A Boundary Scattering Strength Extraction Algorithm for the Analysis of Long-Range Reverberation Data

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This report describes a method for extracting rough boundary scattering strengths from long-range reverberation data generated by scattering from the rough ocean surface and bottom. The method is implemented through modifications and extensions of an existing NRL boundary-range-dependent reverberation model, RASP.

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A BOUNDARY SCATTERING STRENGTH EXTRACTION ALGORITHM FOR THE ANALYSIS OF LONG-RANGE REVERBERATION DATA

1. INTRODUCTION

This report describes the Scattering Strength Extraction Algorithm (SSEA) model, developed to extract boundary acoustic scattering strengths from measured underwater acoustic reverberation time series. Ocean surface scattering strengths have been deduced from short-range data with the results of Chapman and Harris [1] perhaps being the most well known and widely used. The geometry of short-range experiments is simple enough that scattering strengths may be extracted by combining the results of ray traces, beam patterns, and geometric calculations in a sonar equation analysis. For long-range data, the surface is not uniformly ensonified and there are multiple ray paths. These two complications make it impractical to extract scattering strengths from long-range data without the use of a comprehensive computer model. The SSEA model allows us to extract scattering strengths from measured long-range reverberation time series and will enable us to corroborate the scattering strengths deduced from short-range measurements with those from long-range data. It is desirable to corroborate with long-range data for several reasons:

- to demonstrate that the short-range data, typically collected by using impulsive broadband signals, can be used to predict reverberation from all signal types;
- to extend the available scattering strength data base with regard to environmental conditions and signal types.

2. REVERBERATION EQUATIONS

2.1 Short-Range Reverberation

As shown in Fig. 1, the calculation of reverberation levels for short-range geometries is given by

\[ R(t) = \sum_{n} Q(t - T_{out} - T_{back}) b_S(\psi_{S_n}, \phi_{S_n}) b_R(\psi_{R_n}, \phi_{R_n}) \left[ \frac{S(\theta_{out}, \theta_{back}, \psi)}{L_{out} L_{back}} \right] A_n \]  

(1)

Here \( Q(t) \) is the source envelope, and the summation over \( n \) represents an integration of scattering contributions from each surface area element. In the case of long-range reverberation it is necessary to add an additional sum over ray pairs. In this case, however, there is only one incident and one scattered eigenray to each surface patch. The angles \( \psi_{S_n} \) and \( \phi_{S_n} \) are the horizontal and vertical launch angles at the source, and \( b_S \) is the relative beam power in that direction. The same relation holds for the receiver ray arrival angles \( \psi_{R_n} \) and \( \phi_{R_n} \) and for the relative receiver beam power \( b_R \). The horizontal bistatic angle \( \psi \) is the difference between the azimuthal angles of the incident...
and scattered rays. We have included the horizontal bistatic argument in the scattering strength function for completeness in our discussion although the cases that we ultimately consider are independent of $\psi$. For each $n$ in the surface integration, the $n$th surface patch of area $A_n$ is uniformly ensonified and contributes reverberation characterized by specific values for the travel times $t$; the transmission losses $1/L_{\text{out}}$ and $1/L_{\text{back}}$; and the grazing $\theta_{\text{out}}$ and backscatter angles $\theta_{\text{back}}$. Geometrical spreading from source to surface is represented by $1/L_{\text{out}}$, and spreading from surface to receiver is represented by $1/L_{\text{back}}$. We interpret $R(t)$ to be the multi-ping average of the pressure squared, where $t$ represents time with respect to ping transmission.

