Evidence for Strong Electron-Phonon Coupling in Thermal Conductivity of YBa$_2$Cu$_3$O$_{7-\delta}$

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We have measured the ab-plane thermal conductivity ($\kappa$) on single crystal YBa$_2$Cu$_3$O$_{7-\delta}$ ($\delta = 0.08$) in the temperature range $10 \text{ K} \leq T \leq 300 \text{ K}$. Our main focus is on analysis of the slope change in $\kappa(T)$ which occurs just below the superconducting transition. The reduced-temperature ($t=T/T_c$) derivative of the normalized thermal conductivity, $d(\kappa(5)/\kappa(n)/dt)|_{t=1}$, as determined from the data, is rather small, $<1.1$. From measurements of the electrical conductivity on the same specimens and application of the Wiedemann-Franz Law we determine the relative contributions to the heat conduction from the carriers and the lattice, and discuss the normal-state phonon scattering mechanisms. Employing these results we calculate the slope of the lattice thermal conductivity at $T_c$ and infer that the slope of the carrier conductivity must be very large, $>6$. This result implies strong coupling for some of the carriers.
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We have measured the ab-plane thermal conductivity ($\kappa$) on single crystal YBa$_2$Cu$_3$O$_{7-\delta}$ ($\delta \sim 0.08$) in the temperature range 10K $\leq$ T $\leq$ 300K. Our main focus is an analysis of the slope change in $\kappa(T)$ which occurs just below the superconducting transition. The reduced-temperature ($t=T/T_c$) derivative of the normalized thermal conductivity, $d(\kappa^n/\kappa^0)/dt|_{t=1}$, as determined from the data, is rather small, $\leq$ 1.1. From measurements of the electrical conductivity on the same specimens and application of the Wiedemann-Franz Law we determine the relative contributions to the heat conduction from the carriers and the lattice, and discuss the normal-state phonon scattering mechanisms. Employing these results we calculate the slope of the lattice thermal conductivity at $T_c$ and infer that the slope of the carrier conductivity must be very large, $\geq$ 6. This result implies strong coupling for some of the carriers.

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The thermal conductivity, \( \kappa \), in the high-\( T_c \) superconductors is unique in providing information on the transport mechanisms occurring in both the normal and superconducting states. The dominance of the phonon heat transport over that of the carriers and the importance of phonon-carrier scattering in the cuprates are manifested as a sharp upturn in the ab-plane \( \kappa \) for \( T < T_c \). An analysis of this feature offers the potential of revealing information about the superconducting energy gap \( \Delta \) and the strength of the electron-phonon coupling, two fundamental parameters which are currently the focus of much experimental and theoretical research.

In this paper we analyze the slope change in the ab-plane thermal conductivity of two \( \text{YBa}_2\text{Cu}_3\text{O}_{7-\delta} \) (\( \delta \approx 0.08 \)) single crystals. Measurements of both the thermal and electrical conductivities on the same specimens enable us to estimate the carrier and lattice contributions to the total \( \kappa \). The importance of phonon-phonon, phonon-carrier, and phonon-defect scattering are demonstrated. Employing these results we calculate the slope-change in the lattice conductivity at the superconducting transition. We infer that the carrier thermal conductivity decreases sharply below \( T_c \), consistent with strong coupling for some of the carriers.

The \( \text{YBa}_2\text{Cu}_3\text{O}_{7-\delta} \) (YBCO) crystals were grown in zirconia crucibles by a self-decanting flux method described elsewhere, and were subsequently annealed in oxygen at 450-500°C. Both specimens were high-quality, mm-sized (80μm-thick) single crystals with \( \rho(295K) \approx 250 \mu\Omega \text{ cm} \), transition widths \( \Delta T_c \approx 0.5K \), and \( R=0 \) at 92K.
The oxygen deficiency of specimen 1 was estimated as \( \delta = 0.08 \) from measurements of thermoelectric power, performed simultaneously with the thermal conductivity and reported previously.\(^3\) This value is consistent with the value inferred from the anomalous features in magnetization hysteresis loops, which have recently been shown\(^4\) to correlate with c-axis lattice parameter (a measure of oxygen content). The magnetization of specimen 2 implies an oxygen content similar to that of sample 1.

