Feasibility of Measuring Transverse Electric Noise at VLF and LF on an Ice Cap

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ADMINISTRATIVE INFORMATION

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EXECUTIVE SUMMARY

PROBLEM

Investigate the conditions under which transverse electric (TE) noise can be measured on an ice cap.

RESULTS

The amplitudes of TE waves at the surface of an ice cap are evidently strong enough to be measured at lower low frequencies when the ice cap is about 1000 meters thick and at very low frequencies when the ice cap is about 2000 meters thick. Amplitudes are stronger over colder ice. Measurements of temperature in a drill hole would be needed for extrapolating fields to higher heights.
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INTRODUCTION

There is a need for very low frequency (VLF) and low frequency (LF) transverse electric (TE) air-to-air communications. This need exists because transverse magnetic (TM) signals, which are emitted by vertical land-based transmitting antennas, are subject to deep fades due to modal interference. The TE signals are inherently somewhat weak, however, and measurements of VLF and LF TE noise are needed. Since the noise is variable, these measurements are needed as a function of time of day and, at least to some extent, time of year.

Electromagnetic fields as a function of height are described by VLF waveguide mode theory [Budden, 1961]. The TE field at the surface of high conductivity ground is small and in the limit of infinite conductivity vanishes. The conductivities of sea water and rock are too high for the TE noise field to be feasibly measured near such surfaces. The TE field in the earth-ionosphere waveguide increases rapidly as a function of height. Some measurements of TE noise have been made from a balloon. Measurements of only short duration can be made this way, however. The TE field at the surface of thick ice would be expected to be much larger than over ordinary ground, since the conductivity of ice is smaller. The main purpose of this report is to evaluate the magnitude of this effect and hence the feasibility of making measurements of TE noise on an ice cap.

A formulation is also given in this report for the overall reflection coefficient of TE waves as a function of the properties of the ice cap. In principle, if this reflection coefficient, the value of the fields at the surface of the ice, and a model of the ionosphere are sufficiently well known, the measured TE fields could be extrapolated to higher heights. Field et al. [1986] measured TE fields from a rocket at Wallops Island, Virginia. The fields were transmitted from an aircraft using a horizontal trailing wire antenna. The aircraft was far enough away that one mode was dominant but close enough, except on one flight, so that at least one other mode was significant. The fields as a function of height were found to match waveguide mode theory. Peden and Webber [1970] considered the effect of a layer of ice as a lower boundary of the earth-ionosphere waveguide for TE polarization. They found a resonance effect such that the attenuation in the guide was a maximum when the ice thickness was 0.4 times the wavelength in the ice. They attribute this to waveguide propagation in the ice layer.

This report presents, first, the formulations for reflection from a homogeneous layer of ice and for reflection from a stratified layer of ice. It then describes the basics of pure ice theory for complex permittivity as a function of temperature and frequency. Next it summarizes some measurements of conductivity and complex permittivity in Antarctica. It then gives some sample calculations of the total TE field at the surface of a homogeneous layer of ice. Next it summarizes some measurements of ice thickness, and finally, it discusses the temperature profile of an ice cap.
REFLECTION COEFFICIENTS

The reflection coefficient of a plane wave in free space with the electric vector perpendicular to the plane of incidence and incident on a semi-infinite layer of ice with index of refraction, $n_1$, is given by

$$ R = \frac{C - q_1}{C + q_1} $$

where

$$ C = \cos \theta $$

$$ \theta = \text{angle of incidence} $$

$$ q_1 = (n_1^2 - S^2)^{1/2} $$

$$ S = \sin \theta $$

and where $n_1$ is the index of refraction of the ice. For the corresponding case in which the electric vector is parallel to the plane of incidence, the reflection coefficient is given by

$$ R = \frac{Cn_1^2 - q_1}{Cn_1^2 + q_1} $$

The reflection coefficient of a plane wave incident on a finite homogeneous layer of ice of thickness $\Delta z_1$, over ground (see figure 1) is given by

$$ R = R_u + T_d(WR_gWT_u + WR_gWR_dWR_gWT_u + ...) $$

$$ = R_u + T_d W^2 R_g T_u \sum_{j=0}^{\infty} (R_d W^2 R_g)^j $$

$$ = R_u + \frac{T_d W^2 R_g T_u}{1 - R_d W^2 R_g} $$

where

$$ W = \exp (- ikq_1 \Delta z_1) $$

$$ i = (-1)^{1/2} $$

$$ k = \text{wave number} $$

The needed reflection and transmission coefficients are given by Budden [1985].
Figure 1. Reflection from a homogeneous layer of ice.

