LOAD SHARING IN ANTI-AIR WARFARE COORDINATION: CRITERIA AND A SIMULATION TEST PLAN

by

Stephen Hume Kelley

September 1991

Thesis Advisor: Glenn F. Lindsay

Approved for public release; distribution is unlimited
This thesis addresses coordination between ships of a force in anti-air warfare. In support of the need for effective coordination, two coordination schemes are presented. One is based on earliest intercept time and is a candidate for future use. Here, the ship with the earliest projected intercept time is directed to engage the attacker. The second scheme introduces a load sharing feature wherein current magazine inventories are considered. In line with broad goals of AAW coordination, several measures of effectiveness to compare the schemes are introduced and particular attention is given to the utility of these measures of effectiveness. Potential simulation scenarios and input parameters for a comparison of the two schemes are then presented along with some specific suggestions for statistical analysis of the results. The thesis concludes with final remarks about load sharing, measures of effectiveness, and testing procedures.
Load Sharing in Anti-Air Warfare Coordination: 
Criteria and a Simulation Test Plan

by

Stephen Hume Kelley
Lieutenant, United States Navy
B.S., Marquette University, 1984

Submitted in partial fulfillment
of the requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
September 1991

Author: Stephen Hume Kelley

Approved by: Glenn F. Lindsay, Thesis Advisor
William Walsh, Second Reader
Peter Purdue, Chairman
Department of Operations Research
ABSTRACT

This thesis addresses coordination between ships of a force in anti-air warfare. In support of the need for effective coordination, two coordination schemes are presented. One is based on earliest intercept time and is a candidate for future use. Here, the ship with the earliest projected intercept time is directed to engage the attacker. The second scheme introduces a load sharing feature wherein current magazine inventories are considered. In line with broad goals of AAW coordination, several measures of effectiveness to compare the schemes are introduced and particular attention is given to the utility of these measures of effectiveness. Potential simulation scenarios and input parameters for a comparison of the two schemes are then presented along with some specific suggestions for statistical analysis of the results. The thesis concludes with final remarks about load sharing, measures of effectiveness, and testing procedures.
# TABLE OF CONTENTS

I. INTRODUCTION ........................................... 1  
   A. THE NEED FOR COORDINATION .......................... 1  
   B. THE SCOPE OF THIS THESIS ............................ 2  
II. EARLIEST INTERCEPT CONCEPT WITH LOAD SHARING ... 4  
   A. SOME GOALS AND CONSIDERATIONS OF COORDINATION 4  
      1. Battle Space .................................... 5  
      2. Resource Allocation ............................... 7  
   B. TWO COORDINATION SCHEMES .......................... 9  
      1. The Earliest Intercept Scheme .................... 9  
      2. The Earliest Intercept Scheme with Load  
          Sharing ........................................... 11  
   C. TWO EXTREME CASES FOR LOAD SHARING ............ 12  
III. DESCRIPTION OF SAMS .................................. 15  
   A. OVERVIEW ........................................... 15  
      1. Inputs to SAMS ................................... 17  
      2. Output ........................................... 18  
   B. DIFFICULTY IN IMPLEMENTATION ...................... 19  
IV. MEASURES OF EFFECTIVENESS FOR AAW COORDINATION  
    SCHEMES ................................................ 21  
   A. MEASURES RELATING TO SURVIVABILITY ............... 22  
   B. MEASURES RELATING TO SUSTAINABILITY .............. 27  

iv
ACKNOWLEDGMENTS

Many thanks to Roger Stemp and Mary Geer, two people on the Operations Analysis staff who provided me counsel through the turbulent waters of learning. Thanks also to Glenn Lindsay for his patience. The greatest thanks of all go to my wife, Laura, for her support and understanding.
I. INTRODUCTION

This thesis is concerned with force anti-air warfare (AAW) coordination for the U.S. Navy. The Navy is currently conducting research in order to find an efficient method of automating the coordination of multiple ships against an airborne threat. In their proposed work, candidate coordination schemes will be tested in simulation by the Naval Warfare Analysis Department at the Applied Physics Laboratory of The Johns Hopkins University.\[Ref. 1:p. 12\] This report will present a potential modification to a candidate force AAW coordination scheme and examine means by which the modification, as compared to the basic scheme, can be tested.

A. THE NEED FOR COORDINATION

The need for coordination of ships in battle against an airborne threat has probably never been as important as it is today. The need has intensified because of the high level of performance of both offensive and defensive weapons at sea. Anti-ship missiles, even those in possession of third world nations, are fast and deadly. If fired in sufficient numbers, such missiles have the potential to overwhelm the defenses of even the most capable ships.
The integration and speed of modern defensive systems can also be used to illustrate the need for coordination. These automated systems, if left unchecked, can empty a ship's magazines in a matter of minutes, thus leaving the ship's defense severely deteriorated.

Coordination of ships against the air threat is intended to make timely use of all area defense systems, available forcewide, in order to counter the threat.

B. THE SCOPE OF THIS THESIS

As discussed above, this thesis will introduce a modification to a proposed coordination scheme and explore means by which the performance of the modification, as compared with the basic scheme, can be tested.

The attributes of survivability and sustainability and how they can be related to coordination schemes will be discussed in Chapter II. The basic coordination scheme and the modification will also be presented. Chapter II ends with a discussion of two situations which illustrate the extreme conditions for the modification.

An overview of the simulation being used at Johns Hopkins to analyze candidate schemes will be presented in Chapter III. Such considerations as the input needed and the output capabilities will be discussed.

In Chapter IV, some measures of effectiveness which can quantify the hazy concepts of survivability and
sustainability will be explored. Each measure, as it relates to either survivability or sustainability, will be described. The potential for non-independence of the measures will also be investigated.

Chapter V presents a structure for the evaluation of alternate coordination schemes. In order to ensure that only the desired attributes are tested, the control of inputs and simulation scenario parameters will be examined.