The thermal conductivity was measured using a steady-state technique, employing a differential chromel-constantan thermocouple and small resistive heater glued to the specimen with varnish. The temperature difference during measurement, \( \Delta T \), was typically 0.3K-1.0K. Linearity in the \( \Delta T \) response was confirmed throughout the temperature range by varying the heater power. Errors in \( \kappa \) due to heat losses are estimated to be less than 2% except for \( T > 200\)K where radiation losses become more significant. At room temperature we estimate an error in \( \kappa \) of \( \approx 10\% \).

Figure 1 shows \( \kappa(T) \) for both specimens. Near room temperature \( \kappa \approx 8-9 \) W/mK, which agrees with previous measurements on single-crystal YBCO.\(^5\) The most notable feature in these data is the sharp upturn and peak that occur for \( T < T_c \). This behavior is widely observed in the cuprates,\(^1\) and arises from an increase in the phonon mean free path due to the reduced scattering of phonons by carriers as the latter condense into superconducting pairs. An analysis of this slope change is our
principal focus and is discussed in detail below. We first address the behavior of $\kappa$ for $T > T_c$.

The contribution of the carriers to heat conduction may be calculated using the Wiedemann-Franz law (WFL), which states that $\kappa_e/\sigma T = L_0$, where $\kappa_e$ is the electronic component of the thermal conductivity, $\sigma$ is the electrical conductivity, and $L_0 = 2.45 \times 10^{-8}$ WΩ/K² is the Lorenz number. This yields an upper limit estimate of $\kappa_e$ and we find at 100K, $\kappa_e/\kappa = 0.31$ for both specimens.

In Fig. 2 we plot the lattice conductivity, $\kappa_L = \kappa - \kappa_e$, where $\kappa_e$ is determined, as above, from the measured electrical resistivity and the WFL. The slight upturn in $\kappa_L$ at $T \approx 100K$ for specimen 2 is probably associated with superconducting fluctuations, an issue we address in more detail elsewhere. In general, we anticipate three principal sources of scattering for phonons in a metal: defects, free carriers, and other phonons. Phonon-carrier and phonon-defect scattering yield a temperature-independent or weakly decreasing $\kappa_L$ with decreasing temperature at high temperatures. The observed decrease in $\kappa_L$ with increasing $T$ is characteristic of phonon-phonon scattering. The overall change in $\kappa_L$ from 300K down to $T_c$ is, for all specimens, relatively small ($\leq 20\%$) in comparison to that expected for a defect-free insulator ($\geq 3$). Thus with regard to lattice conduction, YBCO appears to be in the complicated regime where all three scattering mechanisms are significant. These observations are consistent with a more quantitative analysis discussed below.
We now address the change in heat transport at the superconducting transition and consider the ratio of the thermal conductivities in the superconducting (s) and normal (n) states, $\kappa^s/\kappa^n$, a quantity which is most readily compared with theory. We may write the temperature derivative of this ratio as,

$$\frac{d}{dt} (\kappa^s/\kappa^n) = \eta \frac{d\alpha}{dt} - (1 - \eta) \frac{d\beta}{dt} + \frac{d\ln \kappa^n}{dt},$$

where $t = T/T_c$ is the reduced temperature, $\alpha = \kappa^s/\kappa^n$, $\beta = \kappa^s/\kappa^n$, and $\eta = \kappa^n/\kappa^n$ is the relative contribution of carriers to the total heat conduction. The normal-state thermal conductivity, $\kappa^n$, is determined, for $T < T_c$, from the experimental data by smooth extrapolation of the normal-state data to several degrees below $T_c$. In Fig. 3 we plot $\kappa^s/\kappa^n$ versus $t$ near $t=1$. The slopes are $d(\kappa^s/\kappa^n)/dt|_{t=1} = -0.4$ (sample 1) and $-1.1$ (sample 2) as indicated by the solid lines. The third term on the right-hand-side of (1) is determined directly from the data at $T \geq T_c$. This term is quite small and thus the slope is determined by a competition between the first two terms which have opposite sign.

