A Comparison of Several Theories which Use Jet Scaling to Predict Liftoff Heights of Turbulent-Jet Flames

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In this paper several previously developed theories for predicting liftoff heights of turbulent-jet flames are examined and compared. The five theories are: a turbulent flame speed theory, a cascade mixing time theory, a strain-rate theory, a scalar dissipation-rate theory and an empirical correlation. Each theory uses far-field scaling of isothermal, turbulent jets. Through a systematic algebraic development, the predictions of the theories are presented in a common form. They are then compared for their ability to predict the experimentally observed linear dependence of liftoff height on nozzle exit velocity. Following this, their ability to correctly predict the relative liftoff heights of different fuels is examined. Finally, the theories are extended to predict blowout and their predictions are compared to experimental results of several fuels.
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Abstract

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List of Symbols

- $a$: velocity gradient at extinction in a counterflow diffusion flame
- $c_i$: constants, $i = 1, 2, 3, ...$
- $c_\nu$: constant with dimensions of kinematic viscosity
- $C_p$: specific heat at constant pressure
- $d$: nozzle diameter
- $d^*$: momentum diameter, $d(\rho_o/\rho_\infty)$
- $D$: molecular diffusivity
- $f_i(\eta)$: function of $\eta$ as defined by a particular equation, $i = 1, 2, 3, ...$
- $F_n(\eta)$: "correction functions" to make theories match experimental results, $n = t, m, s, d$ and $e$.
- $h$: liftoff height
- $k$: thermal conductivity
- $l$: integral length scale
- $r$: radial coordinate
- $S$: maximum laminar flame speed
- $\bar{u}$: mean velocity
- $u'$: rms velocity
- $U_o$: nozzle exit velocity
- $x$: axial distance
- $Y$: fuel mass fraction
- $Y_o$: initial fuel mass fraction (balance is air)
- $\eta$: $r/x$
- $\kappa$: thermal diffusivity, $k/(\rho C_p)$
- $\nu$: kinematic viscosity
- $\rho$: density
- $\sigma$: strain rate
\( \tau_c \) fuel chemical time
\( \tau_f \) cascade mixing time
\( \chi \) scalar dissipation rate, \( 2D|\nabla Y|^2 \)

Other symbols

\( (.)_o \) nozzle exit conditions
\( (.)_\infty \) reservoir conditions
\( (.)_{st} \) stoichiometric conditions
\( (.)_l \) conditions at maximum laminar flame speed
\( (.)_{stab} \) assumed stabilization conditions, either \( (.)_{st} \) or \( (.)_l \)
\( (.)_{qu} \) quenching
\( (.)_{cl} \) centerline
\( (.)_b \) blowout
\( (.)_t \) turbulent flame speed theory
\( (.)_m \) cascade mixing time theory
\( (.)_s \) strain-rate theory
\( (.)_d \) scalar dissipation-rate theory
\( (.)_e \) empirical correlation theory
\( (.)_n \) name, representing \( t, m, s, d \) or \( e \)
\( (\bar{.}) \) time average
1. Introduction

Several theories using far-field scaling of isothermal, turbulent jets have been developed to explain the dependence of the mean liftoff height above the nozzle exit, $\bar{h}$, on the flow parameters. By appropriately grouping the parameters in each theory, the dependence of $\bar{h}$ on the nozzle velocity $U_o$, momentum diameter $d^*$, initial mass fraction of fuel $Y_o$, and chemical and transport properties of the fuels is brought out.

The theories examined here are: the turbulent flame speed theory of Kalghatgi [1], the cascade mixing time theory of Broadwell et al. [2], the strain-rate theory of Miake-Lye and Hammer [3], the scalar dissipation-rate theory of Peters and Williams [4] and a theory developed by Pitts [5]. Each of the above theories is treated in a similar manner. First, through algebraic manipulation, an equation is derived involving $\bar{h}, U_o, Y_o$, fuel parameters, similarity functions of $\eta \equiv r/x$ for velocity, concentration, etc. and what will be called a correction function, $F_n(\eta)$, which is needed to make predictions match experimental results. (Ideally, $F_n(\eta) \equiv 1$ for each theory.) Next, the necessary shape of $F_n(\eta)$ to make the theory match results is found. Finally, setting $F_n(\eta) \equiv 1$, $\eta$ is eliminated from the equation and the true predictions of the theory are given.

The remainder of the paper is organized as follows. First, experimental results of lifted flames are summarized and assumed scaling laws of isothermal jets are given. Next, each of the five theories is analyzed as described in the preceding paragraph. In the Comparisons section the similarities and differences of the theories are summarized.

2. Experimental results and assumed similarity profiles

2.1 Results for lifted jet flames

Kalghatgi [1] has found that for pure fuel issuing from a nozzle $\bar{h}$ is proportional to $U_o$ and independent of $d$ for a given fuel, once $\bar{h}$ reaches $\sim 10d$. In non-dimensional terms
this can be written as
\[
\frac{\bar{h}}{d^*} = \tau_c' \frac{U_o}{d^*},
\]
where \(\tau_c'\) is a chemical time dependent of the fuel. For small dilutions \((Y_o \approx 1)\)
\(\bar{h}/U_o \propto (1/Y_o)\) [3] and so Eq. (1) can be written as
\[
\frac{\bar{h}}{d^*} = \tau_c \frac{U_o Y_{st}}{d^* Y_o}.
\]
For larger dilutions \(\bar{h}/U_o\) was not found to be proportional to \(1/Y_o\), although this may
be due to near-field effects.

