Adaptive Control of Nonlinear and Stochastic Systems--Final Report (Unclassified)

Significant progress was made in a number of aspects of nonlinear and stochastic systems. An important problem in the adaptive estimation of a finite state Markov chain was solved, and significant progress was made on the corresponding, but much more difficult adaptive control problem. Problems of adaptive control with unknown disturbance distributions were solved in the case of incomplete state observations. A study of the adaptive control of bilinear ARMAX models was completed. Discretization procedures for adaptive Markov control processes were designed and analyzed, and problems of adaptive control with unknown disturbance distributions were solved in the case of incomplete state observations. In the area of nonlinear systems, the effect of sampling on linearization for continuous time systems was investigated. The smooth feedback stabilization of nonlinear systems was studied, and a model reference adaptive control scheme for pure-feedback nonlinear systems was developed and studied, and some problems in the linearization of discrete-time nonlinear systems were solved. In addition, some important problems in the areas of discrete event systems, robotics, and discrete time systems were solved.
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1. SUMMARY OF RESEARCH PROGRESS AND RESULTS

During the five years supported by this grant, we have made significant progress both in areas we proposed to investigate and in related areas. In this section, we summarize the progress in those areas that have resulted in publications.

1.1. Stochastic Control.

1.1.1. Stochastic Control of Markov Processes.

We have begun a research program in a major new area involving adaptive estimation and control problems for stochastic systems involving either incomplete (or noisy) observations of the state or nonlinear dynamics. The first class of problems we have studied involves finite state Markov chains with incomplete state observations and unknown parameters; in particular, we have studied certain classes of quality control, replacement, and repair problems. However, we found that work remained to be done for such problems with known parameters; this problem was studied in [23] and [44]. In these papers, we consider partially observable Markov decision processes with finite or countable (core) state and observation spaces and finite action set. Following a standard approach, an equivalent completely observed problem is formulated, with the same finite action set but with an uncountable state space, namely the space of probability distributions on the original core state space. It is observed that some characteristics induced in the original problem due to the finiteness or countability of the spaces involved are reflected onto the equivalent problem. Sufficient conditions are then derived for a bounded solution to the average cost optimality equation to exist. We illustrate these results in the context of machine replacement problems. By utilizing the inherent convexity of the partially observed problem, structural properties for average cost policies are obtained for a two state replacement problem; these are similar to results available for discount optimal policies. In particular, we show that the optimal policy has the “control limit” or “bang-bang” form. The set of assumptions used seems to be significantly less restrictive than others currently available. In [25], necessary conditions are given for the existence of a bounded solution of the average cost optimality equation. We consider in [46] average cost Markov decision processes on a countable state space and with unbounded costs. Under a penalizing condition on the cost for unstable behavior, we establish the existence of a stable stationary strategy which is strong average optimal.
As a prelude to studying adaptive control, the problem of characterizing the
effects that uncertainties and/or small changes in the parameters of a model can
have on optimal policies is considered in [26], [43]. It is shown that changes in the
optimal policy are very difficult to detect, even for relatively simple models. By
showing for a machine replacement problem modeled by a partially observed, finite
state Markov decision process, that the infinite horizon, optimal discounted cost
function is piecewise linear, we have derived formulas for the optimal cost and the
optimal policy, thus providing a means for carrying out sensitivity analyses. This
work is extended in [24] to several other classes of problems, including an inspection
problem with standby units, an optimal stopping problem, input optimization for
infinite horizon programs, and Markov decision processes with lagged information.

We have studied in [27] controlled diffusion processes on an infinite horizon with
three non-standard cost criteria: weighted cost, variance sensitive cost, and overtak-
ing optimality. Under a stability assumption we establish the existence of stationary
Markov controls which are optimal for these criteria in certain classes. Also, under
very general conditions we establish the existence of an $e$-optimal Markov policy for
the weighted criterion.

