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ABSTRACT

This paper addresses a specific reactive-flow configuration, namely, the interaction of a detonation wave with convected homogeneous isotropic weak turbulence (which can be constructed by a Fourier synthesis of small-amplitude shear waves). The effect of chemical heat release on the rms fluctuations downstream of the detonation is presented as a function of Mach number. In addition, for the particular case of the von Karman spectrum, the one-dimensional power spectra of these flow quantities is given.
1. INTRODUCTION. Shock-turbulence interaction is an ubiquitous phenomenon present practically in all high-speed flows of technological importance. It has attracted much attention recently owing to the renewed interest in aerospace planes that will cruise at hypersonic speeds. The propulsion devices of these vehicles will be either the SCRAMJET Engine or even possibly the Oblique Detonation Wave Engine (ODWE), although it must be mentioned at once that the latter device is still a research concept. In the SCRAMJET Engine, the hypersonic free stream is compressed and retarded by the inlet to supersonic Mach numbers ranging from 2 to 6 and allowed to mix with hydrogen in the combustor. The mixing rate at these Mach numbers is known to be greatly reduced owing to as yet little known mechanisms ascribed to compressibility. To enhance the mixing process, appropriately generated shock wave interactions have been proposed (Kumar, Bushnell and Hussaini\(^1\)). In the Oblique Detonation Wave Engine, the stability of the shock-induced detonation wave in the presence of turbulence is still an open question.

Relevant theoretical studies mostly deal with the interaction of a shock wave with a disturbance which is a vorticity wave, an entropy wave or a sound wave (see Ribner\(^2\) and Zang, Hussaini and Bushnell\(^3\) and references cited therein). Such studies explain the fundamental mechanisms at play in shock-turbulence interactions. The numerical simulation of the interaction of an entropy spot with a shock wave (Hussaini, Collier and Bushnell\(^4\)) show that such interactions could be a potent source of turbulence production or enhancement, especially in reacting flows. The first study of shock-vorticity wave interaction in reactive flows appears to be that of Jackson, Kapila and Hussaini\(^5\). They show that exothermicity amplifies the resultant triad of vorticity, entropy and acoustic waves, significantly more so near the critical angle of incidence. These results which are based on linear theory are verified by the numerical solutions of the Euler equations obtained by Lasseigne, Jackson and Hussaini\(^6\). The numerical simulations, apart from enabling some limits to be placed on the validity of the linear theory, throw some light on the transient response, and provide results of purely nonlinear nature.
The present investigation extends the earlier study to include a broad spectrum of waves. Specifically, a convected field of isotropic turbulence is allowed to interact with a detonation wave. Following Ribner\textsuperscript{2,7}, the turbulence is further assumed to be solenoidal. This is probably justified in the light of Morkovin's hypothesis (Bradshaw\textsuperscript{8}, Hussaini, Erlebacher and Sarkar\textsuperscript{9}) that the structure of turbulence (except for jets) for free stream Mach number less than five is similar to the corresponding constant-density flow. In the next section we formulate the problem while in the following two sections we give selected results. Finally, conclusions are given in Section 5.

2. FORMULATION. Consider a three-dimensional field of small disturbances (representative of turbulence in some sense) in an otherwise uniform stream ahead of a reacting shock (detonation). Let this pattern be convected through the detonation at some instant of time. The goal is to determine the nature of the downstream turbulence as a function of the normal upstream Mach number and chemical heat release.

