A COMBINED LUMPED PARAMETER/ONE-DIMENSIONAL BLOWDOWN MODEL FOR THE REGENERATIVE LIQUID PROPELLANT GUN

TERENCE P. COFFEE

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**A Combined Lumped Parameter/One-Dimensional Blowdown Model for the Regenerative Liquid Propellant Gun**

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**Abstract:**
A new model for the regenerative liquid propellant gun has been developed. The model is lumped parameter except for the gun tube, which is one-dimensional. The purpose of the new model is to cover the blowdown phase, after the projectile has exited from the gun tube. The gun tube profiles are then used as input for a thermal management code, which computes the heat transfer to the gun tube.

**Subject Terms:**
- liquid gun propellants; concept VIC; regenerative gun;
lumped parameter model; blowdown; one-dimensional model

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1. INTRODUCTION

Requirements for advanced gun systems, such as the U.S. Army Advanced Field Artillery System (AFAS), include increased firing rates in both burst and sustained firing modes. A serious problem attendant to such capabilities is that of thermal management, which is required to minimize thermal damage and wear to the tube and associated gun parts, to reduce the effects of thermal distortions on gun tube accuracy, and to eliminate safety problems such as charge or projectile cookoff. A comprehensive approach to the problem involves a careful analysis of the influence of firing rate on both the weapon heating problem and mission effectiveness of each of the propulsion options under consideration. In the case of AFAS, this choice now includes solid and liquid propellants.

A two-dimensional (2-D) gun tube heat conduction code was developed especially to address the current thermal management concerns (Chandra and Fisher 1989a, 1989b; Chandra 1990; Conroy 1990; Keller et al. 1991). This code uses the gas temperature, pressure, and velocity output of an interior ballistics (IB) code as its input. It is assumed that the heat transfer process can be uncoupled from the combustion and flow occurring within the gun tube. The interior ballistics code provides a file containing gas temperature, gas pressure, and gas velocity as a function of time and distance along the gun tube. The IB calculation is continued until the gas temperature and pressure decline to near ambient (i.e., the blowdown phase must be included).

For a solid propellant charge, the XNOVAKTC code (Gough 1980, 1990) is used to generate the required profiles. This report discusses the code developed to run the liquid propellant simulations.

2. LIQUID GUN CODES

Most of the regenerative liquid propellant gun modeling at the U.S. Army Ballistic Research Laboratory (BRL), Aberdeen Proving Ground, MD, has used a lumped parameter code developed in-house (Coffee 1985, 1988; Morrison and Coffee 1990). This code has been extensively validated against experimental data (Coffee, Wren, and Morrison 1989,
1990; Wren, Coffee, and Morrison 1990; Coffee et al. 1991). Some special purpose codes have also been developed—a one-dimensional (1-D) model for the liquid reservoir (Coffee 1988) and a 2-D model for the combustion chamber/gun tube (Coffee, to be published). Also, Gough has written several liquid propellant models—a model that is 1-D in the gun tube and lumped parameter elsewhere (Gough 1987), and a complete 1-D model (Wren and Gough 1991). However, the solution terminates at projectile exit.

The new factor required is the modeling of the blowdown phase. After the projectile exits the gun tube, a rarefaction wave will propagate from the muzzle toward the breech. It is possible to model a rarefaction wave using a lumped parameter code. In fact, the lumped parameter code assumes that a rarefaction wave propagates from the chamber toward the muzzle after burn out of the propellant (Morrison and Coffee 1990). However, rather than modeling two rarefaction waves with a lumped parameter code, a 1-D model in the gun tube was developed. For the rest of the system, the lumped parameter representation is sufficiently accurate.

3. NUMERICAL PROCEDURE

In the Concept VIC regenerative liquid gun, the propellant is initially behind two pistons. The inner or control piston moves first, opening up an annular vent. The outer, or injection, piston then slightly trails the inner piston. The motion of the control piston is modulated by a damper region. Propellant is injected between the pistons into the combustion chamber. The combustion takes place in the combustion chamber, and the gas then flows into the gun tube. Unlike solid propellant guns, there is a large area change from the chamber to the tube.

The governing equations from the lumped parameter model are used for the damper, liquid reservoir, combustion chamber, and projectile. Previous work indicates that very little liquid gets into the gun tube, so for simplicity, the fluid in the tube is assumed to be pure gas.

