This report summarizes progress achieved this year both in the more general formulation of our new criterion for nonlinear systems which allows us to tell right away if a chaotic system will exhibit a 1/f spectrum, and in the application and further study of the quantum 1/f effect. The general criterion was applied to a one-dimensional crystal with anharmonic interactions, predicting for the first time a 1/f phonon number spectrum in the chaotic regime at very low frequencies and always when cubic terms are dominant in the potential energy. The quantum 1/f theory was applied to a quartz resonator directly for the first time, providing both an explanation for the observed 1/f frequency fluctuations and optimization means. Our new formula for collector 1/f noise in ultrasmall BJT's was found to agree reasonably with the experiment. Finally, the fractional dimension of band-limited quantum 1/f noise was determined for the first time and found to coincide with the number of octaves considered, in agreement with known experiments. This indicates a quantum chaos nature of 1/f noise in infrared detectors.
Abstract

This report summarizes progress achieved this year both in the more general formulation of our new criterion for nonlinear systems which allows us to tell right away if a chaotic system will exhibit a 1/f spectrum, and in the application and further study of the quantum 1/f effect. The general criterion was applied to a one-dimensional crystal with anharmonic interactions, predicting a 1/f phonon number spectrum in the chaotic regime at very low frequencies and always when cubic terms are dominant in the potential energy. The quantum 1/f theory was directly applied to a quartz resonator for the first time, providing an explanation for the observed quantum 1/f frequency fluctuations and optimization means. Our new formula for collector 1/f noise in ultrasmall BJT’s was found to agree reasonably with the experiment. Finally, the fractional dimension of band-limited quantum 1/f noise was determined for the first time and found to coincide with the number of octaves considered, in agreement with known experiments. This indicates a quantum chaos nature of 1/f noise in infrared detectors.
CONTENTS

Abstract 2

I. Introduction 4

II. General Sufficient Criterion for 1/f Noise in Chaotic Nonlinear Systems 4

III. Quantum 1/f Fluctuation Spectra for Mode Energy and Phonon Number in a Nonlinear Chaotic Chain of Atoms 5

IV. Quantum 1/f Fluctuations in Quartz Resonators 8

V. Experimental Checks on Collector 1/f Noise in BJT’s 9
   V.1 Introduction 9
   V.2 Comparison of the calculated results with experimental data 11

VI. Quantum 1/f Noise is Quantum Chaos: Fractal dimension of Quantum 1/f Noise 15

VII. References 19

VIII. Papers Published During this Grant Period 20

IX. General Quantum 1/f Bibliography 22
I. INTRODUCTION

Progress has been achieved this year in the study of nonlinear systems which generate chaotic 1/f fluctuations, in the application of the Quantum 1/f Theory to various materials used in small and ultrasmall electronic devices, and in the application of the Quantum 1/f Theory to electronic devices. During this year four main achievements are reported. They are our discovery of the general mathematical principle causing the ubiquitous 1/f spectrum in nature, science and technology briefly presented in Sec. II, our application of this principle as a sufficient 1/f criterion to various chaotic nonlinear systems such as the one-dimensional crystal with anharmonic interactions presented as an example in Sec. III, our development of a new method allowing for direct application of the quantum 1/f principle to quartz resonators and to other piezoelectric and ferroelectric systems presented in Sec. IV, a comparison of our improvements in the quantum 1/f collector noise formula in ultrasmall BJTs with the experimental evidence shown in Sec. V, the first calculation of the fractal dimension of the 1/f noise process presented in Sec. VI, and new analytical quantum 1/f mobility fluctuation calculations in semiconductors. My collaborators have been E. Bernardi, T. Chung, A. Först-Chung, L.M.N. Sastri, X. Hu, Jian Xu, and M. Leong. These results will be briefly presented below.