In this paper the turbulence length scale $l_T$ is assumed to be much larger than the thickness of the detonation, so that the detonation can be treated as a discontinuity in an otherwise inert flow. Of course, the detonation is not discontinuity actually but in general consists of a lead shock, an induction zone and an explosion zone. If the thickness of these combined zones $l_D$ is such that $l_D / l_T << 1$, then the details of the reaction scheme are not important, only the overall heat release. This is a restrictive assumption since induction zones can be quite large, but then the present results may not hold under these conditions. The generalized Rankine-Hugoniot relations across this discontinuity provide the proper conditions (see, e.g., Williams\textsuperscript{10}). A result of this analysis is that there exists a minimum value $M_{CJ}$ (called the Chapman-Jouguet Mach number) of the normal Mach number $M$ ahead of the shock for which a steady detonation wave can possibly be sustained. The Chapman-Jouguet Mach number is given by
\[ M_{ci} = \left\{ 1 + (1 + \gamma) \alpha + \sqrt{(1 + (1 + \gamma) \alpha)^2 - 1} \right\}^{1/2}, \tag{1} \]

where \(\gamma\) is the ratio of specific heats (taken here to be 1.4 for all calculations) and \(\alpha\) is the heat release parameter which characterizes the strength of the reaction.

A snapshot of a turbulent velocity field may be represented as a three dimensional Fourier integral or spectrum of plane waves with normal along the (generally oblique) wave vector \(K\). We specify weak turbulence, so that it may be taken as incompressible, even though convected by supersonic flow. The constraint of incompressibility on the fluctuating quantities then dictates that the waves are transverse (Batchelor; Ribner); they are sinusoidal "shear waves".

A single spectral shear wave encountering the shock/detonation wave, seen edge-on, appears as in Figure 3 of Ribner\(^2\) (it may have a velocity component normal to the page). On the downstream side appears a refracted shear wave, a superposed entropy wave, and a pressure (sound) wave. These are related to the upstream shear wave by transfer functions \(\hat{X}, T,\) and \(P\), respectively. More specifically, if the amplitude of the \(u\) velocity component (normal to the shock) of the upstream shear wave is taken as unity, the corresponding component \(u'\) of the refracted shear wave is \(\hat{X}\), the temperature amplitude in the entropy wave is \(T\), and the pressure amplitude in the sound wave is \(P\). These transfer functions depend on the wave inclination angle \(\theta\) and the upstream normal Mach number \(M\). They are derived by a linearized analysis in Jackson, Kapila and Hossaini\(^5\), and reduce to those of Ribner\(^1\) for the special case of zero heat release; i.e., \(\alpha = 0\) (ordinary shock wave).

These single spectral wave relations dictate corresponding relations between upstream and downstream power spectra. These are implicit in the following integrals of power spectra connecting mean square values of velocity, temperature, and sound pressure perturbations (Ribner\(^2,7\)),

\[ \overline{u^2} = \iiint [u \ u] \ d^3K \tag{2a} \]
\[ \overline{u^2} = \iiint |X|^2 [u u] d^3K \]  \hspace{1cm} (2b)

\[ t^2 = \iiint |T|^2 [u u] d^3K \]  \hspace{1cm} (2c)

\[ p^2 = \iiint |P|^2 [u u] d^3K \]  \hspace{1cm} (2d)

where \( d^3K = dK_1 dK_2 dK_3 \), \([u u]\) is the symbol for the spectral density of \( \overline{u^2} \) in wavenumber space \( K \), and the limits of integration are over the entire wavenumber space. For further details the cited references may be consulted.

3. ROOT-MEAN-SQUARE COMPONENTS. For the special case of isotropic homogeneous turbulence, the longitudinal spectral density \([u u]\) has the general form (Batchelor\textsuperscript{11})

\[ [u u] = K^{-2} G(K) \cos^2 \theta \]  \hspace{1cm} (3)

where \( G(K) \) is an arbitrary function of \( K \). Since the turbulence is assumed isotropic, and hence has spherical symmetry, it is convenient to introduce spherical polar coordinates

\[ K_1 = -K \sin \theta, \quad K_2 = K \cos \theta \cos \phi, \quad K_3 = K \cos \theta \sin \phi, \]  \hspace{1cm} (4a)

\[ d^3K = K^2 \cos \theta dK d\phi d\theta. \]  \hspace{1cm} (4b)

