A SINGLE-COMMODITY MINE TRANSSHIPMENT PROBLEM

by

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The Fleet Commanders-in-Chief often request recommendations from Commander, Mine Warfare Command, on schedules for naval mine transshipment. This thesis develops and implements a model, Scheduler for Mine Transshipment (SUMIT), that generates optimal schedules for transporting mines of a single type to suitable staging sites and for laying mines in mine fields. The model considers the number of available air, land and sea assets such as military aircraft, trucks, submarines and ships in finding optimal schedules for mine transshipment. SUMIT is designed to solve problems for scenarios in a limited region of the world that last several days and is based on interconnected, time-expanded mine and mode networks. SUMIT is written in the General Algebraic Modeling System (GAMS) and is a mixed integer linear program in which all integer variables are binary. Ten realistic test problems are solved to demonstrate the viability of SUMIT and to compare the relative efficiencies of two model variants. One variant is on average 87% faster than the other.
ABSTRACT

The Fleet Commanders-in-Chief often request recommendations from Commander, Mine Warfare Command, on schedules for naval mine transshipment. This thesis develops and implements a model, Scheduler for Mine Transshipment (SUMIT), that generates optimal schedules for transporting mines of a single type to suitable staging sites and for laying mines in mine fields. The model considers the number of available air, land and sea assets such as military aircraft, trucks, submarines and ships in finding optimal schedules for mine transshipment. SUMIT is designed to solve problems for scenarios in a limited region of the world that last several days and is based on interconnected, time-expanded mine and mode networks. SUMIT is written in the General Algebraic Modeling System (GAMS) and is a mixed integer linear program in which all integer variables are binary. Ten realistic test problems are solved to demonstrate the viability of SUMIT and to compare the relative efficiencies of two model variants. One variant is on average 87% faster than the other.
THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.
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I. INTRODUCTION

A. PURPOSE

The Fleet Commanders-in-Chief often request recommendations from Commander, Mine Warfare Command (CMWC), who supports mining operations executed by the U.S. Navy, on schedules for naval mine transshipment in the event of war or for mine-laying exercises. Schedules for transporting mines to suitable staging sites and for laying mines in mine fields must be generated to meet the needs of mine warfare, given the number of available air, land, and sea assets (such as military aircraft, trucks, submarines, and ships). Only a limited number of military assets are capable of carrying mines due to special equipment requirements. Furthermore, since most mine transshipment and delivery assets are not solely dedicated to mine warfare, a mine transshipment scheduler must take into account both transshipment and delivery times and mine field and resource priorities. The current methods of planning mine transshipment schedules are manual, with no utilization of computerized optimization algorithms. Recognizing the need to computerize data and algorithms for many mine warfare problems, CMWC has initiated the installation of a computer system, the Geo-Operational Planning and Assessment System (GOPAS), to automate and optimize several aspects of mining operations [Ref. 1].

The purpose of this thesis is to build a computer-based mine transshipment model that can accept data stored by GOPAS and is automatic with respect to formulation and execution. While this thesis focuses on the rapid deployment of mines in regional real-time or exercise scenarios that take two to three days to complete, recommendations are also made for approaching problems of longer duration, such as a few months. The solutions of the Scheduler for Mine Transshipment (SUMIT) model can be used to recommend not only feasible schedules for mine transshipment but, in many cases, optimal (or close to optimal) schedules. While SUMIT recognizes time as a critical factor, its consideration of mine field and resource priorities allows the user to account for intangible aspects of the scenario by including both time and priority in the measure of effectiveness (MOE) used in the model.

B. BACKGROUND

The mine transshipment problem is modeled, using two networks. Nodes in the network represent supply, transshipment, and demand sites. In the network for mine
flow, arcs represent possible transportation routes between nodes in which modes, such as trucks, aircraft, ships, and submarines, carry mines. The network for mode flow is similar to the mine network, except that additional return arcs are needed to represent the return trips of modes back to their home base.

1. Node Description

Nodes can be separated into three broad categories: supply, transshipment and demand. The U.S. Navy has 13 Mobile Mine Assembly Groups (MOMAGs) located throughout the world which store mines and function as supply nodes. These MOMAGs are typically collocated with large U.S. Navy and U.S. Air Force bases which have resources, such as aircraft capable of transporting and/or deploying mines. The mines at the MOMAGs are maintained at six levels of readiness, from disassembled to fully-assembled (ready-to-deploy). If mines are disassembled, MOMAGs have the facilities to prepare mines for delivery. The rate of assembling mines in preparation for deployment is called "build rate" and is measured in mines per hour. Some MOMAGs have the capability of setting up dual parallel assembly lines, thereby doubling the build rate. The U.S. Navy has an inventory of approximately 16 types of underwater mines and the build rate is known for each mine type [Ref. 2].

To decrease delivery times, fully-assembled mines are also pre-staged at locations other than the MOMAGs. They can be stored at bases that have mine delivery air modes, on aircraft carriers and surface ships to be deployed via ship-based air modes, or on submarines to be deployed by the submarine itself. Currently, surface ships do not have the capability to deploy mines but a new class of mine-laying ships, called High-Volume Mine Layers (HVMLs), are planned to fill this gap.

Transshipment nodes are typically U.S. Navy and U.S. Air Force bases with aircraft capable of supporting mine deployment. In this thesis, a military base that (1) has no pre-staged fully-assembled mines in inventory and (2) is not collocated with a MOMAG, is categorized as a transshipment node.

CMWC has developed mine field plans for a variety of scenarios that could occur throughout the world. In fact, the execution of all mine field plans simultaneously would deplete the entire supply of mines. Because the potential demand is much greater than the supply, all mine field plans are prioritized according to the importance of their objective. Approximately 90 percent of the time, mine field plans require only one type of mine and, therefore, generate only single-commodity scheduling problems.

Node data that can be obtained from GOPAS to support the proposed model SUMIT include the number of each mine type stored at every MOMAG at each level.
of readiness, the number of each mine type pre-staged at given bases, the number of each mine type demanded at every mine field, the build rate of every MOMAG for each mine type, the capability for a double assembly line, the priority of the mine field, and the type of node (land or sea).

2. Mode Description

Modes can be separated into two broad categories: transportation modes and delivery modes. Modes that cannot deploy mines in a mine field are categorized as transportation modes. These modes include land modes, such as trucks, and air modes, such as U.S. Air Force C-141 and C-130 cargo aircraft. Delivery modes can deploy mines and include air modes, such as U.S. Navy and U.S. Marine Corps shore-based and ship-based aircraft and U.S. Air Force B-52 bombers, and sea modes, such as submarines. Due to weight and size constraints, most air modes and all land modes can only carry one mine type per trip. Sea modes, which will include HVMLs in the future, are able to carry more than one mine type aboard. Because modes have multiple capabilities, they may also be critical to the success of other missions and can be prioritized according to the scenario. For example, if Anti-Submarine Warfare (ASW) plays a vital role in the scenario, a higher priority should be placed on non-ASW modes to deploy the mines.

Since the mine transshipment problem includes mobile supply sites (aircraft carriers, ships, and submarines), differences arise in how mobile supply sites are incorporated into the network. For this thesis, resources normally classified as modes are treated as nodes if they function as supply nodes and can only store mines. For example, aircraft carriers and surface ships, which store mines but are not capable of laying them, are classified as fixed nodes in the model. The aircraft stationed aboard the vessel function as modes that lay mines. To maintain consistency in the structure of the network, submarines and HVMLs, which are able to deploy mines, are treated as fixed nodes that have dummy modes stationed aboard to deliver mines. In addition, because aircraft carriers and ships may not be located at the on-station point for launch of aircraft at the beginning of the problem, SUMIT allows the transit of aircraft carriers and ships at the beginning and end of the problem. This exception is not required for submarines which travel directly to the mine fields.

Mode data supported by GOPAS include category (transshipment or delivery); dimension (land, air or sea); speed (nautical miles per hour); capacity for each mine type (mines per unit mode); time to load mines (hours); time to unload or deliver mines (hours); total time to refuel, change crews, and conduct routine maintenance or repairs
to prepare modes for their next trip (hours); maximum range (nautical miles); the number of each mode available at each supply and transshipment node; and mode priority.

3. **Arc Description**

   Because the flow of mines between nodes in the network is very structured, the only arc data that require support from GOPAS is the distance between nodes (nautical miles). Arc capacity (mines per trip) and length (time periods) can be calculated from the node data. Based on the data input by the user, SUMIT only forms arcs for mine networks that are directed from supply nodes to supply nodes, supply nodes to transshipment nodes, supply nodes to demand nodes, and transshipment nodes to demand nodes. SUMIT generates the same arcs for mode networks, except the return arcs are also included and are directed back to the home base. In Chapter II, the section entitled "Network Generation Rules" explains in detail other rules used to generate the networks.

C. **SCOPE**

   Other researchers have studied the multi-commodity transshipment problem for mine warfare and for other military applications. Wingate and Zakary [Ref. 3] proposed a continuous variable model for multi-commodity transshipment problems, that could be applied to mine transshipment. However, their model was too general and did not address characteristics unique to the mine transshipment problem, such as the existence of modes (i.e., aircraft carriers and submarines) that also function as supply nodes.

   Collier, Lally, and Puntenney [Refs. 4, 5, 6] developed continuous variable models for military deployment problems from the U.S. Transportation Command (TRANSCOM) using sea and air assets. The mine transshipment problem is smaller with potential supply sites limited to the 13 MOMAGs plus the bases and sea assets at which mines are pre-staged. The proposed model SUMIT is designed for regional problems spanning a time window of two to three days in which the total number of nodes can be about ten, while the TRANSCOM model developed by Puntenney covers movement requests among as many as 22 ports, planning general schedules lasting up to three months. Due to the importance of time in a short-duration mine transshipment problem, sea assets are limited to U.S. Navy vessels, located near the mine fields at the beginning of the problem, that either have the capability to deploy mines or have air assets aboard that are able to lay mines, while sea assets play a prominent role in shipping material in the TRANSCOM problem. Since the mine transshipment problem requires mines to be deployed as soon as possible, it does not include opportune delivery
times, which were used to generate costs for the objective function in the TRANSCOM problem. Finally, data concerning limits on the mine loading and unloading capacities at non-MOMAG supply sites and restrictions on the number of aircraft allowed at transshipment sites at one time is not available in GOPAS.

The scope of this thesis could encompass a multi-commodity transshipment model that optimizes schedules for deploying different mine types in a global scenario. Multi-commodity problems usually involve mixed-integer programs that use extensive amounts of computer resources to find optimal solutions for large networks. Because only one mine type can be transported by a mode for each trip, the complexity of the mine transshipment problem increases. Since very little preliminary groundwork has been laid which specifically addresses schedule optimization for the mine transshipment problem, the scope of this thesis has been narrowed to a single-commodity approach. Because this thesis focuses on regional scenarios that require mine deployment to a couple of mine fields over a period of several days, the use of a single-commodity model is justified by the fact that (1) most mine fields require only one type of mine and (2) the majority of MOMAGs do not supply all 16 types of mines. Furthermore, the transshipment of mines over long distances may take more time than is allotted for the scenario, which then restricts mine supply to sites in the same region of the world as the mine fields. Thus, a global problem could conceivably be divided into several regional subproblems, which could be solved separately, if nodes contained in one regional subproblem were not contained in the other regional subproblems.

D. OUTLINE

This thesis is divided into four chapters. Chapter I is the introduction. Chapter II proposes and describes in detail two model formulations. Chapter III discusses the results of ten test problems that compare the size and speed of the two model formulations and recommends procedures for executing SUMIT in the General Algebraic Modeling System (GAMS) [Ref. 7], a software package. Chapter IV contains conclusions about the methodology described in this thesis, discusses the weaknesses of SUMIT, and recommends future enhancements of SUMIT.
II. MODEL FORMULATION

A. GENERAL DESCRIPTION

Both proposed mine transshipment models contain two separate networks: one for the mines and another for the modes. The mine network is represented by a set of balance equations that controls the flow of mines through all nodes. Two types of variables are contained in the balance equations for the mine network: one variable type represents the shipment of mines from origin nodes to destination nodes via modes and the other represents the inventory of mines at nodes. The mode network is represented by another set of balance equations that controls the flow of modes to and from all nodes. The mine and mode networks are linked by a set of equations that relate the flow variables in the two sets of balance equations. In the linking equations, sending mines via an arc contained in the mine network, forces flow on the corresponding arc in the mode network. In other words, mines cannot flow through the mine network unless there is a mode to carry them. Furthermore, the number of mines flowing through the network is limited by the capacities of the associated modes.

The mine network is directed, meaning that arcs are ordered pairs of nodes. All paths must start at supply nodes, possibly flow through other supply nodes and transshipment nodes, and must arrive at demand nodes. Circuits, which are paths that start and end at the same node, are limited to transfers between supply nodes. Figure 1 illustrates a mine network, where \( s_i \), \( t_i \), and \( d_i \) respectively represent supply, transshipment, and demand at node \( i \). Notice that the path from \( s_2 \) to \( t_4 \) contains two arcs, which indicates that two different types of modes are available to transport the mines. [Ref. 8]

The structure of the mode network is an expansion of the mine network, which has additional arcs returning empty modes back to their origin node. The return arcs are critical because they prevent arcs that represent the same mode leaving at a different time period from being used before the mode has returned from its previous run. Figure 2 depicts a mode network, associated with the mine network of Figure 2.

Time-expansion of nodes in the two networks and in the linking equations is required to allow modes to make multiple sequential trips in either transporting or laying mines. Different arc lengths (which are measured in time periods) also make time-expansion desirable. In essence, time-expansion expands every node over several periods.
of time. The arcs of the time-expanded network connect earlier nodes to later nodes. Figure 3 illustrates a very simple time-expansion of a mine network.

To take into account the build rate of disassembled mines, a MOMAG-supply node is split into two nodes separated by an arc. The MOMAG node (the origin node), which has no arcs entering it, assembles mines, and sends them to the supply node (the destination node). Although dummy modes represent build rate to maintain consistency in the structure of the mine network, they are not included in the mode network since build-rate arcs can be used every period and there is no limit on availability over all time periods. The MOMAG node in the time-expanded mine network can have only one arc leave it per time period and this single arc must arrive at the supply node to which it is connected. Figure 4 illustrates a MOMAG-supply node split into two nodes: $m_2$ and $s_2$.

The following terms are defined to avoid misinterpretation of the problem formulation since they are not part of standard network terminology:
Figure 2. Example of a Mode Network

- Mode group - a set of modes based at the same node that are grouped by their similarity or by their command structure, such as seven P-3 Orion aircraft based at Naval Air Station (NAS) Adak, Alaska, or a squadron of various aircraft capable of carrying mines deployed aboard an aircraft carrier.

- Mode unit - an element of the mode group, such as one P-3 Orion aircraft or a squadron (if the squadron is composed of different aircraft).

- Run - a round trip from a node to a destination via a mode group.

- Mobile node - a node which is not fixed in position (such as aircraft carriers, surface ships and submarines).

- On-station point - the position from which a naval vessel conducts operations.

B. ASSUMPTIONS

Several assumptions were made in formulating the model to find the optimal mine transshipment schedule. Most assumptions reduce the scope of the problem to decrease the size of the network. The model assumptions are:
Figure 3. Example of a Time-Expanded Mine Network

- The model is single-commodity, i.e., only one mine type can be shipped.

- The mine network or the mode network need not be connected. For example, Atlantic-based mines will usually be deployed to mine fields in the Atlantic, Pacific-based mines will usually be laid in mine fields in the Pacific, and there need not be arcs (i.e., routes) connecting the mine and mode networks for the Atlantic fleet to the networks of the Pacific fleet.

- A mode group can have only one destination per run and cannot be split to start more than one run at different times.

- Multiple runs can be made by a mode group to different destinations.

- Two levels of mine readiness are assumed for mines stored at the MOMAG-supply nodes: disassembled and assembled.

- Mobile nodes that cannot deploy mines (such as aircraft carriers and surface ships) are treated as immobile nodes for the duration of the problem. The air modes stationed on the nodes deliver the mines.

- Mobile nodes that cannot deploy mines may transit to an on-station point at the beginning of the problem and to a new destination at the end of the problem. These transit distances must be input by the user. However, no additional transits
may be made to reposition a mobile node at other on-station points. A mobile node can only lay mine fields at demand nodes accessible from the on-station point.

- Mobile nodes that can deploy mines (such as submarines) are also treated as immobile nodes with dummy modes attached to them that function as delivery modes. The dummy modes maintain consistency in the structure of the model.
- Mobile nodes cannot be resupplied.
- Several mine fields located in the same area may be grouped into one demand node.
- All modes must return to their origin node before the problem ends.
- A mode that can either transport or deploy mines will always be classified as a delivery mode to eliminate unnecessary arcs.

C. FORMULATION

1. Indices

The first four of the five following indices are used in the formulations of both models, while the last index applies only to the second proposed model:

- \( n \) - an element of the set of all nodes in the original network, \( N = \{1, 2, \ldots, n, \ldots, n_n\} \), where \( n_n \) is the number of nodes.
• \( \mu \) - an element of the set of all modes in the original network
  \[ M = \{1, 2, \ldots, \mu, \ldots, n_M\} \], where \( n_M \) is the number of modes.

• \( p \) - an element of the set of all time periods used for the time-expansion,
  \[ P = \{0, 1, 2, \ldots, p, \ldots, n_p\} \], where \( n_p \) is the number of time periods plus one.

• \( i \) - an element of the set of integers used for iterative loops,
  \[ I = \{1, 2, \ldots, i, \ldots, n_i\} \],
  where \( n_i = n_p \).

• \( g \) - an element of the set of incompatible arc groups,
  \[ G = \{1, 2, \ldots, g, \ldots, n_g\} \], where
  \( n_g \) is the highest total number of groups, where \( n_g = n_p - 1 \).

The elements in the set \( P \) actually represent the start and end points of time periods
considered by the model. For example, the first period starts at \( p \) equals 0 and ends at
\( p \) equals 1. The extra period starting at \( n_p - 1 \) and ending at \( n_p \) is needed to calculate
constants for mine inventory after the last period of the problem. The procedure for
determining incompatible arc groups is described in the section under “Generation of
Incompatible Arc Groups.”