II. GENERAL SUFFICIENT CRITERION FOR 1/f NOISE IN CHAOTIC NONLINEAR SYSTEMS

In spite of the practical success of our quantum 1/f theory in explaining electronic 1/f noise in most high-tech devices, and in spite of the conceptual success of our earlier classical turbulence approach to 1/f noise, the question about the ultimate origin of nature's omnipresent 1/f spectra remained unanswered. During the last three decades, we have claimed repeatedly that nonlinearity is a general cause of 1/f noise. Our new result proves that nonlinearity always leads to a 1/f spectrum if homogeneity is also present in the equation(s) of motion. Specifically, let the system be described in terms of the dimensionless vector function \( \mathbf{Y}(x,t) \) by the \( m \)th order nonlinear system of differential equation

\[
\Phi(t, x, \mathbf{Y}, \partial \mathbf{Y} / \partial t, \partial \mathbf{Y} / \partial x_1, \ldots, \partial \mathbf{Y} / \partial x_n, \partial^2 \mathbf{Y} / \partial t^2, \partial^2 \mathbf{Y} / \partial x_1^2, \ldots, \partial^m \mathbf{Y} / \partial x_n^m) = 0 \quad (1)
\]
where the vector function $\Phi$ may be nonlinear in any of its arguments. If a number $\theta$ exists such that Eq. (1) implies

$$\Phi[\lambda^\theta t, \lambda x, \lambda y, \partial y/\lambda \partial t, \partial y/\lambda \partial x_1 \ldots \partial y/\lambda \partial x_n, \partial^2 y/\lambda^2 \partial t^2, \partial^2 y/\lambda^2 \partial x_1^2 \ldots \partial^m y/\lambda^m \partial x_n^m] = 0$$

(2)

for any real number $\lambda$, the power spectral density of any chaotic solution for the vector function $Y$ defined by Eq. (1) is proportional to $1/f$.

In conclusion, nonlinearity + homogeneity = $1/f$ noise. The ultimate cause of the ubiquitous $1/f$ noise in nature is the omnipresence of nonlinearities (no matter how weak) and homogeneity. The latter is finally related to rotational invariance and to the isotropy of space. All our four specific theories of $1/f$ chaos in nonlinear systems are just special cases to which our criterion is applicable. They include our magneto-plasma theory of turbulence in intrinsic symmetric semiconductors (1966), our similar theory for metals (1971), the quantum $1/f$ theory (pure quantum electrodynamics, 1975), and the theory of Musha's traffic turbulence (1989). A fifth application was developed in March 1991 and concerns a one-dimensional crystal, i.e., a chain of atoms with slightly anharmonic interaction potentials, which is presented next.

III. QUANTUM 1/F FLUCTUATION SPECTRA FOR MODE ENERGY AND PHONON NUMBER IN A NONLINEAR CHAOTIC CHAIN OF ATOMS

Consider a chain of atoms in the $x$ direction, with a lattice constant $b$ and displacements $q_i$ from the equilibrium position. The equations of motion

$$md^2q_n/dt^2 = A[(q_{n+1} - q_n) - (q_n - q_{n-1})] + B[(q_{n+1} - q_n)^2 - (q_n - q_{n-1})^2] + C[(q_{n+1} - q_n)^3 - (q_n - q_{n-1})^3]$$

(3)

contain anharmonic terms as long as $B$ and $C$ are different from zero. With $\alpha_n = q_{n+1} - q_n$ and $\beta_n = q_n - q_{n-1}$, neglecting the second-order term, we obtain

$$md^2q_n/dt^2 = [(q_{n+1} - q_n) - (q_n - q_{n-1})][A + B(q_{n+1} + q_{n-1}) + C(\alpha_n^2 + \alpha_n \beta_n + \beta_n^2)]$$

(4)
Going over from finite differences to a continuum description, we obtain a differential equation for \( q(x,t) \)

\[
m \ddot{q} = \frac{b^2 \partial^2 q}{\partial x^2} \left[ A + 2Bq + 3Cb^2 \left( \frac{\partial q}{\partial x} \right)^2 \right]. \tag{5}
\]

Performing a Fourier transform with respect to \( x \),

\[
m \ddot{q}_k = -Ab^2 q_k - 2b^2 B \int k'^2 q_{k-k'} dk' \\
+ 3Cb^4 \int \int \int k'k''(k-k'-k'')^2 q_{k-k'}q_{k-k''} \tag{6}
\]

All integrals are from minus infinity to infinity. Substituting \( q_k = u(k,t)e^{ikbt}(A/m)^{1/2} \),

\[
m \ddot{u} = -2b^2 B \int k'^2 u_{k-k'} dk' \\
+ 3Cb^4 \int \int \int k'k''(k-k'-k'')^2 u_{k-k'}u_{k-k''} \tag{7}
\]

Our 1/f chaos criterion requires both nonlinearity and homogeneity, as well as the presence of chaos or of a quasichaotic state. The nonlinearity condition is satisfied unless \( B=0 \) and \( C=0 \), while homogeneity requires the existence of two numbers \( p \) and \( \theta \) such that replacing \( k \) by \( \lambda k \) everywhere except in the integration differentials, and replacing \( t \) by \( \lambda^\theta t \) leaves the equation multiplied by a general factor \( \lambda^p \), i.e., formally invariant. In our case we note that this criterion is satisfied with \( p=2 \) and \( \theta=-1 \) if we neglect the third-order term by setting \( C=0 \). On the other hand, both in the general case and in the \( B=0 \) case the criterion is not satisfied, except for some low-frequency limiting case in which all \( k \) values and frequencies are so small, that we can neglect the term with \( B \) and one of the left hand side terms.