Experiments to determine backscattering strengths are usually designed so that the source and receiver beam patterns and a finite ping duration selects a small, uniformly ensonified surface patch for which $\theta_{\text{out}} = \theta_{\text{back}}$. In this situation, it is only necessary to consider one term in the sum over areas. By taking $10 \log_{10}$ of the summand (contribution from a single surface area patch) of Eq. (1), we get the reverberation sonar equation

$$RL = SL - AG + SS + 10 \log_{10}(A_n) + TL_{\text{out}} + TL_{\text{back}},$$

where

$$SL(t) = 10 \log_{10} [Q(t - T_{\text{out}} - T_{\text{back}})]$$
$$AG = 10 \log_{10} [b_S(\psi_S, \phi_S)b_R(\psi_R, \phi_R)]$$
$$SS = 10 \log_{10} [S(\tilde{\theta})]$$
$$TL_{\text{out}} = 10 \log_{10} [1/L_{\text{out}}]$$
$$TL_{\text{back}} = 10 \log_{10} [1/L_{\text{back}}].$$
2.2 Long-Range Reverberation

The calculation of long-range reverberation levels is similar to that of short range, but with the addition of a summation over contributions from source-receiver multipath scattering pairs. Figure 2 illustrates three possible ray paths out to the nth surface patch in the first convergence zone. The multipath summation $\sum_{i,j}$ for these paths produces nine contributions to the reverberation. In general

$$R(t) = \sum_n \sum_{i,j} Q(t - T_{n,i} - T_{n,j}) b_S(\psi_{S_n,i}; \phi_{S_n,i}) b_R(\psi_{R_n,j}; \phi_{R_n,j}) \left[ \frac{S(\theta_{n,i}, \theta_{n,j}, \psi_{i,j}) A_n}{L_{n,i} L_{n,j}} \right]. \quad (8)$$

The index $i$ denotes a source ray, and the index $j$ denotes a receiver ray. Equation (8) is the basis of an extensive long-range reverberation model, the Range-dependent Active System Performance Prediction Model (RASP), which generates reverberation time series that are used in active system performance prediction. Reference 2 describes this model and its application.

3. SCATTERING STRENGTH EXTRACTION APPROACHES

An initial approach to extracting scattering strengths by use of Eq. (8) employs the fact that, at surface convergence zones, the energetic acoustic rays usually span a small angular range. Thus, if one assumes that at $t$ an average grazing and backscatter angle can be used, the resulting equation, in dB, is

$$RL(t) = SL + SS(\Theta_{ave}) + 10 \log_{10} R_{00}(t), \quad (9)$$

where $R_{00}(t)$ is a model prediction with a 0 dB source level and a 0 dB scattering strength. Comparing $RL(t)$ with the corresponding experimental time series yields average values for the scattering strength for the nominal grazing and backscatter angles. This process was used with some degree of success on both artificial and simulated data. However, surface scattering strengths are very sensitive to grazing angle, and a more refined extraction process is needed to retain the grazing and backscatter angle dependence in the results.
The first refinement considered was an inversion of the matrix equation that results from the discretization of $t$ in Eq. (8). This method was not attempted for several reasons relating to the number of time samples needed to generate an invertible equation.

Ultimately, it was found that an extension of the original method to a two-parameter fit (as opposed to a single parameter 0 dB fit) was tractable as well as sufficiently general to be useful. A parametric form for the scattering strength function is selected, and its parameters are varied to obtain an optimum fit to the measured reverberation time series. The critical aspects of this method are the selection of a physically reasonable scattering strength function with a minimum of fitting parameters and use of the optimization algorithm.

4. SELECTING A FUNCTIONAL FORM FOR SCATTERING STRENGTHS

We have chosen the following two-parameter scattering strength function:

$$S(\theta_{\text{back}}, \theta_{\text{out}}) = \alpha [\sin(\theta_{\text{out}}) \sin(\theta_{\text{back}})]^\gamma.$$  \hspace{1cm} (10)

The scattering strength parameters $\alpha$ and $\gamma$ are implicit functions of sea state and source frequency. Parametric forms for $\alpha$ and $\gamma$ as functions of sea state and frequency can be determined by applying SSEA to data taken at many different frequencies under a variety of sea state conditions. There is no horizontal bistatic angle dependence to our scattering strength function at this point. Applications of the SSEA method to horizontally bistatic source receiver configurations will require the use of parametrized scattering strength functions with a horizontal bistatic angle dependence. The methods and formulations that are introduced below change very little in their details if scattering strength functions with more parameters are introduced; however, the level of computational effort can increase dramatically.