The node and mode indices can be categorized by several subset indices repres-en-ting
the function, dimension or arc position of the node or mode. For example,
MOMAG nodes, which are always origin nodes, can only be positioned on land and
transshipment modes travel on land or by air. This information is critical in constructing
a network that has realistic arcs and meets the assumptions of the model. The following
subindices are subsets of the node set \( N \):

• \( m \) - an element of the set of all MOMAG nodes \( MN \), where \( m \in MN \subseteq N \).

• \( s \) - an element of the set of all supply nodes \( SN \), where \( s \in SN \subseteq N \).

• \( t \) - an element of the set of all transshipment nodes \( TN \), where \( t \in TN \subseteq N \).

• \( d \) - an element of the set of all demand nodes \( DN \), where \( d \in DN \subseteq N \).

• \( l \) - an element of the set of all nodes positioned on land \( LN \), where
  \( l \in LN \subseteq (MN \cup SN \cup TN) \).

• \( c \) - an element of the set of all nodes located on or under the sea \( CN \), where
  \( c \in CN \subseteq (SN \cup DN) \).

• \( i \) - an element of the set of all origin nodes \( IN \) for arcs in the original network,
  where \( i \in IN \subseteq (MN \cup SN \cup TN) \).

• \( j \) - an element of the set of all destination nodes \( JN \) for arcs in the original network,
  where \( j \in JN \subseteq (SN \cup TN \cup DN) \).

Figure 5 indicates relationships between the function subsets \( (MN, SN, TN, and DN) \)
and the dimension subsets \( (LN and CN) \). Figure 6 depicts the relationships between the
function subsets and the arc position subsets \( (IN and JN) \). Notice that the unions of the
four mutually-exclusive function subsets and of the dimension subset are identical to the
node set \( N \). Finally, the union of the arc position subsets also equals \( N \) but their intersection is not empty: \( (IN \cap JN) \equiv (SN \cup TN) \). The only node subset that the user must input in addition to the node set \( N \) is the sea node subset \( CN \). SUMIT assumes all other nodes are land nodes. The proposed model determines the membership of the function subsets from the input data.

The final group of subindices are the following subsets of the mode set \( M \):

- \( \beta \) - an element of the set of all dummy build modes \( BM \) that transfer mines assembled at MOMAG nodes to adjacent supply nodes, where \( \beta \in BM \subseteq M \).
- \( \tau \) - an element of the set of all transportation modes \( TM \) that can only transfer mines to transshipment or land supply nodes, where \( \tau \in TM \subseteq M \).
- \( \delta \) - an element of the set of all delivery modes \( DM \) that can deploy mines into mine fields, where \( \delta \in DM \subseteq M \).
- \( \lambda \) - an element of the set of all modes \( LM \) that travel on land, where \( \lambda \in LM \subseteq (BM \cup TM) \).
- \( \gamma \) - an element of the set of all dummy modes \( CM \) for sea nodes that can deploy mines into mine fields, where \( \gamma \in CM \subseteq DM \).
- \( \alpha \) - an element of the set of all aircraft modes \( AM \) that transport or deliver mines, where \( \alpha \in AM \subseteq (TM \cup DM) \).

The same dummy build mode can be used for all MOMAG nodes if they have the same build rate (the treatment of parallel assembly lines is mentioned in the next section). Figure 7 indicates the relationship between the function and dimension subsets of \( M \). The function subsets (\( BM \), \( TM \), and \( DM \)) are mutually-exclusive sets whose union forms the mode set \( M \). The union of the mutually-exclusive dimension sets (\( LM \), \( CM \), and \( AM \)) also is identical to the mode set \( M \). In addition to listing the mode set \( M \), the user must input all function subsets and must indicate both sea and land subsets. The proposed model assumes that all remaining modes are air modes.

2. Data

The following list of parameters is a brief description of data contained in both model formulations. All data used in both models is fully described in the section entitled "Detailed Data Description for Both Models." The first parameter is not actually contained in the computer implementation of the models but is created to simplify the presentation of the equations located in the sections of this chapter under "Model A" and "Model B." The value of first parameter, \( OBJ_{\mu ij p} \), is not directly input by the user and is calculated by SUMIT from input data.

- \( OBJ_{\mu ij p} \) - the value contributed to the objective function by the mode flow for mode \( \mu \) leaving origin node \( i \) arriving at destination node \( j \) at time period \( p \).
Figure 5. Relationship Between Node Function and Dimension Subsets

- $PAR_{im}$ - the number of time periods with length TPER required to load mode $\mu$ based at origin node $i$, travel to destination node $j$ and unload (or deliver) mines.

- $PRT_{im}$ - the number of time periods with length TPER required to load mode $\mu$ based at origin node $i$, travel to destination node $j$, unload (or deliver) mines, return, and make ground preparations for turnaround.

- $AMT_n$ - the supply or demand of mines at node $n$.

- $XUP_{jm}$ - the upper bound on mine flow variables from origin node $i$ to destination node $j$ via mode $\mu$.

- $ZUP_n$ - the upper bound on mines in inventory at node $n$.

- $MXT_m$ - the maximum total time (h) that mode group $\mu$ can be absent from its origin node $i$ in making all of its runs.

- $MXR_m$ - the maximum number of runs that mode group $\mu$ based at its origin node $i$ is allowed to take.

- $RHX_m$ - the value of the right-hand side of the mine flow balance equation for mines leaving node $n$ at the beginning of time period $p$, where $p$ is either the time period before inventory can start changing or the time period after inventory can stop changing.
Figure 6. Relationship Between Node Function and Arc Position Subsets

- $RHY_{\mu n}$ - the value of the right-hand side of the mode flow balance equation for mode $\mu$ based at origin node $n$ at the beginning of time period $p$.

Of the preceding parameters, only $AMT_n$, $MXT_n$, and $MXR_n$ are input by the user. The remaining parameters are derived from the input data.

3. Decision Variables

Five decision variables are included in the first proposed model, while only the first four decision variables are included in the second proposed model:

- $v$ - the objective value of the model, which is a measure of effectiveness that involves node and mode priorities and return time for arcs selected by the model.
- $x_{iu \mu p}$ - the number of mines sent from origin node $i$ via mode $\mu$ and arriving at destination node $j$ at the end of time period $p$, where $x_{iu \mu p} \geq 0$.
- $y_{iu \mu p}$ - a binary variable that equals 1 if $x_{iu \mu p} > 0$ and equals 0 otherwise.
- $\hat{x}_{iu \mu p}$ - the number of mines remaining in inventory at node $n$ at the end of time period $p$.
- $\hat{y}_{iu \mu p}$ - a binary variable that equals 1 if all of mode units in mode group $\mu$ remained at origin node $i$ during time period $p$ and 0 otherwise.
Figure 7. Relationship Between Mode Function and Dimension Subsets

Mine flow and inventory is represented by positive, continuous variables. Since, for fixed values of $y_{ipm}$, the resulting problem is a single-commodity network flow problem with integer supplies, demands, and bounds on flows, these continuous variables are guaranteed to be integer [Ref. 9]. The value of $y_{ipm}$ indicates whether or not mode $\mu$ is used for the arc from $i$ to $j$ arriving at period $p$ and does not reflect the number of mode units within mode group $\mu$ that are actually needed to carry the mines. Likewise, $y_{ipm}$ indicates whether or not the entire mode group remains at node $i$ during time period $p$. In some cases, the model solution may imply that some, but not all mode units within a mode group, will be empty as they transit. By the assumptions of the model, these empty mode units cannot be diverted to meet other mine field demands. However, in reality, if the scenario warrants it (i.e., squadron integrity is not required for defense purposes), the empty mode units can remain at the origin node or be diverted to accomplish missions, not related to the problem.
4. Model A

Model A, the first model proposed in this thesis, consists of the objective equation, flow balance equations, linking equations, other constraints and bounds on the variables, which are explained in the section after the formulation. XTE and YTE are two sets that represent the variables associated with the arcs that exist in the time-expanded networks. XTE is the set of all variables in the mine network, while YTE is the set of all variables representing flow from origin nodes to destination nodes in the mode network. Rules that explain membership in these two sets are explained in the section under “Flow Existence Rules.” The time periods \( p^1 \), \( p^2 \), and \( p^3 \) are defined as follows for applicable equations in the model formulation:

- \( p^1 = p + 1 \).
- \( p^2 = p + \text{PAR}_{n\mu} \).
- \( p^3 = p - \text{PRT}_{\text{nu}} + \text{PAR}_{nu} \).

Minimize

\[
v = \sum_{i=1}^{n_i} \sum_{j=1}^{n_j} \sum_{p=0}^{n_p} \sum_{\mu \in \text{YTE}} \text{OBJ}_{ijp\mu} y_{ijp\mu}
\]

Subject to

\[
\hat{x}_{np^1} - \hat{x}_{np} + \sum_{j=1}^{n_j} \sum_{\mu \in \text{YTE}} x_{np\mu}^j - \sum_{i=1}^{n_i} \sum_{\mu \in \text{YTE}} x_{inp\mu} = RIX_{np} \quad \forall n \in \overline{D}, p
\]

\[
\hat{x}_{dp^1} - \hat{x}_{dp} - \sum_{i=1}^{n_i} \sum_{\mu \in \text{YTE}} x_{idp\mu} = RHX_{dp} \quad \forall d, p
\]

\[
\hat{y}_{np^1} - \hat{y}_{np} + \sum_{j \in \text{YTE}} y_{np\mu}^j - \sum_{i \in \text{YTE}} y_{inp\mu} = RHY_{np\mu} \quad \forall n, p, \mu
\]

\[
XUP_{ijp\mu} y_{ijp\mu} - x_{ijp\mu} \geq 0 \quad \forall i, j, p, \mu \in \text{YTE}, y_{ijp\mu} \in \text{YTE}
\]
5. Comments about Model A

The following list of comments explains the purpose of the equations given in the preceding model formulation:

- Equation 1 is the equation for the objective value.
- Equations 2 and 3 are mine flow balance equations for every node and time period in which mine flow is possible. Equation 2 is for nondemand nodes and Equation 3 for demand nodes.
- Equation 4 is the mode flow balance equation for every origin node, mode and time period in which mode flow is possible.
- Equation 5 links the mine and mode networks by forcing the mode flow variable \( y_{ijmp} \) to 1 if the mine flow variable \( x_{ijmp} \) is positive.
- Equation 6 is optional and ensures that the total time that a given mode is away from a given base is less than \( MXT_{m} \).
- Equation 7 is optional and ensures that the total number of runs for a given mode from a given base is less than \( MXR_{m} \).
- Equations 8 - 11 ensure that all variables are bounded.

In the computer code for the model, the equal signs are relaxed (i.e., are changed to inequalities) in Equations 1 - 4 to make the model easier to solve. The equal sign in Equation 1 is changed to greater than or equal to \((\geq)\), while the equal signs in Equations 2 - 4 are changed to less than or equal to \((\leq)\).
6. Model B

Model B is an alternate model proposed for SUMIT. It is based on the premise that, by only allowing one mode arc in a group of incompatible mode arcs to be selected by the model, the model can monitor the movement of modes correctly and prevent a mode group from taking-off for its next run before the mode group has returned from its previous run, without using binary variables for mode inventory. The alternate model has one additional index \( g \) and more internal data, eliminates the need for the mode inventory variable \( \hat{j}_{eq} \), replaces Equation 4 with a single equation, and deletes Equation 11. The alternate model makes the same assumptions as the original model. The network generation rules are also the same and the mine and mode flow existence rules are modified. The set, \( I_G \), represents the incompatible arcs derived for group \( g \). The generation of the set \( I_G \) is described more fully in this chapter under “Generation of Incompatible Arc Groups.”

Minimize

\[
\nu = \sum_{i=1}^{n_N} \sum_{j=1}^{n_N} \sum_{p=0}^{n_p} \sum_{\mu \in \text{YTE}} \sum_{\nu \in \text{YTE}} \text{OBJ}_{ijp\nu} y_{ijp\nu} \tag{1}
\]

Subject to

\[
\hat{x}_{np} - \hat{x}_{np} + \sum_{j=1}^{n_N} \sum_{\mu \in \text{YTE}, \nu \in \text{YTE}} x_{njp\mu} - \sum_{i=1}^{n_N} \sum_{\mu \in \text{YTE}, \nu \in \text{YTE}} x_{inp\mu} = RHX_{np} \quad \forall n \in \overline{DN}, p \tag{2}
\]

\[
\hat{x}_{dp} - \hat{x}_{dp} - \sum_{i=1}^{n_N} \sum_{\mu \in \text{YTE}, \nu \in \text{YTE}} x_{idp\mu} = RHX_{dp} \quad \forall d, p \tag{3}
\]

\[
\sum_{j=1}^{n_N} \sum_{\mu \in \text{YTE}, \nu \in \text{YTE} | I_G} y_{ijp\nu} \leq 1 \quad \forall i, \mu, g \tag{12}
\]

\[
XUP_{ijp\nu} y_{ijp\nu} - x_{ijp\nu} \geq 0 \quad \forall i, j, p, \mu \in \text{XTE}, y_{ijp\nu} \in \text{YTE} \tag{5}
\]
\[
\sum_{j=1}^{n_N} \sum_{p \in \mathcal{YTE}} PRT_{ij} y_{ijp} \leq MXT_{ij} \quad \forall i, \mu
\]  
(6)

\[
\sum_{j=1}^{n_N} \sum_{p \in \mathcal{YTE}} y_{ijp} \leq MXR_{ij} \quad \forall i, \mu
\]  
(7)

\[0 \leq x_{ijp} \leq XUP_{ijp} \quad \forall i, j, p, \mu \]  
(8)

\[y_{ijp} \in \{0, 1\} \quad \forall i, j, p, \mu \]  
(9)

\[0 \leq \hat{x}_{np} \leq ZUP \quad \forall n, p \]  
(10)

7. Comments about Model B

All comments from Model A also apply to Model B, except for those that pertain to Equations 4 and 11. The purpose of Equation 12, which replaces Equation 4 in Model A, is to ensure that only one arc in a group of incompatible arcs is selected.

8. Detailed Data Description for Both Models

The data discussed in the next five paragraphs is input by the user and must be manipulated by both models to generate the networks, calculate the data describing the networks, time-expand the nodes to form the time-expanded networks, and develop data for the objective and constraint equations. The input data in the following list is scalar. The first three scalars are required input and the last three are optional (default values will be assumed if no input is given):

- **TEND** - the time (in hours, h) by which all modes transport or deliver mines, meeting all demand, and return to their origin nodes, where \( TEND > 0 \).

- **TPER** - the length of the time period (h) used in the time expansion, where \( 0 < TPER \leq TEND \).

- **RELT** - the value of the zero tolerance for comparing real data, where \( 0 < RELT \leq 0.001 \). i.e., if \(|x - y| < RELT\), then \( x = y \) is assumed.

- **MNLA** - the minimum distance (in nautical miles, nm) that air modes can transport mines to prevent air modes from shipping mines over short routes intended for land modes, where \( MNLA \geq 0 \). The default is 0.

- **MXSS** - the maximum distance (nm) allowed for supply to supply transfers, where \( MXSS > 0 \). The default is the maximum distance between nodes.
- **DISC** - a binary parameter in which 1 means to run the model without checking for disconnected networks and 0 means to check for disconnected networks and to run the model for the first connected network found. The default is 0.

The model converts **TEND** into \( \lceil \frac{TEND}{TPER} \rceil \) time periods since it time-expands the network over time periods of length **TPER**. If **TPER** is relatively small, the round-off error is less when converting times to time periods. However, a smaller time period length also yields more time periods for the problem, which expands the size of the model. Since a large model is harder to solve, a balance must be struck between **TPER** and the number of periods that will be formed. The user must also make **TEND** large enough to result in a feasible solution while making it small enough to cut down the size of the time-expansion.

The data presented in the following list pertains to the node index \( n \) or any of its subindices. For all data pertaining to priorities in this thesis, a **lower value implies a higher priority**, e.g., a demand node with a priority of 1 is more important than a node with priority 2. The purpose of the third parameter \( MXD_n \) is to allow the user to group mine fields that are close together into one node and to account for transit time needed to travel between mine fields. Since a mode group can only have one destination node per run, \( MXD_d \) permits the mode group to travel to all mine fields within a demand node if it has the capacity to carry enough mines. The first parameter is required as input and the last five are optional:

- **AMT** - the supply or demand at node \( n \) (mines), where, by convention, assembled supply and MOMAG disassembled supply amounts are positive and demand amounts are negative. The default is 0.
- **PRN** - the priority of node \( n \), where \( PRN_n > 0 \). The default is the maximum node priority (or 1 if none are input).
- **MXD** - the maximum distance (nm) between mine fields within a demand node \( d \), where \( MXD_d \geq 0 \). The default is 0.
- **BGD** - the distance (nm) that must be transited by the sea node \( c \) to reach its on-station point before launching aircraft to lay mines, where \( BGD_c \geq 0 \). The default value is 0.
- **EDD** - the distance (nm) that must be transited by the sea node \( c \) to reach a new destination by the last period of the problem after it completes its last run, where \( EDD_c \geq 0 \). The default value is 0.
- **SPX** - the transit speed (nm/h) of the sea node, where \( SPX_c > 0 \). If \( BGD_c > 0 \) or \( EDD_c > 0 \), \( SPX_c \) is no longer optional.

The optional parameters \( BGD_c, EDD_c, \) and \( SPX_c \) are not intended to be used for submarines and HVMLs since these modes transit directly to the mine field.
The following list of input parameters describes mode characteristics indexed by mode $\mu$ or any of its subindices. The first seven parameters are required and the last one is optional:

- **$CAP_\mu$** - the capacity of one mode unit of mode $\mu$ (mines), where $CAP_\mu > 0$. $CAP_\mu$ is required for all modes except dummy build modes, whose capacities are computed by the model.
- **$MXL_\mu$** - the maximum distance (nm) that mode $\mu$ can travel from its origin node and be able to return, where $MXL_\mu > 0$. $MXL_\mu$ is required for all modes except the dummy build modes.
- **$SPD_\mu$** - the average transit speed (nm/h) of mode $\mu$, where $SPD_\mu > 0$. $SPD_\mu$ is required for all modes, where the speed of a dummy build mode $\beta$ is its build rate.
- **$TLD_\mu$** - the average amount of time (h) needed to load a mode group and take-off, where $TLD_\mu \geq 0$. The default is 0.
- **$TUL_\mu$** - the average amount of time (h) needed to unload a mode group for transportation modes or deploy the mines for delivery mode groups, where $TUL_\mu \geq 0$. The default is 0.
- **$TGD_\mu$** - the average amount of time (h) needed to spend on the ground after a run before reloading and taking off, where $TGD_\mu \geq 0$. The default is 0.
- **$PRM_\mu$** - the priority of mode $\mu$, where $PRM_\mu > 0$. The default is the maximum mode priority (or 1 if none are input).