A staggered grid is used in the gun tube (Figure 1). The velocities are defined at the solid lines. This vector grid is evenly spaced between the gun tube entrance and the projectile. The scalar quantities (pressure, temperature, and density) are defined midway between the
velocities (dotted lines). The grid divides the tube into scalar control volumes (between the solid lines). The scalar quantities are assumed to be uniform in the scalar control volumes, with a discontinuity at the solid lines. Similarly, there are vector control volumes (between the dotted lines). The velocity is assumed to be uniform within vector control volumes. Additional control volumes are established outside the physical gun tube to help set the boundary conditions (see below). The projectile motion is computed in the usual fashion, based on the pressure at the base of the projectile. As the projectile moves, the grid stretches (conformal grid). Additional points are added to the grid as needed.

Figure 1. The Grid for the 1-D Gun Tube Model.
The governing equations are cast in integral form. The procedure is inherently conservative. That is, if mass, momentum, or energy leaves one control volume through a boundary, it automatically enters the neighboring control volume. For simplicity, an explicit numerical integration scheme is used. Upwind differencing is used for stability.

3.1 Governing Equations. The 1-D continuity equation is (Bird, Stewart, and Lightfoot 1960)

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial x} = 0,$$  \hspace{1cm} (1)

where $\rho$ is the gas density ($g/cm^3$) and $v$ is the gas velocity (cm/s). Now consider some volume in three-dimensional (3-D) space. By the divergence theorem, Equation 1 can be written as

$$\frac{\partial \rho}{\partial t} \int \rho \, dV = - \int \frac{\partial (\rho v)}{\partial x} \, dV$$

$$= - \int (\rho v l) \, n \, dS.$$  \hspace{1cm} (2)

The first two integrals are volume integrals. The last integral is a surface integral. The vector $l$ is the unit vector in the $x$ (down bore) direction, and $n$ is the unit outward normal vector to the surface. So any change in the mass in the control volume is due to mass flux across a boundary.

The energy equation can be written as (Bird, Stewart, and Lightfoot 1960)

$$\rho c_v \frac{\partial T}{\partial t} = -\rho c_v v \frac{\partial T}{\partial x} - T \frac{\partial p}{\partial T} \rho \frac{\partial v}{\partial x},$$  \hspace{1cm} (3)

where $T$ is the temperature (K), $c_v$ is the specific heat at constant volume ($J/g - K$), and $p$ is the pressure (MPa). The viscous heating term, which is almost always small, is eliminated. The thermal conductivity term is also not included. In the gun tube, the temperature is fairly
constant, and the thermal conductivity term is much smaller than the convection terms. Using the Noble-Abel equation, the derivative of $p$ with respect to $T$ with the density constant is simply the pressure over the temperature. Making this substitution and dividing by the specific heat produces

$$
p \frac{\partial T}{\partial t} = -p \nu \frac{\partial T}{\partial x} - \frac{p}{c_v} (\partial v/\partial x).
$$  \hfill (4)

Using the continuity equation, this can be written as

$$
\partial \left( \rho \frac{\partial T}{\partial x} \right) / \partial x = -\partial (\rho v T) / \partial x - \frac{p}{c_v} (\partial v/\partial x).
$$  \hfill (5)

The divergence theorem is applied. The pressure is assumed to be constant in the control volume and can be taken outside the integral. Then

$$
\frac{\partial}{\partial t} \int_v (\rho T) \, dV = -\int_s (\rho v T) n \, dS - (p/c_v) \int_s (v l) n \, dS
$$  \hfill (6)

The basic variable is the mass times the temperature. This can, as before, be changed by convection (first right-hand term). Hot or cold gas can enter through a boundary. The temperature can also be changed through work (second right-hand term). Note that in the last term, the pressure is not evaluated at the boundary, but is the pressure in the control volume.

The momentum equation in the axial direction is (Bird, Stewart, and Lightfoot 1960)

$$
p \frac{\partial v}{\partial t} = -p \nu \frac{\partial v}{\partial x} - \partial p/\partial x.
$$  \hfill (7)

The viscosity terms are not included. Viscosity will only be important for shear stress (perpendicular to the direction of flow). For the present 1-D model, this cannot be resolved. Using the continuity equation, this can be rewritten as

5
\[
\frac{\partial (\rho v)}{\partial t} = -\frac{\partial (\rho v^2)}{\partial x} - \frac{\partial \rho}{\partial x}.
\] 
\hspace{1cm} (8)

Applying the divergence theorem,

\[
\frac{\partial \rho}{\partial t} \int _V (\rho v) \, dV = - \int _{\partial V} (\rho v^2 I + p I) n \, dS.
\] 
\hspace{1cm} (9)

3.2 Grid. Figure 1 shows the grid used in the gun tube. The solid lines represent the vector grid and are labeled from 1 to \(n_t\). The grid points are evenly spaced. Midway between the vector grid is the scalar grid, labeled from 1 to \(n_t - 1\).

