SUSPENDED SUBSTRATE RESONATOR DESIGN

by

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December 1990

Thesis Advisor

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The open-end resonator in suspended substrate transmission line was analyzed in terms of two open-end discontinuities on a line segment. A procedure for computing the resonant frequency and constant of propagation was demonstrated using a full wave analysis. The fringing capacitance was computed using an equivalent length extension model and a transmission line circuit model. The characteristic equation was derived using Galerkin's method applied in the Fourier transform domain. The calculation has been carried out in two increasing orders of approximation, and the results compared. The discontinuity capacitance at the resonator open ends was calculated for a range of line dimensions and substrate dielectric constants. The dispersive propagation constant of suspended substrate line was also calculated by the Galerkin method.
SUSPENDED SUBSTRATE RESONATOR DESIGN

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ABSTRACT

The open-end resonator in suspended substrate transmission line was analyzed in terms of two open-end discontinuities on a line segment. A procedure for computing the resonant frequency and constant of propagation was demonstrated using a full wave analysis. The fringing capacitance was computed using an equivalent length extension model and a transmission line circuit model. The characteristic equation was derived using Galerkin's method applied in the Fourier transform domain. The calculation has been carried out in two increasing orders of approximation, and the results compared. The discontinuity capacitance at the resonator open ends was calculated for a range of line dimensions and substrate dielectric constants. The dispersive propagation constant of suspended substrate line was also calculated by the Galerkin method.
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I. INTRODUCTION

The analysis of planar transmission lines in the Fourier transform domain or spectral domain is superior to many numerical methods in the spatial domain. The analysis in the Fourier transform domain was used earlier by Yamashita and Mittra [Ref. 4] for computation of the characteristic impedance and phase velocity of a microstrip line based on a quasi-TEM approximation. A variational method was used in the Fourier transform domain to calculate the line capacitance from the assumed charge density. This is a low-frequency approximation neglecting longitudinal electric and magnetic fields supported by the strip transmission line having two dielectric media in its cross section, or inhomogeneous line.

As the operating frequency is increased, dispersion characteristics of the inhomogeneous line become important for precise designs. This requirement has led to the full wave analysis of microstrip lines, represented by the work of Denlinger [Ref. 15], who solved the integral equations using a Fourier transform technique. The solution by his method, however, strongly depends on the assumed current distributions on the strip in process of solution. To avoid this difficulty and permit systematic improvement of the solution for the current components to a desired degree of accuracy, a new method was presented by Itoh and Mittra and commonly called the Spectral Domain Approach (SDA). In SDA, Galerkin's method is used to yield a homogeneous system of equations to determine the propagation constant and the characteristic frequencies from which the equivalent circuit is derived.
In each of these methods the Fourier transform is taken along the coordinate axes in the plane of the strip. By virtue of the Fourier transform domain analysis and Galerkin's method, SDA has several features:

- Easy formulation in the form of a pair of algebraic equations.
- Variational nature in determination of the propagation constant.

The Spectral Domain Approach is applicable to the following structures:

- Most planar transmission lines such as microstrip, finline, and coplanar waveguide in multilayer configurations.
- Both open and enclosed structures.
- Slow-wave lines with lossy dielectric materials.
- Resonators of planar configurations.

An efficient formulation of the SDA is achieved in the present work by enclosing the suspended substrate in a metallic shield enclosure. In this way the Fourier transform becomes the finite Fourier transform, calculated as a summation. Singularities of the integrand can be avoided, and the truncated summations have acceptable convergence behavior. As applied here, the analysis assumes loss-free conductors and dielectric media, and also assumes infinitesimal thickness for the strip conductors. The resonant frequencies are obtained by numerically solving the characteristic equation, and equivalent open-circuit capacitances are obtained using the end-effect and the transmission-line models. The details of the analysis method will appear in Chapter II of this thesis.

In Chapter III, discussions are presented concerning the accuracy of the solution for the resonant frequencies, the propagation constants, and the end-effect at the open end of the microstrip structure by calculation of the fringing capacitance. These solutions for various orders of ap-
proximation are compared, based on the assumptions for various current distributions in x and z - directions.
II. SPECTRAL DOMAIN APPROACH FOR SHIELDED SUSPENDED SUBSTRATE RESONATOR

A. INTRODUCTION

The shielded suspended substrate resonator to be analyzed is shown in Figure 1. A rectangular strip of width w and length l is placed on the suspended substrate which is separated from the ground plane by an air gap. The sides and the top of the structure are enclosed with metallic shielding walls. Thus the entire structure is considered to be the suspended microstrip resonator located in enclosed partially filled waveguide. It is assumed that the thickness of the strip is negligible and that all the media and conductors are lossless. The shielding waveguide has dimensions 3.2 mm X 1.575 mm X 100.0 mm, which corresponds to a system in actual practice. For simplicity, the strip is assumed to be symmetrically located.

The operating frequency is chosen to be below cutoff of the shielding waveguide partially filled with substrate material, to avoid interaction with waveguide-mode resonances. From the formula in [Ref. 10] the cutoff frequency for the structure in Figure 1 is shown in Table 1.

<table>
<thead>
<tr>
<th>Dielectric constant ($\varepsilon_r$)</th>
<th>Frequency cutoff (GHZ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2 (duroid)</td>
<td>44.7574</td>
</tr>
<tr>
<td>9.8 (alumina = $Al_2O_3$)</td>
<td>43.3333</td>
</tr>
</tbody>
</table>
B. METHOD OF SOLUTION FOR RESONANT FREQUENCY

1. Field Equations

The analysis of field equations on strip transmission lines of the type considered here has been carried out by Uwano and Itoh [Ref. 2]. These authors treated the problem by applying Galerkin's method of moments in the Fourier transform domain. Their work is summarized in a set of Green's function equations as follows

\[ \tilde{Z}_{zz} \tilde{J}_z + \tilde{Z}_{zx} \tilde{J}_x = \tilde{E}_z \]  \hspace{1cm} (1a)

\[ \tilde{Z}_{xz} \tilde{J}_z + \tilde{Z}_{xx} \tilde{J}_x = \tilde{E}_x \]  \hspace{1cm} (1b)
Where the tilde over the symbols indicates the Fourier transformed quantities. Here, \( \tilde{J}_z \) and \( \tilde{J}_x \) are the transformed z and x components of the currents on the strip conductor. \( \tilde{E}_z \) and \( \tilde{E}_x \) are the components of electric field tangential to the substrate surface. \( \tilde{Z}_{zz} \), \( \tilde{Z}_{zx} \), \( \tilde{Z}_{xz} \), \( \tilde{Z}_{xx} \) are the Green’s impedance functions, and are defined as the following:

\[
\tilde{Z}_{zz} = -\frac{1}{\alpha^2 + \beta^2} \left[ \beta^2 \tilde{Z}_e + \alpha^2 \tilde{Z}_h \right] \tag{1c}
\]

\[
\tilde{Z}_{zx} = -\frac{\alpha \beta}{\alpha^2 + \beta^2} \left[ \tilde{Z}_e - \tilde{Z}_h \right] \tag{1d}
\]

\[
\tilde{Z}_{xz} = \tilde{Z}_{zx} \tag{1e}
\]

\[
\tilde{Z}_{xx} = -\frac{1}{\alpha^2 + \beta^2} \left[ \alpha^2 \tilde{Z}_e + \beta^2 \tilde{Z}_h \right] \tag{1f}
\]

\[
\tilde{Z}_e = \frac{\gamma_{y2} C_{t3} + \gamma_{y3} C_{t2}}{C_{t2} C_{t3} + C_{t1} C_{t3} \gamma_{y2} / \gamma_{y1} + C_{t1} C_{t2} \gamma_{y3} / \gamma_{y1} + \gamma_{y3} / \gamma_{y2}} \tag{1g}
\]

\[
\tilde{Z}_h = \frac{\gamma_{z2} C_{t2} + \gamma_{z3} C_{t3}}{\gamma_{z1} \gamma_{z2} C_{t1} C_{t2} + \gamma_{z1} \gamma_{z3} C_{t1} C_{t3} + \gamma_{z2} \gamma_{z3} C_{t2} C_{t3} + \gamma_{z2}^2} \tag{1h}
\]

\[
C_{t1} = \coth \gamma_{1} h \tag{1i}
\]

\[
C_{t2} = \coth \gamma_{2} t \tag{1j}
\]

\[
C_{t3} = \coth \gamma_{3} d \tag{1k}
\]

\[
\gamma_{i}^{2} = \alpha^{2} + \beta^{2} - k_{i}^{2} \tag{1l}
\]
\[ k_i^2 = \omega^2 \mu_i \varepsilon_i \]  
\( (1m) \)

\[ \gamma_{yl} = \frac{\gamma_l}{y_l} \]  
\( (1n) \)

\[ \gamma_{zl} = \frac{\gamma_l}{z_l} \]  
\( (1o) \)

\[ y_l = j \omega \varepsilon_l \]  
\( (1p) \)

\[ z_l = j \omega \mu_l \]  
\( (1q) \)

\[ \varepsilon_l = \varepsilon_r \varepsilon_\circ \]  
\( (1r) \)

\[ \mu_l = \mu_r \mu_\circ \]  
\( (1s) \)

Where subscripts \( l = 1,2,3 \) refer to the corresponding regions 1,2,3, \( \omega \) is the operating frequency, and \( \varepsilon_\circ \) and \( \mu_\circ \) are the free-space permittivity and permeability, respectively. In this thesis, the Fourier transform is carried out in the bounded region interior to the enclosed resonator. In order to examine the layered structures in Figure 1 with added bounding vertical electric and magnetic walls at \( x = \pm 1.6 mm, z = \pm 50 mm \), the finite Fourier transform should be used [Ref. 20]. There we have the two-dimensional finite Fourier transform pairs

\[ \tilde{f}(\alpha_n, y, \beta_m) = \int_{-1.6}^{1.6} \int_{-50}^{50} f(x, y, z) e^{-j\alpha_n x} e^{-j\beta_m z} dx dz \]  
\( (2a) \)
\[ f(x, y, z) = \frac{1}{4ab} \sum_{\alpha_n} \sum_{\beta_m} f(\alpha_n, y, \beta_m) e^{j\alpha_n x} e^{j\beta_m z} d\alpha_n d\beta_m \]  
\hspace{1cm} (2b)

Where \( \alpha_n \) and \( \beta_m \) are the discrete transform variables for the dominant mode and are defined by, \( \alpha_n = (n - 1/2)\pi/a \) for \( E_z \) even, \(-H_z \) odd(in x) modes, and \( \beta_m = (m - 1/2)\pi/b \) for \( E_z \) even, \(-H_z \) odd(in z) modes. Note that \( b \) is the half length of the resonator and is defined to be 5 times the length of strip, and \( a \) is the half width of the resonator. From (2b) we have

\[ f(x, y, z) = \frac{\pi^2}{(2ab)^2} \sum_{\alpha_n} \sum_{\beta_m} \tilde{f}(\alpha_n, y, \beta_m) e^{j\alpha_n x} e^{j\beta_m z} \]  
\hspace{1cm} (2c)

Equations (1a,b) contain four unknowns \( \tilde{J}_z \), \( \tilde{J}_x \), \( \tilde{E}_z \), and \( \tilde{E}_x \). Two unknowns \( \tilde{E}_z \) and \( \tilde{E}_x \), however, can be eliminated by applying Galerkin's method in the spectral domain. The first step is to expand the unknowns \( \tilde{J}_z \) and \( \tilde{J}_x \) in terms of known basis functions \( \tilde{J}_{ji} \) and \( \tilde{J}_{jk} \)

\[ \tilde{J}_z(\alpha_n, \beta_m) = \sum_{j=1}^{N2} c_j \tilde{J}_{ji}(\alpha_n, \beta_m) \]  
\hspace{1cm} (3a)

\[ \tilde{J}_x(\alpha_n, \beta_m) = \sum_{k=1}^{N1} d_k \tilde{J}_{jk}(\alpha_n, \beta_m) \]  
\hspace{1cm} (3b)

Where \( c_j \) and \( d_k \) are unknown coefficients. The basis functions must be chosen to approximate the true but unknown distributions on the strip.
Since the current is nonzero only on the strip, therefore each basis function must be chosen so that it is nonzero only on the strip. After substituting (3) into (1a,b) and taking inner products of the resulting equations with the basis functions $\tilde{J}_n$ and $\tilde{J}_x$, for different values of $i$, this process yields the equations,

\[
\sum_{j=1}^{N1} K_{ij}^{(1,1)} c_j + \sum_{k=1}^{N2} K_{ik}^{(1,2)} d_k = 0, \quad i = 1,2,3,\ldots,N2. \quad (4a)
\]

\[
\sum_{j=1}^{N1} K_{ij}^{(2,1)} c_j + \sum_{k=1}^{N2} K_{ik}^{(2,2)} d_k = 0, \quad i = 1,2,3,\ldots,N1. \quad (4b)
\]

Where from definition of inner products associated with the Fourier transform defined by (2), equations (4) lead to a set of homogeneous equations forming the solution matrix for the $c_j$ and $d_k$. The matrix elements are

\[
K_{ij}^{(1,1)} = c_j \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \tilde{J}_z(\alpha_n,\beta_m) \tilde{Z}_{zz}(\alpha_n,\beta_m) \tilde{J}_z(\alpha_n,\beta_m) \quad (5a)
\]

\[
K_{ik}^{(1,2)} = d_k \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \tilde{J}_z(\alpha_n,\beta_m) \tilde{Z}_{zx}(\alpha_n,\beta_m) \tilde{J}_x(\alpha_n,\beta_m) \quad (5b)
\]
The right hand side of (1a,b) can be eliminated in Galerkin's process via the application of Parseval's relation, because the current $\tilde{J}_{zi}(x)$, $\tilde{J}_{xi}(x)$ and the field components $\tilde{E}_{zi}(x,D + T)$, $\tilde{E}_{xi}(x,D + T)$ are nonzero in the complementary regions of $x$. Note that the solution matrix is symmetrical. In order to have non-trivial values for $c_k$, $d_j$, the determinant of the matrix must be zero. From this we have the solution for resonant frequency. The accuracy of the solution can be systematically improved by increasing the number of basis function $(N1 + N2)$ and by solving a larger size matrix equation. However, if the first few basis functions are chosen so as to approximate the actual unknown current distribution reasonably well, the necessary size of the matrix can be held small for a given accuracy of the solution, resulting in numerical efficiency. Moreover, the numbers of basis functions for $J_y(z)$ and $J_x(z)$ should be equal [Ref. 18]. Hence the choice of basis functions is important from the numerical point of view. In actual computations for the dominant mode [Ref. 1], $\tilde{J}_{zi}$ and $\tilde{J}_{xi}$ have been chosen to be product functions of the form:

$$\tilde{J}_{zi}(\alpha_n, \beta_m) = \tilde{J}_1(\alpha_n) \tilde{J}_2(\beta_m)$$ (6a)
\[ \tilde{J}_{x_1}(\alpha_n, \beta_m) = \tilde{J}_3(\alpha_n) \tilde{J}_4(\beta_m) \]  \hspace{1cm} (6b)

Note that (6a,b) are the Fourier transforms of

\[ J_{z_1}(x,z) = J_1(x)J_2(z) \]
\[ J_{x_1}(x,z) = J_3(x)J_4(z) \]

Where the functional forms of \( J_1, J_2, J_3, J_4 \), are given in Figure 2. The coordinate forms of the current distributions shown in Figure 2 are the following:

\[ J_1(x) = \frac{1}{2w} \left[ 1 + \left| \frac{x}{w} \right|^2 \right] \]  \hspace{1cm} (7a)
\[ J_2(z) = \frac{1}{l} \cos\left( \frac{\pi z}{2l} \right) \]  \hspace{1cm} (7b)
\[ J_3(x) = \frac{1}{w} \sin\left( \frac{\pi x}{w} \right) \]  \hspace{1cm} (7c)
\[ J_4(z) = \frac{z}{2l^2} \]  \hspace{1cm} (7d)

Taking the Fourier transform of (7) we get:

\[ \tilde{J}_{1}(\alpha_n) = \frac{2 \sin(\alpha_n w)}{\alpha_n w} + \frac{3}{(\alpha_n w)^2} \left[ \cos(\alpha_n w) - \frac{2 \sin(\alpha_n w)}{\alpha_n w} + \frac{2(1 - \cos(\alpha_n w))}{(\alpha_n w)^2} \right] \]  \hspace{1cm} (8a)
\[ \tilde{J}_{2}(\beta_m) = \frac{\pi \cos(\beta l)}{(\frac{\pi}{2})^2 - (\beta l)^2} \]  \hspace{1cm} (8b)
Figure 2. Forms of assumed current distributions.

\[
\tilde{J}_3(x) = \frac{1}{2w} \left[ 1 + \frac{x}{w} x \right] \]

\[
\tilde{J}_3(z) = \frac{1}{l} \cos(\frac{\pi z}{2l})
\]

\[
\tilde{J}_5(x) = \frac{1}{w} \sin(\frac{\pi x}{w})
\]

\[
\tilde{J}_4(z) = \frac{z}{2l^2}
\]

Corresponding to the structure in Figure 1, \( w \) in (7) and (8) is the half width of the strip and \( l \) is the half length of the strip.
2. Procedure of solution for resonant frequency

The field equations (1a,b)

\[ \ddot{Z}_{zz} \ddot{J}_z + \ddot{Z}_{zz} \ddot{J}_x = \ddot{E}_z \]

\[ \ddot{Z}_{xz} \ddot{J}_z + \ddot{Z}_{xz} \ddot{J}_x = \ddot{E}_x \]

were derived assuming the existence of \( J_z \), and any solution derived from (1a,b) must satisfy the equations (1a) and (1b) simultaneously. Since there are four unknowns, \( c_j, d_k, \dot{E}_z, \dot{E}_x \) in (1a,b), they can not be solved in general. We introduce Galerkin’s procedure by taking inner products of (1a) with \( \tilde{J}_{zi} \) and likewise (1b) with \( \tilde{J}_{xi} \). Then, apply the (3a,b),

\[
\sum_{i, \beta} c_j \tilde{J}_{zi} \ddot{Z}_{zz} \ddot{J}_z + d_k \sum_{i, \alpha} \tilde{J}_{xi} \ddot{Z}_{xz} \ddot{J}_x = \sum_{i, \beta} \ddot{E}_z \tilde{J}_{zi} \tag{9a}
\]

\[
\sum_{i, \beta} c_j \tilde{J}_{zi} \ddot{Z}_{xz} \ddot{J}_x + d_k \sum_{i, \alpha} \tilde{J}_{xi} \ddot{Z}_{xx} \ddot{J}_x = \sum_{i, \beta} \ddot{E}_x \tilde{J}_{xi} \tag{9b}
\]

By the virtue of Parseval’s theorem, the right sides of (9) are zero, so we have

\[
\sum_{i, \beta} c_j \tilde{J}_{zi} \ddot{Z}_{zz} \ddot{J}_z + d_k \sum_{i, \alpha} \tilde{J}_{xi} \ddot{Z}_{xz} \ddot{J}_x = 0 \tag{10a}
\]
In second-order approximation, equations (10) form the matrix solution with dimensions $[2 \times 2]$. The matrix is symmetrical. In order to have nontrivial values for $c_j$, $d_k$, the determinant of the matrix must be zero.

