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PROCEEDINGS OF THE
8TH MIT/ONR WORKSHOP ON C³ SYSTEMS

EDITED BY:
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ABSTRACT

This paper has a threefold thrust: (1) a brief survey is presented of the development of approaches to modeling C2/C3 systems as given primarily in this forum. The MIT/ONR Workshop on C3 Systems; (2) an outline of a theory of C3 systems is developed which is compatible with previous efforts and which is rich enough for rigid, yet tractable, analysis; (3) as part of this theory, a procedure is exhibited for integrating subjective and objective/probabilistic/numerical information for C3 system decisionmakers.

1. INTRODUCTION

The C3 problem is a real-world problem and thus, analogous to theories in Chemistry, Physics, or Biology, a proposed theory for a C3 system must be based on empirical, as well as sound, logical considerations. In addition, such a theory following the usual pattern of change for scientific inquiries- will incorporate, overlap to some degree, or otherwise relate with, previously established models. Finally, the author’s own biases and predilections will generally be reflected in the degree of detail granted to the various components of the overall model.

Compatible with the above philosophy, the goal of this paper is the development of a general C3 theory which accounts for a systematic/comprehensive treatment of the combination of subjective information- such as linguistic-based descriptions- with the usual probabilistic or numerical type information. In conjunction with this effort, a literature search was conducted for previous work in this area. In addition to the premier collection of unclassified C2/C3 work- these proceedings over the past eight years- other unclassified sources were also considered, including IEEE publications, Operations Research journals, Psychology publications, and separately published papers and books, among others. A brief survey of that portion of the literature relevant to the task here is presented in the next section. In section 3, general models of warfare and C3 systems are proposed in the form of networks whose nodes represent decision makers/followers. These networks are also assumed to be time-varying. Section 4 is an abridged analysis of intranodal behavior, utilizing both probabilistic and possibilistic processes, analogous to the previous established PACT (Possibilistic Approach to Correlation and Tracking) program in Ocean Surveillance [55].

2. BRIEF SURVEY OF RELEVANT C3 WORK

A now extensive C3 and related discipline literature exists solely within the first seven annual Proceedings of this journal (283 articles). Perhaps because of the great complexity of the overall C3 problem, relatively few papers have been written establishing quantitative models of generic C3 systems. Of course, this does not detract from the progress made for various aspects of the problem proper and for related issues. Foremost among the latter is Surveillance, and in particular, multi-target tracking and data association. To a lesser degree, Data Base Management and Communications within C3 systems have also been extensively treated quantitatively. Similarly, limited portions of the C3 problem proper have been thoroughly advocated under appropriate conditions- including command decision theory, viewed as a possible multiple player statistical decision game involving, typically, threat situations and system effectiveness reflected on the loss or objective functions, as e.g. in [1], or considering players’ mental images of one another together with limited knowledge of rules of play, as in [2]. In a similar vein, distributed or decentralized decision theory appears to be a valuable tool for analyzing C3 systems which may be spread out geographically or otherwise have loose communications structures. (See, e.g., Tenney [3]-[6*] and Sandell [5] for basic results in this direction.) Complexity of distributed decision problems relative to C3 was presented in [6] in the form of NP-completeness. Other general results, including asymptotic forms, may be found in Tsitsiklis’ general work [7].

Hierarchical games and systems were used as models for parts of C3 systems by Castanon [8] and others [9]. Later, Castanon [10] applied rational aggregate theory to linear dynamic state processes to obtain sequential (relative to hierarchy level) solutions of systems with hierarchies defined by behavior tempo having also possible uncertain models. (See also Luh et al. [11] for other aspects of hierarchical systems useful in C3.) Often, C3 systems have been defined as essentially involving the management of military resources. In conjunction with this, a number of papers have considered resource allocation techniques [12],[13], e.g. as the prime characterization of C3 systems. In addition, as mentioned numerous times, C3 analysis requires multidisciplinary usage. For example, Control Theory could be thought of as central to the problem [13*], e.g.]. Many papers have concentrated on the human decision maker-in-the-loop aspect, as a perusal of the last two Proceedings of this journal will show. Such papers can vary in thrust of analysis from input-output conditions of models [14] to various detailed (sow, qualitative, others, quantitative in scope) internally analyzed systems as in [15] or Wohl’s and others’ extended SOR(Sense, Hypothesize,Option,Response) paradigms [16]-[18], related to Lawson’s proposals [19],[20].

