**Title and Subtitle:** Propagation and scattering of microwaves and millimeter waves in snow based on dense random media theory.

**Authors:** Leung Tsang

**Performing Organization:** University of Washington, Seattle, WA 98195

**Sponsoring/Monitoring Agency:** U.S. Army Research Office, P.O. Box 12211, Research Triangle Park, NC 27709-2211

**Abstract:**

The propagation and scattering of electromagnetic waves in snow terrain in the microwave and millimeter wave frequencies are strongly affected by both volume and surface scattering. We have studied the scattering in snow terrain in the frequency range of 5 GHz to 100 GHz. Snow terrain is classified as a dense medium because it is a mixture of ice particles and air with the ice particles occupying an appreciable fractional volume. Volume scattering in snow terrain has been studied with dense media theory. We have validated the dense media theory with Monte-Carlo simulations of pair-distribution functions and by numerical solutions of Maxwell's equations. Also we have performed Monte Carlo simulations of rough surface scattering by the finite element and finite difference methods.
1. ARO Proposal Number: 26384-GS
3. Title of Proposal: "Propagation and Scattering of Microwaves and Millimeter Waves in Snow Based on Dense Random Media Theory"
4. Contract/Grant Number: DAAL03-89-K-0055
5. Name of Institution: University of Washington
6. Author of Report: Leung Tsang

SUMMARY OF RESEARCH

In the following, we summarize the research results for volume scattering and rough surface scattering.

2.1 Volume Scattering in Snow Based on Dense Media Theory

In a dense medium, the particles occupy an appreciable fractional volume. The assumption of independent scattering is not valid for dense media. We have based the dense media scattering theory on the quasicrystalline approximation (QCA) and the quasicrystalline approximation with coherent potential (QCA-CP) for the first moment of the field, and the correlated ladder approximation for the second moment of the field. Correlated cyclical terms have also been included to account for backscattering enhancement. Our recent progress includes dense media theory development, validation of theory by comparison with optical and microwave controlled laboratory experiments, Monte Carlo simulations of pair functions for comparison with the Percus-Yevick approximation, Monte Carlo simulations of dense media scattering based on numerical solution of Maxwell's equations, comparison with extinction data of dry snow from 18 GHz to 90 GHz, and application of neural networks trained with dense media theory for inversion of snow parameters in the Antarctica from satellite passive microwave remote sensing measurements.

2.1.1 Dense Media Theory Development

Dense media radiative transfer equations, including all four Stokes parameters, have been extended to the case of electromagnetic wave propagation in dense discrete random media with multiple species of particles. Multiple species refers to the case when the medium consists of particles of multiple sizes and permittivities. The Percus-Yevick approximation is used to describe the correlations of particle positions. This is an important extension of previous results because the particles in snow are generally governed by particle size distribution. Recent ground-truth measurements indicate that ice grains in snow have a broad size distribution with a standard deviation that can be several times that of the mean. We have recently extended the results to medium to high frequencies and included the effects of Mie scattering from correlated scatterers of multiple.
sizes. Distinct features of dense Mie scattering are illustrated for scattering and extinction in snow from 5 GHz to 94 GHz. The frequency dependence of extinction and scattering can have varied features depending on the particle size distribution and the correlations of particle positions.

2.1.2 Comparison with Optical and Microwave Laboratory Experiments

Laboratory experiments of the measurement of the attenuation rate and bistatic intensities as a function of concentration of particles have been made at both optical and microwave frequencies. One of the interesting features of the experimental data is that both quantities first increase with concentration, and then saturate. Depending on the frequency and the particle size distribution, the scattering and extinction can decrease with a further increase of concentration. We have shown that dense media theory can explain such a feature and is in good agreement with both optical and microwave experiments while the conventional theory of radiative transfer cannot.