The vector grid splits the chamber up into scalar control volumes. The scalar quantities are assumed to be constant within each scalar volume (between solid lines). There is a discontinuity in the scalar quantities at each solid line. Each scalar volume has a volume \(V\) and a gas mass \(M\). The gas in the scalar volume has density \(\rho = M/V\), pressure \(p\), and temperature \(T\).

The dotted lines split up the chamber into vector control volumes. The velocities are assumed to be the same through each vector control volume, and there is a jump in velocity at the dotted lines. Each vector control volume is made up of half of two scalar control volumes. The mass in the vector control volume \(l\) is half the mass in the scalar control volume \(l - 1\) plus half the mass in the scalar control volume \(l\), that is, \(M_v(l) = 0.5 (M(l - 1) + M(l))\). The vector control volumes at the boundary require special consideration (see Section 3.4). If the grid spacing exceeds a user-specified value, \(dx_{\text{max}}\), an additional grid point is added. A maximum number of grid points, \(n_{\text{max}}\), can also be specified.

The grid stretches between the tube entrance and the projectile. The grid velocity \(U(n_t)\) at the projectile equals the projectile velocity. The grid velocity \(U(1)\) at the tube entrance is zero. The other grid velocities are found by linear interpolation. So the grid remains equally spaced as the integration proceeds. At muzzle exit, the grid is attached to the end of the gun tube and all the grid velocities are set equal to zero.
With this formulation, the governing equations can be simplified. Consider the continuity
Equation 2 applied over a scalar control volume. The density is constant within the control
volume, so it can be taken outside of the left-hand integral. The left-hand side then just
becomes density times volume or mass. The right-hand side involves the velocity across the
boundaries. Since the boundaries are also moving, this is the difference between the grid and
gas velocities. The density must also be known on the boundaries. Upwind differencing is
used, which means the density on the boundary is assumed to be the density from the
direction of flow. The direction of flow on a boundary is determined by the sign of the gas
minus grid velocity, defined as

\[ u(i) = v(i) - U(i). \]  \hspace{1cm} (10)

Assuming a positive gas minus grid velocity and using upwind differencing,

\[ \frac{\partial M(i)}{\partial t} = \left( \rho(i-1) \ u(i) - \rho(i) \ u(i+1) \right) A \]  \hspace{1cm} (11)

where \( A \) is the gun tube area (cm$^2$). If instead the quantity \( u(i) \) is negative, the corresponding
boundary densities are \( \rho(i) \) and \( \rho(i+1) \).

Similarly, the energy equation (6) becomes

\[ \frac{\partial [MT]}{\partial t} = \left( \rho(i-1) \ T(i-1) \ u(i) - \rho(i) \ T(i) \ u(i+1) \right) A \]
\[ + \left[ \frac{\rho(i)}{c_p} \right] \left( v(i) - v(i+1) \right) A \]  \hspace{1cm} (12)

The first term on the right-hand side is a convection term, and the appropriate velocity is the
velocity across the boundary. The second term is a work term. The change in temperature is
due to the compression or expansion of the gas and the appropriate velocity is the gas
velocity.
\[
\partial [M_v v](i)/\partial t = \{\rho(i - 1) \ v(i - 1) \ u(i - 1) - \rho(i) \ v(i) \ u(i)\} \ A \\
+ g_o \ \{\rho(i - 1) - \rho(i)\} \ A ,
\]

where \(g_o\) is a conversion constant \((10^7 \text{ gm/s-cm-MPa})\). For this case, the boundaries of a vector control volume are in the middle of the scalar control volumes, so the density and pressure are the same regardless of the direction of flow. However, the velocity must be found on the velocity control volume boundary, and again upwind differencing is used. The form given in Equation 13 is for flow in the positive direction, which will be the case until late in the blow-down phase.

### 3.3 Heat Loss

The loss of energy to the gun tube walls has a noticeable effect on the interior ballistics. To compute heat transfer to the wall, a 2-D code is required. Instead, a correlation from the lumped parameter code is implemented.