Chapter VI provides some final remarks about load sharing, measures of effectiveness, testing procedures and suggestions for future work in this subject.
II. EARLIEST INTERCEPT CONCEPT WITH LOAD SHARING

This chapter will identify two of the primary goals of coordination, define necessary terms, and discuss some of the factors which can be considered in choosing a coordination scheme. Two coordination schemes will then be presented. One scheme which has received attention as a candidate for the Navy is based upon earliest intercept time. The scheme, and how it relates to some basic goals of coordination will be discussed. A second scheme, which is a modification to the earliest intercept scheme, will then be introduced. The modification combines a load sharing feature with the earliest intercept time method. The chapter will conclude with the exploration of two extreme cases involving the load sharing feature.

A. SOME GOALS AND CONSIDERATIONS OF COORDINATION

Two overall goals of Anti-Air Warfare (AAW) coordination are to attempt to increase both the survivability and the sustainability of the force against hostile air threats. Given these goals, two major factors can be considered in the choice of coordination schemes. These factors are the efficient use of battle space, and resource allocation.
1. **Battle Space**

*Battle space* is the distance between the attacker (hostile missile or aircraft) and the center of the friendly force at the time it is determined that the attacker must be engaged. Figure 1 illustrates battle space.

![Figure 1 - Battle Space](image)

An engagement is the physical act of launching a missile salvo at the attacker. A salvo could consist of one, two, three, or more missiles depending on the missile firing doctrine being used. Each missile salvo fired represents a separate engagement.

*Battle space* depends upon a variety of factors, including detection range and identification time, not all of which are in control of the friendly force. Detection
range, or the range at which the attackers are detected, primarily controls battle space. Detection range, itself, depends upon a number of factors such as the type of search radars in use, atmospheric conditions, and the physical size and flight profiles of the attackers. In a simulation, keeping types of radars and atmospheric conditions constant, detection range would depend primarily on the characteristics of the attackers.

Once an attacker has been detected, another factor which affects battle space is the time needed to identify the attacker. The identification process itself is not of concern here. An assumption for the purposes of simulation could allow a constant period of time for identification following the detection of each attacker.

Apart from the considerations discussed thus far, there remain few considerations affecting battle space. The effective use of this available distance, called battle space, becomes critical.

One of the potential purposes of a coordination scheme is to use the available battle space, described above, as efficiently as possible in an attempt to increase the survivability of the force. Survivability can be increased by taking advantage of every firing opportunity at the attacker that battle space allows. Wasted firing opportunities could result in an increase in the number of attackers which penetrate the defense.
2. Resource Allocation

A primary factor relating to sustainability is resource allocation. A reduction of wasted missile resources increases the potential for sustainability. Efficient allocation of missile resources can be described by the following three objectives: 1) every attacker is to be defeated, 2) the fewest number of missiles are to be used to defeat each attack, and 3) assignments are made such that no ship is forced to exhaust magazine inventory unless all ships are nearing zero inventory.

The objective that every attacker is to be defeated is simple in concept. An attacker will be considered defeated if it is destroyed, or is sufficiently damaged to cause it to miss the ships of the force. For study purposes, defeating and destroying the attacker will be synonymous. The objective requires that each attacker is engaged, repeatedly if necessary, until destroyed. Thus, the objective of defeating every attacker ensures that enough missile resources are allocated during the attack to conduct necessary engagements. Unfortunately, the objective may not always be achievable. In an actual battle, the intensity of an attack could saturate a coordination scheme or even an individual ship's missile systems. The saturated scheme or system could then allow some attackers to get through the defense without being destroyed. The existence of any such potential weakness in a coordination scheme can
be discovered in a simulation by incrementing the attack size until the scheme becomes saturated. Such inputs to the simulation will be suggested in Chapter V.

The second resource allocation objective, which was to use the fewest number of missiles to defeat the attack, is intended to prevent the waste of missile resources on redundant engagements (overkill). Overkill can occur when missiles are fired at an attacker which has already been destroyed or when more than one ship engages the same attacker simultaneously. Some coordination schemes intentionally assign multiple ships to simultaneously engage a single attacker in order to improve the probability of kill (Pk). Such intentional overkill reduces sustainability by often wasting missile resources. Sustainability can be improved by using only the fewest number of missiles required, thus depleting inventories no more than necessary.

The third allocation objective, assigning ships to attackers such that no ship is forced to exhaust magazine inventory unless all ships are nearing zero inventory, involves the concept of load sharing. The specific ships which launch the missiles that defeat the attack are of little concern as long as all attackers are destroyed. However, danger can occur when ships begin to exhaust their inventories. While all ships remain capable, each ship supports the defense of the force by defending itself and others as necessary. If a ship exhausts its missile
inventory, it can no longer lend support for the defense of the force. It must also be defended by other ships of the force, increasing the burden on the other ships. This loss of firepower could allow attackers through the defense, causing catastrophic results. Load sharing, then, promotes the depletion of missile inventories evenly throughout the force, preventing any one ship, or ships, from expending magazines prematurely. This results in a force which can defend itself to its greatest potential until it exhausts its missile supply as a whole.

B. TWO COORDINATION SCHEMES

The goals of survivability and sustainability have been presented along with factors which influence their attainment. A force coordination scheme can support the survivability and sustainability goals through the efficient use of battle space and careful resource allocation. One coordination scheme is called the earliest intercept scheme, and the ways in which it relates to the goals and considerations presented, will now be discussed.

1. The Earliest Intercept Scheme

Neglecting efficient resource allocation considerations, a natural choice for Anti-Air Warfare (AAW) coordination is a scheme based upon earliest intercept time. In this scheme, all ships of the force that find a particular unengaged attacker engageable relay their
computed missile intercept time to a designated control ship for comparison. The ship with the earliest projected intercept time is then directed to engage the attacker. The assigned ship will engage the attacker, repeatedly if necessary, until the attacker is destroyed or until the ship can no longer engage the attacker because of physical limitations such as minimum missile range or zero missile inventory.