This liftoff behavior persists until the flame blows out at a maximum velocity \(U_b\)
which is proportional to \(d\), with the constant of proportionality depending on the fuel.

2.2 Assumed scaling laws of isothermal turbulent jets

The self-similar jet profiles which are used in the theories are the following:

\[
\bar{u} = c_1 U_o \frac{d^*}{x} f_1(\eta)
\]

(3)

\[
f_1(\eta) = e^{-c_1' \eta^2}
\]

(3.1)

\[
\bar{Y} = c_2 Y_o \frac{d^*}{x} f_2(\eta)
\]

(4)

\[
f_2(\eta) = e^{-c_2' \eta^2}
\]

(4.1)

\[
u' = c_3 U_o \frac{d^*}{x^2} f_3(\eta)
\]

(5)

\[
f_3(\eta) = e^{-c_3' \eta^2}
\]

(5.1)

\[
\sigma = c_4 U_o \frac{d^*}{x^2} f_4(\eta)
\]

(6)

\[
f_4(\eta) = e^{-c_4' \eta^2}
\]

(6.1)

\[
\chi = c_5 \frac{U_o}{d^*} Y_o^2 \left(\frac{d^*}{x}\right)^4 f_5(\eta)
\]

(7)

\[
l = c_6 x
\]

(8)
The above Gaussian functions (Eqs. (3.1), (4.1), (5.1) and (6.1)) are used only for estimating true predictions of the theories. Functions of any shape may be used in place of the Gaussian functions at the expense of greater complication in stating the final results.

3. Brief development of each of the five theories

In this section the five theories (turbulent flame speed, cascade mixing time, strain rate, scalar dissipation rate and the empirical theory) are examined. The descriptions do not exactly follow the authors' developments for each of the theories. This is done in order to bring out the similarities and differences in the theories. Major deviations from the authors' developments are noted.

3.1 Turbulent flame speed theory

The turbulent flame speed theory assumes that the mean velocity $\bar{u}$ equals the turbulent flame speed $u_t$ at the liftoff location specified by both axial $x$ and radial $r$ coordinates. Also, the liftoff location is assumed to lie along the contour of maximum laminar burning velocity, $Y = Y_l$. $u_t$ is assumed to depend on the flow properties as

$$\frac{u_t}{S} = c'(u'/\nu_l)^{1/2} = c_7(u'x/\nu_l)^{1/2}. \quad (9)$$

There is some experimental support for this expression, although considerable scatter exists [6].

Kalghatgi [1] uses dimensional analysis in his development and eventually arrives at an expression for the turbulent flame speed. The current description starts with the assumption of the turbulent flame speed relation, Eq. (9), and arrives at an equation for $\bar{h}$. Its development is different from that of Kalghatgi in this way and in the use of the correction function described below.

Rather than the rigid requirement $\bar{u} = u_t$ at the liftoff position, in the present discussion we admit that jet turbulence is not homogeneous and isotropic and that there
will be differences at different radial locations \( \eta \). The correction function for this theory is denoted as \( F_t(\eta) \) and we require

\[
\bar{u} = F_t(\eta)u_t, \tag{10}
\]

where \( F_t(\eta) \equiv 1 \) for the turbulent flame speed theory to be exact. Squaring Eq. (10), using Eq. (9), and setting \( x = h \) yields

\[
\bar{u}^2 = S^2 c_i^2 \frac{u' \bar{h}}{\nu_t} F_t^2(\eta).
\]

Using Eqs. (3) and (5),

\[
c_1 U_0 \frac{d^*}{\bar{h}} f_1(\eta) \frac{c_1}{c_3} f_1(\eta) u' = S^2 c_i^2 \frac{u' \bar{h}}{\nu} F_t^2(\eta).
\]

Using Eq. (4) and the assumption \( \bar{Y} = Y_t \) to eliminate \( d^*/\bar{h} \),

\[
\frac{c_i^2}{c_2 c_3} \frac{U_t}{Y_o} \frac{f_1^2(\eta)}{f_2(\eta) f_3(\eta)} = S^2 c_i^2 \frac{\bar{h}}{\nu} F_t^2(\eta).
\]

Finally, solving for \( \bar{h} \),

\[
\bar{h} = c_{t,1} \frac{Y_t}{Y_o} \frac{u_t}{S^2 U_o} \frac{f_1^2(\eta)/(f_2(\eta)f_3(\eta))}{F_t^2(\eta)}, \tag{11}
\]

where the constants have been absorbed into \( c_{t,1} \).

Some important points to note from Eq. (11) are that at a fixed \( \eta \), \( \bar{h} \sim 1/Y_o, \bar{h} \sim U_o \), and \( \bar{h} \) does not depend on \( d^* \). These relations will turn out to be true for all of the theories in this paper.

There are two remaining issues to address regarding the turbulent flame speed theory. The first is whether \( f_1(\eta) \equiv \frac{f_1^2(\eta)}{f_2(\eta)f_3(\eta)} \approx \text{constant} \) as required for agreement with experimental liftoff results that there is a linear relation between \( \bar{h} \) and \( U_o \) for a given fuel. By determining the actual shape of \( f_t(\eta) \) based on experimental measurements of \( f_1(\eta), f_2(\eta) \) and \( f_3(\eta) \), the required correction function \( F_t(\eta) \) can be determined. Alternatively,
once \( f_t(\eta) \) is found. \( F_t(\eta) \) can be set to 1 to show the true turbulent flame speed theory prediction without correction.