The BCS theory of thermal conductivity for weak-coupling superconductors predicts that, in the limiting cases of predominant electron-phonon and electron-defect scattering, respectively, $\alpha$ is a universal function of $\Delta/k_B T$ and for $\Delta = \Delta^{\text{BCS}}$, $d\alpha/dt|_{t=1} = 1.6$ (el-ph.) and 0 (el-def.). For strong coupling and when electron-defect scattering predominates, $d\alpha/dt|_{t=1} = 0$ as in weak coupling. The case of strong coupling and dominant
electron-phonon scattering is, however, much more complicated and depends on the form of the electron-phonon spectral function, \( a^2(\omega)F(\omega) \). For materials such as lead and mercury where these parameters are known from tunneling measurements, the very rapid decrease in \( \kappa_s \) observed experimentally is in reasonable accord with theory. The difference from weak coupling arises both from the more rapid opening of the gap for strong coupling as well as the stronger frequency dependence of the quasiparticle relaxation rate. The value of \( \frac{da}{dt} = 9 \) for lead is the largest known for any material. A slope \( \frac{da}{dt} \) substantially greater than 1.6 may thus be considered evidence for strong electron-phonon coupling.

The slope of the lattice conductivity for weak coupling and predominant phonon-electron scattering can be calculated from the BRT theory. Tewordt and Wolkhausen have modified this theory to include the effects of other scatterers and crystalline anisotropy on the lattice conductivity. In addition to the dependence on \( \Delta/k_BT \), the slope \( \frac{d\beta}{dt} \) depends on the model phonon scattering rate, in the case of YBCO a rate which must, as discussed above, incorporate the combined effects of scattering by carriers, phonons, and defects. For strong coupling, the lattice conductivity very close to \( T_c \) can be calculated from the weak coupling theory, provided the BCS gap is scaled to the appropriate strong coupling value. Thus the temperature dependence of \( \beta \) in this regime is governed by \( \Delta(T) \) which should follow the mean-field expression, \( \Delta(T) = a(1-t)^{1/2} \), independent of
the coupling strength (a is a scaling constant = 1.74Δ(0) for the BCS gap)

It is important to note that the change in the lattice thermal conductivity at Tc generally reflects the coupling of carriers to longitudinal acoustic phonons (presumed responsible for most of the lattice heat conduction) and these phonons may not be the most important for superconductivity. Recent calculations by Peacor et al. suggest that the coupling for these modes in YBCO is rather weak. It is the value of da/dt|_{T=Tc} which holds the most direct information regarding the strength of the carrier-phonon interaction relevant to superconductivity.

We now calculate the lattice conductivity slope from the BRT theory and place limits on the slope of the electronic component for YBCO using our experimental results and Eq. (1). The lattice conductivity is written as,

\[ \kappa_L = \frac{k_B}{2\pi^2 v} \left( \frac{k_B T}{\hbar} \right)^3 \int_0^{\Theta/T} dx \frac{x^4 e^x}{(e^x - 1)^2} \int_0^{\zeta} d\zeta \frac{3}{2} (1 - \zeta^2) \tau(x, \zeta; T) \]  