Although as mentioned above- few papers have attempted to analyze the overall C3 problem quantitatively or qualitatively, those that have, have engendered much controversy. Consider first those qualitatively oriented papers attempting to define or analyze general C3 systems. Lawson [19],[20] was among the first to propose a general theory of C3, based to a degree on analogues with thermodynamic principles, motivated by the classic Manchester equations of force attrition or increase. Later, he emphasized time as a critical factor in all aspects of a C3 system [21], considered briefly C3 sys-
tems from a knowledge-based systems viewpoint, among other items (22], and proposed gener
cic experiments for analyzing C3 systems (23}. Athans also has been active 
in attempting to define the overall C3 problem, beginning 
with the First Workshop (24]-[26] and culminating 
with his view of "expert team of experts" for command-
eres (27]. Other good qualitative overviews of the problem 
may be found in (28]-[31) as well as the short paper 
(32}. See also the more recent comments of Ron (33] 
and Heterk (34). The latter emphasizes expanding 
Lawson's ideas (as mentioned above) to other decisionmakers 
and the interaction in some systematic way of subjective and objective 
information. (This is comparable with section 4 here.) 
Strack (35] has compiled possibly the most far-reaching 
of qualitative analyses of C3 problems in his recent 
report. In a related direction, development of measures 
of-effectiveness (MOE) for C3 systems in general began 
in earnest with Lawson's concern for time/tempo of 
operations (such as in [21]) and Hamon and Brandenberg 
working on internodal and intranodal measures, among 
other topics (36]. Further work in this area has been 
carried out by Bouthonier and Levis (37) (in conjunc-
tion with Levis' organizational approach - see below), 
Linsenmeyer's countermel-easure-oriented MOE paper (38], 
and recently, by Karam and Levis (39].

Recently, two additional approaches have been 
proposed for modeling general C3 systems, which like 
Lawson's earlier proposals are most appropriate for 
large scale system behavior of C3 components typically 
representing men in the field and supplies. Anthony 
(40) proposes four candidate, empirically-verified laws 
resulting from other disciplines as governing C3 systems. 
Mayer (41], somewhat similar to Lawson (3], presents a 
thermodynamics/uncertainty principle approach which 
regulates the more "irreducible primitive" components 
of C3 systems. In addition, Rubin (42], following 
guidelines in (41], under semi-Markov and Markov 
assumptions, derived explicit forms for various sta-
tistics of the processes acting as links among the components of a C3 system. In particular, Lanchester's equations 
were shown to be a special case of this model.

The approach taken in this paper (section 3) 
follows to a degree the general view of Levis et al. 
([43]-[49]. There, a C3 system is considered to be a 
collection of interacting decisionmakers, which as a 
whole, may follow (under appropriate-limiting conditions) 
macroscopic principles (such as Lawson proposes, e.g.). 
However, critical to the analysis is the micro-
scopic analysis of each decision maker or node repre-
senting a unit of decisionmakers acting through cooperation 
as a single individual. The structure of each 
decisionmaker follows the general pattern as the SHOR 
paradigm or variations. Then a quantitative (normative-
descriptive) measure is obtained for each such decision-
maker in the form of the total workload, i.e., entropy 
of all internal random variables connected with decision-
action and choice of related algorithms, involving 
also possible interaction with other decisionmakers 
during this process, as well as accounting for memory. 
By simple summation over all decisionmakers, an overall 
measure of workload G can be obtained. Alter-
natively, the overall joint workload can also be used. 
Anothcr overall performance measure J is assumed obtain-
able, such as effectiveness of overall system in dealing 
with the enemy, so that both G and J are assumed to 
be dependent functionally on G. In a computable manner - on W, 
the internal variables of the decisionmakers. Thus, 
possible tradeoffs or optimizations of G and J 
can be considered relative to W, subject to natural con-
straints on G resulting e.g. from bounded rationality 
invoking G(W) and/or satisfying conditions connected 
with J(W).