To see how the criterion works for \( C=0 \), and to verify that we get indeed a 1/f spectrum in this case, as predicted by our criterion, we set \( \lambda=1/|k| \) and call \( |k'| t = z \)

\[
m \ddot{u} + 2ib(A/m)^{1/2} \ddot{u} = -2b^2 B \int k'^2 u_{k-k'} dk' \tag{8}
\]

Substituting \( u(k,t) = k^{-1}v(k,t) \) we get for \( v \)

\[
m \ddot{v} + 2ib(A/m)^{1/2} \ddot{v} = -2b^2 B \int (k'/k)v(k',z)v(k-k',z)dk'/(k-k') \\
= -2b^2 B \int k''v(kk'',z)v(k-kk'',z)dk''/(1-k'') \tag{9}
\]
We note that $k$ has disappeared from the equation and is present only as a scale factor in the arguments of $v$ on the rhs. Therefore we can expect the existence of solutions $v(k,z)$ of this equation which do not depend on the first argument. Such solutions exhibit "sliding scale invariance", because $t$ and $k$ or $t$ and $x$ provide a scale for each other, with no other scale present.

In certain conditions, instabilities of a solution of Eq. (6) may generate chaos, or turbulence. In a sufficiently large system described by the local dynamical equation (6), in which the boundary conditions become immaterial, homogeneous, isotropic turbulence, (chaos) can be obtained, with a spectral density determined only by Eq. (6). In the absence of instability and chaos, a certain type of random stirring forces can generate a quasichaotic stochastic state which can also be described with our methods familiar from turbulence theory. The stationary autocorrelation function $A(\tau)$ is defined as an average scalar product, the average being over the turbulent ensemble

$$A(\tau) = \langle u(x,t)u(x,t+\tau) \rangle = \int \langle u_k(t)u_k(t+\tau) \rangle dk = \int U(k,z) dk.$$  

(10)

Here we have introduced the scalar

$$U(k,z) = \langle u_k(t)u_k(t+\tau) \rangle$$  

(11)

of homogeneous, isotropic chaos (turbulence), which depends only on $|k|$ and $z=|k|\rho_\tau$, because there is nothing else in Eqs. (6) and (7). All integrals are from minus infinity to plus infinity. Isotropy means here the equivalence of the $+x$ and $-x$ directions. The chain of integro-differential equations for the correlation functions of any order must obey the same sliding-scale invariance which we have noticed in the fundamental dynamical equation (7) above. Therefore, in isotropic, homogeneous, conditions, $u_k(t)$ can only depend on $k$ and $z$. Furthermore, the direct dependence on $k$ must reflect this sliding-scale invariance, and is therefore of the form

$$u_k(z) = |k|^{-1}v(z).$$  

(12)

Indeed, only this form insures that $u_k(z)dk$ and therefore also the corresponding integrals and multiple convolutions in $k$ space have the necessary sliding-scale invariance.
According to the Wiener-Khintchine theorem, the spectral density is the Fourier transform of \( A(t) \),

\[
S_u(f) = \int e^{2\pi if\tau} A(\tau) d\tau = (1/f) \int dk' \int dt' e^{2\pi ifk'\cdot t'} v(z) = C/f, \tag{13}
\]

where we have set \( \tau = t' \), \( k' = k' \), \( z = k \tau = k\tau' \), and the integral

\[
C = \int dk' \int dt' e^{2\pi ifk'\cdot t'} v(z) = \int dk'' \int dt' e^{2\pi ifk''\cdot t'} v(k'') \tag{14}
\]

is independent of \( f \). We have defined the vector \( k'' = t'k' \). This confirms indeed our criterion.

