WRITE(10,38)
38 FORMAT(/7X,'-1, MONOPOLE ELEMENT CURRENTS WITH IDEALIZED', 
A 'SINUSIDAL')
WRITE(10,12)
12 FORMAT(/7X,'DISTRIBUTION AND FOR INFINITE CONDUCTIVITY OF THE')
WRITE(10,13)IFLAG
13 FORMAT(/7X,'MONOPOLE ELEMENT',54X,'IFLAG = ',F14)
IF(CMN.EQ.-1)GOTO 88
WRITE(10,86)FHC
86 FORMAT(/1X,'FREQUENCY IN MEGAHERTZ, FHC',41X,'FHC = ',F8.3)
88 CONTINUE

C****************************************************************************************************
C****************************************************************************************************
RETURN

C-8
SUBROUTINE CISI(CI,CIN,SI,X)

C Standard IBM Fortran Subroutine with slight modifications.
C COSINE INTEGRAL AND SINE INTEGRAL.
X = ARGUMENT (REAL AND POSITIVE).
CI = Ci(x).
SI = Si(x).
CIN = Cin(x).

DATA GAM,P2/.57721566,1.57079632/
A=ABS(X)
IF(A.GT.4.)GO TO 10
IF(A.GT.1.)GO TO 3
IF(A.GT.0.)GO TO 2
CI=0
CIN=0
SI=0
RETURN
2 X2=A*A
SI=X*((.03*X2-1.)*X2/18.+1.)
CIN=.25*X2*((X2/45.-1.)*X2/24.+1.)
GO TO 8
3 Y=(4.-A)*(4.-A)
SI=X*((1.753141E-9*Y+1.568988E-7)*Y+1.374168E-5)*Y+6.939889E-4)
CIN=1.964888E-2*Y+4.395509E-1
C+1.725752E-6*Y+1.185999E-4*Y+4.990920E-3*Y+1.315308E-1)
8 CI=GAM+ALOG(A)-CIN
RETURN
10 SI=SIGN(A)
Z=4./A
U=(((4.048069E-3*Z-2.279143E-2)*Z+5.515070E-2)*Z-7.261642E-2)
W=(((5.108699E-3*Z+2.819179E-2)*Z-6.537283E-2)*Z)
C+3.764000E-4*Z-1.22418E-2)*Z-6.646441E-7)*Z+2.500000E-1
CI=2*(SI*V-Y*U)
SI=Z*(SI*Y*V)+P2
IF(X.LT.0.)SI=-SI
CIN=GAM+ALOG(A)-CI
RETURN
END

SUBROUTINE FRILLS(AN,AK,RK,NK,NEQ,VJ)

FRILLS sets up the voltage column VJ(I).
VJ(I) = voltage column for perfectly conducting wire dipole in
free space using Galerkin's method and sinusoidal bases, and matching the boundary conditions on the surface of the wire.

Using magnetic-frill model for center-fed dipole.

```fortran
REAL*8 DZS,RS1,RS2
COMPLEX EGz,GM,GP,GI(20),VJ(1),GI(1),OST,WST
DATA PI,TP/3.14159265359,6.28318530718/
IDM=20
DO 20 I=1,NEQ
  20 VJ(I)=(0.,0.)
IF(BAR.LE.1.)RETURN
  VJ(I)=(0.,0.)
NSV=NEQ-1
SDK=SIN(DK)
CDK=COS(DK)
BAL=ALOG(BAR)
QST=CMPLX(0,1./4.*BAL*SDK)
BK=AK*BAR
AKS=AK*AK
BKS=BK*BK
LIM=NSV+1
IF(LIM.GT.IDM)LIM=IDM
NPH=6
NPB=2*(NPH/2)
NPP=NPH+1
PHA=.0174533*20.
DPH=PHA/NPH
PH=.0
DO 90 LPH=1,2
  90 VST=DPH*OST/(3.**PI)
G1=1
DO 80 IPH=1,NPP
  80 WF=3.*SGN
IF(IPH.EQ.1.)WF=1.
IF(IPH.EQ.0.)WF=1.
CPH=COS(PH)
IF(IPH.GT.1)GO TO 40
IF(LP1.GT.1)GO TO 40
CPH=COS(DPH/10.)
RS1=2.*AKS*(1.-CPH)
RS2=AKS+BKS-2.*AK*CPH
RB1=DSQRT(RS1)
RB2=DSQRT(RS2)
CALL CISI(CA,CIN,SA,RB1)
CALL CISI(CB,CIN,SB,RB2)
GI(1)=2.*CMPLX(CB-CA,SA-SB)
DO 50 I=2,LIM
  50 DZ=DZ*2
DO 90 IPH=1,NPP
  90 WF=3.*SGN
IF(IPH.EQ.1.)WF=1.
IF(IPH.EQ.0.)WF=1.
CPH=COS(PH)
IF(IPH.GT.1)GO TO 40
IF(LP1.GT.1)GO TO 40
CPH=COS(DPH/10.)
RS1=2.*AKS*(1.-CPH)
RS2=AKS+BKS-2.*AK*CPH
RB1=DSQRT(RS1)
RB2=DSQRT(RS2)
CALL CISI(CA,CIN,SA,RB1)
CALL CISI(CB,CIN,SB,RB2)
GI(1)=2.*CMPLX(CB-CA,SA-SB)
DO 50 I=2,LIM
  50 DZ=DZ*2
DO 90 IPH=1,NPP
  90 WF=3.*SGN
IF(IPH.EQ.1.)WF=1.
IF(IPH.EQ.0.)WF=1.
CPH=COS(PH)
IF(IPH.GT.1)GO TO 40
IF(LP1.GT.1)GO TO 40
CPH=COS(DPH/10.)
```

C-10
SUBROUTINE TPLZ(C,U,VJ,V,Z,IER,IVR,II2,NEQ)