The parameter $CAP_\mu$ can be used in two different ways. If a mode group is composed of the same type of mode unit, the capacity of a mode unit can be input as $CAP_\mu$. But, if the mode group is composed of variety of modes, $CAP_\mu$ should be the total number of mines that the mode group can carry. The user must take this distinction into account when inputting the number of modes stationed at node. For example, if the mode group at a given node is composed of seven P3 Orion aircraft and the user inputs $CAP_\mu$ as the capacity of a mode unit, the number of modes at that node should be seven. However, if the mode group based at a given node is a tactical air squadron and the user inputs $CAP_\mu$ as the total capacity, the number of modes at that node should be one. For dummy sea modes based at mobile sea nodes that can deploy mines, $CAP_\mu$ should equal the supply of the node in which the dummy sea mode is stationed. The three time-related parameters, $TLD_\mu$, $TUL_\mu$, and $TGD_\mu$, are assumed to equal 0 for dummy build modes.

The next list of parameters involves the mode groups based at their origin nodes. The first parameter is required and the last four parameters are optional:

- **$NUM_\mu_i$** - the number of mode units $\mu$ in the mode group based at origin node $i$, where $NUM_\mu_i \geq 0$. $NUM_\mu_i \geq 1$ for at least one mode group containing mode units $\mu$ at every origin node $i$. The default is 0.
• **BGT**<sub>μi</sub> - the earliest time (h) in which mode group μ is available for loading at its origin node i for its first run, where 0 \( \leq BGT_{\mu i} \leq TEND \). The default is 0.

• **EDT**<sub>μi</sub> - the latest time (h) by which mode group μ must return to its origin node i after completing its last run, where 0 \( \leq EDT_{\mu i} \leq TEND \). The default is TEND.

• **MXT**<sub>μi</sub> - the maximum total time (h) that mode group μ can be absent from its origin node i in making all of its runs, where 0 \( \leq MXT_{\mu i} \leq TEND \). The default is TEND.

• **MXR**<sub>μi</sub> - the maximum number of runs that mode group μ based at its origin node i is allowed to take, where 0 \( \leq MXR_{\mu i} \leq \lceil TEND / TPER \rceil \). The default is \( \lceil TEND / TPER \rceil \).

For dummy build modes, **NUM**<sub>m</sub> should equal 2 if the MOMAG node m has dual parallel assembly lines and 1 otherwise. This convention will double the build rate for parallel assembly lines. If the number of parallel assembly lines at MOMAGs are increased in the future, then the user can account for this growth by setting **NUM**<sub>m</sub> equal to the number of parallel assembly lines. For dummy sea modes stationed on mobile sea nodes that can deploy mines, **NUM**<sub>m</sub> should equal 1.

The following optional input parameter pertains to arcs in the mine and mode networks. It allows the user to rule out arcs in the networks that will be selected by the model:

• **XMP**<sub>μij</sub> - a binary parameter in which 1 implies that no arc from origin node i to destination node j via mode group μ shall be allowed in the network. The default value is 0.

Thus, by including **XMP**<sub>μij</sub> if necessary, the user can eliminate arcs that are not allowed because of obscure rules not accounted for in the model.

The final input parameter is required and defines the distance between nodes:

• **DIS**<sub>ij</sub> - the distance (nm) between origin node i and destination node j. The default value is 0.

Distances may be given in a table containing all distances between points. Distances may also be listed separately for arcs that will probably be used in the network. If the distance from node i to node j is input, then the model assumes that it also equals the distance from node j to node i. The former method of input ensures that the model will consider all possible node combinations for the networks, while the latter method is less tedious to enter if the user thinks only a few arcs are needed.

**Two important conventions must be followed to ensure the successful generation of the correct mine and mode networks.** Entering “negative” distances allows the user to describe two properties of the networks without creating additional input parameters.
First, all distances between nodes that are on the same land mass must be less than 
\(-0.5\) to indicate arcs that have potential land routes: \(DIS_{\mu} < -0.5\), where \(l \neq l'\). Second, to identify a MOMAG node and its adjacent supply node, the distance between a MOMAG node and its adjacent supply node must be equal to \(-0.5\): \(DIS_m = -0.5\). This convention ensures that the MOMAG node is connected only to its adjacent supply node.

The next list of parameters is computed by the model, given the input data to describe the original and the time-expanded networks for mode flow and mine flow. These parameters assume that the network has already been generated. (Generation rules will be given in the next section of this chapter under “Network Generation Rules”). The model takes a conservative approach to rounding time periods up or down. For example, when converting time data to time periods, the model rounds up begin times input by the user \((BGT)\) but rounds down end times \((EDT)\). Rounding data in this way ensures that the time periods fall within time bounds input by the user. Likewise, calculations of arrival time periods for one-way trips and return time periods for round trips are rounded up to be on the safe side.

- **NRLT** - the value of the negative zero tolerance allowed when comparing real data, where \(NRLT = -RELT\).
- **SUMA** - the number of arcs in the mine network.
- **ARC_{ij\mu}** - the set that indicates whether or not the arc from node \(i\) to node \(j\) via mode \(\mu\) belongs in the mine and modes networks.
- **ART_{ij\mu}** - a temporary set that is identical to \(ARC_{ij\mu}\).
- **DNO_n** - a temporary parameter that serves two different purposes in the model: (1) to indicate when node \(n\) has already been checked for calculating \(MXM\) in one loop and for calculating \(MXS\) in another loop and (2) to indicate that node \(n\) is included in the first connected network found.
- **NFC\_** - the set of nodes that does not form a component with at least one supply node and one demand node.
- **XTE_{ij\mu}\_** - the set that indicates whether or not the mine arc leaving node \(i\) and arriving at \(j\) at the end of time period \(p\) via mode \(\mu\), belongs in the time-expanded network.
- **YTE_{ij\mu}\_** - the set that indicates whether or not the mode arc leaving node \(i\) and arriving at \(j\) at the end of time period \(p\) via mode \(\mu\), belongs in the time-expanded network.
- **MNU_{ij\mu}\_** - the set of all mine flow balance equations in which at least one arc leaves node \(n\) at the beginning of time period \(p\) or one arc arrives at node \(n\) at the end of time period \(p\).
• **MDU** - the set of all mode flow balance equations in which at least one arc leaves node \( i \) at the beginning of time period \( p \) via mode \( \mu \) or one arc arrives at node \( i \) at the end of time period \( p \) via mode \( \mu \).

• **RHX** - the value of the right-hand side of the mine flow balance equation for mines leaving node \( n \) at the beginning of time period \( p \), where \( p \) is either the time period before inventory can start changing or the time period after inventory can stop changing and

\[
RHX_n = \begin{cases} 
\min (|AMT_n|, ZUP_n) & \text{if } p \text{ is before.} \\
0 & \text{if } p \text{ is after.}
\end{cases}
\]

• **RHY** - the value of the right-hand side of the mode flow balance equation for mode \( \mu \) based at origin node \( n \) at the beginning of time period \( p \), where \( RHY_n \) equals \( I \) if \( p \) is the time period before mode \( \mu \) can start its first run, equals \(-1\) if \( p \) is the time period after it can finish its last run, and equals 0 otherwise. Notice that, since mode flow and inventory variables are binary, \( I \) represents the presence of the supply of mode \( \mu \) at \( n \) when the problem starts and \(-1\) represents the final return of the mode supply at the end of the problem.

• **PAR** - the number of time periods with length \( TPER \) required to load mode \( \mu \) based at origin node \( i \), travel to destination node \( j \) and unload (or deliver) mines, where

\[
PAR_{ij\mu} = \frac{TLD_\mu + TUL_\mu}{TPER} + \begin{cases} 
1 & \text{if } \mu \in BM.
\\
\frac{(|DIS_j| + MXD)}{SPD_\mu TPER} & \text{otherwise.}
\end{cases}
\]

• **PRT** - the number of time periods with length \( TPER \) required to load mode \( \mu \) based at origin node \( i \), travel to destination node \( j \), unload (or deliver) mines, return, and make ground preparations for turnaround, where

\[
PRT_{ij\mu} = \frac{TLD_\mu + TUL_\mu + TGD_\mu}{TPER} + \begin{cases} 
1 & \text{if } \mu \in BM.
\\
\frac{(2|DIS_j| + MXD)}{SPD_\mu TPER} & \text{otherwise.}
\end{cases}
\]

• **IND** - the indegree of (or the number of arcs directed into) node \( n \).

• **OTD** - the outdegree of (or the number of arcs directed out of) node \( n \).

• **BGP** - the earliest time period in which a run can start at origin node \( i \) and arrive at its destination node \( j \) via mode \( \mu \), where \( BGP_{ij\mu} \geq BGT_{ij} / TPER \).

• **EDP** - the latest time period in which a run can start at origin node \( i \) and arrive at its destination node \( j \) via mode \( \mu \), where \( EDP_{ij\mu} \leq EDT_{ij} / TPER \).

• **MXM** - the maximum number of mines that could be sent through node \( n \) if all demand on paths containing node \( n \) were filled.

• **MXS** - the maximum number of mines that could be sent through node \( n \) if all supply on paths containing node \( n \) were sent.

• **OBJ** - the value contributed to the objective function by the mode flow for mode \( \mu \) leaving origin node \( i \) arriving at destination node \( j \) at time period \( p \), where
\[ \text{OBJ}_{i\mu} = [PR_{i} PR_{j} PRM_{\mu} (p + PRT_{i\mu})]^2. \]

- \( ZUP_n \) - the upper bound (mines) on mine inventory for node \( n \), where
  \[ ZUP_n = \min\{MXM_n, MXS_n\}. \]

- \( XUP_{ij\mu} \) - the upper bound on mine flow variables from origin node \( i \) to destination node \( j \) via mode \( \mu \), where
  \[ XUP_{ij\mu} = \min\{ZUP_i, ZUP_j, CAP_{\mu} NUM_{\mu}\}. \]

The parameters \( BGP_{\mu} \) and \( EDP_{\mu} \) are the keys to eliminating unnecessary arcs. For example, if the origin node \( i \) is a transshipment node, the model starts creating outgoing arcs after the earliest period in which mines could have arrived at node \( i \). Likewise, the model does not create arcs that send mines to an origin node in periods after the last possible run could have left the origin node. This requirement cuts down on the number of continuous and binary variables, which increases the speed of model execution.

The parameter \( \text{OBJ}_{i\mu} \) is composed of two parts: the priority parameters and the time parameter. The priority parameters, \( PR_{i}, PR_{j}, \) and \( PRM_{\mu} \), enable the model to consider the relative priorities of nodes and modes during optimization. Because scaling problems may occur if the objective value grows too large, the user should only input relative priorities and the maximum priority should be less than five. Since the model is a minimization, the model tends to select mode flow across arcs for which the priority portion of the objective value is smaller (implying that the priority is higher). The second part of the objective value is the time parameter, \( p + PRT_{i\mu} \). The model will again tend to select mode flow for which the time portion is smaller. Thus, mode flow that starts sooner and has a faster round trip time will be considered more optimal. Because it is the product of the priority parameters and time parameters, the objective value of SUMIT incorporates both priority and time into a measure of effectiveness.

Model B generates nine extra internal parameters and deletes one internal parameter \( MDU_{\mu} \) from the original model. These additional parameters are needed to delineate the set of incompatible arcs \( IG_{\mu} \). Additional parameters generated by Model B are:

- \( IBEG \) - a scalar which is incremented by one in the loop that computes \( GBG_{i\mu} \) and \( GED_{i\mu} \) for arcs in the incompatible arc groups \( IG_{\mu} \), where
  \[ IBEG \geq MNB_{\mu}. \]

- \( MRN_{\mu} \) - the minimum return period \( PRT_{i\mu} \) over all arcs from node \( i \) directed to node \( j \) via mode \( \mu \).
• $MRX_{i\mu}$ - the maximum return period $PRT_{i\mu}$ over all arcs from node $i$ directed to node $j$ via mode $\mu$.
• $MNA_{i\mu}$ - the minimum arrival period $PAR_{i\mu}$ over all arcs from node $i$ directed to node $j$ via mode $\mu$ for which $PRT_{i\mu} = MNA_{i\mu}$.
• $MIN_{i\mu}$ - the minimum begin period $BGP_{i\mu}$ over all arcs from node $i$ directed to node $j$ via mode $\mu$.
• $MXE_{i\mu}$ - the maximum end period $EDP_{i\mu}$ over all arcs from node $i$ directed to node $j$ via mode $\mu$.
• $LSG_{i\mu}$ - the number of incompatible arc groups leaving $i$ via mode $\mu$, where $EDG_{i\mu}$ is $EDG_{i\mu}$ for which $PAR_{i\mu} = MNA_{i\mu}$ and $PRT_{i\mu} = MRA_{i\mu}$ in the equation $LSGL = \min_{j\in N}(EDG^\nu_{j\mu}) + 2 - MNB_{i\mu} - MRN_{i\mu}$.
• $GBG_{i\mu}$ - the group number $g$ of the first incompatible arc grouping $IG_g$ in which the arc from $i$ via mode $\mu$ to $j$ arriving at time period $p$ appears.
• $GED_{i\mu}$ - the group number $g$ of the last incompatible arc grouping $IG_g$ in which the arc from $i$ via mode $\mu$ to $j$ arriving at time period $p$ appears.

9. Network Generation Rules for Both Models

Before discussing the mine and mode flow existence sets $XTE$ and $YTE$, rules for generating the original network are given since membership in the mine or mode networks is a criteria for membership in the flow existence sets. Initially, the set of eligible arcs is assumed to be the power set $N \times N \times M$. Arcs that do not meet the rules for network generation are eliminated from the power set. The following list describes the existence requirements for arcs from origin node $i$ to destination node $j$ via mode $\mu$ that belong to the original network:

• An arc must leave origin node $i$ and arrive at destination node $j$ via mode $\mu$ if $NUM_{i\mu} > 0$, $i \neq j$, $DIS_j \neq 0$, and $MXL_{i\mu} \leq |DIS_j|$.
• Any arc from $i$ to $j$ via $\mu$ is eliminated if $XMP_{i\mu} = 1$.
• Any arc between two supply nodes on land, $i,j \in SN \cap LN$, via a land transportation mode, $\mu \in TM \cap LM$, is eliminated if $DIS_j \geq 0$ or $|DIS_j| \geq MXSS$.
• Any arc between two supply nodes on land, $i,j \in SN \cap LN$, via an air transportation mode $\mu \in TM \cap AM$ is eliminated if $DIS_j < 0$ and one of the following statements are true: $|DIS_j| \geq MNLA$ or $|DIS_j| \geq MXSS$.
• Any arc between two supply nodes, $i,j \in SN$, via mode $\mu$ is eliminated if $i,j \notin LN$ or $\mu \notin TM$.
• Any arc is eliminated between two nodes $i,j$ via land transportation mode $\mu \in TM \cap LM$ if $DIS_j \geq 0$.
• Any arc is eliminated between two land nodes $i,j \in LN$ via air transportation mode $\mu \in TM \cap AM$ if $DIS_j < 0$ and $|DIS_j| \geq MNLA$. 

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Any arc from sea node \( c \) is eliminated if it does not arrive at demand node \( d \).

Any arc directed to either supply node \( s \) or transshipment node \( t \) is eliminated if the mode is a delivery mode \( \delta \).

Any arc arriving at a demand node \( d \) via a nondelivery mode is eliminated.

Any arc leaving a MOMAG node \( m \) and arriving at any node \( j \) for which \( DIS_{imj} \neq -0.5 \) is eliminated.

Any arc entering a MOMAG node \( m \) is eliminated.

Any arc leaving a transshipment node \( i \) and arriving at a supply node \( s \) is eliminated.

Any arc from \( i \) to \( j \) via \( i \) is eliminated if its round trip time \( TPER \times PRT_{ij} \) is greater than the total time allowed \( MXT_i \) or the time difference between \( EDT_i \) and \( BGT_i \).

Eliminate all arcs directed from node \( i \) if node \( i \) has no mines in inventory at the beginning of the problem and no arcs are directed into node \( i \).

Any arc from \( i \) to \( j \) via \( \mu \) is eliminated if \( BGP_{ij} \geq EDP_{ij} \).

Any arc whose origin node \( i \) and destination node \( j \) is not contained in the first connected component found.

10. Generation of Incompatible Arc Groups

The procedure for determining incompatible arc groups for Model B sets up incompatible arc groups around an arc that has a return period equal to \( MRN_i \) and has an arrival period equal to \( MNA_i \). Because such an arc has the minimum return period and the minimum arrival period for mode \( \mu \) leaving node \( i \), the first run associated with this arc has the earliest arrival time period. The number of potential runs for this arc is also greater than (or equal to) the number of potential runs for other arcs that do not meet this criteria because arcs with greater return periods are forced to make fewer runs within the same amount of time. Figure 8 illustrates this point by depicting the relationship between arcs leaving the same node via the same mode arriving at different destinations with different arrival and return time periods. Focusing on this arc, then forms the greatest possible number of incompatible arc groups for mode \( \mu \) leaving \( i \) since it has the most runs and covers all possible time periods since its first run also has the earliest arrival time period.