The second issue is whether the theory correctly predicts the relative liftoff heights of different fuels. This is measured by whether the factor \( Y_l \nu_l/S^2 \) in Eq. (11) agrees with experimental results of slopes of \( \bar{h} \) versus \( U_o \) lines for various fuels (\( \tau'_c \) in Eq. (1)). Addressing this second issue is postponed until the Comparisons section where all five theories are compared.

Returning to the first issue, experimental data is used to approximate the functions, \( f_1(\eta) \), \( f_2(\eta) \) and \( f_3(\eta) \) with Gaussian profiles as indicated in Eqs. (3.1), (4.1) and (5.1). For mean velocity, \( c'_1 = 93.7 \) from data of Chen and Rodi [7]. For fluctuating velocity, \( c'_4 = 40.5 \) is found from curve fitting data of Wygnanski and Fiedler [8] at \( \bar{Y}/\bar{Y}_n = 1/2 \). This curve fit turns out to be about 30% high near the jet edge. \( \eta = 0.2 \), but will nevertheless be used here for simplicity. Finally, for concentration, \( c'_2 = 57 \) [7, 9]. This results in

\[
f_t(\eta) = e^{-(2c'_1-c'_2-c'_4)\eta^2} = e^{-c'_2\eta^2} = e^{-90\eta^2}.
\]

The ratio of \( f_t^{1/2} \) at \( \eta = 0.1 \) and 0.2 (the approximate range of values encountered experimentally in lifted flames) is \( e^{-45\eta^2} = 0.26 \). Therefore, \( F_t(\eta) \) would need to be \( \approx e^{-45\eta^2} \) and the ratio \( \bar{u}/u_t \) would need to vary by a factor of \( \sim 4 \) to make the turbulent flame speed theory match experimental results.

If \( F_t(\eta) \) is set identically equal to 1, the predictions of a true turbulent flame speed theory can be found. Multiplying Eq. (11) by \( Y_l/(Y_0d^*) \),

\[
\frac{Y_l}{Y_o} h \frac{h}{d^*} = c_{t,1} \left( \frac{Y_l}{Y_o} \right)^2 \frac{\nu_l U_o}{S^2 d^*} f_t(\eta).
\]

Solving for \( \eta \) from Eq. (4)

\[
\frac{Y_l}{Y_o} h \frac{h}{d^*} = c_{t,1} \left( \frac{Y_l}{Y_o} \right)^2 \frac{\nu_l U_o}{S^2 d^*} f_t \left( f_2^{-1} \left( \frac{1}{c'_2} \frac{Y_l}{Y_o} \bar{Y}_n \right) \right).
\]
Using the Gaussian functions results in the power law dependence,

\[
\frac{Y_t}{Y_o} = c_{t.2}((\frac{Y_t}{Y_o})^2 \nu_t \frac{U_o}{S^2} d^* )^{1/(1-c'_l/c'_s)}.
\] (14)

The exponent \(E_t \equiv \frac{1}{1-c'_l/c'_s} = -1.73\). Because \(E_t < 0\), the turbulent flame speed theory actually predicts a decreasing liftoff height with increasing nozzle velocity for a fixed \(d^*\) and fuel.

### 3.2. Cascade mixing time theory

In a paper devoted primarily to blowout of flames, Broadwell et al. [2] proposed that lifted flames will be stabilized at a distance such that the mixing time, which is controlled by a turbulence cascade time, \(\tau_f \sim x/u_c\), is equal to a chemical time for the fuel, \(\tau_c.m\).

Their theory is one dimensional because \(\tau_f\) is assumed to have no dependence on \(\eta\). \(\tau_c.m\) is assumed to be proportional to \(\kappa/S^2\) in accordance with simple one-dimensional laminar flame speed theories (e.g., Glassman [10]). A discussion of what value of \(\kappa\) should be used is postponed until the Comparisons section.

For the present description, a correction function \(F_m(\eta)\) will be added to the theory as was done with the turbulent flame speed theory. Using Eq. (3) the liftoff requirement becomes

\[
\frac{\bar{h}^2}{U_o d^*} F_m(\eta) = c_{10} \frac{\kappa}{S^2}.
\] (15)

Assuming the flame stabilizes along the mean stoichiometric contour (an assumption that was not necessary in the original one-dimensional formulation) and using Eq. (4),

\[
\bar{h} = c_{m.1} \frac{Y_{st}}{Y_o} \frac{\kappa}{S^2} \frac{U_o}{F_m(\eta)} 1/f_2(\eta).
\] (16)

As with the turbulent flame speed theory, at constant \(\eta\), \(\bar{h} \sim U_o\), \(h \sim 1/Y_o\) and \(h\) is independent of \(d^*\).
To find the true predicted liftoff height, $F_m(\eta)$ is set to 1 in Eq. (16) to get

$$\frac{Y_{st} \, \ddot{h}}{Y_o \, d^*} = c_m,1 \left( \frac{Y_{st}}{Y_o} \right)^2 \frac{\kappa \, U_o}{S^2 \, d^*} \frac{1}{f_2(\eta)}. \quad (17)$$

Noting the similarity to Eq. (13), we can write

$$\frac{Y_{st} \, \ddot{h}}{Y_o \, d^*} = c_m,2 \left( \frac{Y_{st}}{Y_o} \right)^2 \frac{\kappa \, U_o}{S^2 \, d^*} \frac{1}{f_2(\eta)} \frac{1}{f_2(\eta)}^{1/2}. \quad (18)$$

The Gaussian shape of $f_2(\eta)$ was not used in deriving Eq. (18) because no other functions were involved, and, the exponent, $E_m = 1/2$, is exact. In fact, Eq. (18) can easily be derived from the strictly one-dimensional theory.