Each ship's projected intercept time is computed from the physical location and flight profile of the attacker, the flight characteristics of the missile to be fired, and fire control system and missile launcher availability. Thus, a ship which is currently burdened by ongoing missile engagements would probably submit a later projected intercept time than would an unburdened ship, since an unburdened ship does not have to wait for equipment to become available to support the missile launch.

Survivability is supported by the earliest intercept scheme through the efficient use of battle space. The ship which can intercept the attacker first is the one assigned. This method makes good use of battle space by intercepting attackers at the greatest range possible under a given set of circumstances.

Sustainability is not well supported by the earliest intercept scheme because the missile inventory of individual ships is not considered when assignments are made. Although
the scheme may prove to be an effective method for survivability, ships located closest to the direction from which the attack occurs would likely expend their magazines prematurely. This would occur because their proximity to the threat would cause them to be assigned the vast majority of attackers without regard to their magazine level. Once empty, there is now a reduction in the overall defense of the force. Some consideration for magazine inventory appears to be necessary in order to improve sustainability. (Note that it is possible for attacks to occur from more than one direction, but for simplicity in this study, attacks will only occur from one direction.)

The load sharing modification to the Earliest Intercept coordination scheme is a heuristic approach to the issues discussed. The scheme attempts to make maximum use of battle space, though it will trade-off some battle space to support the efficient allocation of resources.

2. The Earliest Intercept Scheme with Load Sharing

In order to utilize the benefits of the Earliest Intercept approach and address the issue of sustainability (missile allocation), an adjustment considering current magazine inventory can be added to the earliest intercept scheme. Reports to the designated control ship in the force now becomes a ratio of the projected intercept time over that ship's current magazine level. For example, ship i
would report the ratio $T_i/M_i$, where $T_i$ is the projected time to intercept the attacker and the integer $M_i$ is the current missile inventory for its magazine(s). When $M_i$ equals zero, then ship $i$ has no missiles in inventory, and is no longer of value to the scheme and thus not considered. Each time an attacker is identified for engagement by the force, the ship with the smallest ratio is chosen to engage that attacker. The result when missile inventory throughout the force is evenly distributed (the denominators of the ratio are near equal), is a pure earliest intercept time engagement. When there is a disparity between missile inventories, this ratio allows trade-offs between intercept time and load leveling.

There exists a chance that survivability could be reduced while using the load sharing modification. The trade-off of time represents a potential waste of battle space. By giving up the time, and accordingly the distance, one or more opportunities to fire a missile salvo may be given up. This potential decrease in survivability should be examined carefully.

C. TWO EXTREME CASES FOR LOAD SHARING

Two extreme cases for the load sharing feature occur as a result of the positioning of ships relative to the direction from which the attack occurs. Figure 2 illustrates the two cases. It should be noted that, by
symmetry, attacks from any direction are bounded by these cases.

The first extreme case of the load sharing concept occurs when two or more firing ships fall on the same axis with the inbound attacker. The difference in intercept times, not considering fire-control system and missile launcher scheduling, becomes the time-of-flight difference for the more distant ship's missiles to reach the attacker.

The other extreme occurs when missile flight distances approach equality. The equal flight distances could result in having near equal intercept times. In this case, the ship with the greater missile inventory will be assigned to engage the attacker.

In the first case the intercept time difference will reduce the tendency to load share. However, sufficient
missile level disparities between the ships will override the intercept time. The override will cause a ship with a greater missile inventory, but a later intercept time, to be assigned to the attacker. Assigning a ship with a later intercept time could cause the waste of potentially vital battle space.

It remains to be seen whether or not the load sharing feature would severely reduce the survivability performance of the earliest intercept scheme. In Chapter IV, measures of effectiveness will be presented which can be used to determine the value of load sharing.
III. DESCRIPTION OF SAMS

Now that the two schemes have been presented, this chapter briefly describes the Systems Analysis Method by Simulation (SAMS), the simulation program in which the earliest intercept scheme with load sharing concept will be implemented. The first section in this chapter presents an overview of SAMS. The second section discusses a difficulty in programming a coordination scheme such as one based on intercept time.

A. OVERVIEW

The simulation used, called SAMS, is a discrete event simulation developed at The Johns Hopkins University Applied Physics Laboratory by Edward A. Davis and Bruce Bundsen. Its purpose is to simulate the performance of shipboard AAW systems. Intership communications are explicitly modeled in SAMS, permitting evaluation of alternate force coordination schemes. Specifically, emphasis can be placed on engagement control doctrine.[Ref. 1:p.A.23] This is especially useful in the analysis of the earliest intercept scheme with load sharing.

The SAMS simulation models the detect-to-engage sequence of events in a manner similar to the way a real-world Anti-
Air Warfare situation would unfold. That is, SAMS models the following sequence:

- Target detections,
- Formulation of tracks,
- Evaluation of tracks,
- Engagement coordination message flow,
- Weapon assignment, and
- Engagement.

The simulation is modular and object oriented in design. Objects in SAMS simulate the real-world systems which would be employed in an Anti-Air Warfare (AAW) scenario. Examples of the systems which are simulated by objects in SAMS are search radars, and missile launchers. A particular group of specific objects can then be assembled to simulate a given ship's combat system. To simulate the desired systems, information needed by the objects is provided by a data base. The data base contains three categories of information. They are: 1) combat system configurations organized by ship class, 2) parameters and functional relationships of the threat and of the shipboard combat system components, and 3) data on the performance of the defensive systems against the threats.[Ref. 2:p. A.2-22]
1. **Inputs to SAMS**

Inputs to SAMS are divided into four categories:

- those required to characterize a simulation run;
- those required to define cases within a run;
- those required for simulation control and data collection; and
- those required to override the data base.