The \( 1/f \) spectrum obtained by us for the amplitude \( u(t) \) carries over also for the squared amplitude \( u^2(t) \). Indeed, the autocorrelation \( A'(\tau) = \langle u^2(x, t) u^2(x, t+\tau) \rangle \) is given by \( 2A^2(\tau) + A^2(0) \) if we assume the amplitude \( u(t) \) to be well approximated by a Gaussian process. The Fourier transform of \( A^2(\tau) \) is the autocorrelation of \( C/f \), which is \( C^2/f \), if we interpret all \( 1/f \) spectra as the limit of \( \epsilon^{-1} \) spectra for arbitrarily small \( \epsilon \). Therefore, \( S_{u^2}(f) = 2C^2/f + A^2(0)\delta(f) \), where \( \delta(f) \) is the delta function. The energy density and the phonon number density are both proportional to \( u^2 \). This proves that in this case the energy density and the phonon number density are both fluctuating with a \( 1/f \) spectral density if they fluctuate at all, i.e., if the system is either chaotic, or in a quasichaotic state caused by a suitable system of random stirring forces. This confirms for the one-dimensional case the prediction of T. Musha. In three-dimensional piezoelectric crystals similar \( 1/f \) fluctuations of the phonon number are predicted by the quantum \( 1/f \) theory as we show in Sec., and have been observed experimentally in the Brillouin scattering of light by T. Musha.

IV. QUANTUM 1/F FLUCTUATIONS IN QUARTZ RESONATORS

According to the general quantum \( 1/f \) formula, \( \Gamma^{-2} S_\Gamma(f) = 2\alpha A/f \) with \( \alpha = e^2/\hbar c = 1/137 \) and \( A = 2(\Delta j/\varepsilon c)^2/3\pi \) is the quantum \( 1/f \) effect in any physical process rate \( \Gamma \). Setting \( j = dP/dt = P \), where \( P \) is the vector of the dipole moment of the quartz crystal, we obtain for the rate \( \Gamma \) of phonon removal from the main resonator oscillation mode by scattering on a phonon from any other mode of the crystal the spectral density

\[
S_\Gamma(f) = \Gamma^2 4\alpha (\Delta P)^2/3\pi e^2 c^2, \tag{15}
\]
where $(\Delta P)^2$ is the square of the polarization rate change associated with the removal of one of the N phonons present in the main resonator mode. To calculate it, we write the energy $W$ of the resonator and its change in the form

$$W = (N+1/2)\hbar \omega = P^2/2V\chi \omega^2; \quad \Delta W = \hbar \omega = P\Delta P/V\chi \omega^2.$$  \hfill (16)

Here $\chi$ is the susceptibility and $V$ the volume of the quartz crystal. Solving the last equation for $\Delta P$, squaring, and multiplying with the first, we get

$$(\Delta P)^2 = \hbar \omega^3 V\chi/(2N+1), \hfill (17)$$

and

$$S_f = \Gamma^2 4\alpha \hbar \omega^3 V\chi/3\pi e^2 c^2(2N+1)f = \Gamma^2 (2\omega^3 \chi/3\pi c^3 f)(\hbar \omega V/W). \hfill (18)$$

This result is applicable to the fluctuations in the rate of a single, well-defined process. The corresponding frequency fluctuations are given by

$$\omega^{-2} S_\omega (f) = (2\omega^3 \chi/3\pi c^3 f)(\hbar \omega V/WQ^4)(kT/\hbar \omega)^2, \hfill (19)$$

where $Q$ is the quality factor of the single-mode quartz resonator considered, $T$ the temperature.

V. EXPERIMENTAL CHECKS ON COLLECTOR QUANTUM 1/f NOISE IN BJTS

V.1 Introduction

1/f noise in bipolar junction transistors (BJTs) was treated by van der Ziel[1]-[3] who applied a Hooge-type approach similar to Kleinpenning's treatment[4] of pn junctions, and used experimental data to determine the Hooge constant which was in turn compared with the quantum 1/f theory. However, since the BJT is a minority carrier device, it requires the application of the quantum 1/f (Handel) equation[5]-[7] from the beginning, for the correct interpretation of the number of carriers in the denominator of the Langevin noise source.