The gun tube temperature \(T_w\) is assumed to remain constant (infinite sink). Then the heat loss can be represented as

\[
Q_w = 4h_w (T - T_w)/d
\]

where \(T\) is the gas temperature in the tube, \(d\) is the tube diameter (cm), \(h_w\) is the heat transfer coefficient \((\text{j/cm}^2 - \text{K} - \text{s})\), and \(Q_w\) is the heat loss \((\text{j/cm}^2 - \text{s})\) (Bird, Stewart, and Lightfoot 1960). The variation of temperature in the gun tube is ignored. The difficulty is in computing the heat loss coefficient. In the code, the correlation of Sieder and Tate for pipe flow is used. This assumes fully developed flow in a smooth pipe with a nearly constant wall temperature. For highly turbulent flow,

\[
h_w = (k/d) 0.026 \ Re^{0.4} Pr^{13} (\nu/\nu*)^{0.14} ,
\]

where \(k\) is the thermal conductivity of the gas \((\text{j/cm} - \text{K} - \text{s})\), \(Re\) is the Reynolds's number,
Pr is the Prandtl number, and v is the viscosity (poise = g/cm – s). The correlation is for steady-state flow, while the gun conditions are highly transient. More importantly, the correlation has been found to be accurate for Reynold’s numbers up to about $10^5$, while Reynold’s numbers in the gun tube can be two orders of magnitude higher.

If a HAN-based propellant is completely burned, it will form a mixture of CO$_2$, H$_2$O, and N$_2$. Procedures have been developed to find the viscosity and thermal conductivity of a mixture of gases (Coffee and Heimerl 1981). The procedures were developed for the low-density limit. Fortunately, for high temperatures, the low-density limit is very accurate even for very high pressures (Bird, Stewart, and Lightfoot 1960). Fits were made for the viscosity and thermal conductivity of the gases resulting from HAN1845 between 300 K and 3,000 K. Values for other HAN-based propellants are almost the same. A separate low temperature fit was made to determine the viscosity at the gun tube wall $v_w$. The fits are

\[
v = 2.3275 \times 10^{-6} \, T^{0.74884}
\]

\[
v_w = 5.3083 \times 10^{-7} \, T^{0.85508}
\]

\[
k = 1.7974 \times 10^{-4} \, T^{0.91890}.
\]

The Reynold’s number is given by

\[
Re = \rho v dv
\]

and the Prandtl number by

\[
Pr = c_p v / k.
\]

where $c_p$ is the specific heat at constant pressure.
In the lumped parameter code, the correlation is applied to the tube as a whole, using space mean values for $pv$ and $T$. In the 1-D code, the correlation is applied to each scalar control volume.

Note that in the actual thermal management code, a different correlation is used for heat transfer, and the gun tube can change temperature. The effects of these inconsistencies has not yet been studied.

3.4 **Boundary Conditions.** As usual, the flow into the gun tube is assumed to be steady-state isentropic flow (Coffee 1988). This gives the mass flow into the tube. A scalar control volume is set up just to the left of the gun tube ($i = 0$). The density and pressure in this control volume is set equal to the density and pressure just inside the tube (at scalar control volume $i = 1$). The temperature is set equal to the chamber gas temperature, so that mass times temperature is conserved by flow across the boundary. Then the gas velocity at the entrance is given by

$$v(1) = \frac{mf}{\rho(0)A} \quad (19)$$

where $mf$ is the mass flux (g/s).

Before muzzle exit, the velocity at the right-hand side of the tube is just the projectile velocity. For the scalars, a reflection condition is imposed (i.e., the density, temperature, and pressure just outside the region of integration $[i = nt]$ is equal to the scalars just at the end of the physical tube $[i = nt - 1]$).

At muzzle exit, the grid is attached to the end of the gun tube. The flow out of the tube is assumed to be steady-state isentropic flow. If the outflow velocity is larger than the local sound speed, it must be lowered to the sonic velocity (choked flow). The flow will be choked for most of the blowdown phase.

For isentropic flow, the relevant equations are the following. The first law of thermodynamics
\[ h + \frac{v^2}{2g_0} = \text{constant}, \]  

(20)

where \( h \) is the gas enthalpy (J/g) and \( v \) is the gas velocity (cm/s). The isentropic process equation

\[ p \left(1/p - b\right) = \text{constant}, \]  

(21)

where \( p \) is the pressure (MPa), \( \rho \) is the density (g/cm\(^3\)), \( b \) is the covolume (cm\(^3\)/g), and \( \gamma \) is the ratio of specific heats. The equation of state is

\[ p R_s T = p (1 - bp), \]  

(22)

where \( T \) is the temperature (K), and \( R_s \) is the specific gas constant (J/g - K). Also required is the equation for the speed of sound in a gas,

\[ c^2 = g_\gamma \rho \left[p (1 - bp)\right]. \]  

(23)

and the enthalpy equation,

\[ h = c_p T + bp, \]  

(24)

where \( c_p \) is the specific heat at constant pressure (J/g - K).