[Ref. 2:p.A.2-25]

The inputs which characterize a simulation run include:

- launch locations of the attacking elements;
- the number of defending ships, their ship classes, and their positioning.

The inputs which define cases within a run include:

- the number and type of threat for each attacking element;
- the type of engagement coordination used by the defending force, (such as earliest intercept with load sharing); and
- the firing doctrine for the defending ships.

Inputs for the control of the simulation include the number of iterations to be run for each case, the total amount of computer time dedicated to the run, which data to gather during the run, and the format to be used for output.
After executing the desired number of iterations of a specific case, SAMS will continue the simulation run until all defined cases have been completed.

Information in the data base can be overridden for a simulation run or for any case within a run. It is useful to override the data base for parametric studies of defensive capabilities. For instance, a particular coordination scheme may be better suited against targets which fly slower, while another scheme may be better against faster flying targets. Thus, overrides of the data base can be input to vary target speeds in order to explore such capability differences between schemes.

2. Output

The SAMS simulation output can be tailored to the analysis at hand. This enables flexibility in the choice of data to collect. Some examples of the data which can be collected include the numbers and timing of targets which penetrate the defense (penetrators), the number of targets killed (kills), the number of weapons used by the defender during the engagement (firepower), range distributions of target engagement and target kill. [Ref. 2:p. A.2-28] Measures of effectiveness will be discussed in greater detail in the next chapter.
B. DIFFICULTY IN IMPLEMENTATION

Neither the earliest intercept nor the load sharing modification have yet been implemented in SAMS. The difficulty which has been encountered at The Johns Hopkins University Applied Physics Laboratory has been related to engagement scheduling for an entire class of "bidding" schemes. A bidding scheme is one which allows each ship to make independent bids to engage each attacker, and both of the coordination schemes considered here are bidding schemes. For instance, the bid in the earliest intercept time scheme is the computed time of intercept for each ship who finds a particular attacker engageable. The ship with the earliest intercept time "wins" the bid. The load sharing modification also uses a bid, where the bid is a ratio of intercept time and missile inventory.

A problem with such a concept arises from the fact that, because of location or any number of factors, different ships will detect attackers at different times. For instance, if one ship detects an attacker and makes a bid on it before other ships make the detection, then there is no chance for comparison of bids before an assignment is made. Perhaps only seconds after the first ship wins that uncontested bid, another ship, which could achieve an earlier intercept time for that attacker, makes the detection but is not assigned.
The difficulty has delayed implementation of bidding schemes in the simulation. Though the problems are being solved, the delay has precluded the actual analysis of data on the performance of these schemes in this thesis. As a result, only an exploration of potential measures of effectiveness and a structure for comparison will be presented. It is hoped that data will be available from SAMS runs of these schemes in late 1991.
IV. MEASURES OF EFFECTIVENESS FOR AAW COORDINATION SCHEMES

As discussed in Chapter III, SAMS output can be tailored to support the measures of effectiveness of interest. There are a variety of measures of effectiveness that can be used to compare two anti-air warfare coordination schemes. The choice of specific measures of effectiveness to be used in this type of analysis is the subject of continuing discussion among members of the Force Threat Evaluation and Weapon Assignment (FTEWA) working group in the Naval Warfare Analysis and Naval Ship Systems Departments of The Johns Hopkins University Applied Physics Laboratory.[Ref. 3] In this chapter various measures of effectiveness which appear relevant to the goals of coordination in Anti-Air Warfare and recognized by the FTEWA working group will be presented and discussed.

Two primary goals of coordinating the ships of a force in an Anti-Air Warfare scenario are: 1) to improve the force's ability to survive attack, and 2) to improve the ability of the force to sustain that survivability for as long as possible. Measures of survivability and sustainability will be the primary focus of this study.
A. MEASURES RELATING TO SURVIVABILITY

Survivability can be quantified by measures relating to the number and characterization of the attackers which are not destroyed by missile engagements, the number of opportunities to fire at the attackers, and the distance at which the attackers are killed. Data for the following measures of effectiveness can be collected during simulation runs by SAMS:

- The number of penetrators,
- The number of free-riders,
- The number of kills prior to the first penetrator,
- Depth of fire, and
- Ranges of kills.

Each of these five measures will be discussed individually.

A basic measure of effectiveness relating to survivability of the force is a count of penetrators. Penetrators are attackers (targets which are inbound to one or more units of the force) which are not destroyed by missile engagements. (A missile engagement is the physical act of firing a missile at an attacker.) Penetrators are a subset of the overall number of attackers, and may or may not have actually been engaged. They are distinguished only by the fact that they were not destroyed by area defense missiles.
The proportion of attackers which are penetrators can be indicative of the performance of a coordination scheme. During an attack, each penetrator must be dealt with by the secondary, or a ship's self-defense, weapons. The role and effectiveness of such weapons is not the intention of this study and will not be discussed here. Intuitively however, a reduction of the number of penetrators, which are attackers that challenge these defenses, also reduces the number of possible hits made on the force during the attack. Such a reduction in the number of potential hits suggests greater survivability. Thus, a scheme which allows a smaller proportion of penetrators than a competing scheme is desired.

A subset of penetrators is the set of attackers which satisfy the above definition of penetrators, but are not engaged by any area defense missiles. A count of these attackers, called free riders, will also be used in this analysis. The proportion of free riders to penetrators has potential to point out weaknesses in a coordination scheme.

Because a free rider is a penetrator which was never engaged by area defense missiles, it is important to consider why that penetrator was not engaged. In structuring the simulation, it is possible to present unengageable attackers to the defending force. Attackers can be unengageable for a variety of reasons. Some attacker flight profiles could exceed individual ship combat system
capabilities, thereby making the attacker unengageable. The intensity of the attack could be such that the individual ship combat systems become overwhelmed regardless of the coordination scheme used, leaving some of the attackers unengageable. It is also possible that the coordination scheme used does not efficiently assign specific ships to specific attackers. As a result, the proportion of free riders increases as heavily burdened ships become overwhelmed.

