On the Calculation of Accurate Antenna Pointing for Terminals Working with Geosynchronous Communications Satellites

Technical Report 906

M.T. Lane

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Lincoln Laboratory
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ON THE CALCULATION OF ACCURATE ANTENNA POINTING FOR TERMINALS WORKING WITH GEOSYNCHRONOUS COMMUNICATIONS SATELLITES

M.T. LAVE
Group 91

TECHNICAL REPORT 906

11 DECEMBER 1990

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LEXINGTON MASSACHUSETTS
ABSTRACT

Terminals working with communications satellites must have the capability to produce accurate antenna pointing from information provided by the operations center. Usually this takes the form of an (mean) element set for the satellite orbit, which must be propagated by the terminals and from which pointing must be calculated. This report addresses the problem for geosynchronous communications satellites, most non-Soviet communications satellites being in such orbits. First the nature of geosynchronous orbit propagation is summarized, with principal perturbations highlighted, and then the problem of element set transfer is discussed from a practical point of view. Since most operations centers do not maintain element sets for their satellites (but rather receive ephemeris information from another source), this discussion is designed to maximize the accuracy and lifetime of a transferred element set. An example is provided to demonstrate the need for compatible element sets and software, and to show a solution to a timely problem involving element set transfer.

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ACKNOWLEDGMENTS

The author is appreciative of W.W. Ward and W.K. Hutchinson of the Satellite Communications Technology group at MIT Lincoln Laboratory for stimulating an interest in this problem, suggesting a rewarding trip to England to present this report, and reviewing the manuscript.
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1. INTRODUCTION

The implementation of precise antenna pointing algorithms in a satellite communications system presents several problems to be addressed and can even result in unsuccessful experiments. Typically, the business of satellite orbit maintenance and predictions is not of prime concern to the communications operation. Most often the necessary information is acquired from other organizations with interfaces to the operation specified in the charter. Problems arise when ephemeris information is not compatible with the software in the communications operation. This report comments on the problems of satellite orbit determination and prediction from a user's point of view. Hence, the scope is to highlight the important aspects of orbit use so that compatibility issues and accuracy can be tackled head-on to eliminate (as much as possible) these problems. At the very least, it is advisable to be aware of these issues and to understand the trade-offs involved with a particular use of satellite ephemeris information.

Throughout this report, information and examples are centered around the generation and propagation of geosynchronous satellite orbits. That is an area of broad application. Most non-Soviet communications satellites are maintained in such orbits, as are some weather satellites. In fact, this realm is one of the more benign cases for adequately dealing with the problems of compatibility and orbit modeling. Many more serious problems exist with the ephemeris prediction of low-altitude satellites because of the difficulty in predicting solar flux values and atmospheric drag effects.

One of two courses of action is typically implemented in a communications operation in need of satellite ephemeris information: The first generates precise pointing predictions for the communications site's antenna at a satellite orbital analysis center and supplies them at densely spaced intervals. Interpolation algorithms are employed at the communications site to determine the proper antenna pointing at any desired time during the experiment. This method can provide sufficient accuracy but is tedious and time-consuming for both the orbital analysis center and the communications operation. Large files must be provided, and extensive input and output must be adequately defined in order for the information to be useful. Moreover, each file of pointing information is only useful for one particular site. If several sites are involved during a particular experiment, then several different files must be generated and sent out. This is a heavy burden on the orbital analysis center. The advantages of this procedure are that no compatibility issues are encountered and accurate antenna pointing is ensured.

The second course of action sends an element set of some kind to the site so that a propagation-and-prediction algorithm can produce the desired antenna pointing at the desired time. This is certainly preferable for the orbital analysis center since only one element set needs to be generated and it can be used at all sites taking part in the experiment. This method appears to be easier, but compatibility problems can produce large errors, and in fact, the uninitiated site can often fail to produce adequate antenna pointing for a successful experiment. Therefore, it is important
to understand this problem and to consider ways to eliminate (or reduce) its consequences. The remainder of this report only considers this second mode of satellite ephemeris transfer.