Let \( \rho_r, \rho_f, \) and \( T_r, \) be the scalar quantities in the scalar control volume at the end of the gun tube, and \( v_r \) the gas velocity at the vector grid point just before the muzzle exit. The problem is then finding the quantities \( \rho_r, \rho_f, \) \( T_r, \) and \( v_r \) at the throat. The velocity \( v_r \) is assumed to be the velocity exactly at the end of the tube. The scalar quantities are the values in the control volume just outside the tube.
Suppose the throat pressure is known. The throat density is found by the process equation. The temperature is found from the equation of state. Then from Equations 20 and 24,

$$v_t^2 = v_r^2 + 2g_o \left[ c_p (T_r - T_i) + b(P_r - P_i) \right].$$

(25)

Previously, the interest has been in subsonic flow. In this case, the throat pressure can just be taken as the pressure just outside the throat (in this case, as 1 atm). However, when the flow reaches the local speed of sound (in the throat), the flow becomes choked, and no further increase in the pressure drop will have any effect. The gas cannot move away fast enough, and a higher pressure region will exist just outside the gun tube. The pressure at which the flow becomes choked is called the critical pressure.

For an ideal gas, the critical pressure can be found analytically. Let the mach number $M = v/c$. Define the stagnation pressure $p_o$ as the pressure obtained when a gas is decelerated isentropically to zero velocity. Then (Fox and McDonald 1985)

$$p_o = p \left( 1 + (\gamma - 1) \frac{M^2}{2} \right)^{\gamma/(\gamma - 1)}.$$  

(26)

Then the critical pressure $p_c$ is given by

$$p_c = p_o \left( 1 + (\gamma - 1)/2 \right)^{\gamma/(\gamma - 1)}.$$  

(27)

The procedure is to find the critical pressure based on the conditions at the right end of the tube. If the critical pressure is less than the exit pressure, the flow is subsonic, and the exit pressure is used as the throat pressure. If the critical pressure is greater than the exit pressure, the flow is choked, and the critical pressure is used as the throat pressure.

For the Noble-Abel equation of state, the relevant equation becomes transcendental, and the critical pressure cannot be found analytically. Instead, an iterative procedure is used. The
ideal gas law critical pressure is computed as a starting point. Using this as the throat
pressure, $p_i$, $T_i$, $v_i$, and $c_i$ are computed. If the gas velocity $v_i$ is less than the sound
speed, $p_i$ is decreased by an amount proportional to the difference in the velocities. If $v_i$ is
greater than $c_i$, $p_i$ is increased. The ideal gas law critical pressure is a good approximation,
so convergence is rapid.

There is one last problem. The mass flow out of the gun tube should be $p_i v_i A$, where $A$
is the area of the tube. However, the code uses upwind differencing, so the normal procedure
would give a mass flow of $p_i v_i A$. So special logic is included at the end of the tube to obtain
the proper mass flux out.

Late in the blowdown phase, the pressure inside the tube can become lower than
atmospheric pressure. In that case, there is flow into the tube. The gas outside the tube is
assumed to be stagnant, since there is no simple way to compute its velocity.

On the other hand, flow from the chamber into the tube is assumed to always be positive,
since back flow has not previously been important. If the conditions indicate back flow, the
mass flux is just set to zero. This has not been considered important enough to update.

3.5 Time Step. To take a time step, the time derivatives of the lumped parameter
quantities are first evaluated and stored in a vector $Y_{DOT}$. The time step $dt$ is a user-
specified parameter. Copies of the lumped parameter values are stored in a vector $Y$. This is
updated using the Euler formula,

$$ Y = Y + Y_{DOT} \cdot dt . $$

(28)

However, the actual lumped parameter values are not updated, so the code is fully explicit.

To take a tube time step, the boundary conditions are applied to find the velocity at the
tube entrance and at the projectile. Next, the heat loss term is applied to the energy equation,
i.e.,
\[ MT = M T - Q_w V \, d\tau/\alpha. \] (29)

Other than heat loss to the walls, thermal conductivity is neglected.

Then the Lagrange part of the time step is taken. That is, the grid is momentarily assumed to move at the same velocity as the gas. The continuity and energy equations are unchanged, since there is no mass flow through the boundaries. For the momentum equation, the work term is computed

\[ [M, \nu] (i) = g_o \{ \rho (i - 1) - \rho(i) \} \, A. \] (30)

The mass \( M \) in the vector volume remains unchanged. The velocities are updated by

\[ \nu(i) = [M, \nu](i)/M_r(i). \] (31)

The boundary conditions are then updated.