The combat ability of each individual ship's primary missile system is not of direct concern in this study. The efficient coordination of such systems, on the other hand, is the concern. The desire then is to structure the simulation to have no free riders which occur as a result of the attackers exceeding combat system capability. This can be accomplished by using attackers which would individually be considered engageable. The number and direction of the attackers can then be varied in order to expose the strengths and weaknesses of the coordination schemes.

The relationships between attackers (A), penetrators (P), free riders (F), attackers which are engaged (E), and attackers which are destroyed (D), are shown in Figure 3.
Notice that \((E - D)\) represents the set of attackers which were engaged but not destroyed, and \((P - F)\) represents the set of penetrators that were engaged. These are indeed the same set.

The third survivability measure, the number of kills prior to the first penetrator, indicates a coordination scheme's ability to prevent being saturated. A small number of kills prior to the first penetrator suggests that the coordination scheme may be easily saturated. This might occur because the scheme does not efficiently distribute the assignments of attackers to ships. Again, this would cause individual ships to become overwhelmed and unable to make all required engagements. A small number of kills prior to the first penetrator may also be due to the scheme's inability to process the assignments in time. In either
case, by keeping the attack and defense configurations constant and varying only the coordination schemes, we can use this measure to examine any potential differences in efficiency.

**Depth of fire**, the fourth measure, is a count of the number of hypothetical engagements which can be carried out against a particular undestroyed attacker, before it reaches the minimum range of area defense missiles. Each hypothetical engagement includes the assignment of a missile, or missiles, to the attacker, the firing of the missile(s), the flight of the missile(s) to the intercept point, and an evaluation period to determine whether or not the attacker was killed. During the simulation, the depth of fire for a specific attacker can be measured at each ship individually, or can be a composite, force-wide, measure. A force-wide measure is the depth of fire value of the ship with the most firing opportunities (at that specific attacker) of all the ships of the force. Keeping the type of attacker and the defending force composition constant, depth of fire is primarily affected by the speed of a scheme in assigning ships to attackers. The faster a scheme makes assignments, the more likely it is that the depth of fire will be a greater number. Because each engagement has a known (or computable) probability of kill, an increase in depth of fire indicates an increase in the probability that the attacker will be destroyed. Accordingly, greater depth
of fire is desired. In order to make use of depth of fire as a measure of effectiveness, the mean depth of fire against all attackers can be used and the simulation structured so that each attacker will have identical characteristics.

The final survivability measure, ranges of kills, is a list of the ranges from each firing ship to attackers they are engaging when the attackers are killed. Ranges at which attackers are killed can be attributed to many factors. The attacker's flight characteristics are one consideration. A high flying attacker is more likely to be killed at greater range than a low flyer because low flying attackers are not detected at as great a range as high flying attackers. Another consideration, again, is the speed that the scheme makes assignments. Greater ranges of attacker kills are desired and are suggestive of an efficient coordination scheme. As with depth of fire, ranges of kills will be averaged in order to provide a meaningful, composite, measure.

B. MEASURES RELATING TO SUSTAINABILITY

As discussed above, the ability of a force with limited resources to sustain operations for longer periods of time is another goal of a coordination scheme. An attempt to quantify sustainability could include reliability considerations of individual combat systems as well as the
logistical procedures used by the force. However, these factors will be assumed constant for the purposes of this study. Sustainability will be measured primarily by the relative efficiency with which resources (missiles) are allocated. The data for the following measures of effectiveness relating to resource allocation can be gathered during the simulation by SAMS:

- Magazine usage (missiles fired) by each ship,
- Remaining missile inventory for each ship,
- The total number of missiles fired by the force, and
- The number of redundant engagements.

Magazine usage by each ship is a measure which is used to determine whether one or more ships are expending substantially more missiles than other ships in the force. "Substantially more" is a relative term. Utilizing either the remaining missile inventory measure or the magazine usage measure, two coordination schemes might be compared by noting which scheme provides a more equal missile expenditure throughout the force.

Differing magazine usage between ships of the force is not terribly important until one or more of the ships has expended all or nearly all of its missiles. Prior to that situation, each ship retains its own capability. By retaining that capability, each ship can then support the force in whichever coordination scheme is being used.
However, the defensive capability of the force may be reduced when one or more of the ships loses full capability by exhausting its supply of missiles. Ships which are located closest to the attack, or are equipped with more capable missile systems than other ships, are more likely to expend their missile inventories faster than ships which are located farther from the attack or which are less capable. Again, this is not necessarily a bad situation unless a reduction in the defense appears because a ship has expended its entire missile inventory, while other ships in the force have gone under-utilized. Most ships will carry more than enough missiles to destroy a small number of attackers. 

Missile allocation becomes more critical in a large attack or series of small attacks when resupply is infeasible. Because it is the potential reductions in the defense which are the primary concern, a better measure of effectiveness might be the count of missiles remaining on each ship. This measure, called remaining missiles in inventory, can be used to determine whether any ships of the force reach zero missiles in inventory. It is similar to the $M_i$ value discussed in Chapter II. The measure differs only in that it is the final number of missiles in inventory at the end of the simulation run. If, after the attack(s), a ship has empty magazines while other ships retain inventories not close to zero, then a potential weakness in missile allocation may have appeared. This measure, unlike
the measure described above, does not depend on initially equal magazine inventories to be of value. This is important because, in reality, ships would likely enter battle with unequal missile inventories.

The total number of missiles fired by the force can also be used to measure the efficiency of the coordination scheme. As discussed above, the allocation of the fewest resources to defeat the attack, that is, preventing all attackers from becoming penetrators, is desired. A scheme which can defeat an attack by expending fewer missiles is more efficient than a scheme which defeats a similar attack, but requires the expenditure of more missiles. Under no circumstances, however, would the allocation of fewer resources be acceptable if it caused a reduction in performance as measured by penetrators or free riders.