**a. First order approximation**

For the lowest-order approximation, here called first-order, we assume the current only in $z$-direction ($J_x = 0, i = 1, j = 1$), and choose the current distribution $J_z = J_1(x) \times J_2(z)$. Since we have current only in the $z$-direction ($\tilde{J}_z \neq 0, \tilde{J}_x = 0$), then from (10) there remains only the equation:

$$c_1 \sum_{\alpha, \beta} \tilde{J}_{z1} \tilde{Z}_{zz} \tilde{J}_{z1} = 0 \quad (11)$$

By using trial values of frequency, the solution $f_0$ (resonant frequency) is found as the value best satisfying (11)

**b. Second order approximation**

For the second order solution we assume current exists in both directions ($z$ and $x$) and choose the current distribution,

$$\tilde{J}_{z1}(\alpha_n, \beta_m) = \tilde{J}_1(\alpha_n)\tilde{J}_2(\beta_m)$$

$$\tilde{J}_{x1}(\alpha_n, \beta_m) = \tilde{J}_3(\alpha_n)\tilde{J}_4(\beta_m)$$
Since we have current in the z and x-directions \((\tilde{J}_z \neq 0, \tilde{J}_x \neq 0)\), then we have the matrix solution

\[
\sum_{\alpha, \beta} c_{\alpha} \tilde{J}_{z1} \tilde{Z}_{z\alpha} \tilde{J}_{z1} + d_{\alpha} \sum_{\alpha, \beta} \tilde{J}_{x1} \tilde{Z}_{x\alpha} \tilde{J}_{x1} = 0
\]  \hspace{1cm} (12a)

\[
\sum_{\alpha, \beta} \tilde{J}_{x1} \tilde{Z}_{x\alpha} \tilde{J}_{x1} + d_{\alpha} \sum_{\alpha, \beta} \tilde{J}_{x1} \tilde{Z}_{x\alpha} \tilde{J}_{x1} = 0
\]  \hspace{1cm} (12b)

In this case we find \(f_0\) as the frequency for which the determinant of the coefficients vanishes.

C. METHOD OF SOLUTION FOR PROPAGATION CONSTANT

The increasing use of microstrip lines at microwave frequencies has recently created considerable interest in the study of dispersion characteristics of these lines. Until quite recently, much of the work on the microstrip line was based on a TEM analysis. This analysis is employed to calculate the static capacitance of the structure from which the characteristic impedance and the propagation wavenumber are subsequently derived. However, this analysis, which is necessarily approximate, is ineffective for estimating the dispersion properties of the line at higher frequencies [Ref. 14].

1. Field equation

The analysis technique for computing dispersion characteristics of shielded transmission line of the type considered here has been carried out by Mittra and Itoh [Ref. 3]. The formulation of the problem applied
Galerkin's method of moments in the Fourier transform domain and is summarized in a set of Green's function equations like those used above.

\[
\tilde{Z}_{zz}(\alpha_n, \beta_m)\tilde{J}_z(\alpha_n) + \tilde{Z}_{zx}(\alpha_n, \beta_m)\tilde{J}_x(\alpha_n) = \tilde{E}_z(\alpha_n) \quad (13a)
\]

\[
\tilde{Z}_{xz}(\alpha_n, \beta_m)\tilde{J}_z(\alpha_n) + \tilde{Z}_{xx}(\alpha_n, \beta_m)\tilde{J}_x(\alpha_n) = \tilde{E}_x(\alpha_n) \quad (13b)
\]

Where \( \beta \) is the unknown propagation constant, and \( \alpha_n = (n - 1/2) \frac{\pi}{a} \). Coefficients \( \tilde{Z}_{zz}, \tilde{Z}_{zx}, \tilde{Z}_{xz}, \tilde{Z}_{xx} \) are the Green's impedance functions and are defined as shown in the previous section, and

\[
\tilde{J}_x(\alpha_n) = \int_{-1.6}^{1.6} \tilde{J}_x(x)e^{-j\alpha_n x} \, dx \quad (14a)
\]

\[
\tilde{J}_z(\alpha_n) = \int_{-1.6}^{1.6} \tilde{J}_z(x)e^{-j\alpha_n x} \, dx \quad (14b)
\]

\( \tilde{J}_z \) and \( \tilde{J}_x \) are the transforms of strip currents \( J_z \) and \( J_x \). Notice that \( \tilde{E}_z \) and \( \tilde{E}_x \) are unknown since the electric fields \( E_z(x,D + T) \) and \( E_x(x,D + T) \) are unknown for \( w < |x| < a \), though they are zero on the strip. The field equations in (13a,b) are similar to the field equations in (1a,b), but with current distributions assumed uniform in the z-direction and propagation constant \( \beta \) is now the unknown.
2. Procedure of solution for propagation constant

The first step is to expand the unknowns $\tilde{J}_z$ and $\tilde{J}_x$ in terms of known basis functions $\tilde{J}_{zj}$ and $\tilde{J}_{xk}$

\[
\tilde{J}_z(\alpha_n) = \sum_{j=1}^{N2} c_j \tilde{J}_{zj}(\alpha_n) \quad (15a)
\]

\[
\tilde{J}_x(\alpha_n) = \sum_{k=1}^{N1} d_k \tilde{J}_{xk}(\alpha_n) \quad (15b)
\]

Where $c_j$ and $d_k$ are unknown coefficients. The basis functions $\tilde{J}_{zj}$ and $\tilde{J}_{xk}$ must be chosen such that their inverse Fourier transforms are nonzero only on the strip $|x| < w$. After substituting (15) into (13) and taking inner products with the basis functions $\tilde{J}_{zl}$ and $\tilde{J}_{xl}$ for different values of $i$. This process yields the equation,

\[
\sum_{j=1}^{N1} \sum_{k=1}^{N2} K_{ij}^{(1,1)} c_j + \sum_{k=1}^{N2} K_{ik}^{(1,2)} d_k = 0 \quad , i = 1, 2, 3, \ldots, N2. \quad (16a)
\]

\[
\sum_{j=1}^{N1} \sum_{k=1}^{N2} K_{ij}^{(2,1)} c_j + \sum_{k=1}^{N2} K_{ik}^{(2,2)} d_k = 0 \quad , i = 1, 2, 3, \ldots, N1. \quad (16b)
\]

Where from definition of inner products associated with the Fourier transform defined by (14), the matrix elements are
The right hand side of (13) can be eliminated in Galerkin's process via the application of Parseval's relation.

Now the simultaneous equations (16) are solved for the propagation constant $\beta$ at each frequency $\omega$ by setting the determinant of the coefficient matrix equal to zero.

**a. First order approximation.**

We choose of matrix size: $N1 = 0$, $N2 = 1$. The matrix problem reduces to:

$$
c_1 \sum_{\alpha, \beta} \tilde{J}_{z1}(\alpha) \tilde{Z}_{z2}(\alpha, \beta) \tilde{J}_{z1}(\alpha) = 0
$$

(18)
b. Second order approximation.

We choose matrix of size: \( N_1 = 1 \), \( N_2 = 1 \). The matrix to be solved is

\[
\sum_{\alpha, \beta} J_{z1}(\alpha) \tilde{Z}_{z2}(\alpha, \beta) J_{z1}(\alpha) + d_1 \sum_{\alpha, \beta} J_{z1}(\alpha) \tilde{Z}_{xx}(\alpha, \beta) J_{x1}(\alpha) = 0 \quad (19a)
\]

\[
\sum_{\alpha, \beta} J_{x1}(\alpha) \tilde{Z}_{x2}(\alpha, \beta) J_{x1}(\alpha) + d_1 \sum_{\alpha, \beta} J_{x1}(\alpha) \tilde{Z}_{xx}(\alpha, \beta) J_{x1}(\alpha) = 0 \quad (19b)
\]

Typical output values for the moment-method calculation of propagation constant \( \beta \) for various frequencies are shown in Table 3 and Table 4 on page 24, Figure 6 on page 29 and Figure 7 on page 30.

D. METHOD OF SOLUTION FOR FRINGING CAPACITANCE

In this work, the resonant frequency data of the suspended substrate resonators have been used for estimating the end effect at the open ends of suspended substrate lines using the full-wave theory. In order to get the value of fringing capacitance, we use two methods, the equivalent length extension model and the transmission line circuit model.

The descriptions of each method are as follows:

1. Equivalent length extension model

From the dispersion relation the guide wavelength \( \lambda_g = 2 \frac{\pi}{\beta} \) is derived at the resonant frequency \( f_0 \) of the microstrip resonator of length \( l \). Consider the open-circuit resonator whose structure is identical to Figure 1 except that the length is \( l' \) instead of \( l \). The length \( l' \) is determined from the resonant condition of the open circuited line \( l' = \frac{\lambda_g}{2} \).
The hypothetical extension of the suspended substrate line, which accounts for the end effect, is given by $\Delta l = l' - l$. In terms of TEM model for transmission line characteristic impedance is defined by $L$ (Henry/meter) and $C_o$ (Farad/meter), and we have equations:

\[
Z_0 = \sqrt{\frac{L}{C_o}} \quad (20a)
\]

\[
\beta = \omega \sqrt{LC_o} \quad (20b)
\]

From equations (20a) and (20b), the capacitance $C_o$ is defined

\[
C_o = \frac{\beta}{Z_o \omega} \quad (21)
\]

where $C_o$ is capacitance/meter of line. The value of fringing capacitance $\Delta C$ is

\[
\Delta C = \frac{\Delta l C_o}{Z_o} = (l' - l) \frac{\beta}{Z_o \omega} \quad (22)
\]

In this work we calculate the end effect for a single open end, as shown in Figure 3 (for $\frac{\Delta L}{2}$) using the computer program in Appendix D.

2. Transmission line circuit model

The value of fringing capacitance can be calculated by using the approach of the transmission circuit model. The input impedance seen from the center of the strip resonator is calculated using formula as the following [Ref. 12]:

\[
Z_{in} = \frac{Z_c + jZ_o \tan(\beta l)}{Z_o + jZ_c \tan(\beta l)} Z_o \approx 0.0 \quad (23)
\]
Figure 3. End effect on substrate resonator

Where

\[ Z_c = \frac{1}{j\omega_0 \Delta C} \]

and \( \Delta C \) is a fringing capacitance, \( l \) is the half of strip length, \( \beta \) is the propagation constant, obtained as explain in the previous section, and \( Z_0 \) is the characteristic impedance of strip. The formula to be used to calculate characteristic impedance \( Z_0 \) is the following [Ref. 17]:

For \( 0 < W < a \)

\[ Z_0 = \frac{z_1}{\sqrt{\varepsilon_{eff}}} \]

\[ Z_1 = 60[V + R l_n(6G/W + \sqrt{1 + 4(G/W)^2})] \]
Where,

\[ V = -1.7866 - 0.2035(T/G) + 0.4750(2A/G) \]

\[ R = 1.0835 + 0.1007(T/G) - 0.09457(2A/G) \]

\[ \sqrt{\varepsilon_{\text{eff}}} = \frac{\beta c}{\omega} \]

and \( c \) is the free-space velocity of light. This formula has been confirmed by variational calculation [Ref. 19], and \( Z_0 \) is assumed here to vary only slowly with frequency. The computer program for this calculation is shown in Appendix D. In Table 2 is shown the values of \( Z_0 \), the characteristic impedance for different value of \( \varepsilon_r \) and the width of strip.

<table>
<thead>
<tr>
<th>Dielectric constant ((\varepsilon_r))</th>
<th>Width of strip (mm)</th>
<th>Impedance first order ((\beta), Z_0)</th>
<th>Impedance second order ((\beta), Z_0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2</td>
<td>0.711</td>
<td>94.635</td>
<td>94.220</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>79.185</td>
<td>78.789</td>
</tr>
<tr>
<td></td>
<td>1.25</td>
<td>68.963</td>
<td>68.596</td>
</tr>
<tr>
<td>9.8</td>
<td>0.711</td>
<td>70.651</td>
<td>70.204</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>60.533</td>
<td>59.916</td>
</tr>
<tr>
<td></td>
<td>1.25</td>
<td>53.446</td>
<td>52.732</td>
</tr>
</tbody>
</table>

The true value of fringing capacitance \( \Delta C \) can be founded by using trial values in (23) and looking for the minimum value of the input impedance \( Z_{in} \).
III. DISCUSSION OF RESONATOR COMPUTATION

A. INTRODUCTION

The calculation of the suspended stripline resonator in this work used numerical computations in a scale model of millimeter wave integrated circuits. The numerical results presented in this paper have been generated on a main frame IBM 370 using FORTRAN 77 and compiler VS 2 FORTRAN, since this compiler is 10 times faster than WATFOR 77. The solution for resonant frequencies and propagation constant was made by taking the absolute value of the determinant matrix. This approach leads to the minimum value for the system determinant rather than crossing the axis of determinant = 0. All output data together are shown in Appendix A, while the complete data values are shown in Table 3 for $\varepsilon_r = 2.2$ and Table 4 for $\varepsilon_r = 9.8$. The computation results for fringing capacitance using the transmission line circuit model agree with the computation result using the length extension model, as is shown by the very small difference between results of these two methods.

For each of these methods we can see that as the width of strip increases the resonant frequency of a resonator of given length increases as shown in Figure 4 and Figure 5. The complex dispersion behavior of this line system is shown by the decrease of $\beta$ with increasing the width of strip. Also the value of fringing capacitance goes up with width. This situation may be due to the fact that the end region will store more accumulated charge with wider lines of the strip. There will be currents flowing in the end region, corresponding to the extra charge [Ref. 6].
Table 3. THE DATA RESULTS FOR \( \varepsilon_r = 2.2 \)

<table>
<thead>
<tr>
<th>Method of solution</th>
<th>Width of strip, ( w ) (mm)</th>
<th>Resonant frequency, ( f_o ) (GHZ)</th>
<th>Propagation constant, ( \beta ) (Rad/m)</th>
<th>End-effect model, ( \Delta C_1 ) (Farad)</th>
<th>Transmission line model, ( \Delta C_2 ) (Farad)</th>
<th>Error, ( \Delta C_{12} )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>First order</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.711</td>
<td>12.9096</td>
<td>301.8001</td>
<td>0.8050E-14</td>
<td>0.8060E-14</td>
<td>1.E-17</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>13.0237</td>
<td>301.0026</td>
<td>1.0152E-14</td>
<td>1.0167E-14</td>
<td>1.E-17</td>
<td></td>
</tr>
<tr>
<td>1.25</td>
<td>13.0913</td>
<td>300.5175</td>
<td>1.2024E-14</td>
<td>1.2043E-14</td>
<td>2.E-17</td>
<td></td>
</tr>
<tr>
<td><strong>Second order</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.711</td>
<td>12.8582</td>
<td>301.9217</td>
<td>0.8038E-14</td>
<td>0.8048E-14</td>
<td>1.E-17</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>12.9668</td>
<td>301.1930</td>
<td>1.0099E-14</td>
<td>1.0114E-14</td>
<td>1.E-17</td>
<td></td>
</tr>
<tr>
<td>1.25</td>
<td>13.0331</td>
<td>300.7837</td>
<td>1.1906E-14</td>
<td>1.1923E-14</td>
<td>2.E-17</td>
<td></td>
</tr>
</tbody>
</table>

Table 4. THE DATA RESULTS FOR \( \varepsilon_r = 9.8 \)

<table>
<thead>
<tr>
<th>Method of solution</th>
<th>Width of strip, ( w ) (mm)</th>
<th>Resonant frequency, ( f_o ) (GHZ)</th>
<th>Propagation constant, ( \beta ) (Rad/m)</th>
<th>End-effect model, ( \Delta C_1 ) (Farad)</th>
<th>Transmission line model, ( \Delta C_2 ) (Farad)</th>
<th>Error, ( \Delta C_{12} )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>First order</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.711</td>
<td>9.4069</td>
<td>294.5652</td>
<td>2.3461E-14</td>
<td>2.3537E-14</td>
<td>8.E-17</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>9.7084</td>
<td>293.5173</td>
<td>2.7951E-14</td>
<td>2.8050E-14</td>
<td>10.E-17</td>
<td></td>
</tr>
<tr>
<td><strong>Second order</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.711</td>
<td>9.35427</td>
<td>294.7848</td>
<td>2.3477E-14</td>
<td>2.3551E-14</td>
<td>7.E-17</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>9.6251</td>
<td>293.9983</td>
<td>2.7820E-14</td>
<td>2.7915E-14</td>
<td>9.E-17</td>
<td></td>
</tr>
<tr>
<td>1.25</td>
<td>9.7804</td>
<td>293.6181</td>
<td>3.1695E-14</td>
<td>3.1806E-14</td>
<td>1.E-16</td>
<td></td>
</tr>
</tbody>
</table>
B. RESONANT FREQUENCY

For the first order approximation the modified summation in terms of $\alpha$ and $\beta$ to (11) is represented as

$$\sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} G(\alpha_n, \beta_m) = 0$$ (24)

Where $G(\alpha_n, \beta_m)$ is the product form of basis functions and Green's impedance function. The computer programs for first and second order approximation for $J_z$ is shown in Appendix B. These programs calculate the value of the determinant with various values of strip width in order to get the resonant frequency, with dielectric constant for substrate: $\varepsilon_r = 2.2$ for duroid, and $\varepsilon_r = 9.8$ for alumina. The solutions for first order are found:

$$\sum_{n=-30}^{+30} \sum_{m=-3000}^{+3000} J_{z1}^2(\alpha_n, \beta_m) \tilde{Z}_{zz}(\alpha_n, \beta_m) = 0$$ (25)

In the x-direction we take the sum from from $n = -30$ to $n = +30$ and in the z-direction from $m = -3000$ to $m = +3000$. Since a higher limit does not change the result as shown in Table 5.