In this report, Section 2 presents several issues involved in building a propagation theory, including the perturbing forces acting on a geosynchronous satellite and the nature of their effects: the debate over using special or general perturbation propagation theories (that is, numerical, analytical, or semianalytical integration methods); and reference frame considerations. Section 3 follows with a discussion of proper element set transfer, the topic of mean versus osculating element sets, and the process of converting the transferred element set to computed antenna pointing. Section 4 gives an example involving a communications operations center and the transfer of element sets of particular communications satellites to it from an outside source and from it to remote terminals. This example highlights problems that exist when an operations center does not maintain a satellite ephemeris and must rely on transfer of element sets from the outside. It also demonstrates one solution to the problem of compatibility. Finally, key points are summarized and concluding remarks are presented in Section 5.
2. GEOSYNCHRONOUS ORBIT PROPAGATION

The development of an orbit propagation theory to provide antenna pointing information for a communications experiment involves many decisions based upon the accuracy needed and the length of time before a new element set is available. For example, some communications operations centers (and their remote terminals) can tolerate position errors up to 100 mdeg in azimuth and elevation angle over a period of 30 days. Therefore, a simple Keplerian two-body orbit propagation model for the communications satellites is sufficient for the needs of the operations center and its remote terminals. If tighter accuracy constraints are stipulated or longer time spans are required between element sets, then perturbations must be considered in the orbit propagation software. On the other hand, at the Millstone Hill Radar Operations Center there is ongoing work to calibrate sensors to within 1 to 10 mdeg in azimuth and elevation angle and to less than 1 m in range. For this work, even elaborate analytical theories designed to model perturbing effects on the satellite orbits are not accurate enough, and it is necessary to resort to precision numerical integration of the equations of motion.

Such fine modeling of forces that act on satellite orbits are more commonly known as special perturbations, which solve Newton's laws of motion directly without applying any variational principles. These models are computationally expensive, so a less accurate analytical theory is needed for near-real-time applications. The simplification of the satellite's equations of motion using variational principles so that analytical or semianalytical integration is possible is known as general perturbations. Speed is gained, perhaps at the expense of accuracy, but some of the more elaborate analytical theories available today are able to predict geosynchronous orbits on a routine basis to within 20 mdeg in azimuth and elevation angle and 200 m in range over a time interval of 150 days or so. Better accuracy is available for longer prediction time spans if the secular and long-periodic effects are integrated numerically and the short-periodic variations are evaluated analytically. This third type of propagation is referred to as a "semianalytical method," and usually little speed is sacrificed since large step sizes can be used for the numerical integration.

Whether the orbit propagation theory is based upon special or general perturbations, the dominant source of error in precision orbit calculation is not due to gravitational, but to satellite-dependent effects such as solar radiation pressure, atmospheric drag-like influences, and satellite maneuvers. There has been impressive progress in the last 30 years in modeling gravitational perturbations, which makes this last statement true, but satellite-dependent effects are often difficult to model. Moreover, maneuvers can change satellite attitude slightly, which in turn can alter the effects of solar radiation pressure. However, these complicated effects are only on the order of 500 to 1000 m in range (excepting, of course, large orbit changing maneuvers) and can be safely neglected in more crude orbit theories. Nevertheless, most orbital analysis centers attempt to achieve the best possible fits to observed data, and modeling of these effects is usually implicit in the output element set.
The important gravitational effects on geosynchronous orbits include third-body attractions from the Sun and the Moon, latitude-dependent harmonics in the geopotential, and critical longitude-dependent harmonics exhibiting a resonance effect on the orbit.