The final sustainability measure, the number of redundant engagements, is a count of the situations when an attacker is engaged unnecessarily. An unnecessary engagement occurs when a ship engages an attacker that has been evaluated as killed or is already engaged by another ship. It should be noted that some coordination schemes assign multiple ships to engage a single attacker. Such schemes intentionally assign redundant engagements in order to ensure high probabilities of kill. A large number of redundant engagements during an attack, however, reduces sustainability as a result of the wasted resources.
Again, the survivability and sustainability measures just described can be recorded from SAMS runs for each coordination scheme and compared for significant differences in measures of effectiveness. The structure of this evaluation will be described in the following chapter.

C. SOME CONSIDERATIONS OF NON-INDEPENDENCE

Though each of the eight measures relating to survivability and sustainability presented above are specific measures of effectiveness, many are related. This relationship, or non-independence between the measures, could allow some of the measures to be discarded without diminishing the results of the study. Eliminating some of these measures would streamline the data analysis effort after running the simulation. Some potential relationships between the measures of effectiveness relating to survivability will now be discussed.

Free riders are related to penetrators by definition. Because free riders are a subset of penetrators, the number of free riders cannot exceed the number of penetrators. An increase or decrease in the number of penetrators could be caused by a like change in the number of free riders. However, the proportion of penetrators which are free riders would not be expected to remain constant. Penetrators which are not free riders could increase, for instance, if the probability of kill ($P_k$) of a missile against a particular
type of attacker was low. On the other hand, the proportion of free riders could increase if a sufficiently large number of attackers were to overwhelm a coordination scheme. The principal reason to keep both the penetrator and free rider measures would be to investigate the characterization of attackers which eluded area defense missiles. This would be accomplished using the proportion of penetrators which were free riders. The free rider measure could probably be discarded in a study seeking only to know which coordination scheme is "better".

The number of kills prior to the first penetrator is less likely to be related to the number of penetrators than free riders. Though its name implies that it is related to the penetrator measure, the number of kills prior to the first penetrator is not as concerned with the number of penetrators as it is with the potential interarrival times of the penetrators. Interarrival times between penetrators, if there are any penetrators at all, are potentially critical to the contribution a coordination scheme makes to the survivability of the force (a short interarrival time, or small number of kills prior to a penetrator, translates into greater strain on other defenses). It is not likely, then, that this measure would be discarded.

The two other measures which have an obvious potential relationship are depth of fire and ranges of kills. If most kills are occurring at great range, then it is likely that
depth of fire is also great. In other words, the greater the average distance that targets are being killed, the more likely there would be opportunity to shoot at them again if required. Ranges of kills are dependent upon the probability of kill (Pk) of each shot. Such probability is not the concern of this study and, consequently, probability of kill for each shot could be kept constant for simulation purposes. By using a constant Pk for each engagement, the ranges of kills measure would likely be as valuable as the depth of fire measure, and can be observed from simulation runs, where as depth of fire must be computed. If ranges of kills and depth of fire were closely related, then it would be possible to omit depth of fire as a measure.

The potential relationships of the measures relating to sustainability are also of interest. The total number of missiles fired by the force is the sum of the number fired by each force unit. There is little need to check for the relationship between these measures. The total number was included as a summary measure of the efficiency of the force against an attack. However, as discussed above, a better measure concerning efficient missile allocation was the missiles remaining measure. Accordingly, by using the missiles remaining in inventory measure, it would be possible to omit the total number of missiles fired and magazine usage by each ship measures.
The number of redundant engagements is also very likely to correspond with the number of missiles fired by each ship and the total missiles fired measures. By keeping attack size constant, the number of redundant engagements can easily be deduced from either the missile usage measure or the missiles remaining measure.

By checking for these relationships between measures of effectiveness, it is possible to reduce the number of measures from the original eight to three or four potentially independent measures. The measures which appear to have the least redundancy which relate to survivability are:

- The number of penetrators,
- The number of kills prior to the first penetrator, and
- Ranges of kills.

The most promising measure relating to sustainability is:

- Remaining missiles in inventory for each ship.

This reduction in the number of measures to consider would greatly reduce the amount of effort required to set up simulation runs and process the data after the runs. Checking for independence of the measures could be accomplished by making a test run of the simulation and compiling a covariance matrix of the MOE values. Those
measures which appear closely correlated could then be discarded as described above.
V. STRUCTURE OF SIMULATION RUNS TO COMPARE THE AAW COORDINATION SCHEMES

The chapters thus far have presented and discussed two coordination schemes, a simulation program likely to test the schemes, and some measures of effectiveness which relate to survivability and sustainability. This chapter will present a plan for simulation runs to compare the earliest intercept coordination scheme with the earliest intercept scheme modified to include a load sharing feature.

A. BASIS FOR COMPARISON

There are two primary interests for a comparison between these schemes. They are to determine whether the load sharing feature: 1) increases sustainability, and 2) decreases survivability. The load sharing feature is intended to increase sustainability. Accordingly, sustainability must be increased in order to consider load sharing a success. As discussed in Chapter IV, however, sustainability should not be increased at the expense of survivability. Therefore, regardless of performance relating to sustainability, the load sharing scheme should not be considered successful if it causes a significant decrease in survivability.
The evaluation can be composed of simulation runs of various scenarios. For each scenario, like measures of effectiveness for the schemes can then be compared by hypothesis tests for significant statistical differences. From the results of the comparison on each measure, conclusions can then be drawn on the overall performance of the load sharing feature.