For perpendicular polarization

\[ R_u = \frac{C - q_1}{C + q_1} = -R_d \]

\[ T_d = \frac{2C}{C + q_1} \]

\[ T_u = \frac{2q_1}{C + q_1} \]

\[ R_g = \frac{q_1 - q_g}{q_1 + q_g} \]

where \( n_g \) is the index of refraction of the ground. For parallel polarization

\[ R_u = \frac{C n_1^2 - q_1}{C n_1^2 + q_1} = -R_d \]

\[ T_d = \frac{2C n_1^2}{C n_1^2 + q_1} \]

\[ T_u = \frac{2q_1}{C n_1^2 + q_1} \]
\[ R_s = \frac{n_g^2 q_1 - n_1^2 q_g}{n_g^2 q_1 + n_1^2 q_g} \]

In either case
\[ q_g = (n_g^2 - S^2)^{1/2} \]

Ice whose index of refraction varies with depth can be approximated by a sufficiently large number of layers (see figure 2). For \( M \) layers
\[ R_{m-1} = R_u + \frac{T_d W^2 R_m T_u}{1 - R_d W^2 R_m} \]

for \( m = M, \ldots, 1 \), where \( R_M = R_s \), and \( R_o \) is either \( R_{||} \) or \( R_{\perp} \), the desired reflection coefficient for reflection from the ensemble of layers.

\[
\begin{array}{c|c|c|c}
 & n_1 & \Delta z_1 & \\
\hline
n_0 = 1 & & & \\
\hline
n_1 & & & \\
\hline
& & & \\
\hline
& & & \\
\hline
n_{M-1} & & \Delta z_{M-1} & \\
\hline
n_M = n_g & & & \\
\end{array}
\]

Figure 2. Layers of stratified ice cap.

For perpendicular polarization
\[ R_u = \frac{q_{m-1} - q_m}{q_{m-1} + q_m} = - R_d \]
\[ T_d = \frac{2q_{m-1}}{q_{m-1} + q_m} \]
\[ T_u = \frac{2q_m}{q_{m-1} + q_m} \]
For parallel polarization

\[ R_u = \frac{n_m^2 q_{m-1} - n_m^{-2} q_m}{n_m^2 q_{m-1} + n_m^{-2} q_m} = -R_d \]

\[ T_d = \frac{2n_m^2 q_{m-1}}{n_m^2 q_{m-1} + n_m^{-2} q_m} \]

\[ T_u = \frac{2n_m^{-2} q_m}{n_m^2 q_{m-1} + n_m^{-2} q_m} \]

In either case

\[ q_m = (n_m^2 - S^2)^{1/2} \]

\[ W = \exp(-i k q_m \Delta z_m) \]

The index of refraction, \( n \), is the square root of the complex permittivity, \( \epsilon^* \). Conductivity, \( \sigma \), is given by

\[ \sigma = 2 \pi f \epsilon_0 \epsilon'' \]

where \( f \) is the wave frequency, \( \epsilon_0 = 8.85 \times 10^{-12} \) is the permittivity of free space, and \( \epsilon'' \) is such that

\[ \epsilon^* = \epsilon' - i \epsilon'' \]

### THEORY FOR PURE ICE

Values of complex permittivity for pure polycrystalline ice are known as a function of temperature [Auty and Cole, 1952]. The theory is in two parts. The complex permittivity is a function of a relaxation frequency, \( f_r \), which in turn is a function of temperature.