1.1.2 Stochastic Adaptive Estimation and Control.

In [19], [29], and [38], the adaptive estimation of the state of a finite state Markov
chain with incomplete state observations and in which the state transition proba-
bilities depend on unknown parameters is studied. A new adaptive estimation
algorithm for finite state Markov processes with incomplete observations is de-
veloped. This algorithm is then analyzed via the Ordinary Differential Equation (ODE)
Method. That is, it is shown that the convergence of the parameter estimation al-
gorithm can be analyzed by studying an “averaged” ordinary differential equation.
The most crucial and difficult aspect of the proof is that of showing that, for each
value of the unknown parameter, an augmented Markov process has a unique in-
vARIANT measure. New techniques for the analysis of the ergodicity of time-varying
Markov chains are utilized. The convergence of the recursive parameter estimates
is studied, and the optimality of the adaptive state estimator is proved.

We have begun to apply similar methods to adaptive stochastic control problems
with incomplete observations. We have first considered a quality control problem
in which a system, such as a manufacturing unit or computer communications
network, can be in either of two states: good or bad. Control actions available to the inspector/decision-maker are:

(a) produce without inspection,
(b) produce and inspect; or
(c) repair.

Under actions (a) and (b) the system is subject to Markovian deterioration, while a repair puts the unit in the good state by the next decision time. Informative data might become available while producing without inspection, and inspection is not always perfect. Hence the problem is modeled as a partially observed Markov decision process (POMDP). Furthermore, we assume that deterioration of the system depends on an unknown parameter, namely the probability of the state going from the good to the bad state in one time epoch when no repair is done. For the case of known parameters, we have shown (see above) that there is an optimal policy for the infinite horizon average cost criterion that is of the control limit (bang-bang) type. The adaptive stochastic control problem is, however, much more difficult than the adaptive estimation problem, because the presence of feedback causes the system transitions to depend on the parameter estimates and introduces discontinuities.

In [45] and [47], we have analyzed algorithms for this quality control problem in which the parameter estimates are updated only after the system is repaired; such algorithms are analogous to those in which estimates in queueing systems are updated only after each busy cycle. Since the system is returned to the "good" state after repair, one obtains a perfect observation of the state at that time, and our algorithm uses the observation at the next time to estimate the parameter. Hence, we develop parameter estimation techniques based on the information available after actions that reset the state to a known value. At these times, the augmented state process "regenerates," its future evolution becoming independent of the past. Using the ODE method, we show that two algorithms, one based on maximum likelihood and another based on prediction error, converge almost surely to the true parameter value. In addition, we modify the method of Shwartz and Makowski to prove optimality of the resulting certainty equivalent adaptive policy, assuming only the existence of some sequence of parameter estimates converging almost surely to the true parameter value. Again, the discontinuities and partial observations in this problem preclude the direct use of previously existing methods, but we have
been able to generalize the method to problems such as this. Also, we have avoided the very strong standard assumption that the parameter estimates converge almost surely to the true parameter value under any stationary policy. In [39] and [42], we have proved some initial results toward the more difficult analysis of such adaptive control problems, but in which the parameter estimates are updated at every time; in this case, the regenerative structure used in [45], [47] is not present. Also, in [5] our recent results for parameter-adaptive Markov decision processes (MDP's) are extended to partially observable MDP's depending on unknown parameters. These results include approximations converging uniformly to the optimal reward function and asymptotically optimal adaptive policies.

Another aspect of our research on adaptive control has involved systems with unknown disturbance distribution. In [6], we consider adaptive control of stochastic systems in which the disturbance or driving process is a sequence of independent identically distributed random variables with unknown distribution and a discounted reward criterion is used. Three different adaptive policies are shown to be asymptotically optimal, and for each of them we obtain uniform approximations of the optimal reward function. We have also obtained preliminary results on the extension of these results to the situation in which only incomplete or noisy observations of the state are available. In addition, we have in [17] extended the nonparametric results of [6] to problems with incomplete state observations. Our approach combines convergence results for empirical processes and recent results on parameter-adaptive stochastic control problems. The important issue of implementation has been addressed in [18], which presents finite-state discretization procedures for discrete-time, infinite horizon, adaptive Markov control processes which depend on unknown parameters. The discretizations are combined with a consistent parameter estimation scheme to obtain uniform approximations to the optimal value function and asymptotically optimal adaptive control policies.