C MODIFIED VERSION OF SUBROUTINE FURNISHED BY CHARLES KLEIN.
C SOLVES SIMULTANEOUS LINEAR EQUATIONS.
C
C SET IVR = (I OR 0) TO GET (PRINTOUT OR NO-PRINTOUT).
C SET II2 = 1 ON FIRST CALL, WHERE MATRIX INVERSION IS REQUIRED.
C SET II2 = 2 IF MATRIX Z HAS ALREADY BEEN INVERTED ON PREVIOUS CALL.
C NEQ = NUMBER OF SIMULTANEOUS LINEAR EQUATIONS.
C Z(J) IS THE FIRST ROW OF THE TOEPLITZ IMPEDANCE MATRIX
C VJ(J) = INPUT VOLTAGE COLUMN
C C(J) = OUTPUT CURRENT COLUMN
C U(J) AND V(J) ARE WORK ARRAYS OF LENGTH NEQ
C IF IER = 0 , NO ERROR OCCURRED

C***************************************************************

C COMPLEX C(1),U(1),VJ(1),V(1),Z(1)
COMPLEX ALMDA,ALPHA,C1,C2,COEF,FAC,TAU1,V,V1,V2
2 FORMAT(X,5,7,F.4,7X,F15.7,7X,F10.1)
7 FORMAT(5X)
IF(NEO.GT.1)GO TO 8
C(1)=VJ(1)/Z(1)
C00R=CABS(C(1))
GO TO 100
8 IF(II2.NE.1)GO TO 45
N=NEQ-1
IER=0
C***************************************************************

C NORMALIZE INPUT MATRIX
C***************************************************************

TAU1=Z(1)
DO 10 II=1,N
10 Z(II)=Z(II+1)/TAU1
ALMDA=1-Z(1)*Z(1)
U(1)=-Z(1)
I=2
15 KK=I-1
ALPHA= (.0,.0)
DO 20 N=1,KK
LL=I-M
20 ALPHA=ALPHA+U(N)*Z(LL)
ALPHA=(ALPHA+Z(I))
IF(CABS(ALPHA).EQ.0.0)GO TO 130
COEF=ALPHA/ALMDA
ALMDA=ALMDA-C0EF*ALPHA
DO 30 J=1,KK
L=I-J
30 W(J)=U(J)+COEF*U(L)
DO 40 J=1,KK
40 U(J)=W(J)
U(I)+COEF
IF(I.GE.N)GO TO 45
I=I+1
GO TO 15
C************************************************************
C THE FOLLOWING COMPUTES THE ELEMENTS OF THE INVERSE
C*******************************************************************************
45 NH=(NEO-1)/2
FAC=ALMDA*TAU1
NF=NEQ+1
CNOR=.0
DO 90 I=1,NH
IF(I.NE.1)GO TO 55
V(I)=1./FAC
DO 50 J=2,NF
50 W(J)=U(J-1)/FAC
GO TO 70
55 C1=U(I-1)
NPJ=NP-I
C2=U(NPJ)
DO 60 J=1,NH
J=NPJ-JJ
NPJ=NP-J
60 W(J)=W(J-1)+((C1*U(J-1)-C2*U(NPJ))/FAC)
V(I)=U(I-1)/FAC
C*******************************************************************************
C MATRIX MULTIPLY
C*******************************************************************************
70 V1=(.0,.0)
DO 80 J=1,NF
V2=VJ(J)
V=V+V2*W(J)
NPJ=NP-J
80 V1=V1+V2*W(NPJ)
C(I)=V
NPJ=NP-I
C(NPJ)=V1
IF(JVR.LE.0.0)GO TO 90
CA=CABS(V)
IF(CA.GT.CNOR)CNOR=CA
CA=CABS(V1)
IF(CA.GT.CNOR)CNOR=CA
90 CONTINUE
100 IF(JVR.LE.0.0)GO TO 120
C*******************************************************************************
C PRINT OUT THE SOLUTION FOR THE CURRENTS C(J)
C*******************************************************************************
C***********************************************************************
IF(CNOR.LE.0.)CNOR=1.
DO I=1,(NEQ+1)/2
  V=V(I)
  CA=CABS(V)
  CN=CA/CNOR
  PH=.0
  IF(CA.GT.0.)PH=57.29578*ATAN2(AIMAG(V),REAL(V))
  IF(I.NE.((NEQ+1)/2))WRITE(10,2)I,CN,(CA*2),PH
  IF(I.EQ.((NEQ+1)/2))WRITE(10,22)I,04,(CA*2),PH
END DO
120 RETURN
130 IER=I
RETURN
END