Over periods of time on the order of a satellite revolution, typically referred to as the “short period,” the effects due to the Moon and the Sun are dominant and have amplitudes on the order of 2 km and 800 m, respectively. For example, the total perturbation from the Moon’s gravitational force on the semimajor axis and longitude of a geosynchronous satellite is plotted in Figure 1. A strong secular portion is evident in the plot of the effect on longitude, while the perturbation of the semimajor axis exhibits a short-periodic behavior with large amplitude.

![Figure 1. The lunar gravitational perturbation on the semimajor axis and longitude for a geosynchronous orbit.](image)

The effects due to the latitude-dependent zonal harmonics in the Earth’s potential are about an order of magnitude smaller for the geosynchronous orbit, but resonance effects cause a long-periodic behavior on the order of 1000 days with an amplitude reaching 20 km or more. However, resonance is only important for those orbits commensurable with the Earth’s rotation rate. In Table 1 this is demonstrated as the longitude-dependent geopotential effects on the semimajor axis characterized for three satellite orbits with varying degrees of commensurability. The effects on the first two satellites are small and inconsequential, but the perturbation on Lincoln Experimental Satellite 8 (LES-8) soars in both amplitude and period since it is exactly synchronous with the Earth’s rotation. The nature of resonance appears secular over small time frames even though it is
entirely periodic. However, as more harmonics in resonance are allowed to interact, as is the case of the undisturbed geosynchronous orbit, the effect takes on a pseudoperiodic behavior that can be very difficult to model analytically over extremely long periods of time. Indeed, the ability of a simple two-body propagation model to provide adequate antenna pointing over a given period of time largely depends on the longitude position of the geosynchronous satellite and its impending resonance motion. Each of the other perturbations has secular (or long-periodic) components, but the short-periodic effects exhibit a larger amplitude. As long as the size of the short-periodic amplitude of a particular perturbing force is well within the error tolerances set forth for the orbit propagation theory, the entire effect can be neglected over median time spans of, say, 30 days or so. However, a satellite orbit in exact commensurability with the Earth's rotation rate, located near an equilibrium point, is less likely to diverge far from the two-body propagation over median time intervals than an orbit located midway between equilibrium points.

**TABLE 1**

*Longitude-Dependent Effects on the Semimajor Axis of Near-Synchronous Orbits*

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Mean Motion (rev/day)</th>
<th>Period (days)</th>
<th>Amplitude (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IDCSP-16</td>
<td>1.1</td>
<td>4.6</td>
<td>127</td>
</tr>
<tr>
<td>EKran-5</td>
<td>1.054</td>
<td>11.5</td>
<td>252</td>
</tr>
<tr>
<td>LES-8</td>
<td>1.003</td>
<td>1065</td>
<td>20,430</td>
</tr>
</tbody>
</table>

Figure 2 displays the error in propagating the semimajor axis and longitude of a geosynchronous orbit over 150 days for two different analytical propagation models. One is a simple theory lacking resonance effects. The other is a more elaborate theory with resonance and higher-order contributions from the lunar and solar gravitational forces adequately modeled. One can see that over a small time period of 30 days both theories perform adequately, but for the full period of 150 days the simple theory is not very close to the truth model, while the advanced theory covers the slow long-periodic motion nicely. A good overview of analytic modeling of satellite perturbations that the reader might find helpful is given in [1].

A summary of the important perturbative effects on geosynchronous orbits is given in Table 2. Also included is an estimate of the maximum residual error incurred from an analytical solution to the equations of motion under the action of each force... This table is general, and certain satellites may be affected to a greater or lesser extent than that which is tabulated. For example, the force...
from the Moon's gravitational potential exhibits a perturbation in range for geosynchronous orbits with an amplitude of about 2 to 3 km in range and about 1° in azimuth and elevation angle. while this effect can be modeled analytically with an expected error on the order of 100 m in range and 10 mdeg in azimuth and elevation angle. More detail on analytic modeling of perturbations on satellite orbits can be found in the following references: perturbations due to zonal harmonics in the Earth's potential are developed in [2], resonance effects are described in [3], models for the effects of lunar and solar gravitational forces are given in [4], and a good analytical approximation of solar radiation pressure effects is given in [5]. Several more references are listed in those cited works for the interested reader.