We have investigated the adaptive control of stochastic bilinear systems in [2] and [34]. The minimum variance control law for bilinear systems with known parameters is shown to yield in most cases controls with infinite variance; this calls into question the use of the so-called bilinear self-tuning regulators. An adaptive weighted minimum variance controller based upon the cost with weighted control effort is suggested for first order bilinear systems and is shown to yield bounded-
ness of the closed loop system variables under a certain condition on the parameter estimate.

1.2. Nonlinear Systems.

In order to deepen our insight into nonlinear systems, we have also investigated and solved a number of problems in the linearization of discrete-time and discretized nonlinear systems. In [1] and [30], necessary and sufficient conditions for approximate linearizability are given. We also give a sufficient condition for local linearizability. Finally, we present analogous results for multi-input nonlinear discrete-time systems. In [3], necessary and sufficient conditions for local input-output linearizability are given. We show that these conditions are also sufficient for a formal solution to the global input-output linearization problem. Finally, we show that zeros at infinity of the system can be obtained by a particular structure algorithm for locally input-output linearizable systems. Whereas the objective of input-output linearizability is to make the input-dependent part of the output sequence linear in the new input, that of immersion by nonsingular feedback into a linear system (solved in [8], [35]) is to make the output sequence jointly linear in the new input and some analytic function of the initial state. Necessary and sufficient conditions for such immersion are given.

In [4], [31], we characterize the equivalence of single-input single-output discrete-time nonlinear systems to linear ones, via a state-coordinate change and with or without feedback. Four cases are distinguished by allowing or disallowing feedback as well as by including the output map or not; the interdependence of these problems is analyzed. An important feature that distinguishes these discrete-time problems from the corresponding problem in continuous-time is that the state-coordinate transformation is here directly computable as a higher composition of the system and output maps. Finally, certain connections are made with the continuous-time case. We build on these results in [16], [36], [40], [41] in which we investigate the effect of sampling on linear equivalence for continuous time systems. It is shown that the discretized system is linearizable by state coordinate change for an open set of sampling times if and only if the continuous time system is linearizable by state coordinate change. Also, for $n = 2$, we show that even though the discretized system is linearizable by state coordinate change and feedback, the continuous time affine complete analytic system is linearizable by state coordinate change only. Also,
we suggest a method of proof when \( n \geq 3 \).

The papers [7], [32] investigate the global controllability of piecewise-linear (hypersurface) systems, which are defined as control systems that are subject to affine dynamics on each of the components of a finite polyhedral partition. Various new tools are developed for the study of the problem, including a classification of the facets of the polyhedra in the partition. Necessary and sufficient conditions for complete controllability are obtained via the study of a suitably defined controllability connection matrix of polyhedra. In [9] and [37], we investigate the problem of smooth feedback stabilization of nonlinear systems with stable uncontrolled dynamics. We present sufficient conditions for the existence of a smooth feedback stabilizing control that are also necessary in the case of linear systems. Analogous results are established for discrete time systems.

1.3. Deterministic Nonlinear Adaptive Control.

Almost all of the work in the field of deterministic adaptive control is restricted to the study of linear plants. In trying to extend adaptive schemes to nonlinear systems, one is faced with considerable obstacles. The most important of these is the lack of a systematic methodology for nonlinear feedback design. In recent years, considerable effort has been invested in the study of canonical forms for nonlinear systems and in particular the characterization of the class of those systems which are linearizable under the action of the nonlinear feedback group. Equivalence to linear dynamics is a particularly desirable property from the point of view of control synthesis.