The problem of processing and integrating sub-
jective or linguistic-based information occurring with-
in or between decision nodes and stochastic-based data 
in C3 systems is an extension of that for the surveil-
zance problem. In both situations, it may not be appro-
riate to model both types of information stochastically. 
In place of this, a possibilistic or multi-valued logic-
based analysis may be the proper choice. (See [50] for 
motivations, background, and further details.) Zadeh 
([51],[52] originally propounded in these proceedings use 
of possibilities in place of probabilities only, 
decisions that could typically occur in C3 systems.

Similarly, Goodman employed such an approach - using it 
also with the coverage and incidence functions of 
C3 systems (i.e., random sets) - in addressing the data association problem in tracking [53]-

[55]. Other approaches to the modeling of subjective 
information that occur for C3 systems have used forms of 
expert knowledge-based systems (56],[57]. Still others 
have considered use of neural network theory and 
the related area of self-organizing systems for 
C3 analysis such as (58),(59) for background.)

3. OUTLINE OF A C3 THEORY

This section outlines a C3 theory which to some 
extent follows the spirit of Levis et al. ([46],e.g.) 
in considering a C3 system dependent upon its local be-
haviors and analyzing the latter. (See also the discus-
sion in section 2.)

First consider a warfare process. A warfare pro-
cess V is a time-indexed process given for convenience 

\[ V \triangleq (V_t)_{t \geq 0} \]  

(3.1) 

where each \( V_t \) represents the overall warfare situation for some pre-described region at time \( t \). (Note, that the term "process" and likewise all variables to be intro-
duced below are to be interpreted in possibilistic terms 
in general, not necessarily probabilistic. Again, see 
[50] or [52] for background.) In turn, each warfare sit-
uation consists of a collection of C3 systems

\[ V_t \triangleq (C_{t,j} \mid j \in K_{t,2}) \]  

(3.2) 

where \( K_{t,1} \) is some index set and each \( C_{t,j} \) is some C3 
system of interest. These C3 systems may in a sense (to 
be explained) overlap, be subsets of each other, or be 
disjoint, reflecting both the design of the individual 
systems and the choice of levels of analysis. \( V_t \) can 
be partitioned into

\[ V_t = \bigcup_{j \in K_{t,2}} (V_{t,j}) \]  (disjointly)  

(3.3)

where \( K_{t,2} \) is the index set of adversaries in conflict, 

\[ V_{t,j} \triangleq (C_{t,j} \mid j' \in K_{t,1}) \]  

(3.4) 

and

\[ K_{t,1} = \bigcup_{j \in K_{t,2}} (K_{t,1,j}) \]  

(3.5) 

is a corresponding decomposition of index sets. 

Often, \( K_{t,2} = \{(1,2)\} \)  

(3.6) 

where \( 1 \) represents friendly forces and 2 that of hostile 
ones.

In general, each C3 system is represented as a 
type of network through the following ordered quadruple:

\[ C_{t,j} \triangleq (N_{t,j},I_{t,j},O_{t,j},M_{t,j}) \]  

(3.7) 

where

\[ N_{t,j} \triangleq (N_{t,j,k} \mid k \in K_{t,3,j}) \]  