### 3.3 Strain-rate theory

A theory very similar to the above mixing time theory is the maximum strain-rate theory of Miake-Lye and Hammer [3]. Here, however, the liftoff condition is that the large scale strain rate $\sigma$ cannot exceed a maximum strain rate $\sigma_{crit} \sim S^2/\kappa$. Also, $\sigma$ is allowed to have a radial dependence, hence making this a two-dimensional theory. The flame is assumed to lie along a contour on which stoichiometric conditions are most probable. To approximate this condition, it is assumed that $Y = Y_{st}$.

Using a correction function $F_s(\eta)$ the liftoff requirement becomes

$$\sigma = c_4 \, U_o \frac{d^*}{x^2} \, f_4(\eta) = c_{11} \, \frac{S^2}{\kappa} \, F_s(\eta).$$

Solving for $\ddot{h}$,

$$\ddot{h} = c_{s,1} \frac{Y_{st}}{Y_o} \frac{\kappa \, U_o \, f_4(\eta)}{F_s(\eta)}, \quad (19)$$

which is very similar to Eq. (16). The only difference is that the radial strain function $f_4(\eta)$ appears. Therefore, to match experimental liftoff results, $F_s(\eta) = f_4(\eta)/f_2(\eta)$.

Unfortunately, $f_4(\eta)$ has not been experimentally measured. Nor was it completely defined in Ref. [3] where the authors described it as the large-scale strain rate between
large vortical structures. One possibility is to use $\frac{\partial \tilde{s}}{\partial r}$ for $\sigma$. This, however, will not give a Gaussian profile. Instead, in the present paper, a Gaussian profile is used which matches $\frac{\partial \tilde{u}}{\partial r}$ at $r = 0.1$ and $r = 0.2$ to get

$$c'_4 = c'_1 - \log(0.2/0.1)/(0.2^2 - 0.1^2) = 71.$$  

This makes $f_s(\eta) \equiv f_4(\eta)/f_2(\eta) = e^{-c'_s \eta^2}$ where $c'_s = c'_4 - c'_2 = 14$.

The true predictions of the theory are then

$$\frac{Y_{st} \tilde{h}}{Y_o d^*} = c_{s,2} \left( \frac{Y_{st}}{Y_o} \frac{\kappa U_o}{S^2 d^*} \right)^{1/(1-c'_s/c'_2)}. \tag{20}$$

The exponent $E_s \equiv 1/(1 - c'_s/c'_2) = 1.3$, although, due to the approximation in determining $c'_4$, this number should be treated with some skepticism.

### 3.4 Scalar dissipation-rate theory

Peters and Williams [4] have proposed that the liftoff height is determined by extinction of thin flamelets. Through an asymptotic analysis they find that the scalar dissipation rate $\chi$ is the parameter which determines whether a flamelet will be quenched. The criterion for flame stabilization is that the probability of $\chi < \chi_{qu}$ at stoichiometric conditions is greater than some minimum probability, where $\chi_{qu}$ is a limiting scalar dissipation rate for the particular fuel. Using percolation theory, this requirement is simplified to $\bar{\chi}_{st} < \chi_{qu}$. In the present formulation, we will again add a correction function $F_d(\eta)$ and require

$$\bar{\chi}_{st} = \chi_{qu} F_d(\eta). \tag{21}$$

Here, $\bar{\chi}_{st}$ is the average scalar dissipation rate conditional on $Y = Y_{st}$. In order to make use of experimental measurements in jets, $\bar{\chi}_{st}$ must be related to the unconditional average dissipation rate $\bar{\chi}$. For this we assume that $\bar{\chi}_{st} = \bar{\chi} f_7(Y_{st}/\bar{Y}_{st}, \eta)$, and that $\bar{Y} = Y_{st}$. Then $\bar{\chi}_{st} = \bar{\chi} f_7(f_2(\eta), \eta)$. Combining this with Eq. (21),

$$\bar{\chi} = \bar{\chi}_{st}/\tilde{f}_7(\eta) = \chi_{qu} F_d(\eta)/\tilde{f}_7(\eta).$$
Now, assuming \( \hat{f}_7(\eta) \) is not a strong function of \( \eta \) (or equivalently absorbing it into \( F_d(\eta) \)) we arrive at the condition

\[
\tilde{\chi} = c_{12} \chi_{qu} F_d(\eta)
\]  

(22)

for determining the liftoff height.

After that somewhat complicated introduction, we are no ready to proceed using Eqs. (22), (7) and (4) as with the previous theories to get

\[
\bar{h} = c_{d,1} \frac{Y_{st} Y_{st}^2 Y_o}{\chi_{qu} U_o} \frac{f_5(\eta)/f_2^3(\eta)}{F_d(\eta)}.
\]  

(23)

Like the previous theories at constant \( \eta \), \( \bar{h} \sim U_o \), \( h \sim 1/Y_o \) and \( \bar{h} \) is independent of \( d^* \). Unlike the previous theories an additional \( Y_{st}^2 \) appears in the result, but \( Y_{st}^2/\chi_{qu} \) is simply a chemical time relating the relative liftoff heights of different fuels.