2.1.3 Monte Carlo Simulations of Pair-Distribution Functions in Dense Media

In the approaches of dense media theory, the results of propagation and scattering are dependent on the statistics of particle positions. For the case of a high density of particles of finite size, there exists correlations of particle positions as exhibited by the second-order statistics. For the case of macroscopic objects like particles in geophysical terrain, we have used a special case of the Percus-Yevick approximation by assuming that particles do not exert any inter-particle forces aside from the fact that they cannot interpenetrate. It is important to verify the applicability of the Percus-Yevick approximation. We recently have performed Monte Carlo simulations of the pair-distribution functions of dense discrete random media with multiple sizes of particles. Computer experiments are performed on a system with $N$ number of particles. In the Monte Carlo simulations, we used $N$ of the order of 400 to 4000. The pair-distribution functions $g_{ij}(r)$ for two particles of sizes $a_i$ and $a_j$ is proportional to the conditional probability of finding a particle of size $a_j$ at a distance from the origin given that there is a particle of size $a_i$ at the origin. It approaches unity as the separation distance becomes large meaning that the particle positions are independent if they are far apart. For a fractional volume of less than 40%, the pair functions are practically equal to unity for pair separations larger than five diameters. Two methods are used to generate the random positions of particles: the Metropolis shuffling method and the sequential addition method. In the Metropolis shuffling method, the initial realization is created by placing $N$ spheres randomly or periodically inside a cubic cell. To generate new configurations, the particles are shuffled. In each cycle of shuffling, every particle is subject to a random displacement that is not governed by any interparticle force except that it cannot penetrate other particles. In the sequential addition method, the particles are placed in the cubic unit cell randomly disallowing inter-penetration of particles. Each realization is created anew independently of previous realizations. In both methods, the pair functions are calculated by counting pair separations of particles. The Monte Carlo simulations of pair functions by both methods are found to compare very well with the Percus-Yevick approximation.

2.1.4 Monte Carlo Simulations of Extinction Rate of Dense Media Based on Solution of Maxwell's Equations

As discussed in 2.1.2, we have tested analytic dense media theory against controlled laboratory experiments. With the advent of modern computers, it is important to test the dense media theory against Monte Carlo simulations by numerical solution of Maxwell's equations. We have performed the simulations on the extinction rate (coherent wave attenuation rate) of a dense medium consisting of dielectric spheres (3-dimensional
vector problem) occupying up to 25% by volume and size parameter of $ka=0.2$. Maxwell's equations in multiple scattering form are solved iteratively for each realization of spheres, the positions of which are randomly generated as described in 2.1.3 above. Because the coherent wave nature is preserved, we distinguish this type of simulation from the Monte Carlo simulations of photon transport. The computed results are averaged over realizations. Convergence tests of the averaged results for the fixed fractional volume and size of the spheres are performed by varying the number of iterations up to 9 iterations, varying the number of spheres up to 4000 spheres, and varying the number of realizations. A large number of spheres are required in order to have a sufficient amount of incoherent waves in the simulated physical problem. Results of the simulations are compared (Figure 1) with that of the independent scattering approximation, Foldy’s approximation, the quasicrystalline approximation, and the quasicrystalline approximation with coherent potential. As shown in Figure 1, the simulations are in good agreement with the last two approximations.

