A PERFORMANCE MODEL OF THERMAL IMAGING SYSTEMS (TISs) WHICH INCLUDES THE HUMAN OBSERVER’S RESPONSE TO "STATE OF THE ART" DISPLAYS

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This paper presents a model for predicting the performance of thermal imaging systems (TISs). This model combines conventional modeling relationships\(^1\),\(^2\),\(^3\) and recently reported characteristics of display monitors\(^4\),\(^5\) to determine the signal-to-noise ratio (SNR) out of the TIS. Also included are the results of psychophysical experiments which evaluated the capability of a human observer to detect the presence of an object displayed on the same monitor\(^4\). The model is then used to determine the noise equivalent temperature difference (NETD) based on background photon noise limited (BLIP) operating conditions of the TIS. Finally, the minimum detectable temperature difference (MDT) in the scene is determined from the maximum signal-to-noise ratio of the monitor.
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I Introduction

The list of applications for thermal imaging systems (TISs) is forever expanding. It starts on the battlefields of the military for the detection of strategic targets and spans into areas like the surveying of air pollution in our metropolitan areas and medical diagnosis such as the detection of breast cancer. The growing use of these systems is continuously challenging engineers to predict the performance of their systems during the designing stages before the expense of assembling the system and without timely testing of the system under the same conditions of its intended use.

Much work has been done to model the performance of TIS's based on design specifications. A major limitation of these models is their attempt to include the human observer’s ability to detect an object displayed on a monitor. This paper includes the work of Hans Roehrig et al. at the University of Arizona’s Radiology Department. They have experimentally measured both the output and noise characteristics of several "state of the art" display monitors. They also performed several psychophysical experiments which characterize the human observer’s response to various test patterns projected onto the same monitors. They found close correlation between the physically measured signal-to-noise ratio and the psychophysically determined threshold contrasts and "Just Noticeable Differences" (JND's), which is the smallest luminance difference $\Delta L$ that can be displayed on the monitor and still detected by the human observer.
II Background

Figure II.1 depicts the basic application of a TIS. The target of interest is surrounded by a cluttered background. The photon flux difference generated by the temperature difference between the target and its background is considered to be the signal.

![Diagram of a Thermal Imaging System (TIS)](image)

*Figure II.1: A Thermal Imaging System (TIS)*

The optical signal passes through the atmosphere which attenuates the signal and blurs the image. The effects from this atmosphere are continuously changing and can be quite complicated to model. Therefore the influence of the atmosphere will be neglected...
The modeling will predict the capability of the system to detect a simple, stationary target like the square test pattern of Figure II.2 where the target's area is $A_o$ and its temperature is $T_o$, and the temperature of the background is $T_b$. Use of this test pattern will eliminate the need to include functions such as the optical transfer function (OTF) of the lens and the modulation transfer function (MTF) of the electronics which describe the system's response to spatial and temporal frequencies in the object.

The various subsystems of Thermal Imaging Systems (TISs) which are included in this model are shown schematically in the block diagram of Figure II.3. The remaining sections of this background are each dedicated to describing a separate subsystem.
Figure II.3: A Block Diagram of a Thermal Imaging System (TIS)

II.1 The Scene

The typical viewing scene contains a target of interest surrounded by a background. For detection, this target must exhibit an effective temperature difference from its background of sufficient magnitude to distinguish it from normal variations in the background. This temperature difference can be due to an energy exchange with the environment, an emissivity difference, self heating or from the reflection off of other sources.
II.2 The Detection System

The detection system is made up of a combination of imaging optics and a photodetector followed by processing electronics which first amplify the low level analog signal from the detector and then sample it with an analog to digital (A/D) converter. This digital signal can either be sent to a computer for evaluation or to a video monitor where it is displayed for a human user.

The exact nature of this system has varied from the early days of a single detector element scanned to provide a 2-D image of the scene, to the widely used 1-D "linear" detector array requiring only one mirror to perform scanning in the orthogonal direction. Future systems will incorporate current 2-D focal plane detector arrays and eliminate the need for mechanical scanning.

II.3 Digital Recognition

Evaluation of the scene and any targets in it can be accomplished in a purely digital manor with a variety of processing algorithms. The first step includes signal processing such as scaling, rotation and edge enhancement. The next step is to compare the signature of the target to a data base containing various objects of interest.