For estimating the shape of \( f_5(\eta) \), results of Dowling’s [9] point measurements of concentration are used. Using Taylor’s hypothesis along with the mean off-centerline velocity, he states that “the mean value of [the estimated scalar dissipation rate] is almost independent of [\( \eta \)] ...”. Therefore, \( f_5(\eta) \) is approximated as a constant, and \( F_d(\eta) \) would need to be equal to \( f_d(\eta) \equiv f_5(\eta)/f_2^3(\eta) \approx e^{-171\eta^2} \) for the scalar dissipation-rate theory to match experimental results. Peters and Williams approximated \( \tilde{\chi} \) as \( U_o d^* (\partial Y_o / \partial r)^2 \) which makes \( f_5(\eta) \sim \eta^2 e^{-2c_2^2 \eta^2} \) if the Gaussian profile for \( f_2(\eta) \) given by Eq. (4.1) is used. The current discussion will assume that \( f_5(\eta) \approx \text{constant} \), however.

Setting \( F_d(\eta) \equiv 1 \) in Eq. (23) results in

\[
\frac{Y_{st}}{Y_o} \frac{\bar{h}}{d^*} = c_{d,2} \frac{Y_{st}^2}{Y_o} \frac{U_o}{\chi_{qu} d^*} \frac{1/(1 - c_d/c'_2)}{1/4},
\]  

(24)

where \( c'_d = -3c_2^2 \) and therefore \( E_d \equiv 1/(1 - c_d/c'_2) = 1/4 \).

It should be pointed out that Peters and Williams have not stated that their theory should be applicable to diluted fuels where \( Y_o < 1 \). The idea that the quenching scalar dissipation rate \( \chi_{qu} \) is a constant is based in part on the assumption of boundary
conditions of the flamelets being pure fuel on one side and pure oxidizer on the other. However, for comparison with other theories in this paper, the effect of $Y_0$ has been included with the assumption that $\chi_{qu}$ would not change.

One other point to be made concerns the constant $c_5$ in Eq. (7). Although Eq. (7) is expected to be correct in the limit $Re \to \infty$, at Reynolds numbers of 5000 and 16000, Dowling finds $c_5 = 54$ and 32 respectively. This would cause even further disagreement between experimental results and the scalar dissipation-rate theory in this range of Reynolds numbers.

3.5 Empirical correlation proposed by Pitts

Pitts [5] has proposed a correlation for predicting liftoff heights which is based on jet scaling (with some caveats). He proposes that flames stabilize along the contour of maximum laminar burning velocity ($\bar{Y} = Y_l$) at a location where

$$
\bar{u} = \frac{1}{c_\nu} \frac{x}{d^*} x S^2 Y_l^2.
$$

The first difficulty with the correlation is that $c_\nu$ is a constant with dimensions of kinematic viscosity, independent of fuel and oxidizer, which cannot physically be correct. A possible fix to this would be to replace $c_\nu$ with $\nu_l$, although this would change predictions slightly for different fuels.

The second difficulty concerns the factor $Y_l^2$ in Eq. (25). Whether this should be replaced by $(Y_l/Y_o)^2$ or $(Y_i/Y_o)Y_l$ or left as $Y_l^2$ when $Y_o < 1$ is not stated by Pitts. The correlation was developed to match data for pure fuels only, and so this was not a consideration. In the present discussion $Y_l^2$ will be replaced by $(Y_l/Y_o)Y_l$ because the other two choices result in $\bar{u}_{stab}$ depending on $x/d^*$, or, through simple algebraic manipulation, on $Y_o$. It is assumed in jet far-field scaling that the flow is independent of these parameters.
Replacing $Y_i^2$ with $Y_i(Y_i/Y_o)$ in Eq. (25) and adding a correction function $F_e(\eta)$.

$$\bar{u} = \frac{1}{c_\nu} \frac{x}{d^*} x S^2 Y_i(Y_i/Y_o)F_e(\eta). \tag{26}$$

Using Eq. (4) to eliminate $(x/d^*)(Y_i/Y_o)$ better illustrates the assumed dependence of $\bar{u}$ on the flow variables:

$$\bar{u}/S = c_2 Y_i(xS/c_\nu)f_2(\eta)F_e(\eta). \tag{26.1}$$

It is difficult to justify theoretically this relationship, especially with the factor $Y_i$, but at least there is no remaining $x/d^*$ which would put it out of the realm of jet scaling theories.

Using Eq. (3) in Eq. (26),

$$\bar{h} = c_{e,1} \frac{Y_i}{Y_o} \frac{c_\nu}{Y_iS^2} U_o \frac{f_1(\eta)/f_2^2(\eta)}{F_e(\eta)}, \tag{27}$$

where $Y_i$ has been explicitly left in the numerator and denominator. For a linear relation between $U_o$ and $\bar{h}$, $F_e(\eta)$ must equal $f_\epsilon(\eta) = f_1(\eta)/f_2^2(\eta) = e^{-c'_e \eta^2}$, where $c'_e = c'_1 - 2c'_2 = -20$. Pitts effectively used such a correction function by using a slightly higher value of $c'_1$ than had been found in turbulent jet experiments.

The true predictions of the theory are

$$\frac{Y_i}{Y_o} \frac{\bar{h}}{d^*} = c_{e,2} \left(\frac{Y_i}{Y_o}\right)^2 \frac{c_\nu}{Y_iS^2} U_o d^* E_e, \tag{28}$$

where $E_e = 1/(1 - c'_e/c'_2) = 0.74$.