According to Cole [1955] a common characteristic of dielectric and other relaxation theories based on simple molecular or macroscopic models is that they predict a time rate of change of polarization, or other response, which is proportional to the difference of the polarization from its equilibrium value, and that in such cases the complex dielectric constant is expressed by an equation of the form

\[ \epsilon^* = \epsilon_\infty + (\epsilon_{dc} - \epsilon_\infty) / (1 + i \omega \tau_0) \]
This equation is known as the Debye equation. In it $\omega = 2\pi f$, $\tau_0$ is the relaxation time, and $\epsilon_{dc}$ and $\epsilon_{\infty}$ are the values of $\epsilon^*$ in the limit of low and high frequencies, respectively. Both $\epsilon_{dc}$ and $\epsilon_{\infty}$ are real.

Auty and Cole [1952] measured $\epsilon_{\infty}$ to be very nearly 3.10 over the range $-0.1^\circ$ to $-65.8^\circ$C. They measured $\epsilon_{dc}$ to be 91.5, 95.0, 97.4, and 100 at $-0.1^\circ$, $-10.8^\circ$, $-20.9^\circ$, and $-32.0^\circ$C, respectively. However, Peden, Webber, and Chandler [1972] found $\epsilon_{dc} = 84$ for Antarctic ice.

In terms of the relaxation frequency $f_r = 1/2\pi \tau_0$, the Debye equation is

$$\epsilon^* = \epsilon_{\infty} + (\epsilon_{dc} - \epsilon_{\infty}) / (1 + i f / f_r)$$

Measurements of Auty and Cole [1952] give

$$\ln f_r = 33.475 - 6716/T$$

where $f_r$ is in kilohertz and $T$ is in degrees Kelvin.

MEASUREMENTS OF CONDUCTIVITY AND COMPLEX PERMITTIVITY

Crary and Crombie [1972] used 16-kHz signals from Rugby, UK, that propagated over both a long and a short path to calculate the attenuation $\alpha$ resulting from losses in the Antarctic ice sheet. They found $\alpha = 17$ to 31 for the sunlit case. They then used an assumed relationship between $\alpha$ and the conductivity $\sigma$ to deduce a 16-kHz value of $\sigma$ between 10 and 50 $\mu$Sm$^{-1}$.

Webber and Peden [1970] and Peden et al. [1972] made measurements of the vertical magnetic field along the perpendicular bisector of a 34-km antenna at Byrd station in Antarctica to deduce bulk values of the complex permittivity of the ice at several discrete very low frequencies. From an interpolated value of complex permittivity at 16 kHz, they deduced a value of $\sigma = 5$ $\mu$Sm$^{-1}$, a value much lower than that found by Crary and Crombie.

Rogers and Peden [1975] measured the complex permittivity of surrounding ice in a core hole drilled at Byrd Station. Temperature had been measured down to bedrock at a depth of 2164 meters [Gow, Ueda, and Garfield, 1968]. The permittivities were measured down to a depth of 1500 meters, where the ice had since caved in. They used a 3-meter dipole probe and a method of measurement described by Rogers and Peden [1973]. They found that measured relaxation frequencies were 13% to 20% lower than those obtained
by calculation for pure polycrystalline ice and measured temperatures. They concluded that pressure, differences in crystal size, and trace impurities in the form of volcanic ash cannot explain the differences in \( f_r \). They did not consider air bubbles, however, which were reported by Gow, Ueda, and Garfield [1968] to be in the ice.

**COMPUTED SURFACE FIELD VALUES**

Absolute values of the perpendicular component of the electric field at the surface of the ice computed using pure ice theory are given in figures 3–17. The values are normalized to unit incident field. The variables are wave frequency, ice layer thickness, temperature of the ice, and angle of incidence from the vertical. A single homogeneous layer of ice is assumed.

Figures 3–8 give values of the electric field for frequencies from 10 to 60 kHz. The figures are for ice thicknesses of 1,000 and 2,000 meters, temperatures of -10°C, -20°C, and -30°C, and an angle of incidence of 75 degrees. That angle of incidence was chosen to be somewhat representative of earth-ionosphere waveguide eigenangles. In general, the field is stronger for colder ice and for higher frequencies. However, there is also a resonance effect which is due to interference between the wave reflected from the top of the ice and that reflected from the bedrock. The field is a maximum at about 50 kHz in figures 4 and 5, and at about 23 kHz in figures 7 and 8. Hence, the resonance is at lower frequencies for thicker ice.