(2)

where v = 5,000 m/s is the sound velocity, Θ = 400K is the Debye temperature, x = hω/k_BT is the reduced phonon frequency, and τ(x, ζ; T) is the phonon relaxation time. The scattering of phonons by crystal boundaries, point defects, carriers, and other phonons is represented by τ^{-1}(x, T) = ν/d + A_{pd}(xT)^4 + A_{pp}xTg(x, T)(1 - ζ^2)^{1/2} + A_{pp}x^2T^4, where d is the crystallite thickness. Here, g(x, T) is the ratio of the phonon-carrier scattering rates in the normal and superconducting states as defined by BRT, equal to
unity for $T > T_c$ and a universal function of $\Delta/k_B T$ for $T < T_c$. The integration over the variable $\zeta$ describes the anisotropy of the electron-phonon interaction; it is assumed that only the in-plane component of phonon momentum is emitted or absorbed by the carriers. Following reference 17 we take $\Delta = \chi \Delta^{CS}$, where $\chi$ is a constant. The coefficients $A_{pp}, A_{pp}$ are determined self consistently by numerically integrating Eq. (2) to fit the normal-state $\kappa_L$ data and the data for $T \leq 20K$ where $\kappa = \kappa_L$. The normal-state curves for the two specimens are shown as solid lines in Fig. 2. The slope $d\beta/dt|_{t=1}$ is then determined from the calculated $\beta(t)$ curves. In the inset of Fig. 3 we plot $-d\beta/dt|_{t=1}$ versus the gap size $2\Delta(0)/k_BT_c$ for the two parameter sets. A number of experiments suggest the existence of two gaps in YBCO, associated with the CuO$_2$ planes and Cu-O chains, with $2\Delta_p(0)/k_BT_c$ = 5-6 and $2\Delta_{ch}(0)/k_BT_c$ = 2.5-3.5. The two-band expression$^{13}$ for $\kappa_L$ is more complicated than Eq. (2), but we may place a conservative lower limit on $d\beta/dt|_{t=1}$ by using the smallest gap size, $2\Delta(0)/k_BT_c$ = 2.5. This implies $d\beta/dt|_{t=1} = -4.5$ which, from Eq. (1), yields $d\alpha/dt|_{t=1} = 6$. A more reasonable picture$^{13}$ that assigns strong coupling to the planes and weak coupling to the chains, would imply a plane contribution, $d\alpha^p/dt|_{t=1} = 20$. Our main conclusion, $d\alpha/dt|_{t=1} \geq 6$, indicates that strong coupling characterizes some of the carriers in YBCO.

Since our analysis relies on the calculated value of $d\beta/dt|_{t=1}$, and specifically the choice of scattering parameters, we have examined the influence of the latter carefully. The
coefficients employed for sample 1 (2) imply that at T=100K
defect and carrier scattering account, respectively, for
approximately 75% (60%) and 18% (30%) of the lattice thermal
resistivity. Hence \( \frac{d\beta}{dt} \) is determined principally by the
relative weights of the carrier and defect scattering terms. We
find that \( \frac{d\beta}{dt} \) varies by less than 20% when these terms are
varied within acceptable ranges. We also find similar results
when we allow for a substantial deviation from the WFL, as would
be expected for \( T < 0 \) in a pure (electronically speaking)
material or a material with very strong electron-phonon
coupling. Thus we believe that our overall conclusion regarding
\( \frac{d\alpha}{dt} \) will survive a more detailed and specific model.

As mentioned above, in the context of the existing theory
values of \( \frac{d\alpha}{dt} \) larger than 1.6 imply strong electron-phonon
coupling and predominant electron-phonon scattering. Electron-
phonon scattering predominates at \( T_\epsilon \) only in the purest of
conventional materials and one may question whether it is
feasible for this condition to hold in YBCO. The most obvious
reason why it may be reasonable is the higher transition
temperature of YBCO; the relative weight of phonons in the total
scattering of carriers is substantially enhanced at higher
temperatures. Secondly, the strength of the electron-phonon
coupling may be substantially larger in YBCO than in any
conventional material.