4. Comparison of theories

Each of the theories investigated produces a prediction of the form

$$\bar{h} = c_{n,1}(Y_{stab}/Y_o)\tau_{c,n} U_o f_n(\eta). \tag{29}$$

where $Y_{stab}$ is the assumed stabilization concentration, $Y_{st}$ for the mixing time, strain-rate and scalar dissipation-rate theories and $Y_i$ for the turbulent flame speed and empirical
theories. \( \tau_{c,n} \) is a fuel-dependent chemical time and \( f_n(\eta) \) is a function which depends on jet similarity functions. The above relation is a direct result of the fact that the proposed stabilization conditions of each of the theories can be expressed as

\[
\frac{\bar{h}}{\dot{u}_{cl}} \sim \tau_{c,n} f_{n,2}(\eta),
\]

\[ \hat{Y} = Y_{stab}, \]

where \( f_{n,2}(\eta) = f_n(\eta)f_2(\eta) \). By eliminating \( \eta \) from Eq. (29) using Eq. (4), the liftoff prediction can be stated as

\[
\frac{Y_{stab}}{Y_o} \frac{\bar{h}}{d^*} = f_{n,3} \left( \frac{Y_{stab}}{Y_o} \right)^2 \tau_{c,n} \frac{U_o}{d^*}. \tag{30.1}
\]

Approximating \( f_n(\eta) \) and \( f_2(\eta) \) with Gaussian functions results in a simple power law form for the function \( f_{n,3} \)

\[
\frac{Y_{stab}}{Y_o} \frac{\bar{h}}{d^*} = c_{n,2} \left( \frac{Y_{stab}}{Y_o} \right)^2 \tau_{c,n} \frac{U_o}{d^*} E_n. \tag{30.2}
\]

This section is organized into four subsections. First, predictions for a fixed fuel (and fixed chemical and transport parameters) are compared by examining \( f_n(\eta) \). Second, the necessary parameters for comparing different fuels via different expressions for \( Y_{stab}\tau_{c,n} \) are tabulated. The third subsection uses these parameters to compare experimental results for liftoff heights with predictions for different fuels. The last subsection addresses blowout predictions.

### 4.1 Predictions for a single fuel

Table 1 lists the following properties of each theory: expression for \( f_n(\eta) \) in terms of similarity functions \( f_1, f_2, \) etc.; the approximate value of the \( c'_n \) when \( f_n(\eta) \) is approximated as \( e^{-c'_n \eta^2} \); the the exponent in Eq. (30.2), \( E_n \equiv 1/(1 - c'_n/c'_2) \); and the expression for \( Y_{stab}\tau_{c,n} \).
Of the above theories, only the turbulent flame speed theory predicts a decreasing \( \hat{h} \) with increasing \( U_0 \), as evidenced by \( E_n < 0 \). The scalar dissipation-rate theory has the largest, negative \( c'_n \) and consequently predicts a dependence of \( U_0^{0.25} \). Other theories have a more nearly linear dependence in agreement with experimental results. Due to lack of data on the strain-rate profile, the results for the strain-rate theory are less certain than those of the other theories.

Table 1

Summary of Theory Predictions

<table>
<thead>
<tr>
<th>Theory</th>
<th>( f_n(\eta) )</th>
<th>( c'_n )</th>
<th>( E_n )</th>
<th>( Y_{stab} \tau_{c,n} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>turbulent flame speed</td>
<td>( f_1^2/(f_2 f_3) )</td>
<td>90</td>
<td>-1.73</td>
<td>( Y_l(\nu_l/S^2) )</td>
</tr>
<tr>
<td>cascade mixing time</td>
<td>( 1/f_2 )</td>
<td>-57</td>
<td>1/2</td>
<td>( Y_{st}(\kappa/S^2) )</td>
</tr>
<tr>
<td>strain rate</td>
<td>( f_4/f_2 )</td>
<td>14</td>
<td>1.3</td>
<td>( Y_{st}(\kappa/S^2) )</td>
</tr>
<tr>
<td>scalar dissipation</td>
<td>( f_5/f_2^3 )</td>
<td>-171</td>
<td>0.25</td>
<td>( Y_{st}(Y_{st}^2/\chi_{qu}) )</td>
</tr>
<tr>
<td>empirical correlation</td>
<td>( f_1/f_2^2 )</td>
<td>-20</td>
<td>0.74</td>
<td>( Y_l(c_u/(Y_l S^2)) )</td>
</tr>
</tbody>
</table>

4.2 Properties for evaluation of chemical times

In Table 2, values for \( Y_{st} \), \( Y_l \) and \( S \) are taken from Kalghatgi [1]. In accordance with turbulent flame speed correlations, \( \nu_l \) is evaluated at the unburned condition, assumed to be 300 \( K \).

Evaluation of \( \kappa \) is somewhat more difficult. Broadwell et al. used for a value for \( \kappa \) that of air at 2000\( K \) for all the fuels because the hydrocarbons have similar flame temperatures. (Hydrogen's temperature is somewhat higher.) Miake-Lye and Hammer used \( \kappa \) of stoichiometric unburned mixtures at 2200 \( K \), although this required considerable extrapolation for the thermal conductivities of some of the components. More detailed inspection of the laminar flame speed theories reveals that \( \rho \) should be evaluated at the cold, unburned condition. The theory of Mallard and Le Chatelier [10] requires
$C_p$ to be evaluated at a temperature between the flame temperature and the unburned temperature, and $k$ to be evaluated near the flame temperature for the most physical interpretation of the theory. The more complex theory of Zeldovich, Frank-Kamenetskii and Semenov [10] requires that $C_p$ and $k$ be constant. Therefore, in the Table 2, $\beta$ and $C_p$ are evaluated at stoichiometric, unburned conditions, and $k$ is evaluated at the adiabatic flame temperature. Because reliable data for $k$ of the products, $H_2O$ and $CO_2$, could not be found, $k$ for $N_2$ at the flame temperature is used.

