PARTICLE RELAXATION DISTANCES
DOWNSTREAM OF NORMAL
AND OBLIQUE SHOCKS

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A particle equation of motion is coupled with ideal, inviscid shock theory to predict the dynamic velocity bias of laser velocimetry seed within these flow regions. Dimensionless graphs of relaxation distances are presented as an aid for experimental design and data evaluation efforts.
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Introduction

Laser Velocimetry (LV) is based on measuring the velocity of 'seed' particles within a flowfield. Consequently, the particles are assumed to follow fluid streamlines and to model the flowfield velocities. For several primary flow structures, however, such as shock waves, vortices, expansion turns, and turbulent eddies, seed particles do not always model the fluid. They can be centrifuged from vortices and expansion turns, and lag behind turbulent frequencies in both phase and amplitude. In the case of shock waves, the upstream momentum of a particle may be unaffected by the shock itself, relying on drag forces immediately downstream of the shock to slow the particle to the fluid velocity.

An example of particle lag downstream of a shock wave is shown in Fig. 1. In this experiment, LV measurements were made through the boundary layer at the vertex of a Mach 6 roughened plate - compression ramp model using polydispersed silicon oil as seed material. Following the velocity histograms from the freestream towards the plate, the outer measurement stations are above the separation shock, and show the high velocity and low turbulence intensity of the freestream. Just below the shock, only the smallest seed within the continuum of particle size has relaxed to the fluid velocity, smearing the lower velocity distribution of the histogram. Continuing into the boundary layer, particles are further downstream of the shock. Consequently, there is a gradual shift from particles remaining near the upstream velocity, towards relaxing to the downstream fluid velocity. The mean velocity and turbulence intensity statistics from this data both over-estimate the velocity and turbulence intensity of the fluid.

As an aid to the experimental design process and measurement analysis of LV data, a general
solution is presented for estimating the distance which LV seed travels before relaxing to the fluid velocity behind a normal or oblique shock.

**Approach**

Analytical approaches for quantifying particle lag behind shock waves are plentiful, extending back to the late 1960's and early 70's. More recently, Nichols extended the domain of the analysis by coupling a form of the particle equation of motion derived by Maxey and Riley with the drag law of Crowe, which accounts for inertia, compressibility, heat transfer, and rarefaction effects from subsonic to hypersonic Mach numbers. The resulting equation set applied to the uniform, steady-state flow downstream of a shock wave can be written as:

\[
\frac{dv}{dt} = - \frac{3c_d}{4d} \frac{\rho_f}{\rho_p + \rho_f/2} (\overline{v-u}) \left| \overline{v-u} \right| + H \int_0^t \frac{d}{d\tau} \frac{(\overline{v-u})}{(t-\tau)^{1/2}} d\tau
\]

and

\[c_d = f(Re_p, Ms, T_p/T_f)\]

where \(v\) is the particle velocity, \(u\) is the fluid velocity, \(d\) is the particle diameter, \(f\) and \(p\) denote the fluid and particle respectively, \(R\) denotes relative conditions between the fluid and the particle, and \(H\) is the history integral coefficient.

The subsequent general solution to the equation of motion is found by numerical integration, neglecting the effects of the history integral. Although Nichols shows that the integral term can be important when the seed density is near or less than the fluid density, or when the particle relative acceleration is large compared to the particle relative velocity, the term itself is numerically unstable. Unlike the drag term, which dampens out numerical perturbations, perturbations within the numerically evaluated integral term were found to grow unbounded. Consequently, the contribution of the term was either negligible or inseparable from the accumulation of truncation and round-off errors.

This paper extends the utility of prior works by reducing the equation set to three independent similarity parameters, and then providing graphical results over a wide range of independent variables.
Results

A typical result of numerically integrated particle lag is shown in Fig. 2, which represents the velocity lag of various sized seed particles through a normal shock at the Mach 6 test conditions. In this case, the velocities of particles as a function of downstream distance from the normal shock are defined by the upstream Mach number, pressure and temperature, and the particle density and diameter. If the same coordinate system is applied to an oblique shock, the particle equation of motion becomes a coupled set for the two dimensions, and the shock angle also needs to be specified.