In a similar way, we may compute the second order approximation with the same summation limits for $\alpha_n$ and for $\beta_m$. All data outputs for resonant frequency appear in Appendix A.
Figure 4. Resonant frequency versus width of strip for $\varepsilon_r = 2.2$

Table 5. THE COMPARISON FOR RESONANT FREQUENCY IN FIRST ORDER APPROXIMATION

<table>
<thead>
<tr>
<th>$\alpha_n$</th>
<th>$\beta_m$</th>
<th>$\varepsilon_r$</th>
<th>$w$</th>
<th>Resonant Frequency (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n = -30$</td>
<td>$m = -3000$</td>
<td>2.2</td>
<td>1.0</td>
<td>13.023998</td>
</tr>
<tr>
<td>$n = +30$</td>
<td>$m = +3000$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n = -40$</td>
<td>$m = -4000$</td>
<td>2.2</td>
<td>1.0</td>
<td>13.023498</td>
</tr>
<tr>
<td>$n = +40$</td>
<td>$m = +4000$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 5. Resonant frequency versus width of strip for \( \varepsilon_r = 9.8 \)
C. PROPAGATION CONSTANT

The program to be used for computation of the propagation constant is shown in Appendix C. An initial approximation is taken as the following [Ref. 7]:

\[
\beta = \frac{\omega}{c} \sqrt{\frac{\varepsilon_r + 1}{2}} \tag{26}
\]

Where \( c \) is the free-space velocity of light. To find the exact value of propagation constant, we sweep the value of \( \beta \) from 1% to 200% of (26) in order to bring the value of the matrix determinant in (18) to zero. The propagation constant can be identified by the occurrence of a minimum value of the matrix determinant. In this case we take the summation limits from \( n = -40 \) to \( n = +40 \), since a higher limit does not change the result as shown in Table 6.

<table>
<thead>
<tr>
<th>( \alpha_n )</th>
<th>( \varepsilon_r )</th>
<th>( w )</th>
<th>Propagation Constant (( \beta ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>n = -40 to n = +40</td>
<td>2.2</td>
<td>1.0</td>
<td>301.80</td>
</tr>
<tr>
<td>n = -100 to n = +100</td>
<td>2.2</td>
<td>1.0</td>
<td>301.81</td>
</tr>
</tbody>
</table>

(See in Appendix A for the complete results)
Figure 6. Resonant frequency versus propagation constant for $\varepsilon_r = 2.2$
Figure 7. Resonant frequency versus propagation constant for $\varepsilon_r = 9.8$
D. PREDICTION OF FRINGING CAPACITANCE VALUE

The value of fringing capacitance can be found by solving equation (22) for the equivalent length extension model and solving equation (23) for transmission line circuit model. The computer program used to calculate the fringing capacitance are shown in Appendix D. From the results in Table 3 and Table 4 we see that these two methods agree since there is only a small difference in their results.
Figure 8. Fringing capacitance versus width of strip for $\varepsilon_r = 2.2$
Figure 9. Fringing capacitance versus width of strip for $\varepsilon_r = 9.8$
IV. CONCLUSION

From a computational point of view, the calculation using Green's function is very effective to solve the stripline resonator problem. Actually the computed result for the resonant frequency depends on the choice of the definition of the current distributions in the x-direction and the z-direction. In the Table 3, Table 4, Figure 5 and Figure 6 it is seen that the resonant frequencies increases as the width of strip increases. We do not see this phenomenon in a resonator without suspended substrate, as shown in [Ref. 1].

The values of the fringing capacitance found here appear to be reasonable: the values are of the order of $10^{-14}$ Farad. The error between the two methods of calculation is very small, providing a confirmation of the length extension model.
APPENDIX A. OUTPUT DATA

A. RESONANT FREQUENCY

1. First order approximation

- For name \( w = 0.711 \) mm and \( \epsilon_r = 2.2 \)

<table>
<thead>
<tr>
<th>FREQUENCY (GHz)</th>
<th>DETERMINANT'S VALUES</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.909532500000</td>
<td>0.0195708908</td>
</tr>
<tr>
<td>12.909547800000</td>
<td>0.0134981312</td>
</tr>
<tr>
<td>12.909563100000</td>
<td>0.0063938200</td>
</tr>
<tr>
<td>12.909578300000</td>
<td>0.0007518493</td>
</tr>
<tr>
<td>12.909593600000</td>
<td>0.0081374571</td>
</tr>
</tbody>
</table>

- For \( w = 1.0 \) mm and \( \epsilon_r = 2.2 \)

<table>
<thead>
<tr>
<th>FREQUENCY (GHz)</th>
<th>DETERMINANT'S VALUES</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.023622500000</td>
<td>0.0549158342</td>
</tr>
<tr>
<td>13.023684500000</td>
<td>0.0306886621</td>
</tr>
<tr>
<td>13.023746500000</td>
<td>0.0076220483</td>
</tr>
<tr>
<td>13.023808500000</td>
<td>0.0151751861</td>
</tr>
<tr>
<td>13.023870500000</td>
<td>0.0382801555</td>
</tr>
</tbody>
</table>

- For \( w = 1.25 \) mm and \( \epsilon_r = 2.2 \)

<table>
<thead>
<tr>
<th>FREQUENCY (GHz)</th>
<th>DETERMINANT'S VALUES</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.090996700000</td>
<td>0.0922133327</td>
</tr>
<tr>
<td>13.091095900000</td>
<td>0.0595099851</td>
</tr>
<tr>
<td>13.091195100000</td>
<td>0.0294705480</td>
</tr>
<tr>
<td>13.091294300000</td>
<td>0.0022094622</td>
</tr>
<tr>
<td>13.091393500000</td>
<td>0.0342780240</td>
</tr>
<tr>
<td>13.091492700000</td>
<td>0.0658574104</td>
</tr>
<tr>
<td>13.091591800000</td>
<td>0.0986036658</td>
</tr>
</tbody>
</table>

- For \( w = 0.711 \) mm and \( \epsilon_r = 9.8 \)

<table>
<thead>
<tr>
<th>FREQUENCY (GHz)</th>
<th>DETERMINANT'S VALUES</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.406851964275</td>
<td>0.0055711373</td>
</tr>
<tr>
<td>9.406858630475</td>
<td>0.0026056429</td>
</tr>
<tr>
<td>9.406865296675</td>
<td>0.0003598495</td>
</tr>
<tr>
<td>9.406871962875</td>
<td>0.0033253400</td>
</tr>
<tr>
<td>9.406878629075</td>
<td>0.0062908285</td>
</tr>
</tbody>
</table>
- For $w = 1.0 \text{ mm and } \varepsilon_r = 9.8$

<table>
<thead>
<tr>
<th>FREQUENCY (GHZ)</th>
<th>DETERMINANT'S VALUES</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.708079097500</td>
<td>0.1101123587</td>
</tr>
<tr>
<td>9.708178995000</td>
<td>0.0734216453</td>
</tr>
<tr>
<td>9.708278892500</td>
<td>0.0367312736</td>
</tr>
<tr>
<td>9.708378790000</td>
<td>0.0000412434</td>
</tr>
<tr>
<td>9.708478687500</td>
<td>0.0366484453</td>
</tr>
<tr>
<td>9.708578585000</td>
<td>0.0733377923</td>
</tr>
<tr>
<td>9.708678482500</td>
<td>0.1100267979</td>
</tr>
</tbody>
</table>

- For $w = 1.25 \text{ mm and } \varepsilon_r = 9.8$

<table>
<thead>
<tr>
<th>FREQUENCY (GHZ)</th>
<th>DETERMINANT'S VALUES</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.891719662500</td>
<td>0.0013436906</td>
</tr>
<tr>
<td>9.891720994500</td>
<td>0.0009188024</td>
</tr>
<tr>
<td>9.891722326500</td>
<td>0.0004939143</td>
</tr>
<tr>
<td>9.891723658500</td>
<td>0.0000690263</td>
</tr>
<tr>
<td>9.891724990500</td>
<td>0.0003558617</td>
</tr>
<tr>
<td>9.891726322500</td>
<td>0.0007807497</td>
</tr>
<tr>
<td>9.891727654500</td>
<td>0.0012056375</td>
</tr>
</tbody>
</table>

2. Second order approximation

- For $w = 0.711 \text{ mm and } \varepsilon_r = 2.2$

<table>
<thead>
<tr>
<th>FREQUENCY (GHZ)</th>
<th>THE VALUES OF DETERMINANT</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.858237475000</td>
<td>134.7679778482</td>
</tr>
<tr>
<td>12.858239430000</td>
<td>45.1968072590</td>
</tr>
<tr>
<td>12.858241385000</td>
<td>44.3743231685</td>
</tr>
<tr>
<td>12.858243340000</td>
<td>133.9454133199</td>
</tr>
</tbody>
</table>

- For $w = 1.0 \text{ mm and } \varepsilon_r = 2.2$

<table>
<thead>
<tr>
<th>FREQUENCY (GHZ)</th>
<th>THE VALUES OF DETERMINANT</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.966809460000</td>
<td>21.9744171435</td>
</tr>
<tr>
<td>12.966809956000</td>
<td>12.5409965103</td>
</tr>
<tr>
<td>12.966810452000</td>
<td>3.1075768752</td>
</tr>
<tr>
<td>12.966810948000</td>
<td>6.3258416755</td>
</tr>
<tr>
<td>12.966811444000</td>
<td>15.7592591259</td>
</tr>
</tbody>
</table>

- For $w = 1.25 \text{ mm and } \varepsilon_r = 2.2$

<table>
<thead>
<tr>
<th>FREQUENCY(GHZ)</th>
<th>THE VALUES OF DETERMINANT</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.033089704000</td>
<td>95.3389230655</td>
</tr>
<tr>
<td>13.033102202000</td>
<td>36.4297524373</td>
</tr>
<tr>
<td>13.033114700000</td>
<td>168.1980546933</td>
</tr>
</tbody>
</table>

36
For \( w = 0.711 \text{ mm} \) and \( \epsilon_r = 9.8 \)

<table>
<thead>
<tr>
<th>FREQUENCY (GHz)</th>
<th>THE VALUES OF DETERMINANT</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.354219440000</td>
<td>838.0000000000</td>
</tr>
<tr>
<td>9.354244230000</td>
<td>369.8750000000</td>
</tr>
<tr>
<td>9.354269030000</td>
<td>96.0000000000</td>
</tr>
<tr>
<td>9.354293820000</td>
<td>605.0000000000</td>
</tr>
</tbody>
</table>

For \( w = 1.0 \text{ mm} \) and \( \epsilon_r = 9.8 \)

<table>
<thead>
<tr>
<th>FREQUENCY (GHz)</th>
<th>THE VALUES OF DETERMINANT</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.624875000000</td>
<td>2076.6096794134</td>
</tr>
<tr>
<td>9.625000000000</td>
<td>1067.1692908816</td>
</tr>
<tr>
<td>9.625125000000</td>
<td>57.7672394809</td>
</tr>
<tr>
<td>9.625250000000</td>
<td>951.5964768065</td>
</tr>
<tr>
<td>9.625375000000</td>
<td>1960.9218599920</td>
</tr>
</tbody>
</table>

For \( w = 1.25 \text{ mm} \) and \( \epsilon_r = 9.8 \)

<table>
<thead>
<tr>
<th>FREQUENCY (GHz)</th>
<th>THE VALUES OF DETERMINANT</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.780300000000</td>
<td>359.7268520610</td>
</tr>
<tr>
<td>9.780360000000</td>
<td>83.7447038466</td>
</tr>
<tr>
<td>9.780420000000</td>
<td>192.2324952445</td>
</tr>
<tr>
<td>9.780480000000</td>
<td>468.2047453591</td>
</tr>
</tbody>
</table>
### B. PROPAGATION CONSTANT

#### 1. First order approximation

- **For \( w = 0.711 \text{ mm} \) and \( \varepsilon_r = 2.2 \)**

<table>
<thead>
<tr>
<th>FREQUENCY (GHZ)</th>
<th>BETA (RAD/METER)</th>
<th>DETERMINANT</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.909578300</td>
<td>301.793213000</td>
<td>0.006551314</td>
</tr>
<tr>
<td>12.909578300</td>
<td>301.796631000</td>
<td>0.003314178</td>
</tr>
<tr>
<td>12.909578300</td>
<td>301.800049000</td>
<td>0.000016271</td>
</tr>
<tr>
<td>12.909578300</td>
<td>301.803467000</td>
<td>0.003176749</td>
</tr>
<tr>
<td>12.909578300</td>
<td>301.806885000</td>
<td>0.006300699</td>
</tr>
</tbody>
</table>

- **For \( w = 1.0 \text{ mm} \) and \( \varepsilon_r = 2.2 \)**

<table>
<thead>
<tr>
<th>FREQUENCY (GHZ)</th>
<th>BETA (RAD/METER)</th>
<th>DETERMINANT</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.023746500</td>
<td>300.933559227</td>
<td>0.043851649</td>
</tr>
<tr>
<td>13.023746500</td>
<td>300.968062043</td>
<td>0.016535467</td>
</tr>
<tr>
<td>13.023746500</td>
<td>301.002564859</td>
<td>0.010783630</td>
</tr>
<tr>
<td>13.023746500</td>
<td>301.037067675</td>
<td>0.038105642</td>
</tr>
<tr>
<td>13.023746500</td>
<td>301.071570490</td>
<td>0.065430568</td>
</tr>
</tbody>
</table>

- **For \( w = 1.25 \text{ mm} \) and \( \varepsilon_r = 2.2 \)**

<table>
<thead>
<tr>
<th>FREQUENCY (GHZ)</th>
<th>BETA (RAD/METER)</th>
<th>DETERMINANT</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.091294300</td>
<td>300.448130204</td>
<td>0.047616359</td>
</tr>
<tr>
<td>13.091294300</td>
<td>300.482811969</td>
<td>0.023635390</td>
</tr>
<tr>
<td>13.091294300</td>
<td>300.517493734</td>
<td>0.000348146</td>
</tr>
<tr>
<td>13.091294300</td>
<td>300.552175499</td>
<td>0.024334248</td>
</tr>
<tr>
<td>13.091294300</td>
<td>300.586857264</td>
<td>0.048322918</td>
</tr>
</tbody>
</table>

- **For \( w = 0.711 \text{ mm} \) and \( \varepsilon_r = 9.8 \)**

<table>
<thead>
<tr>
<th>FREQUENCY (GHZ)</th>
<th>BETA (RAD/METER)</th>
<th>DETERMINANT</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.406865000</td>
<td>294.290520504</td>
<td>0.218513360</td>
</tr>
<tr>
<td>9.406865000</td>
<td>294.427688289</td>
<td>0.122327343</td>
</tr>
<tr>
<td>9.406865000</td>
<td>294.565216074</td>
<td>0.026102604</td>
</tr>
<tr>
<td>9.406865000</td>
<td>294.702563859</td>
<td>0.070160849</td>
</tr>
<tr>
<td>9.406865000</td>
<td>294.839911643</td>
<td>0.166463009</td>
</tr>
</tbody>
</table>

- **For \( w = 1.0 \text{ mm} \) and \( \varepsilon_r = 9.8 \)**

<table>
<thead>
<tr>
<th>FREQUENCY (GHZ)</th>
<th>BETA (RAD/METER)</th>
<th>DETERMINANT</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.708379000</td>
<td>293.233772058</td>
<td>0.208247936</td>
</tr>
<tr>
<td>9.708379000</td>
<td>293.375522190</td>
<td>0.123411162</td>
</tr>
<tr>
<td>9.708379000</td>
<td>293.517272321</td>
<td>0.038539359</td>
</tr>
<tr>
<td>9.708379000</td>
<td>293.659022453</td>
<td>0.046367465</td>
</tr>
<tr>
<td>9.708379000</td>
<td>293.800772584</td>
<td>0.131309303</td>
</tr>
</tbody>
</table>
### 2. Second order approximation