(3.8)
is the set of all nodes of the network;

\[ I_{t,j}^{d} = \{ I_{t,j,k} | k \in K_{t,3,j} \} \] (3.9)

is the set of all inputs (at t) of the network;

\[ \bar{O}_{t,j}^{d} = \{ \bar{O}_{t,j,k} | k \in K_{t,3,j} \} \] (3.10)

is the set of all outputs (at t) of the network; and

\[ M_{t,j}^{d}(N_{t,j}, J_{t,j,k}^{J}, k | k \in K_{t,3,j}, (J', k') \in K_{t,3,j}, k' (3.11) \]

is the set of all media/environment/noise involving any node in the network with any other node (of any other network), where \( K_{t,3,j} \) is the index set of all nodes for \( C_{t,j} \) and \( K_{t,3,j} \) is an index set representing those possible nodes outside of \( N_{t,j,k} \) to which an initial output can be directed (whether on purpose or due to general radiation patterns, distances, etc.). Thus

\[ K_{t,3,j} = \{(J', k') | J' \in K_{t,2}, k' \in K_{t,3,j} \} \] (3.12)

and

\[ \bar{O}_{t,j,k}^{d} = \{ \bar{O}_{t,j,k,j', k}^{J} | J' \in K_{t,3,j} \} \] (3.13)

is the decomposition of the output at node \( N_{t,j,k} \) into possible outputs directed towards other nodes (for all adversaries).

Hence, \( (I_{t,j}, \bar{O}_{t,j,k}) \) is the input-output pair for node \( N_{t,j,k} \) at t. But the causal or semi-causal relation between inputs and outputs is given as: \( I_{t,j,k} \)

resulting in \( \bar{O}_{t,j,k}^{d} \), for some \( t_{2} \neq t_{1} \) through

\[ H_{t_{1}, t_{2}, j, k}^{d}(N_{t,j,k}, t_{1}, s_{: t_{1}}) \ (3.14) \]

due to processing delays within the node, as some version of the SHOR paradigm is carried out interacting possibly with other decisionmakers, etc. In (3.13), each \( \bar{O}_{t,j,k,j', k}^{J} \) is that output from \( N_{t,j,k} \) directed towards \( N_{t,j', k} \), through medium \( H_{t,j,k,j', k} \). Thus, typically, the additive-like regression relation holds (where again, note that the values involved may be non-numerical in nature—hence the use of \( \Theta \))

\[ I_{t_{2}, j', k}^{f} = f_{t_{2}, t_{1}, t_{2}, j, k, j', k}^{R} \bar{O}_{t_{1}, j_{1}, j_{2}, k_{1}, k_{1}}^{D} \] (3.15)

where \( f \) represents some function and \( R \) some noise, where possibly the constraint

\[ t_{1} \leq t_{2} \leq t_{1}, s_{t_{1}}^{R} \] (3.16)

holds.

Next, each node is internally represented as an ordered quadruple

\[ N_{t,j,k}^{d}(S_{t,j,k}, \bar{S}_{t,j,k}^{D}, S_{t,j,k}^{C}, \bar{S}_{t,j,k}^{D}) \] (3.17)

where \( S_{t,j,k} \) is the true state vector of \( N_{t,j,k} \), possibly unknown to the decisionmaker complex \( D_{t,j,k} \) of \( N_{t,j,k} \) and evolving in time according to possibilistic, or, in particular, probabilistic transition values. Typically, \( S_{t,j,k} \) can contain entries (possibly decoupled) for location and pattern of deployment of individuals

with \( N_{t,j,k} \), number of personnel there, equation of motion parameter values for that portion of the node involved in movement or going to battle, and weapon descriptions, if any weapons are present at the node. Similarly, \( D_{t,j,k} \) is the node's estimate of its own state, while \( S_{t,j,k} \) is the node's estimate of all remaining relevant state vectors outside of the node. Finally, \( D_{t,j,k} \) need not be a decisionmaker(s) in the narrow sense, but may also indicate a follower complex (such as a unit of soldiers ready for combat and following command orders). Use of \( D_{t,j,k} \), possibly with \( N_{t,j,k} \) and \( C_{t,j,k} \), if not vacuous, leads to the basic input-output mentioned around eq. (3.14). (One aspect of this will be given in section 4.)