The scalar control volumes have a new Lagrange volume,

\[ V_r(i) = V(i) + (\nu(i + 1) - \nu(i)) \, A \, dt. \] (32)

The new density is given by

\[ \rho(i) = M(i)/V_r(i). \] (33)
The temperature equation is updated using the work term

\[ [M_T](i) = [M_T](i) - \left[ \frac{p(i)}{c_r} \right] [V_i - V] , \]  

(34)

and the new temperature is now

\[ T(i) = [M_T](i)/M(i) . \]  

(35)

Finally, the grid is moved to its proper position. The values needed on the boundaries can now be found using upstream differencing. Assuming that \( u(l) \) is positive everywhere, the advection part of the equations are

\[ M(l) = M(l) + \{ \rho(l - 1) u(l) - \rho(l) u(l + 1) \} A \]  

(36)

\[ [M_T](i) = [M_T](i) + \{ \rho(l - 1) T(l - 1) u(l) - \rho(l) T(l) u(l + 1) \} A \]  

(37)

\[ [M_v](i) = [M_v](i) + \{ \rho(l - 1) v(l - 1) u(l - 1) - \rho(l) v(l) u(l) \} A . \]  

(38)

The boundary conditions are called again and the scalars are updated. The pressure is obtained from the equation of state.

Next, the grid is updated. The new position of the grid points is found by applying the grid velocity over the time step. The size of the new control volumes is computed. The acceleration on the projectile is found from the pressure at the right-hand scalar control volume. From the acceleration, the new projectile velocity is found. This then allows the new grid velocities to be computed. If the control volumes are longer than a user-specified length, \( dx_{max} \), a new grid point is added. The grid points are rearranged so the grid spacing is uniform. The velocities on the new grid are found by interpolation. The new scalar control volumes are each made up of parts of two old scalar control volumes. The appropriate
fraction of material from each of the old control volumes is put into the new control volumes, and the boundary conditions are updated. Finally, the new values of the lumped parameter variables are updated.

There is a minor numerical problem if the projectile initially has zero offset. In the lumped parameter code, if the projectile is only a short distance downtube, the chamber/tube is treated as one region. This is no longer desirable, since the chamber can have liquid and the tube cannot. So instead, a minimum offset of 0.01 cm is assumed. However, suppose a moderate-sized time step is taken. The mass flux into the tube is constant over the time step. Since the tube volume is tiny, the density and pressure will overshoot, leading to numerical problems. So logic was added to allow a smaller time step if the grid spacing is smaller than \( dx_{\text{max}} \).

4. VALIDATION

4.1 Comparison With Lumped Parameter Model. An initial comparison was made with the lumped parameter gun code (Table 1). This can only be done up to muzzle exit.

Table 1. Comparison Between Lumped Parameter and 1-D Models for Round 77

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<th>( dt ) (ms)</th>
<th>( dx_{\text{max}} ) (cm)</th>
<th>( n_{\text{max}} )</th>
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</table>
As a test case, a recent 14-liter (full charge) firing from the General Electric second generation 155-mm gun was used (Round 77) (Coffee and Wren, to be published). This results in a relatively high muzzle velocity (893 m/s), where the differences in the gun tube model are expected to be more important. The projectile travel is just under 7 m.

Several runs were made to check on the convergence properties. The numerical controls are the time step $dt$, the grid spacing $dx_{max}$, and the maximum number of grid points $n_{max}$. The calculated muzzle velocities and maximum chamber pressures are given. The calculations were performed on a Silicon Graphics 4D/310 GTX workstation. The results show convergence even with a fairly coarse grid.

Figure 2 compares the chamber pressure from the lumped parameter model and from the two grids used with the 1-D model (smallest time steps from Table 1). The profiles are almost overlays. It is only at late times, when the projectile is far downbore, that the 1-D model shows differences from the lumped parameter model. Figure 3 shows the corresponding curves for the projectile velocities.

Figures 4–7 compare the gun tube profiles near the end of the firing cycle at 18 ms. Overall, the two 1-D models are in very close agreement, indicating that grid independence has been reached. The 1-D code shows higher velocities, but the velocity at the projectile is almost the same as from the lumped parameter model. The pressure curve from the 1-D model shows less curvature than the lumped parameter model. However, both models predict a pressure difference of approximately 5 MPa from the tube entrance to the projectile. The density and temperature profiles in the gun tube are somewhat more complicated. There is a low-density region behind the projectile, since the motion of the projectile expands the gas. The pressure equilibrates faster than the density, so the temperature goes up. On the other hand, in a slower projectile case, there is instead a high-density region just behind the projectile. The flow from the chamber tends to increase the density, and the motion of the projectile decreases the density. The competition between these two effects can lead to somewhat complicated behavior near the projectile.
Figure 2. Round 77. Chamber Pressure. Lumped Parameter Model (Line). Coarse Grid (Dot). Finer Grid (Dash).