Values of $\chi_{qu}$ are obtained from the equation of Peters and Williams

$$\chi_{qu} = \chi_{st} \approx 4aY_{st}^2[erfc^{-1}(2Y_{st})]^2,$$

where the velocity gradient at extinction in a counterflow diffusion flame $a$ is taken from the data of Tsuji and Yamaoka [11] for $CH_4$ and $C_3H_8$ and from the computations of Dixon-Lewis et al. [12] for $H_2$. No value has been found for $C_2H_4$.

Also included in Table 2 are experimental results for the slopes of the $\bar{h}$ versus $U_o$ lines from the data of Kalghatgi [1]. The final column gives data on blowout in the form of $d*/U_b$ taken from Broadwell et al. who used the data of Kalghatgi [13]. These last two columns will be compared with predictions in sections 4.3 and 4.4.

**Table 2**

*Fuel properties and experimental results for liftoff and blowout*

<table>
<thead>
<tr>
<th>Fuel</th>
<th>$Y_{st}$</th>
<th>$Y_1$</th>
<th>$S$</th>
<th>$\nu_l$</th>
<th>$\kappa_{st}$</th>
<th>$\chi_{qu}$</th>
<th>$\bar{h}/U_o$</th>
<th>$d^*/U_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CH_4$</td>
<td>0.0549</td>
<td>0.058</td>
<td>0.39</td>
<td>15.9</td>
<td>106</td>
<td>4.9</td>
<td>2.42</td>
<td>46.6</td>
</tr>
<tr>
<td>$C_2H_4$</td>
<td>0.0632</td>
<td>0.073</td>
<td>0.75</td>
<td>15.1</td>
<td>106</td>
<td>—</td>
<td>0.672</td>
<td>19.4</td>
</tr>
<tr>
<td>$C_3H_8$</td>
<td>0.060</td>
<td>0.068</td>
<td>0.45</td>
<td>14.6</td>
<td>102</td>
<td>6.1</td>
<td>1.91</td>
<td>51.2</td>
</tr>
<tr>
<td>$H_2$</td>
<td>0.0284</td>
<td>0.076</td>
<td>3.06</td>
<td>30.1</td>
<td>113</td>
<td>76.1</td>
<td>0.030</td>
<td>0.178</td>
</tr>
</tbody>
</table>
4.3 Comparison of predicted and measured liftoff heights of different fuels

Several points can be noted about the expressions for \( Y_{stab \tau_{c,n}} \) given in Table 1. All contain a \( 1/S^2 \) term, except the scalar dissipation-rate theory. The expressions for the mixing time and strain-rate theories are identical. Also the empirical correlation's expression would agree with that of the turbulent flame speed theory if \( c_v/Y_1 \) were replaced with \( \nu_l \) in Eq. (26.1).

Using the properties in Table 2, the relative predicted liftoff slopes (\( Y_{stab \tau_{c,n}} \)) of the fuels are evaluated and tabulated in Table 3. For ease of comparison, the values have been normalized by the value for methane, as have the experimental results for \( h/U_o \). In addition, the deviation of the prediction from the experimental result is listed in percent. All of the theories correctly predict the relative liftoff heights of different fuels to within \( \pm \sim 30\% \) except for the turbulent flame speed's prediction for hydrogen.

<table>
<thead>
<tr>
<th>Fuel</th>
<th>( h/U_o )</th>
<th>turb. f.s.</th>
<th>mix. time</th>
<th>strain rate</th>
<th>scalar diss.</th>
<th>empirical</th>
</tr>
</thead>
<tbody>
<tr>
<td>( CH_4 )</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( C_2H_4 )</td>
<td>0.278</td>
<td>0.323</td>
<td>0.311</td>
<td>0.311</td>
<td>—</td>
<td>0.270</td>
</tr>
<tr>
<td></td>
<td>(+16%)</td>
<td>(+12%)</td>
<td>(+12%)</td>
<td></td>
<td></td>
<td>(-3%)</td>
</tr>
<tr>
<td>( C_3H_8 )</td>
<td>0.789</td>
<td>0.810</td>
<td>0.788</td>
<td>0.788</td>
<td>1.05</td>
<td>0.749</td>
</tr>
<tr>
<td></td>
<td>(+3%)</td>
<td>(+0%)</td>
<td>(+0%)</td>
<td></td>
<td></td>
<td>(+33%)</td>
</tr>
<tr>
<td>( H_2 )</td>
<td>0.0124</td>
<td>0.0404</td>
<td>0.00896</td>
<td>0.00896</td>
<td>0.00891</td>
<td>0.0162</td>
</tr>
<tr>
<td></td>
<td>(+226%)</td>
<td>(-28%)</td>
<td>(-28%)</td>
<td></td>
<td></td>
<td>(-28%)</td>
</tr>
</tbody>
</table>

4.4 Blowout predictions and experimental results

By assuming that blowout occurs when the flame base reaches some minimum value of \( \eta, \eta_b \), each of the theories can be extended to predict a blowout velocity \( U_b \). This
assumption is equivalent to the criterion that $h_b$ be proportional to the axial distance at which $Y_{\tau l} = Y_{stab}$, where $Y_{stab} = Y_{st}$ for the mixing, strain-rate and scalar dissipation-rate theories, and $Y_{stab} = Y_l$ for the turbulent flame speed and empirical theories.