- **For \( \omega = 1.25 \text{ mm and } t_r = 9.8 \)**

<table>
<thead>
<tr>
<th>FREQUENCY (GHz)</th>
<th>BETA (RAD/METER)</th>
<th>DETERMINANT</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.891724000</td>
<td>292.705621320</td>
<td>0.146004764</td>
</tr>
<tr>
<td>9.891724000</td>
<td>292.850048436</td>
<td>0.069357733</td>
</tr>
<tr>
<td>9.891724000</td>
<td>292.994475552</td>
<td>0.007321405</td>
</tr>
<tr>
<td>9.891724000</td>
<td>293.138902668</td>
<td>0.084032642</td>
</tr>
<tr>
<td>9.891724000</td>
<td>293.283329783</td>
<td>0.160775971</td>
</tr>
</tbody>
</table>

- **For \( \omega = 0.711 \text{ mm and } t_r = 9.8 \)**

<table>
<thead>
<tr>
<th>FREQUENCY (GHz)</th>
<th>BETA (RAD/METER)</th>
<th>DETERMINANT</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.354269000</td>
<td>294.739297051</td>
<td>402.7507924038</td>
</tr>
<tr>
<td>9.354269000</td>
<td>294.762060358</td>
<td>192.4514179721</td>
</tr>
</tbody>
</table>

- **For \( \omega = 1.25 \text{ mm and } t_r = 2.2 \)**

<table>
<thead>
<tr>
<th>FREQUENCY (GHz)</th>
<th>BETA (RAD/METER)</th>
<th>DETERMINANT</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.858241000</td>
<td>301.914079040</td>
<td>264.8257310433</td>
</tr>
<tr>
<td>12.858241000</td>
<td>301.91663866</td>
<td>188.2549021972</td>
</tr>
<tr>
<td>12.858241000</td>
<td>301.91918869</td>
<td>111.6834688237</td>
</tr>
<tr>
<td>12.858241000</td>
<td>301.92174352</td>
<td>35.1114309231</td>
</tr>
<tr>
<td>12.858241000</td>
<td>301.92429834</td>
<td>41.461214998</td>
</tr>
<tr>
<td>12.858241000</td>
<td>301.92685317</td>
<td>118.0344584496</td>
</tr>
<tr>
<td>12.858241000</td>
<td>301.92940799</td>
<td>194.6083099229</td>
</tr>
<tr>
<td>12.858241000</td>
<td>301.93196282</td>
<td>271.1827659171</td>
</tr>
</tbody>
</table>

- **For \( \omega = 1.0 \text{ mm and } t_r = 2.2 \)**

<table>
<thead>
<tr>
<th>FREQUENCY (GHz)</th>
<th>BETA (RAD/METER)</th>
<th>DETERMINANT</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.966810000</td>
<td>301.189557419</td>
<td>47.3499570198</td>
</tr>
<tr>
<td>12.966810000</td>
<td>301.191275018</td>
<td>25.5508907438</td>
</tr>
<tr>
<td>12.966810000</td>
<td>301.192992616</td>
<td>3.7517095664</td>
</tr>
<tr>
<td>12.966810000</td>
<td>301.194710215</td>
<td>18.0475865116</td>
</tr>
<tr>
<td>12.966810000</td>
<td>301.196427814</td>
<td>39.8469974898</td>
</tr>
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</table>

- **For \( \omega = 1.25 \text{ mm and } t_r = 2.2 \)**

<table>
<thead>
<tr>
<th>FREQUENCY (GHz)</th>
<th>BETA (RAD/METER)</th>
<th>DETERMINANT</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.033102000</td>
<td>300.776834261</td>
<td>27.3862465221</td>
</tr>
<tr>
<td>13.033102000</td>
<td>300.778560641</td>
<td>15.066490067</td>
</tr>
<tr>
<td>13.033102000</td>
<td>300.780287021</td>
<td>2.7466685987</td>
</tr>
<tr>
<td>13.033102000</td>
<td>300.782013401</td>
<td>9.5732177036</td>
</tr>
<tr>
<td>13.033102000</td>
<td>300.783739781</td>
<td>21.8931688989</td>
</tr>
</tbody>
</table>

- **For \( \omega = 0.711 \text{ mm and } t_r = 2.2 \)**

<table>
<thead>
<tr>
<th>FREQUENCY (GHz)</th>
<th>BETA (RAD/METER)</th>
<th>DETERMINANT</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.354269000</td>
<td>294.739297051</td>
<td>402.7507924038</td>
</tr>
<tr>
<td>12.354269000</td>
<td>294.762060358</td>
<td>192.4514179721</td>
</tr>
<tr>
<td>FREQUENCY (GHz)</td>
<td>BETA (RAD/METER)</td>
<td>DETERMINANT</td>
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<tr>
<td>----------------</td>
<td>------------------</td>
<td>-------------</td>
</tr>
<tr>
<td>9.625125000</td>
<td>293.904601953</td>
<td>311.5213476632</td>
</tr>
<tr>
<td>9.625125000</td>
<td>293.951446805</td>
<td>120.8819158085</td>
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<td>9.625125000</td>
<td>293.998291657</td>
<td>69.7832174509</td>
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<tr>
<td>9.625125000</td>
<td>294.045136509</td>
<td>260.4740500634</td>
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<tr>
<td>9.625125000</td>
<td>294.091981361</td>
<td>451.1905799777</td>
</tr>
</tbody>
</table>

- For \( w = 1.0 \text{ mm and } r_r = 9.8 \)

<table>
<thead>
<tr>
<th>FREQUENCY (GHz)</th>
<th>BETA (RAD/METER)</th>
<th>DETERMINANT</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.780360000</td>
<td>293.532445612</td>
<td>204.6350496775</td>
</tr>
<tr>
<td>9.780360000</td>
<td>293.561005835</td>
<td>136.5694649559</td>
</tr>
<tr>
<td>9.780360000</td>
<td>293.589566057</td>
<td>68.4983401777</td>
</tr>
<tr>
<td>9.780360000</td>
<td>293.618126279</td>
<td>0.4216756325</td>
</tr>
<tr>
<td>9.780360000</td>
<td>293.646686502</td>
<td>67.6605283907</td>
</tr>
<tr>
<td>9.780360000</td>
<td>293.675246724</td>
<td>135.7482716025</td>
</tr>
<tr>
<td>9.780360000</td>
<td>293.703806947</td>
<td>203.8415537136</td>
</tr>
</tbody>
</table>

- For \( w = 1.25 \text{ mm and } r_r = 9.8 \)
### C. FRINGING CAPACITANCE USING LENGTH EXTENSION MODEL

1. **First order approximation**

<table>
<thead>
<tr>
<th>w (mm)</th>
<th>( \epsilon_r )</th>
<th>Resonant Frequency</th>
<th>Propagation Constant</th>
<th>Fringing Capacitance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.711</td>
<td>2.2</td>
<td>12.9095780</td>
<td>301.800050</td>
<td>0.0000000000000805039</td>
</tr>
<tr>
<td>1.0</td>
<td>2.2</td>
<td>13.0237470</td>
<td>301.002570</td>
<td>0.0000000000001015215</td>
</tr>
<tr>
<td>1.25</td>
<td>2.2</td>
<td>13.0912940</td>
<td>300.517490</td>
<td>0.000000000000120243</td>
</tr>
<tr>
<td>0.711</td>
<td>9.8</td>
<td>9.4068650</td>
<td>294.565220</td>
<td>0.00000000000023461</td>
</tr>
<tr>
<td>1.0</td>
<td>9.8</td>
<td>9.7083790</td>
<td>293.517270</td>
<td>0.00000000000027951</td>
</tr>
<tr>
<td>1.25</td>
<td>9.8</td>
<td>9.8917240</td>
<td>292.994480</td>
<td>0.000000000003185787</td>
</tr>
</tbody>
</table>
2. Second order approximation

<table>
<thead>
<tr>
<th>For ( w = 0.711 \text{ mm} ) and ( \varepsilon_r = 2.2 )</th>
<th>FRINGING CAPACITANCE (FARAAD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RESONANT FREQUENCY</td>
<td>PROPAGATION CONSTANT</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>For ( w = 1.0 \text{ mm} ) and ( \varepsilon_r = 2.2 )</th>
<th>FRINGING CAPACITANCE (FARAAD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RESONANT FREQUENCY</td>
<td>PROPAGATION CONSTANT</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>For ( w = 1.25 \text{ mm} ) and ( \varepsilon_r = 2.2 )</th>
<th>FRINGING CAPACITANCE (FARAAD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RESONANT FREQUENCY</td>
<td>PROPAGATION CONSTANT</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>For ( w = 0.711 \text{ mm} ) and ( \varepsilon_r = 9.8 )</th>
<th>FRINGING CAPACITANCE (FARAAD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RESONANT FREQUENCY</td>
<td>PROPAGATION CONSTANT</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>For ( w = 1.0 \text{ mm} ) and ( \varepsilon_r = 9.8 )</th>
<th>FRINGING CAPACITANCE (FARAAD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RESONANT FREQUENCY</td>
<td>PROPAGATION CONSTANT</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>For ( w = 1.25 \text{ mm} ) and ( \varepsilon_r = 9.8 )</th>
<th>FRINGING CAPACITANCE (FARAAD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RESONANT FREQUENCY</td>
<td>PROPAGATION CONSTANT</td>
</tr>
</tbody>
</table>
D. FRINGING CAPACITANCE USING TRANSMISSION LINE CIRCUIT MODEL

1. First order approximation

- For $w = 0.711$ mm and $\varepsilon_r = 2.2$

<table>
<thead>
<tr>
<th>FREQUENCY (GHZ)</th>
<th>CAPACITANCE (FARAAD)</th>
<th>IMPEDANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.9095783</td>
<td>0.0000000000000000802</td>
<td>0.043897341449</td>
</tr>
<tr>
<td>12.9095783</td>
<td>0.0000000000000000804</td>
<td>0.029423400665</td>
</tr>
<tr>
<td>12.9095783</td>
<td>0.0000000000000000806</td>
<td>0.014949733818</td>
</tr>
<tr>
<td>12.9095783</td>
<td>0.0000000000000000808</td>
<td>0.004763408999</td>
</tr>
<tr>
<td>12.9095783</td>
<td>0.0000000000000000810</td>
<td>0.013996780989</td>
</tr>
<tr>
<td>12.9095783</td>
<td>0.0000000000000000812</td>
<td>0.028469623184</td>
</tr>
</tbody>
</table>

- For $w = 1.0$ mm and $\varepsilon_r = 2.2$

<table>
<thead>
<tr>
<th>FREQUENCY (GHZ)</th>
<th>CAPACITANCE (FARAAD)</th>
<th>IMPEDANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.0237470</td>
<td>0.000000000000000101667</td>
<td>0.000020988378</td>
</tr>
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<td>0.000030099857</td>
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</table>

- For $w = 1.25$ mm and $\varepsilon_r = 2.2$

<table>
<thead>
<tr>
<th>FREQUENCY (GHZ)</th>
<th>CAPACITANCE (FARAAD)</th>
<th>IMPEDANCE</th>
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<tbody>
<tr>
<td>13.0912940</td>
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<tr>
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<td>13.0912940</td>
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</table>

- For $w = 0.711$ mm and $\varepsilon_r = 9.8$

<table>
<thead>
<tr>
<th>FREQUENCY (GHZ)</th>
<th>CAPACITANCE (FARAAD)</th>
<th>IMPEDANCE</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
<td>9.4068650</td>
<td>0.0000000000000002353658</td>
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<td>0.000001775109</td>
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<td>9.4068650</td>
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- For $w = 1.0$ mm and $\varepsilon_r = 9.8$

43
<table>
<thead>
<tr>
<th>FREQUENCY (GHZ)</th>
<th>CAPACITANCE (FARAD)</th>
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<tbody>
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<td>0.0000000000000028040</td>
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<tr>
<td>9.7083790</td>
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</tbody>
</table>

- For \( w = 1.25 \text{ mm and } \epsilon_r = 9.8 \)
<table>
<thead>
<tr>
<th>FREQUENCY (GHZ)</th>
<th>CAPACITANCE (FARAD)</th>
<th>IMPEDANCE</th>
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<tbody>
<tr>
<td>9.8917240</td>
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<tr>
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2. Second order approximation

- For \( w = 0.711 \text{ mm and } \epsilon_r = 2.2 \)
<table>
<thead>
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<tbody>
<tr>
<td>12.8582410</td>
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<tr>
<td>12.8582410</td>
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</tr>
<tr>
<td>12.8582410</td>
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<tr>
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<td>12.8582410</td>
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</table>

- For \( w = 1.0 \text{ mm and } \epsilon_r = 2.2 \)
<table>
<thead>
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<th>FREQUENCY (GHZ)</th>
<th>CAPACITANCE (FARAD)</th>
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<tbody>
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<td>12.9668100</td>
<td>0.0000000000000010114</td>
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<tr>
<td>12.9668100</td>
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- For \( w = 1.25 \text{ mm and } \epsilon_r = 2.2 \)
<table>
<thead>
<tr>
<th>FREQUENCY (GHZ)</th>
<th>CAPACITANCE (FARAD)</th>
<th>IMPEDANCE</th>
</tr>
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<tbody>
<tr>
<td>13.0331020</td>
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<tr>
<td>13.0331020</td>
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<tr>
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<td>13.0331020</td>
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</table>
For $w = 0.711$ mm and $\varepsilon_r = 9.8$

<table>
<thead>
<tr>
<th>FREQUENCY (GHz)</th>
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<tr>
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For $w = 1.0$ mm and $\varepsilon_r = 9.8$

<table>
<thead>
<tr>
<th>FREQUENCY (GHz)</th>
<th>CAPACITANCE (FARAD)</th>
<th>IMPEDANCE</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
<td>9.6251250</td>
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</table>

For $w = 1.25$ mm and $\varepsilon_r = 9.8$

<table>
<thead>
<tr>
<th>FREQUENCY (GHz)</th>
<th>CAPACITANCE (FARAD)</th>
<th>IMPEDANCE</th>
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<tr>
<td>9.7803600</td>
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<td>0.000129946020</td>
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APPENDIX B. PROGRAMS FOR CALCULATION OF RESONANT FREQUENCY

* VARIABLE DEFINITION:
* I = REPRESENT THE REGION 1,2,3 IN RESONATOR
* ALPA = DISCRETE TRANSFORM VARIABLE FOR DOMINANT MODE (x-direction)
* BETA = DISCRETE TRANSFORM VARIABLE FOR DOMINANT MODE (z-direction)
* A = A HALF WIDTH OF RESONATOR DEVICE = 1.6 mm
* D = HEIGHT OF RESONATOR BELOW SUBSTRATE = 0.66 mm
* T = THE THICKNESS OF SUBSTRATE = 0.255 mm
* W = THE WIDTH OF STRIP, CAN BE 0.711 mm, 1.0 mm, or 1.25 mm
* H = THE HEIGHT OF RESONATOR ABOVE SUBSTRATE = 0.66 mm
* AL = THE LENGTH OF STRIPLINE , ASSUME = 10 mm
* AKI = RADIAN FREQUENCY * SQRT OF (PERMITTIVITY*PERMEABILITY)
* MO = PERMEABILITY IN VACCUM = PI*4,E-7
* MR = PERMEABILITY IN A SUBSTRATE ASSUME = 1
* EO = PERMITTIVITY IN VACCUM = (1/(36*PI))*E-9 = 8.85E-12
* ER = PERMITTIVITY IN A SUBSTRATE, CAN BE 2.2 OR 9.8
* FREQ = FREQUENCY OPERATION IN GHZ
* DE = DENOMINATOR OF ZE
* DE = NUMERATOR OF ZE
* DH1,DH2,DH = DENOMINATOR OF ZH
* NH = NUMERATOR OF ZH
* ZE = IMPEDANCE OF ELECTRIC FIELD, eqn (1g)
* ZH = IMPEDANCE OF MAGNETIC FIELD, eqn (1h)
* ZZZ,ZZX,ZXZ,AND ZXX ARE THE GREEN'S IMPEDANCE FUNCTION (1c,d,e,f)
* COT1 = CT1, EQN (1i)
* COT2 = CT2, EQN (1j)
* COT3 = CT3, EQN (1k)
* J1 = EQN (8a)
* J2 = EQN (8b)
* J1 = Jz1 EQN (6a)
* J4 = Jx1 EQN (6b)
* J21 = BASIS FUNCTION OF CURRENT COMPONENT IN Z - DIRECTION
* JX1 = BASIS FUNCTION OF CURRENT COMPONENT IN X - DIRECTION
* K11,K12,K21,K22 = MATRIX ELEMENT
* J1, J3 = ASSUME CURRENT DISTRIBUTION IN X - DIRECTION
* J2, J4 = ASSUME CURRENT DISTRIBUTION IN Z - DIRECTION
* ALL COMPUTATION IN STANDARD "MKS"
A. FIRST ORDER APPROXIMATION

* THIS IS AN EXAMPLE PROGRAM TO COMPUTE THE VALUE
* OF DETERMINANT OF GREEN FUNCTION IN FIRST ORDER
* APPROXIMATION WITH \( w = 0.711 \) mm and \( \text{ER} = 2.2 \)

* MAIN PROGRAM:

