Measurements of surface shear stresses under a three-dimensional turbulent boundary layer using oil-film laser interferometry

Measurements of surface shear stress magnitude and direction are reported for a three-dimensional, pressure driven, turbulent boundary layer around a wing-body junction. Measurements were made using a dual-beam oil film laser interferometer at 56 locations.

An iterative procedure was developed which increased the precision of the data extracted from the data records. Skin friction directions computed using a least square error fit were compared to angles obtained from surface oil flows, hot wire anemometry, and LDV measurements. Also, the magnitude of the skin friction coefficients were compared to independently obtained skin friction coefficients. The data agreed to within experimental error outside the effects from the vortex legs present along the side of the wing-body. No accurate data was available for quantitative comparison under the effects of the vortex, but the magnitudes followed the qualitative trends expected. This method failed badly in the region of large three-dimensional effects and requires further study in this area of application.
Table of Contents

1.0 INTRODUCTION ......................................... 1

2.0 THEORY ............................................... 5

3.0 HISTORICAL DEVELOPMENT ............................. 11

4.0 APPARATUS ........................................... 21

5.0 PROCEDURE ............................................ 28

6.0 DATA REDUCTION ........................................ 30
   6.1 Uncertainty Analysis .................................. 43
   6.2 Curve fitting method .................................. 46
List of Tables

Table 1. Sample data reduction output .................................. 62
Table 2. Skin friction coefficients and comparisons ................. 63
List of Illustrations

Figure 1. Perspective View of the wing-body junction. ...................... 66
Figure 2. TiO₂-Kerosene Oil Flow Visualization ............................ 67
Figure 3. Probability density functions at X/T = -.20 ..................... 68
Figure 4. Two-dimensional oil flow nomenclature .......................... 69
Figure 5. Geometry for optical pathlength difference ..................... 70
Figure 6. Fringe numbering nomenclature .................................. 71
Figure 7. Geometry and notation for unknown oil flow direction ....... 72
Figure 8. Problem area: flow direction near normal to measurement
direction .................................................................................. 73
Figure 9. VPI&SU Low Speed Boundary Layer Wind Tunnel .......... 74
Figure 10. Plywood tunnel floor inserts ....................................... 75
Figure 11. Wooden frame surrounding the wind tunnel .................. 76
Figure 12. Movable platform and traversing rails ......................... 77
Figure 13. Photograph of movable platform mounted on wooden frame .. 78
Figure 14. Photograph of entire apparatus .................................... 79
Figure 15. Side view of laser assembly ........................................ 80
Figure 16. Laser assembly viewed from optical rail axis .................. 81
Figure 17. Laser assembly geometry diagram ................................ 82
Figure 18. Box diagram of hardware components ...................... 83

Figure 19. Photograph of test surface with data location transparency attached ........................................... 84

Figure 20. Sample photodetector output ........................................ 85

Figure 21. Sample photodetector output ........................................ 86

Figure 22. Sample photodetector output ........................................ 87

Figure 23. Sample photodetector output ........................................ 88

Figure 24. Sample “bad” photodetector output ............................. 89

Figure 25. Sample “bad” photodetector output ............................. 90

Figure 26. Skin friction coefficient magnitude vs. time origin error ........ 91

Figure 27. Skin friction coefficient magnitude vs. time origin error ........ 92

Figure 28. Skin friction coefficient magnitude vs. time origin error ........ 93

Figure 29. Non-perpendicularity Geometry and Notation .................. 94

Figure 30. Skin friction coefficient vectors plotted: entire flowfield ........ 95

Figure 31. Skin friction coefficient vectors plotted: front quadrant ........ 96

Figure 32. $C_f$ vs. ID number .............................................. 97

Figure 33. Problem area: oil flow pattern in front of wing leading edge ... 98

Figure 34. Problem area: oil flow pattern in tail region .................... 99

Figure 35. Non-uniform initial oil line thickness ............................. 100
Nomenclature

\[ A \quad \text{Interference signal amplitude} \]

\[ C(x) \quad \text{Product of fringe number and time, } C(x) = N_{\text{eff}}t_{\text{eff}} \]

\[ C_f \quad \text{Skin friction coefficient, } C_f = \frac{c}{q_{\infty}} \]

\[ C_x \quad \text{Skin friction coefficient component in } x' \text{ direction} \]

\[ C_z \quad \text{Skin friction coefficient component in } z' \text{ direction} \]

\[ dt \quad \text{Time origin error} \]

\[ \frac{\partial p}{\partial x} \quad \text{Pressure gradient} \]

\[ e \quad \text{Error on product of fringe number and time} \]

\[ f \quad \text{Lens focal length} \]

\[ h \quad \text{Oil film thickness} \]

\[ h_k \quad \text{Change in thickness corresponding to pathlength change of one wavelength} \]

\[ i \quad \text{Data record element} \]

\[ k \quad \text{Zero crossing number} \]
\( n_o \) Index of refraction in oil

\( n \) Normal coordinate

\( N \) Fringe number

\( q_{\infty} \) Dynamic pressure, \( q_{\infty} = \frac{\rho U_{\infty}^2}{2} \)

\( Re_\theta \) Reynolds number based on momentum thickness

\( s \) Streamwise coordinate

\( t \) time

\( T \) Maximum wing thickness, 7.17cm

\( T_o \) Temperature of air flow

\( T_{\text{nom}} \) Nominal temperature for oil viscosity

\( u \) Velocity component in \( x' \) direction

\( u_f \) Particle velocity on oil-air interface

\( U_{\text{inf}} \) Undisturbed freestream velocity

\( w \) Velocity component in \( z' \) direction

\( x_o \) Leading edge of oil film

\( x \) Direction of measurement

\( x', y', z' \) Cartesian coordinates (reference points defined in figure 1)

\( \alpha \) Direction of flow with respect to the \( x' \)-axis

\( \alpha_d \) Measurement direction with respect to the \( x' \)-axis

\( \alpha_{\text{vis}} \) Flow direction measured from oil flow visualization

\( \alpha_{\text{w/u}} \) Flow direction measured from hot-wire and LDV data

\( \beta \) Error in oil film leading edge

\( \gamma \) Angle between measurement direction and flow direction

Nomenclature
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon$</td>
<td>Pressure gradient correction factor</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>Laser beam incidence angle, in air</td>
</tr>
<tr>
<td>$\theta_o$</td>
<td>Laser beam incidence angle, in oil</td>
</tr>
<tr>
<td>$\lambda_{air}$</td>
<td>Laser wavelength in air</td>
</tr>
<tr>
<td>$\lambda_o$</td>
<td>Laser wavelength in oil</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Oil viscosity</td>
</tr>
<tr>
<td>$\mu_{num}$</td>
<td>Nominal oil viscosity</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density of air</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Shear stress or skin friction at oil-air interface</td>
</tr>
<tr>
<td>$\tau'$</td>
<td>Corrected shear stress</td>
</tr>
</tbody>
</table>
Subscripts

\(0\) Related to the oil film leading edge

\(1\) "Upstream" laser beam location

\(2\) "Downstream" laser beam location

\(\text{eff}\) Effective (computed) value

\(o\) Property of the oil

\(\infty\) Free stream flow values
1.0 INTRODUCTION

Fluid flowing over a surface creates a shear stress which acts on that surface according to Newton's Law of Viscosity. If this shear stress, referred to as skin friction, is known over the entire surface, it can be integrated over the surface to determine the skin friction component of the total drag acting on that surface. The other major drag component, referred to as pressure drag or form drag, is due to the boundary layer effect on the pressure distribution and also depends on the shape and orientation of a body in a flow.

Due to the importance of skin friction in determining the total drag on such surfaces as aircraft, ship hulls, etc., many methods have been devised to measure this parameter. The most direct method is the floating beam balance which measures the force exerted by the flow over a small area of the test section. However, it is very sensitive to misalignment of the test area with respect to the surrounding surface and to pressure gradients, and is generally complicated, time consuming, and expensive to use. Other methods are based on near-wall velocity-
profile similarity assumptions or property analogies. For example, in the well known "Law of the Wall", the similarity velocity profile scales on the wall shear stress and can be used to determine the skin friction from measured velocity profiles. Similarly, the analogy between heat transfer and skin friction is often exploited to provide a means of measurement using heated metal films. These are just a few of the more popular techniques. All of these methods, however, are either difficult to use, expensive, intrusive, or indirect and depend on assumptions or analogies. Thus, the search continued for a direct, inexpensive, nonintrusive method which can be easily implemented.

Tanner and Blows (1976) developed such a method known as Oil Film Laser Interferometry based on standard oil lubrication theory and laser interferometry. Although the method was both inexpensive and non-intrusive, it was very time consuming. Monson and Higuchi (1981), Monson, Driver, and Szodruch (1981), and Monson (1984) simplified the method and reduced the time necessary to obtain the data. Applying it to a variety of complex flows produced results in good agreement with those obtained by traditional methods. They also developed equations to measure an unknown oil viscosity by using this method with gravity as the only force on the oil. Westphal (1985,1986) further improved the method by automating the data reduction procedure. In addition, he investigated several parameters and set up guidelines for determining bad data records. In subsequent studies, Kim and Settles (1988) applied the method to supersonic flow past a wedge and Cooke (1988) applied the method to three-dimensional flow past a wing-body junction with pressure gradients.
This study continues Cooke's study by employing oil-film laser interferometry to investigate air flow around a wing-body junction and includes an exploration of skin friction behind the tail and in areas around a line of low shear. The wing is composed of a 3:2 elliptical nose and a NACA 0020 tail mounted at zero angle of attack (Figure 1). The momentum thickness Reynolds number for the conditions in this study was $Re_a = 6700$ based on the undisturbed freestream velocity $U_{ref} = 27.7m/s$ and the momentum thickness of the approach boundary layer 15.1cm upstream of the wing leading edge. The surface oil flow visualization in Figure 2 illustrates the primary features of the flowfield. The flow includes a three-dimensional, pressure-driven, turbulent boundary layer, which is symmetric about the wind tunnel centerline, and is dominated by a horseshoe vortex formed around the nose of the wing (Devenport and Simpson, 1988b). This vortex brings high momentum flow down along the side of the wing, into the test wall boundary layer and out away from the model. A line of low shear develops at the outer edge of the horseshoe vortex due to the effect of the upward flow of the vortex at this location (Devenport and Simpson, 1988a). Figure 3 shows the probability density functions of the U component velocity fluctuations measured in the vicinity of the vortex upstream of the nose of the wing at $x'/T = -0.20$. The double-peaked probability density functions near the wall indicate that the vortex is subject to intense, large-scale, low-frequency unsteadiness. Exact details of this unsteadiness are currently unknown. This particular flow has been the subject of intense investigation by several researchers at Virginia Polytechnic Institute and State University. Surface pressure

INTRODUCTION
fluctuations, hot wire and laser Doppler anemometer velocity measurements, and
flow visualizations have been obtained (Agarwal and Simpson, 1989, Devenport

In this thesis, the basic principles of oil lubrication theory and light
interference will be discussed as they apply to the measurement of skin friction.
An historical development of the oil film laser interferometer technique will follow. The hardware and procedure used in this study will then be discussed, followed by an analysis of the final results and major sources of error. Conclusions and recommendations will then be presented.
2.0 THEORY

Oil-film laser interferometry was developed from two basic principles -- oil lubrication theory and laser interferometry. Both principles were well known and accepted within the scientific community long before Tanner and Blows (1976) combined them to measure skin friction.

Oil lubrication theory states that a thin oil film will flow over a smooth surface under the action of the shear stress created by air flowing over the surface. Squire (1962) developed the basic equations for oil flowing under the action of a shear stress. In a two dimensional, laminar airflow with a constant shear stress and no shear gradients, pressure gradients, or surface curvature, the oil film will assume a linear profile (Figure 4). Starting with Newton's Law of Viscosity

\[ \tau = \mu \frac{\partial u}{\partial y} \bigg|_{y=0} \]  

(1)

and noting that, for the linear profile

\[ \tau = \mu \frac{\partial u}{\partial y} \]
\[ \frac{\partial u}{\partial y} = \frac{u_F - 0}{h - 0} \]  
(2)

and

\[ u_F = \frac{x - x_0}{t} \]  
(3)

where \( t \) is the time it would take an oil particle to travel from \( x_0 \) to \( x \) at speed \( u_F \). Tanner and Blows (1976) derived the simple relation between the shear stress and the time dependent oil-film thickness at a given location, \( x \), for such a flow

\[ \tau = \mu \frac{x - x_0}{h(t,x)t} \]  
(4)

The measurement location, \( x \), and the oil-film leading edge, \( x_0 \), are fixed for a particular run and can be accurately measured. The time of flowing, \( t \), can be measured directly or calculated from the data record; the latter method alleviates several problems and will be discussed later. The oil viscosity as a function of temperature can be obtained from the manufacturer's equation. Thus, if the time dependent oil-film thickness can be accurately determined, the time-averaged shear stress over the area between \( x \) and \( x_0 \) can be obtained from equation (4).

The time dependent oil film thickness can be measured accurately using a laser interferometer. This is accomplished by directing a laser beam onto an oil film flowing over a partially reflective glass surface as depicted in Figure 5. Part of the laser beam is specularly reflected from the air-oil interface B, resulting in
beam BE, and the remainder of the beam is transmitted into the oil film as beam BC. A similar reaction occurs at the oil-surface interface C, creating beams CD and CJ. At the oil-air interface, part of beam CD is transmitted as beam DF and part is reflected. This process will continue, but due to losses from the multiple reflections, beams other than BE and DF can be neglected. Since BE and DF are parallel and coherent, they interfere with each other, and the resulting interference pattern is detected by the receiving optics. As the oil film thins due to the action of the shear stress acting upon it, the differences in path length between beams BE and DF decrease. This path length difference changes the relative phases of beams BE and DF at parallel wavefronts causing the intensity of the combined beams to vary as a sinusoid with a decaying period (Figure 6). As the oil film continues to thin, the path length difference goes through full interference cycles -- varying from maximum constructive interference through maximum destructive interference and back to maximum constructive interference. This cycle, referred to as one “fringe,” corresponds to a change in the path length difference of one laser wavelength. Note that, due to a difference in indices of refraction between the air and the oil, the wavelength of the laser in the oil differs from the wavelength in air according to the relation

\[ \lambda_o = \frac{\lambda_{air}}{n_o} \]  

When light travelling in a medium with refractive index \( n_1 \) encounters an interface with a material of refractive index \( n_2 \) where \( n_2 < n_1 \) both transmission and reflection occur, but the phase of the reflected beam is shifted \( 180^\circ \). If
When $n_2 > n_1$, then no phase shift occurs. Note that the phase of the transmitted beam is never shifted, regardless of $n_1$ and $n_2$.