C***********************************************************************
SUBROUTINE TSPAR
***********************************************************************
SUBROUTINE TSPAR(AK,DK,NEQ,Z)
C
TSPAR sets up the impedance matrix Z(J).
C Z(J) = First row of impedance matrix for perfectly conducting
C thin-wire dipole in free space, using Galerkin's method with
C overlapping sinusoidal basis functions and matching the
C boundary conditions on the surface of the wire.
C AK = k*a, where k = 2*pi/lambda and a = wire radius.
C DK = k*d, where d = segment length.
C NEO = number of simultaneous linear equations.
C
REAL*8 DZS,RS
COMPLEX CEM,CEP,EM,EPD,END2,EPD2,Z11,Z22,G11,Ol1
DIMENSION CID(90),SID(90),CH(90),CP(90),SM(90),SP(90)
DATA GA,P2/.577215664, 1.57079632/
DATA ETA,PI/376.727,3.14159/
IDN-90
1 FORMAT(3X,'MUST INCREASE DIMENSIONS IN SUBROUTINE TSPAR')
2 FORMAT(3X,'ACTUAL DIMENSION IDM = ',I5,6X,
2 'REQUIRED DIMENSION MAX2 = ',15)
IF(NEQ.LE.0)RETURN
MAX2=NEQ+2
DO 14 I=1,NEQ
14 Z(I)=(0.,0.)
16 IF(MAX2.LE.IDM)GO TO 16
WRITE(10,1)
WRITE(10,1)IDM,MAX2
10 RETURN
16 TDK=DK
S11=0
S13=TDK
S21=DK
S23=3.*DK
DO 20 N=1,MAX2
I=I+1
DZ=I+DK
CID(N)=COS(DZ)
SID(N)=SIN(DZ)
20 CONTINUE
C-13
20  EID(N)=CMPLX(CID(N),SID(N))
    CDK=COS(DK)
    SDK=SIN(DK)
    EPD=CMPLX(CDK,SDK)
    END=CMPLX(CDK,-SDK)
    BPD=EPD+BPD
    EBP=END+END
    CPH=2.*CDK+END
    CPE=2.*CDK+EPD
    AK2=AK*AK
    CSS=ETA/(8.*PI*SDK*SDK)
    NPH=6
    NPH2=(NPH/2)
    NPH2+1
    PFA=0.0174533*20.
    DPH=PFA/NPH
    PH=0
    DO 100 JPH=1,2
    CST=DPH*ETA/(24.*PI*SDK*SDK)
    C22=DPH/(3.*PI)
    SGH=1.
    DO 80 JPH=1,NPP
    CPH=COS(PH)
    SPH=SIN(PH)
    IF(IPH.GT.1)GO TO 30
    IF(JPH.GT.1)GO TO 30
    PHO=DPH/10.
    CPH=COS(PHO)
    SPH=SIN(PHO)
30  RS=2.*AK2*(1.-CPF)
    RK=DSQRT(RS)
    WF=3.*SGN
    IF(IPH.EQ.1)VF=1.
    IF(IPH.EQ.NPP)VF=1.
    WST=WFP*CST
    W22=WFP*C22
    DO 40 N=1,MAX2
    IF(N.EQ.1)GO TO 30
    ARG=RS*PD
    CALL CISI(CN(N),CIN,SP(N),ARG)
    EN(N)=CMPLX(CN(N),-SN(N))
    IF(N.GT.1)GO TO 38
    CH(1)=CP(1)
    SM(1)=SP(1)
    EN(1)=EP(1)
    GO TO 40
38  ARG=RS/ARG
    CALL CISI(CH(N),CIN,SM(N),ARG)
    EN(N)=CMPLX(CH(N),-SN(N))
    CONTINUE
    R=4.*(-CM(2)+2.*CP(1)-CP(2))
    A=2.+CID(3)+CM(3)+2.*CP(1)-2.*CP(2)+CP(3))
    B=2.*SID(3)+(-SM(3)+2.*SN(2)-2.*SP(2)+SP(3))
    E=4.*(-SM(2)-2.*SP(1)+SP(2))
    C=2.*CID(3)+(-SM(3)+2.*SN(2)-2.*SP(1)+2.*SP(2)-SP(3))
SUBROUTINE ZSURF(AK,CMN,FNC,ZS)

C******************************************************************************
C SUBROUTINE ZSURF
C******************************************************************************

C ZSURF calculates the surface impedance ZS for thin wire.
C
C AK = k*a where k = 2*pi/lambda and a = wire radius.
C
C CMN = conductivity of wire (megamhos/meter)
C
C FNC = frequency (megahertz).

C******************************************************************************

COMPLEX BES,BES1,ZS
DATA ETA,SQT,TP/376.72727,1.41421356,6.2831853/
SQSVE=1.66*SQRT(CMN/TP/FNC/8.85433)

C-15
IF(X.GT.8.) GO TO 50
T=X/8.
T2=T*T
T4=T2*T2
BER=(((.901E-5*T4+.122552E-2)*T4-.08349609)*T4
+2.641914)*T4-32.363456)*T4+113.77778)*T4-64.)*T4+1.
BEI=(((.11346E-3*T4-.01103667)*T4+.52185615)*T4
+2.567638)*T4+72.817777)*T4-113.77778)*T4+16.)*T4
BERP=X*T2*(((.194E-5*T4+.45957E-3)*T4-.02609253)*T4
+2.66047849)*T4-6.0681481)*T4+14.222222)*T4-4.)*BEI
BER-PX*((((.4609E-4*T4-.379386E-2)*T4+14677204)*T4
+2.3116751)*T4+11.377778)*T4-10.666667)*T4-5.)*BEI
S-CMPLX(BER,BEI)
DESIm. 77107*CNPLX(BERP-BEIP, BERP+BEIP)
50 XP=.70710681*X
X1=1./X
F=((-0.459205*X1+.390625E-2)*X1+.08838835)*X1+1.
T=((-0.3960359*X1-.0625)*X1-.08838835)*X1-.39269907*XP
BES=F*CMPLX(COS(T),SIN(T))
F=((-1.19290231*X1+.063515625)*X1-.26516505)*X1+1.
T=((-1.160097*X1-.1875)*X1+.26516505)*X1+1.1780972*XP
BES=F*CMPLX(COS(T),SIN(T))
100 ZS=CMPLX(1.,-1.)*ETA*BES/BES1/SQ/ SQSWE
RETURN
END
APPENDIX D

COMPUTER PROGRAM WAIT-SURTEES FOR THE INPUT IMPEDANCE OF A MONOPOLE ELEMENT ON A DISK GROUND PLANE ABOVE FLAT EARTH
COMPUTER PROGRAM WAIT-SURTEES

by

JACK II. RICHMOND

December 29, 1989

INTRODUCTION

Appendix I presents Richmond's computer program WAIT-SURTEES FOR together with the subroutines CISI, FRILLS, TPLZ, TSPAR and ZSURF. This FORTRAN program calculates the impedance of a vertical monopole antenna centered on a circular disk over the flat lossy earth. This program combines the following:

a) Richmond's moment method for the impedance $Z_\infty$ of a vertical monopole antenna on an infinite ground plane; and

b) The theory of Wait and Surtees for the change $\Delta Z$ in the antenna impedance, where $\Delta Z = Z_f - Z_\infty$ and $Z_f$ denotes the impedance of the vertical monopole on a finite circular ground plane over the flat earth.