In the first step of the procedure, incompatible arcs groups are generated for arcs from \( i \) via mode \( \mu \) to \( j \) arriving at the end of time period \( p \) with return periods of \( MRN_i \) and with arrival periods of \( MNA_i \). Incompatible arcs for an arc from node \( i \) arriving at node \( j \) by the end of time period \( p \) are those that leave \( i \) before the mode group can return to \( i \). Then, any arc that leaves node \( i \) after period \( p - MNA_i \) and before
period $p + MRN_{i} - MNA_{i}$, is not compatible. Adding $MNA_{i}$ to both sides implies that incompatible arcs arrive at node $j$ after period $p$ and before period $p + MRN_{i}$. The procedure starts at $IBEG = MNB_{i}$ for the first group, iterates through a loop that calculates $GBG_{iup}$ and $GED_{iup}$, increments $IBEG$ by one, and repeats the calculations until the loop for group $LSG_{i}$ is completed. $GBG_{iup}$ is then the minimum group number such that the following bounds are true for arrival time period $p$: $IBEG \leq p < IBEG + MRN_{i}$. Likewise, $GED_{iup}$ is the maximum group number for which the bounds are true. Figure 9 illustrates the formation of incompatible arc groups for arcs with minimum arrival and return periods using this procedure.

The next step of this procedure then determines arc incompatibility for all remaining arcs (which have return time periods greater than $MRN_{i}$ or arrival time periods greater than $MNA_{i}$). It selects an incompatible arc by comparing the time period that the arc leaves node $i$ and the time period in which it returns to node $i$ with the take-off and return time periods of the incompatible arc groups formed in the first step. Once again, the procedure initializes $IBEG$ to $MNB_{i}$ for the first group and iterates through the loop to calculate $GBG_{iup}$ and $GED_{iup}$ until group number $LSG_{i}$ is checked. The bounds for period $p$ are more complex in this step because the arrival time periods are not equal to $MNA_{i}$ or the return time periods are not equal to $MRN_{i}$. $GBG_{iup}$ is the
minimum group number such that the following bounds are true for arrival time period \( p \):

\[
p \geq IBEG - MNA_{i\mu} - PAR_{ij\mu} \text{ and }
\]

\[
p < IBEG + (2 \times MRN_{i\mu}) - MNA_{i\mu} + PRT_{ij\mu} - PAR_{ij\mu}.
\]

Likewise, \( GED_{i\mu} \) is the maximum group number for which the bounds hold true. Figure 10 illustrates this step in generating incompatible arcs groups.

Although it appears that all incompatible groups have been generated, one more step is required because in some situations two arcs in \( IG_z \) may be compatible. This situation occurs when the earliest arc in a group to leave for one destination is compatible with the latest arc in the same group to leave for another destination. Figure 11 illustrates the situation when two compatible arcs are in the same group. To avoid compatibility within incompatible arc groups, the compatible arc \textit{that does not have the minimum return period} \( PRT_{ij\mu} \) \textit{and the minimum arrival period} \( PAR_{ij\mu} \) is removed from that group. Removing this arc does not change its relationship with other incompatible arcs because it does not have the minimum return and arrival period and must have already been included in previous incompatible arc groups.
The final step of the procedure is to eliminate the arcs added to $IG_2$ in the second step that are actually compatible with arcs placed in $IG_1$ in the first step. Loop through all incompatible arc groups $IG_g$, where $1 \leq g \leq LSG_{IG_1}$, change $GBG_{IG_g}$ to the maximum of $GBG_{IG_g}$ and $g+1$ if the following is true:
- $PRT_{ij} > MRN_{ij}$ or $PRT_{ij} > MRN_{ij}$ and
- $p \geq MNB_{ij} + (g-1) + MRN_{ij} - MNA_{ij} + PAR_{ij}$.

Change $GED_{ij}$ to the minimum of $GED_{ij}$ and $g-1$ if the following is true:
- $PRT_{ij} > MRN_{ij}$ or $PRT_{ij} > MRN_{ij}$ and
- $p < MNB_{ij} + (g-1) + MRN_{ij} - MNA_{ij} + PAR_{ij} - PRT_{ij}$.

Figure 11 illustrates the final step of the procedure that generates incompatible arc groups.

11. Flow Existence Rules

Once the mine and mode networks have been generated, membership in the time-expanded network for Model A is simple to express. Rules for membership in the existence sets are as follows:
- $x_{ij}$ is in $XTE$ if the arc from $i$ to $j$ via mode $\mu$ is part of the original network and $BGP_{ij \mu} \leq p \leq EDP_{ij \mu}$. 

30
Figure 11. Illustration of the Final Step in Generating Incompatible Arc Groups

- \( y_{ij \mu} \in \text{YTE} \) if the arc from \( i \) to \( j \) via mode \( \mu \) is part of the original network and \( BGP_{ij \mu} \leq p \leq EDP_{ij \mu} \).

The mine and mode flow existence sets for the Model B are very similar to those of the original model:

- \( x_{ij \mu} \in \text{XTE} \) if the arc from \( i \) to \( j \) via mode \( \mu \) is part of the original network and \( BGP_{ij \mu} \leq p \leq EDP_{ij \mu} \).
- \( y_{ij \mu} \in \text{YTE} \) if the arc from \( i \) to \( j \) via mode \( \mu \) is part of the original network and \( BGP_{ij \mu} \leq p \leq EDP_{ij \mu} \).
- \( y_{ij \mu} \in \text{IG} \) if \( y_{ij \mu} \in \text{YTE} \) and \( GBG_{ij \mu} \leq g \leq GED_{ij \mu} \).
III. COMPUTATIONAL RESULTS

A. COMPUTER IMPLEMENTATION OF PROPOSED MODELS

GAMS, General Algebraic Modeling System, [Ref. 7] was selected for several reasons to implement the proposed models into computer code and to compare the relative efficiencies of the models. GAMS is a software package that incorporates FORTRAN-based solvers for optimization problems. The advantages of GAMS in developing optimization models are that GAMS allows changes to be made simply, takes care of mundane details such as array sizes, generates all equations needed to solve the problem based on the algebraic expressions given in the model formulation, and uses relational databases to organize the data. Also, GAMS outputs the size of the problem in terms of the number of individual constraints (called "equations" by GAMS) and the number of discrete and continuous variables as well as information on the efficiency of the model in terms of the number of iterations and the computer resources needed to solve the problem. Such output can be used to choose the fastest model of the two proposed models for SUMIT.

B. COMPARISON OF MODELS

Since Model B eliminates the need for tracking mode inventory, it should have a little over half the number of binary variables generated by Model A in most cases. Fewer binary variables should make the alternate model easier to solve and, thus, faster than the original model. To test this hypothesis, ten different problems were solved by both models. Table I summarizes the input data for the ten problems. Since having more than three transshipment nodes for a regional problem is rare, the number of transshipment nodes is set at two for all ten problems. Also, the number of node-mode combinations, which is the number of positive $NUM_{iw}$ parameters input by the user, is included rather than the number of modes. The number of node-mode combinations is more representative of the size of the model since all mode types are rarely stationed at the same origin node. To keep this thesis unclassified, the input data for the ten problems do not represent actual data, but are realistic approximations of actual data. The resulting output concerning model efficiency is expressed in Table 2. Both models arrived at the same optimal solution for all ten problems, which demonstrates the validity of Model B. The problems were executed on an Amdahl 5990 mainframe using
GAMS/ZOOM, where ZOOM is one of the packages available with GAMS for mixed integer programs.

Table 1. INFORMATION ABOUT SCENARIOS FOR TEN TEST PROBLEMS

<table>
<thead>
<tr>
<th>Model Number</th>
<th>End time</th>
<th>Number of Periods</th>
<th>Number of Nodes</th>
<th>Number of MOMAG/Supply Nodes</th>
<th>Number of Demand Nodes</th>
<th>Number of Node-Mode Combinations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
<td>10</td>
<td>10</td>
<td>6</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>10</td>
<td>11</td>
<td>6</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
<td>10</td>
<td>11</td>
<td>5</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>26</td>
<td>6</td>
<td>10</td>
<td>6</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>30</td>
<td>6</td>
<td>11</td>
<td>6</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>27</td>
<td>7</td>
<td>9</td>
<td>4</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>7</td>
<td>30</td>
<td>6</td>
<td>12</td>
<td>7</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>75</td>
<td>15</td>
<td>12</td>
<td>7</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>9</td>
<td>45</td>
<td>7</td>
<td>11</td>
<td>5</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>10</td>
<td>26</td>
<td>7</td>
<td>10</td>
<td>6</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>Average</td>
<td>37</td>
<td>8</td>
<td>11</td>
<td>6</td>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>

Several important conclusions about the relative efficiencies of the two models can be made, based on the information presented in Table 2. For all ten problems, Model B produced fewer constraints than Model A, reducing the number of equations by an average of 15%. Both models have the same number of continuous variables because both models contain the same flow balance equations to track mine flow and mine inventory. Table 2 shows that, on average, Model B has fewer binary variables (by 39%), fewer iterations (by 87%), and smaller work parameters (by 76%). The work parameter controls the number of nodes waiting to be checked during the “branch and bound” portion of execution. If the work parameter is set too small, ZOOM will terminate execution and ask the user to reset the work parameter. Thus, for problems similar to the test problems, the sixth column of Table 2 suggests that setting the work parameter at 10,000 would suffice. In short, the data in Table 2 indicates that, for problems similar to the ten problems tested, Model B yields fewer constraints, binary variables, and iterations and smaller work parameters.
Table 2. RESULTS OF TEN TEST RUNS TO COMPARE BOTH MODELS

<table>
<thead>
<tr>
<th>Problem and Model Numbers</th>
<th>Number of Constraints</th>
<th>Number of Continuous Variables</th>
<th>Number of Discrete Variables</th>
<th>Number of Iterations</th>
<th>Work Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A</td>
<td>160</td>
<td>102</td>
<td>96</td>
<td>167,646</td>
<td>15,614</td>
</tr>
<tr>
<td>1B</td>
<td>136</td>
<td>102</td>
<td>54</td>
<td>41,641</td>
<td>5,319</td>
</tr>
<tr>
<td>2A</td>
<td>237</td>
<td>169</td>
<td>144</td>
<td>155,958</td>
<td>4,568</td>
</tr>
<tr>
<td>2B</td>
<td>211</td>
<td>169</td>
<td>94</td>
<td>4,153</td>
<td>100</td>
</tr>
<tr>
<td>3A</td>
<td>179</td>
<td>118</td>
<td>114</td>
<td>213,304</td>
<td>2,361</td>
</tr>
<tr>
<td>3B</td>
<td>155</td>
<td>118</td>
<td>69</td>
<td>57,593</td>
<td>217</td>
</tr>
<tr>
<td>4A</td>
<td>75</td>
<td>39</td>
<td>43</td>
<td>23,663</td>
<td>1,403</td>
</tr>
<tr>
<td>4B</td>
<td>56</td>
<td>39</td>
<td>22</td>
<td>3,707</td>
<td>257</td>
</tr>
<tr>
<td>5A</td>
<td>144</td>
<td>92</td>
<td>90</td>
<td>106,720</td>
<td>3,283</td>
</tr>
<tr>
<td>5B</td>
<td>125</td>
<td>92</td>
<td>59</td>
<td>8,053</td>
<td>824</td>
</tr>
<tr>
<td>6A</td>
<td>137</td>
<td>96</td>
<td>84</td>
<td>149,176</td>
<td>5,634</td>
</tr>
<tr>
<td>6B</td>
<td>122</td>
<td>96</td>
<td>57</td>
<td>36,735</td>
<td>2,330</td>
</tr>
<tr>
<td>7A</td>
<td>86</td>
<td>50</td>
<td>47</td>
<td>187,531</td>
<td>4,119</td>
</tr>
<tr>
<td>7B</td>
<td>66</td>
<td>50</td>
<td>26</td>
<td>4,407</td>
<td>543</td>
</tr>
<tr>
<td>8A</td>
<td>142</td>
<td>100</td>
<td>81</td>
<td>318,461</td>
<td>5,161</td>
</tr>
<tr>
<td>8B</td>
<td>116</td>
<td>100</td>
<td>49</td>
<td>43,308</td>
<td>1,455</td>
</tr>
<tr>
<td>9A</td>
<td>143</td>
<td>96</td>
<td>87</td>
<td>401,748</td>
<td>7,525</td>
</tr>
<tr>
<td>9B</td>
<td>127</td>
<td>96</td>
<td>55</td>
<td>26,773</td>
<td>1,041</td>
</tr>
<tr>
<td>10A</td>
<td>83</td>
<td>45</td>
<td>50</td>
<td>150,445</td>
<td>3,178</td>
</tr>
<tr>
<td>10B</td>
<td>62</td>
<td>45</td>
<td>27</td>
<td>9,128</td>
<td>474</td>
</tr>
<tr>
<td>Average A</td>
<td>139</td>
<td>91</td>
<td>84</td>
<td>187,465</td>
<td>5,195</td>
</tr>
<tr>
<td>Average B</td>
<td>118</td>
<td>91</td>
<td>51</td>
<td>23,550</td>
<td>1,256</td>
</tr>
<tr>
<td>Percent Reduction</td>
<td>15%</td>
<td>0%</td>
<td>39%</td>
<td>87%</td>
<td>76%</td>
</tr>
</tbody>
</table>

Of the ten problems executed by Model A, problem nine had the highest number of iterations and required 12 minutes of CPU time to reach optimality. When problem nine was solved by Model B, CPU time was cut to 0.75 minutes. Problem seven was the most dramatic illustration of the reduction in CPU time, while problem three was the least dramatic. For problem seven, the CPU time for Model A was 3.5 minutes, which
was reduced to only 0.1 minutes when solved by Model B. The CPU time taken by Model A and by Model B to solve problem three was 7.5 minutes and two minutes respectively. The fact that Model B produces fewer constraints and fewer binary variables means that it requires fewer iterations and a smaller work parameter to reach optimality. This reduction in size and increase in speed, cut the CPU time by two-thirds in the worst-case situation of the ten problems tested.

C. GAMS IMPLEMENTATION OF SUMIT

The implementation of SUMIT and a sample problem in GAMS is given in Appendix A. The output from the first run of the problem in Appendix A is contained in Appendix B. The input for and the output from the second, and final run, are contained in Appendices C and D respectively. Appendix E discusses several aspects of the GAMS implementation of SUMIT, including a description of SUMIT's output.

Because large mixed integer problems are hard to solve, the user should limit the number of time periods to ten if the problem has more than ten nodes. However, since it is difficult to judge the end time and time period length for a new problem, the user should follow the recommendations given in Appendix E to find appropriate values for the end time and time period length. Notice that the efficiency data in Table 2 does not include the initial runs discussed in Appendix E because these runs are very quick.

Other mixed integer program solvers are also available with GAMS. The solver, XA, solved the first run of sample problem in Appendix A in only 18,859 iterations, while, as shown in Appendix B, ZOOM required 76,589 iterations. Thus, XA seems to be superior to ZOOM in solving mine transshipment problems using the SUMIT model.

A way to attack large problems is to allow SUMIT to divide the large problem into several subproblems. SUMIT does not require that the mine or the mode network be connected. In other words, a path need not exist between all nodes in each network. If the network can be divided into multiple connected components, then SUMIT can split the problem into separate subproblems, one for each component. For networks that are large enough to tax the computer's disk space, running SUMIT for each connected component may solve a problem that could not otherwise be solved. By setting the scalar DISC equal to 0, SUMIT will run the optimization on the first connected component of the network that it finds and will notify the user of the nodes not contained in the first component. The user must then rerun SUMIT for the nodes that were not in the first connected component.
The sample program given in Appendix A is an example of a large problem that was divided into subproblems by SUMIT. Appendix B contains the first solution output by SUMIT. SUMIT tells the user to rerun the program for nodes not included in the first component found. If there are at least one supply node and one demand node in this list that have not already been included in components already solved, the user should rerun SUMIT for the nodes indicated. The user may rerun SUMIT without deleting excessive amounts of input data by setting (1) $NUM_m$ equal to 0 for every node and mode combination that was included in the mine network for the first run of SUMIT and (2) $AMT_r$ equal to 0 for all demand already filled by the first run. Appendix C contains the input portion of SUMIT for the second run and illustrates this simple change. Notice that, since the second problem was smaller, the end time $TEND$ and time period length $TPER$ could be decreased.

In many cases, finding the proven optimal solution is very expensive in computer time and memory, so GAMS provides an option that allows the user to specify an acceptable "optimality gap." During the branch and bound portion of execution, ZOOM finds an upper bound and a lower bound on the objective value for the optimal solution. ZOOM terminates execution and reports an "INTEGER SOLUTION" if it has found a feasible integer solution in which the absolute distance between these two bounds divided by the lower bound is less than the optimality gap, called OPTCR. The integer solution reported when ZOOMS halts execution for this reason is not a proven optimal solution. In fact, when running the ten test problems using Model B, the solver found the optimal solution, without proving optimality, by using the GAMS default value of OPTCR (0.1) in six cases. Optimal solutions were also found, but not proven, for three of the remaining four problems by using an OPTCR of 0.01. Appendix E contains recommendations for setting an appropriate OPTCR.

D. REDUCTION OF MINE FLOW AND INVENTORY IN BOTH MODELS

As stated previously, both models have the same number of continuous variables since they both contain the same flow balance equations for mine flow. Creating "reasonable" mine flow and inventory variables decreases the number of continuous variables needed in the model. In the computer implementation of SUMIT, applying the network generation and flow existence rules ensures that only "reasonable" variables for mine flow and inventory are used. In the first portion of the "Pre-Model Manipulation" section of the program (see Appendix A), the set parameter $ARC_{m}$ indicates the arcs that make sense for mine flow in the mine network. In the part of this
section preceding the generation of incompatible arc groups, the set parameter $XTE_{uv}$
indicates the times for which mine flow associated with the arcs in $ARC_{uv}$ can exist.
Likewise, the set parameter $MNU_{v}$ shows the only time periods in which potential
changes in mine inventory may occur. Tables 3 and 4 illustrate the reduction of mine
flow and inventory, respectively, for the ten test problems. They compare the potential
number of variables (mine flow and inventory) with the number of variables actually
created by SUMIT. Using the power set $N \times N \times M$ inflates the number of potential
variables because the number of node-mode combinations can be estimated from the
input data. Thus, the value for $N \times M$ is replaced by the number of node-mode combi-
nations used earlier in Table 1. This swap is indicated by $N-M$ in Tables 3 and 4 and
provides a more accurate estimation of the number of potential variables.