Figure 3. Round 77. Projectile Velocity. Lumped Parameter Model (Line). Coarse Grid (Dot). Finer Grid (Dash).
Figure 4. Round 77. Gun Tube Gas Velocity, Time = 18 ms. Lumped Parameter Model (Line), Coarse Grid (Dot), Finer Grid (Dash).

Figure 5. Round 77. Gun Tube Gas Pressure, Time = 18 ms. Lumped Parameter Model (Line), Coarse Grid (Dot), Finer Grid (Dash).
Figure 6. Round 77. Gun Tube Gas Density, Time = 18 ms. Lumped Parameter Model (Line). Coarse Grid (Dot). Finer Grid (Dash).

Figure 7. Round 77. Gun Tube Gas Temperature, Time = 18 ms. Lumped Parameter Model (Line). Coarse Grid (Dot). Finer Grid (Dash).
Comparisons were also made for a 30-mm system, a 105-mm system, and smaller charge 155-mm systems. The agreement between the lumped parameter code and the 1-D code was better than that reported above.

4.2 Comparison With Theory. Experimental data for the blowdown phase is not available for liquid propellant guns. So a comparison is made with a theoretical model developed by Comer (1950).

The Comer model assumes that the propellant is totally burned before the projectile starts moving. The gas then expands adiabatically until muzzle exit. It is assumed that the pressure follows a standard Lagrange gradient. A rarefaction wave develops and moves back toward the breech. Comer developed expressions for the profiles from the breech to the rarefaction wave, as well as a formula for the location of the rarefaction wave. There is not a general formula for the profiles from the rarefaction wave to the muzzle. Comer uses expansions in the quantity charge over projectile mass, and ignores second-order terms, so his expressions are only approximate. The model requires as input the projectile muzzle velocity and the average temperature of the gas in the tube at muzzle exit.

In order to mimic the assumptions of the Comer model in the 1-D code, the mass flow from the chamber to the tube is set to zero. The gun tube is chosen as 60 cm long, with a projectile offset of 10 cm. At time = 0, the gun tube is filled with hot gas at a pressure of 500 MPa. The resulting charge is 762.8 g, and the projectile mass was set at 5,000 grams (charge/mass ratio = 0.25).

The 1-D code was run with a maximum grid spacing of 5 cm, 2 cm, and 1 cm. Muzzle exit takes place at 1.06 ms. Figure 8 compares the velocity profiles some time after muzzle exit. The sharp bend in velocity near the tube exit is an artifact of the model. In the model, steady-state choked flow is set up instantaneously. This leads to a jump in velocity from the grid point just inside the tube to the grid point at the muzzle exit. In fact, there should be some delay in reaching steady-state conditions. Otherwise, agreement is good, although not as good as the previous case. Figure 9 compares the pressure curves. By this point in time, the pressures are low. Again, agreement is reasonable.

The Corner model is compared to the 1-D simulation with a grid spacing of 1 cm (Figures 10–15). The muzzle velocity and average gun tube temperature at muzzle exit are used as input into the Corner model. At muzzle exit, the 1-D code does show a linear velocity profile. The pressure profiles are fairly flat and agreement is good. Figure 12 compares the velocity profiles some time after muzzle exit. The corner model has a sharp location for the rarefaction wave, while the 1-D code shows a more gradual curvature. The agreement near the tube entrance is still excellent, and the location of the rarefaction wave is about correct. The pressure curve shows similar behavior. At a later time (Figures 14 and 15) the agreement is still good. So the blowdown model is qualitatively showing the proper type of behavior.

5. TEST CASE

The thermal management code was first applied to a simulation of the firing of Unicharge propellant in a 23-liter, 52-caliber cannon, with a projectile travel of 6.9 m. The zone six Unicharge produced a muzzle velocity of about 945 m/s with the 43.54 kg (96 pound) M549 projectile. To make a fair comparison, a liquid propellant simulation was developed that mimicked as closely as possible the Unicharge simulation.

As a starting point, the 14-liter case mentioned in Section 4.1 is used. The projectile offset was reduced to essentially zero (0.01 cm), and the travel extended slightly to 6.9 m. The projectile mass was decreased slightly to the M549 projectile mass. The projectile resistance profiles were changed to that used in the Unicharge simulation. Air shock in front of the projectile was not included. Finally, the damper was adjusted until a muzzle velocity near 945 m/s was obtained.

Again, a series of convergence tests were made (see Table 2). For these tests, the integration was carried out to 50 ms. Muzzle exit occurred at around 17.6 ms.