* FUNCTION: \( F1(\text{ALPA}, \text{BETA}, \text{FREQ}) = \text{ZZZ} \) * \( JZ1 \)**2

COMPLEX \( KK, K11, F1 \)
INTEGER \( N, M, L, I \)
REAL \( A, W, D, T, H, M0, E0, \text{ER, PI, MAGKK, AL, FREQ, ALPA, BETA} \)
PARAMETER (\( \text{PI} = 3.141592654 \), \( E0 = 8.85E-12 \))
PARAMETER (\( H = 0.66E-03 \), \( T = 0.255E-03 \), \( D = 0.66E-03 \))
PARAMETER (\( M0 = \text{PI} \times 4. \text{E-7} \), \( \text{AL} = 0.01 \), \( A = 1.6E-03 \))
PARAMETER (\( \text{ER} = 2.2 \), \( W = 0.711E-03 \))
* NOTE: PARAMETER \( W \) CAN BE 0.711, 1.0 OR 1.25 mm
* NOTE: PARAMETER \( \text{ER} \) CAN BE 2.2 OR 9.8

DO 15 FREQ = 12.9075, 12.9125, 3.34E-4
\( KK = \text{CMPLX} \) (0.0,0.0)

* START SUMMATION IN \( \text{ALPA} \) FROM \( \text{N} = -30 \) TO \( \text{N} = +30 \)
DO 25 L = 1, 61
\( N = L - 31 \)
\( \text{ALPA} = (N-0.5) \times \text{PI} / \text{A} \)
\( K11 = \text{CMPLX} \) (0.0,0.0)

* START SUMMATION IN \( \text{BETA} \) FROM \( \text{M} = -3000 \) TO \( \text{M} = +3000 \)
DO 35 I = 1, 6001
\( M = I - 3001 \)
\( \text{BETA} = (M-0.5) \times \text{PI} / (5. \times \text{AL}) \)
\( K11 = K11 + F1(\text{ALPA}, \text{BETA}, \text{FREQ}) \)
35 CONTINUE
\( KK = KK + K11 \)
25 CONTINUE
MAGKK = CABS((KK))
WRITE(6, 100) FREQ, MAGKK
100 FORMAT(‘’, 3X, F25.12, 3X, F35.10)
15 CONTINUE
STOP
END

* FUNCTION SUBPROGRAM F1

COMPLEX FUNCTION F1(\( \text{ALPA}, \text{BETA}, \text{FREQ} \))
COMPLEX \( \text{ZE}, \text{ZH}, \text{DE}, \text{NE}, \text{NH}, \text{COT1}, \text{COT2}, \text{COT3}, \text{GM1}, \text{GM2}, \text{GM3} \)
COMPLEX \( \text{YE}, \text{YH}, \text{DH1}, \text{DH2}, \text{DH}, \text{Z21}, \text{Z22}, \text{Z2}, \text{JZ1} \)
REAL \( G1, G2, G3, \text{AK1, AK2, AKK1, AKK2, GM1, GM2, GM3, COTT1, COTT2, COTT3} \)
REAL \( \text{YYE}, \text{YH}, \text{ALPA}, \text{BETA}, \text{FREQ, AL, H, D, T, W, A, M0, PI, ER, E0} \)
REAL \( \text{X, Y, X1, X11, X12, X13, X14, X15, X1, X21, X22, X2, JJ1} \)
PARAMETER (PI = 3.141592654, E0 = 8.85E-12)
PARAMETER (H = 0.66E-03, T = 0.255E-03, D = 0.66E-03)
PARAMETER (MO = PI*4.E-7, AL = 0.01, A = 1.6E-03)
PARAMETER (W = 0.71E-03, ER = 2.2 )
* NOTE : PARAMETER W CAN BE 0.711, 1.0 OR 1.25 mm
* NOTE : PARAMETER ER CAN BE 2.2 OR 9.8

YYE = -1. / (2.E+9*PI*FREQ*E0)
YE = CMPLX ( 0.0,YYE )
YYH = 2.E+9*PI*MO*FREQ
YH = CMPLX(0.0,YYH)
AK1 = MO*E0*((2.E+9*PI*FREQ)**2)
AK2 = MO*E0*ER*((2.E+9*PI*FREQ)**2)
AKK1 = ALPA**2 + BETA**2 - AK1
AKK2 = ALPA**2 + BETA**2 - AK2
G1 = SQRT(ABS(AKK1))
G3 = G1
IF (AKK1 .LT. 0.0) THEN
GM1 = CMPLX(0.0,G1)
GM3 = GM1
GMM1 = G1*H
GMM3 = G3*D
COTT1 = -1. / TAN(GMM1)
COTT3 = -1. / TAN(GMM3)
COT1 = CMPLX(0.0,COTT1)
COT3 = CMPLX(0.0,COTT3)
ELSE
GM1 = CMPLX(G1,0.0)
GM3 = GM1
GMM1 = G1*H
GMM3 = G3*D
COTT1 = 1. / TANH(GMM1)
COTT3 = 1. / TANH(GMM3)
COT1 = CMPLX(COTT1,0.0)
COT3 = CMPLX(COTT3,0.0)
END IF
G2 = SQRT(ABS(AKK2))
IF (AKK2 .LT. 0.0) THEN
GM2 = CMPLX ( 0.0,G2 )
GMM2 = G2*T
COTT2 = -1. / TAN(GMM2)
COT2 = CMPLX ( 0.0,COTT2 )
ELSE
GM2 = CMPLX(G2,0.0)
GMM2 = G2*T
COTT2 = 1. / TANH(GMM2)
COT2 = CMPLX ( COTT2,0.0 )
END IF
DE = COT2*COT3 + COT1*COT3*(GM2/GM1)/ER + COT1*COT2 + (GM3/GM2)*ER
NE = ((GM2/ER)*COT3 + GM3*COT2)*YE
NH = GM2*COT2 + GM3*COT3
DH1 = GM1*GM2*COT1*COT2 + GM1*GM3*COT1*COT3
DH2 = GM2*GM3*COT2*COT3 + GM2*2
DH = DH1 + DH2
ZH = (NH / DH)*YH
ZE = NE / DE

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X1 = -1./(ALPA**2 + BETA**2)
ZZZ1 = CMPLX ( X1, 0.0 )
ZZZ2 = (BETA**2)*ZE + (ALPA**2)*ZH
ZZZ = ZZZ1*ZZZ2
X = ALPA*W/2.
J11 = 2.*SIN(X)/X
J12 = 3./X**2
J13 = COS(X)
J14 = J11
J15 = (2.**(1. -COS(X)))/X**2
J1 = J11 + J12*(J13 - J14 + J15)
Y = BETA*AL / 2.
J21 = PI * COS(Y)
J22 = (PI/2.)**2 - Y**2
J2 = J21 / J22
JJ1 = J1*J2
JZ1 = CMPLX(JJ1,0.0)
F1 = ZZZ*JZ1*JZ1
RETURN
END
B. SECOND ORDER APPROXIMATION

THIS IS AN EXAMPLE PROGRAM TO COMPUTE THE VALUE OF DETERMINANT OF GREEN FUNCTION IN SECOND ORDER APPROXIMATION WITH \( W = 0.711 \) mm and \( ER = 2.2 \)

**MAIN PROGRAM:**

* * *

`F1(ALPA,BETA,FREQ) = ZZZ * JZ1 ** 2`
`F2(ALPA,BETA,FREQ) = ZZX * JZ1 * JX1`
`F3(ALPA,BETA,FREQ) = ZXZ * JZ1 * JX1`
`F4(ALPA,BETA,FREQ) = ZXX * JX1 ** 2`

COMPLEX K11,K22,K33,K44,KK1,KK2,KK3,KK4
COMPLEX KKK1,KKK2,KK
COMPLEX F1,F2,F3,F4
INTEGER M,N,I,L
REAL MAGKK,A,W,D,T,H,M0,EO,ER,PI,AL,ALPA,BETA,FREQ
PARAMETER ( PI=3.141592654, EO=8.85E-12 )
PARAMETER ( H=0.66E-03, T=0.255E-03, D=0.66E-03 )
PARAMETER ( M0=PI*4.E-7, AL=0.01, A =1.6E-03 )
PARAMETER ( ER=2.2, W = 0.711E-03 )

* NOTE: PARAMETER W CAN BE 0.711, 1.0, OR 1.25 mm
* NOTE: PARAMETER ER CAN BE 2.2 OR 9.8

DO 15 FREQ = 12.8582191,12.8582687,2.48E-6
KK1 = CMPLX( 0.0,0.0 )
KK2 = CMPLX( 0.0,0.0 )
KK3 = CMPLX( 0.0,0.0 )
KK4 = CMPLX( 0.0,0.0 )

* START SUMMATION IN ALPA FROM N=-30 TO N=+30
DO 25 L = 1, 61
N = L - 31
ALPA = (N-0.5)*PI/A
K11 = CMPLX( 0.0,0.0 )
K22 = CMPLX( 0.0,0.0 )
K33 = CMPLX( 0.0,0.0 )
K44 = CMPLX( 0.0,0.0 )

* START SUMMATION IN BETA FROM M=-3000 TO M=+3000
DO 35 I = 1, 6001
M = I - 3001
BETA = (M-0.5)*PI/(5.*AL)
K11 = K11 + F1(ALPA,BETA,FREQ)
K22 = K22 + F2(ALPA,BETA,FREQ)
K33 = K33 + F3(ALPA,BETA,FREQ)
K44 = K44 + F4(ALPA,BETA,FREQ)
35 CONTINUE
KK1 = KK1 + K11
KK2 = KK2 + K22
KK3 = KK3 + K33
KK4 = KK4 + K44
25 CONTINUE
KKK1 = KK1*KK4
KKK2 = KK2*KK3
KK = KKK1 - KKK2
MAGKK = CABS((KK))
WRITE(6,100) FREQ, MAGKK
100 FORMAT( ',2X,F16.12,3X,F20.10) 15 CONTINUE
STOP
END

* FUNCTIONS SUBPROGRAM F1, F2, F3, F4 *

COMPLEX FUNCTION F1(ALPA,BETA,FREQ)
COMPLEX ZE, ZH, DE, NE, NH, COT1, COT2, COT3
COMPLEX GM1, GM2, GM3, YE, YH, DH1, DH2, DH
COMPLEX ZZZ1, ZZZ2, ZZZ, JZ1
REAL G1, G2, G3, AK1, AK2, AKK1, AKK2, GMM1, GMM2, GMM3, COTT1, COTT2, COTT3
REAL YYE, YYH, ALPA, BETA, FREQ, AL, H, T, D, W, A, M0, PI, ER, EO
REAL X, Y, X1, J11, J12, J13, J14, J15, J11, J22, J2, JJ1
PARAMETER ( PI=3.141592654, ER=2.2, EO=8.85E-12 )
PARAMETER ( H=0.66E-03, T=0.255E-03, D=0.66E-03 )
PARAMETER ( M0=PI*4. E-7, AL=0.01 )
PARAMETER ( A=1.6E-03, W=0.711E-03 )
* NOTE : PARAMETER W CAN BE 0.711, 1.0, OR 1.25 mm
* NOTE : PARAMETER ER CAN BE 2.2 OR 9.8

YYE = -1./ (2.E+9*PI*FREQ*EO)
YE = COMPLX (0.0,YYE)
YYH = 2.E+9*PI*M0*FREQ
YH = COMPLX (0.0,YYH)
AK1 = M0*EO*((2.E+9*PI*FREQ)**2)
AK2 = M0*EO*ER*((2.E+9*PI*FREQ)**2)
AKK1 = ALPA**2 + BETA**2 - AK1
AKK2 = ALPA**2 + BETA**2 - AK2
G1 = SQRT(ABS(AKK1))
G3 = G1
IF (AKK1 .LT. 0.0) THEN
GM1 = COMPLX (0.0,G1)
GM3 = GM1
GMM1 = G1*H
GMM3 = G3*D
COTT1 = -1./TAN(GMM1)
COTT3 = -1./TAN(GMM3)
COT1 = COMPLX(0.0,COTT1)
COT3 = COMPLX(0.0,COTT3)
ELSE
GM1 = COMPLX (G1,0.0)
GM3 = GM1
GMM1 = G1*H
GMM3 = G3*D
COTT1 = 1./TANH(GMM1)
COTT3 = 1./TANH(GMM3)
COT1 = COMPLX (COTT1,0.0)
COT3 = COMPLX (COTT3,0.0)
END IF
G2=SQR(ABS(AKK2))
IF (AKK2 .LT. 0.0) THEN
GM2 = CMPLX ( 0.0, G2 )
GMM2 = G2*T
COTT2 = -1./TAN(CMM2)
COT2 = CMPLX ( 0.0, COTT2 )
ELSE
GM2 = CMPLX ( G2, 0.0 )
GMM2 = G2*T
COTT2 = 1./TANH(GMM2)
COT2 = CMPLX ( COTT2, 0.0 )
END IF
DE = COT2*COT3+COT1*COT3*(GM2/GM1)/ER+COT1*COT3*(GM3/GM2)*ER
NE = ((GM2/ER)*COT3+GM3*COT2)*YE
NH = GM2*COT2 + GM3*COT3
DH1 = GM1*GM2*COT1*COT2 + GM1*GM3*COT1*COT3
DH2 = GM2*GM3*COT2*COT3 + GM2**2
DH = DH1+DH2
ZH = (NH / DH) * YH
ZE = NE / DE
X1 = -1./ (ALPA**2 + BETA**2 )
ZZZ1 = CMPLX ( X1, 0.0 )
ZZZ2 = (BETA**2)*ZE + (ALPA**2)*ZH
ZZZ = ZZZ1*ZZZ2
X = ALPA*W/2.
Y = BETA*AL / 2.
J11 = 2.*SIN(X)/(X)
J12 = 3./ (X**2)
J13 = COS(X)
J14 = J11
J15 = (2.*(1. -COS(X))) / (X**2)
J1 = J11+J12*(J13-J14+J15)
J21 = PI*COS(Y)
J22 = (PI/2.)**2 - (Y**2)
J2 = J21/J22
JJ1 = J1*J2
JJ2 = CMPLX ( JJ1,0.0 )
F1 = ZZZ*JJ1*JJ1
RETURN
END

COMPLEX FUNCTION F2(ALPA,BETA,FREQ)
COMPLEX ZE,ZH,DE,NE,NH,COT1,COT2,COT3
COMPLEX GM1,GM2,GM3,YE,YH,DH1,DH2,DH
COMPLEX ZZX,JZ1,JX1
REAL G1,G2,G3,AK1,AK2,AKK1,AKK2,GM1,GM2,GM3,COTT1,COTT2,COTT3
REAL YYE,YYH,ALPA,BETA,FREQ,AL,H,T,D,W,A,M0,PI,ER,EO
REAL X,Y,J11,J12,J13,J14,J15,J1,J21,J22,J2,JJ1
REAL J41,J42,J4,J31,J32,J3,J4,ZZX1
PARAMETER ( PI=3.141592654, ER=2.2, EO=8.85E-12 )
PARAMETER ( H=0.666E-03, T=0.255E-03, D=0.666E-03 )
PARAMETER ( M0=PI*4.E-7, AL=0.01 )
PARAMETER ( A=0.16E-02, W=0.711E-03 )
* NOTE : PARAMETER W CAN BE 0.711, 1.0, OR 1.25 mm
* NOTE : PARAMETER ER CAN BE 2.2 OR 9.8
YYE = -1. / ((2.E+9*PI*FREQ*EO)
YE = CMPLX (0.0,YYE)
YYH = 2.E+9*PI*MO*FREQ
YH = CMPLX (0.0,YYH)
AK1 = MO*EO*((2.E+9*PI*FREQ)**2)
AK2 = MO*EO*ER*((2.E+9*PI*FREQ)**2)
AKK1 = ALPA**2 + BETA**2 - AK1
AKK2 = ALPA**2 + BETA**2 - AK2
G1 = SQRT(ABS(AKK1))
G3 = G1
IF (AKK1 .LT. 0.0) THEN
GM1 = CMPLX (0.0,G1)
GM3 = GM1
GMM1 = G1*H
GMM3 = G3*D
COTT1 = -1./TAN(GMM1)
COTT3 = -1./TAN(GMM3)
COT1 = CMPLX (COTT1,0.0)
COT3 = CMPLX (COTT3,0.0)
ELSE
GM1 = CMPLX (G1,0.0)
GM3 = GM1
GMM1 = G1*H
GMM3 = G3*D
COTT1 = 1./TANH(GMM1)
COTT3 = 1./TANH(GMM3)
COT1 = CMPLX (COTT1,0.0)
COT3 = CMPLX (COTT3,0.0)
END IF
G2=SQR1T(ABS(AKK2))
IF (AKK2 .LT. 0.0) THEN
GM2 = CMPLX (0.0,G2)
GMM2 = G2*T
COTT2 = -1./TAN(GMM2)
COT2 = CMPLX (0.0,COTT2)
ELSE
GM2 = CMPLX (G2,0.0)
GMM2 = G2*T
COTT2 = 1./TANH(GMM2)
COT2 = CMPLX (COTT2,0.0)
END IF
DE = COT2*COT3+COT1*COT3*(GM2/GM1)/ER+COT1*COT2+(GM3/GM2)*ER
NE = ((GM2/ER)*COT3+GM3*COT2)*YE
NH = GM2*COT2 + GM3*COT3
DH1 = GM1*GM2*COT1*COT2 + GM1*GM3*COT1*COT3
DH2 = GM2*GM3*COT2*COT3 + GM2**2
DH = DH1+DH2
ZH = (NH/DH)*YH
ZE = NE/DE
ZZX1 = -1.*(ALPA*BETA)/(ALPA**2+BETA**2)
ZZX = ZZX1*(ZE-ZH)
X = ALPA*W/2.
Y = BETA*AL/2.
J31 = 2.*PI*Sin(X)
J32 = X**2 - PI**2

