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**13. ABSTRACT (Maximum 200 words)**

The following is a summary list of accomplishments under this contract: (1) ICAM was established in August of 1987. (2) An interactive computer system was developed. (3) Over 85 research papers were produced. (4) Over 33 students were supported in part by the contract. (5) More than 12 PhD's and 5 MS students were produced. (6) Seven Post Doctoral Associates were supported. (7) More than 65 short term visitors came to Virginia Tech (16 of these were long term visitors). (8) Over 100 scientists attended the conference on Numerical Methods for Partial Differential Equations.
FINAL REPORT

on

AN INTEGRATED RESEARCH PROGRAM FOR THE MODELING, ANALYSIS AND CONTROL OF AEROSPACE SYSTEMS
(Contract No. F49620-87-C-0116)

for

1 September 1987 - 30 September 1991

by

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Blacksburg, Virginia 24061

March 3, 1992
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I. PREFACE

This FINAL REPORT report contains a summary of the activities supported under the DARPA Contract F49620-87-C-0116 during the period from September 1, 1987 to September 30, 1991. This project is concerned with the development of an integrated research program for the modeling, analysis and control of aerospace systems. In addition to the usual goal of production of research papers, the project was concerned with the training of new mathematical scientists and engineers and the coordination of an active exchange program between scientists.

The principal investigators were Professors John A. Burns, Department of Mathematics, Eugene M. Cliff, Aerospace and Ocean Engineering and Terry L. Herdman, Department of Mathematics.

The proposed statement of work contained three essential items:

A) The establishment of an interdisciplinary center for the theoretical and computational study of innovative control technology for aerospace vehicles and flexible structures.

B) The development of a research program in modelling, analysis and control of aerospace systems. This aspect of the project involved the development of new computational algorithms for identification, control and simulation of dynamical systems that govern fluid flows, structural vibrations and flight mechanics.

C) The coordination of an active exchange program between graduate students and various scientists and engineers.

The Interdisciplinary Center for Applied Mathematics (ICAM) was formed in August 1987 to promote and facilitate interdisciplinary research and graduate education in applied mathematics at Virginia Tech. This center has provided the mechanism needed to promote the studies of control technology for aerospace vehicles and flexible structures. In addition, ICAM helped organize the active visitors program and the conference on numerical partial differential equations.
II. BRIEF SUMMARY OF ACCOMPLISHMENTS

The following list contains a summary of the various activities and accomplishments that were funded in part by this contract. A detailed description of the research projects, personnel supported, students produced and other activities are presented in the following sections.

ICAM was established in August of 1987
Developed an interactive computer system
Over 85 research papers were produced
Over 33 students were supported in part by this grant
More than 12 Ph.D.s and 5 M.S. students were produced
7 Post Doctoral Associates were supported
More than 65 short term visitors came to Virginia Tech
16 of these were long term visitors
Over 100 scientists attended the conference on Numerical Methods for Partial Differential Equations
III. DESCRIPTION OF PROJECT

COMPUTING FACILITIES

The local computing environment is built around a DEC GPX-II Graphics Workstation. The GPX-II with three 159 MB hard disks performs a file server function on a local Ethernet. A VAX 2000 with a 70 MB hard disk and several personal computers are connected to the GPX-II via an Ethernet. A LNO3 laser printer acts as the printing device for the system. The system runs under DEC Ultrix; we have installed the GKS graphics system and language compilers (FORTRAN and C). The DELIGHT.MIMO software, which links a sophisticated non-smooth optimization package to some linear system software, is on the system. The package was kindly furnished by Professor E. Polak, Electrical and Computer Engineering Department, University of California-Berkeley.

RESEARCH

The basic research goal of this project is the study of control designs for hybrid aerospace systems. The procedure for designing high performance controllers for complex aerospace systems includes:

1. Developing accurate mathematical models of the physical problem, and

2. Constructing computational algorithms specifically suited for control design and analysis.

Step 2 is needed since computational algorithms developed for simulation may not be suitable for control design and optimization. This problem occurs, for example, when the governing equations are coupled partial functional differential equations of mixed type. Our approach is to combine fluid/structure dynamics and control in one model.