2.1.5 Comparison with Extinction Measurements of Dry Snow Between 18 GHz and 90 GHz

We have also applied the dense media Mie theory for comparison with the extinction measurements of dry snow. Hallikainen et al. measured the extinction behavior of different types of dry snow at four frequencies: 18 GHz, 35 GHz, 60 GHz and 90 GHz. Thus, it is important to match the data for a sample over the entire frequency range with one set of physical parameters. The standard deviation of grain-size radius in snow can be several times larger than the mode radius showing that snow has a broad grain-size distribution. The maximum grain diameter is between 3 mm to 4 mm with the number ratio of large grains to grains with mean size of about 1 to 300. In Figure 2, we show the comparison with the snow data for sample 9 with the ground truth of mean grain diameter $= 1$ mm and snow density $= 0.385$ gm/cc. This corresponds to a fractional volume of 0.423. The reported measurements of extinction are: 10.2 dB/m, 58.8 dB/m, 247.2 dB/m and 304.0 dB/m, respectively, at frequencies of 18 GHz, 35 GHz, 60 GHz and 90 GHz. Comparisons are made with QCA-PY-Mie (Quasicrystalline approximation, Percus-Yevick and Mie scattering) theory. In Figure 2, we make a comparison with data based on a histogram size distribution of 15 sizes with a mean grain radius of 0.0479 cm and fractional volume of 0.4134. We have also assumed large grains between 3 mm to 4 mm with the number ratio to grains of mean size of about 1 to 300. The agreement is good. It shows that with size distribution, the QCA-PY-Mie theory can explain the frequency dependence of the extinction of dry snow between 18 GHz and 90 GHz. The results of independent scattering are also shown for comparison.

2.1.6 Inversion of Snow Parameters from Satellite Passive Microwave Remote Sensing Measurements with Artificial Neural Networks Trained with Dense Media Scattering Theory

We have performed the inversion of snow parameters from passive microwave remote sensing of measurements with a neural network trained with the dense media radiative transfer theory. The basic idea is to use the input-output pairs generated by the dense media scattering model to train the neural network. We have performed inversion of three parameters: mean-grain size of ice particles in snow, snow density, and snow temperature from five brightness temperatures: 19 GHz V polarization, 19 GHz H polarization, 22 GHz V polarization, 37 GHz V polarization and 37 GHz H polarization. It is shown that the neural network works very well on synthetic testing data. We have also used the neural network to invert the SSMI (Special Sensor Microwave Imager) satellite data over the Antarctica region.
We first use the dense media theory to compute the brightness temperatures for a half-space snow medium for the five channels using different combinations of input parameters of the mean-grain size of ice particles in snow, snow density, and snow temperature. About 1000 sets of input-output pairs are generated in this manner that are well distributed in mean-grain size of ice in snow, snow density, and snow temperature. On a VAX 3500 workstation, it takes about 2 hours cpu time to calculate all five-channel brightness temperatures for these 1,000 cases based on dense medium radiative transfer theory. These are used as training data for the neural network. Using the error backpropagation algorithm on these sets results in a set of weighting coefficients. The cpu time for the training of 10,000 iterations with these 1,000 sets of training data by this backpropagation algorithm is about 31 hours on a VAX 3500. Thus, the training time can be large. But once the training is complete, the actual inversion of parameters is done speedily. The neural network then is tested by a set of simulated testing data which is also generated by the passive dense medium theory and is randomly distributed in mean-grain size of ice particles in snow, snow density, and snow temperature. After 10,000 iterations in the training, the absolute percentage error for mean-grain size is less than 10%; the absolute percentage error for snow density is less than 10%; the absolute error for physical temperature is less than 3 degrees K. We have also used the 5-channel brightness temperatures from SSMI data as inputs to the neural network. An advantage of using the neural network method here is that the amount of SSMI data is voluminous and the neural network can process these data speedily with trained weighting coefficients. In this manner, we processed 30,000 sets of 5-channel brightness temperature SSMI data over the Antarctica region in 10 cpu minutes on a VAX 3500 workstation and produced contour plots of the three physical parameters: mean-grain size of ice particles in snow, snow density, and snow temperature for the Antarctica region.