None of the reflections which result in beams BE and DF cause a phase change. Therefore, when the difference in path length between beams BE and DF is an integral number of wavelengths, a constructive maximum will occur. The path length difference, $\Delta l$, is (refer to Figure 5 for geometry and notation)

$$\Delta l = n_o(BCD) - BI$$

However, ID and BH represent successive positions of the same wavefront (Jenkins and White, 1957). Therefore, the beams HD and BI contain the same number of wavelengths

$$n_o(HD) = BI$$

and the path length difference becomes

$$\Delta l = n_o(BCH) + n_o(HD) - BI = n(BCH)$$

By extending DC until it intersects a vertical line through B, and noting that GC = BC due to equal angles of incidence and reflection at point C, the final form of the path length is

$$\Delta l = n_o(GC + CH) = n_o(GH) = n_o[2h \cos(\theta_o)]$$

As stated previously, a maxima will occur when the path length difference is an integral multiple of wavelengths
\[ m\lambda = 2n_0 h \cos \theta_o \quad m = 1, 2, 3, \ldots \] (10)

Thus, a change in the path length of \( \lambda_0 \) corresponds to a change in oil film thickness of

\[ h_A = \frac{\lambda}{2n_0 \cos \theta_o} \] (11)

where

\[ \theta_o = \sin^{-1} \left( \frac{\sin \theta_i}{n_o} \right) \] (12)

This change in height is referred to as a thickness change of one "fringe." By recording the interference level as a function of time and allowing the oil film to thin to less than one fringe in height, the height of the oil film at any time can be computed by simply counting the fringes backward (peaks labeled as "N = " in Figure 6) beginning with the final fringe, to the desired time. The height of the oil film at that time is simply the number of interference fringes, \( N \) (referred to as the fringe number), times the "fringe thickness," \( h_i \) \( (t_n, x) = h_i N_i \). Thus, the oil lubrication equation becomes

\[ \tau = \mu \frac{x - x_0}{h_i N t} \] (13)

Since, \( h_i, x - x_0, \mu, \) and \( \tau \) are constant for a given test run, the product of fringe number, \( N \), and time, \( t \), must also be a constant, \( C(x) \), at a given location.
\[ C(x) = \text{constant} = Nt \] (14)

Note that most interferometers reflect one of the interfering beams off of a stationary reference and measure the change in distance of the test surface often to less than one micron. Thus, interferometers are generally very sensitive to vibration of the test surface. However, the system used to measure thinning of an oil film is relatively insensitive to vibrations of the test surface since the interference occurs due to the distance between the oil-air interface and the oil-surface interface. Thus, if the test surface is shifted upward a small amount during vibration, the oil film will also be shifted upward by the same amount. While this changes the actual distance travelled by the laser beams from the laser to the receiving optics, it will not affect the path length difference which is what causes the interference to occur.
3.0 HISTORICAL DEVELOPMENT

The first oil film laser interferometer, designed and built by Tanner and Blows (1976), consisted of a He-Ne laser beam directed through a beamsplitter to create two laser beams a fixed distance apart; this distance was measured directly. A drop of oil was then placed on the test section and the beams were directed onto the test section where the upstream beam was visually positioned at the oil drop's leading edge, $x_0$. Thus, only the downstream beam was used to measure the thinning of the oil film. For this reason, it is referred to as a single beam system. The photodetector output from the receiving optics was initially monitored on a strip chart recorder, and the positively sloped zero crossings were read off manually to denote the interference fringes. Note that, using the positively sloped zero crossings to identify the fringes yields fringe numbers which will be integers plus 1/4 (i.e. 1.25, 2.25, etc.) as a result of equation (10). Later, an analog fringe-detection circuit and computer were used to identify the zero crossings.
Two separate data reduction methods were applied to this single beam system. First, the oil was allowed to thin to less than one fringe, as mentioned earlier, and an effective time of flowing was computed using a best fit to equation (14). (For clarity, when the time or fringe number is calculated instead of being measured directly, an “eff” subscript will be used to denote an effective quantity as opposed to a measured quantity.) This procedure resulted in very long test runs—some as long as 60 minutes. As an alternative, the time of flowing was referenced to the time the wind tunnel was started, and the effective fringe numbers, $N_{\text{eff}}$ were calculated from equation (14). This allowed for shorter test runs since the oil did not have to thin to less than one fringe. However, this method underpredicts the actual time of flowing due to the initial slope of the oil film. This time discrepancy was shown to be $\frac{\mu}{\alpha \tau}$ where $\alpha$ is the initial slope of the oil film leading edge (Tanner and Blows, 1981), and was computed to be typically between 2 seconds and 5 seconds for the specific flow conditions studied.

Tanner’s method had several drawbacks that prevented its widespread use. First, it often entailed very long test runs—especially for low shear flows. Second, visually locating the oil film leading edge was found to be a difficult task in practice and the leading edge was also subject to move slightly due to oil spreading after it had been located with the upstream laser beam and before the wind tunnel was started. Furthermore, the effective flow time of the oil differs from the length of time the wind tunnel is operating due to transient effects and the initial slope of the leading edge of the oil drop.

HISTORICAL DEVELOPMENT
Monson and Higuchi (1981) were able to eliminate the major problems with Tanner's system by positioning both beams downstream of the leading edge and using both to record data. Monson perpendicularly polarized the two beams and separated the signals they carried with polarizing filters. Since both beams were used to obtain data, this is referred to as a dual-beam system. The time of the signal peaks, instead of the positively sloped zero crossings used by Tanner and Blows (1976), were used to denote the fringe numbers to eliminate difficulties arising from any drift in the mean signal level. Therefore, Monson and Higuchi's fringe numbers were integers. Noting that \( \tau \) (taken as an average over the area between the two beams), \( \mu \), and \( x_0 \) are identical for both beams, the leading edge position can be eliminated algebraically by solving one of the following equations for \( x_0 \) and substituting it into the other.

\[
\tau = \mu \frac{x_1 - x_0}{h \lambda N_{\text{eff1}} t_{\text{eff1}}} \quad (15)
\]

\[
\tau = \mu \frac{x_2 - x_0}{h \lambda N_{\text{eff2}} t_{\text{eff2}}} \quad (16)
\]

After simplification, Monson's equation for a dual-beam laser interferometer becomes

\[
\tau = \mu \frac{x_2 - x_1}{h \lambda (N_{\text{eff2}} t_{\text{eff2}} - N_{\text{eff1}} t_{\text{eff1}})} \quad (17)
\]
Thus, the distance between the two laser beams is required, but the leading edge need not be located. Also, Monson ran each individual test run only long enough to identify 20 “good” fringes (smooth, easily identifiable, etc.) on the downstream beam, recording his output on a strip-chart recorder. He then developed equations to calculate the effective time based upon the elapsed time for 20 fringes on the downstream beam to occur, and for the fringe numbers for each beam based on the elapsed time for 10 fringes from each beam to occur (Monson and Higuchi, 1981). This considerably reduced the time necessary to obtain the data, eliminated all the problems related to locating the leading edge of the oil film, and allowed the time origin to be computed without having to let the oil film thin to less than one fringe.

Monson and coworkers applied their dual beam system to a wide variety of flows and derived equations and correction terms for various flow conditions. They showed that if the direction of measurement (denoted by an imaginary line connecting the two laser beams) were different than the actual direction of the skin friction, the interferometer system would measure the component of skin friction in the direction of measurement. In addition, they derived first order correction terms for shear gradients, pressure gradients, linear temperature gradients, and body forces. Up to this point in the development of the Oil Film Laser Interferometer, “bad” data records were rejected subjectively upon visual inspection of the strip-chart recorder output. Uneven signal visibility and obvious noisiness were considered the main reason for rejection of bad data records.
Murphy and Westphal (1985) and Westphal and Bachalo (1986) were the next to improve the system. They obtained a more equal signal visibility between the two beams than Monson by spatially resolving the laser beams. However, Westphal's most important contribution was to automate the data reduction procedure. Instead of using a strip chart recorder, he converted the photodetector output, using an analog-to-digital converter, to a digital signal which was stored directly on a computer hard disk. He wrote FORTRAN routines to identify fringes from the data record and a two-variable least squares error minimization routine to obtain the best fit to equation (14). He defined the effective time and fringe numbers as

\[
N_{\text{eff}} = N_0 - k
\]

\[
t_{\text{eff}} = t_0 + t_k
\]

where \(k\) is the chronological fringe number within the data record (see Figure 6), \(t_k\) is the time of the \(k^{th}\) positively sloped zero crossing referenced to the beginning of the data record, \(N_0\) is a constant to give the actual height, in \(h_i\)'s, and \(t_0\) is the time origin or effective time of oil flowing. Note that, if transient effects and the initial slope of the oil film leading edge are negligible, \(t_0\) is identical to the time the wind tunnel was started. The error minimization routine iterates on \(N_0\) and \(t_0\) to find the minimum normalized rms error of the constant \(C(x)\)

\[
e = \frac{\sqrt{(C^2) - (\overline{C})^2}}{\overline{C}}
\]
It is important to note that previously, \( N_{\text{eff}} \) and \( t_{\text{eff}} \) were calculated using only two or three distinct, widely spaced fringes. Using the power and speed of the computer, Westphal used all of the fringes to calculate \( N_{\text{eff}} \) and \( t_{\text{eff}} \). In addition, Westphal computed a time origin \( t_0 \) from each data record, thus producing two values for the time origin. Since the time origin is a property of the entire oil flow, it is independent of the measurement location \((x_1, x_2)\). Realizing this, Westphal determined that if the difference exceeded a certain level, then at least one of the data records was “bad.” The criterion he established for rejection was

\[
dt = \frac{|t_{01} - t_{02}|}{t_{\text{eff}}(\text{at } k = 1)} > .02
\]

Westphal also established a maximum allowable error on the constant \( C(x) \) for each beam, defined in equation (20). This criterion limited the error to \( e < .004 \); if either beam failed to meet this requirement, the data record was considered unacceptable. In practice, he found most good runs to have \( e < .003 \). These rejection criteria were based upon a maximum allowable error of \( \pm 5\% \) on the final skin friction result for a single run. This accounted for variations in oil viscosity, beam separation measurements \((x_2 - x_1)\), etc. These criteria will produce a result within a \( \pm 5\% \) uncertainty only for a two dimensional, laminar flow with no pressure or shear gradients and no gravitational effects such as the flow Westphal studied.

It is important to note here that Westphal (1986) defines the variable \( \lambda_0 \) to be one wavelength in the oil (which is correctly defined in equation (5)), but
then gives an equation for a thickness variation (referred to here as \( h_a \)) of the oil film corresponding to a change of one \( \lambda_o \) and uses the same variable name, \( \lambda_o \), for this change in height. However, his subsequent analysis is consistent with the definition of \( \lambda_o \) as the change in height. For clarity, \( h_a \) is used in this study.

Cooke (1988) applied Oil-Film Laser Interferometry to flow past a wing body junction. He used the programs written by Westphal (1986) combined with a linear pressure gradient correction term developed by Monson, Driver, and Szodruch (1981). He determined the skin friction as if no pressure gradient existed, and then modified the result as follows

\[
\tau' = \frac{\tau}{1 - \varepsilon}
\]

where

\[
\varepsilon = \frac{\lambda N_{eff} B}{2n_o \tau \cos \theta_o} \cdot \frac{\partial p}{\partial x}
\]

Murphy and Westphal (1985) showed that for sufficiently thin oil films (less than 20 microns), this system would measure the time-averaged shear stress in a three-dimensional flow. For oil films thicker than 20 microns, they showed that surface waves may appear at the oil-air interface depending on the particular flow conditions.

Cooke did not actually measure both the magnitude and direction of this shear stress in his experiments. Instead, he pre-determined the direction of flow from surface oil flow patterns produced by earlier researchers working on the
same flow. Knowing the flow angle, he aligned the laser beams with the direction of flow and measured the skin friction magnitude. He also experimented with various scanning devices which could take data at several closely spaced data points, but did not find an acceptable device. He also experimented with measuring the oil-film thickness from below the test section. This requires the thickness of the glass plate to be measured and subtracted from the thickness measured with the oil on the test section. His final decision was to use a dual-beam system mounted above the test section. The system was placed on a wooden frame which surrounded the wind tunnel but was not in contact with it. This eliminated the possibility of transmitting vibrations from the wind tunnel motor to the interferometer system.

With the exception of Cooke, this method had only been applied to well understood flows such as two-dimensional flows, axisymmetric flows, flows with one-dimensional pressure gradients, etc. Cooke applied this method to a complex three-dimensional flow, but relied upon surface oil flow visualizations to determine the flow angles and then aligned the laser beams with the flow.

For the situation in which the flow angle is unknown, Monson, Driver, and Szodruch (1981) derived an important result for two dimensional flow. Figure 7 illustrates the geometry and notation used to derive the flow angle and shear stress for this situation. For a constant shear stress and constant oil viscosity (i.e. constant temperature), the oil lubrication equations at points \((x'_{1}, z'_{1})\) and \((x'_{2}, z'_{2})\) (refer to Figure 7 for notation and geometry) are
\[ \tau = \frac{\mu(s_1 - s_0)}{h_1 t_1} \]  \hspace{1cm} (24)

\[ \tau = \frac{\mu(s_2 - s_0)}{h_2 t_2} \]  \hspace{1cm} (25)

Since the flow direction is unknown, the streamwise variables, \( s_0 \), \( s_1 \), and \( s_2 \) must be solved for in terms of the measurement direction variables \( x_0 \), \( x_1 \), and \( x_2 \)

\[ s_1 - s_0 = \frac{x_1 - x_0}{\cos \gamma} \]  \hspace{1cm} (26)

\[ s_2 - s_0 = \frac{x_2 - x_0}{\cos \gamma} \]  \hspace{1cm} (27)

Substituting into equations (21) and (22) and eliminating \( x_0 \) as previously done yields

\[ \tau = \frac{\mu(x_2 - x_1)}{h_2 t_2 - h_1 t_1} \cdot \frac{1}{\cos \gamma} \]  \hspace{1cm} (28)

Thus, the total shear \( \tau \) is unknown since \( \gamma \) is unknown, but \( \tau_x = \tau \cos \gamma \) which gives the final result

\[ \tau_x = \frac{\mu(x_2 - x_1)}{h_2 t_2 - h_1 t_1} \]  \hspace{1cm} (29)

Thus, oil-film laser interferometry measures the component of the shear stress in the direction of an imaginary line connecting the laser beams on the test surface.
(known as the measurement direction). It is very important that the initial line of oil be placed perpendicular to the measurement direction as will be shown in Chapter 7.

In this thesis, measurements were initially taken in the $x$ and $-z$ directions (using the nomenclature of Figure 1) so the measurements in each direction could be combined vectorially to obtain the magnitude and direction of the shear stress at each point. However, when the angle between the flow and the $x$-axis was less than approximately $20^\circ$ and the laser interferometer was aligned in the $-z$ direction, the oil did not flow over both data beams (see Figure 8). Therefore, data could not be obtained in the $-z$ direction, and the angles and magnitudes of the actual shear stress could not be determined.

Since both principal axis directions could not be used, measurements were taken in three directions far enough apart to provide acceptable spatial resolution. Data were obtained at $0^\circ$, $-45^\circ$, and $-60^\circ$ with respect to the wind tunnel axis. Note that measurements in only two directions are sufficient to obtain the skin friction magnitude and direction. Measurements in the third direction were included to provide redundancy. The rejection criteria developed by Westphal ($e < .004$) were applied to this data and most good runs were observed to have $e < .003$ for each beam, in agreement with his findings.
4.0 APPARATUS

All the experiments from this study were done in the VPI&SU Low-Speed Boundary-Layer Wind Tunnel (Figure 9). The tunnel is an open circuit tunnel with a rectangular test section 8m long x .91m wide x .26m high (over the central portion). Air is taken into a centrifugal blower through a filter. Upstream of the test section is a fixed-setting damper, a plenum, a honeycomb section, seven screens, and a 4:1 contraction ratio nozzle which leads to the test section. In the absence of the wing, a nearly zero pressure gradient exists in the test section. At the inlet to the test section, the flow is uniform to within .5% in the spanwise direction and 1% in the vertical direction with a .2% turbulence intensity at 27m/s. The flow is tripped as it enters the test section by a .63cm step, and is accelerated to test speed by a further 1.5:1 contraction produced by the shape of the upper wall. The wind tunnel is the same as that used by Cooke (1988).

The upper wall of the wind tunnel is made of plexiglas reinforced with aluminum channel. The side tunnel walls are made of glass and lined internally
with 6.4mm thick plexiglas plates. When the wing is mounted in the test section, the plexiglas liners near the wing are removed to minimize the effects of blockage induced pressure gradients. This effectively widens the test section by 12.7mm in the area between a location 330mm upstream of the leading edge of the wing and 203mm downstream of the trailing edge of the wing. These distances were chosen to make the side walls follow, very approximately, a streamline produced by the wing in an unbounded flow. The corners produced by the removal of the plexiglas liners are taped over adhesive tape to reduce added turbulence and recirculation due to the surface discontinuity. The streamlines followed by the side walls were calculated using two-dimensional potential flow theory.

The floor of the test section is made of 19mm thick fin-form plywood. The area immediately surrounding the permanent wing mount has been cut away so different materials may by used for the actual test surface. The test area inserts used in this study consist of two parts. The first is made of 3/4" finished plywood secured flush with the surrounding floor by countersunk bolts located at the edges. A cutout section with a 1/2" lip holds the second piece -- a 1/4" thick Denton Vacuum MLBS-30 partially reflective glass plate. The glass plate, chosen by Cooke to match the intensity of the interfering beams, has a 30% reflective coating on the test side. Cooke took data only in front of the wing, using the plywood insert shown in Figure 10(a) The present study investigated positions in front, along the side, and behind the wing. Therefore, two additional plywood inserts were constructed, with cutouts as shown in Figures 10(b) and 10(c).
The wing used in this study consists of a 3:2 elliptical nose upstream of the maximum thickness and a NACA 0020 tail. The maximum thickness is 7.17 cm, the chord is 30.48 cm, and the height of the wing is 22.9 cm. This height allowed for a 3.7 cm gap between the top of the wing and the wind tunnel roof to prevent the formation of a junction vortex at the top of the wing. Such a vortex could have interfered with the flow further downstream on the test wall. Sandpaper strips (120 grade) located 10 mm upstream of the maximum thickness ensured a constant turbulence transition point on the wing surface.

The plywood insert and glass plate were secured flush with the surrounding tunnel floor and the wing was bolted to a permanent mount in the tunnel floor. The wing was mounted with its leading edge 1.39m downstream of the tunnel throat and its chord parallel to the tunnel centerline. Small gaps between the inserts and the surrounding floor were covered with Scotch brand “Magic Mending” tape to provide a smooth surface continuity.