As a final example, it is appropriate to consider an alternate way of visualizing the individual effects on geosynchronous orbit propagation. Two sets of data were simulated for LES-8 with a special-perturbations propagation routine using an accurate force model. The first set covered a 10-day span, and the second was lengthened to 100 days. The data were in the form of equally spaced metric observations (azimuth, elevation angle, range, and range rate) from a particular site. The force model that generated the data included a full gravitational model for the Earth, Moon, and Sun, and also a precise model of nongravitational effects such as atmospheric drag, tides, and solar radiation pressure. This exact model is routinely used with true radar metric data on LES-8 to fit arcs to within the inherent noise of the observations. This typically means orbits are fit

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**Figure 2. Analytical theories compared to numerical propagation.**
### TABLE 2
Major Perturbations in Range for Geosynchronous Orbits Over a 30-Day Propagation

<table>
<thead>
<tr>
<th>Perturbation</th>
<th>Size (m)</th>
<th>Analytic Model Error (m)</th>
<th>Nature of Effects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moon</td>
<td>2500</td>
<td>100</td>
<td>Periodic</td>
</tr>
<tr>
<td>Sun</td>
<td>800</td>
<td>10</td>
<td>Periodic</td>
</tr>
<tr>
<td>Oblate Geoid</td>
<td>200</td>
<td>10</td>
<td>Periodic</td>
</tr>
<tr>
<td>Resonance</td>
<td>3000</td>
<td>50</td>
<td>Secular</td>
</tr>
<tr>
<td>Radiation pressure</td>
<td>800</td>
<td>50</td>
<td>Periodic</td>
</tr>
<tr>
<td>Drag-like force</td>
<td>500</td>
<td>20</td>
<td>Periodic</td>
</tr>
<tr>
<td>Combined forces</td>
<td>3000 to 5000</td>
<td>200</td>
<td></td>
</tr>
</tbody>
</table>

To an accuracy of 10 m in range and 10 mdeg in azimuth and elevation angle. These data were used to test an analytical force model, and later isolated models reduced by one force at a time. The root-mean-squared (rms) errors of the converged residuals from the orbit fits are displayed in Table 3, where one can see more clearly the size of isolated effects. For example, it is seen that if resonance is ignored over 10 days, the error is small, while if it is ignored over 100 days the error has grown to appreciable levels (over 1° in azimuth, 0.16° in elevation angle, and 7 km in range). The other effects make a smaller contribution but are nevertheless real for accurate theories. On the other hand, this table also demonstrates that (even over 100 days) models can be designed to fit arcs of data accurate to within 20 mdeg in azimuth and elevation angle and 200 m in range. These experiments have also been verified with real radar data, and it is safe to assume that the truth model used here accurately represents an actual geosynchronous orbit. Of equal importance is the fact that the computations required for analytical theories can run quickly on floating-point machines for near-real-time use.

Once it is decided which perturbing forces will be included in the theory and which type of propagation theory is to be applied (numerical, analytical, or semianalytical integration), it is important to settle on the reference frame for calculating the Earth-fixed antenna pointing such as azimuth and elevation angle and range. This is typically decided by the representation of Greenwich sidereal time and the calculation of the ecliptic inclination (if it is used). Most of this is well known by those who build theories to calculate antenna pointing predictions, but it is safe to reiterate the hazards of not paying attention to this detail. For example, if an element set is received for a particular satellite and it is referred to the equinox of 1950, and the Greenwich sidereal time in the prediction program is referred to the equinox of date, the error in the calculation of the argument of
### TABLE 3
Converged rms Error of Numerically Simulated Data and Analytical Theories with Different Force Models

