Three-Wave Envelope Solitons:

Can the Speed of Light in the Fiber be Controlled?

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Theory predicts that three-wave envelope solitons (TWES) can be generated in dual-mode optical fibers by injecting two copropagating light waves or a light wave and a flexural acoustic wave. The mechanism of the three-wave interaction is the recently observed intermodal stimulated forward Brillouin scattering. The velocity of the TWES can be controlled by changing the pump power. Using 200 mW pump power for a typical dual-mode fiber, the average speed of the light pulse in the fiber can be made as low as $3 \times 10^4$ m/sec.

1. INTRODUCTION

A recent growth of interest in dual-mode (DM) optical fibers for a variety of nonlinear switching and modulation schemes [1-3] stems from the long interaction lengths and from the two nondegenerate copropagating optical modes (at the same optical frequency) offered by these fibers.

This letter presents theoretical results suggesting that the speed of light in optical fibers can be controlled by using a nonlinear resonant interaction between two copropagating light waves and an acoustic wave in a DM optical fiber. The second order nonlinearity involved in this process is the intermodal forward stimulated Brillouin scattering (FSBS) in DM optical fibers. FSBS has been observed recently for the first time [4]. In our quantitative examples we use the fiber and wave parameters from that experiment.
2. PHYSICS OF THREE-WAVE ENVELOPE SOLITONS

In an optical fiber, two waves having frequencies $\omega_{1,2}$ and wavevectors $k_{1,2}$ can parametrically generate the third wave with frequency $\Omega$ and wavevector $K$ determined by

$$\Omega = \omega_1 - \omega_2, \quad K = k_1 - k_2. \quad (1)$$

In a single-mode optical fiber, this kind of process is possible only for light waves propagating in opposite directions. In a dual-mode fiber, the $LP_{01}$ and $LP_{11}$ modes and a flexural acoustic wave satisfy the phase matching conditions (1), making possible the three-wave interaction of copropagating light waves [6]. For stimulated Brillouin scattering (SBS), only one light wave (which is called the pump) is injected into the fiber and the other two waves are amplified from spontaneous levels.

In this letter we consider the case when two of the three waves are injected into the fiber as shown in Fig. 1. They can be two light waves with slightly different frequencies $\omega_1 > \omega_2$ or a light wave and an acoustic wave. A beating signal generates, via electrostriction (or the strain-optical effect), a flexural acoustic wave (or the second light wave). After this wave is amplified, a coupled nonlinear field distribution appears in the fiber. In general, this nonlinear complex can propagate with widely varying speeds having acoustic and light wave velocities as two extremes for the cases when the power of the acoustic or light waves is dominant.

The detailed evolution of the waves in the fiber depends on the amplitude and phase modulation of the injected waves. This process is described by a time-dependent set of nonlinear equations:

$$\left( \frac{\partial}{\partial t} + V_l \frac{\partial}{\partial x} \right) A_{1,2} = \mp \alpha_l A_{2,1} U, \quad (2)$$

$$\left( \frac{\partial}{\partial t} + V_a \frac{\partial}{\partial x} + \Gamma \right) U = \alpha_a A_2 A_1$$

where $x, t$ are space and time variables, $V_l$ is the group velocity of the light waves, $V_a$ is the group velocity of the acoustic wave, $A_{1,2}(t, x)$ are real slowly varying amplitudes of the light waves, $U(t, x)$ is a real slowly varying amplitude of the flexural wave, and $\Gamma$ is an acoustic wave phenomenological absorption [6]. The relative phase of the waves $\varphi = \varphi_1 - \varphi_2 - \varphi_3$ is assumed to be equal to $\pi/2$ or $3\pi/2$ throughout the interaction. The coupling coefficients $\alpha_l$ and $\alpha_a$ are given in [6]. We neglect the absorption of light.
waves since we use a short fiber (≤ 1 km).