Comment statements have been inserted in the main computer program and in each subroutine to assist the user. Only a few brief additional comments will be required in this Introduction.

In calculating $Z_\infty$, the monopole is divided into segments of equal length and the unknown current distribution is expanded in overlapping sinusoidal basis functions. Thus, $I(z)$ is taken to be piecewise sinusoidal. The magnetic-frill model is employed (rather than the slice-generator model), and boundary matching is enforced on the surface of the monopole rather than on the axis. The wire radius is assumed to be much smaller than the wavelength. The wire monopole may be assigned perfect conductivity or finite conductivity as desired. With Galerkin's method, the calculated impedance $Z_\infty$ is believed to be accurate for short, medium and long...
monopoles. If the monopole length exceeds 6 wavelengths, however, one may wish to increase the dimensions (IDM=99) in the main program and subroutine TSPAR.

In the theory of Wait and Surtees, the monopole is assumed to have a sinusoidal current distribution. (It does not appear difficult to generalize this to a piecewise-sinusoidal distribution, but we have not attempted this.) Since the current distribution departs significantly from the sinusoidal form when the monopole length exceeds one-half wavelength, $\Delta Z$ and $Z_f$ may begin to lose reliability as the monopole length increases. We have not investigated this possible problem.

Appendix II presents the output data generated by WAIT-SURTEES.FOR on a VAX computer with the same input data indicated in Appendix I. This output shows excellent agreement with the original results obtained in 1979 on a DATACRAFT computer. This indicates that no additional double-precision operations are required for VAX operation.

Richmond has shown that WAIT-SURTEES.FOR is useful even for a monopole antenna on a circular disk in free space. [J. H. Richmond, "Monopole Antenna on Circular Disk," IEEE Trans., Vol. AP-32, pp. 1282-1287, December 1984]. For this case, set ER=1 and SIG=0 in the main program.

In WAIT-SURTEES.FOR the monopole is centered on a circular disk which may lie on the surface of the earth, or it may be located any distance above the earth surface. In the main program $H_DL$ denotes the height of the circular disk above the flat earth, measured in free-space wavelengths.

For a monopole on a circular disk on the surface of the earth, Richmond has shown satisfactory agreement between WAIT-SURTEES.FOR and Richmond's moment method (which enforces the boundary conditions to determine the current distributions on the monopole and the disk). See: [J. H. Richmond, "Monopole Antenna on Circular Disk Over Flat Earth," IEEE Trans., Vol. AP-33, pp. 633-637, June 1985.]

In the output data of Appendix II, the resistance $R_{FIN}$ is plotted as the dashed-line curve of Figure 5 in [Richmond, 1985]. The reactance $X_{FIN}$ in Appendix II should have been plotted as the dashed-line curve of Figure 6 in [Richmond, 1985]. By mistake, however, the dashed-line curve of Figure 6 shows the output of WAIT-SURTEES.FOR for a monopole on a circular disk in free space.
Appendix I. WAIT-SURTEES.FOR

WAIT-SURTEES.FOR  WAIT.1

C IMPEDANCE OF MONOPOLE AT CENTER OF CIRCULAR DISK ON FLAT EARTH.
C SEE: WAIT AND SURTEES, "IMPEDANCE OF TOP-LOADED ANTENNA OF
C ARBITRARY LENGTH OVER A CIRCULAR GROUNDED SCREEN," J. APPL. PHYS.,
C VOL. 25, PP. 553-555, MAY 1954.
C LINK CISI;FRILLS;PFIL;TPFAR;FZSURF
COMPLEX COG,CGJ,CGA,CGB,DELZ
COMPLEX EC,EGJ,EJH,EJS,ETJ,ETD,ETH
COMPLEX CM,GP,QI,Q2,Q3,QC
COMPLEX X11,X12,DFIN,FIN,RS,SSG
COMPLEX CJ(99),QI(99),Q2(99),Q3(99),M(99),E(99)
DATA ETA,F1,FT376.730366239,3.14159265359,6.28318530718/
DATA E0,U0/8.85418533677E-12,1.25663706144E-6/
DATA P2/1.57079632679/
DATA IDM/99/
C AI,AL,AM = RADIUS OF MONOPOLE WIRE IN (INCHES,WAVELENGTHS,METERS).
C BAR = RATIO OF OUTER RADIUS AND INNER RADIUS OF COAXIAL FEED.
C BL,BM = OUTER RADIUS OF CIRCULAR DISK (WAVELENGTHS,METERS).
C CMN = CONDUCTIVITY (MEGAMHOS/METER) OF MONOPOLE WIRE.
C CMm = -1. FOR PERFECTLY CONDUCTING MONOPOLE.
C ER = RELATIVE PERMITTIVITY OF EARTH.
C FMC = FREQUENCY (MEGAHERTZ).
C HL, HM = LENGTH OF MONOPOLE (WAVELENGTHS,METERS).
C SIG = CONDUCTIVITY OF EARTH (MHOS/METER).
C WAVM = WAVELENGTH IN FREE-SPACE (METERS).
2 FORMAT(IX,8F11.5)
5 FORMAT(1X0)
AL= .003
BAR = 3.
CMN= -1.
ER=4.
FMC=300.
HL=.229
SIG= .001
WAVM=300./FMC
INW=1
OMEG=TP=FMC*1.66
EC=CMPLX(ER,-SIG/(OMEG*EO))
ETA2=ETA/CSQRT(EC)
AK=TP=AL
HK=TP=HL
SK=2.*MK
EJS=CMPLX(COS(SK),SIN(SK))
CHK=COS(HK)
SHK=SIN(HK)
EJH=CMPLX(CHK,SHK)
NS=15.*HL
IF(NS.LT.6)NS=6
IF(NS.GT.IDM)NS=IDM
IG=NS/2
HS=2*IG
N=NS-1
SUM=*H,SUM=RETURN()
Z=SUM(0.,0.)
C ZS = SURFACE IMPEDANCE OF MONOPOLE WIRE.
C DK=HK/IG
CDM=COS(DK)
SDK=SIN(DK)
C ZJ(N) = FIRST ROW OF IMPEDANCE MATRIX FOR DIPOLE IN FREE SPACE,
C USING GALERKIN'S METHOD WITH OVERLAPPING SINEUSOIDAL BASIS FUNCTIONS.
C (DIPOLE LENGTH = TWICE THE MONOPOLE LENGTH, USING IMAGE THEORY.)
C CALL TSPAR(AJK,D,N,ZJ)
IF(CIHM.LT.0.)GO TO 52
G7=2.*(DK-CDK*SDK)
C 3
D-5
GL = SDK - DX + CDK
FH = 4 * PI + RK + SDK + SDK
LJ(1) = RJ(1) + 2 * GY + GH
LJ(2) = RJ(2) + 2 * GY + GH