This comparison can be accomplished in a digital computer with the use of algorithms such as the Fast Fourier Transform FFT and "Matched" filtering to perform a correlation which will result in a peak output from an auto correlation when the stored and input target match.
Another approach includes transferring the enhanced image and the many filters to the optical domain via 2-D Spatial Light Modulators (SLMs). This approach utilizes the instantaneous Fourier Transform properties of a lens to provide a real time two dimensional correlation which could theoretically scan through more filters in a given time period.

These digital recognition systems require such a large database of comparative filters that they aren't practical at this point in time. Therefore they won't be the focus of this paper but references 7 and 8 are offered for interested readers.

II.4 Psychophysical Recognition

Standard TISs utilize the capabilities of the Human Observer to evaluate the target in a scene. The signal from the detector system typically has a raster structure with TV type frame rates. This signal is projected on a display device such as a CRT, an Image Intensifier Tube or a Light Emitting Diode Array. These display devices typically provide adjustable video gain and background level to improve the image contrast and brightness.

The observer's task is to detect an object or point on the display as a potential target of interest. Once detected the observer must recognize the object as a member of a class of targets, ie. a person, vehicle etc. Utilizing all of his past training, experience and image interpretation skills, the observer must then identify the target as a particular member of the class of objects ie. a model M60, T62, or M551 tank.
III Characterization of TISs

To analyze the limitations of a TIS one starts by modeling the performance of each component based on its design parameters. The individual performance and limitations of each device must then be cascaded to predict the performance of the system as a whole. To relate the various devices one must be familiar with the conversion relationships between unit quantities since the outputs of the stages are different. For instance; the scene’s output signal is represented in number of photons, the photons are converted into electrons in the detector, whose output can be a current or analog voltage level. The A/D converter outputs a digital series of voltages, and the monitor’s frame buffer interprets these digital grey levels (GLs) as voltage levels applied to the electron gun and the final output is luminance.

A more convenient method of comparison would include a unitless figure of merit which could be defined for each component.

III.1 Signal to Noise Ratio

A solution to the above problem is to find the signal and noise outputs of each stage and cascade these values to determine a signal-to-noise ratio (SNR) of the total TIS. The signal to noise ratio is a unitless characteristic which provides a theoretical limit for the performance of a system\(^1\). Mathematically, the SNR per pixel (SNRp) is the ratio of the mean value of the output signal (x) of a single pixel over the image to the standard deviation (σ) "noise" of the same signal.
\[ SNR_p = \frac{signal}{noise} = \frac{x}{\sigma} \] (III.1)

The SNR is used to determine figures of merit such as the noise equivalent temperature difference (NEΔT)\(^1\) which is a widely used measure of a TIS's ability to discriminate a small signal from a background of noise. The NEΔT is simply the temperature difference between the target and background of Figure II.2 which produces a SNR of one.

### III.1.A The Influence of the Human Observer

When including the response of the human observer in a model of a TIS one should recognize that the temperature differences in the scene are displayed to the observer as luminance differences. Although the human observer cannot determine the absolute luminance level projected from the display, contrast differences can be detected quite consistently. As the luminance difference (\(\Delta L\)) becomes small the detectability is dependant on the presence of noise in the signal. In order for the signal \(\Delta L\) to be perceived it has to larger than the noise \(\sigma\) by some factor \(k_o\).\(^9\)

\[ \Delta L = k_o \sigma \] (III.2)

where \(\Delta L = L_{\text{target}} - L_{\text{background}}\) and \(k_o\) can be interpreted as a threshold SNR.

Rosell and Willson\(^10\) did research in the area of human perceptions and found that a SNR of at least 3.1 is needed for a 50% probability that a human observer will detect an object. To increase this probability of detection to \(-\) 100% a SNR \(\geq\) 5.3 is needed. Roehrig et al. performed psychophysical experiments using a US Pixel monitor and found that for
their system $k_0 = 7.78$. However, a word of caution is necessary since Roehrig et al. used a "contrast detail" pattern which presented a series of objects simultaneously. Here the human observer is not asked if an object is present; rather the objects are always present and he is asked which one can be seen.