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J3 = J31/J32
J41 = COS(Y)/(Y)
J42 = SIN(Y)/(Y**2)
J4 = J41-J42
JJ4 = J3*J4
JX1 = CMPLX( JJ4,0.0 )
J11 = 2.*SIN(X)/(X)
J12 = 3./X**2
J13 = COS(X)
J14 = 2.*SIN(X)/(X)
J15 = (2.**(1.*COS(X)))/(X**2)
J1 = J11+J12*(J13-J14+J15)
J21 = PI*COS(Y)
J22 = (PI/2)**2-Y**2
J2 = J21/J22
J11 = J1*J2
JZ1 = CMPLX (J1,0.0 )
F2 = ZZX*JX1*JZ1
RETURN
END

COMPLEX FUNCTION F3(ALPA,BETA,FREQ)

COMPLEX ZE,ZH,DE,NE,NH,COT1,COT2,COT3
COMPLEX GM1,GM2,GM3,YE,YH,DH1,DH2,DH
COMPLEX ZXZ,JZ1,JX1
REAL Gi ,G2,G3,AK1 ,AK2,AKK1,AKK2,GMM1,GMM2,GMM3,COTT1,COTT2,COTT3
PARAMETER ( PI=3.141592654, ER=2.2, EO=8.85E-12 )
PARAMETER ( H=0.66E-03, T=-0.255E-03, D=0.66E-03 )
PARAMETER ( MO=PI*4.E-7, AL=-0.01 )
PARAMETER ( A=1.6E-03, W=0.711E-03 )

* NOTE : PAREMETER W CAN BE 0.711, 1.0, OR 1.25 mm
* NOTE : PAREMETER ER CAN BE 2.2 OR 9.8

YYE = -1./(2.E+9*PI*FREQ*EO)
YE = CMPLX (0.0,YYE)
YYH = 2.6E+9*PI*M0*FREQ
YH = CMPLX (0.0,YYH)
AK1 = M0*EO*(((2.6E+9*PI*FREQ)**2)
AK2 = M0*EO*ER*(((2.6E+9*PI*FREQ)**2)
AKK1 = ALPA**2 + BETA**2 - AK1
AKK2 = ALPA**2 + BETA**2 - AK2
G1 = SQRT(ABS(AKK1))
G3 = G1
IF (AKK1 .LT. 0.0) THEN
GM1 = CMPLX (0.0,G1 )
GM3 = GM1
GMM1 = G1*H
GMM3 = G3*D
COTT1 = -1./TAN(GMM1)
COTT3 = -1./TAN(GMM3)
COTT1 = CMPLX(0.0,COTT1)
COTT3 = CMPLX(0.0,COTT3)
ELSE

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GM1 = CMPLX ( G1,0.0 )
GM3 = GM1
GMM1 = G1*H
GMM3 = G3*D
COTT1 = 1./TANH(GMM1)
COTT3 = 1./TANH(GMM3)
COT1 = CMPLX ( COTT1,0.0 )
COT3 = CMPLX ( COTT3,0.0 )
ENDIF
G2 = SQRT(ABS(AKK2))
IF (AKK2 .LT. 0.0) THEN
GM2 = CMPLX ( 0.0,G2 )
GMM2 = G2*T
COTT2 = -1./TAN(GMM2)
COT2 = CMPLX ( 0.0,COTT2 )
ELSE
GM2 = CMPLX ( G2,0.0 )
GMM2 = G2*T
COTT2 = 1./TANH(GMM2)
COT2 = CMPLX ( COTT2,0.0 )
ENDIF
DE = COT2*COT3+COT1*COT3*( GM2/GM1) /ER+COT1*COT2+( GM3/GM2)*ER
NE = (((GM2/ER)*COT3+GM3*COT2)*YE
NH = GM2*COT2 + GM3*COT3
DH1 = GM1*GM2*GM1*GM3*COT3
DH2 = GM2*GM3*COT2*GM2*COT3 + GM2**2
DH = DH1+DH2
ZH = (NH / DH) * YH
ZE = NE / DE
ZX = -(1.0*(ALPA*BETA))/(ALPA**2 + BETA**2)
ZXZ = ZX*Z
X = ALPA*W/2.
Y = BETA*AL / 2.
J11 = 2.*SIN(X)/(X)
J12 = 3./(X**2)
J13 = COS(X)
J14 = J11
J15 = (2.0*(1.0-COS(X))) / (X**2)
J1 = J11+J12*(J13-J14+J15)
J21 = PI*COS(Y)
J22 = (PI/2.)**2 - (Y**2)
J2 = J21/J22
J21 = J1*J2
J21 = CMPLX ( J1,0.0 )
J31 = 2.*PI*SIN(X)
J32 = X**2 - PI**2
J3 = J31 / J32
J41 = COS(Y)/(Y)
J42 = SIN(Y)/(Y**2)
J4 = J41-J42
J54 = J3*J4
JX1 = CMPLX ( J4,0.0 )
F3 = ZXZ*J21*JX1
RETURN
END
COMPLEX FUNCTION F4(ALPA,BETA,FREQ)

COMPLEX ZE,ZH,DE,NE,NH,COT1,COT2,COT3
COMPLEX GM1,GM2,GM3,YE,YH,DH1,DH2,DH
COMPLEX ZXX1,ZXX2,ZXX,JX1

REAL G1,G2,G3,AK1,AK2,AKK1,GMM1,GMM2,GMM3,COTT1,COTT2,COTT3
REAL YYE,YYH,ALPA,BETA,FREQ,AL,H,T,D,W,A,M0,PI,ER,E0
REAL X,Y,X1,J31,J32,J3,J41,J42,J4,JJ4

PARAMETER ( PI=3.141592654, ER=2.2, E0=8.85E-12 )
PARAMETER ( H=0.66E-03, T=0.255E-03, D=0.66E-03 )
PARAMETER ( M0=PI*4.4E-7, AL=0.01 )
PARAMETER ( A=1.6E-03, W=0.711E-03 )

NOTE : PAREMETER W CAN BE 0.711, 1.0, OR 1.25 mm
NOTE : PAREMETER ER CAN BE 2.2 OR 9.8

Y YE = -1./(2.E+9*PI*FREQ*EO)
YE = CMPLX (0.0,Y YE)
YH = 2.E+9*PI*M0*FREQ
YH = CMPLX (0.0,YH)
AK1 = MO*EO*((2.E+9*PI*FREQ)**2)
AK2 = MO*EO*ER*((2. E+9J*PI*FREQ)**2)
AKK1 = ALPA**2 + BETA**2
AKK2 = ALPA**2 + BETA**2 - AK1
AKK3 = ALPA**2 + BETA**2 - AK2
G1 = SQRT(ABS(AKK1))
G3 = G1
IF (AKK1 .LT. 0.0) THEN
GM1 = CMPLX ( 0.0,G1 )
GM3 = GM1
GMM1 = G1*H
GMM3 = G3*D
COTT1 = -1./TAN(GMM1)
COTT3 = -1./TAN(GMM3)
COT1 = CMPLX(0.0,COTT1)
COT3 = CMPLX(0.0,COTT3)
ELSE
GM1 = CMPLX ( G1,0.0 )
GM3 = GM1
GMM1 = G1*H
GMM3 = G3*D
COTT1 = 1./TANH(GMM1)
COTT3 = 1./TANH(GMM3)
COT1 = CMPLX ( COTT1,0.0 )
COT3 = CMPLX ( COTT3,0.0 )
ENDIF
G2=SQRT(ABS(AKK2))
IF (AKK2 .LT. 0.0) THEN
GM2 = CMPLX ( 0.0,G2 )
GMM2 = G2*T
COTT2 = -1./TAN(GMM2)
COT2 = CMPLX ( 0.0,COTT2 )
ELSE
GM2 = CMPLX ( G2,0.0 )
GMM2 = G2*T
COTT2 = 1./TANH(GMM2)
COT2 = CMPLX ( COTT2,0.0 )
ENDIF
DE = COT2*COT3+COT1*COT3*(GM2/GM1)/ER+COT1*COT2+(GM3/GM2)*ER
NE = ((GM2/ER)*COT3+GM3*COT2)*YE
NH = GM2*COT2 + GM3*COT3
DH1 = GM1*GM2*COT1*COT2 + GM1*GM3*COT1*COT3
DH2 = GM2*GM3*COT2*COT3 + GM2**2
DH = DH1+DH2
ZH = (NH / DH) * YH
ZE = NE / DE
X1 = -1./(ALPA**2 + BETA**2)
ZXX1 = CMPLX(X1,0.0)
ZXX2 = (ALPA**2)*ZE + (BETA**2)*ZH
ZXX = ZXX1+ZXX2
X = ALPA*W/2.
Y = BETA*AL/2.
J31 = 2.*PI*SIN(X)
J32 = X**2 - PI**2
J3 = J31 / J32
J41 = COS(Y)/(Y)
J42 = SIN(Y)/(Y**2)
J4 = J41-J42
JJ4 = J3*J4
JX1 = CMPLX(JJ4,0.0)
F4 = ZXX*JX1*JX1
RETURN
END
APPENDIX C. PROGRAMS FOR CALCULATION OF
PROPAGATION CONSTANT

VARAIBLE DEFINITION:

I = REPRESENT THE REGION 1,2,3 IN RESONATOR
ALPA = DISCRETE TRANSFORM VARIABLE FOR DOMINANT MODE (x-direction)
BETA = DISCRETE TRANSFORM VARIABLE FOR DOMINANT MODE (z-direction)
A = A HALF WIDTH OF RESONATOR DEVICE = 1.6 mm
D = HEIGHT OF RESONATOR BELOW SUBSTRATE = 0.66 mm
T = THE THICKNESS OF SUBSTRATE = 0.255 mm
W = THE WIDTH OF STRIP, CAN BE 0.711 mm, 1.0 mm, or 1.25 mm
H = THE HEIGHT OF RESONATOR ABOVE SUBSTRATE = 0.66 mm
AL = THE LENGTH OF STRIPLINE, ASSUME = 10 mm
AKI = RADIUS FREQUENCY * SQRT OF (PERMITTIVITY*PERMEABILITY)
MO = PERMEABILITY IN VACUUM = PI*4.E-7
MR = PERMEABILITY IN A SUBSTRATE ASSUME = 1
EO = PERMITTIVITY IN VACUUM = (1/(36*PI))*E-9 = 8.85E-12
ER = PERMITTIVITY IN A SUBSTRATE, CAN BE 2.2 OR 9.8
FREQ = FREQUENCY OPERATION IN GHZ
DE = DENOMINATOR OF ZE
DE = NUMERATOR OF ZE
DH1, DH2, DH = DENOMINATOR OF ZH
NH = NUMERATOR OF ZH
ZE = IMPEDANCE OF ELECTRIC FIELD, eqn (1g)
ZH = IMPEDANCE OF MAGNETIC FIELD, eqn (1h)
ZZZ, ZZX, ZXZ, AND ZXX ARE THE GREEN'S IMPEDANCE FUNCTION (1c,d,e,f)
COT1 = CT1, eqn (1i)
COT2 = CT2, eqn (1j)
COT3 = CT3, eqn (1k)
J1 = EQN (8a)
J2 = EQN (8b)
JJ1 = Jz (6a)
JJ4 = Jx (6b)
JZ1 = BASIS FUNCTION OF CURRENT COMPONENT IN Z - DIRECTION
JX1 = BASIS FUNCTION OF CURRENT COMPONENT IN X - DIRECTION
K11,K12,K21,K22 = MATRIX ELEMENT
J1, J3 = ASSUME CURRENT DISTRIBUTION IN X - DIRECTION
J2, J4 = ASSUME CURRENT DISTRIBUTION IN Z - DIRECTION
ALL COMPUTATION IN STANDARD "MKS"
A. FIRST ORDER APPROXIMATION

* This is an example program to compute constant propagation in first order approximation with \( W = 0.711 \) and \( ER = 2.2 \)

* MAIN PROGRAM :

* 

FUNCTION : \( F1(\text{ALPA}, \text{BETA}, \text{FREQ}) = \text{ZZZ} \ast \text{JZ1} \ast 2 \)

COMPLEX \( F1, K11 \)
REAL \( B0, BO0, \text{MAGKK}, \text{ALPA}, \text{BETA}, \text{MO} \)
REAL \( A, D, T, H, \pi, W, \text{FREQ}, \text{OMG}, E0 \)
INTEGER \( N, L \)
PARAMETER ( \( \pi = 3.141592654, \text{ER} = 2.2, \text{E0} = 8.85 \times 10^{-12}, H = 0.66 \times 10^{-3} \) )
PARAMETER ( \( T = 0.255 \times 10^{-3}, D = 0.66 \times 10^{-3}, \text{MO} = \pi \times 4 \times 10^{-7}, \text{A} = 0.16 \times 10^{-2} \) )

* NOTE: PARAMETER \( W \) can be 0.711 mm, 1.0 mm, 1.25 mm
* NOTE: PARAMETER \( ER \) can be 2.2 or 9.8

\( \text{FREQ} = 12.9095783 \)
\( \text{OMG} = \text{FREQ} \times \pi \times 2 \times 10^9 \)
\( B0 = (\text{OMG} / 3 \times 10^8) \times \sqrt{\frac{(\text{ER} + 1)}{2}} \)

* START SWEEP THE BETA VALUE FROM 84% TO 90%
DO 20 \( BO0 = 0.84, 0.9, 1.2 \times 10^{-3} \)
BETA = \( B0 \ast BO0 \)
K11 = .0,0,0)

* START SUMMATION IN ALPA FROM \( N = -40 \) TO \( N = +40 \)
DO 10 \( L = 1, 81 \)
N = \( L - 41 \)
ALPA = \( (N - 0.5) \ast \pi / A \)
K11 = \( K11 + F1(\text{ALPA}, \text{BETA}, \text{FREQ}) \)
CONTINUE
MAGKK = \( \text{CABS}((K11)) \)
WRITE(6,100) \( \text{FREQ}, \text{BETA}, \text{MAGKK} \)
FORMAT(' ',3X,\( \text{F14.9}, 3X, \text{F18.9}, 3X, \text{F15.9} \))
CONTINUE
STOP
END

* FUNCTION SUBPROGRAM \( F1 \)

COMPLEX FUNCTION \( F1(\text{ALPA}, \text{BETA}, \text{FREQ}) \)
COMPLEX \( \text{GM1}, \text{GM2}, \text{GM3}, \text{COT1}, \text{COT2}, \text{COT3}, \text{DE}, \text{ZE}, \text{DH}, \text{ZH}, \text{NE}, \text{NH}, \text{DH1}, \text{DH2}, \text{YE} \)
COMPLEX \( \text{JZ1}, \text{ZZZ1}, \text{ZZZ2}, \text{ZZZ}, \text{YH} \)
REAL \( A, D, T, H, \pi, \text{ER}, W, \text{FREQ}, \text{OMG}, \text{E0}, G1, G2, G3, \text{COTT1}, \text{COTT2}, \text{COTT3}, \text{YYE} \)
REAL \( \text{YYH}, \text{GMM1}, \text{GMM2}, \text{GMM3}, \text{ALPA}, \text{AKK1}, \text{AKK2}, \text{X1}, \text{X} \)
REAL \( \text{J11}, \text{J12}, \text{J13}, \text{J14}, \text{J15}, \text{J1}, \text{M0}, \text{AK1}, \text{AK2}, \text{BETA} \)
PARAMETER ( \( \pi = 3.141592654, \text{ER} = 2.2, \text{E0} = 8.85 \times 10^{-12}, H = 0.66 \times 10^{-3} \) )
PARAMETER ( \( T = 0.255 \times 10^{-3}, D = 0.66 \times 10^{-3}, \text{MO} = \pi \times 4 \times 10^{-7}, \text{A} = 0.16 \times 10^{-2} \) )
PARAMETER (W=0.711E-3)

NOTE: PARAMETER W CAN BE 0.711 MM, 1.0 MM, 1.25 MM

NOTE: PARAMETER ER CAN BE 2.2 OR 9.8

OMG = FREQ*PI*2.E+9
YVE = -1./ (OMG*EO)
YF = CMPLX (0.0,YVE)
YH = OMG*MO
YH = CMPLX (0.0,YH)
AK1 = (OMG**2)*MO*EO
AK2 = (OMG**2)*MO*EO*ER
AKK1 = ALPA**2 + BETA**2 -AK1
AKK2 = ALPA**2 + BETA**2 -AK2
G1 = SQRT(ABS(AKK1))
G3 = G1

IF (AKK1 .LT. 0.0) THEN
GM1 = CMPLX (0.0,G1)
GM3 = CMPLX (0.0,G1)
GMM1 = G1*H
GMM3 = G3*D
COTT1 = -1./TAN(GMM1)
COTT3 = -1./TAN(GMM3)
COT1 = CMPLX (0.0,COTT1)
COT3 = CMPLX (0.0,COTT3)
ELSE
GM1 = CMPLX (G1,0.0)
GM3 = CMPLX (G1,0.0)
GMM1 = G1*H
GMM3 = G1*D
COTT1 = 1./TANH(GMM1)
COTT3 = 1./TANH(GMM3)
COT1 = CMPLX (COTT1,0.0)
COT3 = CMPLX (COTT3,0.0)
END IF