A wooden frame was constructed around the wind tunnel to support the laser assembly. On top of this structure, Cooke secured the laser assembly such that it was directly above the test location. In order to provide access to the additional downstream locations investigated in this study, Cooke’s structure was extended two and a half feet in the streamwise direction (see Figure 11 for the final structure). With Cooke’s setup, the time required to move between locations was about 45 minutes. This large relocation time occurred because the laser assembly had to be physically picked up and moved, carefully re-aligned with the flow direction, and clamped in place. To reduce this relocation time, a movable
platform was mounted on top of the wooden structure on a rail system (Figure 12). The movable platform could slide along the rails in the streamwise (top set of rails) and spanwise (bottom set of rails) directions, thus allowing the laser assembly mounted on top of the movable platform to be relocated without changing the alignment angle. This was done by attaching two parallel metal angles to the bottom of the platform. These angles subsequently fit into an additional set of oppositely oriented angles with the same spacing, forming a set of rails. Another, orthogonally oriented pair of angles was bolted to the bottom of the first set of rails and fit into a pair of angles rigidly mounted atop the wooden frame.

To allow for easy rotation of the laser assembly with respect to the platform, a 1/2" thick "U-shaped" aluminum piece was attached to the laser assembly base plate. One inch holes were drilled in both the platform and the U-shaped fixture, and a 1" bolt through both holes firmly attached the laser assembly to the movable platform. The laser beams were aligned with the three different measurement directions in a similar manner. Three 1/4" holes in the platform, when aligned with a 1/4" hole in the laser assembly base plate, oriented the laser beams at 0°, 45°, or 60° with respect to the wind tunnel centerline. Wheels mounted to the underside of the laser assembly base plate further simplified the rotation.

The platform was made of 3/4" birch plywood with a 1/4" aluminum plate affixed to the corner with a 1" bolt to provide additional strength. Two 90° arcs cut through the platform and aluminum plate gave the laser beams clear access
to the test section at any angle between 0° and 90° with respect to the tunnel centerline. Figure 13 shows the movable platform mounted on the rail system above the wooden frame. Figure 14 shows the author applying oil to the test surface and depicts the relative positions of the wooden frame, rail system, movable platform, and laser assembly with respect to the wind tunnel.

The basic laser interferometer system used was that developed by Cooke. Figure 15 shows the laser assembly from the side and Figure 16 shows the laser assembly looking down the optical rail axis. Two parallel optical rails were firmly mounted on a common 1/4" aluminum base plate, and all optical components were mounted and aligned on these rails. A Spectra Physics Model 105-1 5 mW He-Ne laser ($\lambda = 6328\text{Å}$) was mounted such that lateral and vertical adjustments could easily be made. The laser beam paths shown in Figure 17 are described as follows. The laser beam first passes through a beam expander and focusing assembly consisting of a 20X objective lens and an output lens with a focal length $f = 301\text{mm}$. This assembly is mounted on the same optical rail as the laser beam. The beam is then reflected from a plane mirror, mounted on the same optical rail as the laser, to the beamsplitter, a 6mm thick optical window, which is mounted on the second optical rail. The two beams are then directed down to the test surface via an adjustable plane mirror. Although the beamsplitter creates a pathlength difference which prevents both beams from being simultaneously focused on the test surface, this difference is not crucial. Rather, focal lengths were adjusted such that the laser beams have approximately the same diameter at the test surface -- one focused slightly above the test surface and the other
focused the same distance below it. The returning beams are reflected from a plane mirror similar to the one used for locating the beams on the test surface, and are then re-focused through a convex lens with focal length \( f = 401 \text{mm} \). These focused beams are split spatially by two plane mirrors mounted 90 degrees apart, and are again reflected to finally impinge on United Detector Technology silicon PiN-10DP photodiodes operating in photovoltaic mode. The photodiode amplifier circuits were the same as those used by Cooke (1988). The spot size of the laser beams on the photodiodes was controlled by an adjustable iris. The iris opening used in these experiments was approximately 1.5mm to 2mm.

Each photodiode signal was amplified to approximately 2.5 volts and subsequently offset to allow bi-polar analog-to-digital conversion. This offset adjustment was calibrated through use of a voltmeter while in complete darkness (lights and laser turned off). The voltmeter responded too slowly to be useful during data acquisition. A Krohn-Hite model 3202 fourth order Simple RC filter was operated in the low-pass mode at a cutoff frequency of 20 Hz to eliminate noise from vibration of the tunnel motor, etc. The signal was monitored in real time on an oscilloscope; data acquisition was initiated when interference variations were detected. A Data Translation DT 2801-A analog-to-digital converter board installed in an IBM PC XT operating at 10 Hz recorded the digital signal to a computer floppy disk for later analysis. Figure 18 shows a block diagram of the components as they were set up for this study.

The oil used was Dow Corning 200 fluid (manufactured by Dow Corning Corporation, Midland, MI), a 50 centi-Stoke nominal viscosity silicone oil with a
nominal temperature of 25° Celsius. The oil was chosen based on Cooke’s recommendation for the shear levels involved. The temperature was monitored from a thermometer attached to the inside of the test section approximately two feet downstream of the tail of the model. The manufacturer’s equation for oil viscosity-temperature dependence was used to determine the oil viscosity for each run

\[ \mu(T_o) = \frac{\mu_{nom}}{e^{0.0146(T_o - T_{nom})}} \]  \hspace{1cm} (30)
5.0 PROCEDURE

A transparency with the measurement locations was attached to the bottom of the glass plate to assist in positioning the laser beams in the proper location (Figure 19). The laser beams were positioned such that the measurement location (marked on the transparency) was directly between the two, since the average shear stress between the two beams is actually measured. The laser and the lights were turned off and the zero level for the photodetectors was set to -1.25 volts. The laser was then turned on and fine adjustments were made to the receiving optics alignment.

The beam spacing was then measured directly by placing a piece of paper on the test section and measuring the distance between the upstream edges of the laser beams with a caliper. These measurements were accurate to .05 mm. The beam spacing used in these experiments was 4.35mm. The test section and surrounding area were then cleaned using commercial glass cleaner and lens paper. The oil was kept in a syringe to protect it from dust while still allowing
easy application. The oil was applied from the syringe to the edge of a taped razor blade (the tape was to prevent scratching the reflective surface of the glass plate). The razor blade was then pressed down on the test section between one and two beam spacings ahead of the upstream beam, creating a line of oil perpendicular to the measurement direction. The tunnel was then started. Once the tunnel reached operating conditions, the dynamic pressure \( q_{\infty} = \frac{1}{2} \rho U_{\infty}^2 \) was measured using a pitot-static tube mounted in the wind tunnel throat. The temperature was also recorded; it was normally between 24°C and 26°C. When the intensity of the downstream beam, viewed in real time on an oscilloscope, reached a smooth oscillation, the lights in the lab were turned off (to prevent the photodetectors from picking up stray light) and the computer program, OILAD32 (see Appendix A), which recorded the data to a computer disk was begun. The amount of time between when the tunnel reached the designated dynamic pressure and when the beginning of the data recording was typically between one and two minutes. When the data was finished recording, the lights were turned on and the initial temperature was verified.

Figures 20, 21, 22, and 23 are examples of typical raw data records obtained using this method. Figures 24 and 25 (upstream and downstream data records for the same test run) show an example of a data record that cannot be reduced. The most likely reason for the signal degradation in Figure 25 is that the receiving optics gradually shifted over the test run due to vibration.
6.0 DATA REDUCTION

Several different data reduction methods were tried in searching for the best. The first method was the same as that used by Cooke. Cooke recorded data in a fashion similar to the procedure mentioned previously and used Westphal's data reduction codes, modified to account for the pressure gradient, to reduce the data he obtained. Although the error criterion determined by Westphal produced results within \( \pm 5\% \) uncertainty only for his simpler flow, his criteria were also applied to this flow.

The second method used the derivative of the oil film height with respect to time to determine the skin friction since the time rate of change of the oil-film height is the directly measured quantity. Rearranging equation (1) to isolate the height

\[
h = \frac{\mu x}{\tau t}
\]  

and taking the derivative of the oil film height with respect to time yields
\[ \frac{\partial h}{\partial t} = -\frac{\mu x}{\tau t^2} \] (32)

Since \( \frac{\partial h}{\partial t} \) is inherently negative for an oil film thinning under the action of a shear stress, multiplying through by -1 and taking the logarithm of each side yields a linear equation

\[ \log\left(-\frac{\partial h}{\partial t}\right) = \log\left(\frac{\mu x}{\tau}\right) - 2 \log(t) \] (33)

\[ y = l + mx \] (34)

By evaluating \( \frac{\partial h}{\partial t} \) as a backward finite difference and substituting \( h = h_iN \), discrete values can be determined for \( \log\left(-\frac{\partial h}{\partial t}\right) \) and \( \log(t) \). Thus the equation is in the form of a straight line with slope -2. Using a linear least squares fit, the constant term, \( \log\left(\frac{\mu x}{\tau}\right) = b \), and the related uncertainties for each beam were determined. The slopes for most data records were between -2 and -2.2. The errors on the constant terms were about 3% on average. Solving for the skin friction and eliminating \( x_0 \) as before, the final equation becomes

\[ \tau = \mu \frac{[x_2 - 10^{(b_1-b_2)}x_1]}{10^{b_1} - 10^{b_2}} \] (35)

The values of the constants \( b_1 \) and \( b_2 \) were fairly close, thus their position in the denominator makes the skin friction very sensitive to these values. In addition, by taking 10 to the power of \( b_1 \) and \( b_2 \), any errors associated with these
constants are increased accordingly and degrade the usefulness of the calculated skin friction. This process results in an unacceptably high uncertainty of as much as 50% on the final result.

The third method was actually a modification of the first method. It was realized that more information could be obtained from each data record. Most notably, the negatively sloped zero crossings were used. These points were not used in the calculation of $C(x)$ and therefore can be used to obtain another value of $C(x)$ and shear stress using data from the same test run. In addition, the data was shifted up by .1 rms of each signal and the positive and negative zero crossings were used to calculate two more values of the shear stress. The data was then shifted down from the original level by .1 rms of each signal and the process was repeated. Thus, it was possible to obtain six values of the skin friction from a single test run, which translated into 18 possible independent values of skin friction at every location, in each of the three directions.

Applying the third method produced large amounts of scatter among the skin friction values. In fact, measurements taken at the same location and in the same direction varied so much that any effort to combine them with measurements in other directions at the same location sometimes resulted in uncertainties on the final values in excess of 100%.

The fourth and final method extended the third method, using another piece of physical information, to reduce the scatter among the skin friction values calculated at a given point in a given direction. In reviewing the output from the reduction program and seeking a method of reducing the uncertainty of the
results, a pattern between the magnitude of the skin friction and the time origin error, $dt$, within each run was ascertained. Many plots were made of skin friction magnitude versus $dt$ at a given location and angle. Figures 26, 27, and 28 are typical of these plots. The data points from each individual run are connected with straight lines. For the large majority of test runs, the magnitude of the skin friction component is almost a linear function of $dt$ within each run. Different test runs at the same angle do not fit onto the same line, or even necessarily have slopes of the same sign. However, in well over half of the locations viewed, if the lines formed by connecting the values from each test run were extended to $dt = 0$, the values converged.

It was realized that, since the time origin is a property of the particular flow, and therefore independent of $x_1$ or $x_2$, the time origin for each beam should be identical. Westphal's program included the option to either input the time origin for each beam, or to allow the program to calculate the time origin for each beam. By varying the number of zeroes used from each beam, it was normally possible to obtain a time origin error within the stated criteria ($dt < .02$). In fact, in most cases, it was possible to obtain $dt$ values less than .01. Once this had been accomplished by interactively iterating on the number of zeroes for each beam, an average of the upstream and downstream beam time origins was input as the time origin for each beam. Thus, the time origin was forced to be the same for each beam and the time origin error was identically zero. This process usually required between 3 and 5 iterations. If the errors on the constants $C(x_1)$ and $C(x_2)$ were still below the acceptable level ($e < .004$), then the skin friction values

DATA REDUCTION
computed were used in determining the magnitude and direction of the skin friction at that location. A typical example of this iteration process is shown in Table 1.

The first column of Table 1 gives the number of zeroes used from the upstream beam / downstream beam. The second column shows the skin-friction coefficients obtained in the first reduction iteration. The top line, denoted "0 = ", is the reduction using the positive sloped zero crossings. The second line, denoted "M = ", is the value obtained by subtracting .1 of the signal rms from the average signal intensity and using the positive sloped zero crossings. The third line, denoted "P = ", is the value obtained by adding .1 of the signal rms value to the signal average and using the positive sloped zero crossings. In the next three lines, the first letter again denotes the value used for the signal mean and calculated the skin friction using the negative sloped zero crossings.

In Table 1, part (a), the data records (upstream and downstream beams) were reduced using 20 zero crossings from each beam and the time origins were calculated by the computer to obtain a best fit to equation (14). Note that the lowest value of the skin friction coefficient was .00175 and the highest value was .00187. Also, some of the time origin errors (dt) were above the acceptable limit of .02. Therefore, the data were reduced again using 10 zero crossings on the upstream beam. The output is shown in Table 1, part (b). Again, the computer was allowed to calculate individual time origins for each skin friction coefficient. Reducing the data using these parameters produced time origin errors within the acceptable limits. Therefore, 10 zero crossings were used on the upstream beam
and 20 zero crossings were used on the downstream beam. The time origin was fixed at 75.5 (the average value) for each beam for the final run. Using these parameters produced the output in part (c). Notice that the maximum difference in the skin friction coefficient values for the final run is much less than 1% of the skin friction coefficient. This method increases the precision of the skin friction coefficient, but not necessarily the accuracy. It is possible that another combination of the number of zero crossings used from each beam could also produce an acceptable, but different, skin friction coefficient.

Once all of the individual data records had been reduced with $dt = 0$, factors such as the uncertainty in the angle between the oil-film leading edge and the measurement direction, and the effects of non-zero shear gradients had to be accounted for.

The analysis at the end of Chapter 3, in which equation (29) was developed for unknown flow angles, was based on the perpendicularity of the initial oil line and the measurement direction. Since the oil line was applied to the surface by hand, this angle could not be precisely ensured. The effects of errors in angular position may be determined through an analysis of Figure 29. This figure depicts an oil flow with an unknown direction $\gamma$ with respect to the measurement direction and an oil line which deviates from being perpendicular to the measurement direction by an angle $\beta$. The distance travelled by an oil particle from the leading edge to point $x_1$ is $s_1 - s_{01}$ and the distance to $x_2$ is $s_2 - s_{02}$. Thus, the basic shear stress equations are
\[ \tau' = \frac{\mu(s_1 - s_0)}{h_1 t_1} = \frac{\mu[(x_1 - x_0) \cos \gamma + (x_1 - x_0) \sin \gamma \tan(\gamma - \beta)]}{h_1 t_1} \]  

(36)

\[ \tau' = \frac{\mu(s_2 - s_0)}{h_2 t_2} = \frac{\mu[(x_2 - x_0) \cos \gamma + (x_2 - x_0) \sin \gamma \tan(\gamma - \beta)]}{h_2 t_2} \]  

(37)

Eliminating \( x_0 \) as before and reducing yields

\[ \tau' = \frac{\mu(x_2 - x_1)}{h_2 t_2 - h_1 t_1} \left[ \frac{1}{1 + \tan \gamma \tan \beta} \right] \cdot \frac{1}{\cos \gamma} \]  

(38)

and

\[ \tau' x = \tau' \cos \gamma = \tau_x \left[ \frac{1}{1 + \tan \gamma \tan \beta} \right] \]  

(39)

where \( \tau_x \) is the shear stress component in the measurement direction calculated from equation (17) and \( \tau' \) is the total shear stress (with correction for angle \( \beta \)).

However, \( \beta \) is an unknown error angle so the total shear stress cannot be computed. The effect of this error factor increases as the angle between the measurement direction and the flow angle, \( \gamma \) increases. When the measurement angle and flow angle coincide (\( \gamma = 0 \)) the error factor is 1.0, and therefore the angle \( \beta \) has no effect. Since this effect was not considered until after the data had been obtained, the uncertainty in the angle \( \beta \) was not minimized. However, under normal conditions (i.e. sitting at a desk), the error in this angle is less than \( \pm 1^\circ \) if care is taken. However, for purposes of data reduction, this error was assumed to be less than \( \pm 3^\circ \) to account for the limited working space in the wind.
tunnel while applying the oil (see Figure 14). The uncertainty due to this angle was included in the data reduction program LS.FOR.