Overall (real- or vector-valued) performance measures \( J_{t_{1}, j_{1}, j_{2}, j_{3}, ..., k} \), can be constructed for each \( C_{t,j} \) system \( C_{t,j} \), generally through some function, such as addition, numerical averaging, or retaining the joint form of local performance measures at each node. Thus, e.g., one could have

\[ J_{t_{1}, j_{1}, j_{2}, j_{3}, ..., k}^{d} = \varepsilon (J_{t_{1}, j_{1}, j_{2}, j_{3}, ..., k}) \] (3.18)

where each \( J_{t_{1}, j_{1}, j_{2}, j_{3}, ..., k} \) is considered a function of the internal decision variable possibility functions of \( D_{t,j,k} \) through the relation

\[ J_{t_{1}, j_{1}, j_{2}, j_{3}, ..., k} = \varepsilon (\varepsilon (Y_{t_{1}, t_{2}, t_{3}, j_{1}, ..., j_{k}^{(N_{t,j,k, t_{1}})}), \varepsilon (Y_{t_{1}, t_{2}, t_{3}, j_{1}, ..., j_{k}^{(N_{t,j,k, t_{1}})}))) \] (3.19)

where \( Y_{t_{1}, t_{2}, t_{3}, j_{1}, ..., j_{k}^{(N_{t,j,k, t_{1}})}} \) is a collection of internal variables of \( D_{t,j,k} \) operating over time interval \( [t_{1}, t_{2}] \); similarly for the inputs \( I_{t_{1}, t_{2}} \); and where \( \varepsilon \) is an appropriately chosen function. Quote marks surround the expectation since possibility functions may be involved, in which case a possibilistic measure of central tendency replaces ordinary probabilistic expectation [50].

Thus, as mentioned earlier, one can then determine tradeoffs between various performance measures of a given \( C_{t,j} \) or even of \( C \), through admissible possibility functions, here, corresponding to \( W \).

It is of some interest to determine if under reasonable conditions, as the number of nodes increases indefinitely, behaving in some "random" manner, that the proposed thermodynamic-type \( C \) models can be obtained as limiting cases of the model presented here. At present, work is being carried out in this direction. Further details of the general theory presented here will be presented in a later publication. For the present, analysis will concentrate on intranodal use of subjective and objective information, in order to obtain the basic input-output equations.

4. COMBINATION OF EVIDENCE AT NODES

In this section, some quantitative results are derived for intranodal behavior of a \( C \) system.

Consider any node \( N_{t,j,k}^{d}(I_{t,j,k}, S_{t,j,k}) \) with internal variable set \( W_{t,j,k}^{d}(I_{t,j,k}, S_{t,j,k}) \) and possibly additional input set of variables during processing time \( [t_{1}, t_{2}] \). \( I_{t,j,k}^{d}(I_{t,j,k}, S_{t,j,k}) \), as well as original input set \( I_{t,j,k} \). Without loss of generality, suppose subjective components of the relevant quantities below are indicated by primes as superscripts, while objective/possibilistic ones are denoted by superscripted double primes.
With all of this established, the basic question arises as to the behavior of $\phi(0,1)$ as more and more of the probabilistic information is used in terms of the discretization procedure, i.e., what is $\lim_{p \to \infty} \phi(0,1)^p$?

The following theorem has an analogue for the PACT application ([50], Chap. 9); but differs somewhat in structure from the forms presented there.

**Theorem**

Suppose that all constructions hold as presented in (3.28) for any index $p$, where for convenience $f$ is assumed to be also bounded. Suppose also the following:

1. $\phi$ as a function of two arguments possesses continuous second order derivatives in some neighborhood of $(0,0)$.

2. $\phi_{or}$ is an Archimedean t-conorm, i.e., for the two argument case, for example,

$$\phi_{or}(x,y) > x, \forall 0 < x < 1.$$  

(Many t-conorms are Archimedean and indeed it can be shown that arbitrary t-conorms can be written as affine types of mixtures (called ordinal sums) of Archimedean and the non-Archimedean t-conorm max. Again, see [50], Chap. 2.3.)