<table>
<thead>
<tr>
<th>LES-8</th>
<th>10 Days/100 Days</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Az (mdeg)</td>
</tr>
<tr>
<td>Full-force model</td>
<td>3/15</td>
</tr>
<tr>
<td>Simple lunar theory</td>
<td>5/16</td>
</tr>
<tr>
<td>No resonance</td>
<td>9/1189</td>
</tr>
<tr>
<td>No radiation pressure</td>
<td>4/31</td>
</tr>
<tr>
<td>No drag</td>
<td>4/347</td>
</tr>
</tbody>
</table>

the right ascending node can be about 0.5°. An error of this size can, of course, get an experiment off to a bad start.
3. ELEMENT SET TRANSFER

Once the propagation theory is well in place, it is important to consider how element sets are to be updated in the system. Typically, precision metric tracking data are collected on the satellite, and fitting procedures are applied to update the orbital element set. However, this fit is useful only for the theory that is used for the fit procedure. In the case of a special-perturbations propagator the osculating element set is fit to the data, and in the case of a general-perturbations propagator with some analytical integration involved, a mean element set is fit to the data. The difference between the two is fundamental. The osculating element set is the true position of the satellite at a given time, and a mean element set often refers to the average of a number of osculating element sets. Analytical theories resort to mean element sets because the method of integration of the equations of motion requires simplification of the differential equations. Either some of the more slowly moving variables are to be considered constant or else the entire element set is transformed so that the equations are simplified when the system is changed to this new coordinate frame. A well-known example of this is the classical theory of canonical transformations, which has evolved into the more modern generalized method of averaging.

Generally, it is best to assume that two different software packages that predict satellite orbits and antenna pointing are incompatible. This is true even if the design philosophy is the same for both theories. Slight variations such as the use of different precision for the constants such as $\pi$ or the radius of the Earth can cause numerical inconsistencies, and so conversion algorithms must be applied in order to use an element set generated by one theory in the other theory. Thus, the problem is more far-reaching than which forces are included in the model and which inertial frame is adopted: rather, critical design issues are involved such as the method of propagation, the types of higher-order terms included, and the truncation or simplification applied.

Once mean elements are calculated for an analytical theory, the transformation from the mean to the osculating element set (so that antenna pointing can be calculated) is the direct problem and requires evaluating the analytical solution with the mean elements. The inverse problem of determining the mean given the osculating element set is more difficult and usually requires iteration. An important point to make in this connection is that

The accuracy of a converted element set is only as good as the least accurate theory involved in the conversion.

Suppose that a satellite operations center applies a special-perturbations propagator to fit precise tracking data so that the output element set is accurate to within 10 m in range and 5 mdeg in azimuth and elevation angle. Now if this osculating element set is inverted to a mean element set that is compatible for a simple two-body propagation theory, then the accuracy of that propagator is no better than if it were used to fit the tracking data. That is, the elaborate models involved with the production of the osculating element set at the orbital analysis center or the accuracy attained...
(such as it may be) will not be of particular importance to users of the simpler theory. Moreover, if the osculating element set from the special-perturbations routine is sent directly to the simple two-body theory without modification or inversion, then much worse errors could result.

A more subtle problem exists in the area of inverting precise osculating element sets to mean element sets compatible with an analytical theory. For reference, suppose this procedure can be interpreted mathematically by the relation

$$\mathbf{z}_O(t) = F(\mathbf{z}_M(t)).$$

where \(\mathbf{z}_O\) denotes an oscillating element set and \(\mathbf{z}_M\) denotes the mean element set. There is some time-dependent function \(F\) that relates the two. The direct problem is to simply evaluate \(F\) at \(\mathbf{z}_M(t)\) for a given time \(t\), and the inverse problem of determining \(\mathbf{z}_M(t)\) given \(\mathbf{z}_O(t)\) can be solved by iteration using Newton's method or some other root-finding procedure. If the inverse problem is solved for \(t\), then there is no guarantee that \(\mathbf{z}_M\) will propagate accurately for other times \(t\). Indeed, if several important short-period perturbations (such as from the Moon and the Sun) are ignored in \(F\), then there can exist a bias in the solution for \(\mathbf{z}_M\) that can be as large as the amplitude of the neglected perturbations. This bias can be largely reduced if several osculating element sets are used to solve for \(\mathbf{z}_M\) in a least-squares fashion. Thus, it is important to make sure that if a mean element set is converted from an oscillating element set, then the solution is valid over the time interval before the next updated oscillating element set is available.