The geometry for the shear gradient analysis (see Figure 7) is similar to that for the misalignment angle, $\beta$, analysis. In all previous analyses, the shear stress was assumed to be constant and was denoted simply as $\tau$. Since the shear stress is now assumed to be variable, $\tau_1$ and $\tau_2$ are used to represent the shear stresses at points $(s_1, n_1)$ and $(s_2, n_2)$, respectively.

$$\tau'_1 = \frac{\mu (s_1 - s_{01})}{h_1 t_1} \quad (40)$$

$$\tau'_2 = \frac{\mu (s_2 - s_{02})}{h_2 t_2} \quad (41)$$

Furthermore, expanding the shear stress in a Taylor series about point $(s_1, n_1)$ yields

$$\tau' = \tau'_1 + \frac{\partial \tau}{\partial x'} (x' - x'_1) + \frac{\partial \tau}{\partial z'} (z' - z'_1) + ... \quad (42)$$

Keeping only the first order terms, writing the above equation for $\tau(s_2, n_2) = \tau_2$ and substituting for $\tau_1$ and $\tau_2$ from equations (40) and (41) yields

$$\frac{\mu (s_2 - s_{02})}{h_2 t_2} = \frac{\mu (s_1 - s_{01})}{h_1 t_1} + \frac{\partial \tau}{\partial x'} (x'_2 - x'_1) + \frac{\partial \tau}{\partial z'} (z'_2 - z'_1) \quad (43)$$

Making the following geometrical substitutions

DATA REDUCTION
\[ s_1 - s_{01} = \frac{x_1 - x_0}{\cos \gamma} \]  
\[ (44) \]

\[ s_2 - s_{02} = \frac{x_2 - x_0}{\cos \gamma} \]  
\[ (45) \]

\[ x_2' - x_1' = (x_2 - x_1) \cos \alpha_d \]  
\[ (46) \]

\[ z_2' - z_1' = (x_2 - x_1) \sin \alpha_d \]  
\[ (47) \]

Solving for \( x_0 \), substituting into equations (40) and (41), and multiplying each by \( \cos \gamma \) to obtain the shear stress component in the direction of measurement \( x \) yields

\[ \tau_{1x}' = \frac{\mu(x_2 - x_1)}{h_2t_2 - h_1t_1} \left[ 1 - \frac{(h_2t_2)}{\mu} \left( \frac{\partial \tau}{\partial x'} \cos \alpha_d + \frac{\partial \tau}{\partial z'} \sin \alpha_d \right) \right] \]  
\[ (48) \]

\[ \tau_{2x}' = \frac{\mu(x_2 - x_1)}{h_2t_2 - h_1t_1} \left[ 1 - \frac{(h_1t_1)}{\mu} \left( \frac{\partial \tau}{\partial x'} \cos \alpha_d + \frac{\partial \tau}{\partial z'} \sin \alpha_d \right) \right] \]  
\[ (49) \]

However, the laser beams were positioned such that the data point was midway between the two beams. Thus, since only the linear terms were retained in the Taylor series expansion, the value of the shear stress at the data point is taken as the average of \( \tau_1 \) and \( \tau_2 \)

\[ \tau_x' = \tau_x \left[ 1 - \frac{h_1t_1 + h_2t_2}{2} \frac{\cos \gamma}{\mu} \left( \frac{\partial \tau}{\partial x'} \cos \alpha_d + \frac{\partial \tau}{\partial z'} \sin \alpha_d \right) \right] \]  
\[ (50) \]
Rearranging equations (15) and (16) as follows

\[ h_1 t_1 = h_N \alpha_{eff} t_{eff} = \frac{\mu(x_1 - x_0)}{\tau} \]  

(51)

\[ h_2 t_2 = h_N \alpha_{eff} t_{eff} = \frac{\mu(x_2 - x_0)}{\tau} \]  

(52)

shows that the constants \( h_1 t_1 \) and \( h_2 t_2 \) are proportional to the distance from each of the beams to the leading edge of the oil film. Thus, the effects of non-zero shear gradients can be minimized by keeping the distance from the upstream beam to the leading edge \((x_1 - x_0)\) as small as possible. This minimization, however, is limited by the fact that \( x_1 \) must be far enough away from \( x_0 \) to obtain sufficient fringes for data reduction. Westphal (1986) recommends using at least 10 fringes for the upstream beam calculations. Decreasing the distance between the beams \((x_2 - x_1)\) also reduces the distance between the downstream beam and the oil film leading edge, which reduces the effect of non-zero shear gradients. Therefore, the minimum laser beam separation required to maintain spatial resolution should be used.

If the locations at which data were taken composed an orthogonal grid, the shear gradients could be obtained through an iterative procedure. However, since this was not the case, a method was needed to determine the shear gradients with the given data. Noting that \((\frac{\partial \tau}{\partial x'} \cos \alpha_y + \frac{\partial \tau}{\partial z'} \sin \alpha_y)\) from equation (50) represents the shear gradient in the direction of measurement \((\frac{\partial \tau}{\partial x'}\) ), an approximation of the shear gradient can be obtained from the data records.
Referring to the linear oil-film profile in Figure 4, the following result may be obtained via similar triangles at time $t_1$

$$\frac{h_2(t_1) - h_1(t_1)}{x_2 - x_1} = \frac{h_2(t_1)}{x_2 - x_0} = \frac{h_1(t_1)}{x_1 - x_0}$$  \hspace{1cm} (53)

where $h_1$ and $h_2$ are the heights of the oil film at $x_1$ and $x_2$ at the same time. The easiest time to determine the height of the oil film is at one of the zero crossings, but the upstream and downstream beams do not cross the zero axis at the same times. Therefore, the height of the upstream beam was determined at one of its zero crossings, and the height of the downstream beam was found by linear interpolation at the same time. Once $h_1$ and $h_2$ are known, the distances $x_1 - x_0$ and $x_2 - x_0$ can be determined from equation (53). Thus, the shear at the upstream and downstream locations can be determined as follows

$$\tau_1 = \frac{\mu(x_1 - x_0)}{h_1 t_1}$$ \hspace{1cm} (54)

$$\tau_2 = \frac{\mu(x_2 - x_0)}{h_2 t_2}$$ \hspace{1cm} (55)

The shear gradient was then taken as

$$\frac{\partial \tau}{\partial x} = \frac{\tau_2 - \tau_1}{x_2 - x_1}$$ \hspace{1cm} (56)

Having reduced the individual data records and accounted for uncertainty on these values, the skin friction values that met the error criteria were combined.
to give the magnitude and direction using program LS.FOR (Appendix C). Program LS combines the acceptable skin friction measurements in a least square error fit of the data to obtain the $x'$ and $z'$ components of the skin friction coefficient. Each of these skin friction coefficients should satisfy the following equation

$$C_{m,i}(d_{i}) = C_{x'} \cos \alpha_{d,i} + C_{z'} \sin \alpha_{d,i}$$  (57)

$C_{x'}$ and $C_{z'}$ were subsequently chosen to minimize the square of the errors

$$S = \sum_{i=1}^{9} (C_{m,i} - C_{x'} \cos \alpha_{d,i} - C_{z'} \sin \alpha_{d,i})^2 = \text{minimum}.$$  (58)

To obtain the values of $C_{x'}$ and $C_{z'}$ which yield a minimum for $S$, the partial derivatives of $S$ with respect to $C_{x'}$ and $C_{z'}$ were set equal to zero

$$\frac{\partial S}{\partial C_{x'}} = 0 = \sum_{i=1}^{9} (-C_{m,i} \cos \alpha_{d,i} + C_{x'} \cos^2 \alpha_{d,i} + C_{z'} \cos \alpha_{d,i} \sin \alpha_{d,i})$$  (59)

$$\frac{\partial S}{\partial C_{z'}} = 0 = \sum_{i=1}^{9} (-C_{m,i} \sin \alpha_{d,i} + C_{x'} \cos \alpha_{d,i} \sin \alpha_{d,i} \cos \alpha_{d,i} + C_{z'} \sin^2 \alpha_{d,i})$$  (60)
With the following definitions

\[ A = \sum_{i=1}^{9} \cos^2 \alpha_{d,i} \]  
(61)

\[ B = \sum_{i=1}^{9} \cos \alpha_{d,i} \sin \alpha_{d,i} \]  
(62)

\[ D = \sum_{i=1}^{9} \sin^2 \alpha_{d,i} \]  
(63)

\[ E = \sum_{i=1}^{9} C_{m,i} \cos \alpha_{d,i} \]  
(64)

\[ F = \sum_{i=1}^{9} C_{m,i} \sin \alpha_{d,i} \]  
(65)

these equations may be written in matrix notation

\[
\begin{bmatrix}
A & B \\
B & C
\end{bmatrix}
\begin{bmatrix}
\{C_x'\} \\
\{C_z'\}
\end{bmatrix} =
\begin{bmatrix}
\{E\} \\
\{F\}
\end{bmatrix}
\]  
(66)

The values of \( C_x' \) and \( C_z' \) which best fit the data \( C_{m,i} \) are then given by
These components were then combined to give the magnitude and direction of the skin friction coefficient

\[ C_f = \sqrt{C_{x'}^2 + C_{z'}^2} \]  

(69)

\[ \alpha = \tan^{-1}\left( \frac{C_{z'}}{C_{x'}} \right) \]  

(70)

6.1 Uncertainty Analysis

Program LS averaged all six values from a single run together and computed 95% uncertainty bounds on the averaged value from each run. Thus, for the general case in which there were three test runs taken in each of the three measurement directions, LS first computed nine skin friction coefficients along with their uncertainties. Then the uncertainty in each measurement due to the leading edge perturbation angle \( \beta \) was added to the original uncertainty by taking the square root of the sum of the squares. This produced nine skin friction coefficients and their related uncertainties -- \( C_{m_i} \) and \( \delta C_{m_i} \), \( i = 1,2,...,9 \). The least squares fit was
then performed as described above to obtain \( C_x \) and \( C' \). These values were considered the "baseline" values.

To determine the uncertainty in \( C_x \) and \( C' \) due to each of the individual measurements, the first value was perturbed by an amount equal to its uncertainty

\[
C_{m,1}' = C_{m,1} + \delta C_{m,1}
\]  

and the least square error fit was performed again. The differences between the \( C_x \) and \( C' \) obtained from the perturbed calculation and the baseline case were taken as the uncertainties in \( C_x \) and \( C' \) due to the uncertainty in the first measured skin friction coefficient \( C_{m,1} \). Each of the remaining eight values were similarly perturbed to obtain the uncertainties in \( C_x \) and \( C' \) due to the uncertainty in each of the original values \( C_{m,i} \). These nine uncertainties were then combined with the least square error \( S \), by taking the square root of the sum of the squares to produce the final uncertainties on \( C_x \) and \( C' \) of \( \delta C_x \) and \( \delta C' \), respectively. For comparisons in Table 2, the magnitude and angle were necessary, and thus the uncertainties needed to be expressed in terms of the magnitude and angle. Therefore, \( \delta C_x \) and \( \delta C' \) were combined as follows to obtain \( \delta C_f \) and \( \delta \alpha \)

\[
\delta C_f = \sqrt{\left( \frac{\partial C_f}{\partial C_x} \right)^2 (\delta C_x)^2 + \left( \frac{\partial C_f}{\partial C'} \right)^2 (\delta C')^2}
\]  

where

**DATA REDUCTION**

44
\[
\frac{\partial C_f}{\partial C_x'} = \frac{C_x'}{\sqrt{C_x'^2 + C_z'^2}} \tag{73}
\]

from equation (69), and

\[
\frac{\partial C_f}{\partial C_z'} = \frac{C_z'}{\sqrt{C_x'^2 + C_z'^2}} \tag{74}
\]

from equation (69)

\[
\delta \alpha = \sqrt{\left(\frac{\partial \alpha}{\partial C_x'}\right)^2 (\delta C_x')^2 + \left(\frac{\partial \alpha}{\partial C_z'}\right)^2 (\delta C_z')^2} \tag{75}
\]

where

\[
\frac{\partial \alpha}{\partial C_x'} = \frac{-C_z'}{C_x'^2 + C_z'^2} \tag{76}
\]

from equation (70), and

\[
\frac{\partial \alpha}{\partial C_z'} = \frac{C_x'}{C_x'^2 + C_z'^2} \tag{77}
\]

from equation (70).
6.2 Curve fitting method

An alternative method for reducing the data was developed based on curve fitting the entire data record. This method uses every element in the data record, instead of only using the zero-crossings or signal peaks, to obtain the skin friction. Knowing that the intensity of the returning laser beams varies sinusoidally as a function of the oil film height and that a change in height of $h_A$ corresponds to one period, the intensity may be written as

$$i_i = A \sin \left( \frac{2\pi h}{\lambda} + \phi \right)$$  \hspace{1cm} (78)

where $A$ is the amplitude of the signal. As $t \to \infty$, $h \to 0$, and $\frac{i_i}{A} \to 1$. Thus, $\phi = \frac{\pi}{2}$ and the expression for the intensity becomes

$$\frac{i_i}{A} = \cos \left( \frac{2\pi h}{h_A} \right)$$  \hspace{1cm} (79)

Solving equation (4) for $h$ and substituting yields

$$\frac{i_i}{A} = \cos \left( \frac{2\pi k}{t} \right)$$  \hspace{1cm} (80)

where

$$k = \frac{\mu(x - x_0)}{\tau t}$$  \hspace{1cm} (81)
The constant $k$ cannot be found by simply solving equation (80) for $k$ and evaluating the expression for all $\frac{i}{A}$ since $\cos^{-1}\left(\frac{i}{A}\right)$ oscillates between -1 and 1 while $t$ increases continually. Multiple values of $k$ yield the same value of $\frac{i}{A}$ in equation (80) since cosine is a multiple-valued inverse function. Thus, it is necessary to add $2\pi$ to the value of $k$ for each successive period. In addition, unless the oil is allowed to thin to less than one fringe in height, the multiple of $2\pi$ to be added is unknown. After careful inspection of this phenomenon, the following equation was derived which can be iteratively solved for $k$

$$k = \frac{t}{2\pi} S \left[ \cos^{-1}\left(\frac{i}{A}\right) + 2\pi SP \right]$$  \hspace{1cm} (82)$$

where

$$S = \begin{cases} +1 & \text{for } m < \frac{k}{t} < m + \frac{1}{2} \\ -1 & \text{for } m + \frac{1}{2} < \frac{k}{t} < m + 1 \end{cases}$$  \hspace{1cm} (83)$$

and

$$P = \text{integer portion of} \left(\frac{2k}{t} + \frac{1}{2}\right)$$  \hspace{1cm} (84)$$

The value of $S$ may be represented analytically by
The iteration on equation (82) is a two variable iteration (on k and \( t_0 \) since the time \( t \) in equation (82) is referenced to the time origin \( t_0 \).

However, the current method uses only the data elements at the zero crossings to determine the skin friction component from each data record. This results in using only 20 or 30 data elements of the 4000 recorded. Since the curve fitting procedure uses every data element recorded, it is likely to reduce the total number of data elements necessary and thereby reduce the amount of time necessary to record data.
The locations at which data were obtained fall into six basic groups plus a few miscellaneous points. Four of the groups, consisting of points 10-17, 20-27, 31-38, and 41-49, were arranged such that all the points within a group are located on a line extending away from the wing-body. The lowest numbered data point in each of these groups was furthest away from the wing body, and the rest count inward toward the wing-body. Points 1-9 and 61-66 were located in the nose region and are all located outside the line of low shear. Points 51 and 52 were located far from the wing-body along the side. Most of the remaining data points were located in the plane of symmetry of the wing-body.

Skin friction coefficient vectors measured at the various positions in the wing-body junction flow are plotted in Figures 30 and 31, and are presented in tabular form in Table 2. The first three columns of Table 2 contain data location information -- identification number, and the \( x' \), \( z' \) coordinates non-dimensionalized by the maximum wing thickness \( T \). The next two columns,
labeled $C_f$ and $\alpha$, contain the magnitude and direction of the skin-friction coefficients obtained using the oil film interferometry method discussed in the previous chapter. The initial height of the oil films ($h = h_{i}N_{0,2}$) used in this study were between $10\mu m$ and $20\mu m$.

The column labeled $\delta_{\mu}$ contains flow directions which were measured directly from $TiO_2$-Kerosene surface oil flow visualizations similar to that represented in Figure 2. To obtain these angles, a tangent was drawn to the oil streaks at the point of interest, and the angle of the tangent with respect to the tunnel centerline was measured. These angles could only be determined to within $\pm 3^\circ$ in most locations with significantly higher uncertainties ($\pm 10^\circ$) in the nose region (points 13-17). The angles measured from the surface oil flow visualization agree to within the experimental error of the angles measured using the Oil-Film Laser Interferometry technique in other areas away from the line of low shear. Near the line of low shear, the angles show the greatest disagreement. The heights of the oil films used in these flows were less than $20 \mu m$. However, the oil used in the surface oil flow visualization was several times thicker, and varied in thickness inversely with the shear. Thus, these angles are more likely to be affected by pressure gradients.