Table 3. MINE FLOW REDUCTION

<table>
<thead>
<tr>
<th>Model Number</th>
<th>Power Set N x N-M</th>
<th>Number of ARCs</th>
<th>Percent Reduction</th>
<th>Power Set N x N-M x P</th>
<th>Number of XTEs</th>
<th>Percent Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80</td>
<td>12</td>
<td>85%</td>
<td>800</td>
<td>54</td>
<td>93%</td>
</tr>
<tr>
<td>2</td>
<td>99</td>
<td>14</td>
<td>86%</td>
<td>990</td>
<td>94</td>
<td>91%</td>
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<tr>
<td>3</td>
<td>88</td>
<td>13</td>
<td>85%</td>
<td>880</td>
<td>69</td>
<td>92%</td>
</tr>
<tr>
<td>4</td>
<td>80</td>
<td>12</td>
<td>85%</td>
<td>480</td>
<td>22</td>
<td>95%</td>
</tr>
<tr>
<td>5</td>
<td>110</td>
<td>21</td>
<td>81%</td>
<td>660</td>
<td>59</td>
<td>91%</td>
</tr>
<tr>
<td>6</td>
<td>72</td>
<td>15</td>
<td>79%</td>
<td>504</td>
<td>57</td>
<td>89%</td>
</tr>
<tr>
<td>7</td>
<td>108</td>
<td>9</td>
<td>92%</td>
<td>648</td>
<td>26</td>
<td>96%</td>
</tr>
<tr>
<td>8</td>
<td>84</td>
<td>7</td>
<td>92%</td>
<td>1260</td>
<td>49</td>
<td>96%</td>
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<td>9</td>
<td>88</td>
<td>10</td>
<td>89%</td>
<td>616</td>
<td>55</td>
<td>91%</td>
</tr>
<tr>
<td>10</td>
<td>80</td>
<td>12</td>
<td>85%</td>
<td>560</td>
<td>27</td>
<td>95%</td>
</tr>
<tr>
<td>Average</td>
<td>89</td>
<td>13</td>
<td>86%</td>
<td>740</td>
<td>51</td>
<td>93%</td>
</tr>
</tbody>
</table>

The reduction in the number of variables for mine flow and inventory also affects
the number of discrete variables because mine flow and mode flow are linked by the
linking equations. In fact, the number of discrete variables (which is the number of
variables for mode flow) created by Model B always equals the number of mine flow
variables. Thus, by only including reasonable mine flow variables, SUMIT considers
only logical mode flow variables. Decreasing the number of mode flow variables reduces
the number of discrete variables, which increases the speed of execution for SUMIT.
Table 4.  MINE INVENTORY REDUCTION

<table>
<thead>
<tr>
<th>Model Number</th>
<th>Power Set N x N-M</th>
<th>Number of MNUs</th>
<th>Percent Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>47</td>
<td>57%</td>
</tr>
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IV. CONCLUSIONS

This thesis developed a single-commodity mine transshipment model, called SUMIT, for the rapid deployment of mines in a region of the world over a time window of two to three days. It considered two possible versions of SUMIT for computer implementation into GAMS. For ten problems tested, Model A and Model B arrived at the same optimal solution, but differed in relative efficiencies. For the size of the problems tested, Model B appeared to be faster and is, therefore, the model of choice for the implementation of SUMIT into GAMS.

Transshipment models involving integer programs tend to be difficult to solve, even for small problems. When limited to a region of the world spanning a few days, the mine transshipment problem is small enough to attempt integer programming. This thesis explored a viable approach for solving small mine transshipment problems using integer programs by replacing the binary mode networks, which was done in Model A, with sets of incompatible arc groups for every mode, which was done in Model B. The results of the ten test problems in this thesis demonstrate the superiority of using incompatible arc groups in the size and speed of the model for problem comparable to the test problems.

In addition to providing optimal or nearly optimal schedules for mine transshipment, SUMIT can be used as one of many tools for making decisions concerning mine warfare. SUMIT can give insight to logistical planners at CMWC in deciding the quantity and location of pre-staged mines, the quantity of mines that should be fully assembled at MOMAGs, and the MOMAGs that should receive resources for dual assembly lines to decrease the amount of time needed to deploy sets of mine fields for the most likely scenarios. In preparation for mine deployment, SUMIT can be run for different combinations of nodes and modes to indicate where modes should be based to decrease deployment time. Finally, SUMIT can be used to demonstrate the advantages of building new mine laying platforms, such as the HVMLs, by not only illustrating the time that could be saved in mine deployment for short-term regional scenarios, but also indicating which non-mine warfare platforms would be allowed to perform other critical missions.

Several enhancements could be made to SUMIT to improve its user friendliness and expand the size of the problems that it can manage:
• Develop a “front end” program in another language to read user input from a data file and printout the GAMS program for execution. The front end program would read data from GOPAS and allow users to use SUMIT without directly interfacing with GAMS.

• Develop a program in another language to read the GAMS output file and construct schedules that are more readable.

• Allow a mode group from a given node to be split between destinations when appropriate.

• Determine whether or not allowing more than two levels of mine readiness would be practical. This could be accomplished by splitting the MOMAG nodes so that each level of readiness for disassembled mines is represented by a MOMAG node and by putting one arc between MOMAG nodes for each level of readiness for disassembled mines and the associated supply node for assembled mines.

• Include constraints that take into account the limits placed on mine loading and unloading capacities at non-MOMAG supply sites and the restrictions on the number of aircraft allowed at transshipment sites at one time. Currently, this data is not available in GOPAS.

Finally, the problem discussed in this thesis could be expanded for follow-on research in two ways:

• Develop a model that encompasses world-wide scenarios of longer durations. This problem could be approached by adapting the integer rounding technique developed by Puntenney to SUMIT [Ref. 6].

• Develop a multi-commodity transshipment model by applying the work of Collier, Lally, and Puntenney [Refs. 4, 5, 6].
APPENDIX A. SAMPLE PROGRAM, FIRST RUN

$TITLE  *** Scheduler for Mine Transshipment (SUMIT) ***
*-----DOCUMENTATION------------------------------------------------------------------------
* Software to Optimize Schedules for Mine Transshipment
* Developed for Commander, Mine Warfare Command
* Written by Tammy L. Glaser, LT, USN
* Complete documentation contained in thesis
* 06 September 1991
*-----GAMS AND DOLLARS CONTROL OPTIONS------------------------------------------------------
* Do not change the following options.
$OFFUPPER OFFSYMREF OFFSYMLIST
OPTIONS SOLPRINT = OFF, LIMROW = 0, LIMCOL = 0;
* Increase the following options only when recommended in the
* solution report of the output.
OPTIONS RESLIM = 10000, ITERLIM = 10000000, WORK = 10000;
* Set OPTCR as recommended on page 72 of the thesis documentation.
OPTIONS OPTCR = 0.00001;
*-----DEFINITIONS AND DATA---------------------------------------------------------------------
* See Chapter 2 of thesis documentation to explain data in detail.
SCALARS
* The following scalars must be set to values greater than 0.
  TEND  end time of problem (h) /48/
  TPER  length of time period (h) /6/
  RELT  zero tolerance for comparisons of real data /0.001/
* The following scalars are optional (enter 0 for default value).
  MNLA  minimum distance for air modes over land routes (nm) /50/
  MXSS  maximum distance for transfers between supply nodes (nm) /100/
  DISC  1 to run entire problem and 0 to split into subproblems /0/;
SETS
* The first two sets must have the element 0 in them. Note that
* N must contain all elements in CN(N), found in the optional section.
* M must contain all elements in DM(M) (and TM(M), BM(M), CM(M), and
* LM(M), which are found in the optional section). See page 7 of
* thesis documentation to explain the MOMAG-supply node split.
  N  nodes  /0, M8GQ, S8GQ, M9RP, S9RP, M10JA, S10JA, M11SC, S11SC, ANDERSNAFB, NASADAK, MCASIWKNI, NASJAX, CVNA, CVNB, SSN688A, SSN688B, MINEFIELD1, MINEFIELD2, MINEFIELD3, MINEFIELD4/
  M  modes  /0, TRUCK, C130, C141, P3, B52, TACAIRD, SSN688DM, DBRM/
  DM(M) delivery modes /P3, B52, TACAIRD, SSN688DM/
* P must start with 0 and end with the value of the number of time
* periods plus one. G and I must start with 1. G must end with
* the number of time periods, while I must end with the square of
* the number of nodes (nonzero elements in N).
  P  time periods /0 * 9/
  G  group numbers /1 * 8/
  I  iteration numbers /1 * 400/
* The following sets are optional. Enter 0 between slashes for sets
* not needed for the problem. If CN(N) is empty (i.e., initialized to

41
nodes are assumed to be land nodes. If there are no transshipment
modes or disassembled mines, enter 0 between slashes for TM(M) or
BM(M). All modes are assumed to be air modes if CM(M) and LM(M)
are initialized to 0.

\begin{itemize}
\item CN(N) sea nodes /CVNA, CVNB, SSN688A, SSN688B/
\item TM(M) transportation modes /TRUCK, C130, C141/
\item BM(M) dummy build rate modes /DBRM/
\item CM(M) dummy sea modes /SSN688DM/
\item LM(M) land modes /TRUCK/
\end{itemize}

* Do not change the following evolution set.
E mode evolution /TLD, TUL, TGD/;

* Do not change the following set aliases:
ALIAS (N,N1,N2,N3,N4);
ALIAS (M,M1,M2,M3);
ALIAS (P,P1);
ALIAS (I,I1);

PARAMETERS
* The following parameter is required for node N:
\begin{itemize}
\item AMT(N) supply (+) or demand (-) at node N (mines)
\item /M8GQ 200
\item S8GQ 25
\item M9RP 159
\item M10JA 270
\item S10JA 95
\item M11SC 100
\item S11SC 40
\item ANDERSNAFB 89
\item CVNA 161
\item CVNB 100
\item SSN688A 50
\item SSN688B 40
\item MINEFIELD1 -195
\item MINEFIELD2 -249
\item MINEFIELD3 -199
\item MINEFIELD4 -201/
\end{itemize}

* The following parameters are optional (leave blank line with no
slashes for default values):
\begin{itemize}
\item PRN(N) priority of node N
\item /MINEFIELD1 2
\item MINEFIELD2 1
\item MINEFIELD3 3
\item MINEFIELD4 1/
\item MXD(N) maximum distance between mine fields at node N (nm)
\item /MINEFIELD2 50/
\item BGD(N) transit distance for sea node N at beginning (nm)
\item /CVNA 33
\item CVNB 75/
\item EDD(N) transit distance for sea node N at end (nm)
\item /CVNB 75/
\end{itemize}
SPX(N) transit speed for sea node N (nm per h)
/CVNA 25
CVNB 25/

* The following parameters are required for mode M:
CAP(M) capacity of mode unit in mode group M (mines per mode)
/TRUCK 20
C130 20
C141 20
P3 8
B52 22
TACAIR 51
SSN688DM 50/

MXL(M) maximum distance that mode M may travel (nm)
/TRUCK 150
C130 3000
C141 3000
P3 1500
B52 3000
TACAIR 500
SSN688DM 200/

SPD(M) transit speed of mode M (nm per h)
/TRUCK 30
C130 380
C141 410
P3 350
B52 480
TACAIR 450
SSN688DM 15

* The following parameter is optional (leave blank line for default): PRM(M) priority of mode M
/TRUCK 1
C130 3
C141 3
P3 1
B52 1
TACAIR 1
SSN688DM 3

* The following parameter is required for mode M at node N: NUM(N,M) number of mode M at node N (modes)
/M8GQ. DBRM 1
S8GQ. TRUCK 1
M9RP. DBRM 2
S9RP. C130 4
S9RP. P3 6
M10JA. DBRM 1
S10JA. P3 6
M11SC. DBRM 1
S11SC. TRUCK 3
S11SC. C130 4
The following parameters are optional (leave blank line for default):

- **BGT(N,M)**: earliest beginning time for mode M at node N (h)
  
- **EDT(N,M)**: latest end time for mode M at node N (h)
  
- **MXT(N,M)**: maximum total time mode M can be absent from node N (h)
  
- **MXR(N,M)**: maximum number of runs for mode M from node N
  
- **XMP(N,N1,M)**: 1 means eliminate runs from nodes N to N1 via mode M

The following two parameters are required.

* READ PAGE 22 OF THESIS DOCUMENTATION, WHICH explains WHEN NEGATIVE DISTANCES MUST BE USED.

**TABLE DIS(N,N1)**: distance between node N and node N1 (nm)

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<th>S8GQ</th>
<th>M9RP</th>
<th>S9RP</th>
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<th>S10JA</th>
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* M11SC S11SC ANDERSNAFB NASADAK MCASIWKNI

| M8GQ | 7501 | 7501 | -30  | 2939  | 1631  |

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+         NASJAX  CVNA  CVNB  SSN688A  SSN688B
M8GQ      7485     1566  7789  2405     7812    |
S8GQ      7485     1566  7789  2405     7812    |
M9RP      7768     1580  8020  2881     8152    |
S9RP      7768     1580  8020  2881     8152    |
M10JA     7032     849   7293  2144     7414    |
S10JA     7032     849   7293  2144     7414    |
M11SC     -124     6175  294   4899     420     |
S11SC     -124     6175  294   4899     420     |
ANDERSNAFB 7500     1583  7801  2421     7831    |
NASADAK   4209     2088  7801  777      4546    |
MCASIWKNI 5950     267   6216 1067     6305    |
NASJAX    0       6188  351   4900     366     |
CVNA      6188     0     6445 1324     6546    |
CVNB      366      6445 0      5164     327     |
SSN688A   4900     1324  5164  0       5249    |
SSN688B   366      6546 327    5249     0       |
MINEFIELD1| 4869     1344  5146 110      5220    |
MINEFIELD2| 6051     234   6294 1244     6413    |
MINEFIELD3| 7660     1786  7817 3073     8023    |
MINEFIELD4| 463      6578 500    5270     174     |

+         MINEFIELD1  MINEFIELD2  MINEFIELD3  MINEFIELD4
M8GQ      2459     1819   2207  7797     |
S8GQ      2459     1819   2207  7797     |
M9RP      2909     1725   897   8152     |
S9RP      2909     1725   897   8152     |
M10JA     2170     1015  1203  7414     |
S10JA     217C     1015  1203  7414     |
M11SC     4866     6032  7607  548      |
S11SC     4866     6032  7607  548      |
ANDERSNAFB 2455     1800  2188  7812    |
NASADAK   744      2032  3850 4551     |
MCASIWKNI 1090     352   2053 6332     |
NASJAX    4869     6051  7660 463      |
CVNA      1344     234   1786 6578     |

45
TABLE TEV(M,E) time needed to complete evolution E for mode M (h)

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<th>TUL</th>
<th>TGD</th>
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</table>

* Input is complete. Do not change any statements after this line *

**---PRE-MODEL MANIPULATIONS---**

SCALAR

- NRLT: negative zero tolerance for comparisons of real data
- STOP: stopping criteria for loop that finds connected component
- SUMA: number of arcs in the mine network
- IBEG: counter that is incremented in incompatible arc group loop;

SETS

- NO(N): dummy node /0/
- MN(N): momag nodes
- SN(N): supply nodes
- TN(N): transshipment nodes
- DN(N): demand nodes
- LN(N): land nodes
- M0(M): dummy mode /0/
- AM(M): air modes
- ARC(N,N1,M): arcs in network from node N to node N1 via mode M
- ART(N,N1,M): temporary version of ARC
- DNO(N): arcs in first connected component found
- NFC(N): nodes that do not form a complete component
- XTE(N,N1,P,M): arcs in time-expanded network for period P
- MNU(N,P): mine flow balance equation used for N at period P;

* Verify correctness of the inputted data.

ABORT$(RELT LE 0) "ERROR: RELT must be positive.", RELT;
ABORT$(RELT GE 0.00100001) "ERROR: RELT must be less than 0.001.", RELT;
ABORT$(TEND LE RELT) "ERROR: TEND must be positive.", TEND;
ABORT$(TPER LE RELT) "ERROR: TPER must be positive.", TPER;
ABORT$(TPER GE (TEND + RELT)) "ERROR: TPER must be less than TEND.",

TPER, TEND;
ABORT$((MNLA + RELT) LE 0) "ERROR: MNLA must be nonnegative.", MNLA;
ABORT$((MXSS + RELT) LE 0) "ERROR: MXSS must be nonnegative.", MXSS;
ABORT$((ABS(DISC - 1) GE RELT) AND (ABS(DISC) GE RELT))
AND (ABS(DIS(N,N1)) GE RELT)) = YES;
* Eliminate all arcs as indicated by user.
ARC(N,N1,M)$(ABS(XMP(N,N1,M) - 1) LE RELT) = NO;
* Eliminate all arcs between supply nodes on land via land modes
* that have nonnegative distances or have distances greater than MXSS.
ARC(N,N1,M)$( (SN(N) * SN(N1) * LN(N) * LN(N1) * TM(M) * LM(M))
  $((DIS(N,N1) GE NRLT) OR ((MXSS - ABS(DIS(N,N1))) LE RELT))) = NO;
* Eliminate all arcs between supply nodes on land via air trans-
* portation when the arcs are over land and absolute distances are
* less than Mnla or their absolute distances are greater than MXSS.
ARC(N,N1,M)$( (SN(N) * SN(N1) * LN(N) * LN(N1) * TM(M) * AM(M))
  $(((MNLA - ABS(DIS(N,N1))) GE RELT) AND (DIS(N,N1) LE NRLT))
  OR ((MXSS - ABS(DIS(N,N1))) LE RELT))) = NO;
* Eliminate all arcs between two supply nodes in which at least
* one node is on land or that use nontransportation modes.
ARC(N,N1,M)$(SN(N) * SN(N1) * (NOT (LN(N) * LN(N1) * TM(M)))) = NO;
* Eliminate all arcs between nodes via land transportation modes
* for which the distance is nonnegative.
ARC(N,N1,M)$( (TM(M) * LM(M))$((DIS(N,N1) GE NRLT)) = NO;
* Eliminate all arcs between land nodes via air transportation
* modes when the arcs are over land and absolute distances are
* less than Mnla.
ARC(N,N1,M)$((LN(N) * LN(N1) * TM(M) * AM(M))
  $(((MNLA - ABS(DIS(N,N1))) GE RELT) AND (DIS(N,N1) LE NRLT))) = NO;
* Eliminate all arcs from sea nodes to nondemand nodes.
ARC(N,N1,M)$(CN(N) * (NOT DN(N1))) = NO;
* Eliminate all arcs to either supply or transshipment nodes
* via delivery modes.
ARC(N,N1,M)$((SN(N1) + TN(N1)) * DM(M)) = NO;
* Eliminate all arcs to demand nodes via nondelivery modes.
ARC(N,N1,M)$((DN(N1) * (NOT DM(M)) = NO;
* Eliminate all arcs leaving MOMAG nodes to nodes for which
* the distance is not equal to -0.5.
ARC(N,N1,M)$(MN(M)$((ABS(DIS(N,N1) + 0.5) GE RELT)) = NO;
* Eliminate all arcs arriving at MOMAG nodes.
ARC(N,N1,M)$MN(N1) = NO;
* Eliminate all arcs from transshipment nodes to supply nodes.
ARC(N,N1,M)$TN(N) * SN(N1)) = NO;
*
* Verify that only one arc leaves each MOMAG node.
ABORT$(S MAX(N$SN(N), SUM(N1,M)ARC(N,N1,M), 1)) GE 1.5)
"ERROR: At least one MOMAG node has more than one arc leaving it. 
"MOMAG nodes and the current network is listed as follows:", MN, ARC;