The mean square components of the previous section can now be written as

\[ \overline{u^2} = 2 \int_0^{2\pi} G(K) dK \int_0^\pi d\phi \int_0^\pi \cos^3 \theta d\theta \]  \hspace{1cm} (5a)

\[ \overline{t^2} = 2 \int_0^{2\pi} G(K) dK \int_0^\pi d\phi \int_0^\pi |X|^2 \cos^3 \theta d\theta \]  \hspace{1cm} (5b)

\[ t^2 = 2 \int_0^{2\pi} G(K) dK \int_0^\pi d\phi \int_0^\pi |T|^2 \cos^3 \theta d\theta \]  \hspace{1cm} (5c)

\[ p^2 = 2 \int_0^{2\pi} G(K) dK \int_0^\pi d\phi \int_0^\pi |P|^2 \cos^3 \theta d\theta. \]  \hspace{1cm} (5d)
The actual form of \( G(K) \) is not needed when determining the \( \text{rms} \) components, since it will cancel out when forming ratios. The \( \text{rms} \) components, in percent of freestream velocity, are now defined by (Ribner\textsuperscript{7})

\begin{align*}
\text{lateral velocity:} & \quad \% \left( \frac{\bar{v}^2}{\bar{u}^2} \right)^{1/2} \\
\text{longitudinal velocity:} & \quad \% \left( \frac{\bar{u}^2}{\bar{u}^2} \right)^{1/2} \\
\text{temperature:} & \quad \% \left( \frac{\bar{t}^2}{\bar{u}^2} \right)^{1/2} \\
\text{pressure:} & \quad \% \left( \frac{\bar{p}^2}{\bar{u}^2} \right)^{1/2}
\end{align*}

where \( \% \) means the percent of the preshock longitudinal component of turbulence to the mean velocity of the free stream, and \( r = M / M_{CJ} \). Note that the \( \text{rms} \) components are independent of the preshock spectra, so long as it is consistent with isotropy.

Figure 1 gives the variation of the \( \text{rms} \) components with \( M / M_{CJ} \), the ratio of the upstream Mach number to the Chapman-Jouguet Mach number, for various values of the heat release parameter \( \alpha \). Here the preshock turbulence intensity is 1\% of freestream (\( \% = 1 \)), and the \( \text{rms} \) pressure fluctuation is measured far downstream of the detonation wave (\( x = \infty \)). Figure 1a corresponds to the results of Ribner\textsuperscript{2} for \( \alpha = 0 \), and is provided as a reference case for \( \alpha > 0 \). One can see from Figures 1b,c that as \( \alpha \) is increased so do all the \( \text{rms} \) components, with the greatest changes occurring for \( 1 < M / M_{CJ} < 2 \). As \( M / M_{CJ} \to \infty \), the \( \text{rms} \) values are independent of \( \alpha \). Thus, the effect of heat release is to increase the turbulence levels, with the greatest changes occurring around the Chapman-Jouguet Mach number. Note that the \( \text{rms} \) pressure fluctuation becomes unbounded as \( M \to M_{CJ} \); this behaviour will be analyzed in a future manuscript.

The noise generated by the detonation-turbulence interaction is measured on the acoustic scale in decibels, given by
\[ dB = 20 \log_{10} \left\{ r \frac{\overline{p^2}}{\overline{u^2}} \right\}^{1/2} P_{\text{ref}} \], \quad P_{\text{ref}} = 2 \times 10^{-10} \text{ atm} \quad (6) \]

when the post shock ambient pressure is taken to be 1 atm. Figure 2 gives the variation of the noise in decibels with \( M/M_{\text{ CJ}} \) for a preshock turbulence intensity of 1%. As in Figure 1, the effect of heat release is to increase the noise to extreme levels, with the greatest changes occurring around the Chapman-Jouguet Mach number.