A possible design methodology, applicable to linearizable systems, is to build a controller for the nonlinear system by designing one for the equivalent linear system and utilizing the transformation (from linear to nonlinear) along with its inverse. This approach has already been applied to the design of automatic flight-control systems for aircraft of significant complexity. The chief drawback of this method is that it relies on all the states being measured and on an exact cancellation of nonlinear terms in order to get linear behavior. Consequently, in the case where the plant contains unknown or uncertain parameters, adaptation is desirable in order to robustify, i.e., make asymptotically exact, the cancellation of nonlinear terms. A major difficulty here is that the linearizing transformation, being a function of the system parameters, is itself unknown, and hence, the above design approach does not
allow for a straightforward incorporation of an adaptive controller. The extension of parameter adaptive algorithms developed for linear systems to "linearizable" ones becomes, therefore, an important problem.

In a study [12], [33], that seems to be among the first of its kind, we restricted our attention to "pure feedback" systems, a special class of nonlinear systems which arise as a canonical form of linearizable dynamics. We presented an adaptive algorithm and the design of a model reference adaptive controller for this class of problems. An interesting feature of our adaptive scheme is that it updates estimates of the feedback and coordinate transformation required to linearize the system. Under some mild technical conditions, we established global convergence of the output error for all initial estimates of the parameter vector lying in an open neighborhood of the true parameters in the parameter space. Also, in simulation studies, the performance of the algorithm was excellent. At first sight, this model might seem as a fairly restricted class of nonlinear plants. One should keep in mind, though, that not only does this model cover a wide range of interesting real life applications, but, in addition, the pure-feedback form may be viewed as a canonical form of feedback linearizable nonlinear systems.

1.4. Other Related Research.

We have also made progress in a number of other related areas of research. In the area of robotics, the problem of selecting joint space trajectories for redundant manipulators is considered in [13]. Solutions which allow secondary tasks to be performed by the arm simultaneously with end-effector motions may be selected in a number of ways. An algorithm to accomplish this by means of conditions on a scalar function of the joint variables is introduced and analyzed in [13]. In [14], the problem of the distribution of dynamic loads for multiple cooperating manipulators, is considered. Methods of load distribution, which allow desired object motion while selecting loads desirable for alleviating manipulator dynamic loads, are developed. The motion and internal loads induced on an object manipulated by two or more robotic mechanisms are considered in [28]. In particular, for a desired motion trajectory of the object, the question of load distribution among the arms is analyzed, with particular attention given to the internal loading of the object. A new representation of the load distribution problem is given by the introduction of a particular "non-squeezing" pseudoinverse, which is shown to possess properties
which expose the underlying structure of the problem. It is expected that by using this pseudoinverse, new insight will be gained, and necessary analysis simplified, in a number of aspects of multiple manipulator research. A number of these aspects are detailed and illustrated using a two armed example.

In the area of discrete event dynamic systems, we have designed algorithms for supervisor synthesis problems with partial observations [15]. These algorithms provide a good suboptimal solution to the problem; in addition, they involve new classes of automata which are of interest in their own right. However, these solutions are often too restrictive, and in [20] we have studied a more general class of solutions. These give rise to another interesting class of supervisors, but they are computationally much more difficult. In [21], we discuss the computation of supremal controllable and normal sublanguages. We derive formulas for both supremal controllable sublanguages and supremal normal sublanguages when the languages involved are closed. As a result, those languages can be computed without applying recursive algorithms.

Periodic orbits of the matrix Riccati equation are studied in [10]; it is shown that periodic solutions are bounded if and only if the span of their range does not intersect the orthogonal complement of the controllable subspace of the associated linear system. In [11], a discrete-time, linear, time-invariant control system with a fixed time delay in the feedback loop is considered; simple necessary and sufficient conditions for feedback stabilization are developed. Based on a minimax criterion, we define in [22] the concept of equalizability for a nonlinear, discrete-time communication channel. Sufficient conditions for a channel to be equalizable, via a finite memory equalizer, are also derived.

2. PUBLICATIONS

Journal Publications


effect of sampling on linear equivalence and feedback linearization, Systems & Control Letters 13 (1989), 373–381.


[27] M.K. Ghosh and S.I. Marcus, Infinite horizon controlled diffusion problems with


Other Publications


3. RESEARCH PERSONNEL ASSOCIATED WITH THE RESEARCH EFFORT

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4. PAPERS PRESENTED


1989, Urbana, IL.