The column labeled $C_{f,log}$ contains skin friction values at most of the locations examined in this thesis. These skin friction values were obtained by fitting the semi-logarithmic region of mean velocity profiles (measured using a laser anemometer) to the log law for equilibrium boundary layers (Devenport and Simpson, 1989) according to the equation
$$u^+ = 2.5 \log_e y^+ + 5.2$$ (86)

where $$y^+ = \frac{yu_r}{v}$$, $$u_r = \sqrt{\frac{\tau_w}{\rho}}$$, and $$u^+$$ was taken to be $$\frac{\sqrt{u'^2 + w'^2}}{u_r}$$. However, since the log law is a two-dimensional formulation, this method was not expected to yield reliable values of $$C_f$$ under the vortex legs, but provide a qualitative basis for comparison.

The skin friction coefficients obtained by fitting the semi-log region and those obtained by oil film interferometry are plotted as a function of ID number for comparison in Figure 32. In the region in front of the nose, both sets of skin friction coefficients exhibited the same trend, although the $$C_{f_{\log}}$$ values exhibit much greater fluctuations. Near the maximum wing thickness, the trends are also alike, $$C_f$$ increases as the wing is approached, with a drop in the magnitude near the line of low shear (located in the proximity of points 23 and 24). In this area, however, the values of $$C_{f_{\log}}$$ are much higher than those measured using oil film interferometry. In the regions along the side and rear of the wing, the trends are again similar, and the discrepancy between the magnitudes decreases with distance downstream. Notice that these discrepancies between $$C_f$$ and $$C_{f_{\log}}$$ increase rapidly inside the line of separation. This trend is expected since the log law is not expected to hold under the vortex leg. Furthermore, the differences between the two methods is smaller downstream, where the vortex strength has decreased, than upstream.
The close agreement away from the vortex, adds credence to both methods for such a flow. However, since the $C_{f_{log}}$ values can only be considered as a qualitative guide under the vortex, the similar trends exhibited by the $C_f$ and the $C_{f_{log}}$ methods is encouraging. Sources of error due to three-dimensional effects will be discussed later in this chapter.

The column labeled $\alpha_{w/u}$ contains flow angles obtained by combining $x'$ and $x'$ velocity components which were measured by hot-wire anemometry (points 1-9) and laser Doppler velocimetry (points 10-27 and 61-66). The angles for points 10-27 were computed from the following equation

$$\alpha_{w/u} = \tan^{-1}\left( \frac{w}{u} \right)$$

(87)

The angles for points 61-66 were measured by Olcmen (1990) in a similar manner. Velocity component data were not available at other locations in the flow. The height at which the velocity components were measured at each location are presented in the final column, labeled $h_{w/u}$. The angles given at locations 1 and 2 are somewhere in the peak between the two legs of a Johnston (1960) hodograph plot. Therefore, the actual angles at these locations are at least as large as those given in Table 2, and probably slightly larger. The rest of the $\alpha_{w/u}$ data were located on the near-wall leg of the Hodograph plot and can therefore be considered accurate as near the wall as is known. Uncertainties were available only for points 61-66, and tend to be very large near the wall (characteristic of LDV angle measurements near the wall). At points 61-66, the LDV angles and the oil-film interferometry angles agreed to within the
experimental uncertainties given. The velocity-component angles and the laser interferometry angles follow the same general trend -- increasing as the flow approaches the wing, attaining a maximum near points 10-17, and decreasing around the side of the wing, as expected. However, the oil film interferometry angles exhibited much larger variations than the velocity component angles. However, as noted previously, as the wall is approached, the uncertainties in the LDV angles becomes very large -- even at 150 \( \mu m \). No angles were available for comparison with the oil-film interferometry angles which were measured at the same height.

At several points in the flow (marked with a D in Table 1), data could not be obtained. A separation point exists along the centerline at \( x'/T = -0.47 \). Points 29 and 30 are between this point and the model nose. In this region, the direction of flow is from the nose towards the separation point, and the flow direction changes rapidly away from the centerline (Figure 33). As a result, the oil does not flow over both of the laser beams. The situation could be remedied by significantly decreasing the beam separation. Points 39 and 40 are located behind the tail of the model. In this region, the oil was observed to diverge from the tunnel centerline (Figure 34). This divergence likewise prevented data from being obtained in this region since the oil did not flow over both of the laser beams.

At points marked with an A, B, or C in Table 1, data could not be obtained in one of the three directions of measurement. This occurred when the angle of measurement was nearly normal to the measurement direction, as
depicted in Figure 8. This data loss was not critical, but the uncertainties quoted are questionable since the largest source of uncertainty was normally the error of resulting from the least squares fit.

Three dimensional effects are illustrated in Figure 30. Note that downstream of the maximum wing thickness the flow speed and direction vary gradually. The exception to this is the data point furthest downstream and closest to the tunnel centerline (point 33). At this location, the flow diverges sharply from the centerline creating large shear gradients. At points upstream of and near the maximum wing thickness three-dimensional effects are larger due to the flow acceleration around the wing and are very significant close to the wing (points 13-17) effects. The flow angles change significantly over the distance between the laser beams, and the streamline radii of curvature are on the order of the distance between the laser beams. Since the laser beams were positioned such that the measurement location was located midway between the laser beams, the laser beams impinged on the test surface at different locations when aligned in each of the three measurement directions. The flow varied significantly over the distance between the laser beams. Therefore, skin friction measurements in the different directions (0°, -45°, and -60°) were measuring skin friction in significantly different flows. Attempts to combine these data produced absurd results. The angles predicted from the oil interferometer reduction method for points 13-17 were between 0° and 25° although the actual oil pattern was observed after the test runs to be approximately 50° to 60°.

DISCUSSION
Another possible source of error is the variation in thickness of the initial oil line. In all of the previous analyses, it was assumed that the initial oil line was of uniform thickness, and that as the oil thinned, lines parallel to the oil film leading edge were also of uniform thickness. If one end of the initial oil line is thicker than the other end (as shown in Figure 35) and the angle $\gamma$ between the measurement direction and the flow angle is significant, the initial conditions along the oil film leading edge are not identical. Therefore, $x_0$ cannot be assumed identical for each of the laser beams and cannot be eliminated from the oil lubrication equations. However, during and after test runs in which the initial oil line was observed to have uneven thickness, fringes of constant thickness, visible in white light, were qualitatively observed to be very close to parallel to the oil film leading edge by the time the oil thinned enough to begin obtaining data. Thus, this non-uniform oil thickness was observed to be a transient effect which subsides quickly.

The pressure gradients encountered in this flow were found to have minimal effects on the oil-film interferometry method used. The largest pressure gradient effects occurred in the region near points 13-17. When the pressure gradient correction term was included for these points, the skin friction value produced from a given test run differed only in the fifth significant digit. Therefore, the pressure gradient effects were not included in the reduction of the remaining points.

The error induced by the unknown leading edge angle, $\beta$ was on the order of 3% to 5% on the individual test runs. The least square error fitting procedure
tended to minimize this effect so that the uncertainty in $\beta$ of $\pm 3^\circ$ had little effect on the final results. The largest source of error was normally the disagreement between measurements in the three different directions when combined in the least square error fit.
8.0 CONCLUSIONS

In this study, improvements were made to the basic dual beam oil film laser interferometer system which increased the ease of applicability and decreased the uncertainty of the final results for a three-dimensional flow. This improved system was used to produce skin friction coefficients around a wing-body (hull-appendage) junction. In the course of this investigation, limitations of this system were encountered and methods for minimizing these limitations were discussed.

The laser assembly was secured in one of three pre-set directions atop a movable platform which in turn, was mounted on a perpendicular set of rails. This mounting decreased the time required to re-locate the laser assembly at new data point from 45 minutes to less than 5 minutes. In addition, an iterative procedure for accurately determining the time origin for each oil flow was developed.
The current interferometer system was not able to obtain data in the centerline of the wake or in front of the wing's leading edge due to the flow patterns. In addition, this method produced very poor results (angles off by more than 40° at some locations) in the presence of large three-dimensional effects.

It was not possible to make direct comparisons to vouch for the validity of these results. The intent of this study was not to confirm other results, but to produce needed skin friction values in a complex flow. It was possible to obtain flow angles from surface oil flow visualizations, hot wire data, and LDV data and compare these angles to the angles computed for the skin friction coefficients. The angles computed in this study agreed well with angles from the other methods in areas in which the flow was nearly two-dimensional. In areas where large three-dimensional effects were present, all of the angles generally disagreed with each other since all the angles were measured at different heights in the flow and the flow angle varies rapidly near the wall in these regions.

Skin friction data were available at some locations in the flow, was not quantitatively correct under the vortex, and the data inside the line of separation could only be considered qualitatively. Away from the line of separation, the values produced by the two methods agreed to within experimental uncertainty.

Significant improvements in the current system could be obtained by reducing the beam spacing ($x_2 - x_1$) and the distance to the leading edge of the oil film ($x_1 - x_0$). These distances, to a large extent, determine the applicability of the method to the flow. If the flow changes significantly over these distances, then the method will not work. Thus, reducing these distances will increase the
complexity of the flow that can be studied using oil-film laser interferometry. Furthermore, since this system measures the average shear stress over the area between the beams, reducing \((x_2 - x_1)\) will make the final result approach the actual point value more closely.

The author has several general suggestions for increasing the performance of the present system for application to very complex flows, such as the one in the present study. Some method or mechanical device to increase the accuracy of the angle of the initial oil line will reduce the error due to this effect. Although this effect was generally very small compared to the other errors involved, in order to obtain very low uncertainty data, this effect can be greatly reduced if considered beforehand. Also, since knowledge of the shear gradient is necessary, taking measurements on an orthogonal grid may prove useful. Furthermore, a method to make fine adjustments of the movable platform would be helpful, and a more solid support structure than the wooden frame used in this study should be constructed.

In further studies, closer investigation of the curve fitting method of data reduction should have high priority. The reduction in time necessary to obtain enough data for data reduction could potentially be as significant as the reduction in time which occurred in moving from a single beam method to a dual beam method. Also, especially important if the curve fitting method is used, the photodetectors should be operated in the photoconductive mode instead of the photovoltaic mode since the photovoltaic mode may distort the shape of the interference signal.
Bibliography


Table 1. Sample data reduction output

Part (a): First reduction iteration
\[
\begin{array}{cccc}
N_1/N_2 & 0 & M & P \neg浼
\hline
20/20 & 0.00176 & 0.00175 & 0.00185 & 0.00185 & 0.00187 \\
20/20 & 0.00246 & 0.00249 & 0.00247 & 0.00199 & 0.00202 \\
20/20 & 0.00293 & 0.00302 & 0.00331 & 0.00323 & 0.00328 \\
\end{array}
\]

Part (b): Second reduction iteration
\[
\begin{array}{cccc}
N_1/N_2 & 0 & M & P \neg浼
\hline
10/20 & 0.00169 & 0.00169 & 0.00180 & 0.00178 & 0.00183 \\
10/20 & 0.00259 & 0.00272 & 0.00186 & 0.00197 & 0.00180 \\
10/20 & 0.00293 & 0.00298 & 0.00331 & 0.00323 & 0.00328 \\
\end{array}
\]

Part (c): Final reduction iteration (\(dt = 0\))
\[
\begin{array}{cccc}
N_1/N_2 & 0 & M & P \neg浼
\hline
10/20 & 0.00176 & 0.00176 & 0.00176 & 0.00176 & 0.00176 \\
10/20 & 0.00256 & 0.00268 & 0.00251 & 0.00201 & 0.00203 \\
10/20 & 0.00317 & 0.00321 & 0.00329 & 0.00320 & 0.00323 \\
\end{array}
\]
Table 2. Skin friction coefficients and comparisons

<table>
<thead>
<tr>
<th>ID #</th>
<th>X/T</th>
<th>Z/T</th>
<th>$C_f$ (±%)</th>
<th>$\alpha$ (°)</th>
<th>$\alpha_{sy}$ (°)</th>
<th>$C_{pf}$</th>
<th>$\alpha_{sy}$ (°)</th>
<th>$h_{pf}$ (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1.03000</td>
<td>-0.25000</td>
<td>0.00141 ± 9.1%</td>
<td>-16.2 ± 2.5°</td>
<td>-18°</td>
<td>.0016</td>
<td>-9.3°</td>
<td>255</td>
</tr>
<tr>
<td>2</td>
<td>-1.03000</td>
<td>-0.50000</td>
<td>0.00167 ± 5.7%</td>
<td>-19.8 ± 2.0°</td>
<td>-23°</td>
<td>.0017</td>
<td>-14.2°</td>
<td>255</td>
</tr>
<tr>
<td>3</td>
<td>-0.66700</td>
<td>-0.50000</td>
<td>0.00191 ± 5.2%</td>
<td>-55.4 ± 2.7°</td>
<td>-36°</td>
<td>.00155</td>
<td>-29.6°</td>
<td>280</td>
</tr>
<tr>
<td>4</td>
<td>-0.66700</td>
<td>-0.75000</td>
<td>0.00222 ± 7.9%</td>
<td>-39.1 ± 4.4°</td>
<td>-34°</td>
<td>.00195</td>
<td>-26.2°</td>
<td>230</td>
</tr>
<tr>
<td>5</td>
<td>-0.32900</td>
<td>-1.01100</td>
<td>0.00287 ± 2.7%</td>
<td>-26.9 ± 1.4°</td>
<td>-31°</td>
<td>.0036</td>
<td>-27.3°</td>
<td>255</td>
</tr>
<tr>
<td>6</td>
<td>0.00900</td>
<td>-0.87900</td>
<td>0.00388 ± 3.1%</td>
<td>-16.1 ± 1.3°</td>
<td>-32°</td>
<td>.0042</td>
<td>-31.8°</td>
<td>205</td>
</tr>
<tr>
<td>7</td>
<td>0.02800</td>
<td>-1.13200</td>
<td>0.00357 ± 3.7%</td>
<td>-13.3 ± 1.5°</td>
<td>-23°</td>
<td>.0039</td>
<td>-24.1°</td>
<td>255</td>
</tr>
<tr>
<td>8</td>
<td>0.36100</td>
<td>-0.98700</td>
<td>0.00444 ± 4.7%</td>
<td>-8.0 ± 1.5°</td>
<td>-7°</td>
<td>.0051</td>
<td>-22.6°</td>
<td>230</td>
</tr>
<tr>
<td>9</td>
<td>0.36500</td>
<td>-1.24400</td>
<td>0.00371 ± 2.0%</td>
<td>-4.7 ± 1.6°</td>
<td>-10°</td>
<td>.0050</td>
<td>-1.8°</td>
<td>255</td>
</tr>
<tr>
<td>10</td>
<td>-0.45781</td>
<td>-0.88723</td>
<td>0.00259 ± 4.4%</td>
<td>-33.2 ± 2.2°</td>
<td>-34°</td>
<td>.0024</td>
<td>-36.3°</td>
<td>100</td>
</tr>
<tr>
<td>11</td>
<td>-0.30802</td>
<td>-0.75470</td>
<td>0.00283 ± 6.0%</td>
<td>-49.1 ± 3.4°</td>
<td>-44°</td>
<td>.00265</td>
<td>-50.7°</td>
<td>100</td>
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<tr>
<td>12</td>
<td>-0.19568</td>
<td>-0.65531</td>
<td>0.00339 ± 4.6%</td>
<td>-56.8 ± 2.6°</td>
<td>-46°</td>
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<td>255</td>
</tr>
<tr>
<td>13A</td>
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<td>-0.58905</td>
<td>0.00547 ± 9.3%</td>
<td>-10.8 ± 4.4°</td>
<td>-10°</td>
<td>.0021</td>
<td>-59.3°</td>
<td>330</td>
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<tr>
<td>14A</td>
<td>-0.08333</td>
<td>-0.55592</td>
<td>0.00605 ± 13.2%</td>
<td>-16.8 ± 6.7°</td>
<td>-10°</td>
<td>.0025</td>
<td>-57.5°</td>
<td>255</td>
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<tr>
<td>15A</td>
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<td>-0.52279</td>
<td>0.00721 ± 16.6%</td>
<td>-14.9 ± 7.2°</td>
<td>-16°</td>
<td>.0058</td>
<td>-69.0°</td>
<td>430</td>
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<tr>
<td>16A</td>
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<td>-0.48965</td>
<td>0.00814 ± 7.0%</td>
<td>-25.8 ± 3.5°</td>
<td>-10°</td>
<td>.0078</td>
<td>-72.6°</td>
<td>90</td>
</tr>
<tr>
<td>17A</td>
<td>0.02900</td>
<td>-0.45652</td>
<td>0.01046 ± 7.0%</td>
<td>-25.8 ± 3.5°</td>
<td>-10°</td>
<td>.0078</td>
<td>-72.6°</td>
<td>90</td>
</tr>
<tr>
<td>18A</td>
<td>0.06646</td>
<td>-0.42339</td>
<td>0.01046 ± 7.0%</td>
<td>-25.8 ± 3.5°</td>
<td>-10°</td>
<td>.0078</td>
<td>-72.6°</td>
<td>90</td>
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<td>19</td>
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<td>20</td>
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<td>-1.32500</td>
<td>0.00382 ± 8.3%</td>
<td>-3.2 ± 0.7°</td>
<td>-2°</td>
<td>.0045</td>
<td>-3.9°</td>
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<td>0.00401 ± 5.4%</td>
<td>-2.0 ± 5.4°</td>
<td>-1°</td>
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<td>-6.9°</td>
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<td>-2°</td>
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<td>-6.5°</td>
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<td>-11.3°</td>
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<td>-0.92500</td>
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<td>-8.3 ± 1.6°</td>
<td>-10°</td>
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<td>-15.0°</td>
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<td>0.75000</td>
<td>-0.85000</td>
<td>0.00532 ± 9.4%</td>
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<td>-21°</td>
<td>.0078</td>
<td>-21.9°</td>
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<td>-16°</td>
<td>.0098</td>
<td>-19.9°</td>
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<td>-0.70000</td>
<td>0.00740 ± 6.6%</td>
<td>-24.9 ± 2.6°</td>
<td>-10°</td>
<td>.0100</td>
<td>-16.7°</td>
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<td>-0.86000</td>
<td>0.00000</td>
<td>0.01046 ± 7.0%</td>
<td>-25.8 ± 3.5°</td>
<td>-10°</td>
<td>.0078</td>
<td>-72.6°</td>
<td>90</td>
</tr>
<tr>
<td>29D</td>
<td>-0.30000</td>
<td>0.00000</td>
<td>0.01046 ± 7.0%</td>
<td>-25.8 ± 3.5°</td>
<td>-10°</td>
<td>.0078</td>
<td>-72.6°</td>
<td>90</td>
</tr>
<tr>
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<td>0.00000</td>
<td>0.01046 ± 7.0%</td>
<td>-25.8 ± 3.5°</td>
<td>-10°</td>
<td>.0078</td>
<td>-72.6°</td>
<td>90</td>
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<td>-4°</td>
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<tr>
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<td>-0.80000</td>
<td>0.00250 ± 2.8%</td>
<td>0.4 ± 1.7°</td>
<td>-6°</td>
<td>.0026</td>
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<tr>
<td>34</td>
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<td>-0.60000</td>
<td>0.00286 ± 3.3%</td>
<td>-5.6 ± 1.6°</td>
<td>-7°</td>
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<tr>
<td>35</td>
<td>4.46100</td>
<td>-0.50000</td>
<td>0.00292 ± 4.5%</td>
<td>-3.8 ± 1.6°</td>
<td>-7°</td>
<td>.0028</td>
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A - No data in 0° direction
B - No data in 60° direction
C - No data in 45° direction
D - No data available (See Discussion Section)
<table>
<thead>
<tr>
<th>ID #</th>
<th>X/T</th>
<th>Z/T</th>
<th>C_f</th>
<th>( \alpha )</th>
<th>( \alpha_{90} )</th>
<th>C_{fmax}</th>
<th>a_{w/s}</th>
<th>h_{w/s} \mu m</th>
</tr>
</thead>
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<tr>
<td>36</td>
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<td>-0.40000</td>
<td>0.00324 ± 7.1%</td>
<td>1.2° ± 1.9°</td>
<td>-9°</td>
<td>0.0032</td>
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<td>-0.30000</td>
<td>0.00309 ± 5.3%</td>
<td>-11.9° ± 1.8°</td>
<td>-9°</td>
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<td>38B</td>
<td>4.46100</td>
<td>-0.20000</td>
<td>0.00342 ± 3.0%</td>
<td>-35.4° ± 1.8°</td>
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<td>0.00249 ± 2.4%</td>
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<td>19.2° ± 2.6°</td>
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<td>0.00321 ± 1.8%</td>
<td>11.9° ± 3.9°</td>
<td>5°</td>
<td>0.0034</td>
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<td>-0.78015</td>
<td>0.00361 ± 3.5%</td>
<td>-1.7° ± 1.8°</td>
<td>-2°</td>
<td>0.0038</td>
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<td>-4.5° ± 1.7°</td>
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<td>3.18700</td>
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<td>7.9° ± 3.6°</td>
<td>3°</td>
<td>0.0042</td>
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<td>3.18700</td>
<td>-0.48015</td>
<td>0.00417 ± 1.6%</td>
<td>4.3° ± 3.5°</td>
<td>6°</td>
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<td>2.71950</td>
<td>-1.56030</td>
<td>0.00280 ± 1.8%</td>
<td>10.4° ± 2.7°</td>
<td>8°</td>
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<tr>
<td>51B</td>
<td>1.78470</td>
<td>-1.48750</td>
<td>0.00333 ± 3.7%</td>
<td>6.6° ± 3.9°</td>
<td>6°</td>
<td></td>
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</tr>
</tbody>
</table>