CALL FRILLS(AK, BAR, DK, H, CJ)
DO 60 I = 1, N
   R = 1 + IABS(IG - I)
   VJ(I) = CJ(N)
60 CALL TPLZ(CJ, U, VJ, W, EJ, ZEPW, IP, 112, N)
CALL TPLZ(CJ, U, VJ, W, EJ, ZEPW, IP, 112, N)

C SOLVE THE SIMULTANEOUS LINEAR EQUATIONS TO DETERMINE THE CURRENTS CJ(N)
C ON THE WIRE DIPOLE IN FREE SPACE. (THE IMPEDANCE MATRIX IS TOEPLITZ.)
CALL TPLZ(CJ, U, VJ, W, EJ, ZEPW, IP, 112, N)

C ZINF = IMPEDANCE OF MONOPOLE ANTENNA OVER INFINITE GROUND PLANE.
C ZINF = 5 / CJ(1G)
WRITE (6, 2) AL, HL, ZINF
WRITE (15, 2) AL, HL, ZINF
WRITE (6, 5)
WRITE (15, 5)

C ZINF - IMPEDANCE OF MONOPOLE ON INFINITE CIRCULAR DISK.
C ZFIN = IMPEDANCE OF MONOPOLE AT CENTER OF FINITE CIRCULAR DISK.
C DELZ = ZFIN - ZINF.
C CALCULATE DELZ USING THE FORMULA OF WAIT AND SURETTES.
CALL CISI(CP, CIN, SP, 2 * (RJ + HK))
CALL CISI(CM, CIN, SM, 2 * (RJ + HK))
Q1 = CMPLX(CP, P2 - SP) * EJS + CMPLX(CM, P2 - SM) / EJS
CALL CISI(CP, CIN, SP, RQ + SK + HK)
CALL CISI(CM, CIN, SM, RQ + BK - HK)
Q2 = CMPLX(CP, P2 - SP) * EJS + CMPLX(CM, P2 - SM) / EJS - CMPLX(CB, P2 - SB)
CALL CISI(CI, CIN, SI, 2 * BQ)
DELZ = Q1 - 4 * CMPLX(CP, P2 - SI)
DELZ = ZINF + DELZ
ZFIN = ZINF + DELZ
WRITE (15, 2) BL, ZFIN
WRITE (6, 2) BL, ZFIN
WRITE (15, 2) BL, ZFIN
400 CALL EXIT
END
SUBROUTINE CISI(CI,CIN,SI,X)

Standard IBM Fortran Subroutine with slight modifications.

COSINE INTEGRAL AND SINE INTEGRAL.

X = ARGUMENT (REAL AND POSITIVE).

CI = CI(x).

SI = SI(x).

CIN = Cin(x).

DATA GAM.PZ/.57721566,1.57079632/

A=ABS(X)

IF(A.GT.4.)GO TO 10

IF(A.GT.1.)GO TO 3

IF(A.GT.0.)GO TO 2

CI=0

CIN=0

SI=0

RETURN

2 X2=A*A

SI=SI+((.03*X2-1.)*X2/18.+1.)

CIN=CIN+25*X2*(X2/45.-1.)*X2/24.+1.)

GO TO 8

3 Y=(4.-A)*(4.-A)

SI=SI+(((1.753143E-9*Y+1.58888E-7)*Y+1.37416E-5)*Y+9.9999E-4)

C=1.96488E-2*Y+4.39550E-1

CIN=CIN+(((1.38698E-10*Y+1.58499E-8)*Y

C+1.72575E-6)*Y+1.18599E-4)*Y+4.99092E-3)*Y+1.31530E-1)

8 CI=GAM+ALOG(A)-CIN

RETURN

10 SI=SIN(A)

Y=COS(A)

Z=4./A

U=((((((4.048069E-3*Z-2.279143E-2)*Z+5.51507E-2)*Z+7.26162E-2)

C*2+4.905766E-2)*Z+1.332519E-3)*Z+2.314617E-2)*Z+1.34958E-5)*Z

C+6.250011E-2)*Z+2.383989E-10

V=((((((5.10889E-3*Z+2.81917E-2)*Z+6.53723E-2)*Z

C+1.90203E-2)*Z+4.40041E-2)*Z+7.94555E-3)*Z+2.60129E-2)*Z

C-3.764000E-4)*Z+3.12214E-2)*Z+6.44444E-7)*Z+2.56000E-1

C=Z*(51V-Y*U)