PARAMETERS
PAR(N1,M) time periods to travel from N to N1 via M one-way
PRT(N1,M) time periods to travel from N to N1 via M round trip
IND(N) in degree of (number of arcs entering) node N
OTD(N) out degree of (number of arcs leaving) node N
BGP(N,N1,M) earliest time to include arc from N to N1 via M
EDP(N,N1,M) latest time to include arc from N to N1 via M
MXM(N) maximum number of mines through N to fill demands
MXS(N) maximum number of mines through N from supply nodes
ZUP(N) upper bound on mine inventory at N
XUP(N,N1,M) upper bound on mine flow from N to N1 via M
RHX(N,P) fixed value on mine inventory at N at period P
MIN(N,M) minimum return period PRT from N via M
HMRX(N,M) maximum return period PRT from N via M
MINA(N,M) minimum arrival period PAR from N via M if PRT=MRN
MNBN(M,N) minimum begin period BGP from N via M
MNXE(N,M) maximum end period EDP from N via M
LSGN(M,N) last group number needed from N via M
GBGN(N1,P,M) first group number from N to N1 via M at P
GEDN(N1,P,M) last group number from N to N1 via M at P;

* Initialize or set default values for certain parameters.
* Convert times to time periods.

\[ \text{CAP}(M) = \text{FLOOR}(\text{CAP}(H)); \]
\[ \text{PRN}(N) = 0; \]
\[ \text{PRM}(M) = 0; \]
\[ \text{PRN}(N) = \text{MAX}(1, \text{SMAX}(1, \text{PRN}(N1))); \]
\[ \text{PRM}(M) = \text{MAX}(1, \text{SMAX}(1, \text{PRM}(M))); \]
\[ \text{MRX}(N,M) = 0; \]
\[ \text{BGT}(N,M) = 0; \]
\[ \text{SPX}(N) = 0; \]
\[ \text{BGT}(N,M) = 0; \]
\[ \text{EDD}(N) = 0; \]
\[ \text{EDT}(N) = 0; \]
\[ \text{SUM}(N) = 0; \]
\[ \text{EDT}(N,M) = 0; \]
\[ \text{HXT}(N,M) = 0; \]
\[ \text{WXRC}(N) = 0; \]
\[ \text{GBG}(N,N1,P,M) = 0; \]
\[ \text{GED}(N,N1,P,M) = 0; \]
\[ \text{HXR}(N,H) = \text{FLOOR}(\text{MRX}(N,M)); \]
\[ \text{MXT}(N,H) = \text{FLOOR}(\text{HXT}(N,M) / \text{TPER}); \]
\[ \text{HXT}(N,M) = 0; \]
\[ \text{MXT}(N,M) = 0; \]
\[ \text{MXR}(N,M) = 0; \]
\[ \text{MXD}(N,M) = 0; \]
\[ \text{GBG}(N,N1,P,M) = 0; \]
\[ \text{GED}(N,N1,P,M) = 0; \]
\[ \text{MXR}(N,M) = 0; \]
\[ \text{MXD}(N,M) = 0; \]
\[ \text{GBG}(N,N1,P,M) = 0; \]
\[ \text{GED}(N,N1,P,M) = 0; \]

* Calculate capacity for dummy build rate modes.

\[ \text{CAP}(M) = \text{FLOOR}(\text{SPD}(M) \times \text{TPER}); \]

* Verify the correctness of the inputted data.

\[ \text{STOP} = \text{TEND} + 1; \]
\[ \text{ABORT}((\text{ABS}(\text{CARD}(P) - 2 - \text{TEND}) \geq \text{RELT}) \text{ GE RELT}) \]
  "ERROR: The set P must contain integers from 0 to 'STOP'",
  "(see the next line for the value of 'STOP').", STOP, P;
\[ \text{ABORT}((\text{ABS}(\text{CARD}(G) - \text{TEND}) \geq \text{RELT}) \text{ GE RELT}) \]
  "ERROR: The set G must contain integers from 1 to 'TEND'.", TEND, G;
\[ \text{STOP} = (\text{CARD}(N) - 1) \times 2; \]
\[ \text{ABORT}((\text{ABS}(\text{CARD}(I) - \text{TEND}) \geq \text{RELT}) \text{ GE RELT}) \]
  "ERROR: The set I must contain integers from 1 to 'STOP'",
  "(see the next line for the value of 'STOP').", STOP, I;
\[ \text{ABORT}((\text{SHIN}(N, \text{PRN}(N)) \geq \text{RELT}) \text{ LE 0}) \]
  "ERROR: PRN(N) must all be nonnegative.", PRN;
\[ \text{ABORT}((\text{SHIN}(N, \text{MXD}(N)) \geq \text{RELT}) \text{ LE 0}) \]
  "ERROR: MXD(N) must all be nonnegative.", MXD;
\[ \text{ABORT}((\text{SHIN}(N, \text{BGD}(N)) \geq \text{RELT}) \text{ LE 0}) \]
  "ERROR: BGD(N) must all be nonnegative.", BGD;
\[ \text{ABORT}((\text{SHIN}(N, \text{SPX}(N)) \geq \text{RELT}) \text{ LE 0}) \]
"ERROR: EDD(N) must all be nonnegative.", EDD;
ABORT$((SMIN(N, SPX(N)) + RELT) LE 0)
"ERROR: SPX(N) must all be nonnegative.", SPX;
ABORT$((SUM(N$(((BGD(N) GE RELT) OR (EDD(N) GE RELT))
AND (SPX(N) LE RELT)), 1) GE 0.5)
"ERROR: If BGD(N) or EDD(N) is inputted, SPX(N) must be positive.",
BGD, EDD, SPX;
ABORT$(SMIN(MS(NOT M0(M)), CAP(M)) LE 0.5)
"ERROR: CAP(M) must all be positive.", CAP, CM;
ABORT$((SMIN(M, PRM(M)) + RELT) LE 0)
"ERROR: PRM(M) must all be nonnegative.", PRM;
ABORT$((SMIN(M, PRM(M) + RELT)) LE 0)
"ERROR: PRM(M) must all be nonnegative.", PRM;
ABORT$((SUM(N$(((BGD(N) GE RELT) OR (EDD(N) GE RELT))
AND (SPX(N) LE RELT)), 1) GE 0.5)
"ERROR: BGD(N) or EDD(N) is inputted, SPX(N) must be positive.",'n
* Adjust BGT and EDT if nonzero BGD and EDD is inputted. Convert
* BGT and EDT to time periods.
BGT(N,M)$(NOT (NO(N) + M0(M)))
= CEIL(((BGD(N) / SPX(N))$((SPX(N) GE RELT) AND (NUM(N,M) GE 0.5))
+ BGT(N,M)) / TPER);
EDT(N,M)$(NOT (NO(N) + M0(M)))
= FLOOR(((EDD(N) / SPX(N))$((SPX(N) GE RELT) AND (NUM(N,M) GE 0.5))
+ EDT(N,M)) / TPER);

* Calculate PAR taking into account load, travel, and unload/
* delivery times. Calculate PRT taking into account load,
* travel, unload/delivery, return travel, and ground times.
PAR(N,Nl,M)$(ARC(N,N1,M)$((SPD(M) GE RELT))
= CEIL(((TEV(M, 'TLD') + TEV(M, 'TUL')) / TPER) + 1$BM(M)
+ ((ABS(DIS(N,N1)) + MXD(N1)) / (TPER * SPD(M)))$((NOT BM(M)))
+ (TPR(M) / TPER) + (TPR(M) / TPER);

PRT(N,N1,M)$(ARC(N,N1,M)$((SPD(M) GE RELT))
= CEIL(((TEV(M, 'TLD') + TEV(M, 'TUL') + TEV(M, 'TGD')) / TPER)
+ (((2 * ABS(DIS(N,N1)) + MXD(N1)) / (TPER * SPD(M)))$((NOT BM(M))
+ 1$BM(M));

* Eliminate all arcs from N to N1 via M if PRT is greater than
* MXT or (EDT - BGT). If node N has no supply and no incoming arcs,
* eliminate all arcs directed from node N.
ARC(N,N1,M)$((MXT(N,M) - PRT(N,N1,M)) LE NRLT)
OR (((PRT(N1,N,M) - (EDT(N, M) - BGT(N,M)) GE RELT))) = NO;
ARC(N,N1,M)$((SUM(N$(((BGD(N) GE RELT) OR (EDD(N) GE RELT))
AND (SPX(N) LE RELT)), 1) LE 0.5)
AND (ABS(AMT(N)) LE 0.5)) = NO;
ART(N,N1,M) = ARC(N,N1,M);

* In a loop, take the following steps until a loop in which
* no arcs are deleted from the mine network is made:
Calculate SUMA, IND, and OTD. Initialize DNO and MXM.

Calculate EDP and MXM by checking all possible paths in the network. Reinitialize DNO and MXS. Calculate BGP and MXS by checking all possible paths in the network. (In these loops, DNO indicates whether or not MXM or MXS has already been updated for origin node of the arc being checked.) Calculate EDP for arcs leaving MOMAG nodes.

Eliminate arcs from N to N1 via M for which PRT is greater than (EDP - BGP).

If node N has no supply and no incoming arcs from other nodes, eliminate all arcs directed from node N.

If DISC equals 0, then set DNO to 'yes' for all nodes in the first connected component found. If DISC equals 1, then set DNO to 'yes' for all nodes (except dummy node 0). Eliminate arcs whose nodes are not contained in the first component found.

```
LOOP(T1$(ABS(SUMA - SUM((N,N1,M)$ART(N,N1,M), 1)) GE RELT),
SUMA = SUM((N,N1,M)$ART(N,N1,M), 1);
IND(N) = SUM((N1,M)$ART(N1,N,M), 1);
OTD(N) = SUM((N1,M)$ART(N,N1,M), 1);
DNO(N) = YES;
MXM(N) = 0;
LOOP(N$(DNO(N)$((IND(N) + OTD(N)) GE 0.5)),
DNO(N) = NO;
LOOP((N1,M)$ART(N1,N,M),
EDP(N1,N,M) = MIN(EDT(N1,M), TEND) - PRT(N1,N,M) + PAR(N1,N,M);
MXM(N1)($(DNO(N1) + DNO(N)) = ABS(AMT(N)) + MXM(N1);
DNO(N1) = NO;
LOOP((N2,M1)$ART(N2,N1,M1),
EDP(N2,N1,M1) = MAX(EDP(N2,N1,M1),
   MIN(EDP(N1,N,M) - PAR(N1,N,M), MIN(EDT(N2,M1),TEND)
   - PRT(N2,N1,M1) + PAR(N2,N1,M1)));
MXM(N2)$(DNO(N2) + DNO(N)) = ABS(AMT(N)) + MXM(N2);
DNO(N2) = NO;
LOOP((N3,M2)$ART(N3,N2,M2),
EDP(N3,N2,M2) = MAX(EDP(N3,N2,M2),
   MIN(EDP(N2,M1) - PAR(N2,N1,M1),
   MIN(EDT(N3,M2), TEND)
   - PRT(N3,N2,M2) + PAR(N3,N2,M2)));
MXM(N3)$(DNO(N3) + DNO(N)) = ABS(AMT(N)) + MXM(N3);
DNO(N3) = NO;
LOOP((N4,M3)$ART(N4,N3,M3),
EDP(N4,N3,M3) = MAX(EDP(N4,N3,M3),
   MIN(EDP(N3,N2,M2) - PAR(N3,N2,M2),
   MIN(EDT(N4,M3), TEND)
   - PRT(N4,N3,M3) + PAR(N4,N3,M3)));
MXM(N4)$(DNO(N4) + DNO(N)) = ABS(AMT(N)) + MXM(N4);
DNO(N4) = NO)));
DNO(N1)$($(NOT DN(N1)) = YES);
BGP(N,N1,M)$ART(N,N1,M)
   = PAR(N,N1,M)$((IND(N) LE 0.5) + TEND$(IND(N) GE 0.5)
   + BGT(N,M);
DNO(N) = YES;
MXS(N) = 0;
LOOP(N$(((IND(N) LE 0.5) AND (OTD(N) GE 0.5)),
   MXS(N)$DNO(N) = AMT(N);
```
DNO(N) = NO;
LOOP((N1,M)$ART(N1,N1,M),
    MXS(N1)$DNO(N1) = MXS(N1) + AMT(N1)
+ AMT(N1)$NOT DN(N1));
DNO(N1) = NO;
LOOP((N2,M1)$ART(N2,N2,M1),
    BGP(N2,N2,M1) = MIN(BGP(N2,N2,M1), PAR(N2,N2,M1)
+ BGP(N2,N2,M1)$AMT(N2) LE 0.5) + BGT(N2,N2,M1));
    MXS(N2)$DNO(N2) = MXS(N2) + AMT(N2)$NOT DN(N2));
DNO(N2) = NO;
LOOP((N3,M2)$ART(N3,N3,M2),
    BGP(N3,N3,M2) = MIN(BGP(N3,N3,M2), PAR(N3,N3,M2)
+ BGP(N3,N3,M2)$AMT(N3) LE 0.5) + BGT(N3,N3,M2));
    MXS(N3)$DNO(N3) = MXS(N3) + AMT(N3)$NOT DN(N3));
DNO(N3) = NO;
LOOP((N4,M3)$ART(N4,N4,M3),
    BGP(N4,N4,M3) = MIN(BGP(N4,N4,M3), PAR(N4,N4,M3)
+ BGP(N4,N4,M3)$AMT(N4) LE 0.5) + BGT(N4,N4,M3));
    MXS(N4)$DNO(N4) = MXS(N4) + AMT(N4)$NOT DN(N4));
DNO(N4) = NO));
DNO(N)$IND(N) GE 0.5) = YES;
EDP(N1,M)$DISC LE 0.5) = MIN(EDP(N1,M),
CEIL(MIN(MXS(N),MNX(N))) / (CAP(M) * NUM(N,M)));
ART(N1,M)$DISC LE 0.5) = MIN(ART(N1,M),
((SUM(N1,M)$DISC LE 0.5) OR (DISC LE 0.5)) = NO;
* Initialize DIS, DNO and STOP if DISC equals 0.
DIS(N1)$DISC LE 0.5) = 0;
LOOP(N$DISC LE 0.5),
    DIS(N1)$ART(N1,N1,M) = 1);
DNO(N)$DISC LE 0.5) = YES$(ABS(SMIN((N1,N2,fl)$NOT DN(N1)) * ART(N1,N2,M), ORD(N1))
- ORD(N))$LE RELT);
STOP$DISC LE 0.5) = SUM((N1,N1)$DNO(N), DIS(N1,N1) + DIS(N1,N));
* Starting with the first nondummy node in network found, set
* DNO to 'yes' for any node which (1) is on a path with the first
* node found or (2) is on a backtrack path from other nodes on a
* path with the first node found. Stop when all arcs in the network
* have been checked.
LOOP(I$STOP GE 0.5) AND (DISC LE 0.5)),
    LOOP(N$DNO(N),
    LOOP(N$DISC LE 0.5) OR (DISC LE 0.5),
        DNO(N1) = YES;
    DIS(N1)$DISC LE 0.5) = 0;
    DNO(N1)$DISC LE 0.5) = 0);
    STOP = SUM((N1,N1)$DNO(N), DIS(N1,N1) + DIS(N1,N));
    DNO(N)$NOT DN(N) = YES;
    ART(N1,N1)$NOT (DNO(N) = DNO(N1))) = NO;
* Ensure that the first component found contains both supply and
* demand. If not, repeat the process for the next component.
OR (SUM(N$(DNO(N) * SN(N)), AMT(N)) LE 0.5)$DISC 0.5) = 0;
NFC(N)$DNO(N)) SUM(N1$(DNO(N1) * SN(N1)), AMT(N1)) LE 0.5)
OR (SUM(N2$(DNO(N2) * DNO(N2)), ABS(AMT(N2)) LE 0.5)) = YES;
ART(N,N1,M)($(NOT (NFC(N) * NFC(N1))) * ARC(N,N1,M) = YES);

* Ensure that total supply is greater than total demand
* in generated network.
ABORT$((SUM(N$(DNO(N) * DN(N)), ABS(AMT(N))
- SUM(N$(DNO(N) * (MN(N) + SN(N))), AMT(N)) GE 0.5)
"ERROR: Total demand (-) exceeds total supply (+)",
"in generated network.", DNO, AMT;

* Reset the following parameters if N is not in the first connected
* component found.
ART(N,N1,M) = ARC(N,N1,M);
ARC(N,N1,M)$NOT (DNO(N) * DNO(N1)) = NO;
MXT(N,M)$NOT DNO(N)) = 0;
MNR(N,M)$NOT DNO(N)) = 0;
BGT(N,M)$NOT DNO(N)) = 0;
EDT(N,M)$NOT DNO(N)) = 0;
PRT(N,N1,M)$NOT ARC(N,N1,M)) = 0;
BGP(N,N1,M)$NOT ARC(N,N1,M)) = 0;
EDP(N,N1,M)$NOT ARC(N,N1,M)) = 0;
NUM(N,M)$SUM(N1$PRT(N,N1,M), 1) LE 0.5) = -1 * NUM(N,M);