A - No data in 0° direction
B - No data in 60° direction
C - No data in 45° direction
D - No data available (See Discussion Section)
Illustrations
Figure 1. Perspective View of the wing-body junction.
Figure 2. \( TiO_2 \)-Kerosene Oil Flow Visualization: \( Re = 6700 \)
Figure 3. Probability density functions at $X/T = -0.20$
Figure 5. Geometry for optical pathlength difference
Figure 8. Problem area: flow direction near normal to measurement direction
Figure 9. VPI&SU Low Speed Boundary Layer Wind Tunnel
Figure 10. Plywood tunnel floor inserts

Illustrations
Figure 11. Wooden frame surrounding the wind tunnel
Figure 12. Movable platform and traversing rails
Figure 13. Photograph of movable platform mounted on wooden frame
Figure 14. Photograph of entire apparatus
Figure 15. Side view of laser assembly
Figure 16. Laser assembly viewed from optical rail axis
Figure 17. Laser assembly geometry diagram
Figure 18. Box diagram of hardware components
Figure 19. Photograph of test surface with data location transparency attached
Figure 21. Sample photodetector output: Point 05, Angle 60, Take 3, downstream beam.
PT 11, ANGLE 45, TAKE 3
BEAM 1 (UPSTREAM LOCATION)

Light intensity (arbitrary units)
(Thousands)

0 1 2 3 4 5 6
0 1 2 3

Figure 22. Sample photodetector output: Point 11, Angle 45, Take 3, upstream beam
Figure 23. Sample photodetector output: Point 11, Angle 45, Take 3, downstream beam
Figure 24. Sample "bad" photodetector output: Point 28, Angle 45, Take 1, upstream beam.
PT 28, ANGLE 45, TAKE 1

BEAM 1 (UPSTREAM LOCATION)

Sample "bad" photodetector output: Point 28, Angle 45, Take 1, downstream beam
Figure 26. Skin friction coefficient magnitude vs. time origin error: Point 01, angle 60
Figure 27. Skin friction coefficient magnitude vs. time origin error: Point 04, angle 60
Figure 28. Skin friction coefficient magnitude vs. time origin error: Point 31, angle 60
Figure 29. Non-perpendicularity Geometry and Notation: Error in angular position of oil film leading edge
Figure 30. Skin friction coefficient vectors plotted: entire flowfield
Figure 34. Problem area: oil flow pattern in tail region
Appendix A. PROGRAM OILAD32.BAS

1000 '  
1005 DEFINT A-Z  
1010 'OIL-FILM INTERFEROMETER A/D PROGRAM, VERSION 3.2 082587  
1020 '  
1030 'A/D PROGRAM FOR TWO BEAMS & INPUT CHNLS, W/O SCANNER & EXT TRIG  
1040 GOSUB 20000  
1050 'INPUT PARAMETERS FOR DATA FILE AND A/D CONVERSION  
1080 CLS:PRINT:PRINT 'OIL A.D PROGRAM & DATA FILE PARAMETERS':PRINT  
1120 PRINT 'A/D PROGRAM FOR DUAL BEAM LASER INTERFEROMETER ':PRINT  
1140 PRINT INPUT*: 'LOCATION OF MEASUREMENTS '*X!  
1145 INPUT*: 'Z-LOCATION OF MEASUREMENTS '*Z!  
1150 PRINT INPUT*: REF PRESSURE-Q (IN. WATER) '*UREF!  
1155 PRINT INPUT*: NUMBER OF DATA SAMPLES '*NUM.CONV  
1156 PRINT INPUT*: DATA SAMPLING RATE IN HERTZ '*;HERTZ!  
1160 PRINT INPUT*: DISC FILE FOR RESULTS '*;DS  
1180 PRINT:PRINT 'Ensure <CAPS LOCK> is on before continuing'  
1190 PRINT:PRINT:PRINT 'Press any key to continue'  
1200 AS=INKEYS AS = "" THEN 1200  
1300 CLOSE 1:OPEN DS FOR OUTPUT AS #1  
1310 PRINT#:"X = ";X!  
1320 PRINT#:"Z = ";Z!  
1330 PRINT#:"Q = ";UREF!  
1400 PERIOD# = (1#/(2*a*HERTZ#))*1000000a#  
1450 TICKS# = CINT(PERIOD# = 1.25 = 32768#) + 32768#  
1460 PERIOD# = TICKS# = 1.25#  
1500 START CHANNEL = 1:END CHANNEL = 2  
1600 NUM.CONV = ((END CHANNEL - START CHANNEL + 1)*NUM.CONV)*2  
2000 GOSUB 21000  
3060 PARM = (NUM.CONV-4)  
4000 FOR ADDRESS = 0 TO PARM STEP 4:TPRINT = PEEK(ADDRESS) + PEEK(ADDRESS + 1)*256!  
4005 TPRINTB = PEEK(ADDRESS + 2) + PEEK(ADDRESS + 3)*256!  
4010 PRINT#:TPRINT,TPRINTB  
4020 NEXT  
4030 END  
20000 '  
20010 'Initialise A/D converter  
20020 '  
20030 BASE ADDRESS = &H2EC  
20040 COMMAND REGISTER = BASE ADDRESS + 1  
20050 STATUS REGISTER = BASE ADDRESS + 1  
20060 DATA REGISTER = BASE ADDRESS  
20070 COMMAND WAIT = &H4  
20080 WRITE WAIT = &H2  

Appendix A. PROGRAM OILAD32.BAS 101
20090 READ.WAIT = &H5
20100 CSTOP = &HF
20110 CCLEAR = &H1
20120 CERROR = &H2
20130 CCLOCK = &H3
20140 CSAD = &HD
20150 CRAD = &HE
20160 EXT.CLOCK = &H40
20170 EXT.TRIGGER = &H80
20180 CDMA = &HI0
20190 DUMMY = 5
20200 DMACHannel = 1
20210 DMAMODE = &H45
20220 BASEREG = 2
20230 COUNTREG = 3
20240 PAGEREG = &H83
20250 STARTPAGE = &H3000
20260 DEF SEG = STARTPAGE
20270 DMABASEH = &H0
20280 DMABASEH = &H0
20290 DMAPAGE = STARTPAGE/&H1000
20300 'Stop and clear the DT2801.
20310 OUT COMMAND.REGISTER, CSTOP
20320 TEMP = INP(DATA.REGISTER)
20330 WAIT STATUS.REGISTER, COMMAND.WAIT
20340 OUT COMMAND.REGISTER, CCLEAR
20350 CLS:PRINT:PRINT"SETTING UP INTERFACE"
20360 'Get A:D gain.
20370 PRINT:PRINT "Gain of A D (1,2,4 or 8) (2 for +/5v)";GAIN.CODE
20380 GAIN.CODE = LOG(GAIN.CODE)/LOG(2)
20390 'Get A:D channel.
20400 PRINT:PRINT "Signal A (Front Beam)-USE A:D CH #1:";S.CHANNEL:PRINT
20410 E.CHANNEL = S.CHANNEL
20420 PRINT-SIGNAL B (REAR BEAM) ON CH#2"
20430 PRINT:PRINT:PRINT"Press any key to continue"
20440 AS = INKEYS AS = "" THEN 20510
20450 RETURN
20500 'Instruct the A/D to take data
20510 WAIT STATUS.REGISTER, COMMAND.WAIT
20520 OUT COMMAND.REGISTER, CSAD
20530 WAIT STATUS.REGISTER, WRITE.WAIT, WRITE.WAIT
20540 OUT DATA.REGISTER, GAIN.CODE
20550 WAIT STATUS.REGISTER, WRITE.WAIT, WRITE.WAIT
20560 OUT DATA.REGISTER, START.CHANNEL
20570 OUT DATA.REGISTER, WRITE.WAIT, WRITE.WAIT
20580 OUT DATA.REGISTER, END.CHANNEL
20590 OUT DATA.REGISTER, WRITE.WAIT, WRITE.WAIT
20600 OUT DATA.REGISTER, WRITE.WAIT, WRITE.WAIT
20610 OUT DATA.REGISTER, DMAMODE
20620 OUT DATA.REGISTER, DUMMY
20630 OUT DATA.REGISTER, DMABASEL
20640 OUT DATA.REGISTER, DMABASEH
20650 OUT DATA.REGISTER, DMAPAGE
20660 OUT DATA.REGISTER, DUMMY
20670 OUT DATA.REGISTER, COMMAND.WAIT
20680 OUT COMMAND.REGISTER, CCLOCK
20690 PH# = INT(TICKS#/256)
2070 PL# = TICKS*-PH#*256
20710 PERIODH = PH#
20720 PERIODL = PL#
20730 WAIT STATUS.REGISTER, WRITE.WAIT, WRITE.WAIT
20740 OUT DATA.REGISTER, PERIODL
20750 WAIT STATUS.REGISTER, WRITE.WAIT, WRITE.WAIT
20760 OUT DATA.REGISTER, PERIODH
20770 DMACOUNT = (NUM.CONV"2")-1
20780 DMACOUNTL = DMACOUNT-DMACOUNT*256
20790 DMACOUNTL = DMACOUNT-DMACOUNT*256
20800 OUT 11, DMAMODE "set DMA mode
20810 OUT 12,0 "clear byte flip-flop
20820 OUT BASEREG, DMABASEL "set DMA memory base address

Appendix A. PROGRAM OILAD32.BAS 102
21290 OUT BASEREG,DMABASEH  
21300 OUT COUNTREG,DMACOUNTL  
21310 OUT COUNTREG,DMACOUNTH  
21320 OUT PAGEREG,DMAPAGE  
21330 OUT 10,DMACHANNEL  
21340 WAIT STATUS REGISTER,COMMAND,WAIT  
21350 STATUS = INP(STATUS REGISTER)  
21360 IF (STATUS AND &H80) THEN GOTO 22000  
21370 WAIT STATUS REGISTER,COMMAND,WAIT  
21380 OUT COMMAND REGISTER,CRADE-CDMA+COMMAND  
21390 WAIT STATUS REGISTER,COMMAND,WAIT  
21400 STATUS = INP(STATUS REGISTER)  
21410 IF (STATUS AND &H80) THEN GOTO 22000  
21420 RETURN  
22000 '  
22010 ' Error handling for A/D  
22020 '  
22030 PRINT "FATAL BOARD ERROR"  
22040 PRINT "STATUS REGISTER VALUE IS \"HEXS(STATUS)\" HEXDECIMAL"  
22050 PRINT:GOSUB 22100  
22060 PRINT "ERROR REGISTER VALUES ARE:"  
22070 PRINT " BYTE 1 - \"HEXS(ERROR1)\" HEXDECIMAL:"  
22080 PRINT " BYTE 2 - \"HEXS(ERROR2)\" HEXDECIMAL"  
22090 PRINT:GOTO 22240  
22100 '  
22110 ' Read the error register  
22120 OUT COMMAND REGISTER,CSTOP:TEMP = INP(DATA REGISTER)  
22130 WAIT STATUS REGISTER,COMMAND,WAIT  
22140 OUT COMMAND REGISTER,CERROR  
22150 WAIT STATUS REGISTER,READ,WAIT  
22160 ERROR1 = INP(DATA REGISTER)  
22170 WAIT STATUS REGISTER,READ,WAIT  
22180 ERROR2 = INP(DATA REGISTER)  
22190 STOP  
22200 '  
22210 ' Illegal Status Register  
22220 PRINT "FATAL ERROR - ILLEGAL STATUS REGISTER VALUE:"  
22230 PRINT "STATUS REGISTER VALUE IS \"HEXS(STATUS)\" HEXDECIMAL"  
22240 STOP
Appendix B. PROGRAM OILCF.FOR

```
CCC PROGRAM OILCF

C
C VERSION 6 INPUTS NZEROB AND NZEROA MAXIMUMS
C AND PRINTS TO FOR EACH BEAM
C NOTE: PRINTER IS REQUIRED
C
CCC REDUCES DIGITAL OUTPUT OF PHOTODIODE TO A COEFFICIENT OF
CCC SKIN FRICTION BY USE OF OIL LUBRICATION THEORY.
CCC
CCC USES THE FOLLOWING SUBROUTINES:
CCC
CCC SMITHD-SMOOTHES RAW DIGITAL OUTPUT OF PHOTODIODE
CCC SOILFM-IDENTIFIES FRINGES OF PHOTODIODE OUTPUT
CCC SLUBFT-APPLIES LUBRICATION THEORY TO DATA
CCC CLFCFCL-CALCULATES THE WALL SHEAR FOR THE OIL FILM
CCC
CCC THIS PROGRAM IS BASED ON SUBROUTINE USED BY DR. WESTPHAL
CCC AT NASA-AMES AND GIVEN IN NASA TM-88216 "IMPROVED
CCC SKIN FRICTION INTERFEROMETER"
CCC
CCC
CCC THIS PROGRAM READS THE INPUT DATA FILE AS A SEQUENTIAL
CCC ACCESS FILE (FILE #1), PARAMETERS ARE READ IN
CCC FROM THE SCREEN. OUTPUT FILES WRITTEN ARE:
CCC #12-SMOOTHED INPUT DATA (INTEGER COUNTS)
CCC #13 & #14-BEAM A&B ZEROES AND TIME LOCATIONS
CCC SET DIMENSION, REALS, INTEGERS, COMMONS, AND DATA
CCC IMPLICIT DOUBLE PRECISION (A-H)
CCC IMPLICIT DOUBLE PRECISION (O-Z)
CCC INTEGER IRAWA(5000),IRAWB(5000),NZA(6),NZB(6),ICHK(6),FORM
CCC CHARACTER*18 SOURCE.OUTFIL
CCC COMMON-FITCOM,CPN,FITASLFIT,STFIT,ZERO(200),NZERO,RMSERR
CCC EXTERNAL FITERR
CCC DOUBLE PRECISION MUNOM,NOIL,MINUS,ERRA(6),ERRB(6),DT(6),CFM(6),ERO
CCC D0203.
CCC ERRa(K) = 0.DO
CCC NUL = 0
CCC WRITE(*,*) INPUT NUMBER OF ZEROES TO USE FOR BEAM A
CCC READ(*,*)NA
CCC WRITE(*,*) INPUT NUMBER OF ZEROES TO USE FOR BEAM B
CCC READ(*,*)NB
CCC
CCC OPEN INPUT AND OUTPUT FILES
CCC
CCC INPUT FILE #1—RAW DIGITAL PHOTODIODE OUTPUT
```
WRITE(*,11)
11 FORMAT(5X,ENTER FILE #1,DATA)
OPEN(1,FILE = ' ',STATUS = 'OLD')
INQUIRE(1,NAME = SOURCE)
OPEN(9,FILE = 'LPT1',STATUS = 'OLD')
WRITE(*,SOURCE)