SI=SI+(SI+Y*V)*P2

IF(X.LT.-1)SI=-SI

CIN=GAM+ALOG(A)-CI

RETURN

END
SUBROUTINE FRILLS(AK, BAR, DX, NEQ, VJ)
FRILLS sets up the voltage column VJ(I).
VJ(I) = voltage column for perfectly conducting wire dipole in
free space using Galerkin's method and sinusoidal bases, and
matching the boundary conditions on the surface of the wire.
Using magnetic-frill model for center-fed dipole.
REAL*8 DZ, RS1, RS2
COMPLEX EGI, GM, GP, GI(20), VJ(1), GI, QST, WST
DATA PI, TP/3.141592653589, 6.28318530718/
IDM=20
DO 20 I=1,NEQ
VJ(I)=(0., 0.)
VJ(1)=(1., 0.) RETURN
VJ(I)=(0., 0.)
NS=NEQ+1
SDK=SIN(DX)
CDK=COS(DX)
BAL=Aalog(BAR)
QST=CHPLX(0., 1.)/(4.*BAL+SDK)
BK=AK*BAR
RS=AK*AK
LIM=NS+1
IF (LIM.GT.IDM)LIM=IDM
NPH=6
NPH=2*(NPH/2)
NPP=NPH+1
PHA=0.0174533*20.
DPH=PHA/NPH
PH=0
DO 90 LPN=1,2
WST=DPH*QST/(3.*PI)
SGN=-1.
DO 80 IPH=1,NPP
W=3.*SGN
IF (IPH.EQ.1) W=1.
CPH=COS(PH)
IF (IPH.GT.1) GO TO 40
IF (IPH.GT.1) GO TO 40
CPH=COS(PH)
IF (IPH.GT.1) GO TO 40
IF (IPH.GT.1) GO TO 40
CPH=COS(PH)
CPH=CPH*(LIM/10.)
40 RS1=2.*AKS*(1.-CPH)
RS2=AKS*BKS*AKS*2.*AK*BK*CPH
RH1=DSQRT(RS1)
RH2=DSQRT(RS2)
CALL CI1SI(CA, CIN, SA, RH1)
CALL CI1SI(CR, CIN, SB, RH2)
G1((I)=2.*CHPLX(CB-CA, SA-SB)
DO 50 I=1,2
50 GZ=CMPLX(C2-Cl, 81-52)
ZGZ=CMPLX(COS(DZ), VIS(WP)))
GI(I)=W*WST+(1I)-CDK*GI(I))
IF (NEQ.LE.1) GO TO 78
K1=0
DO 60 I=2,NEQ
K1=K1+1
K2=K1+1
K3=K2+1
IF (K3.GT.IDM) GO TO 60
GP=GI(K1)-2.*CD*K(G2)+GI(K3)
VJ(1)=VJ(1)+WF*WST*GP
60 CONTINUE
78 SGN=-SGN
80 PH=PH+DPH
DPH=(P1-PNA)/NPH
90 PH=PNA
VJ(1)=2.*VJ(1)
RETURN
END
SUBROUTINE TPLZ (C, U, VJ, W, IER, IMR, II2, NEQ)

C MODIFIED VERSION OF SUBROUTINE FURNISHED BY CHARLES KLEIN.
C SOLVES SIMULTANEOUS LINEAR EQUATIONS.
C SET IMR = 1 OR 0) TO GET (PRINTOUT OR NO-PRINTOUT).
C SET II2 = 1 ON FIRST CALL, WHERE MATRIX INVERSION IS REQUIRED.
C SET II2 = 2 IF MATRIX 2 HAS ALREADY BEEN INVERTED ON PREVIOUS CALL.
C NEQ = NUMBER OF SIMULTANEOUS LINEAR EQUATIONS.
C II(J) IS THE FIRST ROW OF THE TOEPLITZ IMPEDANCE MATRIX
C VJ(J) = INPUT VOLTAGE COLUMN
C C(J) = OUTPUT CURRENT COLUMN
C U(J) AND W(J) ARE WORK ARRAYS OF LENGTH NEQ
C IF IER = 0, NO ERROR OCCURRED
C COMPLEX C(1), D(1), VJ(1), W(1), Z(1)
C COMPLEX ALMDA, ALPHA, C1, C2, COEF, FAC, TAU1, V, V1, V2
2 FORMAT(1X, 15, F10.3, F15.7, F10.1)
5 FORMAT(1X)
1 FORMAT(1X, 15, F10.3)
8 FORMAT(1X, 15, F10.3)
10 FORMAT(1X, 15, F10.3)
15 FORMAT(1X, 15, F10.3)
20 FORMAT(1X, 15, F10.3)
30 FORMAT(1X, 15, F10.3)
40 FORMAT(1X, 15, F10.3)
45 FORMAT(1X, 15, F10.3)
50 FORMAT(1X, 15, F10.3)
55 FORMAT(1X, 15, F10.3)

C THE FOLLOWING COMPUTES THE ELEMENTS OF THE INVERSE
45 NH = (NEQ+1)/2
FAc = ALMDA*TAU1
NP = NEQ+1
CHN = 0
C DO 90 I = 1, NH
IF (I.LT.1) GO TO 55
W(I) = 1./FAC
DO 50 J = 1, WQ
50 W(J) = W(J-1)/FAC
GO TO 70
DO 55 J = 1, NEQ
C1 = U(J-1)
NP1 = NP - 1
C2 = U(NP1)
C = 50 J = 1, WQ
55 IF (I.EQ.N) GO TO 45
I = I + 1
GO TO 15