This approach leads to models of the form:

\[ z(t) = Az(t) + Ly(t) + Bu(t) \]
\[ q(t) = Fq(t) + Gz(t) \]
\[ y(t) = Cq(t) + Dz(t) \]

where the parameters A, L, B, F, G, C, and D are partial functional or ordinary differential operators. For example,
A might be a partial differential operator which describes the elasticity of the structure while $F$ might be a quasi-linear partial differential operator which describes the fluid dynamics. The rest of the parameters govern the coupling between fluid and structural dynamics and are highly dependent on the geometry of the structure. In the next few pages we describe some of the projects supported in part under this contract.

AEROELASTIC MODELING

For the past four years we have been studying issues related to the design of high performance controllers for complex aerospace systems in which the elastic motions of the structure are coupled with the motions of the surrounding fluid. Typically, the equations of motion of such systems involve coupled differential and singular integral equations, requiring a considerable amount of mathematical analysis to provide the necessary dynamic modeling, estimation, computation and control. In particular, we have studied the flutter problem for a three-degrees-of-freedom "typical" airfoil section in two-dimensional, incompressible flow (the so-called Theodorsen problem) where the dynamic model describing the composite system can be formulated as a system of singular neutral functional differential equations. Motivated by this model we have addressed the following questions: i) (Well-Posedness) Under what conditions can singular neutral equations be considered in the framework of linear infinite-dimensional systems? ii) (Approximation) Using the abstract approximation framework available for linear infinite-dimensional systems (i.e., Trotter-Kato type theorems) how can we construct robust, highly accurate computational schemes for the control and identification of systems governed by singular neutral functional differential equations?

Our study of the well-posedness of functional differential equations, including the aeroelastic system discussed above, has been in the context of functional analytic semigroup theory. The applicability of the semigroup approach for the development of approximation techniques for parameter estimation and optimal control for such system requires the identification of an appropriate state-space which not only yields the $C_0$-semigroup but also allows one to obtain a dissipative estimate for the infinitesimal generator for the semigroup. This requirement is due to the convergence of the numerical schemes being established by using the Trotter-Kato semigroup approximation theorem. This dissipative estimate for the aeroelastic system with state-space $\mathbb{R}^8 \times L_2$ is guaranteed, however, finding the equivalent norm on that space, which is needed to give the dissipative estimate, is not
straightforward. For retarded functional differential equations and atomic neutral functional differential equations on the state space $\mathbb{R}^n \times L_2, g$, a weight function, a well-developed theory is available. However, for singular neutral systems, such as our aeroelastic system, a characterization of $g$ that is needed to obtain the dissipative estimate was unknown. Our investigation shows that the selection of $g$ plays a key role in the development of the semigroup formulation for our system.

We have established that the finite delay version of the aeroelastic system generates a $C_0$-semigroup on the product space $\mathbb{R}^7 \times L_2, g$ and showed that the infinitesimal generator of that semigroup does, indeed, satisfy a dissipative estimate. Here the function $g$ is defined by $g(s) = [1 + (-s)^{-1}]^\frac{1}{2}$. This framework was extended to include more general singular neutral functional differential equations. In the general case we allow a weight function $g$ satisfying (i) $g(s) > 0, g(\cdot) \in L_1$, (ii) for any given $c > 0$, there is a $\tau > 0$ such that $g(s) \geq cg(s)$ on $[-\tau, 0]$. We developed an approximation scheme and applied it to two examples to illustrate examples to indicate the feasibility of our approach. One example is a scalar equation with inconsistent initial data, no smoothness on the solution at $t = 0$, the other example is a two-dimensional system which exhibits characteristics similar to those one observes in the aeroelastic system.

We have completed an in-depth study of the derivation of the equations for the aeroelastic system assuming that the circulation history belongs to a weighted $L_2$ space. We established that the assumption that the "past history" of the derivation of the total circulation function belongs to $L_2, g$, same $g$ that provided the dissipative estimate, is sufficient to assure the validity of the Söhnken's inversion formula for the solution of the airfoil equation. Therefore, we have obtained compatibility of the validity of the inversion formula and the well-posedness of the resulting neutral system. The resulting state space $\mathbb{R}^7 \times L_2, g$ and the numerical techniques described above provides a suitable setting for control design for the aeroelastic system.