Figures 16–19 compare the profiles for the three different grid sizes (with the smaller time step) for the profiles at 25 ms. The coarse grid is not completely accurate, but the medium and fine grids show excellent agreement.
Figure 10. **Simple Blowdown. Gun Tube Gas Velocity, Time = 1.06 ms. One-Dimensional Model (Line). Corner Theory (Dot).**

Figure 11. **Simple Blowdown. Gun Tube Gas Pressure, Time = 1.06 ms. One-Dimensional Model (Line). Corner Theory (Dot).**
Figure 12. Simple Blowdown, Gun Tube Gas Velocity, Time = 1.5 ms. One-Dimensional Model (Line), Corner Theory (Dot).

Figure 13. Simple Blowdown, Gun Tube Gas Pressure, Time = 1.5 ms. One-Dimensional Model (Line), Corner Theory (Dot).
Figure 14. Simple Blowdown. Gun Tube Gas Velocity, Time = 2.0 ms. One-Dimensional Model (Line). Corner Theory (Dot).

Figure 15. Simple Blowdown. Gun Tube Gas Pressure, Time = 2.0 ms. One-Dimensional Model (Line). Corner Theory (Dot).
Table 2. Comparison Between Lumped Parameter and 1-D Models for Thermal Management Test Case

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Figure 16. Test Case: Gun Tube Gas Velocity, Time = 25 ms. Coarse Grid (Line). Medium Grid (Dot). Fine Grid (Dash).
Figure 17. Test Case. Gun Tube Gas Pressure, Time = 25 ms. Coarse Grid (Line). Medium Grid (Dot). Fine Grid (Dash).

For the thermal management simulation (Keller et al. 1991), the integration was carried out to 500 ms. This was to ensure that the gas in the gun tube cooled to near ambient conditions. Significant heat transfer can occur long after muzzle exit as long as the gas in the tube is still hot. For this simulation, the fine grid was used with a time step of 0.0001 ms. The grid refinement tests reported in Table 2 indicate that this is actually a finer grid and a much smaller time step than required for numerical convergence.

Figure 20 shows the pressures at both ends of the gun tube as a function of time. The pressure essentially reaches atmospheric pressure shortly after 100 ms. Figure 21 shows the corresponding average gas temperature in the gun tube. The temperature increases as the pressure goes up, since the gas is being compressed and hence heated. Once the pressure equilibrates with the outside air, further gas temperature decreases are very gradual.

Figure 22 shows the velocity at the right-hand end of the tube simulation. This is the projectile velocity until muzzle exit and the muzzle gas velocity afterwards. For comparison, the space mean sound speed in the tube is also shown. At muzzle exit, by coincidence, the projectile velocity is almost the same as the local speed of sound. So there is no jump in
Figure 20. Test Case. Pressure at Left of Tube (Line). Pressure at Right of Tube (Dot).

Figure 21. Test Case. Average Gun Tube Gas Temperature.
velocity as steady-state choked flow is established. The flow stays choked until a little after 100 ms. When the flow unchokes, the velocity drops more rapidly. Because of inertia, the pressure in the tube actually becomes less than 1 atm, and the exit velocity becomes negative. Gas then flows in and out of the tube in a gradually damped fashion.

Figure 23 shows the velocity profiles in the tube starting shortly after muzzle exit. The sharp increase in velocity just at the exit is an artifact of the model, since the exit velocity instantaneously reaches the steady-state choked flow velocity. The rarefaction wave propagates very slowly down tube, since the local gas velocity is almost at the speed of sound. Figure 24 shows the velocity profiles at more widely spaced time intervals. The rarefaction wave is damped as it propagates downstream. This is the expected behavior (Comer 1950). Figures 25 and 26 show the corresponding pressure profiles.
Figure 23. Test Case. Gun Tube Gas Velocity. Time = 20 ms (Line), 25 ms (Dot), 30 ms (Dash), 35 ms (Dot-Dash).

Figure 24. Test Case. Gun Tube Gas Velocity. Time = 40 ms (Line), 50 ms (Dot), 60 ms (Dash), 70 ms (Dot-Dash).
Figure 25. Test Case. Gun Tube Gas Pressure. Time = 20 ms (Line), 25 ms (Do), 30 ms (Dash), 35 ms (Dot-Dash).

Figure 26. Test Case. Gun Tube Gas Pressure. Time = 40 ms (Line), 50 ms (Dot), 60 ms (Dash), 70 ms (Dot-Dash).

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6. CONCLUSIONS

A regenerative liquid propellant gun code which is 1-D in the barrel and lumped parameter elsewhere was developed. Before muzzle exit, the new code agrees very well with the entirely lumped parameter model. After muzzle exit, the results are qualitatively as expected. Combined with a thermal management code to predict wall temperature, the model allows the detailed simulation of barrel heating for liquid propellant guns.
REFERENCES


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