B. THE SIMULATION SCENARIOS

As discussed in Chapter IV, it is necessary to be aware of the sometimes sensitive nature of each of the measures of effectiveness. Care must be used to ensure that the coordination schemes alone are being tested. For example, presenting attackers in the simulation which are individually unengageable to the defending force would provide little insight into the performance of the coordination schemes. The scenarios should also be simple in nature. The simplicity will aid in the isolation of causes and effects regarding the performance of the schemes. The simulation runs to compare the two schemes can be structured as follows. All runs would consist of three ships in a column formation. Because of the large relative speed difference, between ships and modern aircraft, the ships can be fixed in position throughout the simulation. The lead ship and the trailing ship of the column will be designated as the missile firing ships. These ships will
defend themselves, each other, and the third ship (in the center of the column). Initial spacing between ships will be 5,000 yards. In order to detect whether ship spacing has any effect on scheme performance, spacing will be varied in successive runs to 2,500 yards and 10,000 yards respectively.

The attack direction will be varied in order to determine whether or not attack direction changes the performance of the coordination schemes. Attacks will be made against the column of ships (the force) from one of three possible directions. One attack direction should be from 000 degrees relative from the center ship, another from 045 degrees relative, and the third from 090 degrees relative. The 000 and 090 directions will test the extreme cases for the load sharing feature as discussed in Chapter II. Attack sizes can also be varied. Attack sizes of 15, 30, and 60 attackers should provide enough stress on the schemes to uncover potential limitations in the schemes. Figure 4 illustrates attack directions and ship spacing. Coordination scheme performance might also be affected by the type of attackers against which it must perform. The types of attackers can be generic in nature and will be described by their flight profiles. They are defined as follows: Type 1) low and slow, Type 2) low and fast, and Type 3) high diver (fast). The actual speed and altitude profiles can be made as specific as the security
classification of the study will allow. Each attack should consist of like attackers, and each attacker, if alone, should be engageable to the force. Again, engageability is a function of missile system capability versus attacker flight characteristics.

The following table is a summary of the variable scenario inputs which have been discussed.

Table I  SCENARIO INPUTS

<table>
<thead>
<tr>
<th>Ship spacing:</th>
<th>2,500yds.</th>
<th>5,000yds.</th>
<th>10,000yds.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attack directions:</td>
<td>000</td>
<td>045</td>
<td>090 deg R</td>
</tr>
<tr>
<td>Number of attackers:</td>
<td>15</td>
<td>30</td>
<td>60</td>
</tr>
<tr>
<td>Attacker types:</td>
<td>Type 1</td>
<td>Type 2</td>
<td>Type 3</td>
</tr>
</tbody>
</table>
There are three cases for each of the four factors being varied. A complete set of data for all combinations would require $3^4$, or 81, different runs of the simulation for each scheme.

For each set of conditions, repeated simulation of attacks would yield differing results since the simulation randomizes the occurrence of such variables as attacker arrivals, targeted ships, and range of detection. Replications made during each run permit the computation of mean and variance values for each measure. Replication also allows an appeal to the Central Limit Theorem for use of test statistics which approximate normality for sample means. A run for each combination of the scenario inputs described above (for each coordination scheme) would provide sufficient data to go on to the comparison of the schemes.

C. HYPOTHESIS TESTING

The hypothesis test on each measure of effectiveness will consist of a null hypothesis and a two-tailed alternate hypothesis. Tests for equality of means, proportions and variance can be done on selected measures.

1. How the Measures can be Tested

The measures described in Chapter IV are related to either survivability or sustainability. It is desired to know whether ad sharing increases sustainability without
decreasing survivability. The tests must then be constructed to provide this information.

**a. Measures Relating to Survivability**

As discussed earlier, we are interested in any potential decrease in survivability resulting from load sharing. A potential decrease in survivability could be indicated by an increase in the numbers of penetrators or free-riders. Accordingly, we can test for any such increase in these measures when the load sharing scheme is used.

The mean number of penetrators for each scheme can be compared for equality. The null hypothesis, $H_0$, for all tests for equality of means discussed in this section, will state $\mu_1 = \mu_2$. The alternate hypothesis, $H_1$, will state $\mu_1 \neq \mu_2$. The mean number of free-riders can also be compared between the schemes in a like manner. However, the free-rider measure is closely related to the penetrator measure and may be omitted.

In general it is expected that the number of penetrators will be small. It then may be difficult to distinguish any differences between the schemes for this measure. As discussed in Chapter IV, it could be beneficial to test the proportion of attackers which are penetrators. Because of the relationship between penetrators and attackers, more information is captured in the proportion. A test on a proportion, similar to the test on the mean,
would look for an increase in the proportion of attackers which are penetrators between the schemes.

Another potential decrease in survivability could be indicated by an decrease in either the mean number of kills prior to the first penetrator measure or the mean range of kills measure. Again, by comparing each mean with its counterpart, differences in the means can be detected.

b. Measures Relating to Sustainability

The measures which relate to sustainability will also need to be tested for differences between the schemes. This is because the object of load sharing is to increase sustainability through a greater efficiency of missile allocation.

An increase in sustainability could be indicated by a decrease in the mean of the total number of missiles fired measure, a decrease in the mean of the number of missiles fired by each ship measure, or a decrease in the mean of the number of redundant engagements measure. A sustainability increase could also be indicated by an increase in the mean number of missiles remaining on each ship. All of these measures would not be required to test for a potential reduction in missile usage. For this reason, only one or two of the measures need to be tested for equality of means. As discussed in Chapter IV, the number of missiles remaining measure seems to be the best
candidate for equality of means. It may be interesting, however, to also include the total missiles fired measure in a test for equality of means.

A final test which may be of interest for all the survivability and sustainability measures discussed above would be a test for equal variance. The variability of the measures could prove to be insightful prior to drawing any conclusions about which scheme performs better. A scheme which performs slightly better than the other in a test on the mean may have more variability. That extra variability may not be desirable.

Tests for equality of means, proportions and variance will now be described.