G2 = SQRT(ABS(AKK2))
IF (AKK2 .LT. 0.0) THEN
GM2 = CMPLX (0.0,G2)
GMM2 = G2*T
COTT2 = -1./TAN(GMM2)
COT2 = CMPLX (0.0,COTT2)
ELSE
GM2 = CMPLX (G2,0.0)
GMM2 = G2*T
COTT2 = 1./TANH(GMM2)
COT2 = CMPLX (COTT2,0.0)
END IF

DE = COT2*COT3+COT1*COT3*(GM2/GM1)/ER+COT1*COT2+(GM3/GM2)*ER
NE = (GM2*COT3/ER+GM3*COT2)*YF
ZE = NE/DE
NH = GM2*COT2+GM3*COT3
DH1 = GM1*GM2*COT1*COT2+GM1*GM3*COT1*COT3
DH2 = GM2*GM3*COT2*COT3+GM2**2
DH = DH1 + DH2
ZH = (NH/DH)*YH
X1 = -1./(ALPA**2 + BETA**2)
ZZZ1 = CMPLX (X1,0.0)
ZZZ2 = (BETA**2)*ZE + (ALPA**2)*ZH
ZZZ = ZZZ1*ZZZ2
X = ALPA*W/2.
J11 = 2.*SIN(X)/(X)
J12 = 3./X**2
J13 = COS(X)
J14 = J11
J15 = 2.*((1.0-COS(X))/(X**2)
J1 = J11+J12*(J13-J14+J15)
JZ1 = CMPLX ( J1, 0.0 )
F1 = ZZZ*JZ1**2
RETURN
END
B. SECOND ORDER APPROXIMATION

* THIS IS AN EXAMPLE PROGRAM TO COMPUTE CONSTANT PROPAGATION
* IN SECOND ORDER APPROXIMATION WITH W = 0.711 AND ER = 2.2

* MAIN PROGRAM :

* F1(ALPA,BETA,FREQ) = ZZ * JZ1 ** 2
* F2(ALPA,BETA,FREQ) = ZX * JZ1 * JX1
* F3(ALPA,BETA,FREQ) = ZX * JZ1 * JX1
* F4(ALPA,BETA,FREQ) = ZX * JX1 ** 2

COMPLEX F1,F2,F3,F4
COMPLEX KK,K11,K22,K33,K44
COMPLEX KKK1,KKK2
REAL MAGKK,B00,B0,ALPA,BETA,MO
REAL A,D,T,H,PI,ER,W,FREQ,OMG,E0
INTEGER N,L
PARAMETER ( PI=3.141592654, EO=8.85E-12, H=0.66E-03 )
PARAMETER ( T=0.255E-03, D=0.66E-03, M0=PI*4.E-7, A=0.16E-02 )
PARAMETER ( W=0.711 ,ER=2.2 )
* NOTE : PARAMETER W CAN BE 0.711 , 1.0 , OR 1.25
* NOTE : PARAMETER ER CAN BE 2.2 OR 9.8

FREQ = 12.858241
OMG = FREQ*PI*2.E+9
B0 = (OMG/3.E08)*SQRT((ER+I)/2)

* START SWEEP THE BETA VALUE FROM 88.2% TO 89.7%
DO 20 B00 = 0.882,0.897,7.5E-6
BETA = B0*B00
K11 = (0.0,0.0)
K22 = (0.0,0.0)
K33 = (0.0,0.0)
K44 = (0.0,0.0)

* START SUMMATION IN ALPA FROM N=-40 TO N=+40
DO 10 L = 1, 81
N = L - 41
ALPA = (N - 0.5)*PI/A
K11 = K11 + F1(ALPA,BETA,FREQ)
K22 = K22 + F2(ALPA,BETA,FREQ)
K33 = K33 + F3(ALPA,BETA,FREQ)
K44 = K44 + F4(ALPA,BETA,FREQ)
10 CONTINUE

KKK1 = K11*K44
KKK2 = K22*K33
KK = KKK1-KKK2
MAGKK = CABS((KK))
WRITE(6,100) FREQ,BETA,MAGKK
100 FORMAT(' ',3X,F14.9,3X,F18.9,3X,F15.9)
20 CONTINUE
FUNCTIONS SUBPROGRAM F1, F2, F3, F4

COMPLEX FUNCTION F1(ALPA, BETA, FREQ)
COMPLEX GM1, GM2, GM3, COT1, COT2, COT3, DE, ZE, DH, ZH, NE, NH, DH1, DH2, YE
COMPLEX YH, JZ1, ZZZ1, ZZZ2, ZZZ
REAL A, D, T, H, PI, ER, W, FREQ, OMG, E0, G1, G2, G3, COT1, COT2, COT3, YYE
REAL YYH, GMM1, GMM2, GMM3, ALPA, AKK1, AKK2, X1, X
REAL J11, J12, J13, J14, J15, J1, M0, AK1, AK2, BETA
PARAMETER ( PI=3.141592654, E0=8.85E-12, H=0.66E-03 )
PARAMETER ( T=0.255E-03, D=0.66E-03, M0=PI*4. E-7, A=0.16E-02 )
PARAMETER ( W=0.711, ER=2.2 )
* NOTE : PARAMETER W CAN BE 0.711, 1.0, OR 1.25
* NOTE : PARAMETER ER CAN BE 2.2 OR 9.8

OMG = FREQ*PI*2. E+9
YYE = -1. /(OMG*E0)
YE = CMPLX (0.0, YYE)
YYH = OMG*M0
YH = CMPLX (0.0, YYH)
AK1 = (OMG**2)*M0*E0
AK2 = (OMG**2)*M0*E0*ER
AKK1 = ALPA**2 + BETA**2 - AK1
AKK2 = ALPA**2 + BETA**2 - AK2
G1 = SQRT(ABS(AKK1))
G3 = G1
IF (AKK1 .LT. 0.0) THEN
GM1 = CMPLX (0.0, G1)
GM3 = GM1
GM1 = G1*H
GM3 = G3*D
COT1 = -1./TAN(GMM1)
COT3 = -1./TANH(GMM3)
COT1 = CMPLX (0.0, COT1)
COT3 = CMPLX (0.0, COT3)
ELSE
GM1 = CMPLX (G1, 0.0)
GM3 = GM1
GM1 = G1*H
GM3 = G3*D
COT1 = 1./TANH(GMM1)
COT3 = 1./TANH(GMM3)
COT1 = CMPLX (COT1, 0.0)
COT3 = CMPLX (COT3, 0.0)
END IF
G2 = SQRT(ABS(AKK2))
IF (AKK2 .LT. 0.0) THEN
GM2 = CMPLX (0.0, G2)
GM2 = G2*T
COT2 = -1./TAN(GMM2)
COT2 = CMPLX (0.0, COT2)
ELSE
GM2 = CMPLX (G2, 0.0)
END IF
GMM2 = G2*T
COTT2 = 1./TANH(GMM2)
COT2 = CMPLX ( COTT2, 0.0 )
END IF

DE = COT2*COT3+COT1*COT3*(GM2/GM1)/ER+COT1*COT2+(GM3/GM2)*ER
NE = (GM2*COT3/ER+GM3*COT2)*YE
ZE = NE/DE
NH = GM2*COT2 + GM3*COT3
DH1 = GM1*GM2*COT1*COT2 + GM1*GM3*COT1*COT3
DH2 = GM2*GM3*COT2*COT3 + GM2**2
DH = DH1 + DH2
ZH = (NH/DH)*YH
X1 = -1./((ALPA**2 + BETA**2)
ZZZ1 = CMPLX (X1,0.0)
ZZZ2 = (BETA**2)*ZE + (ALPA**2)*ZH
ZZZ = ZZZ1*ZZZ2
X = ALPA**W/2.
J11 = 2.*SIN(X)/(X)
J12 = 3./(X**2)
J13 = COS(X)
J14 = J11
J15 = 2.*(1.0-COS(X))/(X**2)
J1 = J11+J12*(J13-J14+J15)
J21 = CMPLX ( J1,0.0 )
F1 = ZZZ*J21**2
RETURN

END

COMPLEX FUNCTION F2(ALPA,BETA,FREQ)
COMPLEX GM1,GM2,GM3,COT1,COT2,COT3,DE,ZE,DH,ZH,NE,NH,D11,DH2,YE
COMPLEX YH,JZI,JX1,ZZX
REAL A,D,T,H,PI ,ER,W,FREQ,OMG,E0,G1,G2,G3,COTT1,COTT2,COTT3,YE
REAL YYH,GMM1,GMM2,GMM3,ALPA,AKK1,AKK2,X,ZZX1,BETA
REAL J31,J32,J3,J11,J12,J13,J14,J15,J1,M0,AK1,AK2
PARAMETER ( PI=3.141592654, E0=8.85E-12, H=0.66E-03 )
PARAMETER ( T=0.255E-03, D=0.66E-03, M0=PI*4. E-7, A=0.16E-02 )
PARAMETER ( W=0.711 ,ER=2.2 )
* NOTE : PARAMETER W CAN BE 0.711 , 1.0 , OR 1.25
* NOTE : PARAMETER ER CAN BE 2.2 OR 9.8
OMG = FREQ*PI*2.E+9
YYE = -1./OMG
YE = CMPLX (0.0,YYE)
YYH = OMGE0
YH = CMPLX (0.0,YYH)
AK1 = (OMG**2)*M0*E0
AK2 = (OMG**2)*M0*EO*ER
AKK1 = ALPA**2 + BETA**2 - AK1
AKK2 = ALPA**2 + BETA**2 - AK2
G1 = SQRT(ABS(AKK1))
G3 = G1
IF (AKK1.LT.0.0) THEN
GM1 = CMPLX (0.0,G1)
GM3 = GM1
GMM1 = G1*H
GMM3 = G3*D

COTT1 = -1. /TAN(GMM1)
COTT3 = -1. /TAN(GMM3)
COT1 = CMPLX ( 0.0, COTT1 )
COT3 = CMPLX ( 0.0, COTT3 )
ELSE
GM1 = CMPLX ( G1,0.0 )
GM3 = GM1
GMM1 = G1*H
GMM3 = G1*D
COTT1 = 1. /TANH(GMM1)
COTT3 = 1. /TANH(GMM3)
COT1 = CMPLX ( COTT1,0.0 )
COT3 = CMPLX ( COTT3,0.0 )
END IF
G2 = SQRT(ABS(AKK2))
IF (AKK2 .LT. 0.0) THEN
GM2 = CMPLX ( 0.0,G2 )
GMM2 = G2*T
COTT2 = -1. /TAN(GMM2)
COT2 = CMPLX ( 0.0,COTT2 )
ELSE
GM2 = CMPLX ( G2,0.0 )
GMM2 = G2*T
COTT2 = 1. /TANH(GMM2)
COT2 = CMPLX ( COTT2,0.0 )
END IF
DE = COT2*COT3+COT1*COT3*(GM2/GM1)/ER+COT1*COT2*(GM3/GM2)*ER
NE = (GM2*COT3/ER+GM3*COT2)*YE
ZE = NE/DE
NH = GM2*COT2 + GM3*COT3
DH1 = GM1*GM2*COT1*COT2 + GM1*GM3*COT1*COT3
DH2 = GM2*GM3*COT2*COT3 + GM2**2
DH = DH1 + DH2
ZH = (NH/DH)*YH
ZZX1 = -1. * (ALPA*BETA)/(ALPA**2+BETA**2)
ZZX = ZZX1*(ZE-ZH)
X = ALPA*W/2.
J11 = 2.*SIN(X)/(X)
J12 = 3./(X**2)
J13 = COS(X)
J14 = J11
J15 = 2.*(1.0-COS(X))/(X**2)
J1 = J11+J12*(J13-J14+J15)
J31 = 2.*PI*SIN(X)
J32 = X**2 - PI**2
J3 = J31/J32
JZ1 = CMPLX ( J1,0.0 )
JX1 = CMPLX(J3,0.0)
F2 = ZZX*JZ1*JX1
RETURN
END

COMP1PLEX FUNCTION F3(ALPA,BETA,FREQ)
COMP1LEX GM1,GM2,GM3,COT1,COT2,COT3,DE,ZE,DH,ZH,NE,NH,DH1,DH2,YE
COMP1LEX YH,JZ1,JX1,ZX
REAL A,D,T,H,PI,ER,W,FREQ,OMG,E0,G1,G2,G3,COTT1,COTT2,COTT3,YE
REAL YHY,GMM1,GMM2,GMM3,ALPA,AKK1,AKK2,X,BETA
REAL J11,J12,J13,J14,J15,J1,J31,J32,J3,M0,AK1,AK2
PARAMETER (PI=3.141592654, E0=8.85E-12, H=0.66E-03 )
PARAMETER ( T=0.255E-03, D=0.66E-03, M0=PI*4.E-7, A=0.16E-02 )
PARAMETER ( W=0.711, ER=2.2 )
* NOTE: PARAMETER W CAN BE 0.711, 1.0, OR 1.25
* NOTE: PARAMETER ER CAN BE 2.2 OR 9.8

OMG = FREQ*PI*2.E+9
YHY = -1./(OMG*E0)
YEH = CMPLX (0.0,YHY)

YHY = CMPLX (0.0,YHY)
AK1 = (OMG**2)*M0*E0
AK2 = (OMG**2)*M0*E0*ER
AKK1 = ALPA**2 + BETA**2 - AK1
AKK2 = ALPA**2 + BETA**2 - AK2
G1 = SQRT(ABS(AKK1))
G3 = G1
IF (AKK1 .LT. 0.0) THEN
GM1 = CMPLX (0.0,G1 )
GM3 = GM1
GMM1 = G1*H
GMM3 = G3*D
COTT1 = -1./TAN(GMM1)
COTT3 = -1./TAN(GMM3)
COT1 = CMPLX (0.0,COTT1)
COT3 = CMPLX (0.0,COTT3)
ELSE
GM1 = CMPLX (G1,0.0)
GM3 = GM1
GMM1 = G1*H
GMM3 = G3*D
COTT1 = 1./TANH(GMM1)
COTT3 = 1./TANH(GMM3)
COT1 = CMPLX (COTT1,0.0)
COT3 = CMPLX (COTT3,0.0)
END IF

G2 = SQRT(ABS(AKK2))
IF (AKK2 .LT. 0.0) THEN
GM2 = CMPLX (0.0,G2)
GMM2 = G2*T
COTT2 = -1./TAN(GMM2)
COT2 = CMPLX (0.0,COTT2)
ELSE
GM2 = CMPLX (G2,0.0)
GMM2 = G2*T
COTT2 = 1./TANH(GMM2)
COT2 = CMPLX (COTT2,0.0)
END IF

DE = COT2*COT3+COT1*COT3*(GM2/GM1)/ER+COT1*COT2+(GM3/GM2)*ER
NE = (GM2*COT3/ER+GM3*COT2)*YEH
ZE = NE/DE
NH = GM2*COT2 + GM3*COT3
DH1 = GM1*GM2*COT1*COT2 + GM1*GM3*COT1*COT3
DH2 = GH2*GM3*COT2*COT3 + GM2**2
DH = DH1 + DH2
ZH = (NH/DH)*YH
ZXZ = -1.*(ALPA*BETA)/(ALPA**2+BETA**2)
ZXZ = ZXZ*(ZE-ZH)
X = ALPA*W/2.
J31 = 2.*PI*SIN(X)
J32 = X**2 - PI**2
J3 = J31/J32
JX1 = CMPLX(J3,0.0)
J11 = 2.*SIN(X)/(X)
J12 = 3./(X**2)
J13 = COS(X)
J14 = J11
J15 = 2.*(1.0*COS(X))/(X**2)
J1 = J11+J12*(J13-J14+J15)
JZ1 = CMPLX (J1,0.0)
F3 = ZXZ*JZ1*JX1
RETURN
END

COMPLEX FUNCTION F4(ALPA,BETA,FREQ)

COMPLEX GM1,GM2,GM3,COT1,COT2,COT3,DE,ZH,DH,ZH,NE,NH,DH1,DH2,YE
COMPLEX YH,JX1,ZXX1,ZXX2,ZXX
REAL A,D,T,H,PI,ER,W,FREQ,OMG,E0,G1,G2,G3,COTT1,COTT2,COTT3,YE
REAL YYH,GMM1,GMM2,GMM3,ALPA,AKK1,AKK2,AKK4
REAL J31,J32,J3,J0,X,X1,BETA
PARAMETER ( P1=3.141592654, E0=8.85E-12, H=0.66E-03 )
PARAMETER ( T=0.255E-03, D=0.66E-03, MO=PI*4.5E-7, A=0.16E-02 )
PARAMETER ( W=0.711, ER=2.2 )
* NOTE : PARAMETER W CAN BE 0.711, 1.0, OR 1.25
* NOTE : PARAMETER ER CAN BE 2.2 OR 9.8