Figures 9 and 10 also give the electric field values as a function of frequency for thicknesses of 1,000 and 2,000 meters. These values are all for an ice temperature of -20°C, however, and for angles of incidence of 60, 70, and 80 degrees. The field is stronger for steeper angles of incidence, which would correspond to higher order waveguide modes. Again, a resonance effect is evident. The field maximizes at about 45 to 50 kHz in figure 9 and at 20 to 23 kHz in figure 10. Hence, again the resonance is at lower frequencies for thicker ice.

Figure 11 shows the electric field values as a function of ice layer thickness. The curves are for 15, 20, and 30 kHz. All three curves are for an ice temperature of -20°C and an angle of incidence of 75 degrees. The resonance effect is such that the field maximizes at 1600 meters at 20 kHz and at about 1200 meters at 30 kHz.

Electric field values are given as a function of temperature in figures 12–14. These values are for 15, 20, and 30 kHz and for ice thicknesses of 1,000 and 2,000 meters. The angle of incidence is 75 degrees in all three figures. Clearly, the fields are stronger for colder ice.

Figures 15–17 show the electric field as a function of angle of incidence for frequencies of 15, 20, and 30 kHz, ice layer thicknesses of 1,000 and 2,000 meters, and a temperature of -20°C. These figures also show that the field at the surface of the ice would be stronger for higher order modes for the same strength of the incident field.
Figure 3. Total TE field at surface of the ice cap; $T = -10^\circ C$ ($f_r = 2.86$ kHz), $\theta = 75$ degrees.

Figure 4. Total TE field at surface of the ice cap; $T = -20^\circ C$ ($f_r = 1.04$ kHz), $\theta = 75$ degrees.
Figure 5. Total TE field at surface of the ice cap; 
$T = -30^\circ$C ($f_r = 0.35$ kHz), $\theta = 75$ degrees.

Figure 6. Total TE field at surface of the ice cap; 
$T = -10^\circ$C ($f_r = 2.86$ kHz), $\theta = 75$ degrees.
Figure 7. Total TE field at surface of the ice cap; \( T = -20^\circ C \) \((f_r = 1.04 \text{ kHz}), \theta = 75 \text{ degrees}\).

Figure 8. Total TE field at surface of the ice cap; \( T = -30^\circ C \) \((f_r = 0.35 \text{ kHz}), \theta = 75 \text{ degrees}\).
Figure 9. Total TE field at surface of the ice cap; 
T = -20°C (fᵣ = 1.04 kHz), ice thickness = 1000 m.

Figure 10. Total TE field at surface of the ice cap; 
T = -20°C (fᵣ = 1.04 kHz), ice thickness = 2000 m.
Figure 11. Total TE field at surface of the ice cap; $T = -20^\circ C$ ($f_r = 1.04$ kHz), $\theta = 75$ degrees.

Figure 12. Total TE field at surface of the ice cap; frequency = 15 kHz, $\theta = 75$ degrees.
Figure 13. Total TE field at surface of the ice cap; frequency = 20 kHz, $\theta = 75$ degrees.

Figure 14. Total TE field at surface of the ice cap; frequency = 30 kHz, $\theta = 75$ degrees.
Figure 15. Total TE field at surface of the ice cap; frequency = 15 kHz, $T = -20^\circ C$ ($f_r = 1.04$ kHz).

Figure 16. Total TE field at surface of the ice cap; frequency = 20 kHz, $T = -20^\circ C$ ($f_r = 1.04$ kHz).
MEASUREMENTS OF ICE THICKNESS

Ice thickness can be determined by radar or seismic measurements or by the gravitational method. The gravitational method is based on the large density contrast between rock and ice, which is generally in a ratio of about three to one [Bogorodsky, Bentley, and Gudmansen, 1985]. It requires a calibration at a station where the thickness of the ice has been determined by another method. It also requires very accurate altimeter measurements. Its advantage is ease of use on the ground, but it is not as accurate as the other two methods.