3. GENERATION OF THREE-WAVE ENVELOPE SOLITONS

Let us show that equations (2) have soliton solutions. To simplify the derivation, we neglect the absorption (Γ = 0). We are searching for the solitary impulse propagating with a constant velocity $V$ [7]:

$$A_1 = A_0 \cosh^{-1} \Phi, \quad A_2 = A_0 \tanh \Phi, \quad U = U_0 \cosh^{-1} \Phi, \quad \Phi = \frac{x - x_0 - V't}{\Delta}$$

(3)

where $\Delta$ determines the spatial width of the soliton, $U_0$ is the maximum amplitude of the acoustic wave, $A_0$ is the maximum amplitude of the light waves, and $x_0$ specifies an initial position of the soliton. Solution (3) represents pulses of the light wave with the higher frequency $\omega_1$ and the acoustic wave. The second light wave has a constant amplitude $A_0$ almost everywhere, except for the vicinity of the center of the soliton ($\Phi = 0$), where it has a local dip; thus, the relative phase $\psi$ has a $\pi$-jump at the center of the soliton. More general TWES solutions have been discussed in references [8-10, 12]. We call the wave with the lower frequency $\omega_2 < \omega_1$ a pump wave since it has constant amplitude $A_0$ at the entrance to the fiber (assuming $x/\Delta \gg 1$ when $\Phi \approx 0$) and supplies energy into the fiber. This is different from SBS processes where the pump wave has higher frequency [5,6].

Substitution of solitary-like solution (3) into the set of equations (2) gives unique values of $\Delta$ and $\delta \equiv \Delta/V$ as functions of the amplitudes $A_0$, $U_0$:

$$\Delta = \frac{V_1 - V_a}{U_0 \alpha_l(1 + R)}, \quad \delta = \frac{\Delta}{V} = \frac{V_1 - V_a}{U_0 \alpha_l(V_1 R + V_a)}$$

(4)

where $R = \frac{\alpha_s A_2^2}{\alpha_l U_0^2}$ is the normalized ratio of the power densities of acoustic and light waves. The soliton velocity $V$ is uniquely expressed as a function of the amplitudes

$$V = \frac{V_1 R + V_a}{1 + R}$$

(5)

However, since $\Delta(A_0, U_0)$ behaves nonmonotonously as a function of $A_0$ and $U_0$, the plot of $V$ versus $\delta$ has two branches (see Fig. 3). The two branches correspond to light
(\(V_i \geq V \geq 2V_a\)) or acoustic (\(2V_a \geq V \geq V_a\)) dominated power:

\[
V = \frac{V_i + V_a \pm \sqrt{(V_i - V_a)^2 - 4\tau_{NL}^{-2}\delta^2V_iV_a}}{2 \times (1 + \tau_{NL}^{-2}\delta^2)}
\]

where \(\tau_{NL}^2 = 1/A_0^2\alpha_I\alpha_a\) is a nonlinear time scale.

The TWES can be generated by injection of all three waves modulated at the entrance to the fiber (\(x = 0\)) in accordance to expressions (3). This type of boundary conditions for the TWES generation is the exact reproduction of the soliton fields at the entrance as it would propagate from the left-hand part of the fiber (\(x < 0\)). However, for a case when acoustic (\(V \sim V_a\)) or light (\(V \gg V_a\)) power is dominant, it is sufficient to inject only two of the three waves into the fiber to generate the TWES [9].

For example, soliton generation by two laser beams [10] is possible for the case of light dominated power (\(V \gg V_a\)) if we switch on the pump wave with the lower frequency \(\omega_2\) and then modulate this signal by a "dark" pulse with the \(\pi\) phase shift in the middle. Simultaneously the "bright" light pulse with the higher frequency \(\omega_1 > \omega_2\) has to be launched into the fiber. These modulations are described by expressions (3) for \(A_{1,2}(x = 0, t)\); the ratio \(\delta\) to \(\tau_{NL}\) has to give \(V \gg V_a\) for a "+" sign in equation (6) [10]. The corresponding amplitude and phase modulation of the input signals is shown in Fig. 2. This mechanism of soliton generation has been used for soliton experiments in stimulated Raman scattering in a \(para-H_2\) medium [11]. Another way to generate the TWES (the case of acoustic dominated power \(V \simeq V_a\)) is to launch acoustic envelope \(U(x, t)\) described by (3) with \(V \sim V_a\) and \(\Delta \simeq V_i/\alpha U_0\) and then inject the constant laser pump wave \(A_2 = A_0\) in the fiber [9]. So, the two solutions with the different signs in expression (6) approximately correspond to the two mechanisms of soliton generation.
4. NUMERICAL EXAMPLE

