New results have been obtained on nonlinear wave equations and plate equations. These results include exact controllability, strong and uniform stabilization, structural damping, quadratic optimal control problem, Riccati equation, robustness with respect to nonlinear uncertainties, on numerical aspects of the operator Riccati Equation. Both boundary control and point control problems have been considered.
INCREASING THE MARGIN OF STABILITY OF ARBITRARILY
FINITE MODES OF FLEXIBLE LARGE SPACE
STRUCTURES WITH DAMPING

Submitted to:

Air Force Office of Scientific Research/X00
Building 410
Bolling Air Force Base
Washington, DC 20332

Attention:

Program Manager
Control Theory Program

Submitted by:

I. Lasiecka
Professor

R. Triggiani
Professor

Proposal No. AM-DOD/AFOSR-4420-89
February 1991
This final report summarizes the principal investigators’ achievements on the research project during the period December 1, 1989 through August 31, 1991 and provides a list of publications under this grant since 1988. These include new results for wave equations and plate equations, linear and nonlinear, on the following problems: exact controllability, strong and uniform stabilization, structural damping, quadratic optimal control problem, Riccati equations, robustness with respect to nonlinear uncertainties, and numerical aspects thereof. Both boundary control and point control problems are considered.
INCREASING THE MARGIN OF STABILITY OF ARBITRARILY FINITE MODES OF FLEXIBLE LARGE SPACE STRUCTURES WITH DAMPING

SECTION I

1. Premise. The present technical report covers the period of research December 1, 1989 - August 31, 1991, by the Principal Investigators (P.I.s) and their graduate students. As such, it should be viewed as a successor of the previous report (Report No. UVA/525701/AM90/101) of the same grant No. AFOSR-87-0321A, submitted by the P.I.s on February 1990 to the AFOSR, Control Theory Program, and covering the period September 1, 1988 - November 30, 1989. We find more useful and more informative, however, to provide in Section II of the present document a complete list of research publications under Grant AFOSR-87-0321A, appeared since 1988, even though our comments in Section I will refer only to the later grant period December 1, 1989 - August 31, 1991.

2. Summary of research activities, December 1, 1989 - August 31, 1991. During this period, research efforts by the P.I.s and their students have centered on boundary control for waves and beams/plate equations with various boundary conditions, as well as corresponding point control problems. More precisely, major themes of research performed under the grant include the following problems:

(i) control problems with optimal quadratic cost over a finite/infinite time horizon and corresponding differential/algebraic Riccati equations

(ii) uniform stabilization in the case of infinite time/algebraic Riccati equations of (i)

(iii) numerical approximations of Riccati equations and corresponding gain operators, optimal solution, optimal control, etc.

(iv) exact controllability/uniform (strong) stabilization of linear and (non-linear) models of wave and plate equations
(v) interior point control for wave and plate models.

Regarding research areas (i) and (ii) an outstanding open problem of physical relevance was solved in [L-T.6], which refer to Riccati Algebraic Equations and related optimal control problem, when the input-solution map is unbounded. A highlight in this area was the publication by the P.I.s of the Springer-Verlag Lectures Notes #166 on Riccati equations, Ref [L-T.1].

Regarding area (iii), several numerical approximations works for Riccati equations were completed [L-T.5], [L.2], [L.3]. A graduate student (Erik Henrickson) is presently testing actual numerical computations in his Masters’ Thesis.

As to area (iv) of controllability/stabilization, new contributions by the P.I.s are [L-T.12], [L-T.13], [L-T.14] for linear plates, [L.6] through [L.14] for nonlinear wave and plates. Moreover, the research papers by the Ph.D. students listed in § 2.3 of Section II are all in this area. The question regarding the minimal number of boundary controls was settled in particular.

Finally, work on point control, area (v), was completed in [T.8]-[T.10], to show advantages/limitations of this mode of control.

3. Conclusion: It is felt that research efforts by the P.I.s and their graduate students offer a useful contribution to Air Force problems and needs in the area of control and stabilization of flexible structures.

SECTION II

List of Work by the Co-Principal Investigators (I. Lasiecka and R. Triggiani) and their Ph.D. students related to AFOSR Grant 87-0321, appeared since 1988 (or to appear).