* Set XTE equal to 'yes' for all arcs from N to N1 via M that
* are part of the mine network if P is less than or equal to EDP
* and greater than or equal to BGP. Set MNU to 'yes' if at least
* one arc exists from N to N1 (or from N1 to N) via mode M at time
* period P.
XTE(N,N1,P,M) = YES$(ARC(N,N1,M)$((ORD(P) - (EDP(N,N1,M) + 1)) LE RELT)
AND ((ORD(P) - BGP(N,N1,M)) GE RELT)));
MNU(N,P)$NOT NO(N)) = YES
\$(SUM((N1,M), 1$(XTE(N1,N,P,M) + XTE(N1,P + PAR(N1,N1,M),M)) GE 0.5);

* Calculate ZUP, XUP, and RHX using AMT, MXS and MXM.
ZUP(N)$NOT NO(N)) = ABS(AMT(N))$DN(N) + MIN(MXS(N), MXM(N)$NOT DN(N));
XUP(N,N1,M)$ARC(N,N1,M) = MIN(ZUP(N), ZUP(N1), CAP(M) * NUM(N,M));
RHX(N,P)$DNO(N)$ABS(ORD(P) - SHIN(P1$MNU(N1,P1), ORD(P1))) LE RELT))
= ABS(AMT(N)) - 2$(ABS(AMT(N)) LE RELT);
RHX(N,P)$DNO(N)
\$(ABS(ORD(P) - 1 - SMAX(P1$MNU(N1,P1), ORD(P1))) LE RELT)) = -1;
RHX(N,P)$DNO(N)$RHX(N,P) GE RELT)) = MIN(RHX(N,P), ZUP(N));

* Calculate the following parameters in preparation for forming
* incompatible arc groups.
MRN(N,M)$DNO(N)$NUM(N,M) GE 0.5) = SHIN(N1$ARC(N1,N1,M), PRT(N1,N1,M));
MNR(N,M)$DNO(N)$NUM(N,M) GE 0.5) = SMAX(N1$ARC(N1,N1,M), PRT(N1,N1,M));
MNR(N,M)$DNO(N)$NUM(N,M) GE 0.5) = SHIN(N1$ARC(N1,N1,M)
\$(ABS(PRT(N1,N1,M) - MRN(N,M)) LE RELT)), PAR(N,N1,M));
MN1(N,M)$DNO(N)$NUM(N,M) GE 0.5) = SMAX(N1$ARC(N1,N1,M), BGP(N1,N1,M));
MNR(N,M)$DNO(N)$NUM(N,M) GE 0.5) = SMAX(N1$ARC(N1,N1,M), EDP(N1,N1,M));
MXE(N,M)$DNO(N)$NUM(N,M) GE 0.5) = SMAX(N1$ARC(N1,N1,M)
\$(ABS(PRT(N1,N1,M) - MRN(N,M)) LE RELT))

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AND (ABS(PAR(N,N1,M) - MNA(N,M)) LE RELT)), EDP(N,N1,M));
LSG(N,M)$(DNO(N)$(LSG(N,M) GE RELT)) = 2 + LSG(N,M) - MNB(N,M) - MRN(N,M);
LSG(N,M)$(DNO(N)$(MRR(N,M) - (MXE(N,M) - MNB(N,M))) GE RELT)
AND (SUM(N1$ARC(N,N1,M), 1) GE 1.5)) = 1;

* Calculate GBG and GED. This loop is explained in depth on page 27 of the thesis documentation.

LOOP((N,N1,M)$((DNO(N) NOT HN(N))$((NUI(N,M) GE 0.5)
AND ((MRX(N,M) - 1 GE RELT) OR ((ABS(MRX(N,M) - 1) LE RELT)
AND (SUIICN$ARC(N,N2,M), 1) GE 1.5))))),
LOOP(P$XT(N,N1,P,M),
IBEG = MNB(N,M);
LOOP((G$((LSG(N,M) - ORD(G)) GE NRLT),
GBG(N,N1,P,M)$(GBG(N,N1,P,M) LE RELT)
AND (ABS(PRT(N,N1,M) - MRN(N,M)) LE RELT)
AND (ABS(PAR(N,N1,M) - MNA(N,M)) LE RELT))
= ORD(G)$(((ORD(P) - 1 - IBEG) GE NRLT)
AND ((IBEG + MRN(N,M) - ORD(P) + 1) GE RELT));
GED(N,N1,P,M)$(ABS(PRT(N,N1,M) - MRN(N,M)) LE RELT)
AND (ABS(PAR(N,N1,M) - MNA(N,M)) LE RELT))
= MAX(ORD(G)$((ABS(IBEG + MRN(N,M) - ORD(P) + 1) GE RELT)
AND ((ORD(P) - 1 - IBEG) GE NRLT), GED(N,N1,P,M));
GBG(N,N1,P,M)$(GBG(N,N1,P,M) LE RELT)
AND (ABS(PRT(N,N1,M) - MRN(N,M)) GE RELT)
OR (ABS(PAR(N,N1,M) - MNA(N,M)) GE RELT))
= ORD(G)$(((IBEG + 2 * MRN(N,M) - MNA(N,M)
- ORD(P) + PRT(N,N1,M) - PAR(N,N1,M)) GE NRLT)
AND ((ORD(P) - 1 + PAR(N,N1,M)
- IBEG + MNA(N,M)) GE NRLT));
GED(N,N1,P,M)$(ABS(PRT(N,N1,M) - MRN(N,M)) GE RELT)
OR (ABS(PAR(N,N1,M) - MNA(N,M)) GE RELT))
= MAX(ORD(G)$(((IBEG + 2 * MRN(N,M) - MNA(N,M)
- ORD(P) + PRT(N,N1,M) - PAR(N,N1,M)) GE NRLT)
AND ((ORD(P) - 1 + PAR(N,N1,M)
- IBEG + MNA(N,M)) GE NRLT)), GED(N,N1,P,M));
IBEG = IBEG + 1));

LOOP((G,N,M)$((LSG(N,M) - ORD(G)) GE NRLT),
LOOP((N1,P)$((GBG(N,N1,P,M) GE RELT) AND ((ABS(PRT(N,N1,M)
- MRN(N,M)) GE RELT) OR (ABS(PAR(N,N1,M) - MNA(N,M)) GE RELT))
AND ((ORD(P) - PAR(N,N1,M) - MNB(N,M) - ORD(G) - MRN(N,M)
+ MNA(N,M)) GE NRLT)),
GBG(N,N1,P,M) = MAX(GBG(N,N1,P,M), ORD(G) + 1)));

LOOP((G,N,M)$((LSG(N,M) - ORD(G)) GE NRLT),
LOOP((N1,P)$((GED(N,N1,P,M) GE RELT) AND ((ABS(PRT(N,N1,M)
- MRN(N,M)) GE RELT) OR (ABS(PAR(N,N1,M) - MNA(N,M)) GE RELT))
AND ((MNB(N,M) + ORD(G) + MRN(N,M) - MNA(N,M)
- ORD(P) - PRT(N,N1,M) + PAR(N,N1,M)) GE RELT)),
GED(N,N1,P,M) = MIN(GED(N,N1,P,M), ORD(G) - 1)));

*--- MODEL FOR SUMIT -----------------------------

POSITIVE VARIABLES
X(N,N1,P,M) flow of mines from N to N1 via mode M at time period P
Z(N,P) mines inventory at N at time period P;
* fix upper bounds on positive variables; set some positive inventory variables to be constant
X.UP(N,N1,P,M)$XTE(N,N1,P,M) = XUP(N,N1,M);
Z.UP(N,P) = ZUP(N);

FREE VARIABLE
V objective value;

BINARY VARIABLE
Y(N,N1,P,M) 1 (0) if path (not) used from N to N1 via M at P;

EQUATIONS
OBJV objective value
MNFB(N,P) flow balance equations for mine network
LINK(N,N1,P,M) link equations between mine and mode networks
MDSK(N,M,G) mode schedule equations for incompatible arcs
MDPG(N,M) constraint to limit total time that M is from N
MDRG(N,M) constraint to limit total runs that M is from N;

OBJV..
V =G= SUM((N,N1,P,M)$XTE(N,N1,P,M), (PRN(N) * PRN(N1) * PRM(M) *
  (ORD(P) - 1 + PRT(N,N1,M))) ** 2 * Y(N,N1,P,M));

MNFB(N,P)$MN(N,P) * DNO(N))..
0 =G= (1$DN(N) - 1$(NOT DN(N)))
  * (Z(N,P)($((RHX(N,P) LE 0.5) AND (RHX(N,P) GE -1.5))
  - Z(N,P + 1)($RH(N,P) + 1) GE -0.5) + RHX(N,P)$RH(N,P) GE 0.5))
  - SUM((N1,P,M),$XTE(N1,N,P,M)
  - X(N,N1,P + PAR(N,N1,M),M)$XTE(N,N1,P + PAR(N,N1,M),M));

LINK(N,N1,P,M)$XTE(N,N1,P,M)..
0 =L= XUP(N,N1,M) * Y(N,N1,P,M) - X(N,N1,P,M);

MDSK(N,M,G)$DNO(N)$((LSG(N,M) - ORD(G)) GE NRLT))..
1 =G= SUM((N1,P,M)$((ORD(G) - GB(N,N1,P,M)) GE NRLT)
  AND ((GED(N,N1,P,M) - ORD(G)) GE NRLT)), Y(N,N1,P,M));

MDPG(N,M)$((NOT (NO(N) * MO(M))))$((TEND - MXT(N,M)) GE RELT))..
SUM((N1,P)$XTE(N1,N,P,M), PRT(N,N1,M) * Y(N,N1,P,M) =L= MXT(N,M);

MDRG(N,M)$((DNO(N) * (NOT (NO(N) * MO(M))))$((MXR(N,M) GE RELT))..
SUM((N1,P)$XTE(N1,N,P,M), Y(N,N1,P,M) =L= MXR(N,M);

MODEL SUMIT /OBJV,MNFB,MDSK,LINK,MDPG,MDRG/;

SOLVE SUMIT USING MIP MINIMIZING V;

*-----SOLUTION REPORT-------------------------------------------------------------------------
DISPLAY " The following arcs were considered by SUMIT.", ARC;

DISPLAY " *** All times in given in hours, incremented by TPER. ***";

PARAMETER
TIMECOMP(N,* ) Time in which laying of mine field N was completed;
TIMECOMP(N,'TIME_DONE')$DN(N) * DNO(N))
  = SMAX((N1,P,M)$Y.L(N1,N,P,M) GE RELT), TPER * (ORD(P) - 1));
OPTION TIMECOMP: 1;
DISPLAY TIMECOMP;

PARAMETER

MOMAGREP(N,*)  MOMAG Schedule Report  (Start Time is 0);
MOMAGREP(N,'START. TIME')$((MN(N)*DNO(N))
$(SMAX((N1,P,M)$XTE(N,N1,P,M), X.L(N1,N1,P,M)) GE RELT))
= TPER * SHIN((N1,P,M)$((X.L(N1,N1,P,M) GE RELT), ORD(P) - 2));
MOMAGREP(N,'STOP. TIME')$((MN(N)*DNO(N))
$(SMAX((N1,P,M)$XTE(N,N1,P,M), X.L(N1,N1,P,M)) GE RELT))
= TPER * SMAX((N1,P,M)$((X.L(N1,N1,P,M) GE RELT), ORD(P) - 1));
MOMAGREP(N,'NUM. MINES')$((MN(N)*DNO(N))
= SUM((N1,P,M)$((X.L(N1,N1,P,M) GE RELT), X.L(N1,N1,P,M));
OPTION MOMAGREP: 1:1:1;
DISPLAY MOMAGREP;

PARAMETER

SCHEDULE(N,M,G,N1,*) Schedule for Mine Transshipment by Origin Base;
LOOP(N$((DNOCN)*NOT MN(N))$(SUM((N1,P,M), X.L(N1,N1,P,M)) GE RELT)),
LOOP(M$(NUM(N,M) GE 0.5),
IBEG = 1;
LOOP((P,N1)$((X.L(N1,N1,P,M) GE RELT),
LOOP(G$(ABS(ORD(G) - IBEG) LE RELT),
SCHEDULE(N,M,G,N1,'NO. MODES')
= CEIL(X.L(N1,N1,P,M) / CAP(M));
SCHEDULE(N,M,G,N1,'NO. MINES') = X.L(N1,N1,P,M);
SCHEDULE(N,M,G,N1,'LOAD. TIME')
= TPER * (ORD(P) - 1 - PAR(N1,N1,M));
SCHEDULE(N,M,G,N1,'ETA. ORIG')
= (TPER * (ORD(P) - 1 - PAR(N1,N1,M))) + TEV(M,'TLD');
SCHEDULE(N,M,G,N1,'ETA. DEST')
= (TPER * (ORD(P) - 1)) - TEV(M,'TUL');
SCHEDULE(N,M,G,N1,'AVAIL. TIME')
= TPER * (ORD(P) - 1 + PRT(N1,N1,M) - PAR(N1,N1,M));
IBEG = IBEG + 1));
OPTION SCHEDULE: 1:4:1;
DISPLAY SCHEDULE;
SCHEDULE(N,M,G,N1,'NO. MODES') = 0;
SCHEDULE(N,M,G,N1,'NO. MINES') = 0;
SCHEDULE(N,M,G,N1,'LOAD. TIME') = 0;
SCHEDULE(N,M,G,N1,'ETA. ORIG') = 0;
SCHEDULE(N,M,G,N1,'ETA. DEST') = 0;
SCHEDULE(N,M,G,N1,'AVAIL. TIME') = 0);

PARAMETER

MINEINV(N,*) The number of mines in left in inventory;
MINEINV(N,'MINES. LEFT')$((DNO(N)*(MN(N) + SN(N)))
= SUM((N1,P,M)$((X.L(N1,N1,P,M) GE 0.5) OR (X.L(N1,N1,P,M) GE 0.5)),
X.L(N1,N1,P,M) - X.L(N1,N1,P,M)) + AMT(N);
OPTION MINEINV: 0;
DISPLAY MINEINV;

SET

NEXTRUN(N,*) Nodes that should be included in the next run of SUMIT;
NEXTRUN(N,'NEXT. RUN')$((NOT (NOT(N) + DNO(N) + NFC(N)))

56
\[ (\text{SMAX}((\text{N}1, \text{M}), \text{ART}(\text{N}, \text{N}1, \text{M}), \text{ABS}((\text{NUM}(\text{N}, \text{M}))) \geq 0.5) \]
\[ \text{OR} (\text{AMT}(\text{N}) \leq -0.5)) = \text{YES}; \]

\text{LOOP} (I \in (\text{ABS}(\text{ORD}(I) - 1) \leq 0.5)
\text{AND} (\text{SUM}((\text{N} \times \text{NEXTRUN}(\text{N},'\text{NEXT. RUN'})) \ast \text{DN}(\text{N}), \text{AMT}(\text{N})) \leq -0.5)
\text{AND} (\text{SUM}((\text{N} \times \text{NEXTRUN}(\text{N},'\text{NEXT. RUN'}), 1) \geq 0.5)));

\text{DISPLAY} " Change NUN(N,M) and DN(N) to 0 for all nodes not in ",
" NEXTRUN(N) and rerun SUMIT to finish the solution. ", NEXTRUN;

\text{LOOP} (I1 \in (\text{ABS}(\text{ORD}(I1) - 1) \leq 0.5)
\text{AND} (\text{SUM}((\text{N} \times \text{NEXTRUN}(\text{N},'\text{NEXT. RUN'}), \text{AMT}(\text{N})) \leq -0.5)));
\text{AMT}(\text{N}) = \text{AMT}(\text{N}) \times \text{NEXTRUN}(\text{N},'\text{NEXT. RUN'});
\text{OPTION AMT: 0;}
\text{DISPLAY} "WARNING: There is not enough supply to meet demand ",
" for the next run of SUMIT. ", AMT);
APPENDIX B. SAMPLE OUTPUT, FIRST RUN

COMPILATION TIME = 0.950 SECONDS

MODEL STATISTICS

<table>
<thead>
<tr>
<th>Blocks of Equations</th>
<th>Single Equations</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
<td>Blocks of Variables</td>
<td>Single Variables</td>
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<td>4</td>
<td>213</td>
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<tr>
<td>Non Zero Elements</td>
<td>Discrete Variables</td>
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GENERATION TIME = 1.190 SECONDS

EXECUTION TIME = 7.500 SECONDS

SOLVE SUMMARY

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<tr>
<th>Model</th>
<th>SUMIT</th>
<th>Objective V</th>
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<td>Type</td>
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<td>Direction</td>
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<td>Minimize</td>
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<td>Normal Completion</td>
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<tr>
<td>Model Status</td>
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Resource Usage, Limit

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<tr>
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ZOOM / XMP --- Version 2.1 Jun 1988

Courtesy of Dr. Roy E. Marsten,
Department of Management Information Systems,
University of Arizona,
Tucson Arizona 85721, U.S.A.

No options file found - using defaults.

Work space needed (estimate) -- 67966 words.
Work space available -- 67966 words.
Maximum obtainable -- 294142 words.

The LU factors occupied 804 slots (estimate 3068).
The branch and bound tree contained 4550 nodes (max. 10000 nodes).

Iterations: Initial LP 193, Time: 0.26
Heuristic 3214, 8.21
Branch and bound 73061, 149.45
Final LP 121, 0.14

**** REPORT SUMMARY:
0 NONOPT
0 INFEASIBLE
0 UNBOUNDED

---- 815 THE FOLLOWING ARCS WERE CONSIDERED BY SUMIT.