3. The corresponding generating function $h$ to $\phi_{or}$, see Proof below for discussion) has a continuous second order derivative in some neighborhood $[1-\epsilon,1]$ of 1, $0 < \epsilon < 1$.

Then,

$$\lim_{p \to \infty} \phi(0,1) = \phi_{or}(0,1).$$  

where $0 < x < 1$

and $\phi_{or}$ is given in (3.46) in terms of the ordinary expectation of also nondecreasing function $\omega$ of $F(Z',Z''|1)$, with respect to $f(Z',Z''|1)$ now formally a random vector corresponding to p.d.f. $f$ is given in (3.40).

**Proof:**

A. A relatively deep theorem from the theory of probabilistic metric spaces ([60]) shows first that any given Archimedean t-norm, say $\phi$, has an essentially unique generating function $h:[0,1] \to [0,1]$, where $[0,1]$ denotes the positive real line with 0 annexed. That is, $h$ is continuous nonincreasing with

$$h(1)=0; \quad h(0) = \infty$$  

such that for all positive integers $n$ and all $0 < x_1, \ldots, x_n < 1$,

$$\phi_h(x_1, \ldots, x_n) = h^{-1}(\min_{j=1}^n h(x_j)).$$  

The definition of an Archimedean t-norm is dual to that in (3.32):

$$\phi(x,y) > x, \forall 0 < x < 1.$$  

where as in (3.26), (3.27)

$$\phi_{or}(0,1) = \phi_{or}(0,1)$$  

and it is assumed that $\phi_{or}$ represents a compound combination of $\phi_{or}$ applied to probabilistic information followed by $\phi_{or}$ applied to subjective information. In general, the two t-conorms may be different ([50], Chap. 10).
Although any pair of t-norm and t-conorm need not be DeMorgan, any t-conorm can be expressed as the DeMorgan transform of some corresponding t-norn. Furthermore, if one is Archimedean, then so is its DeMorgan transform. Thus, one can let $\phi_{or}$ in (3.32) be written as, for all positive integers $n$, etc.

$$\phi_{or}(x_1,\ldots,x_n) = \phi_{or}(1-x_1,\ldots,1-x_n)$$

where

$$\phi_h(x) = \frac{1}{h(1-x_j)} \left( \min(h(0),x) \right), \quad j=1$$

using assumption $2$.

B. From assumptions 1 and 3,

$$\phi(x,y) = \frac{\phi(x)\phi(y) + 0(y^2; x)}{2h(0)} \quad (3.39)$$

where

$$h(0) = \phi_h(x)$$

and

$$h(1-z) = c_h \cdot z + 0(z^2), \quad (3.41)$$

where

$$c_h = (d(z)/dz)|_{z=1} > 0.$$  

$1 < c_h$ denotes the usual "order of" relation, and $x,y,z$ are arbitrary such that for $c_1,c_2$ fixed

$$0,1 < c_2 < c_1.$$  

C. For any $0 < x_1,\ldots,x_n < 1$, using (3.37) and (3.41),

$$\phi_{or}(x_1,\ldots,x_n) = \phi_h(c_h \cdot x_1^0(x_2^0)) \quad (3.44)$$

Apply (3.39) to (3.31) and (3.28), and then replace each $x_j$ in (34) by $0(0^2; z')$ with index $z'$ in $D'$ replacing $j$, $j=1,\ldots,n$. This yields

$$\phi_{or}(0(0^2; z'), z') = \phi_h(c_h \cdot x_1^0(x_2^0)) \quad (3.45)$$

and

$$z' \in D', \quad O(0,0^2; z') \quad (3.45)$$

The main result then follows using (3.45) in (3.30), where

$$\lim_{D' \rightarrow \infty} \phi_{or}(0(0^2; z'), z') = \phi_h(c_h \cdot x_1^0(x_2^0)) \quad (3.46)$$

Thus, up to essentially increasing transforms, the approach taken here to combining subjective information with objective information involves taking an expectation of the latter and using a multi-valued logical procedure on the former in a unified way, up to the level of discretization/renement used for the probabilistic information. As a final remark, it should be noted that most common t-norms and t-conoms satisfy the rather mild analytic conditions required in the hypotheses of the theorem and max can also be used in place of an Archimedean form for $\phi_{or}$, with appropriate modifications of the proof. (See Appendix A for an important example of this.)