Bogorodsky, Bentley, and Gudmansen [1985] quote various sources that report depths of ice at several locations in Antarctica. The ice thickness was calculated from radar reflection measurements to be 2800 meters at South Pole Station. This was in agreement with seismic reflection data. Along a ground traverse in Queen Maud Land, radar reflection echoes were received from depths as great as 3500 meters. In the interior of East Antarctica, the measured maximum ice thickness, determined by using radar, was 4540 meters.

The core hole to bedrock at Byrd Station is 2164 meters deep [Gow, Ueda, and Garfield, 1968]. Webber and Peden [1970] say a depth of 2570 meters has been reported.
for Old Byrd Station, roughly 10 km from the drill site, and 2335 meters at a location approximately 40 km from Old Byrd in the direction of Little America. The latter two values were deduced from a combination of seismic and gravity data associated with a traverse between Old Byrd and Little America.

Drewry, Jordan, and Jankowski [1982] say that 50% of the Antarctic ice sheet has been covered by radio echo-sounding flight tracks on a 50 to 100 km square grid. They give the average ice thickness of West Antarctica as 1780 meters and of East Antarctica as 2630 meters.

Bogorodsky, Bentley, and Gudmansen [1985] give a contour map of the subglacial elevation of Greenland. It seems likely therefore that ice thickness data for much of Greenland is also available.

TEMPERATURE PROFILE

Direct measurements in a drill hole would seem to be necessary in order to know the temperature profile in the ice. The temperature at a depth of about 15 meters is at the mean annual temperature of the air. The temperature just above bedrock tends to be warmer due to heating from the earth's interior. In between, the temperature is not necessarily a linear profile from one to the other, however.

In at least several drill holes, the temperature decreases for the first 50 meters or more to several hundred meters (negative gradient) and then increases with increasing depth [Bogorodsky, Bentley, and Gudmansen, 1985]. These authors do not explain this phenomenon. However, by analogy with an interpretation of results from drill holes in rock in nonglacial locations [Monastersky, 1991], one possible explanation is that the air temperature was colder one to several hundred years ago and heat from the surface and the bedrock has not yet warmed the colder ice in between.

SUMMARY AND CONCLUSIONS

Conditions under which TE noise can be measured over an ice cap have been described. A formulation for the strength of the TE field as a function of the thickness and temperature profile of the ice has been given. Some thicknesses of ice in Antarctica have been quoted, and an indication of the availability of further information on ice thickness has been included.

The theory of the complex dielectric constant of pure ice as a function of temperature and frequency has been summarized. In it a relaxation frequency is a function of temperature and the complex dielectric constant is, in turn, a function of wave frequency and the relaxation frequency.

Sample calculations for the case of one homogeneous layer of ice have been presented. From these it is seen that, in general, the TE field is stronger at the surface of
colder ice. It is also seen that there is a resonance condition for various combinations of ice thickness, temperature, frequency, and angle of incidence. For example, for an ice thickness of 1000 meters, a temperature of -10° to -30°C, and an angle of incidence of 75 degrees from the vertical the resonance is at lower LF. For a 2000-meter layer for the same conditions the resonance is at VLF.

The ratio of total TE field to the incident field is evidently large enough that TE noise fields could be measured at the surface of an ice cap. If the measurement site were located near a deep drill hole in which the temperature has been measured as a function of depth, the field could be approximately extrapolated to higher heights in the waveguide.
REFERENCES


Feasibility of Measuring Transverse Electric Noise at VLF and LF on an Ice Cap

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There is an existing need for very low frequency (VLF) and low frequency (LF) transverse electric (TE) air-to-air communications. However, TE noise cannot be measured at ground level on highly conducting or even moderately conducting ground, and routine measurements from an aircraft would be prohibitively expensive.

The amplitudes of TE waves at the surface of an ice cap are evidently strong enough to be measured at LF when the ice cap is about 1000 meters thick and at VLF when the ice cap is about 2000 meters thick. Amplitudes are stronger over colder ice. Measurements of temperature in a drill hole would be needed for extrapolating fields to higher heights.
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