CCC
CCC OUTPUT FILE #12—SMOOTHED INPUT DATA (INTEGER)
WRITE(*,12)
12 FORMAT(SX,'ENTER FILE #1,DATA')
OPEN(12,FILE = ' ',STATUS = 'NEW')

CCC
CCC OUTPUT FILE #13—BEAM A ZEROES AND THEIR LOCATIONS
WRITE(*,13)
13 FORMAT(5X,'ENTER FILE #13,OUTPUT')
OPEN(13,FILE = ' ',STATUS = 'NEW')

CCC
CCC OUTPUT FILE #14—BEAM B ZEROES AND THEIR LOCATIONS
WRITE(*,14)
14 FORMAT(5X,'ENTER FILE #14,OUTPUT')
OPEN(14,FILE = ' ',STATUS = 'NEW')

CCC
READ IN FREQUENCY OF DATA TO COMPUTE THE DELTA-TIME
CCC BETWEEN DATA POINTS (FREQ IN Hertz)
WRITE(*,11)
11 FORMAT(5X,'ENTER FREQUENCY (IN Hertz) OF DATA TO COMPUTE DELTA-T')
READ(*,FREQ)
DELT = ID0-FREQ

CCC
READ IN PARAMETER VALUES FOR THE FOLLOWING:
CCC
CCC PARAMETERS FOR THE SMTHPD subroutine
CCC NRAWMX—NUMBER DATA PTS, LENGTH OF IRAWA & IRAWB ARRAYS
CCC MRAWMX—NUMBER DATA PTS, LENGTH OF IRAWA & IRAWB ARRAYS
CCC MODEP—TYPE OF SMOOTHING, BETWEEN 0 & 1: 0—FLAT CURVE, 1—NO SMOOTH
CCC OF TIMES 3 PT SMOOTHING IS USED IF 1-10
WRITE(*,50)
50 FORMAT(5X,'ENTER FACTOR.MODEP')
READ(*,FACTOR,MODEP)

CCC
CCC PARAMETERS FOR THE SOILFM subroutine
CCC MINFA&B—MIN ARGUMENT FOR DATA REDUCTION (MIN DIMENSION OF RAW DATA)
CCC MAXFA&B—MAX ARGUMENT FOR DATA REDUCTION (MAX DIMENSION)
CCC NINTA&B—MIN # OF PTS IN ONE INTERVAL USED TO FIND ZERO CROSSING
CCC LINE, GREATER THAN 1
CCC RMSFA&B—FRACTION OF RMS TO USE FOR TESTING IF PT IS IN REGION
CCC NZMAXA&B—AMT OF STORAGE FOR ZERO( ) IN CALLING PROGRAM
WRITE(*,51)
51 FORMAT(5X,ENTER MINFA,MAXFA,NINTA,RMSFA,NZMAXA & -B VALUES)
READ(*,MINFA,MAXFA,NINTA,RMSFA,NZMAXA,MINFB,MAXFB,NINTB,
& RMSFB,NZMAXB)

CCC
CCC PARAMETERS FOR THE SLUBFT subroutine
CCC MODEA&B—SPECIFIES IF INITIAL TIME BASE TO BE FIT OR AN INPUT,
CCC MODE = 1 TSTFP IS INPUT MODE = 2 TSTFP FOUND FROM FIT
CCC TSTFP & -B ARE THE TIME BASE FOR FIT
WRITE(*,52)
52 FORMAT(5X,ENTER MODEA,MODEB,TSTFP,TSTFPB)
READ(*,MODEA,MODEB,TSTFP,TSTFPB)

CCC
CCC PARAMETERS FOR THE FRICTION CALCULATION
CCC TCOIL—OIL TEMP IN DEG CELSIUS
CCC TCNOM—NOMINAL OIL TEMP FOR USE IN DOW CORNING EQN IN DEG CEL
CCC MVCNOM—NOMINAL OIL VISCOSITY FOR USE IN DOW CORNING EQN
CCC OIL INDEX OF REFRACTION
CCC WLLSER—LASER WAVELENGTH IN METERS, 0.6328X10-6 FOR HE-NE
CCC ANGIN—LASER BEAM INCIDENCE ANGLE IN DEG
CCC DXBEAM—BEAM SEPARATION IN METERS FROM FRONT TO REAR BEAM
WLLSER = 0.6328D-06
WRITE(*,*) ENTER BEAM SPACING IN mm
READ(*,DXYBEAM)
DXBEAM = DXBEAM*1000.D0
MUNOM = .0479538D0
NOIL = 1.40125D0
WRITE(*,*) 'ENTER ANGLE OF INCIDENCE IN DEGREES'
READ(*,*)ANGIN
TCNOM = 25.D0

CCC

CCC FLOW PARAMETERS
CCC QE-DYNAMIC PRESSURE OF FLOW, IN. WATER
CCC READ QE FROM SCREEN
WRITE(*,3)
3 FORMAT(5X,'ENTER QE(DYNAMIC PRESS) IN in. water')
READ(*,*)QE
CCC CONVERT QE FROM IN. WATER TO N/M
QE = QE*248.9460171D0
CCC READ OIL TEMP FROM SCREEN
WRITE(*,35)
35 FORMAT(5X,'ENTER TCOIL (OIL TEMP IN DEG C) AND PRESSURE &GRADIENT (IN N/M)
READ(*,*)TCOIL,DPDX
CCC NUMBER OF DATA POINTS TO PROCESS
WRITE(*,4)
4 FORMAT(5X,'ENTER NUMBER OF DATA POINTS TO PROCESS')
READ(*,*)NRAWMX
ITYP = 1
READ(1,30)X,XVAL
READ(1,30)Y,YVAL
READ(1,30)Q,QVAL
30 FORMAT(A2,F5.2)

CCC READ IN DIGITAL INPUT DATA SIGNAL FROM A/D CONVERTER
CCC FOR THE PHOTODIODE OUTPUT
DO 40 I = 1,NRAWMX
READ(1,*),END = 4000(IRAWA(I),IRAWB(I))
CONTINUE

CCC DO CALCULATIONS
CCC CALL SUBROUTINES TO:
CCC 1) SMOOTH OUTPUT OF PHOTODIODE
CCC 2) I.D. FRINGES
CCC 3)FIT LUBRICATION THEORY TO DATA
CCC 4)CALCULATE CF BASED ON TYPE OF OIL
CCC

DO THE FOLLOWING SUBROUTINES FOR EACH BEAM
CCC
CCC DO SMOOTHING FOR BOTH BEAM A&B AND WRITE THE SMOOTHED
CCC OUTPUT TO FILE #12
CCC
4000 CONTINUE
CLOSE(I)
WRITE(*,100)
100 FORMAT(5X,'ENTRY')

CALL SMTHPD(IRAWA,NRAWMX,FACTOR,MODEP)
CALL SMTHPD(IRAWB,NRAWMX,FACTOR,MODEP)
WRITE(*,35)
55 FORMAT(5X,'SMTHPD CHECK FOR BEAMS A&B-NOW WRITE DATA TO #12')
DO 600 I = 1,NRAWMX
WRITE(12,*)IRAWA(I),IRAWB(I)
CONTINUE
CLOSE(12)

OPEN(2,FILE='SOURCE-DAT'.STATUS = 'NEW')
WRITE(2,6674)SOURCE,'OUT'
REWIND 2
READ(2,*),OUTFIL
CLOSE(2)

OPEN(3,FILE = OUTFIL,STATUS = 'NEW')

Appendix B. PROGRAM OILCF.FOR
CCC CONTINUE CALCULATIONS FOR ONE BEAM AT A TIME,
CCC FIRST FOR BEAM A:
CCC

54 CALL SOILFM(IRAWA,NRAWMX,MINFA,MAXFA,NINTA,AVGA,RMSA,RMSFA,
&ZERO,NZMAXA,NZERO,ITYP)
THICKA = WLLSER*REAL(NZEROA)
C
C MAX NZEROA = NA
C
IF(NZERO.GT.NA) THEN
L = 0
DO 700 K = NZERO-NA+1,NZERO
L = L + 1
700 ZERO(L) = ZERO(K)
NZERO = NA
ENDIF
WRITE(*,56)
56 FORMAT(5X,'SOILFM A CHECK-AVG,RMS,NZERO')
WRITE(*,AVGA,RMSA,NZERO)
IF(NZERO.EQ.0) THEN
WRITE(*,*)'NZEROA = 0 FOR THIS CASE'
WRITE(*,ITYP)'NZEROA = 0
NZEROB = 0
AERR = 0.00
BERR = 0.00
GO TO 6665
END IF
NZEROA NZERO
DO 500 1 = 1,NZERO
ZERO(I) = (ZERO()DELT)+ .0500
WRITE(14,ZERO(I))
500 CONTINUE
BZEROA = ZERO(I)
CALL SLUBFT(ZERO,NZERO,.MODEA,CFNFPA,ASLFPAP,TSTFPA,ERROR)
AERR = error
WRITE(*,57)CFNFPA,ASLFPAP,TSTFPA
57 FORMAT(5X,'SLUBFT CHK, END OF BEAM A CHK-CFN FP,ASL FP, TST FP, ERROR')
C
C SECOND FOR BEAM B- REAR BEAM
C
CALL SOILFM(IRAWB,NRAWMX,MINFB,MAXFB,NINTB,AVGB,RMSB,RMSFB,
&ZERO,NZMAXB,NZERO,ITYP)
THICKB = WLLSER*NZEROB
C
C MAX NZEROB = NB
C
IF(NZERO.GT.NB) THEN
L = 0
DO 710 K = NZERO-NB+1,NZERO
L = L + 1
710 ZERO(L) = ZERO(K)
NZERO = NB
ENDIF
WRITE(*,59)
59 FORMAT(5X,'SOILFM B CHECK-AVG,RMS,NZERO')
WRITE(*,AVGB,RMSB,NZERO)
IF(NZERO.EQ.0) THEN
WRITE(*,*)'NZEROB = 0 FOR THIS CASE'
WRITE(*,ITYP)'NZEROB = 0
NZEROA = 0
NZEROB = 0
BERR = 0.00
GO TO 6665
END IF
NZEROB NZERO
DO 7010 I = 1,NZERO
ZERO(I) = (ZERO(I)*DELT) + .05D0
WRITE(14,'(ZERO(I))')
7010 CONTINUE
BZEROB = ZERO(I)
CALL SLUBFT(ZERO,NZERO,.MODEB,CFNFPB,ASLFPB,TSTFPB,ERROR)
BERR = ERROR