The detection capabilities of the human observer can also be described in terms of a threshold contrast ($C_t$):

$$C_t = \frac{\Delta L}{L_b} = \frac{k_o \sigma}{L_b}$$  \hspace{1cm} (III.3)

where $L_b$ is the luminance of the background and $L_o/\sigma$ is defined as the display signal-to-noise ratio ($SNR_D$). From the model of vision proposed by Rose\textsuperscript{11} the $SNR_D$ of an object displayed with a small contrast ($C$) is given by:

$$SNR_D = C \sqrt{\frac{N}{2} SNR_p} \geq k_o$$  \hspace{1cm} (III.4)

where $SNR_p$ is the signal to noise per pixel and $N$ is the number of pixels contained in the object. Substituting (III.4) into (III.3) results in an equation for the threshold contrast as a function of the $SNR_p$ for the observer to "detect" a target:

$$C_t = \sqrt{\frac{2}{N} \frac{k_o}{SNR_p}}$$  \hspace{1cm} (III.5)
III.1.B The Influence of Each Component

As stated earlier, the goal is to determine the SNR of each component and to cascade them to determine a SNR of the total system. Refer to Table III.1 for a listing of each component in the TIS. The second column contains the factor which describes the influence of the device. The "Type of PDF" is the probability density function which best statistically models the signal and noise of the component. The mean (x) and standard deviation (σ) relate to the signal and noise out of each device.

Following statistical methods, the effects of two uncorrelated variables can be added whose mean values (x₁ & x₂) and standard deviations (σ₁ & σ₂) are known:

\[ x_{1,2} = x_1 + x_2 \]  \hspace{1cm} (III.6)

\[ σ_{1,2} = \sqrt{σ_1^2 + σ_2^2} \]  \hspace{1cm} (III.7)

if two stages are cascaded the statistics are:

\[ x_{12} = x_1 x_2 \]  \hspace{1cm} (III.8)

\[ σ_{12} = \sqrt{x_2^2 σ_1^2 + x_1^2 σ_2^2} \]  \hspace{1cm} (III.9)

where \( x_{12} \) is the mean and \( σ_{12} \) is the standard deviation of stages 1 and 2 cascaded.
<table>
<thead>
<tr>
<th>Component of the TIS</th>
<th>Parameter which influences signal</th>
<th>Type of PDF</th>
<th>Mean ((x))</th>
<th>Standard Deviation ((\sigma))</th>
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<td>(\sqrt{G})</td>
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<td>Non Linear</td>
<td>(L=KE^n)</td>
<td>(\sigma_{out}=nKE^{n-1}\sigma_{in})</td>
</tr>
<tr>
<td></td>
<td>Imaging System: Electron Beam</td>
<td>Poisson</td>
<td>(\bar{N}_e)</td>
<td>(\sqrt{\bar{N}_e})</td>
</tr>
<tr>
<td></td>
<td>Phosphor Gain</td>
<td>Poisson</td>
<td>(G_p)</td>
<td>(\sqrt{G_p})</td>
</tr>
<tr>
<td></td>
<td>Phosphor Grain</td>
<td></td>
<td></td>
<td>(\sigma_{pg})</td>
</tr>
</tbody>
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*Table III.1: Different Components and Their Influence on a TIS*
III.1.C The SNR of Each Subsystem

Objects in the scene are assumed to be blackbodies which are perfect emitters of thermal electromagnetic radiation. The average photon flux \( \phi_p \) of these objects is related to their temperature \( T \) via the Stefan Boltzmann law for photon flux exitance \( M_p \):

\[
\bar{\phi}_p = A_\sigma M_p = A_\sigma \sigma_p T^3
\]

where \( A_\sigma \) is the object area and \( \sigma_p = 1.52 \times 10^{11} \text{ photons sec}^{-1} \text{ cm}^{-2} \text{ K}^{-5} \). This value \( \phi_p \) predicts the average rate of emitted photons over the total spectral range. The exact photon flux of infrared photons can be found by integrating Planck's Law over the wavelength range of interest\(^3\). For this model (III.10) is a fine approximation.