Our chosen form for the scattering strength function is consistent with the small-angle form of the Raleigh perturbative scattering strength for an ocean surface described by a Pierson-Moskowitz spectrum:

$$S_{\text{Rayleigh}}(\theta_{\text{out}}, \theta_{\text{back}}) \approx \frac{8.1 \times 10^{-3}}{16\pi} (\theta_{\text{out}} \theta_{\text{back}})^2.$$ \hspace{1cm} (11)

It is also consistent with the Chapman and Harris empirical fit to surface backscatter [3] derived from monostatic ($\theta_{\text{out}} \approx \theta_{\text{back}}$) data

$$\alpha(f, v) (\theta)^\gamma(f, v),$$ \hspace{1cm} (12)

where $f$ and $v$ are the acoustic frequency and wind speed, respectively. Thus the choice of a two-parameter fit of the form of Eq. (10) for the scattering strength function has its basis in both theory and experiment.

A three-parameter form of the scattering strength function, $\alpha (\theta_{\text{out}} \theta_{\text{back}})^\gamma + \beta$, was examined because of its possible relation to scattering that originated from a subsurface layer of volumetric inhomogeneities. The resulting data fits were not acceptable at low angles since negative linear scattering strengths were predicted.

Linear forms for the scattering strength function like $\alpha \theta + \gamma$ were also considered in the expectation that the scattering strength function would be well enough approximated when the scattering angles contributing the the reverberation vary over a small range. Moreover, a linear scattering strength form simplifies the least-squares inversion process described in the next section.
so that it reduces to a matrix inversion. The angle domain over which linearly extrapolated scattering strengths are useful was found to be too narrow for practical applications. Rather, the logarithm of the scattering strength behaves linearly over a large range of the logarithm of scattering angles.

5. SCATTERING STRENGTH EXTRACTION ALGORITHM

To determine the scattering strength parameters \( \alpha \) and \( \gamma \), we use a least-squares approach. The RASP reverberation model is used to generate the average reverberation time series \( R(t, \alpha, \gamma) \) resulting from a given choice of the scattering parameters. The error that we minimize is the time integral of the squared difference between this trial series and the actual data \( R_M(t) \):

\[
E(\alpha, \gamma) = \left( \frac{1}{t_1 - t_2} \right) \int_{t_1}^{t_2} \frac{[R(t, \alpha, \gamma) - R_M(t)]^2}{R_M^2(t)} \, dt.
\]  

(13)

It is important to note that the integrand is actually the square of the relative error. An absolute error condition was used in the initial implementations of the method. Because scattering strengths and transmission losses range over several orders of magnitude in the integration window, it is not always possible to determine the scattering strength parameters with an absolute error minimization.

The error is minimized by choosing \( \alpha \) and \( \gamma \) so that

\[
\frac{\partial E(\alpha, \gamma)}{\partial \alpha} = 0 \quad \text{and} \quad \frac{\partial E(\alpha, \gamma)}{\partial \gamma} = 0.
\]

Because the scattering strength is linear in \( \alpha \), and the reverberation is linear in the scattering strength, we can solve the extremum condition for \( \alpha \),

\[
\alpha(\gamma) = \frac{\int_{t_1}^{t_2} \left[ \frac{R_M(t)}{R_M^2(t)} \right] \, dt}{\int_{t_1}^{t_2} \left[ \frac{R^2(t,1,\gamma)}{R_M^2(t)} \right] \, dt}
\]

(14)

and then we are left with the nonlinear equation for \( \gamma \):

\[
\int_{t_1}^{t_2} \left[ \frac{\alpha(\gamma)R(t,1,\gamma) - R_M(t)}{R_M^2(t)} \right] \frac{\partial R(t,1,\gamma)}{\partial \gamma} \, dt = 0.
\]

(15)

Note that \( R(t,1,\gamma) \) and \( \partial R(t,1,\gamma)/\partial \gamma \) are equivalent to model reverberation time series predictions with the scattering strengths given by \( (\theta_{out},\theta_{back})^7 \) and \( (\theta_{out}\theta_{back})^7 \ln(\theta_{out}\theta_{back}) \), respectively. There may be many solutions to Eq. (15) in a long-range multipath case, but usually one solution is well isolated from the rest, which is commensurate with the expected range of values of scattering strength for the appropriate wind speed and frequency.