In the most elementary model[8] of a BJT, the collector current $I_C$ arises from minority carriers injected from the emitter into the base, which diffuse across the width $X_B$ of the base and are then all swept across the reverse-biased collector junction by the built-in field of the
junction. If we neglect the usually small leakage current of the collector junction and the small fraction of the carriers recombining in the base, we get for a n+pn BJT:

\[ I_c = A q D_n \left[ n_0 B \exp \left( q V_{BE} / kT \right) / X_B \right], \]

(1)

where \( A \) is the cross sectional area of the base, \( q = -e \) is the charge of the minority carriers in the base, \( D_n \) their diffusion coefficient in the base, \( n_B(0) = n_0 B \exp( q V_{BE} / kT) \) is the electron concentration at the limit of the emitter space charge region, \( V_{BE} \) is the applied base - emitter voltage, and \( X_B \) is the width of the base. The expression in rectangular brackets is the electron concentration gradient calculated with the boundary condition of a vanishing electron concentration at the limit of the collector space charge region. We assume the base to be much narrower than the electron diffusion length \( L_n = \sqrt{D_n t} \), \( X_B << L_n \), but sufficiently wide to avoid ballistic electron transport across the base. Usually \( X_B \) is a fraction of a micron.

Quantum 1/f fluctuations of the collisional cross sections of the electrons in the base will yield fluctuations of the diffusion constant, and of the mobility \((\delta D_n / D_n = \delta \mu / \mu)\):

\[ \delta I_c = A q (\delta D_n) \left[ n_0 B \exp(q V_{BE} / kT) / X_B \right]. \]

(2)

The corresponding spectral density of fractional fluctuations \( I_c^2 S_{Ic} \) is

\[ I_c^2 <(\delta I_c)^2>_f = D_n^{-2} <(\delta D_n)^2> = \mu^{-2} <(\delta \mu)^2> = \alpha_n / fN. \]

(3)

In the last step our quantum 1/f equation\[5\]-\[7\] was used, where \( N \) is the number of carriers which define the scattered, or diffused, current leaving the base and emerging in the collector, while \( \alpha_n = \mu A_n \) is the effective quantum 1/f noise coefficient, or Hooge constant. The number of electrons \( N \) is thus determined by the effective lifetime \( \tau_c \) of the electrons, which will be slightly lower than the lifetime in the unbounded collector material, due to the collector lead contact processes, and due to lateral surface recombination. Indeed, we can write \( N = \tau_c I_c / q \).

Thus we finally obtain the spectral density of the collector current fluctuations:

\[ S_{Ic} = \alpha_n I_c q / (f \tau_c). \]

(4)

in which \( \tau_c \) is the effective lifetime of the majority carriers in the collector. This expression is simpler, but similar to the expression derived earlier, with the important difference that now we have a lifetime of the carriers in the denominator, while before it was the usually much
smaller diffusion time \( \tau_d = X_B g^2 / D_n \) of the electrons in the base. Eq. (4) also implies that in narrow-base BJTs of various base-widths \( \alpha_n \) will be constant, as in other devices, rather than \( \alpha_n / \tau_d \). In the following section we show that this expression is in good agreement with the experimental data in BJTs with 1/f collector noise spectra.

V.2 Comparison of the Calculated Results with Experimental Data

1. The effective quantum 1/f noise coefficient, or Hooge constant, \( \alpha_n \)

We consider the following scattering processes\(^3\) in the calculation of the quantum 1/f noise coefficient \( \alpha_H \):

a) Normal collision processes (Impurity scattering, Optical scattering and Acoustical phonon scattering).

b) Intervalley scattering; there are two types intervalley processes, i.e., g-processes which include Umklapp, and f-processes\(^14\).

From these points of view, we obtain the current spectral density in the form of Eq. (4):

\[
S_{IC}(f) = \alpha_n \left[ I_{C} q / (f T_C) \right] = \alpha A_n \left[ I_{C} q / (f T_C) \right]
\]

For the case a), the normal collision processes:

\[
\alpha_n = \alpha A_n = \alpha 4 \Delta v^2 / (3 \pi c^2) = 4 \alpha (k T/m_0 \pi c^2) = 4.69 \times 10^{-10}, \quad (5)
\]

where we used \( \alpha = \mu_0 c e^2 / 2h = 1/137 \) (the fine structure constant), \( c = 3 \times 10^8 \text{ m/sec}, k = 1.38 \times 10^{-23} \text{ J/K}, T = 300 \text{ K}, \) and \( m_0 = 9.1 \times 10^{-31} \text{ kg} \).