The mean number of emitted photons \( N_p \) can be found by dividing the photon flux by two times the bandwidth \( \Delta f \) of the system:

\[
\bar{N}_p = \frac{\phi_p}{2 \Delta f}
\]

The standard deviation \( \sigma \) from \( N_p \) is a measure of the uncertainty or "noise" present in the photon emission. The emission of photons can be accurately estimated using the Poisson\(^{14} \) probability density function where \( N_p \) is the "mean" number of photons emitted in a period of time \( \chi_1 \), with a standard deviation of \( \sigma_1 = \sqrt{N_p} \). Using these values the SNR for the scene is:
The average number of photons in the image plane of the imaging system \((N_{pl})\) is determined by the characteristics of the objective lens.

\[
\overline{N}_{pl} = M(NA)^2 \tau \overline{N}_p
\]  

where \(M\) is the magnification of the imaging system \((A_{image}/A_{object})\), \(NA\) is the numerical aperture, \((NA)^2\) is the coupling efficiency and \(\tau\) is the transmittance function of the lens.

Each of these predict the probability of an event happening. The throughput efficiency is the probability of a photon in the object plane being imaged onto the detector plane. The \(\tau\) is the probability that a photon striking the lens will be transmitted. These noises follow a binomial probability density\(^{14}\). When cascaded, the parameters of the imaging optics have a mean value of:

\[
x_2 = (NA)^2 \tau
\]  

and a standard deviation of:

\[
\sigma_2 = \sqrt{(NA)^2 \tau (1 - (NA)^2 \tau)}
\]

The cascaded SNR of the scene and the imaging optics is then:

\[
SNR_{12} = \frac{x_{12}}{\sigma_{12}} = \frac{\overline{M\overline{N}}_p (NA)^2 \tau}{\sqrt{\overline{M\overline{N}}_p (NA)^2 \tau}} = \sqrt{\frac{MA_o \sigma_p T^3 (NA)^2 \tau}{2\Delta f}}
\]
Detector noise can be generated by several mechanisms. The conversion of the flux noise in the scene creates shot noise, this has been included. Johnson noise is due to thermal current fluctuations and can be minimized by cooling the imaging and detector assembly. With skillful designing manufacturers have been able to eliminating the 1/f noise in photovoltaic detectors. This leaves the background photon noise limiting the performance of the detector. This situation is termed BLIP (background-limited photodetector) performance. The average number of photons ($N_{pd}$) on the detector is:

$$N_{pd} = \frac{A_d}{M A_o} N_{pt} = \frac{A_d \alpha_p T^2 (NA)^2 \tau}{M2\Delta f}$$  \hspace{1cm} (III.17)

where $A_d$ is the area of the detector element, $M$ is the magnification, $A_o$ is the object area, $MA_o$ is the area of the image in the detector plane and $N_{pt}$ is the average number of photons in the image plane. The average output current of the BLIP detector will be:

$$i = \eta \phi_{pd} q = \eta (N_{pd} 2 \Delta f) q$$  \hspace{1cm} (III.18)

where $\phi_{pd}$ is the photon flux on the detector and $q$ is the charge carried by one photoelectron. The probability that a photoelectron is produced when a photon is incident on the detector is $\eta$, the quantum efficiency. The rms noise current out of the BLIP detector will be:

$$i_{rms} = \sqrt{2qi\Delta f} = 2q\Delta f \sqrt{\eta N_{pd}}$$  \hspace{1cm} (III.19)

the ratio of (III.18) and (III.19) yields the SNR out of the photodetector:
Noise is added in the amplifier when the gain deviates from the expected gain. The gain of the amplifier follows a Poisson probability density function\textsuperscript{4}, with a mean value of $G$ and a standard deviation of $\sigma = \sqrt{G}$. The resulting SNR after cascading the amplifier’s response to (III.20) is:

$$ SNR_{1-4} = \frac{x_{1-4}}{\sigma_{1-4}} = \frac{\eta N_{P_d} 2\Delta f q G}{\sqrt{\eta N_{P_d} 2\Delta f q G(1+2\Delta f q G)}} = \sqrt{\frac{\eta N_{P_d} 2\Delta f q G}{1+2\Delta f q G}} $$(III.21)

if $2\Delta f q G \gg 1$:

$$ SNR_{1-4} = \sqrt{\eta N_{P_d}} = \sqrt{\frac{\eta A_d \sigma_p T^3(NA)^2 \tau}{M2\Delta f}} $$

notice that the SNR is independent of the gain of the amplifier.