C THE FOLLOWING NORMALIZE INPUT MATRIX
TAU1 = Z(1)
DO 10 I = 1, N
10 Z(I) = Z(I)/TAU1
ALMDA = 1. - Z(1)*Z(1)
U(1) = Z(1)
I = 2
20 ALMDA = ALMDA + U(M+1)*Z(LL)
ALMDA = (ALMDA + U(M+1)*Z(LL)
IF (CABS(ALMDA).EQ. 0) GO TO 130
ALMDA = ALMDA - ALMDA*COEF*ALMDA
DO 30 J = 1, KK
30 U(J) = U(J) - COEF*U(J-1)
GO TO 70
55\nW(J) = W(J+1)
U(J) = COEF
IF (I.GE.N) GO TO 45
I = I + 1
GO TO 15

C THE FOLLOWING COMPUTES THE ELEMENTS OF THE INVERSE
45 NH = (NEQ+1)/2
FAc = ALMDA*TAU1
NP = NEQ+1
CHN = 0
C DO 90 I = 1, NH
IF (I.LT.1) GO TO 55
W(I) = 1./FAC
DO 50 J = 1, WQ
50 W(J) = W(J-1)/FAC
GO TO 70
DO 55 J = 1, NEQ
C1 = U(J-1)
NP1 = NP - 1
C2 = U(NP1)
C = 50 J = 1, WQ
55 IF (I.EQ.N) GO TO 45
I = I + 1
GO TO 15
60 W(J) = M(J-1)*(C1*U(J-1) - C2*U(NFJ))/FAC
   W(J) = U(J-1)/FAC
C MATRIX MULTIPLY
70 V= (0.,0.)
   V1= (0.,0.)
   DO 80 J=1,NEQ
   V2=V(J)
   V=V+V2*W(J)
   NFJ=NFJ+1
80 V1=V1+V2*W(NFJ)
   C(I)=V
   NP1=NP1-1
   C(NP1)=V1
   IF (IMR.LE.0.) GO TO 90
   CA=CABS(V)
   IF (CA.GT.CNOR) CNOR=CA
   CA=CABS(V1)
   IF (CA.GT.CNOR) CNOR=CA
90 CONTINUE
100 IF (IMR.LE.0.) GO TO 120
C PRINT OUT THE SOLUTION FOR THE CURRENTS C(J)
   WRITE(6, 5)
   IF (CNOR.LE.0.) CNOR=1.
   DO 110 I=1,NEQ
   V=C(I)
   CA=CABS(V)
   CN=CA/CNOR
   PH= 0
   IF (CA.GT.0.) PH=57.29578*ATAN2 (AIMAG(V), REAL(V))
110 WRITE(6,2)1, CN, CA, PH
   WRITE(6, 5)
120 RETURN
130 IER=1
   RETURN
C
SUBROUTINE TSPAR(AK,DK,NEQ,2)
TSPAR sets up the impedance matrix Z(J).

Z(J) = First row of impedance matrix for perfectly conducting thin-wire dipole in free space, using Galerkin's method with overlapping sinusoidal basis functions and matching the boundary conditions on the surface of the wire.

AK = k*a, where k = 2*pi/lambda and a = wire radius.

DK = k*d, where d = segment length.

NEQ = number of simultaneous linear equations.

REAL'S DIS,RS COMPLEX EID(90),EM(90),EP(90),Z(1)
COMPLEX CID,CEM,END,EPD,EPD2,D11,D22,G11,G11
DIMENSION CID(90),EMID(90),CP(90),SM(90),SP(90)
DATA GAM,P2/.517215664,1.57079632/
DATA ETA,PI/376.727,3.14159/
IDM=90
1 FORMAT(3X, 'MUST INCREASE DIMENSIONS IN SUBROUTINE TSPAR')
2 FORMAT(3X, 'ACTUAL DIMENSION IDM = ',I5)