For the complete system described above the evolution equation for the circulation on the airfoil was coupled to the rigid-body dynamics of the airfoil to obtain the infinite delay singular neutral system of the form

$$\frac{d}{dt} \left[ A\mathbf{x}(t) + \int_{-\infty}^{0} A(s)\mathbf{z}(t + s)ds \right] = B\mathbf{x}(t) + \int_{-\infty}^{0} B(s)\mathbf{z}(t + s)ds + f(t)$$
as the mathematical model. The 8 x 8 matrix $A$ is singular, the 8 x 8 matrix function $A(s)$ is weakly singular, $A_{88}(s) = \left(\frac{(Us - 2)}{Us}\right)^{\frac{1}{2}}$ and $f$ represents a forcing term (possible control) for the system. The state of the system includes the past history of $\Gamma$ where $\Gamma$ denotes the total airfoil circulation. This past history of $\Gamma$ may not be observable for the entire past time $(-\infty, 0)$. However, one would be able to observe this past history over a finite time interval say $[-r, 0]$. Since the kernel function $A(s)$ is not integrable on $(-\infty, 0)$ it is not possible to address the infinite delay problem as a finite delay problem by mapping the interval $(-\infty, 0)$ to some finite interval $[-T, 0]$. On the other hand the "finite delay" version

$$\frac{d}{dt} \left[Ax(t) + \int_{-r}^{0} A(s)x(t + s)ds\right] = Bx(t) + \int_{-r}^{0} B(s)x(t + s)ds + f(t)$$

has been extensively studied. During this period we have investigated the possibility of taking advantage of our results for the finite delay system to study the infinite delay system. We developed an approximation technique for the infinite delay system on an interval of the form $[0,T]$ using the approximation techniques we developed for the finite delay system. In order to accomplish this we have made use of the special structure found in the aeroelastic system. In particular, the representation of the aeroelastic system allows one to view the system as having an integrodifferential Volterra component and a singular integral component. We have shown that the infinite delay singular integral component can be approximated using the corresponding finite delay singular integral component. Our results include error estimates for the approximation. Our two step approximation procedure starts with truncating the infinite delay term. The approximation is dependent on the specified initial data. The desired error bound together with the time interval $[0,T]$ on which one wishes to find the solution dictates the necessary truncation. The second step consists of employing the approximation scheme established for the associate finite delay system.

**PARAMETER IDENTIFICATION**

It has been shown that fractional order operators can provide accurate models for viscoelasticity damped structures. The governing equation for such systems has the
form of a partial-integral equation. These systems can be viewed as neutral functional differential equations having a structure common to the mathematical models described above for aeroelastic systems. For our study of viscoelastic systems we focused our attention to the task of developing a parameter identification scheme for these systems. As a first step, we investigated the effectiveness of quasilinearization for parameter identification in both linear and nonlinear parabolic partial differential equations. Our study included the heat equation where the unknown parameter was the spatially varying diffusion coefficient and the nonlinear Burgers' equation.

During this period we developed a scheme for identification of unknown parameters in parabolic partial differential equations. Our study included theoretical detail and careful numerical testing. We established the Fréchet differentiability of the solution with respect to the unknown parameter which was shown to be a primary result necessary to ensure local convergence of the approximation scheme used to find the unknown parameter. Our numerical examples that we present demonstrate the rapid convergence and accuracy of the algorithm. Our efforts to date involving the use of quasilinearization in nonlinear partial differential equations have only been numerical ones. However, the success of our numerical testing leads us to believe that our quasilinearization scheme will provide an effective approach to parameter identification for certain nonlinear equations.

CONTROL ALGORITHMS FOR INFINITE DIMENSIONAL SYSTEMS

The development of practical control algorithms for infinite dimensional systems described by partial differential equations requires that approximations be introduced at some point in the analysis and design. A typical engineering approach to such problems is to first introduce some type of finite dimensional "design model" and then use this approximate model to design the control. Another approach to this problem involves the use of "truncated balanced realizations" to construct the finite dimensional design models. We compared and investigated these two approaches on several control problems governed by partial differential equations. We discovered an interesting, and somewhat surprising, result. When the two approaches were applied to a problem in the control of a thermal processes, it was found that the standard finite element scheme was an order of magnitude more robust than the "truncated balance realization" method. Moreover, the balanced finite element model produced a pre-conditioned system that was extremely robust. These discoveries have considerable impact on the conditioning of the numerical algorithms used in control design. To be useful, however,
it is essential that one have a practical method for computing these condition numbers. We have discovered that certain Galerkin methods are more robust than the "standard" methods often used in engineering practice.