4. ONE-DIMENSIONAL POWER SPECTRUM. The axisymmetry of isotropic turbulence allows us to introduce cylindrical coordinates

\[ K_1 = K_1, \quad K_2 = K_r \cos \phi, \quad K_3 = K_r \sin \phi, \quad (7a) \]
\[ d^3K = K_r \, d\phi \, dK_r \, dK_1. \quad (7b) \]

Substituting these into the mean square components of section 2, and noting that a first integration with respect to \( d\phi \) yields a factor 2\( \pi \), the mean square components (5) become

\[ \overline{u^2} = 2\pi \int_0^{\infty} [u \, u \,] K_r \, dK_r \, dK_1 = \int_0^{\infty} \Phi_u(K_1) \, dK_1 \quad (8a) \]
\[ \overline{u'^2} = 2\pi \int_0^{\infty} |X| \, [u \, u \,] K_r \, dK_r \, dK_1 = \int_0^{\infty} \Phi_{u'}(K_1) \, dK_1 \quad (8b) \]
\[ \overline{\tau^2} = 2\pi \int_0^{\infty} |T| \, [u \, u \,] K_r \, dK_r \, dK_1 = \int_0^{\infty} \Phi_{\tau}(K_1) \, dK_1 \quad (8c) \]
\[ \overline{p'^2} = 2\pi \int_0^{\infty} |P| \, [u \, u \,] K_r \, dK_r \, dK_1 = \int_0^{\infty} \Phi_{p'}(K_1) \, dK_1. \quad (8d) \]

The one-dimensional spectra \( \Phi_u, \Phi_{u'}, \Phi_{\tau}, \) and \( \Phi_{p'} \) are defined in terms of integrals over \( K_r \). These involve the longitudinal power spectral density \([u \, u \,]\) and the respective transfer functions; they are given by
\[ \Phi_i(K_1) = 2\pi \int_0^\infty |\Gamma_i|^2 [u \ u] K_r \ dK_r, \]  

(9a)

where

\[ \Gamma_i = 1, X, T, P, \quad i = u, u', \ell', p'' \]  

(9b)

respectively.

For the evaluation of the one-dimensional spectra, the longitudinal spectral density \([u u]\) of the input turbulence must be specified. Here we chose the von Karman spectral model (see Ribner\(^2\)), defined as

\[ [u u] = \frac{B K_r^2}{2\pi (1 + K_1^2 + K_r^2)^{17/6}}, \]  

(10)

where

\[ B = \frac{55}{18\pi a}, \quad a = 1.3390 \]

The normalizing constant \(a\) is chosen so that of \(\overline{u^2}\) in (8a) is consistent with the von Karman model, i.e.,

\[ B^{-1} = \int_0^\infty \int_{-\infty}^\infty \frac{K_r^2}{(1 + K_1^2 + K_r^2)^{17/6}} K_r \ dK_r \ dK_1. \]  

(11)

In terms of the von Karman spectral model, the one-dimensional spectra are now defined as

\[ \frac{\Phi_i(K_1)}{\overline{u^2}} = B \int_0^\infty \frac{|\Gamma_i|^2 K_r^3}{(1 + K_1^2 + K_r^2)^{17/6}} dK_r, \quad i = u, u', \ell', p''. \]  

(12)

Figures 3 and 4 display the (normalized) one-dimensional power spectra calculated from (12). For the actual numerical procedure, however, the above equation is re-expressed in a cylindrical coordinate system (Ribner\(^2\)) which reduces the infinite integral to a finite one. The normalization for \(\Phi_u\)
and $\Phi_{u'}$ is $\overline{u'^2}$, the normalization for $\Phi_{\nu}$ is with respect to the ambient temperature, and the normalization for $\Phi_{p''}$ is with respect to the ambient pressure. Figure 3 corresponds to the results of Ribner\textsuperscript{2} for $M = 1.25$ and $\alpha = 0$ (hence, $M_{CF} = 1$), and is provided as a reference case for when $\alpha > 0$. Figure 4(a-d) displays $\Phi_i$, $i = u'$, $t'$, and $p''$ (just behind the detonation wave and far downstream), respectively, for various values of the heat release. In particular, Figure 4a shows that the effect of increasing the heat release on the longitudinal component of post-shock turbulence is minimal as compared to the same effect on the temperature and pressure fluctuations. Figures 4b and 4d show that increasing $\alpha$ significantly increases the temperature and far downstream pressure one-dimensional power spectras, respectively, while decreasing the near downstream pressure one-dimensional power spectra (Figure 4c). All cases show the same asymptotic decay (the Kolmogorov $K_1^{-5/3}$ law) beyond $K_1 = 3$.