2.1. Numerical approximation of control problems and dynamics. Linear, quadratic
boundary control problems and related Riccati equations

2.1.1 Work by I. Lasiecka and R. Triggiani

Book

Review paper

Research papers


2.1.2 Work by I. Lasiecka and A. Manitius


2.1.3 Work by I. Lasiecka and J. Sokolowski

...
2.1.4 Work by I. Lasiecka


2.2 Exact controllability and uniform stabilization of wave/plate equations.

Linear case

2.2.1. Work by I. Lasiecka and R. Triggiani


### 2.2.2 Work by I. Lasiecka


### 2.2.3 Work by S. Chen and I. Lasiecka


### 2.2.4 Work by S. Chen and R. Triggiani


2.2.5 Work by R. Triggiani


Non Linear case

2.2.6 Work by I. Lasiecka and R. Triggiani


2.2.7 Work by I. Lasiecka


2.3 Work by the P.I.s and their Ph.D. students

a) Ph.D. Granted

a₁) Dept. of Mathematics, University of Florida:
Jerry Bartolomeo, (Ph.D. December 1989; Advisor: R. Triggiani)

a₂) Dept. of Applied Mathematics, University of Virginia:
Norman Ourada (Ph.D. July 1990; Advisor: R. Triggiani)
Elizabeth Bradley (Ph.D. July 1991; Advisor: I. Lasiecka)

b) Ph.D. Students (partial list)

Mary Ann Horn (Ph.D. expected summer 1992; Advisor: I. Lasiecka)
Daniel Tataru (Ph.D. expected summer 1992; Advisors: I. Lasiecka and R. Triggiani)
Christine McMillan (Ph.D. expected summer 1993; Advisor: R. Triggiani)
Research papers


M. A. Horn and I. Lasiecka, The Euler-Bernoulli plate is exactly controllable via bending moments only, to appear in Springer-Verlag Lectures Notes in Control & Information Sciences, Proceedings on IFIP Workshop held at Irsee, Germany, April 1990.

M. A. Horn, Exact controllability of the Euler-Bernoulli plate via bending moments only in the space of optimal regularity, Journal of Mathematical Analysis & Applications, to appear.

M. A. Horn, Uniform stabilization of the Euler-Bernoulli plate with feedback acting via bending moments only, Differ. & Integr. Eqts., to appear.


Presently, C. McMillan is writing up a paper on the Differential Riccati equation for plate equations with boundary controls.
Remarks

1) Two of the above Ph.D. students, Mary Ann Horn and Daniel Tataru, have participated in the 1991 SIAM Student Paper Competition (see attached announcement). It was recently announced (see attached letters) that:

(i) Mary Ann Horn has been selected as one of the three winners of the competition, for her paper "Uniform stabilization of the Euler-Bernoulli plate with feedback acting via bending moments only."

(ii) Daniel Tataru has been selected for honorable mention for his paper "Boundary value problems for first order Hamilton-Jacobi equations."

Both of them have been honored at a special ceremony to be held during the forthcoming ICIAM 91 conference in Washington, DC.

2) To date (February 6, 1992), D. Tataru has received offers for Post-Doctoral/academic positions from the Mathematics Department of the following Institutions: U. of Chicago, M.I.T., Princeton, Harvard, Yale, U. of Minnesota, Purdue U., U. of California S. Diego, Cornell U., Kansas State U., Northwestern U. and has cancelled interviews with some other universities.

3) Mary Ann Horn has been recently awarded:

(i) a two/three years NSF Postdoctoral fellowship in residence at the School of mathematics, University of Minnesota;

(ii) a one-year Post-doctoral fellowship at the Institute of Mathematics and its Applications, University of Minnesota, during the 92-93 year in control theory.

As a result of (i) and (ii), she is giving up pursuing offers for academic positions in some Mathematics Depts. (e.g. U. of Hawaii; Auburn Univ.).

4) Both Mary Ann Horn and Christine McMillan have been selected by the Association of Women in Mathematics (AWM) to participate at a special Workshop by the AWM held during the A.M.S. Annual Meeting in Baltimore, Maryland, January 8-11, 1992, where both have presented their research work.