2.2 Rough Surface Modeling Based on Numerical Simulations of One-Dimensional and Two-Dimensional Surfaces Using the Finite-Element Method (FEM) and Finite-Difference Time-Domain Method (FDTD) and the Extended Boundary Condition Method

Monte Carlo simulations of scattering by random rough surfaces were customarily done with the integral equation method. We have recently applied the method of finite element and the method of finite difference to the problem of numerical solutions of rough surface scattering. The attractiveness of such a method is the banded nature of the resulting matrix equation. We have completed the studies for a 1-D random rough surface with Dirichlet boundary condition, Neumann boundary condition, and penetrable medium. For the case of a penetrable medium, we have studied the case of an incident plane wave of TE polarization and calculated the mean reflected intensity and the mean transmitted scattered intensity. The numerical results are compared against the tapered wave integral equation approach (TWIE). The CPU time and the memory storage requirements for the FEM are much less than that of the TWIE method for cases when the number of horizontal sampling points is much larger than the number of vertical sampling points in the region of discretization. The percent error in conservation of energy for the FEM is shown to be less than 0.4% for all the examples treated. It is also found that a large surface length is required in the TWIE method to have a narrow incident angular spectrum to accurately predict the transmitted scattered intensity, whereas a relatively small surface length is sufficient in the FEM. Recently, we have also extended the FEM formalism to penetrable but lossy media for both TE and TM cases.

All these previous methods use frequency domain and calculated time-harmonic solutions. Recently, we also calculated the results of a finite-difference time-domain (FDTD) approach to the Monte Carlo simulation of random rough surfaces. The advantage of the FDTD approach is that the need of matrix inversion of any kind has been completely
eliminated. In addition, one can also study transient scattering by random rough surfaces. On the other hand, the frequency domain solution at different frequencies can also be reconstructed from the transient solution. We have studied a one-dimensional perfect electric conducting random rough surface with Gaussian statistics. The Gaussian surface profile is replaced by a stair-case approximation. FDTD is then applied to the scattered field instead of the total field. We studied both the transient and steady state responses of the rough surface. The steady state solution is compared with the frequency domain solution obtained from the finite-element code we developed and is found to be in good agreement.

All previous numerical simulations were made on a one-dimensional random rough surface. Very little work has been done for two-dimensional rough surface scattering. We applied the extended boundary condition (EBC) method to calculate the exact solution of the problem. The attraction of the method is that there are only two propagating Floquet modes for each wavelength of the surface length. Our recent calculations show that a surface area of 25 square wavelengths is sufficient to predict the incoherent scattered intensity away from the specular direction. Thus, a matrix inversion of size only 361 by 361 is needed for two-dimensional rough surface scattering in the EBC method. Convergence and accuracy of the method is demonstrated by varying the number of evanescent waves. We also show the comparison of one-dimensional results using the FEM and of two-dimensional results using the EBC method. It is found that the mean scattered intensity from two-dimensional rough surfaces differs from that of one dimension for the case of large slope.

2.3 Publications and Graduate Theses

The following is a list of journal publications and graduate theses completed under the support of the ARO Grant.

A. Journal Publications


B. Graduate Theses Completed


Figure 1. Extinction rate normalized to free space wavenumber as a function of fractional density of scatterers. Plots show calculations based on independent scattering, Foldy's formula, QCA-PY, QCA-CP-PY, and Monte Carlo simulations. Other parameters are \( \varepsilon_r = 3.2\varepsilon_0 \) and \( k\alpha = 0.2 \).

Figure 2. Comparison between QCA-PY-Mie theory and independent scattering with snow extinction data at 18 GHz, 35 GHz, 60 GHz, and 90 GHz of Hallikainen et al. [18] with ground truth of mean grain diameter of 1 mm and snow density = 0.385 gm/cc. The theory is based on the histogram with mean grain diameter of 0.958 mm and fractional volume \( f = 0.4134 \) that corresponds to snow density of 0.376 gm/cc. The permittivity of ice grains chosen are \( \varepsilon_r = 3.2\varepsilon_0 + i\varepsilon_0 \) with \( \varepsilon' = 0.007, 0.009, 0.011, \) and 0.014, respectively, at 18 GHz, 35 GHz, 60 GHz and 90 GHz which corresponds to a salinity of 0.12 parts per thousand.