The slope \( \frac{d\alpha}{dt} \) for strong coupling and predominant
electron-phonon scattering is directly proportional to the
product of the derivative, \( \frac{d(\Delta/k_B)^2}{dt} \), and a frequency average of the quasiparticle relaxation rate, \( \Gamma(\omega) \). Employing the mean-field expression for the gap at \( T_c \), a gap size of \( 2\Delta(0)/k_BT_c = 6 \) would enhance \( \frac{d(\Delta/k_B)^2}{dt} \) by roughly a factor \((6/4.1)^2 \approx 2\) over that of lead, for example. It is generally true that the more strongly \( \Gamma(\omega) \) decreases with frequency, the larger is the slope \( \frac{d\alpha}{dt} \). A strong frequency dependence can arise from sharply peaked structure in the coupling function, \( \alpha^2(\omega)F(\omega) \). In this regard we note that there is substantial evidence from Raman scattering\(^2\) that some carriers couple strongly to optical phonons, especially those modes yielding peaks in the phonon density of states \( F(\omega) \) near energies of 15meV and 45meV. Recent tunneling results\(^2\) on Bi\(_2\)Sr\(_2\)CaCu\(_2\)O reveal such structure in the derived coupling spectrum (with \( \lambda = 3.5 \)), and similar results for YBCO are anticipated.\(^2\) Thus it is possible that the contribution to \( \frac{d\alpha}{dt} \) from \( \Gamma(\omega) \) is also large. As more information about \( \alpha^2(\omega)F(\omega) \) becomes available it will be possible to examine these hypotheses in more quantitative fashion.

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1. For a recent review of thermal conductivity in the cuprates, see C. Uher, J. Supercond. 3, 337 (1990).
8. A monotonic behavior of $\kappa^n$ for $T < T_c$ is confirmed by the work of Palstra et al. [Phys. Rev. Lett. 64, 3090 (1990)] where the thermal conductivity of a YBCO single crystal was measured in a field of 10 Tesla, sufficient to suppress $T_c$ by about 5K.
14. V. Ambegaokar and L. Tewordt, Phys. Rev. 134, A805 (1964); V. Ambegaokar and J. Woo, Phys. Rev. 139, A1818 (1965);
20. It is implicitly assumed in the use of this integral equation that the relaxation-time approximation is valid. This means that Normal phonon-phonon collisions are neglected, a good approximation in crystals that are not too perfect (see, e.g. ref. 7, Ch. 8).
Figure Captions

Fig. 1  Thermal conductivity versus temperature for two YBCO single crystals.

Fig. 2  Lattice conductivity ($\kappa_L$) versus temperature. $\kappa_L$ was calculated by subtracting, from the data in Fig. 1, $\kappa_n$ as determined from the measured electrical resistivity and the WFL. The solid lines are calculated from Eq. (2) with $\Delta=0$ and parameters for sample 1 (2):

$A_{pd}=1.73\times 10^3$ (2.80$\times 10^3$) s$^{-1}$K$^{-4}$, $A_{pp}=6.0\times 10^7$ (1.4$\times 10^7$) s$^{-1}$K$^{-1}$, $A_{pp}=40$ (19) s$^{-1}$K$^{-4}$ (see text for details).

Fig. 3  The normalized thermal conductivity near $T_c$ plotted versus reduced temperature. Inset: values of the the reduced-temperature derivative of the normalized lattice thermal conductivity, $d\theta/dt|_{\xi_c}$, plotted versus gap size, as calculated from Eq. (2) using the same parameters as for the normal-state curves in Fig. 2.
Fig. 1

The figure shows the thermal conductivity $\kappa$ in units of $\text{W/mK}$ as a function of temperature $T$ in Kelvin (K). Two curves are plotted, labeled as 1 and 2, which appear to exhibit a peak at a certain temperature, followed by a decrease.
\[ \kappa_L (W/mK) \]

\[ T(K) \]