Consider parameters given in Table 1 - they are close to the parameters of the experiment [4] and are discussed in [6]. Fig. 3 shows the resulting soliton velocity $V$ and width $6\Delta$ versus the time duration $\delta$. Inspection of Fig. 3 reveals that for 200 mW light wave power and pulse time duration of several $\mu$sec to several msec, the average speed of the light pulse in the fiber can be as low as $3 \times 10^4$ m/sec. This value is limited by the lifetime of the TWES which is approximately equal to the attenuation time of the acoustic wave (3.5 msec).

5. SUMMARY

The results of this paper suggest that three-wave envelope solitons (TWES) can be generated by injecting two copropagating light waves or one light wave and one acoustic wave into a dual mode optical fiber. This allows one to control the speed of light pulses in the fiber by adjusting the power of a light wave. The mechanism of interaction is the recently observed intermodal stimulated forward Brillouin scattering. For 200 mW light wave power and pulse time duration of several $\mu$sec to several msec, the average speed of the light pulse in the fiber can be as low as $3 \times 10^4$ m/sec. This value is limited by the lifetime of the TWES which is approximately equal to the attenuation time of the acoustic wave (3.5 msec).

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### TABLE 1. Numerical Values of Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
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<tr>
<td>Acoustic Velocity</td>
<td>( V_3 )</td>
<td>( 5.76 \times 10^3 ) m/sec</td>
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<tr>
<td>Light Power</td>
<td>( W_l )</td>
<td>200 mWatts</td>
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<tr>
<td>Acoustic Power</td>
<td>( W_a )</td>
<td>( \sim 10^{-12} W_l / R ) mWatts</td>
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<td>Laser Wave Length</td>
<td>( \lambda )</td>
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<td>Overlap Integrals</td>
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<td>Dimensionless</td>
</tr>
<tr>
<td>Refractive Index</td>
<td>( n )</td>
<td>1.5</td>
<td>Dimensionless</td>
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<tr>
<td>Refractive Index Difference</td>
<td>( \Delta n )</td>
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<tr>
<td>Fiber Density</td>
<td>( \rho )</td>
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<tr>
<td>Fiber Cladding Radius</td>
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<tr>
<td>Acoustic Wave Frequency</td>
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<tr>
<td>Acoustic Wave Absorption</td>
<td>( 1/\Gamma )</td>
<td>3.5 msec</td>
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<tr>
<td>Young's Modulus</td>
<td>( E )</td>
<td>73 GPa</td>
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Taranenko and Kazovsky: Three-Wave Envelope Solitons ...

References


FIGURE CAPTIONS

Fig. 1. (a). The block-diagram of the proposed experiment with injection of two light waves. (b). The block-diagram of the proposed experiment with injection of a light wave and an acoustic wave.

Fig. 2. Amplitudes and phases of the two waves injected into the fiber.

Fig. 3. The TWES velocity $V$ (thick lines) and width $6\Delta$ (thin lines) versus the temporal duration of the injected light signals. Curves 1 and 5 correspond to the soliton with light dominated power (the fast solution) for 200 mW of the pump and curves 2 and 6 for 50 mW of the pump. Curves 3 and 7 correspond to the soliton with acoustically dominated power (the slow solution) for 200 mW of the pump and curves 4 and 8 for 50 mW of the pump.
\( \varphi_2(t,0) \)

\( \varphi_1(t,0) \) or \( \varphi_3(t,0) \)

\( A_2(t,0) \)

\( A_1(t,0) \) or \( A_3(t,0) \)