**ACTIVE CONTROL OF SMART MATERIALS**

During the past ten years considerable progress has been made in the development and understanding of new and complex materials. These materials are the essential building blocks and an enabling technology for many current and future DOD missions. Shape memory alloys, piezoelectric materials and fiber optics are being used to develop new actuators and sensors for active control of "smart structures" and for structural-acoustics interaction problems. There are numerous DOD applications of such technologies. Although considerable progress has been made in the area of material science, many of these breakthroughs are based on experimental and simulation studies. There is a tremendous void in the development of useful models and corresponding practical computational algorithms for control design of such systems. The fact that the construction of practical computational algorithms for control of these new materials is lagging behind the development of experimental and simulation tools could severely limit the scope of future DOD missions. We addressed the problem of developing practical, fast and robust computational algorithms for the identification of systems involving these new materials. All of the computational algorithms were based on mathematical models that accurately reflect the physics of the systems.

We initiated an investigation into the use of these shape memory alloys as actuators in active control designs. These alloys are best described by thermo-mechanical models consisting of coupled (and nonlinear) hyperbolic and parabolic partial differential equations. The development of computational algorithms for designing controllers for such systems is an immensely complex problem and the subject of several ongoing research projects. In addition to the obvious difficulties related to the nonlinearities, the basic thermo-elastic coupling often gives rise to nonstandard mathematical models and leads to several problems in developing computational algorithms for control. Therefore, the computational methods for controlling a linear thermo-elastic system may be viewed as a first step toward the ultimate nonlinear problem. With this motivation in mind, we considered the problem of controlling a class of coupled partial differential equations that describe the linearized motions of a thermo-mechanical structure. The basic approach was to combine approximation theory with state space modelling to develop convergent computational...
algorithms for LQR control designs. Our results were extremely encouraging.

OPTIMAL MANEUVERS

There is considerable interest in increased agility for fighter aircraft. The belief is that all-aspect missiles put a premium on a first-shot capability. One mechanism for increasing agility is the use of thrust-generated moments to control aircraft attitude. This is to be investigated with the DARPA-sponsored X-31A research vehicle. We conducted a study of optimal maneuvers with mathematical models of aircraft dynamics.

In particular, we have examined optimal rigid-body angular rate control via an approximate dynamic model, which admits analytical solutions of the optimality conditions. The analysis reveals that over a large range of boundary conditions, there are, in general, several distinct external solutions. Second-order necessary conditions have been investigated to establish local optimality of candidate minimizers. Global optimality of the external solutions has been studied.

We have also studied the optimal angular rate problem using an exact dynamic model. Numerical solutions of optimality conditions were obtained which corroborate and extend the findings of the approximate problem. The qualitative feature of multiple external solutions is retained and as before, certain external solutions do not satisfy the Jacobi necessary condition. The choice of minimizing solution has been narrowed down to two sub-families of extremals. A locus of Darboux points was obtained which demarcates the domains over which these two sub-families are globally minimal.

The above studies look at minimum control effort families of external solutions. As a next step, we examined the minimum time control of angular rates, with prescribed hard bounds on available control. Existence of singular subarcs in time-optimal trajectories was explored. Qualitative features exhibited by the minimum-effort problem are preserved. In addition, the control space was deformed to allow roll control and its effect on external solutions was investigated.

In the final phase, we introduced kinematics into the optimal control problem. Minimum time attitude control of a rigid-body were investigated with prescribed hard bounds on available control. A preliminary numerical survey of first-order necessary conditions reveals that there are several distinct external solutions. The character of external solutions depends on whether the pitch or the yaw motion
assumes the dominating role in controlling the roll motion. Moreover, certain spatial symmetries have been identified. Specific maneuvers such as: roll around the velocity vector, and fuselage pointing have been analyzed.

FLUID FLOW CONTROL

We have made considerable progress on a simple fluid flow control problem. The problem is concerned with the control of Burgers' Equation. We have extended some of the basic finite dimensional ideas to this non-linear partial differential equation model. Burgers' Equation is an excellent test problem. We have applied various state feedback schemes (including "linearization followed by standard LQG theory") and investigated the properties of the corresponding non-linear closed-loop systems. The major tasks here were the development of fast computational algorithms and the analysis of the closed-loop system. As to be expected, several numerical difficulties occur in problems with high Renold's number. All of these issues were addressed and feedback laws were computed that not only enhanced stability, but reduced shock formation.
IV. LABORATORY INTERACTION

In addition to the research described above, we had considerable interaction with Laboratory personnel at several laboratories including:

* Air Force Astronautics Laboratory, Edwards Air Force Base;
  - Dr. Alok Das
  - Capt. Doug DeHart