The A/D converter samples the analog signal out of the amplifier and provides a digital equivalent which represents a grey level (GL) that can be input to the monitor’s frame buffer. An A/D with 8 bits resolution would provide GLs 0 through 255 from the input voltage, with GL 0 being zero volts and GL 255 corresponding to the maximum voltage signal out of the amplifier. Digitization noise is generated from the fact that a range of analog values are mapped to the same digital value. For example, with a full range of 10 volts from the output of the amplifier and 8 bits A/D, each GL represents a range of 39 mV.
Having this range causes an uncertainty in finding the original voltage value since, an output value of GL = 1 can be generated by an input voltage of exactly 39 mV or as high as 77 mV. This digitization noise manifests itself as "contouring" to the human observer, he sees adjacent digitization steps in the image instead of the noise from the scene.

The uncertainty can be described statistically in terms of a uniform distribution. The uniform distribution has a mean of $a/2$ and a standard deviation of $\sigma = a^2/\sqrt{12}$, where $a$ is the difference between neighboring quantization levels (1 GL). From this definition the digitization noise $.29$ GL and adds quadratically to the systems noise using (III.7)

$$\sigma_{1-5}^2 = \sigma_{1-4}^2 + \sigma_5^2$$  \hspace{1cm} (III.23)

This quantization noise will be considered insignificant if its magnitude is less than 1% of the other noises, $\sigma_{1-5} \leq 1.01 \sigma_{1-4}$:

$$\sigma_{1-4}^2 + \sigma_5^2 \leq 1.02 \sigma_{1-4}^2$$ \hspace{1cm} (III.24)

i.e.: if the systems noise $\sigma_{1-4}$ is greater than $7.07 \sigma_5$ or $2.08$ GL, the A/D noise can be neglected. and the SNR out of the A/D converter will be the same as the input:

$$SNR_{1-5} = SNR_{1-4}$$ \hspace{1cm} (III.25)

In real life "noisy" applications this condition is met.

The next step is to model the display monitor. A typical CRT monitor is composed of several components. The "monitor electronics" consist of a frame buffer which excepts digital inputs, a D/A converter which converts the digital signal to an analog voltage level
and an electron gun which converts the voltage into a stream of electrons. The electrons from the gun are then imaged onto a phosphor screen which emits photons.

The relationship between the voltage level into the electron gun and the number of electrons released is a non linear one. A result of this non linearity can be seen in the characteristic curve of the monitor. Figure III.1 shows the characteristic curve of a US Pixel monitor measured by Hans Roehrig et al.\textsuperscript{5}.

![Characteristic Curve](image)

*Figure III.1: The Characteristic Curve for a US Pixel Monitor (reprinted from Browne\textsuperscript{5})*

A typical expression for the characteristic curve is\textsuperscript{5}:

$$L = KE^n$$  \hspace{1cm} (III.26)

where $L$ is the output luminance per pixel, $K$ is a constant and $E$ represents the grid drive voltage. The exponent $n$ is device dependent, and is usually between 2 and 3. The
experimentally determined \( n \) for the US Pixel monitor is 2.9.\(^4\)

The relation between the noise into the monitor from the proceeding components \((\sigma_{1.5})\) and the noise in the output signal is characterized by the derivative of (III.26)\(^5\):

\[
\sigma_{out} = nKE^{n-1}\sigma_{in} \tag{III.27}
\]

neglecting the noise generated by the monitor itself an equation for the SNR per pixel out of the monitor can be formed by combining (III.26), (III.27) and (III.22):

\[
SNR_{out} = \frac{L}{\sigma_{out}} = \frac{KE^n}{nKE^{n-1}\sigma_{in}} = \frac{1}{n}SNR_{in} \tag{III.28}
\]

\[
SNR_p = \frac{1}{n}SNR_{1.5} = \frac{1}{2.9}\sqrt{\frac{\eta A_d \sigma_p T^3(NA)^2 \tau}{M2A_f}} \tag{III.29}
\]