Using Eq. (4) to eliminate $h$ from Eq. (29),

$$\frac{d^*}{U_b} = \left(\frac{c_{n,1}}{c_2}\right)\left(\frac{Y_{stab}}{Y_o}\right)^2 \tau_{c,n} f_n(\eta_b)/f_2(\eta_b).$$

(32)

Taking $\eta_b$ to be the same for all fuels results in the blowout prediction

$$\frac{d^*}{U_b} = c_{n,3} \left(\frac{Y_{stab}}{Y_o}\right)^2 \tau_{c,n}.$$  

(33)

Values of $d^*/U_b$ can be compared with $Y_{stab}^2 \tau_{c,n}$ for each theory. These are shown in Table 4, where the values have again been normalized by those of $CH_4$.

Only the authors of the mixing time theory and the strain-rate theory have actually proposed this scheme for predicting blowout. Pitts, in his empirical correlation development, began by assuming that blowout occurs when the stabilization contour reaches its maximum radius $r_{ma}$: this is equivalent to choosing a particular value of $\eta_b$. He then, however, found an empirical expression for the local velocity at blowout which is inconsistent with Eqs. (25) and (26). Effectively, he eliminated $Y_l$ from Eq. (26.1) in his discussion of blowout which then changed $\tau_{c,e}$ to $c_u/S^2$. In the current discussion, the expression of $\tau_{c,e}$ consistent with Eq. (26.1) is used.

All of the theories make reasonable predictions about blowout except in the case of hydrogen where the turbulent flame speed and empirical theories are far off. This is due to these theories using the assumption that $Y_{stab} = Y_l$ rather than $Y_{st}$ as the remaining theories assume. For hydrogen, $Y_l$ is very different from $Y_{st}$, and the choice will affect predictions greatly.
Table 4
Comparison of blowout predictions and experimental results

<table>
<thead>
<tr>
<th>Fuel</th>
<th>$d^*/U_b$</th>
<th>turb. f.s.</th>
<th>mix. time</th>
<th>strain</th>
<th>scalar diss.</th>
<th>empirical</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CH_4$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$C_2H_4$</td>
<td>0.416</td>
<td>0.407</td>
<td>0.358</td>
<td>0.358</td>
<td>—</td>
<td>0.341</td>
</tr>
<tr>
<td></td>
<td>(-2%)</td>
<td>(-14%)</td>
<td>(-14%)</td>
<td></td>
<td>(-18%)</td>
<td></td>
</tr>
<tr>
<td>$C_3H_8$</td>
<td>1.10</td>
<td>0.950</td>
<td>0.861</td>
<td>0.861</td>
<td>1.15</td>
<td>0.879</td>
</tr>
<tr>
<td></td>
<td>(-14%)</td>
<td>(-22%)</td>
<td>(-22%)</td>
<td>(+5%)</td>
<td>(-20%)</td>
<td></td>
</tr>
<tr>
<td>$H_2$</td>
<td>0.00382</td>
<td>0.0529</td>
<td>0.00464</td>
<td>0.00460</td>
<td>0.0213</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(+1300%)</td>
<td>(+21%)</td>
<td>(+21%)</td>
<td>(+20%)</td>
<td>(+460%)</td>
<td></td>
</tr>
</tbody>
</table>

5. Conclusions

Five theories predicting turbulent jet flame liftoff, each utilizing self-similar properties of isothermal jets, have been investigated. The theories are: a turbulent flame speed theory, a cascade mixing time theory, a maximum strain-rate theory, a limiting scalar dissipation-rate theory and an empirical theory. Each predicts a relation between $\bar{h}$ and flow variables of the form

$$\bar{h} = c_{n,1} (Y_{stab}/Y_o) \tau_{c,n} U_o f_n(\eta),$$

(29)

with different expressions for $Y_{stab}$, $\tau_{c,n}$ and $f_n(\eta)$. By using the equation for the mean concentration field and by expressing the similarity functions as Gaussian, $\eta$ was eliminated from Eq. (29) to arrive at the relation

$$\frac{Y_{stab}}{Y_o} \bar{h} = c_{n,2} (Y_{stab}/Y_o)^2 \tau_{c,n} U_o \frac{d^*}{d^*} E_n.$$

(30.2)

(The one-dimensional cascade mixing time theory did not require the assumption of Gaussian functions to arrive at Eq. (30.2).)
The five theories were compared for their ability to correctly predict the experimentally observed linear dependence of $\bar{h}$ on $U_o$ by examining the exponent $E_n$. They were also compared for their ability to predict the relative liftoff heights of different fuels by examining the expression for $Y_{slab}^{\tau c,n}$. Finally, the theories were extended to predict blowout and compared to experimental results.

Because the theories have much in common it is difficult to conclude which one is the best. The turbulent flame speed, cascade mixing time and scalar dissipation-rate theories all predict highly nonlinear relations between $\bar{h}$ and $U_o$. The strain-rate field in turbulent jets has not been measured and as a result the strain-rate theory cannot be properly evaluated. The empirical correlation has no physical basis. The predictions of relative liftoff heights of different fuels were all within $\pm 30\%$ for all of the theories and all of the fuels except the turbulent flame speed's prediction for hydrogen. Blowout predictions of all the theories are within $\pm 25\%$ except for the turbulent flame speed and empirical theories' predictions for hydrogen.

The similarities in the theories illustrate their limitations in understanding lifted diffusion flames. To make significant progress in understanding the liftoff mechanism, more detailed physical models need to be developed.

6. Acknowledgement

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References