* Air Force Armament Laboratory, Eglin Air Force Base;
  - Dr. J. Cloutier

* NASA Langley Research Center;
  - Mr. J. Elliott
  - Dr. C. Gracey
  - Dr. R. Ou

* Wright Laboratory (FIGC), Wright-Patterson Air Force Base;
  - Mr. Jerry Jenkins
  - Mr. Charles Suchomel

* Wright Laboratory (FI3G), Wright-Patterson Air Force Base:
  - Mr. Jon Lee

* Wright Laboratory (FFMG)
  - Mr. Tom Cord
  - Mr. Kevin Langen

We saw this interaction as an important part of our research and educational program.
V. VISITORS PROGRAM

The research program was significantly enhanced by the outside visitors that helped create conditions for interactions among mathematicians, scientists, engineers and computer scientists. During the period of this contract we provided partial support for many of the more than 65 short term visitors. These visits have already produced several joint papers and precipitated many new interdisciplinary research problems.

LIST OF VISITORS

LONG TERM VISITORS

H.T. Banks - University of Southern California
Dennis W. Brewer - University of Arkansas
Martin Brokate - University of Kaiserslautern
Ingo Eichenseher - University of Augsburg, Germany
Richard H. Fabiano, Jr. - Brown University
Mitchell Feigenbaum - Rockefeller University
Elena Fernandez - Instituto de Desarrollo Tecnológico, para la Industria Química
Wilfred M. Greenlee - University of Arizona
Israel Gohberg - Tel Aviv University
Karl H. Hoffmann - University of Augsburg
Franz Kappel - University of Graz
Gunther H. Peichl - University of Graz
Dominique Pelletier - Ecole Polytechnique of Montreal
Janos Turi - University of Texas at Dallas
George Weiss - Brown University
Klaus H. Well - Deutsche Forschungs- und Versuchsanstalt für Luft- und Raumfahrt e.V., Oberpfaffenhöfen, Germany.

SHORT TERM VISITORS

Juergen Ackermann - Institute of Flight Systems Dynamics
Richard Babarsky - James Madison University
Ivo Babuska - University of Maryland
J.P. Boris - US Naval Research Laboratory
M. Breitner - Technical University of Munich and German Aerospace Research Establishment
T. J. Bridges - Worcester Polytechnic Institute
Roger Brockett - Harvard University
Roland Bulirsch - Technical University of Munich
Peter Castro - Eastman Kodak, Rochester, NY
Kenneth L. Cooke - Pomona College
John Dennis - Rice University
Richard E. Ewing - University of Wyoming
Robert E. Fennell - Clemson University
George J. Fix - University of Texas Arlington
David Fox - University of Minnesota
Avner Friedman - University of Minnesota
Revaz Gamkrelidze - Moscow, Russia
James G. Glimm - State University of New York at Stoney Brook
Werner Grimm - Deutsche Forschungs- und Versuchsanstalt für Luft- und Raumfahrt e.V., Oberpfaffenhöfen, Germany.
Jack Hale - Georgia Tech
James Holloway, University of Michigan
Kazufumi Ito - Brown University
Elizabeth Jessup - Knoxville, TN
Franz Kappel - Universit"at Graz
R. Baker Kearfott - University of Southwestern Louisiana
C. Tim Kelley - North Carolina State University
Belinda King - Clemson University
Art Krener - University of California at Davis
Irena Lasiecka - University of Virginia
Alan J. Laub - University of California-Santa Barbara
Zhang Liu - University of Minnesota
Dahlard L. Lukes - University of Virginia
John Lund - Montanta State University
E. F. Mischenko - Moscow, Russia
J. D. Murray - Oxford University
Stephen G. Nash - George Mason University
R. A. Nicolaides - Carnegie Mellon University
Stanley J. Osher - University of California-Los Angeles
Gunther H. Peichl - Universit"at Graz
Robert Pego - University of Maryland
Dominique Pelletier - University of Montreal
C. Peskin - Courant Institute, New York University
Samuel M. Rankin, III - Worcester Polytechnic Institute
M. Reed - Duke University
Gary Rosen - University of Southern California
Ekahkr Sachs - North Carolina State University
Ricardo Sanchez-Pena - University of Buenos Aires
Lee A. Segel - Weizmann Institute of Science, Rehovot, Israel
George Sell - Army High Performance Computing Research Center, Minneapolis, Minnesota
James Sochacki - ISC, LaRamie, Wyoming
Felippe de Souza - Instituto Tecnologico de Aeronautica
Jason L. Speyer - University of Texas
Eitan Tadmor - Tel Aviv University
Roberto Triggiani - University of Virginia
Janos Turi - Worcester Polytechnic Institute
Homer F. Walker - Utah State University
Robert C. Ward - Oak Ridge National Laboratories
Joseph C. Watkins - University of Southern California
W. Wedig - University of Karlsruhe
Helena S. Wisniewski - Lockheed Corporation
Alastair D. Wood - Dublin City University, Dublin, Ireland
Bostwick Wyman - Ohio State University
David M. Young - University of Texas at Austin
Songmu Zheng - University of Utah
Enrike Zuazua - Courant Institute of Mathematical Sciences,
                New York University
VI. OTHER ACTITIES