This equation models the temporal and spatial SNR's per pixel. It would be considered a temporal SNR\(_p\) if one pixel were monitored over time and the noise was the standard deviation of the luminance about the average value. The spatial SNR\(_p\) is determined from scanning the screen and finding how the luminance varies from pixel to pixel. If both sources of noise are present the standard deviations add as in (III.7):

\[
\sigma_{total} = \sqrt{\sigma_{temporal}^2 + \sigma_{spatial}^2} \tag{III.30}
\]

The components considered above all produce a temporal noise which is equal to its spatial noise.
The monitor creates noise of its own. As seen on Table III.1 the electron beam has shot noise which is temporal, there can be both temporal and spatial variance in the phosphor’s gain and phosphor’s granularity produces spatial noise. The temporal noise of the monitor is found by cascading the electron beam’s shot noise to the phosphor’s gain:

\[ \sigma_{\text{temporal}} = \sqrt{G_p N_e + N_e G_p} = \sqrt{2G_p N_e} \]  

(III.31)

The spatial noise of the monitor is found by adding the phosphor’s noises in quadratically:

\[ \sigma_{\text{spatial}} = \sqrt{G_p + \sigma_{pk}^2} \]  

(III.32)

Roehrig et al. measured both the temporal and spatial noise of the monitor\(^4\). The results are found in Figure III.2. As the graph shows, the spatial noise is much larger than the temporal noise.

![Figure III.2: The measured Temporal and Spatial Noise of a US pixel Monitor](image)

(reprinted from Tinglan\(^4\))
From their measurements they determined the $\text{SNR}_p$ out of the US Pixel Monitor to be $\approx 30$ where the input signal was a noiseless computer generated grey level. This would be considered the "monitor noise limited" case.

The noise from the monitor will add to that of the system:

$$\sigma_{\text{total}} = \sqrt{\sigma^2_{1-s} + \sigma^2_m}$$

(III.33)

When the noise into the monitor is much greater than the monitor noise $\sigma_{1-s} \gg \sigma_m$ the system is operated under background photon noise limited (BLIP) conditions and (III.29) determines the signal-to-noise ratio per pixel ($\text{SNR}_p$) of the system.

### III.2 Noise Equivalent Temperature Difference (NE$\Delta$T)

As discussed briefly above, a figure of merit used to describe TIS's is the NE$\Delta$T. The NE$\Delta$T by definition is the blackbody target-to-background temperature difference which is just large enough to provide a SNR per pixel of 1$^1$. The target should be an extended source large enough to fill the NA of a detector element assuring a good signal response. The basic expression for NE$\Delta$T is$^1$:

$$\text{NE} \Delta \text{T} = \frac{\Delta T}{\text{SNR}}$$

(III.34)

Operating under BLIP conditions a more useful expression can be derived from (III.29). Since the signal of interest is the temperature difference $\Delta T$ will replace $T$.
for small \( \Delta T \)'s where \( \Delta T = T_{\text{target}} - T_{\text{background}} \), setting \( \text{SNR}_p = 1 \) and solving for \( \Delta T \) leaves:

\[
\text{NEAT} = \Delta T = \frac{(2.9^2(M2\Delta f))^{1/3}}{\eta A_{\phi} \sigma_p (NA)^2 \tau}
\]

Figure III.3 is a typical setup to determine this \( \Delta T \). The blackbody source in the object plane starts at the ambient background temperature. The baseline noise signal of the system is measured by a detector at the output of the monitor. The temperature of the blackbody is increased until the signal measured at the detector equals the noise. The \( \text{NEAT} \) is the difference between the final and initial temperatures of the blackbody. Note that the detector is monitoring only one pixel of the image, this is a measure of the temporal \( \text{SNR}_p \). Since the temporal noise of the monitor is small the system's noise will dominate and the BLIP approximation is a proper one.
The NEAT is a value which quantifies the baseline noise signal of a thermal imaging system. It isn’t a very useful quantity because it doesn’t predict whether or not the target will be detected. On the other hand, it can be useful as a tool to compare two different designs, where the one with the smaller NEAT is the better design.

### III.3 Minimum Detectable Temperature Difference (MDT)

The minimum detectable temperature difference (MDT) is similar to the NEAT, but it goes one step farther and incorporates the response of the user as seen in Figure III.4. The MDT is the required target-to-background temperature difference for a human observer to detect a square target as a function of the target’s size. This MDT is a figure of merit which characterizes the system’s ability to resolve point sources. Note that since the user observer’s the entire image on the display the monitor’s spatial noise is dominant.