5. CONCLUSIONS. The interaction of a detonation wave with a convected field of weak isotropic turbulence (which can be constructed by a Fourier synthesis of single, small-amplitude shear waves) has been presented. The effect of exothermicity is to amplify the rms fluctuations downstream of the detonation, with the greatest changes occurring around the Chapman-Jouguet Mach number. However, the asymptotic values for increasing Mach number are unaffected by heat release due to combustion. The far downstream noise generated by the interaction increases substantially with exothermicity, with the greatest changes occurring when the heat release parameter increases from zero to unity. The minimum value of the noise level occurs for Mach numbers about 1.5 times the Chapman-Jouguet Mach number, while for Mach numbers less than this the far downstream noise can reach levels that far exceed those that in still air could permanently damage the ear. For the particular case of the von Karman spectrum, the one-dimensional power spectra of these quantities have been given. In all cases, the one-dimensional power spectra display the Kolmogorov decay and are unaffected by exothermicity. Finally, we comment here that the theory
holds only under the assumption of the reaction zone thickness being much smaller than the turbulence length scale. This is a restrictive assumption since induction zones can be quite large, but then the present results may not hold under these conditions. However, we believe that the true value of this manuscript is not in the ability to predict downstream turbulence levels for general kinetics, but rather provide some insight as to the effect of heat release as a function of Mach numbers greater than the Chapman Jouguet Mach number on the downstream turbulence levels and noise, while at the same time providing the numerical modeler a test case (in the limit of fast chemistry) for their code.

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Figure 1. Plot of the $rms$ components versus $M/M_{CJ}$ for a preshock turbulence intensity of 1%. (a) $\alpha = 0$, (b) $\alpha = 1$, and (c) $\alpha = 5$. The numbers correspond to: (1) $rms$ temperature, (2) $rms$ far-downstream pressure, (3) $rms$ longitudinal, and (4) $rms$ lateral components, respectively.
Figure 2. Plot of the noise, measured in decibels, versus $M/M_{ CJ}$ for a preshock turbulence intensity of 1%.
Figure 3. REFERENCE CASE: Shock-turbulence interaction ($\alpha = 0$). Plot of the (normalized) one-dimensional power spectra versus the wavenumber $K_1$ for $M = 1.25$. The numbers corresponds to the one-dimensional power spectra of the: (1) longitudinal component of pre-shock turbulence, (2) longitudinal component of post-shock turbulence, (3) $10^5 \times$ temperature fluctuation, (4) $10^3 \times$ pressure fluctuation just downstream of the shock, and (5) $10^5 \times$ pressure fluctuation far downstream of the shock. (Recovery of Figure 6 of Ribner (1987)).
Figure 4A. Plot of (a) the (normalized) longitudinal component of post-shock turbulence, and (b) the (normalized) temperature fluctuations, for $M/M_{CJ} = 1.25$ and various values of $\alpha$ governing strength of detonation wave. For appropriate scalings see Figure 3.
Figure 4B. Plot of (c) the (normalized) pressure fluctuations just behind the shock, and (d) the (normalized) pressure fluctuations far downstream of the shock, for $M/M_{cj} = 1.25$ and various values of $\alpha$ governing strength of detonation wave. For appropriate scalings see Figure 3.
This paper addresses a specific reactive-flow configuration, the interaction of a detonation wave with convected homogeneous isotropic weak turbulence (which can be constructed by a Fourier synthesis of small-amplitude shear waves). The effect of chemical heat release on the r.m.s. fluctuations downstream of the detonation is presented as a function of Mach number. In addition, for the particular case of the von Karman spectrum, the one-dimensional power spectra of these flow quantities is given.