CONFERENCE ON NUMERICAL SOLUTIONS OF PARTIAL DIFFERENTIAL EQUATIONS SEPTEMBER 24-27, 1988

One long term goal of this project is to develop theoretical and computational tools for attacking complex fluid/structural control problems. As an important step, we organized a conference on numerical solutions of partial differential equations. The goal of this conference was to bring together several experts in numerical analysis and control theory and have them present a survey of recent progress in their area of expertise. The conference attracted over 100 attendees. In addition to support from DARPA, partial support was obtained from the Air Force Office of Scientific Research, the Virginia Tech Department of Mathematics and the Interdisciplinary Center for Applied Mathematics.

Invited Speakers

I. Babuska - University of Maryland
H. T. Banks - Brown University
R. E. Ewing - University of Wyoming
G. J. Fix - University of Texas Arlington
J. G. Glimm - Courant Institute of Mathematical Sciences New York University
R. A. Nicolaides - Carnegie Mellon University
S. J. Osher - University of California Los Angeles
D. L. Russell - University of Wisconsin/Virginia Polytechnic Institute and State University
E. Tadmor - Tel Aviv University

Participants Supported

Robert Acar - University of Oklahoma
Krishan M. Agrawal - Virginia State University
Christoph Börgers - University of Michigan
Moysey Brio - University of Arizona
Christopher L. Cox - Clemson University
M. Desai - Brown University
Donald Estep - Georgia Tech
Ben G. Fitzpatrick - Brown University
Ali Hajjafar - The University of Akron
S. I. Hariharan - The University of Akron
Lisheng Hou - Carnegie Mellon University
Belinda B. King - Clemson University
Kevin L. Kreider - Ames Laboratory
John Lund - Montana State University
Amnon J. Meir - Carnegie Mellon University
Milan Miklavcic - Michigan State University
Hae-Soo Oh - University of North Carolina Charlotte
David Rebnord - Brown University
Peter I. Reichmann - The Catholic University of America
Hisham Sarsour - North Carolina State University
Jingyou Sun - University of Texas Arlington
Hillel Tal-Ezer - ICASE
H. Tran - Brown University
G. Wade - Brown University
Junping Wang - Cornell University
Yun Wang - Brown University
Qi Yu Zhang - Carnegie Mellon University
VII. PERSONNEL

PRINCIPAL INVESTIGATORS

John A. Burns - Professor, Department of Mathematics
Eugene M. Cliff - Professor, Aerospace and Ocean Engineering
Terry L. Herdman - Professor, Department of Mathematics

ASSOCIATE INVESTIGATORS

William T. Baumann, Electrical Engineering,
Yuriko Renardy, Department of Mathematics
David Russell, Department of Mathematics
Janos Turi, Department of Mathematics, University of Texas at Dallas

POST DOCTORAL FELLOWS

Joseph Z. Ben-Asher - Ph.D., Virginia Tech
Deborah Brandon - Ph.D., Carnegie Mellon
Wenceslao Cebuhar - Ph.D., Harvard
Guenter Leugering, Ph.D., Technische Hochschule Darmstadt
Robert E. Miller - Ph.D., Virginia Tech
Gunther Peichl - Ph.D. University of Graz
Moshen Tadi - Ph.D., Virginia Tech