OMG = FREQ*PI*2.E+9
YYE = -1./(OMG*E0)
YE = CMPLX (0.0,YYE)
YYH = OMG*MO
YH = CMPLX (0.0,YYH)
AK1 = (OMG**2)*MO*E0
AK2 = (OMG**2)*MO*E0*ER
AKK1 = ALPA**2 + BETA**2 - AK1
AKK2 = ALPA**2 + BETA**2 - AK2
G1 = SQRT(ABS(AKK1))
G3 = G1
IF (AKK1 .LT. 0.0) THEN
GM1 = CMPLX (0.0,G1)
GM3 = GM1
GMM1 = G1*H
GMM3 = G3*D
COTT1 = -1./TAN(GMM1)
COTT3 = -1./TAN(GMM3)
COT1 = CMPLX (0.0,COTT1)
COT3 = CMPLX (0.0,COTT3)
ELSE
G41 = CMPLX (G1,0.0)
GM3 = GM1
GMM1 = G1*H
GMM3 = G1*D
COTT1 = 1./TANH(GM1)
COTT3 = 1./TANH(GMM3)
COT1 = CMPLX ( COTT1, 0.0 )
COT3 = CMPLX ( COTT3, 0.0 )
END IF
G2 = SQRT(ABS(AKK2))
IF (AKK2 .LT. 0.0) THEN
GMM2 = G2*T
COTT2 = -1./TAN(GM2)
COT2 = CMPLX ( 0.0, COTT2 )
ELSE
GMM2 = G2*T
COTT2 = 1./TANH(GM2)
COT2 = CMPLX ( COTT2, 0.0 )
END IF
DE = COT2*COT3+COT1*COT3*(GM2/GM1)/ER+COT1*COT2+(GM3/GM2)*ER
NE = (GM2*COT3/ER+GM3*COT2)*YE
ZE = NE/DE
NH = GM2*COT2 + GM3*COT3
DH1 = GM1*GM2*COT1*COT2 + GM1*GM3*COT1*COT3
DH2 = GM2*GM3*COT2*COT3 + GM2**2
DH = DH1 + DH2
ZH = (NH/DH)*YH
X1 = -1./((ALPA**2 + BETA**2)
ZXX1 = CMPLX(X1, 0.0)
ZXX2 = (ALPA**2)*ZXX1 + (BETA**2)*ZH
ZXX = ZXX1*ZXX2
X = ALPA**W/2.
J31 = 2.*PI*SIN(X)
J32 = X**2 - PI**2
J3 = J31/J32
JX1 = CMPLX(J3, 0.0)
F4 = ZXX*JX1**2
RETURN
END
APPENDIX D. PROGRAMS FOR CALCULATION OF FRINGING CAPACITANCE

A. TRANSMISSION LINE CIRCUIT MODEL

* THIS IS AN EXAMPLE PROGRAM TO COMPUTE THE FRINGING CAPACITANCE
* IN SECOND ORDER APPROXIMATION WITH W = 0.711 MM AND ER = 2.2
*
COMPLEX ZIN
REAL MAGZIN,OMG,Z0,BETA,AL,C1,C,FREQ,T,ZZ,PI,ZZIN
PARAMETER ( PI=3.141592654, Z0=94.088, AL=0.005 )
PARAMETER ( BETA = 301.921744 )
* NOTE : FILL IN PARAMETERS BETA AND ZO FROM TABLE 1 OR TABLE 2.
DO 10 C1 = 0.8058,0.8062,1.E-6
C = C1*1.E-14
DO 15 FREQ = 12.858241,12.858241
OMG = 2*PI*FREQ*1.E9
T = TAN(BETA*AL)
ZZ = OMG*C*ZO
ZZIN = -1.*(1-ZZ*T)*Z0/(ZZ+T)
ZIN = CMPLX(0.0,ZZIN)
MAGZIN = CABS((ZIN))
WRITE(6,100) FREQ,C,MAGZIN
100 FORMAT( 2X,F16.12,2X,F25.23,2X,F14.12)
15 CONTINUE
10 CONTINUE
END

B. END EFFECT EXTENSION MODEL

* THIS IS AN EXAMPLE PROGRAM TO COMPUTE THE FRINGING CAPACITANCE
* IN SECOND ORDER APPROXIMATION WITH W = 0.711 MM AND ER = 2.2
*
REAL FO,BETA,LL,L,PI,DELTA,CO,OMG,Z0,CP
PARAMETER ( Z0=94.088, L=0.01, PI=3.141592654 )
PARAMETER ( FO=12.858241, BETA=301.921744 )
* NOTE : PARAMETERS FO, BETA, AND ZO CAN VARIES DEPEND ON THE CHOICE
* OF APPROXIMATION
OMG = 2.*PI*FO*1.E9
LL = PI/BETA
DELTA = (LL-L)/2.
CO = BETA/(OMG*Z0)
CP = CO+DELTA
WRITE(6,100) FO,BETA,CP
100 FORMAT(' ',3X,F10.7,5X,F9.5,3X,F30.29)
END
APPENDIX E. PROGRAM FOR CALCULATION OF
CHARACTERISTIC IMPEDANCE OF SUSPENDED SUBSTRATE LINE

* THIS IS AN EXAMPLE PROGRAM TO COMPUTE THE DYNAMIC
  CHARACTERISTIC IMPEDANCE OF SUSPENDED SUBSTRATE LINE
* IN SECOND ORDER APPROXIMATION WITH W=1.25 & ER=9.8
*
REAL EEFF,OMG,FREQ,BETA,W,G,ER,T,A,Z0,Z11,Z1,V,R
PARAMETER ( W=1.25E-3, G=1.575E-3, ER=9.8, T=0.255E-3 )
PARAMETER ( A=3.2E-3, BETA = 293.61813, PI=3.141592654 )
*
NOTE : PARAMETER A IN FIGURE 1 IS 2A = 3.2E-3 METER

FREQ = 9.78036
V = -1.786572035*(T/G)+0.4750*(A/G)
R = 1.0835+0.1007*(T/G)-0.09457*(A/G)
Z11 = 6*G/W + (1+4*((G/W)**2))**0.5
Z1 = 60*(V+R*LOG(Z11))
OMG = FREQ*PI*2.1E+9
EEFF = (BETA*3.E+8/OMG)**2
Z0 = Z1/(EEFF**0.5)
WRITE(6,25) Z0
25 FORMAT( ' ',5X,F9.3)
STOP
END
APPENDIX F. SOME MATHEMATICAL CALCULATION

These equations are used in computer programs

1. The equations \((I_p,q,m)\) on page 7

\[
Y_1 = j\omega \varepsilon_o \quad Z_1 = j\omega \mu_o \quad K_1^2 = \omega^2 \mu_o \varepsilon_o
\]  
(1)

\[
Y_2 = j\omega \varepsilon_o \varepsilon_r \quad Z_2 = j\omega \mu_o \mu_r \quad K_2^2 = \omega^2 \mu_o \mu_r \varepsilon_o \varepsilon_r
\]  
(2)

\[
Y_3 = j\omega \varepsilon_o \quad Z_3 = j\omega \mu_o \quad K_3^2 = \omega^2 \mu_o \varepsilon_o
\]  
(3)

Note that \(Y_1 = Y_3, \ Z_1 = Z_3, \ K_1^2 = K_3^2\)

2. The equations \((I_n,o)\) on page 7

\[
\gamma_1 = \frac{\gamma_1}{Y_1} = \frac{\sqrt{\alpha^2 + \beta^2 - K_1^2}}{j\omega \varepsilon_o} \quad \gamma_1 = \frac{\gamma_1}{Z_1} = \frac{\sqrt{\alpha^2 + \beta^2 - K_1^2}}{j\omega \mu_o}
\]  
(4)

\[
\gamma_2 = \frac{\gamma_2}{Y_2} = \frac{\sqrt{\alpha^2 + \beta^2 - K_2^2}}{j\omega \varepsilon_o \varepsilon_r} \quad \gamma_2 = \frac{\gamma_2}{Z_2} = \frac{\sqrt{\alpha^2 + \beta^2 - K_2^2}}{j\omega \mu_o \mu_r}
\]  
(5)

\[
\gamma_3 = \frac{\gamma_3}{Y_3} = \frac{\sqrt{\alpha^2 + \beta^2 - K_3^2}}{j\omega \varepsilon_o} \quad \gamma_3 = \frac{\gamma_3}{Z_3} = \frac{\sqrt{\alpha^2 + \beta^2 - K_3^2}}{j\omega \mu_o}
\]  
(6)

3. The equation \((1g)\) on page 6

\[
ZE = \frac{NE}{DE}
\]  
(7)

Where :

\[
NE = \gamma_2 C_t 3 + \gamma_3 C_t 2
\]  
(8)

\[
NE = \frac{\sqrt{\alpha^2 + \beta^2 - K_2^2}}{j\omega \varepsilon_o \varepsilon_r} \times C_t 3 + \frac{\sqrt{\alpha^2 + \beta^2 - K_3^2}}{j\omega \varepsilon_o} \times C_t 2
\]  
(9)

\[
NE = \frac{1}{j\omega \varepsilon_o} \left[ \frac{C_t 3}{\varepsilon_r} \sqrt{\alpha^2 + \beta^2 - K_2^2} + C_t 2 \sqrt{\alpha^2 + \beta^2 - K_3^2} \right]
\]
\[ NE = \frac{1}{j\omega_0} \left[ \frac{Ct_3}{\varepsilon_r} \times \gamma_2 + Ct_2 \times \gamma_3 \right] \]  
\[ \text{(10)} \]

Defined \( \frac{1}{j\omega_0} = YY!_\alpha \), then

\[ YYE = \frac{1}{j\omega_0} = j\left[ \frac{-1}{\omega \varepsilon_0} \right] \]

\[ YE = \text{Complex}(0.0, YYE) \]  
\[ \text{(11)} \]

\[ DE = DE1 + DE2 \]

\[ DE1 = Ct_2Ct_3 + Ct_1Ct_3 \left( \frac{\sqrt{\alpha^2 + \beta^2 - K_2^2}}{j\omega_0 \varepsilon_r} \right) \left( \frac{\sqrt{\alpha^2 + \beta^2 - K_1^2}}{j\omega_0} \right) \]  
\[ \text{(12)} \]

\[ DE2 = Ct_1Ct_2 \frac{\gamma_3}{\gamma_1} + \left( \frac{\sqrt{\alpha^2 + \beta^2 - K_3^2}}{j\omega_0} \right) \left( \frac{\sqrt{\alpha^2 + \beta^2 - K_2^2}}{j\omega_0 \varepsilon_r} \right) \]  
\[ \text{(13)} \]

Since \( \gamma_3 = \gamma_3 \), simplicity (12) and (13)

\[ DE1 = Ct_2Ct_3 + Ct_1Ct_3 \left( \frac{\sqrt{\alpha^2 + \beta^2 - K_2^2}}{\sqrt{\alpha^2 + \beta^2 - K_1^2}} \right) \times \frac{1}{\varepsilon_r} \]

\[ \text{(14)} \]

\[ DE1 = Ct_2Ct_3 + Ct_1Ct_3 \frac{\gamma_2}{\gamma_1} \times \frac{1}{\varepsilon_r} \]

\[ DE2 = Ct_1Ct_2 \frac{\sqrt{\alpha^2 + \beta^2 - K_3^2}}{\sqrt{\alpha^2 + \beta^2 - K_2^2}} \times \varepsilon_r \]

\[ \text{(15)} \]

\[ DE2 = Ct_1Ct_2 \frac{\gamma_3}{\gamma_2} \times \varepsilon_r \]

\[ DE = Ct_2Ct_3 + Ct_1Ct_3 \times \frac{\gamma_2}{\gamma_1} \times \frac{1}{\varepsilon_r} + Ct_1Ct_2 \times \frac{\gamma_3}{\gamma_2} \times \varepsilon_r \]  
\[ \text{(16)} \]

In the programs use a symbols:

\[ \text{COT1, COT2, COT3 for } Ct_1, Ct_2, Ct_3, \text{ and } \]
\[ \text{GM1, GM2, GM3 for } \gamma_1, \gamma_2, \gamma_3. \]

4. The equation (11) on page 6

\[ Ct_1 = \text{Coth} \gamma_1 h = \text{Coth} \left\{ \sqrt{\alpha^2 + \beta^2 - K_1^2} \times h \right\} \]  
\[ \text{(17)} \]

\[ \text{IF } (\alpha^2 + \beta^2 - K_1^2) < 0.0, \text{ then } \gamma_1 \text{ is complex. We defined } \gamma_1 \text{ as } G_1, \text{ so} \]
\[ C_{t1} = \coth jG_1h = \frac{1}{\tanh jG_1h} \] (18)

\[ C_{t1} = \frac{1}{j \tan G_1h} = \frac{-j}{\tan G_1h} \] (19)

\[ C_{t1} = \text{Complex}(0.0, -\frac{1}{\tan G_1h}) \] (20)

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\[ C_{t1} = \coth G_1h = \frac{1}{\tanh G_1h} \] (21)

\[ C_{t1} = \text{Complex}(\frac{1}{\tanh G_1h}, 0.0) \] (22)

The same process for \( C_{t2} \) and \( C_{t3} \) in equation (1j) and (1k).

5. The equation (1h) on page 6

\[ ZH = \frac{NH}{DH} \] (23)

Where:

\[ NH = \gamma_{z2}C_{t2} + \gamma_{z3}C_{t3} \] (24)

\[ NH = \frac{\sqrt{\alpha^2 + \beta^2 - K_2^2}}{j \omega \mu_o \mu_r} \times C_{t2} + \frac{\sqrt{\alpha^2 + \beta^2 - K_3^2}}{j \omega \mu_o} \times C_{t3} \] (25)

\[ NH = \frac{1}{j \omega \mu_o} \left[ \frac{C_{t2}}{\mu_r} \sqrt{\alpha^2 + \beta^2 - K_2^2} + C_{t3} \sqrt{\alpha^2 + \beta^2 - K_3^2} \right] \]

We assumed that the value of \( \mu_r = 1.0 \), then we have

\[ NH = \frac{1}{j \omega \mu_o} \left[ C_{t2} \times \gamma_{z2} + C_{t3} \times \gamma_{z3} \right] \] (26)

Write \( NH \) in a symbolic form,

\[ NH = \frac{1}{j \omega \mu_o} \times [\&\&\&] \] (27)

\[ DH = \gamma_{z1} \gamma_{z2} C_{t1} C_{t2} + \gamma_{z1} \gamma_{z3} C_{t1} C_{t3} + \gamma_{z2} \gamma_{z3} C_{t2} C_{t3} + \gamma_{z2}^2 \] (28)

\[ DH = \frac{\sqrt{\alpha^2 + \beta^2 - K_1^2}}{j \omega \mu_o} \times \frac{\sqrt{\alpha^2 + \beta^2 - K_2^2}}{j \omega \mu_o \mu_r} \times C_{t1} C_{t2} \] (29)

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\[ DH2 = \frac{\sqrt{\alpha^2 + \beta^2 - K_1^2}}{j\omega \mu_o} \times \frac{\sqrt{\alpha^2 + \beta^2 - K_2^2}}{j\omega \mu_o} \times C_1 C_3 \]  

(30)

\[ DH3 = \frac{\sqrt{\alpha^2 + \beta^2 - K_2^2}}{j\omega \mu_o \mu_r} \times \frac{\sqrt{\alpha^2 + \beta^2 - K_3^2}}{j\omega \mu_o} \times C_2 C_3 \]  

(31)

\[ DH4 = \left[ \frac{\sqrt{\alpha^2 + \beta^2 - K_2^2}}{j\omega \mu_o \mu_r} \right]^2 \]  

(32)

Simplify (29 to 32):

\[ DH1 = \frac{1}{(j\omega \mu_o)^2} \times \sqrt{\alpha^2 + \beta^2 - K_1^2} \times \sqrt{\alpha^2 + \beta^2 - K_2^2} \times C_1 C_2 \]  

(33)

\[ DH2 = \frac{1}{(j\omega \mu_o)^2} \times \sqrt{\alpha^2 + \beta^2 - K_1^2} \times \sqrt{\alpha^2 + \beta^2 - K_3^2} \times C_1 C_3 \]  

(34)

\[ DH3 = \frac{1}{(j\omega \mu_o)^2} \times \sqrt{\alpha^2 + \beta^2 - K_2^2} \times \sqrt{\alpha^2 + \beta^2 - K_3^2} \times C_2 C_3 \]  

(35)

\[ DH4 = \frac{1}{(j\omega \mu_o)^2} \times \sqrt{\alpha^2 + \beta^2 - K_2^2} \times \sqrt{\alpha^2 + \beta^2 - K_2^2} \]  

(36)

\[ DH = \frac{1}{(j\omega \mu_o)^2} \times (DH1 + DH2 + DH3 + DH4) \]  

(37)

Write DH in a symbolic form,

\[ DH = \frac{1}{(j\omega \mu_o)^2} \times [SSS] \]  

(38)

We defined in the program that \(K_1^2 = K_3^2 = AK1 \) and \(K_2^2 = AK2\):

\[ \sqrt{\alpha^2 + \beta^2 - K_1^2} \times \sqrt{\alpha^2 + \beta^2 - K_3^2} = AKK1 \]  

\[ \alpha^2 + \beta^2 - AK_1 = AKK1 \]  

(39)

and

\[ \sqrt{\alpha^2 + \beta^2 - K_2^2} \times \sqrt{\alpha^2 + \beta^2 - K_2^2} = AKK2 \]  

\[ \alpha^2 + \beta^2 - AK_2 = AKK2 \]  

(40)

From these equations we have the value of \(ZH\) as the following:
\[ ZH = \frac{\frac{1}{j\omega \mu_o}}{\left(\frac{1}{j\omega \mu_o}\right)^2 \times [SSS]} = j\omega \mu_o \times \frac{[& & &]}{[SSS]} \quad (41) \]

Defined \( j\omega \mu_o = YYH \)

\( YH = Complex(0.0, YYH) \quad (43) \)

So

\[ ZH = YH \times \frac{[& & &]}{[SSS]} \]
LIST OF REFERENCES


17. Yong-hui Shu, Xiao-xia Qi, Yun-yi Wang, "Closed form equation for Shielded Suspended Substrate Line", Appendix of [Ref 9]


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