5. CONCLUSIONS

An attempt at modeling the overall $C^3$ problem as a network of nodes has been outlined. Key to this is the local modeling, i.e., the modeling of input-output behavior at each node. A procedure was presented, analogous to the PACT algorithm for multi-target data association which treats in a unified manner subjective and objective information. Future efforts will elaborate further on both global and local aspects of combining such information.

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REFERENCES

In the following references, abbreviations are made for the Proceedings of the MIT/ONR Workshop on $C^3$ Systems by use of the notation $C^3(k)$ or $C^3(k,i)$, where $k$ corresponds to the number of the Proceedings and $i$ denotes the volume (in Roman numerals), if more than one was issued. Years are omitted for such references, noting the correspondences:

1978 $\leftrightarrow$ k=1
1981 $\leftrightarrow$ k=4
1984 $\leftrightarrow$ k=7
1979 $\leftrightarrow$ k=2
1982 $\leftrightarrow$ k=5
1985 $\leftrightarrow$ k=8
1980 $\leftrightarrow$ k=3
1983 $\leftrightarrow$ k=6

4. Tenney, R.R., "Distributed decision making with limited communication", $C^3(2,IV)$, 830-868.
15. Kleinman, D.L., Pattipati, K.B., "Results toward developing a model of human decision making in $C^3$ systems", $C^3(3,III)$, 29-64.
36. Harmon, S., Brandenburg, R., "Command, Control and Communications (C3) systems model and measures of effectiveness (MOE's)", C(4)(IV), 181-206.
40. Anthony, R.W., "Holistic patterns in command, control, communication and information systems", C(7)(1), 71-78.
57. Dillard, R.A., "Integration of Artificial Intelligence into tactical C3 systems", C(7)(1), 79-84.

APPENDIX A

A large and conveniently parameterized family of De'organs transform pair of Archimedean t-norms and t-conorms is due to Frank originally (see [50], Chap.2.3 for additional discussion) and satisfies uniquely the modular relation
\[ \phi_{OR}(x,y) = (x+y-\phi(x), y) \text{ all } 0 < x, y \leq 1. \] (A-1)

The solution is given as, using parameter-index s,
\[ \phi_{s}(x_{1}, x_{2}, \ldots, x_{n}) = \log_{s} \left( \frac{1}{n} \sum_{i=1}^{n} s^{-1} \right) \] (A-2)

where 0 < s < \infty otherwise any real number. In a limiting sense, it is natural to define for the non-Archimedean parameter max,
\[ \phi_{s}(x_{1}, x_{2}, \ldots, x_{n}) = \max \left( \phi(x_{1}, x_{2}, \ldots, x_{n}) \right) \] (A-3)

and note the special cases \( s = 1 \) in (A-2), (A-3):
\[ \phi_{s}(1, x_{2}, \ldots, x_{n}) = \max \left( \phi(x_{2}, \ldots, x_{n}) \right) \] (A-5)

and when \( s \) is treated as a special case. For all \( s > 0 \), with \( s \) also treatable as a special case. For all \( s > 0 \), generator function \( h_{s} \) and (3.38), (3.40), (3.42) become
\[ h_{s}(x) = \log(s^{x} - 1) \] (A-7)

It then follows that Frank's family satisfies the hypotheses of the theorem in section 3, for all \( s > 0 \), with \( s \) also treatable as a special case. For all \( s > 0 \), generator function \( h_{s} \) and (3.38), (3.40), (3.42) become
\[ h_{s}(x) = \log(s^{x} - 1) \] (A-7)

\[ h_{s}(x) = \log(s^{x} - 1) \] (A-7)