GRADUATE STUDENTS SUPPORTED

Joseph Ben-Asher--Ph.D. Aerospace Engineering, June 1988
Marwan Bikdash--Ph.D. Student/Electrical Engineering
Spiro Bocvarov--Ph.D. Student/Aerospace & Ocean Engineering
Philip Bushong--Ph.D. Student/Aerospace & Ocean Engineering
B. Charrier--M.S. Student/Aerospace & Ocean Engineering
Rajiv Chowdhry--Ph.D. Student/Aerospace & Ocean Engineering
Diane DeWalt--Ph.D. Student/Aerospace & Ocean Engineering
Patricia Hammer--Ph.D. Student/Mathematics
Scott Hansen--Ph.D. Student/Mathematics
David Hill--Ph.D. Student/Mathematics
Scott Inch--Ph.D. Student/Mathematics
Makoto Ishigaki--Ph.D. Student/Mathematics
Sungkwon Kang--Ph.D. Student/Mathematics
Ruta Kher--M.S. Student/Mathematics
Renjith Kumar--Ph.D. Student/Aerospace & Ocean Engineering
Gyou Bong Lee--Ph.D. Student/Mathematics
Hamadi Marrekchi--Ph.D. Student/Mathematics
Craig Martell--M.S. Student/Aerospace & Ocean Engineering
Kimberly Oates--M.S. Student/Mathematics
Robert Miller -- Ph.D. Student/Mathematics
Richard Oleksuk -- Ph.D. Student/Electrical Engineering
John Ong -- Ph.D. Student/Mathematics
Marek Rakowski -- Ph.D. Student/Mathematics
John Rogowsky -- Ph.D. Student/Electrical Engineering
Reuben Spies -- Ph.D. Student/Mathematics
Kiran Suwal -- M.S. Student/Aerospace & Ocean Engineering
Mohsen Tadi -- Ph.D. Student/Engineering Science & Mechanics
Brian Thompson -- M.S. Student/Aerospace & Ocean Engineering
Pangiotas Tsiotras -- Ph.D. Student/Aerospace & Ocean Engineering
Gopal Vasudevan -- Ph.D. Student/Aerospace & Ocean Engineering
Xiaohong Zhang -- Ph.D. Student/Mathematics
Lan Zhang -- Ph.D. Student/Mathematics

Undergraduate Students
Jennifer Moore -- B.S. Student/Aerospace & Ocean Engineering

SUPPORT PERSONNEL

Nancy D. Smith -- Research Administrative Officer
Ginger Clayton -- Research Administrative Officer
Debbie F. Journell -- Research Administrative Officer
Kenneth P. Hinson -- Computer Systems Senior Engineer
Brett M. Marcus -- Programmer
Jennifer Moore -- Programmer
Tina Burrell -- Programmer
Christine Freeman -- Office Specialist
Kenneth P. Hinson -- Computer Systems Senior Engineer
Robin C. Endelman -- Programmer
Alice Bell -- Secretary
Michelle Boyd -- Secretary
IX. DEGREES GRANTED

**Ph.D. Degrees Completed**

Joseph Ben-Asher--Ph.D. - 1988, Aerospace Engineering  
Rajiv Chowdhry--Ph.D. - 1989, Aerospace & Ocean Engineering  
Patricia Hammer--Ph.D. - 1990, Mathematics  
David Hill--Ph.D. - 1989, Mathematics  
Scott Inch--Ph.D. - 1992, Mathematics  
Sungkwon Kang--Ph.D. - 1990, Mathematics  
Renjith Kumar--Ph.D. - 1989, Aerospace & Ocean Engineering  
Gyou Bong Lee--Ph.D. - 1991, Mathematics  
Robert Miller--Ph.D. - 1988, Mathematics  
Gopal Vasudevan--Ph.D. - 1989, Aerospace & Ocean Engineering

**M.S. Degrees**

Ruta Kher--M.S. -1987, Mathematics  
Craig Martell--M.S. - 1991, Student/Aerospace & Ocean Engineering  
Kimberly Oates--M.S. - 1991, Mathematics  
Richard Oleksuk--M.S. - 1991, Electrical Engineering  
Kiran Suwal--M.S. - 1989, Aerospace & Ocean Engineering
IX. PUBLICATIONS

The following papers were supported in part under this contract:


35. S. Chen and R. Triggiani, "Gevrey class semigroups arising from elastic systems with gentle dissipation: the case $0<\alpha<\frac{1}{2}$," Proceeding Americ. Math. Soc. vol 110 (1990) 401-415.


63. J.A. Burns and Yuh-Roung Ou, "Control of Oscillatory Forces on a Circular Cylinder by Rotation", in preparation.