Consider the following examples of element set conversion: Three different (analytical) theories were selected to demonstrate the principal errors in converting an oscillating (truth) element set to a mean element set compatible with a particular theory. Theory 1 is an advanced analytical theory. Theory 2 is an analytical theory that does not include lunar and solar short-periodic terms, and Theory 3 is an analytical theory that includes lunar and solar short-periodic terms but no resonance effects. An oscillating element set for a geosynchronous orbit was chosen and propagated ahead 10 days as a truth reference trajectory. A mean element set was fit for each theory, and then a propagation was made 10 days ahead. The residuals in semimajor axis from each theory's propagation to the reference trajectory are shown in Figures 3 and 4. In Figure 3, a single point inversion was used to invert the oscillating into a mean element set. Notice that the advanced theory has small residuals (on the order of 200 m and of a periodic nature). This is consistent with the discussion given in the first section and reflects the overall error in the analytical theory. Theory 3 has a secular degradation because of the lack of resonance in the force model; however, the short-periodic errors displayed are of a similar size as for Theory 1. There is no secular error displayed for Theory 2 in Figure 3 because resonance effects are accounted for, but the large (1-km amplitude) error is due to the lack of modeling short-period lunar and solar effects. Because a single oscillating point was used for the inversion, a bias appears in the residuals as large as the actual effect that was not modeled at the epoch of the inverted element set. This can be removed by using the advanced theory (Theory 1) to generate 20 points spaced over 1 day and performing a least-squares fit to the Theory 2 propagator. This will average out the short-period effects in the
resulting mean element set for Theory 2 and remove the bias. However, this will not remove the large amplitude in the residuals, as shown in Figure 4. If the large secular effect in the residuals of Theory 3 (shown in Figure 3) are to be removed, then a sampling of osculating elements over the entire 10-day propagation must be provided and the Theory 3 mean element set must be fit to these points. This will minimize (in the least-squares sense) the amount of the largest deviation from the truth orbit.

Figure 3. Residuals in semimajor axis for inverting a single osculating element set to a mean element set for each of three theories, propagating each analytical theory ahead, and comparing to the truth reference orbit.

If mean element sets are sent to communications terminal sites from an orbital analysis center, then either the theory that generated these element sets must be used to calculate antenna pointing at the sites or else they must be converted to mean element sets for the desired propagator. Usually, the only point in common for two different mean element sets valid for two different theories is the osculating element set, and a conversion routine can be designed to go through the osculating
Figure 4. Residuals in semimajor axis for Theory 1 and 2, where the mean element set for Theory 2 was obtained by implementing a least-squares fit to 20 osculating points generated by Theory 1 over one day.
set. However, the error in this conversion includes the error from both theories' calculation of the osculating element set. Moreover, both theories must be available to the conversion routine so that the old theory can apply its direct transformation to compute the osculating element set and the new theory can invert that result to compute a mean element set.

The amount of literature on element set transfer is sparse, except in restricted circulations and internal memoranda; therefore, it is difficult to suggest a large number of accessible works that can provide more information on this subject. But [6] may be of interest to the reader and may lend additional insight.