6. RESULTS FROM ANALYZING THE SIMULATED DATA

Figure 3 shows a model prediction of surface and bottom reverberation for a typical deep ocean environment. The signal was a 250 Hz CW with a pulse duration of 10 s, and Chapman and Harris surface scattering strengths for a 17 knot wind were used. Note that outside of the surface
convergence zones the reverberation is dominated by bottom backscatter so that our analysis of surface backscattering strengths must be confined to regions near the convergence zone. Figures 4 and 5 show the results of applying SSEA to several small time windows on and about the second convergence zone peak (at 150 s). The behavior of the time series is quite different in each of these windows. Reverberation in Fig. 3 is roughly monotonically increasing in the earliest window (140 to 150), monotonically decreasing in the latest window (150 to 160), and rising and then falling behavior in the largest window (140 to 160). The near constancy of the scattering strength parameters extracted from these three windows indicates a degree of robustness in the ability of SSEA to deconvolve the complicated convergence zone transmission loss behavior and the simple monotonic scattering strength function from the reverberation time series.

7. NOTES ON THE APPLICATION OF SSEA TO SURFACE REVERBERATION

The example application given above demonstrates the efficacy of SSEA under ideal conditions. In this section we indicate a few of the outstanding issues that have arisen in its application to actual data.

Because the convergence zone time intervals in which the surface reverberation dominates over bottom reverberation are short, it is important that time averaging in the data preprocessing be kept to a minimum. Preprocessing time averaging windows that are a significant fraction of the convergence zone width have been found to significantly alter the scattering strength parameters, even when the averaging process was emulated by the reverberation calculation.

Depending on the signal type it has in some cases been enlightening to filter the reverberation time series into Doppler bins and compute scattering parameters for each of these bins separately.

It is important to note that SSEA assumes that any differences between the experimental data and the model prediction are due to the scattering strength. If a significant amount of time has elapsed between the pings used to produce a ping-averaged set for analysis, then variations in parameters such as the bottom loss, the sound speed profile, or the source/receiver beam pattern orientations, may not be accurately represented by the modeling. These inaccuracies in the modeling will manifest themselves in the extracted scattering strengths. For this reason it is strongly recommended that pings be back to back and in nonoverlapping frequency bands to get useful ping averaging.

8. SUMMARY

We have developed a long-range scattering strength extraction algorithm. It has been successfully tested on modeled reverberation time series generated with specific scattering strength functions and has been shown to recover the original scattering function.

The SSEA model represents a new capability for extracting detailed information on the scattering processes from the analysis of long-range reverberation data. It is still being refined to provide improved accuracy and to account for more details of the signal-dependent signal processing methods used. A short-range bistatic model is also under development. This report focuses on the use of SSEA to extract surface backscatter parameters, but it is equally applicable to the extraction of bottom backscatter parameters. In fact Lambert’s Law for bottom backscatter has the same form as our chosen parametric scattering strength function.
Fig. 3 - Reverberation using a Chapman and Harris surface backscatter function for 250 Hz and 17 knots
Fig. 4 - Comparing Chapman and Harris scattering strengths with those extracted by SSEA for various time intervals that bracket the peak of the convergence zone.
Fig. 5 - Comparing Chapman and Harris scattering strengths with those extracted by SSEA for various time intervals preceding and following the peak of the convergence zone.
9. ACKNOWLEDGMENTS

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10. REFERENCES