IF(NEQ.LE.0)RETURN
MAX2=NEQ+2
DO 14 I=1,NEQ
14 IF(MAX2.LE.IDM)GO TO 16
WRITE(6,1)IDM,MAX2
RETURN
16 TDK=-DK
S11=0.
S13=TDK
S21=DK
S23=1.*DK
DO 20 N=1,MAX2
1=N-1
DI=1.*DK
CID(N)=COS(DI)
SID(N)=SIN(DI)
20 EID(N)=CMPLX(CID(N),SID(N))
CDK=COS(DK)
SDK=2.*CDK
EID=CMPLX(CDK,SDK)
END=CMPLX(CDK,-SDK)
EID2=EID*EID
END2=END*END
CEM=2.*CDK*END
CDR=COS(DF)
SOR=SIN(DF)
EPD=CMPLX(CDR,SOR)
EMD=CMPLX(CDR,-SOR)
EPD2=EPD**2
EMD2=EMD**2
AK=AK+AK
CSS=ETA/(8.*PI*SDK*SDK)
NPH=6
NPH=2.*NPH/2
NPP=NPH+1
PHA=0.9174533*20.
DPH=PHA/NPH
FPH=0.
DO 100 JPH=1,2
C22=DPH*ETA/(24.*PI*PI*SDK*SDK)
C22=DPH/(3.*PI)
SGN=-1.
DO 80 JPH=1,NPH
CPH=COS(PH)
SPP=SIN(PH)
IF(JPH.GT.1)GO TO 30
IF(JPH.GT.1)GO TO 30
PNO=DPH/10.
CPH=COS(PH)
10
D-12
30 SPH=SIN(PH0)
RH=AK+SPH
RM=2.*MAX2*(1.-CPH)
RR=JSQRT(RS)
WF=1.*SGN
IF(1*PH.EQ.1)WF=1.
IF(1*PH.EQ.NP)WF=1.
WST=WF*CST
W2=W*F2*C2
DO 40 N=1,MAX2
I=1
D1=DX
D2=DZ
R=DSQRT(RS+D1)
ARG=RI
IF(N.EQ.1)ARG=RX
CALL CISI(CP(N),CIN,SP(N),ARG)
EP(N)=CMPLX(CP(N),-SP(N))
IF(N.GT.1)GO TO 38
CM(1)=CP(1)
SN(1)=SP(1)
EM(1)=EP(1)
GO TO 20
38 ARG=RS/ARG
CALL CISI(CM(N),CIN,SM(N),ARG)
EN(N)=CMPLX(CM(N),-SM(N))
40 CONTINUE
R=4.-(CM(2)+2.*CP(1)-CP(2))
A=2.*CID(3)+(-CM(3)+2.*CM(2)-2.*CP(1)+2.*CP(2)+CP(3))
B=2.*SID(3)+(-SM(3)+2.*SM(2)+2.*SP(1)+2.*SP(2)-SP(3))
X=4.*SM(2)-2.*SP(2)
C2=-CM(2)+(-SM(3)+2.*SM(2)-2.*SP(1)+2.*SP(2)-SP(3))
D2=-SID(3)+(-CM(3)+2.*CM(2)-2.*CP(1)+2.*CP(2)+CP(3))
Z(1)=Z(1)+2.*CMPLX(R,X)
IF(N.EQ.1)GO TO 70
R=2.*CID(2)+(-CM(3)+3.+CM(2)-4.*CP(1)+3.*CP(2)-CP(3))
A=2.*SID(2)+(-SM(3)+2.*SM(2)+2.*SP(1)+2.*SP(2)-SP(3))
B=2.*CID(4)+(-CM(3)+2.*CM(2)-2.*CP(1)+2.*CP(2)-CP(3))
G=2.*CID(4)+(-SM(4)+2.*SM(3)-SM(2)+2.*SP(1)+2.*SP(2)-SP(3))
X=2.*SID(4)+(-CM(4)+2.*CM(3)-2.*CP(1)+2.*CP(2)-CP(3))
J=2.*SID(4)+(-CM(4)+CM(3)-CM(2)+CP(2)-2.*CP(3)+CP(4))
S(2)=S(2)+2.*CMPLX(R,X)
IF(N.EQ.2)GO TO 70
SI=DX
DO 60 M=1,N-1
M2=M-2
M1=M-1
M2=N+2
M1=N+1
CPA=CP(M2)-2.*CP(M1)+CP(N)
CPB=-2.*CP(M1)-CP(M2)+CP(N)
CPB=CP(N2)-2.*CP(N1)+CP(N)
CMC=CM(N2)-2.*CM(N1)+CM(N)
CMN=CM(N2)-2.*CM(N1)+CM(N)
CMC=CM(N2)-2.*CM(N1)+CM(N)
CMN=CM(N2)-2.*CM(N1)+CM(N)
SPA=SP(M2)-2.*SP(M1)+SP(N)
SPB=-2.*SP(M1)-SP(M2)+SP(N)
SPB=SP(N2)-2.*SP(N1)+SP(N)
SPA=SP(N2)-2.*SP(N1)+SP(N)
SMA=SM(N2)-2.*SM(N1)+SM(N)
SMB=SM(N2)-2.*SM(N1)+SM(N)
SMB=SM(N2)-2.*SM(N1)+SM(N)
X=2.*CID(N2)+(-CP(N)-CM(N)+CP(N1)+CM(N2)+2.*SID(N1)+SP(N2)-SM(N))
IF(N.GT.3)GO TO 80
I=I+1
D-13
X = (C + D) * (E - F) * (G + H)

L = (I + J) * (K - L) * (M + N)

IF (N > 3) THEN X = X * (P - Q) * (R + S)

Z = Z * (T - U) * (V + W)

RETURN

END
SUBROUTINE ZSURF(AK, CHM, FMC, ZS)

ZSURF calculates the surface impedance ZS for thin wire.

AK = k*a where k = 2*pi/lambda and a = wire radius.

CHM = conductivity of wire (megohms/meter)

FMC = frequency (megahertz).

DATA ETA, SQ, TF, SQSWE, 1.66, SQRT(CHM/TF/FMC/B.85433)

X = AK * SQSWE
IF(X.GT.8.) GO TO 50

ZS = 8.

T = 2.

T2 = T2 * T

BEN = (((-0.901E-5*T4+.325322E-2)*T4-.8349609)*T4

2-2.6149164*T4-32.363456*T4+11.377778*T4-4.*T4+1.

BZ = (((-113462-3*T4-.01103667)*T4+.52185615)*T4

2-10.5676584*T4+72.4177779*T4-113.777778*T4+15.)*T2

BERP = x**T2 * (((-0.394E-5*T4+.459578e-3)*T4-.02609253)*T4


BEIP = x(((-4609E-4*T4-.4693866-2)*T4+.14677204)*T4

2-2.3117152)*T4+11.377778)*T4+10.4666667)*T4+5.

BESE = CMLX(BERP, BEI).

BESE1 = .707107*CMLX(BERP-BEIP, BERP-BEIP)

GO TO 100

50 XP = .70710681*X

X1 = 1. / X

P = (-.0459205*X1+.39956256E-2)*X1+.09938835*X1+1.

T = (-.04603559*X1-.0625)*X1-.08838835*X1-.39269907*XP

RE = F-CMLX(COS(T), SIN(T))

F = (-.11290211*X1+.03156255*X1-.26516505)*X1-1.

T = (+.1160097*X1+.1875)*X1+.26516505*X1+1.1780972*XP

CHM = F-CMLX(COS(T), SIN(T))

100 ZS = CMLX(1.,-1.)*ETA*BESE/BESE1/SQ/SQSWE

RETURN

END
### Appendix II. Output data from Wait-Surtees.

<table>
<thead>
<tr>
<th>AL</th>
<th>HL</th>
<th>RINF</th>
<th>XINF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00300</td>
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