---- 815 SET ARC ARCS IN NETWORK FROM NODE N TO NODE N1 VIA MODE M

TRUCK C130 P3 B52 TACAIR SSN688DM DBRM

M8GQ .S8GQ YES
S8GQ .ANDERSNAFB YES
M9RP .S9RP YES
S9RP .MCASIWKNI YES
S9RP .MINEFIELD3 YES
M10JA .S10JA YES
S10JA .MINEFIELD2 YES
S10JA .MINEFIELD3 YES
ANDERSNAFB.NASADAK YES
ANDERSNAFB.MCASIWKNI YES
ANDERSNAFR.MINEFIELD1 YES
ANDERSNAFB.MINEFIELD2 YES
ANDERSNAFB.MINEFIELD3 YES
NASADAK .MINEFIELD1 YES
MCASIWKNI .MINEFIELD2 YES
CVNA .MINEFIELD2 YES
SSN688A .MINEFIELD1 YES

---- 817 *** ALL TIMES IN GIVEN IN HOURS, INCREMENTED BY TPER. ***

---- 824 PARAMETER TIMECOMP TIME IN WHICH LAYING OF MINE FIELD N WAS COMPLETED

TIME DONE
MINEFIELD1 36.0
MINEFIELD2 36.0
MINEFIELD3 36.0

---- 837 PARAMETER MOMAGREP MOMAG SCHEDULE REPORT (START TIME IS 0)
STOP TIME NUM MINES

59
### M8GQ Schedule

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<th>NO. MODES</th>
<th>NO. MINES</th>
<th>LOAD. TIME</th>
<th>ETD. ORIG</th>
<th>ETA. DEST</th>
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### M9RP Schedule

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### M10JA Schedule

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### S8GQ Schedule

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<tr>
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<td>7.0</td>
<td>10.0</td>
<td>12.0</td>
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<tr>
<td>1.0</td>
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<td>12.0</td>
<td>13.0</td>
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### S9RP Schedule

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<td>6.0</td>
<td>7.0</td>
<td>11.7</td>
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<td>6.0</td>
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### S10JA Schedule

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### ANDERSONAFB Schedule

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### CVNA TACAIR Schedule

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<td>17.7</td>
<td>18.0</td>
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<tr>
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<td>18.0</td>
<td>19.0</td>
<td>23.7</td>
<td>24.0</td>
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</table>
### Schedule for Mine Transshipment by Origin Base

<table>
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<th>SSN688A</th>
<th>SSN688DM</th>
<th>NO. MODES</th>
<th>NO. MINES</th>
<th>LOAD. TIME</th>
<th>ETD. ORIG</th>
<th>ETA. DEST</th>
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<tbody>
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### Parameters

#### Mines Left

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<tr>
<td>M9RP</td>
<td>31</td>
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<tr>
<td>M10JA</td>
<td>216</td>
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<td>S10JA</td>
<td>5</td>
</tr>
<tr>
<td>CVNA</td>
<td>8</td>
</tr>
</tbody>
</table>

### Change

```
883 change Num(N,M) and Dn(N) to 0 for all nodes not in NextRun(N) and rerun SUMIT to finish the solution.
```

### Set

```
883 set NEXTRUN nodes that should be included in the next run of SUMIT
```

### Next Run

<table>
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<tr>
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<th>YES</th>
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<tbody>
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<tr>
<td>S11SC</td>
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<tr>
<td>NASJAX</td>
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<td>SSN688B</td>
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<tr>
<td>MINEFIELD4</td>
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</table>

### File Summary

```
**** File Summary for User 8693P

Input TEST1 GAMS A
Output TEST1 LISTING A

Execution Time = 1.370 seconds
```
APPENDIX C. SAMPLE PROGRAM, SECOND RUN

$TITLE * ** Scheduler for Mine Transshipment (SUMIT) ** *
*-----DOCUMENTATION----------------------------------------------
* Software to Optimize Schedules for Mine Transshipment
* Developed for Commander, Mine Warfare Command
* Written by Tammy L. Glaser, LT, USN
* Complete documentation contained in thesis
* 06 September 1991
*-----GAMS AND DOLLARS CONTROL OPTIONS--------------------------
* Do not change the following options.
$OFFUPPER OFFSYMREF OFFSYMLIST
OPTIONS SOLPRINT = OFF, LIMROW = 0, LIMCOL = 0;
* Increase the following options only when recommended in the
* solution report of the output.
OPTIONS RESLIM = 10000, ITERLIM = 10000000, WORK = 10000;
* Set OPTCR as recommended on page 72 of the thesis documentation.
OPTIONS OPTCR = 0.00001;
*-----DEFINITIONS AND DATA---------------------------------------
* See Chapter 2 of thesis documentation to explain data in detail.
SCALARS
* The following scalars must be set to values greater than 0.
TEND end time of problem (h) /27.0/
TPER length of time period (h) /1.2/
RELT zero tolerance for comparisons of real data /0.001/
* The following scalars are optional (enter 0 for default value).
MNLA minimum distance for air modes over land routes (nm) /50/
MXSS maximum distance for transfers between supply nodes (nm) /100/
DISC 1 to run entire problem and 0 to split into subproblems /0/;
SETS
* The first two sets must have the element 0 in them. Note that
* N must contain all elements in CN(N), found in the optional section.
* M must contain all elements in DM(M) (and TM(M), BM(M), CM(M), and
* LM(M), which are found in the optional section). See page 7 of
* thesis documentation to explain the MOMAG-supply node split.
N nodes /0, M8GQ, S6GQ, M9RP, S9RF, M10JA, S10JA,
  M11SC, S11SC, ANDERSNAFB, NASADAK,
  MCAS1WKIN, NASJAX, CVNA, CVNB, SSN688A,
  SSN688B, MINEFIELD1, MINEFIELD2,
  MINEFIELD3, MINEFIELD4/;
M modes /0, TRUCK, C130, C141, P3, B52, TACAIR,
  SSN688DM, DBRM/;
DM(M) delivery modes /P3, B52, TACAIR, SSN688DM/;
* P must start with 0 and end with the value of the number of time
* periods plus one. G and I must start with 1. G must end with
* the number of time periods, while I must end with the square of
* the number of nodes (nonzero elements in N).
P time periods /0 * 23/;
G group numbers /1 * 22/;
I iteration numbers /1 * 400/;
* The following sets are optional. Enter 0 between slashes for sets
* not needed for the problem. If CN(N) is empty (i.e., initialized to
* 0), nodes are assumed to be land nodes. If there are no transshipment
* modes or disassembled mines, enter 0 between slashes for TM(H) or
* BM(H). All modes are assumed to be air modes if CM(H) and LM(H)
* are initialized to 0.

  CN(N) sea nodes /CVNA, CVNB, SSN688A, SSN688B/
  TM(H) transportation modes /TRUCK, C130, C141/
  BM(H) dummy build rate modes /DBRM/
  CH(H) dummy sea modes /SSN688DM/
  LM(H) land modes /TRUCK/

* Do not change the following evolution set.
  E mode evolution /TLD, TUL, TGD/;

* Do not change the following set aliases:
  ALIAS (N,N1,N2,N3,N4);
  ALIAS (M,M1,M2,M3);
  ALIAS (P,P1);
  ALIAS (I,I1);

PARAMETERS

* The following parameter is required for node N:
  AMT(N) supply (+) or demand (-) at node N (mines)
  /M8GQ 200
  S8GQ 25
  M9RP 159
  M10JA 270
  S10JA 95
  M11SC 100
  S11SC 40
  ANDERSNAFB 89
  CVNA 161
  CVNB 100
  SSN688A 50
  SSN688B 40
  MINEFIELD1 0
  MINEFIELD2 0
  MINEFIELD3 0
  MINEFIELD4 -201/

* The following parameters are optional (leave blank line with no
* slashes for default values):
  PRN(N) priority of node N
  /MINEFIELD1 2
  MINEFIELD2 1
  MINEFIELD3 3
  MINEFIELD4 1/

  MXD(N) maximum distance between mine fields at node N (nm)
  /MINEFIELD2 50/

  BGD(N) transit distance for sea node N at beginning (nm)
  /CVNA 33
  CVNB 75/

  EDD(N) transit distance for sea node N at end (nm)
  /CVNB 75/
SPX(N)  transit speed for sea node N (nm per h)
/CVNA  25
CVNB  25/

* The following parameters are required for mode M:

CAF(M)  capacity of mode unit in mode group M (mines per mode)
/TRUCK  20
C130  20
C141  20
P3  8
B52  22
TACAIR  51
SSN688DM  50/

MXL(M)  maximum distance that mode M may travel (nm)
/TRUCK  150
C130  3000
C141  3000
P3  1500
B52  3000
TACAIR  500
SSN688DM  200/

SPD(M)  transit speed of mode M (nm per h)
/TRUCK  30
C130  380
C141  410
P3  350
B52  480
TACAIR  450
SSN688DM  15
DBRM  9/

* The following parameter is optional (leave blank line for default):

PRM(M)  priority of mode M
/TRUCK  1
C130  3
C141  3
P3  1
B52  1
TACAIR  1
SSN688DM  3
DBRM  2/

* The following parameter is required for mode M at node N:

NUM(N,M)  number of mode M at node N (modes)
/M8GQ.DBRM  0
S8GQ.TRUCK  0
M9RP.DBRM  0
S9RP.C130  0
S9RP.P3  0
M10JA.DBRM  0
S10JA.P3  0
M11SC.DBRM  1
S11SC.TRUCK  3
S11SC.C130  4
ANDERSNAFB.B52  0
ANDERSNAFB.C130  0
NASADAK.P3  0
MCASIWKNI.TACAIR  0
NASJAX.P3  6
CVNA.TACAIR  0
CVNB.TACAIR  1
SSN688A.SSN688DM  0
SSN688B.SSN688DM  1/

* The following parameters are optional (leave blank line for default):
  BGT(N,M)  earliest beginning time for mode M at node N (h)
  /SSN688A.SSN688DM  10/
  EDT(N,M)  latest end time for mode M at node N (h)
  /CVNA.TACAIR  24/
  MXT(N,M)  maximum total time mode M can be absent from node N (h)
  MXR(N,M)  maximum number of runs for mode M from node N
  /ANDERSNAFB.B52  2
  ANDERSNAFB.C130  2
  S9RP.C130  2/
  XMP(N,N1,M)  1 means eliminate runs from nodes N to N1 via mode M
  /S9RP.MINEFIELD2.P3  1/

* The following two parameters are required.
* READ PAGE 22 OF THESIS DOCUMENTATION, WHICH EXPLAINS WHEN NEGATIVE
* DISTANCES MUST BE USED.

TABLE DIS(N,N1) distance between node N and node N1 (nm)

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APPENDIX D.  SAMPLE OUTPUT, SECOND RUN

COMPILATION TIME = 0.950 SECONDS

MODEL STATISTICS

BLOCKS OF EQUATIONS  6  SINGLE EQUATIONS  123
BLOCKS OF VARIABLES  4  SINGLE VARIABLES  154
NON ZERO ELEMENTS  446  DISCRETE VARIABLES  51

GENERATION TIME = 1.170 SECONDS

EXECUTION TIME = 7.440 SECONDS

SOLVE SUMMARY

MODEL SUMIT
TYPE MIP
SOLVER ZOOM

**** SOLVER STATUS  1 NORMAL COMPLETION
**** MODEL STATUS   1 OPTIMAL
**** OBJECTIVE VALUE 158139.0000

RESOURCE USAGE, LIMIT 105.820 10000.000
ITERATION COUNT, LIMIT 66392 10000000

ZOOM / XMP --- Version 2.1 Jun 1988

Courtesy of Dr Roy E. Marsten,
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University of Arizona,
Tucson Arizona 85721, U.S.A.

No options file found - using defaults.

Work space needed (estimate) -- 62593 words.
Work space available -- 62593 words.
Maximum obtainable -- 246526 words.

The LU factors occupied 552 slots (estimate 2101).

The branch and bound tree contained 4106 nodes (max. 10000 nodes).

Iterations: Initial LP 118, Time: 0.12
Heuristic 253, 0.62
Branch and bound 65947, 104.94
Final LP 74, 0.07

**** REPORT SUMMARY:
0 NONOPT
0 INFEASIBLE
0 UNBOUNDED

--- 815
THE FOLLOWING ARCS WERE CONSIDERED BY SUMIT.

--- 815 SET ARC ARCS IN NETWORK FROM NODE N TO NODE N1 VIA MODE M

TRUCK C130 P3 TACAIR SSN688DM DBRM

M11SC .S11SC YES
S11SC .NASJAX YES YES
NASJAX .MINEFIELD4 YES
CVNB .MINEFIELD4 YES
SSN688B. MINEFIELD4 YES

--- 817
*** ALL TIMES IN GIVEN IN HOURS, INCREMENTED BY TPER. ***

--- 824 PARAMETER TIMECOMP TIME IN WHICH LAYING OF MINE FIELD N WAS COMPLETED

TIME. DONE
MINEFIELD4 19.2

--- 837 PARAMETER MOMAGREP MOMAG SCHEDULE REPORT (START TIME IS 0)

STOP. TIME NUM. MINES
M11SC 3.6 21.0

--- 860 PARAMETER SCHEDULE SCHEDULE FOR MINE TRANSSHIPMENT BY ORIGIN BASE

S11SC.TRUCK NO. MODES NO. MINES LOAD. TIME ETD. ORIG ETA. DEST AVAIL. TIME
1. NASJAX 2.0 40.0 1.0 5.2 13.2
2. NASJAX 2.0 21.0 3.6 4.6 8.8 16.8

--- 860 PARAMETER SCHEDULE SCHEDULE FOR MINE TRANSSHIPMENT
<table>
<thead>
<tr>
<th>Origin Base</th>
<th>No. Modes</th>
<th>No. Mines</th>
<th>Load Time</th>
<th>ETD Orig</th>
<th>ETA Dest</th>
<th>Avail Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>NASJAX. P3</td>
<td>1. Minefield 4</td>
<td>5.0</td>
<td>40.0</td>
<td>7.2</td>
<td>8.2</td>
<td>10.5</td>
</tr>
<tr>
<td></td>
<td>2. Minefield 4</td>
<td>3.0</td>
<td>21.0</td>
<td>15.6</td>
<td>16.6</td>
<td>18.9</td>
</tr>
<tr>
<td>CVNB. TACAIR</td>
<td>1. Minefield 4</td>
<td>1.0</td>
<td>51.0</td>
<td>3.6</td>
<td>4.6</td>
<td>5.7</td>
</tr>
<tr>
<td></td>
<td>2. Minefield 4</td>
<td>1.0</td>
<td>49.0</td>
<td>9.6</td>
<td>10.6</td>
<td>11.7</td>
</tr>
<tr>
<td>SSN688B. SSN688DM</td>
<td>1. Minefield 4</td>
<td>1.0</td>
<td>40.0</td>
<td>11.7</td>
<td></td>
<td>25.2</td>
</tr>
</tbody>
</table>

--- 860 Parameter Schedule SCHEDULE FOR MINE TRANSSHIPMENT BY ORIGIN BASE

--- 874 Parameter MineInv THE NUMBER OF MINES IN LEFT IN INVENTORY

Mines Left

** File Summary for User 8693P

Input: TEST2 GAMS A
Output: TEST2 LISTING A

Execution Time = 1.140 Seconds
APPENDIX E. GAMS IMPLEMENTATION OF SUMIT

A. SUMIT OUTPUT

The output of SUMIT is composed of standard GAMS reports and of data displayed by the program. The standard GAMS reports include an echo print of the program, compilation time, model statistics (i.e., number of constraints and continuous and discrete variables), and solve summary. The echo print of the program was deleted from Appendix B since the program is given in Appendix A. The data output by SUMIT was also edited to make Appendix B fit within the page margins. The data displayed by the program include the arcs generated for the mine network (which has the same arcs as the mode network), the time in which the mine field deployment was complete, the MOMAG schedule report, the schedules for mine transshipment, the mine inventory at the end of the problem, and the nodes that should be included in the next run of SUMIT. SUMIT prints out the mine network to allow the user to verify that all arcs in the mine and mode networks are appropriate. The MOMAG schedule report contains the times in which each MOMAG must start and stop assembling mines and the number of mines that each MOMAG must build. SUMIT then prints out separate mine transshipment schedules for each origin node that reports the following information: origin node, mode group, run number for the node-mode combination, destination node, total number of mines carried by the mode group, number of mode units needed, the time to start loading the mode group, estimated time of departure from the origin node, estimated time of arrival at the destination node, and the time in which the mode is available for loading for its next flight. It also warns the user if total supply is less than total demand for nodes recommended in the next run.

B. END TIME AND TIME PERIOD LENGTH

To find appropriate values for the end time and time period length for a new problem, the user should take the following steps: the user should initially execute GAMS using the relaxed version of ZOOM, which is faster since it allows binary variables assume any values between zero and one. To indicate the relaxed version of ZOOM, the user must change the problem identifier "MIP" to "RMIP" in the "SOLVE" statement, which is found on line 821 of the program (see Appendix A, page 55). The goal of making these quick runs is to identify the latest end time with an appropriate number of periods in which the solver reports an infeasible solution ("INFEASIBLE") in the
section of the output labeled "SOLUTION REPORT." Then, the user should run SUMIT using an end time equal to the infeasible end time plus one extra time period with binary requirements enforced (replacing "RMIP" with "MIP" in the "SOLVE" statement). In most cases, adding an extra time period will make the solution feasible. If the problem is not feasible when the binary variables are enforced, the "SOLUTION REPORT" portion of the output will contain the phrase "INTEGER INFEASIBLE". If the solution is integer infeasible, the user should first verify that there is enough supply to meet the demand for the mine network output by the SUMIT. If the supply is sufficient, then the user should rerun the "MIP" version of ZOOM after increasing the end time by one extra time period until a feasible solution is found. Since the purpose of these final runs is to find the first integer feasible solution, the user may set the OPTCR at 1.0 to find the solution as quickly as possible.

C. OBTAINING NEARLY OPTIMAL SOLUTIONS

A general rule of thumb is recommended to obtain nearly optimal solutions for SUMIT. When the user makes the first run of the "MIP" version of ZOOM, the user should set OPTCR equal to 0.10 in line 16 of the program, rerun SUMIT, and verify that the reported solution is satisfactory. If the number of iterations is less than 10,000, which is given in the "SOLUTION REPORT" portion of the output indicated by "ITERATION COUNT", rerunning SUMIT with a very small OPTCR, such as 0.01, will probably yield an optimal solution within a reasonable number of iterations, but will not usually prove it. If the number of iterations is greater than 10,000, the user may rerun SUMIT at an OPTCR between 0.01 and 0.1 to find a solution that is optimal or close to optimal, but not proven. Proving optimality by using an OPTCR less than 0.01 is not recommended since this may result in an excessive solution time and excessive disk storage requirements.
LIST OF REFERENCES


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