---

**Figure III.4: An experimental setup to determine the MDT of a TIS**
Recall that the detection capabilities of the human observer can be described in terms of a threshold contrast ($C_t$):

$$C_t = \frac{\Delta L}{L_b}$$

including (III.26) and (III.10) the threshold contrast can be written in terms of the temperature differences in the scene:

$$C_t = \frac{\Delta L}{L_b} = \frac{\Delta E^n}{E^n_b} = \frac{\Delta T^3}{T^3_b}$$

where $\Delta L = L_{\text{target}} - L_{\text{background}}$, $n$ is the nonlinear exponent (2.9 for the US Pixel Monitor) and $\Delta T = T_{\text{target}} - T_{\text{background}}$.

![Graph showing threshold contrast vs. SNR for 2 different object sizes on the US Pixel Monitor.](reprinted from Tinglan)
Figure III.5 was generated from the psychophysical experiments of Roehrig et al.\textsuperscript{4}. The curve represents the threshold contrast as a function of the SNR per pixel (SNRp) out of the display for various object sizes. Recall from above that the SNRp for the US Pixel monitor with dominant spatial noise was 30. This is the theoretical limit for the TIS. If the system isn't "monitor-noise" limited, the SNRp is determined from the background noise limited (BLIP) operating conditions and (III.29). In the experiment, computer generated white noise of various standard deviations was added to the monitor noise to simulate the effects of the TIS.

The response depicted in Figure III.5 can be modeled using (III.5) repeated here for the reader's conveyance:

\[
C_i = \sqrt{\frac{2k_o}{N SNRp}}
\]  

(III.5)

where \(k_o\) is the threshold SNR and has been experimentally determined to be 7.78, N is the number of pixels of the target, and the SNRp is the SNR per pixel out of the TIS.

The minimum detectable temperature difference (MDT) can be calculated by combining (III.37) and (III.5):

\[
MDT = \left[\frac{\sqrt{2k_o}}{\sqrt{nSNRp}}T_b^{3n} + T_b^{3n}\right]^{\frac{1}{3n}} - T_b
\]

(III.38)

for a background temperature of 300 K and the US Pixel monitor with \(n=2.9\), the MDT is
plotted for a range of possible system signal to noise ratios in Figure III.6. Each curve represents a different object size (5x5, 10x10 or 15x15 pixels). As discussed above the theoretical limit would be for the "monitor noise limited case" seen to the right of the curve with $SNR_p = 30$. As the figure shows, the minimum detectable temperature difference increases from this value with the addition of noise to the system, decreasing SNR. Also, the temperature difference required to detect an object decreases with larger objects.

![Figure III.6: The MDT as a function of $SNR_p$ for three different sized objects](image)

The MDT determines the required temperature difference from the background for the detection of a simple, impractical, square target. A more useful measure of a TIS's performance is the minimum resolvable temperature difference (MRT) which is a function of spatial frequency$^1$. Psychophysical experiments could be performed to determine the
threshold contrast as a function of spatial frequency by replacing the square test target of Figure II.2, with bar patterns such as Figure III.7. The observer would determine the maximum frequency which can be detected for each of a range of contrasts. The Rose model of (III.5) would have to be revisited to include this spatial frequency dependence. Finally, the optical transfer function (OTF) of the imaging system and modulation transfer function (MTF) of the electrical components which were rightfully eliminated above would have to be included.

Figure III.7: A sample test pattern to determine the MRT

(reprinted from Lloyd)
IV Summary

This paper started with a description of a typical thermal imaging system (TIS). The sections which followed discussed the influence of the human observer and each separate component on the signal and noise out of the TIS. Equations for the signal to noise ratio through the system are formulated based on both conventional design parameters and recently measured values which model the performance of a "state of the art" US Pixel monitor. Models for the noise equivalent temperature difference (NEAT) and the minimum detectable temperature difference (MDT) which includes the response of the human observer to the thermal imaging system were formed. Finally, suggestions for future work to include a spatial frequency dependence of the target and determine the minimum resolvable temperature difference (MRT) were offered.
References


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