Although six parameters are required to define the problem, numerical solutions can be expressed in terms of the upstream normal Mach number, the particle Reynolds number, and the particle to fluid density ratio. First, note that the tangential velocity of the particle will remain constant behind the shock wave since the tangential velocity of the fluid does not change across the shock. Therefore, by orienting the axes to be normal and tangential to the oblique shock, the velocity lag normal to the shockwave is de-coupled from the constant tangential velocity. Numerical solutions are then one-dimensional with the shock angle and upstream Mach number replaced by the upstream normal Mach number.

\[ M_{tn} = M_{t} \sin(\theta) \]  \hspace{1cm} (3)

The variable set is further reduced by dimensional analysis. By using Sutherland’s viscosity law for closure and restricting the analysis to air, application of the II theorem shows that the particle lag normal to the shock can be fully described in terms of four similarity parameters. The large number of solutions required to present results over the domain of the problem are generated using 4th order Runge-Kutta-Fehlberg (RKF) numerical integration. The particle equation of motion is stiff for small particle diameters; consequently, the RKF method is ideally suited to this case since it calculates the local stiffness of the equation at each time step and adjusts the step size accordingly.

For upstream normal Mach numbers up to 6.0, upstream particle Reynolds numbers from $3.2 \times 10^4$ to $3.2 \times 10^5$, and for upstream density ratios from $10^1$ to $10^7$, the downstream normal distance at which the particles relax to within three percent of the fluid velocity can be interpolated from Figs. 3. These curves are calculated for an upstream temperature of 300 K, however, the results do not vary greatly for other temperatures. For lower temperatures the
presented graphs are conservative, over-estimating the relaxation distance from 0 to 28 percent when the temperature is decreased to 50 K. For temperatures as high as 1000 K, the distance is under-estimated by less than 5 percent.

For fixed flowfield conditions, individual curves can be interpreted as the relaxation distance for seed of a specific density over a range of particle diameters. As the diameter is decreased towards a nominal particle Reynolds number of 1.0, the curves show a corresponding decrease in particle lag. As the diameter is decreased further, rarefaction effects cause the curves to level off towards a minimum value. Consequently, decreasing the particle diameter in order to shorten the relaxation distance will not be successful in this region. On the other hand, particle lag can be still be reduced by using less dense seed, since this corresponds to dropping to a lower curve.

For a constant upstream Reynolds number and density ratio, increasing the upstream Mach number increases the initial velocity ratio behind the shock, as well increasing the downstream fluid density and viscosity. For weak shocks, The higher initial velocity ratio causes the particle lag to increase rapidly from x/d=0.0 at Mach 1.0 towards a peak value at about Mach 1.1. Increasing the Mach number further shows that the higher downstream density and viscosity become dominant, causing the particle lag to decrease nearly linearly. Therefore, for Mach numbers between 1.0 and 1.1, the relaxation distance should be interpolated between x/d=0.0 and the graphical value at Mach 1.1. Relaxation distances for Mach numbers between 1.1 and 6.0 can be found by interpolating between the three graphs.

References


4Maxey, M.R. and Riley, J.J., "Equation of Motion for a Small Rigid Sphere in a Nonuniform

LV MEASURED AXIAL VELOCITY HISTOGRAMS

$M_\infty = 6$

$Re_\infty = 73 \times 10^6 / m$

Rough Surface, 22° Ramp

Fig. 1) Measured particle lag downstream of a shock.
PARTICLE LAG DOWNSTREAM OF A NORMAL SHOCK

\[ M_1 = 6.0 \]
\[ P_1 = 6117 \, \text{Pa} \]
\[ T_1 = 63 \, \text{K} \]
\[ \rho_p = 935 \, \text{Kg/m}^3 \]

**Fig. 2**) Representative computational prediction of particle lag.
PARTICLE LOCATION FOR 3% VELOCITY BIAS

\[ M_{1n} = 1.1 \]

\[ \rho_p/\rho_1 = 10 \]

Fig. 3a) General results for an upstream normal Mach number of 1.1
PARTICLE LOCATION FOR 3% VELOCITY BIAS

\[ M_{1n} = 3.0 \]

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\[ \frac{x_n}{d} \]

\[ 10^{-4} \quad 10^{-3} \quad 10^{-2} \quad 10^{-1} \quad 1 \quad 10 \quad 10^2 \quad 10^3 \quad 10^4 \quad 10^5 \quad 10^6 \]

\[ \text{Re}_D \]

\[ 1 \times 10^7 \]

\[ 1 \times 10^6 \]

\[ 1 \times 10^5 \]

\[ 1 \times 10^4 \]

\[ 1000 \]

\[ 100 \]

\[ \rho_p/\rho_1 = 10 \]

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Fig. 3b) General results for an upstream normal Mach number of 3.0
PARTICLE LOCATION FOR 3% VELOCITY BIAS

\( M_{1n} = 6.0 \)

Fig. 3c) General results for an upstream normal Mach number of 6.0