For the case b), the intervalley scattering + umklapp scattering 1/f noise (g-processes):

\[
\alpha_n = \alpha A_n = \alpha 4 \Delta v^2 / (3 \pi c^2) = \alpha 4 (\hbar \Delta k / m) ^2 / 3 \pi c^2 = 5.86 \times 10^{-7}, \quad (6)
\]

where \( \Delta v = \Delta p / m = \hbar \Delta k / m, |\Delta k| = 0.8 (2\pi / a), \) and \( a = 5.4 \text{ Å} \) for Silicon\(^15\). The g-processes include Umklapp back to the original B.Z., and the conduction effective mass\(^13\) \( m = 0.26 m_0 \).

Comparing the \( \alpha_n = 5.86 \times 10^{-7} \) with reference \(14\) Fig. 8 where the result was from the exact calculation, we find out that they are pretty close. We would like to point out that \( \alpha_n \neq \alpha_{\text{intervalley}} \) which is only an approximation due to the high \( \alpha_{\text{intervalley}} \) comparing with \( \alpha_{\text{impurity}} \) and \( \alpha_{\text{acoustic}} \).
2. The experimental data compared to the Hooge parameters $\alpha_{\text{Hn}}$

i. In n+-p-n bipolar transistors:

$$S_{\mu}(f)/\mu^2 = \alpha_{\mu}/fN.$$  \hspace{1cm} (7)

This can be written in terms of the diffusion constant, $D = kT\mu/q$

$$S_{\sigma}(f)/D^2 = \alpha_{\sigma}/fN$$  \hspace{1cm} (8)

which yields\[12\]

$$S_{IC}(f) = \frac{\alpha_{\text{Hn}} q_{IC} D_n}{f w_B^2} \ln\left[ \frac{N(0)}{N(w_B)} \right],$$  \hspace{1cm} (9)

where $\alpha_{\text{Hn}}$ is the Hooge parameter for electrons, $f$ is the frequency, $D_n$ is the diffusion constant for electrons, $w_B$ is the width of the transistor base region, $N(0)$ is the electron concentration for unit length at the emitter side of the base, and $N(w_B)$ is the electron concentration for unit length at the collector side of the base.

If we introduce the ratio

$$\frac{N(0)}{N(w_B)} \leq \frac{v_n D_n/w_B}{D_n/w_B}$$  \hspace{1cm} (10)

where $v_n$ is the saturation velocity of the electrons in the base region, and the diffusion time

$$\tau_{dn} = w_B^2/2D_n = 1/2f_T$$  \hspace{1cm} (11)

where $f_T$ is the upper cut off frequency of the BJT, then

$$S_{IC}(f) = \frac{\alpha_{\text{Hn}} q_{IC} D_n}{f 2\tau_{dn}} \ln\left[ \frac{v_n+(D_n/w_B)}{D_n/w_B} \right] = \frac{\alpha_{\text{Hn}}}{f} q_{IC} f_T \ln\left[ \frac{V_{n+D_n/w_B}}{D_n/w_B} \right].$$  \hspace{1cm} (12)

ii. In p+-n-p bipolar transistors:

$$S_{IC}(f) = \frac{\alpha_{\text{Hg}} q_{IC} D_p}{f w_B^2} \ln\left[ \frac{P(0)}{P(w_B)} \right].$$  \hspace{1cm} (13)

where
\[
\frac{P(0)}{P(W_B)} = 1 + \frac{v_{cp}W_B}{D_p}, \text{ and } \tau_{dp} = \frac{w_b^2}{2D_p}, \quad (14)
\]

and where \( v_{cp} \approx 10^7 \text{ cm/s in Si} \) is the saturation velocity of holes in the base. For details see[ ], and the following comparisons in table I, where a refers to the normal scattering quantum 1/f calculation and b to the more likely case of g-type intervalley- umklapp scattering. It is more likely to have this case since it has a much larger quantum 1/f effect, and will mask the smaller contribution calculated for case a. Also, the experimentally noticed strong increase in 1/f noise if the transistors are c cut from single-crystals so that the current flows along an [100] - like direction confirms this and can not be explained without the quantum 1/f theory. The experimental values in Table I are much closer to the values calculated for the case b replacing \( \tau_{nd} \) with the lifetime in the collector, \( \tau_c \). This corresponds to the inclusion of a corrective factor \( \tau_{nd}/\tau_c \) in the derivation of the collector quantum 1/f noise spectral density. However, a proportionality of \( S_{1c} \) to \( f_T \) is noticed in many transistors, contrary to our suggestion of replacing \( \tau_{nd} \) with \( \tau_c \). The present formula containing \( \tau_c \) is applicable to ultrasmall BJTs with a very narrow base region for which most of the life time of carriers diffusing from the emitter is spent in the collector. This subject requires further theoretical and experimental study to determine the exact limits of applicability of the new formula.
Table 1: The experimental data vs Hooge parameters*.