2. Testing for Equality of Means

One of the three tests is a test for equality of means. The null hypothesis will state that the mean for all replications of that measure corresponding to one coordination scheme is equal to the mean for that measure corresponding to the other scheme, \( \mu_1 = \mu_2 \). The two way alternate hypothesis will state that the mean values of that measure are not equal, \( \mu_1 \neq \mu_2 \).

When testing a hypothesis for the difference of two means where \( \sigma_1 \) and \( \sigma_2 \) are unknown but assumed equal, a t-test (using the t statistic) is called for. In this case, standard deviation of the data is not known and, in fact,
may not be equal between the schemes. The uncertainty about this potential inequality could lead to first testing for the equality of \( \sigma_1 \) and \( \sigma_2 \). This is not necessary, however, because a special test can be used. The test, called the Aspin-Welch test, is a modification of the t-test. It treats

\[
t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \tag{1}
\]

as if it had a t distribution with degrees of freedom given by

\[
v = \frac{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}{\left(\frac{1}{n_1} + \frac{1}{n_2}\right) - 2} \tag{2}
\]

[Ref. 4:pp. 616-618].

In this case, \( n_1 \) and \( n_2 \) represent the number of replications for each run of the simulation. This simplifies the equations because \( n_1 = n_2 = n \).

Unfortunately, even given a value for \( \alpha \), a critical region cannot be computed in advance because the formula for degrees of freedom uses the observed variance.
3. Testing for the Equality of Proportions

If it is desired to test for equality of the proportions between the two schemes, a common proportion can be estimated by

$$\hat{p} = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}.$$  \hspace{1cm} (3)

which is a weighted average of the sample proportions. As stated before however, \(n_1 = n_2 = n\), so the common proportion can be simplified to

$$\hat{p} = \frac{1}{2}(p_1 + p_2).$$  \hspace{1cm} (4)

This can then be used to estimate the standard deviation of the difference between the proportions. The estimate is computed as

$$\sqrt{\frac{2(p)(1-p)}{n}}.$$  \hspace{1cm} (5)

With a chance of type I error, \(\alpha = 0.05\), then the two tailed test will reject the null hypothesis, \(p_1 = p_2\), if

$$\left| \frac{p_1 - p_2}{\sqrt{\frac{2(p)(1-p)}{n}}} \right| > 1.96$$  \hspace{1cm} (6)

[Ref. 4: pp. 606-607].

45
4. Testing for Equality of Variance

The third test compares the ratio of observed variances to the F distribution.

\[
\frac{s_1^2}{s_2^2} = F_{v_1, v_2}
\]  

(7)

where \( v_1 = v_2 = n - 1 \). The critical values can then be computed for the two tailed test as \( F_{1-a/2} \) and \( F_{a/2} \). Values of the ratio which fall outside of the region bounded by these critical values indicate that the hypothesis of equal variability should be rejected. [Ref. 4: pp. 623-624]
VI. FINAL REMARKS

This chapter will discuss some final thoughts regarding the load sharing idea, measures of effectiveness, testing procedures, and recommendations for continued work.

A. HOPES FOR AND CONCERNS ABOUT THE LOAD SHARING CONCEPT

The load sharing concept, as it has been described in modifying the earliest intercept scheme, is not limited to the modification of only the earliest intercept scheme. It may be proven that earliest intercept is not effective enough to warrant the purchase of the systems necessary to implement it. If so, then load sharing can be compared in the modification of a more economically feasible scheme.

The intent of load sharing is to allow the force to fight area defense battles with sustained capability for as long as resources will permit. However, it is understood that load sharing may give up potentially crucial time and space where time and space may be the most precious of all commodities. It is for this reason that the interest arose to explore the relative value of load sharing in a coordination scheme.
B. THE CONTINUING DISCUSSION OF MEASURES OF EFFECTIVENESS

There are numerous measures of effectiveness regarding area defense in AAW. The Force Threat Evaluation and Weapons Assignment (FTEWA) working group has identified 22 primary measures.[Ref. 3]

It can be argued, as in Chapter IV, that many measures of effectiveness regarding area defense are redundant. Of the 22 measures identified by the FTEWA working group, only nine were mentioned in this thesis. Of the nine measures, four or five could easily be omitted as being redundant. It is hoped that the explanations accompanying the surviving measures is insightful.

C. SOME THOUGHTS ON TESTING PROCEDURES

The use of hypothesis testing in this study is based on a desire to determine whether one AAW coordination scheme outperforms another with regard to specific measures of effectiveness. Another method available, which would provide greater detail into the differences in performance, would be to compute confidence intervals on the difference of two means. Such confidence intervals could provide some additional measure of the degree in which the schemes differ.[Ref. 5]
D. FINAL REMARKS

It is hoped that the work presented here will be useful to those interested in selecting an area defense coordination scheme for force AAW. Recommendations for continued analysis of this subject include the use of data generated by SAMS, using the schemes and testing structure presented here, to examine any possible value in load sharing. Additionally, analysis of variance could be used to determine which inputs, such as attacker types or ship spacing, are most critical to the success of a load sharing, or any, coordination scheme.
LIST OF REFERENCES


<table>
<thead>
<tr>
<th></th>
<th>Initial Distribution List</th>
</tr>
</thead>
</table>
| 1. | Defense Technical Information Center  
    | Cameron Station  
    | Alexandria, Virginia 22304-6145                                                 |
| 2. | Library, Code 52  
    | Naval Postgraduate School  
    | Monterey, California 93943-5002                                                 |
| 3. | Dr. Glenn F. Lindsay, Code OR/  
    | Naval Postgraduate School  
    | Monterey, California 93943-5000                                                 |
| 4. | LCDR William Walsh, Code OR/Wa  
    | Naval Postgraduate School  
    | Monterey, California 93943-5000                                                 |
| 5. | Dr. Edward A. Davis  
    | Naval Warfare Analysis Department  
    | Applied Physics Laboratory  
    | Johns Hopkins Road  
    | Laurel, Maryland 20723-6099                                                     |