In summary, in order to support proper transfer, it is essential to understand the theory used to generate the element set (as well as its reference frame) and the theory used in the antenna site's propagation software. The received element set must be properly converted: several-point conversion is more reliable over time than single-point inversion. Often large biases due to neglected perturbations can result with the single-point inversion, and element sets that have not been converted will only work well in the theory that generated them.
4. EXAMPLE

A communications operations center was recently faced with ephemeris transfer problems of the nature described in this report. This center does not maintain element sets on its satellites, but receives ephemeris information from an outside source. For the first 2 1/2 years of operation, the center (and its remote terminals) was able to communicate with its satellites without trouble by inserting the received element set into its own simple two-body theory propagator and generating pointing information. A closed-loop tracking system helped the communications operations center's terminals lock onto the satellite even if initial errors reduced the received signal. This method was adequate for pointing errors less than 100 mdeg over 30-day intervals.

A data-system upgrade was implemented at the source of the element sets during the third year of operation. The orbit-fitting procedure was changed, and the communications operations center was not able to achieve the same results with the new element sets. For simplicity, the initial method of producing element sets will be referred to as the "old method" and the upgrade will be referred to as the "new method." Figure 5 demonstrates the difference in azimuth and elevation angle as predicted by the simple two-body theory propagation software using old- and new-method element sets as input (where it is kept in mind that an old-method element set was able to predict the closed-loop tracking azimuth and elevation angle with errors up to about 30 to 50 mdeg over 30 days). For the predictions, the deviation of the new- from the old-method element set shows that the new method is reasonably good at first, but does not allow the two-body propagator at the communications operations center to predict accurately over the long term. An analysis of the problem was initiated by a Lincoln Laboratory team, and it was found that the entire problem related back to compatibility issues. However, the main purpose of the investigation was to allow the communications operations center (and its remote terminals) to use the new-method element sets in the same way as before.

The first task in solving this problem was to find out how the new-method element sets were generated and what reference frame was used. The team learned that the new-method are true osculating element sets from a special-perturbations fit to precision tracking data and that they are referred to the mean equinox and equator of date. Independent tracking data were collected by Lincoln Laboratory's L-band radar at the Millstone Hill Radar Operations Center, and an element set was generated by a Lincoln Laboratory special-perturbations propagation-and-fit routine so that the output osculating element set was referred to the mean equinox and equator of date. This element set agreed to within a small amount with that produced by the new method. Since the communications operations center's propagator is a simple two-body theory with a desired lifetime of 30 days between element set updates, it was necessary to convert the osculating set to the simple theory. The reference frame for this propagator is also the mean equinox and equator of date, and so no precession was needed in the conversion.
In order to avoid the large biases that would result if a direct inversion to this single osculating element set was implemented, it was decided to generate more osculating element sets equally spaced over 30 days and to perform a least-squares fit to the two-body mean element set propagation. Any propagation model could be used to generate these element sets, but it is necessary to account for large-periodic variations (such as from the Moon and the Sun) so that the biases can be removed. In order for the conversion routine to run in near real time, it was decided to use an accurate analytical propagator to generate the sample data, and this required generating a mean element set compatible with its theory. A single-point inversion was used, but the large biases were avoided since the large short-periodic perturbations due to the Moon and Sun are accounted for in the analytical theory. Once this was done, 120 points of sample data were generated evenly spaced over 30 days, and the two-body mean element set was fit to these data. The result is now completely compatible with the communications operations center’s propagation-and-pointing routine, and the only real input to that process is a single osculating element set (consisting of six elements together with their epoch). The error in this procedure is determined by the combined errors from each of the theories involved. For this example, there is the initial error from the osculating element set (which should be very small), the error from the generation of the 120 points of sample data (which can be off by as much as 1 km in position), and finally the error from the two-body theory used to point to the satellite. This last source of error is dominant. The residual errors in the predicted
antenna pointing for the communications satellites should be no more than 50 mdeg in azimuth and elevation angle and 10 km in range over 30 days. The flow of this conversion algorithm is shown in Figure 6.

Figure 6. Flow diagram for conversion from an osculating element set to a simple two-body theory mean element set.