<table>
<thead>
<tr>
<th>Current $I_c$ (mA)</th>
<th>BJT No. 1</th>
<th>$S_{lc}^{exp}$ (A²/Hz)</th>
<th>$\tau_{dn}$ (sec)</th>
<th>$\alpha_{Hn}$ (sec)</th>
<th>$\tau_c$ (sec)</th>
<th>Theory $S(\tau_c)$ (A²/Hz)</th>
<th>Theory-A $S(\tau_{dn})$ (A²/Hz)</th>
<th>Theory-B $S(\tau_{dn})$ (A²/Hz)</th>
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</thead>
<tbody>
<tr>
<td>0.1</td>
<td>No. 1</td>
<td>9.00E-23</td>
<td>3.50E-10</td>
<td>5.80E-05</td>
<td>1.0E-7</td>
<td>2.15E-23</td>
<td>2.68E-20</td>
<td>a: 7.51E-26, b: 9.39E-23</td>
</tr>
<tr>
<td></td>
<td>No. 2</td>
<td>1.10E-22</td>
<td>2.00E-11</td>
<td>8.30E-10</td>
<td>1.0E-7</td>
<td>3.76E-25</td>
<td>4.69E-19</td>
<td></td>
</tr>
<tr>
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<td>3.50E-10</td>
<td>7.00E-09</td>
<td>1.0E-7</td>
<td>4.29E-23</td>
<td>5.37E-20</td>
<td>a: 1.50E-25, b: 1.88E-22</td>
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<td>2.00E-11</td>
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<td>7.51E-25</td>
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</tr>
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<td>3.50E-10</td>
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*BJTs
No.1-MRF90 has $f_T = 450$MHz
No.2-NEC57867 has $f_T = 8$GHz
VI. QUANTUM 1/F NOISE IS QUANTUM CHAOS: FRACTAL DIMENSION OF QUANTUM 1/F NOISE

Quantum 1/f noise in a physical quantity such as a current \( j \), a cross section, a process rate, or a kinetic coefficient, such as the mobility, is represented by an expression of the form

\[
\frac{\delta j}{\langle j \rangle} = a \sum_{k, \lambda} |b(k, \lambda)|^2 \cos(ckt + \gamma_{k\lambda})
\]  

where the sum is over all electromagnetic modes labeled by their wave vector \( k \) and polarization \( \lambda \), with \( k = |k| \). Eq. (15) is deterministic and the particle-specific random initial phases \( \gamma_{k\lambda} \) are present in each term as initial conditions, like the initial phases describing a turbulent fluid are present in each Fourier component of the perfectly deterministic velocity field. The difference between the mentioned classical chaos (turbulence) case and the quantum 1/f chaos we are introducing here becomes evident when we recall that \( J \) is in fact a probability current density! Can we calculate a spectral density of the fluctuations \( \delta j \) of this probability current and claim that it represents the expectation value of the spectral density of the observed quantum 1/f fluctuations? Our rigorous derivation in second quantization\(^7\)\(^{18-20}\) tells us we can, provided we divide the single-particle result by \( N \) in the case of bosons and \( N-1 \) for fermions. Here \( N \) is the number of scattered particles used to define what we call scattered current \( j \) or scattering cross section \( \sigma \). Encouraged by this result, we have undertaken an effort to determine the fractal dimension of quantum 1/f noise given by Eq. (15) and restricted to an observed frequency interval from \( c_k = v_o = 2\pi f_0 \) to \( \Lambda = 2\pi F \), where \( v_0 = 2\pi /T \) is determined by the duration \( T \) of the 1/f noise measurement. Our objective is to determine the fractal dimension of quantum 1/f noise and to compare it with measurements of the fractal dimension of physical 1/f noise in HgCdTe MWIR photodiodes\(^2\)\(^1\)\(^\text{21}\). Calculating the spectral density, replacing the summation in Eq. (15) by an integral, and going back to the original fluctuations, we obtain an equivalent representation of the fluctuations\(^7\)\(^{18-20}\)

\[
\frac{\delta j}{\langle j \rangle} = 2(\alpha A)^{1/2} \int_{v_0}^{\Lambda} \cos(\epsilon t + \gamma_{\epsilon}) d\sqrt{\epsilon}
\]