With this procedure in place, the errors from the use of the converted new-method element set in the antenna pointing predictions (compared with closed-loop tracking) are greatly reduced. Figure 7 shows the errors in azimuth and elevation angle predicted by the two-body theory propagator with the mean element set produced by this method as compared to using an old-method element set. One can see that this conversion has enabled the operations center (and its terminals) to use the new-method element sets with the same degree of accuracy as before. The problem has been solved.
Figure 7. Residuals in azimuth and elevation angle predictions from the use of old and converted new-method element sets in the propagator at the communications operations center.
5. SUMMARY

The perturbation problem for geosynchronous orbits is a highly complex system of differential equations involving many degrees of freedom. Some of the dominant forces are those due to the Sun and Moon, the gravitational potential of the Earth, and important satellite-dependent effects such as solar radiation pressure, maneuvers, and drag-like effects. A precision model is possible through a computationally expensive numerical integration algorithm, but less accurate analytical and semianalytical models are also good over shorter time spans. Most surprising, however, is that for most geosynchronous satellite orbits, the simple two-body theory of motion is adequate for moderate error tolerances over time spans of 30 days or so. Nevertheless, if these errors are to be reduced dramatically, then some kind of perturbations modeling must be attempted.

If the satellite ephemeris is not maintained at the tracking site and the element set is received from another establishment, then compatibility issues are important and can even be quite critical. It should be mentioned here that of all the sources of error involved with element set transfer, the subject of compatibility has probably been the most ignored. Sometimes much detail is mapped out in interface control documents regarding the required accuracy of the supplied element set while almost no requirement is made that the element set be compatible with the on-site orbital-element propagator. This is on the verge of the absurd, since almost all accuracy is lost in the transfer, but it should be pointed out that this is a historical problem, not one of negligence. Not long ago, the data that updated satellite element sets were only accurate to within 1 to 10 km in range and 100 mdeg in azimuth and elevation angle, and so such compatibility issues were largely hidden. Intensive calibration efforts along with hardware upgrades have made it possible to calculate precision orbits today. One point is very clear: It does not take much effort to produce pointing that aims the antenna in the general vicinity of a satellite, but to improve the theory to beat down the pointing errors requires extensive effort. At the very least, it is required that issues involving compatibility and accuracy be coordinated and resolved as much as possible to achieve more ambitious goals.

Finally, an example of a solution to a compatibility problem has been presented. The problem encountered with the communications operations center demonstrates that it is not as crucial to change propagation software, which may have required extensive effort and cost, as it is to ensure element set compatibility. The effects of this demonstration can reach beyond this one example. There are compatibility issues to be raised in just about any center that relies on ephemeris information sent from an outside source. At the Millstone Hill Radar Operations Center, for example, tight correlations are made against the entire satellite catalogue, and incompatibilities often are the cause of most of the miscorrelations. In conclusion, it should be stressed that antenna pointing errors can be reduced to be as small as desired, but subtle issues must be addressed, and a well-thought-out plan must be implemented.
REFERENCES


On the Calculation of Accurate Antenna Pointing for Terminals Working with Geosynchronous Communications Satellites

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Terminals working with communications satellites must have the capability to produce accurate antenna pointing from information provided by the operations center. This takes the form of an element set for the satellite orbit, which must be propagated by the terminals and from which pointing must be calculated. This report addresses the problem for geosynchronous communications satellites, most non-Soviet communications satellites being in such orbits. First the nature of geosynchronous-orbit propagation is summarized, with principal perturbations highlighted, and then the problem of element set transfer is discussed from a practical point of view. Since most operations centers do not maintain element sets for their satellites (but rather receive ephemeris information from another source), this discussion is designed to maximize the accuracy and lifetime of a transferred element set. An example is provided to demonstrate the need for compatibility of element sets and software, and to show a solution to a timely problem involving element set transfer.

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