where \( \alpha = 1/137 \) is Sommerfeld's fine structure constant and \( \alpha A = (2\alpha/3\pi)(\Delta \nu/c)^2 \) is the bremsstrahlung coefficient, or infrared exponent, of the process which generates the current \( j \), \( \Delta \nu \) being the velocity change of the charged particles in the process. Repeating the fluctuations
periodically outside the observational interval $T$, we can represent Eq. (16) through an equivalent Fourier series

$$\delta j/\langle j \rangle = 2\alpha A \sum_{n=1}^{M} \cos(n\epsilon_{0}t + \gamma_{n})/(n\epsilon_{0})^{1/2} = x(t)$$  \hspace{1cm} (17)$$

which is similar to Eq. (15), with the terms of the same frequency grouped together, and the terms lumped together in harmonics with random phases $\gamma_{n}$, with $M=\Lambda/\epsilon_{0}$. To this expression we have applied the Takens-Grassberger-Procaccia analysis\[22\], by creating a time series $x_{i}=x(t_{i})$ with $1<i<N$. We consider this series as a one-dimensional sample of the quantum 1/f process. We calculate the correlation function $C_{1}(M)(r)$ which is $N^{-2}$ times the number of data pairs $(x_{i}, x_{j})$ separated by a distance less than $r$. Next we form groups of $d$ consecutive data $(x_{i}, x_{i+1}, ..., x_{i+d-1})$. Considering them as vectors in a $d$-dimensional Euclidean space, we again calculate

$$C_{d}(M)(r) = N^{-2} \sum_{1 \leq i < j \leq N} \theta[r - \|x_{i} - x_{j}\|],$$  \hspace{1cm} (18)$$

where $\theta(x)$ is the step function (zero for negative, 1 for positive, and 1/2 for null arguments). Finally we plot the curves $\log_{10}C_{d}(M)(r)$ as a function of $r$, with $d=0, 1, 2, ...$ as a parameter. We notice (Fig. 1 for $M=1$ and Fig. 2 for $M=2$ and 3 where for $M=2$ the numbers $d=1, 2, 3, 4$, have been replaced by $a, b, c, d$) that the slope of the curves is just $d$, and increases as $d$ is increased from curve to curve up to $M$, the number of terms in Eq. (17). We graphed the case $M=1$ (a single term in Eq. (17) in Fig 1 and $M=2$ and 3 in
\[ \log C_d^{(M)}(r) \]

---

1. \( d=1, \Delta t=0.05, E_0=0.333, N=1440, M=1, \Delta t=0.2, N=28 \)
2. \( d=2, L=360 \)
3. \( d=3, L=240 \)
4. \( d=4, L=180 \)

**Fig. 1**
\[ \log C_d^{(M)}(r) \]

- \( d=1, E0=0.349, N=720, M=2, \delta r=0.4, NM=14 \)
- \[ a \] \( M=3 \)
- \( 2 \) \( d=2, L=360, M=3 \)
- \( b \) \( M=3 \)
- \( 3 \) \( d=3, L=240, M=2 \)
- \( c \) \( M=3 \)
- \( 4 \) \( d=4, L=180, M=2 \)
- \( d \) \( M=3 \)

**Fig. 2**
Fig. 2. After $d=M$, the slope does no longer increase. This maximal slope gives the fractal dimension of the quantum $1/f$ process with $M$ terms. Our conclusion at this point is that quantum $1/f$ noise in a frequency interval corresponding to $M$ terms in Eq. (18) is chaotic with a fractal dimension $M$. Had it been stochastic rather than chaotic, the slope of the curves would have kept increasing indefinitely.

Experimentally, Fote, Kohn, Fletcher and McDonough found a fractal dimension of $d_M=10$ for $1/f$ noise measured in HgCdTe MW infrared detectors in the interval from $f_0=10^{-2}$Hz to $F=10$ Hz. Noticing that $F/f_0=10^3=2^{10}$, we expect $M=10$, which yields also $d_M=10$. This nice agreement between theory and experiment indicates that the measured $1/f$ noise is a form of (deterministic) chaos, whereas the other models which competed with the quantum $1/f$ theory incorrectly described it as a stochastic phenomenon. Furthermore, this agreement represents an independent verification of the quantum $1/f$ theory, independent from tests based on the predicted magnitude and spectral dependence of $1/f$ noise.

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