Appendix B. PROGRAM OILCF.FOR
WRITE(*.60)
60  FORMAT(5X,SLUBFTR B')
WRITE(*,701)CFNFPB,ASLFPB,TSTFPB
701  FORMAT(5X,SLUBF CHK, END OF BEAM CHK-CFNFP,ASLFPB,TSTFPB',
&/,F12.4,1X,F12.4,1X,F12.4)
II = 0
AVGTFB = ((TSTFPB + ZERO(NZEROB)-(TSTFPB + ZERO(1)))/2D0
3000  II = II + 1
TFCHKB = TSTFPB + ZERO(II)
  IF(TFCHKB.EQ.AVGTFB) GOTO 3200
  IF(TFCHKB.LT.AVGTFB) GOTO 3000
TFBPCT = (AVGTFB-(TSTFPB + ZERO(II)))/AVGTFB
ENBAVG = (CFNFPB-FLOAT(II))-TFBPCT
GOTO 3500
3200  ENBAVG = CFNFPB-FLOAT(II)
3500  WRITE( ,3550)AVGTFB,TFCHKB,TFBPC,ENBAVG
3550  FOR.%IAT(2X,'AVG EFFECTIVE TIME,CHECK FOR AVG EFF TIME',
&',&',F12.4,1X,F12.4,1X,F12.4)
CCC  CALCULATE C(X2)-C(X1) FOR THE SHEAR CALCULATION
DELNT = I DO'(ASLFPB-ASLFPA)
WRITE(' I2080)DELNT,aerr,berr
1080  FOR.MAT(2X,'DELTA CX CHK-DELNT,aerr,berr'/3(2X,FI 2.4))
CCC  DO ROUTINE TO CALCULATE THE SHEAR
CCC  THEN USE SHEAR TO FIND FRICTION COEFFICIENT
CALL CFCLC(TCOIL,TCNO,M,MUNOM%,NOIL,WLLSER,ANGIN,DXBEAM,
CDPDX,ENBAVG,DELNT,TAU,TAU
BAR,EPSI,IER)
CF=TAUBAR/QE
CCC  CCC  CCC  WRITE OUTPUT DATA TO SCREEN
CCC  OUTPUT DATA IS TAU,BAR.CF,QE,AVG,RM4S,NZERO,CFNFPB,TSTFPB
CCC  4010 FORMAT(5X,TAUBAR=',F16.8,1,5X,'CF=',F16.8,/.5X.*QE=',
&F16.9)
WRITE(*,401I0)TAUBAR,CF,QE
WRITE('  .4020)AVG,RMISA,N ZEROA,CFNFPA,TSTFPA.ASLFPA
WRITE(,')HEIGHT OF OIL FILM = ',CFNFPA-WLLSER-IE6.' microns'
6665 CONTINUE
DENOM = (TSTFPA-AZERO1 + TSTFPB+BZERO1)*.5D0
IF(ITYP.LT.6) THEN
CFM(ITYP) = CF
ERRA(ITYP) = AERR
ERRB(ITYP) = BERR
TOA(ITYP) = TSTFPA
TOB(ITYP) = TSTFPB
NZA(ITYP) = NZEROA
NZB(ITYP) = NZEROB
DT(ITYP) = ABS(TSTFPA-TSTFPB)/DENOM
ENDIF
IF(ITYP.EQ.6) THEN
ERRA(6) = AERR
ERRB(6) = BERR
TOA(6) = TSTFPA
TOB(6) = TSTFPB
NZA(6) = NZEROA
NZB(6) = NZEROB
DT(6) = ABS(TSTFPA-TSTFPB)/DENOM
CFM(6) = CF
CFAVG = (CFM(2)+CFM(3)+CFM(4)+CFM(5)+CFM(6))/6.0D0
PLUS = DMAX1(CFM(2),CFM(3),CFM(4),CFM(5),CFM(6),CFM(6))-CFAVG
MINUS = CFAVG - DMIN1(CFM(2),CFM(3),CFM(4),CFM(5),CFM(5),CFM(6))
DO 199 IJ = 1,6
199  EROOT(IJ) = SQRT(ERRA(IJ)*ERRA(IJ) + ERRB(IJ)*ERRB(IJ) + DT(IJ)*
1DT(IJ))
WRITE(*,6666)
DO 201 IJ = 1,6
IF(IJ.EQ.1)ASSIGN 6667 TO FORM
IF(IJ.EQ.2)ASSIGN 6668 TO FORM
IF(IJ.EQ.3)ASSIGN 6669 TO FORM
IF(IJ.EQ.4)ASSIGN 6670 TO FORM
Appendix B. PROGRAM OILCF.FOR 108
IF(JJ.EQ.5) ASSIGN 5671 TO FORM
WRITE(5,5671)NZA(JJ),NZB(JJ),CFM(JJ),ERRA(JJ),ERRB(JJ),DT(JJ)
WRITE(*,5673)CFAVG,PLUS,MINUS
WRITE(9,5666)
DO 202 JJ = 1,6
IF(JJ.EQ.5) ASSIGN 5671 TO FORM
WRITE(9,5667)NZA(JJ),NZB(JJ),CFM(JJ),ERRA(JJ),ERRB(JJ),DT(JJ)
WRITE(9,5668)
C
WRITE(9,5669)
HEIGHT OF OIL FILM = CFNFPB*WLLSE*1E6; microns
C
*** check for bad cf values ***
DO 204 IJ = 1,6
ICHK(IJ) = 0
DO 205 IJ = 1,6
IF(NZB(IJ).EQ.0) ICHK(IJ) = 1
IF(dt(IJ),GT.02) ICHK(IJ) = 1
IF(ERRA(IJ),GT.004) ICHK(IJ) = 1
IF(ERRB(IJ),GT.004) ICHK(IJ) = 1
IF(NZA(IJ).LT.10) ICHK(IJ) = 1
IF(NZB(IJ).LT.10) ICHK(IJ) = 1
205 IF(ICHK(IJ).EQ.1) NUL = NUL + 1
GO TO 5000
ENDIF
ITYP = ITYP + 1
GO TO 54
C
***** WRITE TO OUT FILE *****
C
5000 CLOSE (3)
CLOSE (13)
CLOSE (14)
6666 FORMAT(24X,CF',10X,'ERRA';7X,'ERRB';8X,'DT';)!
6667 FORMAT(1X,20X,':F12.8,F11.5,F16.5)
6668 FORMAT(1X,20X,':F12.8,F11.5,F16.5)
6669 FORMAT(1X,20X,':F12.8,F11.5,F16.5)
6670 FORMAT(1X,20X,':F12.8,F11.5,F16.5)
6671 FORMAT(1X,20X,':F12.8,F11.5,F16.5)
6672 FORMAT(1X,20X,':F12.8,F11.5,F16.5)
6673 FORMAT(1X,20X,':F12.8,F11.5,F16.5)
6674 FORMAT(1X,20X,':F12.8,F11.5,F16.5)
6675 FORMAT(1X,20X,':F12.8,F11.5,F16.5)
4030 FORMAT(2X,'AVGB=',F16.8,/,2X,'RMSA=',F16.8,/,2X,'NZEROA=',
&14,/,2X,CFNFPB=',F16.8,/,2X,TSTFPB=',F16.8,/,2X,
&ASLFPA=',F16.8)
STOP
END
C
CCC
CCC
CCC
**********SUBROUTINES************
CCC
CCC
SUBROUTINE SMTHPD(IRAWDT,NRAWMX,FACTOR,MODEP)
CCC
CCC
IMPLICIT DOUBLE PRECISION (A-H)
IMPLICIT DOUBLE PRECISION (O-Z)
SUBROUTINE SOILFMV(IRAWDT,NRAWMX,MINFIT,MAXFIT,NINTMN,AVG,RMS, & RMSFCT,ZERO,NZEMAX,NZERO,ITYP)
CCC SUBROUTINE SOILFMV—FRINGE I.D. ALGORITHM, FINDS ZERO CROSSINGS
CCC OF EACH FRINGE
IMPLICIT DOUBLE PRECISION (A-H)
IMPLICIT DOUBLE PRECISION (O-Z)
DOUBLE PRECISION AVG,RMS,ZERO(NZEMAX)
INTEGER IRAWD(T(NRAWMX))
IF(MINFIT.LT.I) MINFIT = 1
IF(MAXFIT.GT.NRAWMX) MAXFIT = NRAWMX
IF((NINTMN.LT.I).OR.(NINTMN.GT.10)) GOTO 990
AVG = 0.0DO
RMS = 0.0DO
DO 300 I = MINFIT,MAXFIT
IND = I-MINFIT-1
AVG = AVG - (DBLE(IRAWDT(I))-AVG)/DBLE(IND)
300 CONTINUE
DO 305 I = MINFIT,MAXFIT
IND = I-MINFIT-1
DEVIA = DBLE(IRAWDT(I))-AVG
RMS = RMS + (DEVIA*DEVIA*RMS)/DBLE(IND)
305 CONTINUE
RMS = DSORT(RMS)
IF(ITYP.EQ.2.OR.ITYP.EQ.5) AVG = AVG - .1*RMS
IF(ITYP.EQ.3.OR.ITYP.EQ.6) AVG = AVG + .1*RMS
NZERO = 0
IENFL = 0
NIN = 0
DO 500 I = MINFIT,MAXFIT
IF(DABS(DBLE(IRAWDT(I))-AVG)<.0.0) GOTO 410
500 CONTINUE
IF(IENFL.EQ.1.AND.(NIN.GE.NINTMN)) GOTO 420
402 CONTINUE
IF(IENFL.EQ.0) IMIN = 1
IMAX = I
NIN = IMAX-IMIN+1
IENFL = 1
SUBROUTINE SLUBFT TO FIT DATA TO LINEAR LUBRICATION THEORY

DOUBLE PRECISION (A-H, O-Z)
DOUBLE PRECISION PARAM(2), DELTA(2), ZEROP(200)
COMMON FITCOM, CFNFIT, ASLF, TSTFIT, ZERO(200), NZERO, RMSERR
EXTERNAL FITERR
ABSERR = 0.00250
TOL = 0.0002
ERRMAX = 0.05
ITMAX = 100
NZERO = NZEROP

DO 3 I = 1, NZERO
  3  ZERO(I) = ZEROP(I)
  CFNFIT = DBLE(ZERO(I))
  IF (MODEL.EQ.1) TSTFIT = TSTFP
  IF (MODEL.EQ.2) TSTFIT = 0.00
  DELTA(1) = 1.0
  DELTA(2) = ZERO(2) - ZERO(1)
  PARAM(1) = CFNFIT
  PARAM(2) = TSTFIT
  OLDERR = FITERR(MODEL, PARAM)
  ITNO = 0
  5  ITNO = ITNO + 1
  CALL GRIDLS(MODEL, PARAM, DELTA, DUM, FITERR)
  WRITE(*, 2000) ITNO, DUM
  2000 FORMAT(ITERATION NO. = , I5, 'ERROR = ', F12.5)
  CFNFIT = PARAM(1)
TSTFIT = PARAM(2)
IF(DUM.LT.ABSERR.)AND.(DABS(DUM-OLDERR).LT.TOL))GOTO 20
IF(ITNO.GT.ITMAX)AND(DUM.LT.ERRMAX))GOTO 20
OLDERR = DUM
GOTO 15
20 CFNP = CFNFP
ASLFIT = ASLFIT
TSTFP = TSTFIT
ERROR = DUM
GOTO 999
990 CONTINUE
STOP SCCFIT: NO CONVERGENCE
999 RETURN
END
DOUBLE PRECISION FUNCTION FITERR(MODELF,PARAM)
IMPLICIT DOUBLE PRECISION (A-H)
IMPLICIT DOUBLE PRECISION (O-Z)
DOUBLE PRECISION PARAM(2)
COMMON FITCOM CFN,FITAS,FIT.TSTFIT,ZERO(200),NZERO,RMSERR
CFN = PARAM(1)
IF(MODELF.EQ.2) TSTFIT = PARAM(2)
SC = 0.
DO 1 I = 1,NZERO
CI = (CFN(DBLE(I)))*(ZERO(I)-TSTFIT)
SC = SC + (CI-SC) DBLE(I)
1 CONTINUE
RMSERR = DSQRT(SCC-SC'SC)
ASLFIT = SC
FITERR = RMSERR/ASLFIT
RETURN
END
SUBROUTINE GRIDLS
SUBROUTINE CFCLC-TO CALCULATE THE FRICTION COEFF
Appendix B. PROGRAM OILCF.FOR
SUBROUTINE CFCLC(TCOIL,TCNOM,MUNOM,NOIL,WLLSER,ANGIN,DXBEAM,
&DPDX,ENBAVG,DELNT,TAU,TAUBAR,EPSI,IER)
IMPLICIT DOUBLE PRECISION (A-H)
IMPLICIT DOUBLE PRECISION (O-Z)
DOUBLE PRECISION PI,MUOIL,OPOIL,MUNOM,NOIL
PARAMETER (PI = 3.141592650)
INTEGER IER
IER = 1
IF((TCOIL.LT.0.0) OR (TCOIL.GT.100.0)) GOTO 990
IF((TCNOM.LT.200.0) OR (TCNOM.GT.200.0)) GOTO 990
IF((MUNOM.LT.0.0) OR (MUNOM.GT.1.0)) GOTO 990
IF((WLLSER.LT.0.0) OR (WLLSER.GT.10.0)) GOTO 990
IF((NOIL.LT.0.0) OR (NOIL.GT.5.0)) GOTO 990
IF((ANGIN.LT.0.0) OR (ANGIN.GT.90.0)) GOTO 990
IF((DXBEAM.LT.0.0) OR (DXBEAM.GT.1.0)) GOTO 990
IF((DELNT.LT.-1.0) OR (DELNT.GT.1.0)) GOTO 990
MUOIL = MUNOM*DEXP(0.0146*(TCOIL-TCNOM))
ANGOIL = DASIN(DSIN(PI*ANGIN 180.0)/NOIL)
OPOIL = WLLSER/(2.0*NIL*DCOS(ANGOIL))
TAU = MUOIL*DXBEAM*DELNT/NOIL
EPSI = (OPOIL*DPDX*ENBAVG)/TAU
TAUBAR = TAU/(1.0-EPSI)
GOTO 999
990 IER = 0
999 RETURN
END
Appendix C. PROGRAM PMCF.FOR

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
IMPLICIT CHARACTER*(A-H,O-Z)
CHARACTER PT,ANG,T,END'4,OLUTFIL(9)*17,REDFIL(9)*17,END24,PRE-7,
PRE2*7

DIMENSION CFOO(6),CF4S(6),CF60(6),CO(3),C45(3),C60(3),DC0(3),DC45(3),
DC60(3),DCX0(3),DCX45(3),DCX60(3),DCZ0(3),DCZ45(3),DCZ60(3),HT0
(23),HT45(3),HT60(3),AXWT(9),AZWT(9),CM(9),COSA(9),COSG(9),SINA(9),
3Q(9),PX(9),PZ(9),QX(9),QZ(9),TX(9),TZ(9),AM'*AT(9,10),DUM'(9,10)

OPEN(1,FILE = 'DATFILDAT',STATUS = "NEW")

1000 FORMAT(A2)
1010 FORMAT(IX,A7,3A2,A4)
1020 FORMAT(AI8)
1030 FORMAT(IX.'POINT NUMBER'.A3,':'/IX.'NO'.13,SX,'N45',13,5X,'N60',10 = I3,'MAGNITUDE = ',F11.7,'ANGLE = ',F7.1,' degrees';19X,'2-
45F.',%,7,-/-',F7.I,' degrees')
1035 FORM4AT(10X,'BAR',F11.7,20X.'CZBAR',F11.7,
1036 FORMAT(10X,DCX0(',I1,')',F11.7,'%',17X,DCZ0(',I1,), = ',F11
1037 FORMAT(10X,DCX45(',I1,')',F11.7,'%',17X,DCZ45(',I1,), = ',F11
1038 FORMAT(10X,DCX60(',I1,')',F11.7,'%',17X,DCZ60(',I1,), = ',F11
1039 FORMAT(10D8.1)

C **** INITIALIZE VARIABLES *****

R2D = 180.0-*ATAN(1.)
P1 = 4.*ATAN(1.)
NO = 0
N45 = 0
N60 = 0
SIGCX = 0
SIGCZ = 0
DELX0 = 0
DELZ0 = 0
DELX45 = 0
DELZ45 = 0
DELX60 = 0
DELZ60 = 0
DO 301 K = 1,3
CO(K) = 0
DC0(K) = 0
C45(K) = 0
DC45(K) = 0
C60(K) = 0
301 DC60(K) = 0
COS45 = COS(PI*2.5)
SIN45 = COS45
COS60 = COS(PI/3.)
SIN60 = SIN(PI/3.)
BETA = 3.*PI/180.

***** GENERATE NINE FILENAMES *****

WRITE('**')' ENTER POINT NUMBER'
READ(*,1000) PT
WRITE('**')' ENTER INITIAL GUESS FOR ALPHA'
READ(*,10) ALPHA
ALPHA = ALPHA/R2D
END = 'OUT'
END2 = 'RED'
PRE = 'C.OUT'
PRE2 = 'C.RED'
K = 1
ANG = '0'
I DO 1 1 =1,3
IF(1.EQ.1)T = 'T1'
IF(1.EQ.2)T = 'T2'
IF(1.EQ.3)T = 'T3'
WRITE(1,1010)PRE2,PT,ANG,T,END2
10 WRITE(1,1012)PRE,PT,ANG,T,END
IF(K.EQ.1)THEN
   ANG = '45'
ELSE
   ANG = '60'
ENDIF
K = K-1
IF(K.LT.4) GO TO 1
REWIND 1
DO 20 1 =,9
READ(1,1020) REDFIL(1)
20 READ(1,1020)
OUTFIL(1)
CLOSE (1)

***** READ IN CF VALUES *****

DO 40 1 = 1,3
10 = 0
OPEN(1,FILE = REDFIL(1),STATUS = 'OLD',IOSTAT = ION)
IF(ION.NE.0) THEN
   WRITE('**')' REDFILE'.I,' NOT FOUND'
   ELSE
   READ(1,*)HTO(I)
   ENDIF
   CLOSE(1)
   J = 1
   OPEN(1,FILE = OUTFIL(1),STATUS = 'OLD',IOSTAT = ION)
   IF(ION.NE.0) GO TO 39
29 10 = 10 + 1
30 READ(1,1040)CF00(I0),ERRA,ERRB,DT
   J = J + 1
   IF(ERRA.GT..004.OR.ERRB.GT..004.OR.ERRA.LT..00001.OR.ERRB.LT..0000
1. OR.DT.GT..00001) THEN
   IF(J.LT.7) THEN
      GOTO 30
   ENDIF
   ENDFI
   IF(J.LT.7) GOTO 29
   IF(10.LT.2) GOTO 39
   NO = NO + 1
   HTO(NO) = HTO(I)
   DO 35 JJ = 1,10
      CF00(JJ) = CF00(JJ)*((1. - COS(-ALPHA)*HTO(NO))
35 CO(NO) = CO(NO) + CF00(JJ)
   CO(NO) = CO(NO).10
   FACTOR = (1.-HTO(NO)*COS(-ALPHA))

Appendix C. PROGRAM PMCF.FOR
DO 36 JJ = 1,10
36 DCO(N0) = DCO(N0) + (CF00(JJ) - C0(N0))*2
DC0(N0) = SQRT(DC0(N0)/(I0-1))
39 CLOSE(1)
40 CONTINUE
DO 60 I = 4,6
44 J = 0
OPEN(I,FILE = REDFIL(I),STATUS = 'OLD',IOSTAT = ION)
IF(ION.NE.0) THEN
  WRITE(*,*) REDFILE,I, 'NOT FOUND'
ELSE
  READ(I,*)HT45(I-3)
ENDIF
CLOSE(I)
J = I
OPEN(I,FILE = OUTFIL(I),STATUS = 'OLD',IOSTAT = IONM)
IF(IONM.NE.0) GO TO 59
49 I45 = J45 - 1
50 READ(1,1040)CF45(I45),ERRA,ERRB,DT
   J = J + 1
IF(ERRA.GT.0.04 OR. ERRB.GT.0.04 OR. ERRA.LT.0.0001 OR. ERRB.LT.0.0001 OR. DT.GT.0.0001) THEN
   IF(J.LT.7) THEN
     GOTO 50
   ENDIF
ENDIF
IF(J.LT.7) GOTO 49
IF(145.LT.2) GOTO 59
N45 = N45 + 1
HT45(N45) = HT45(I-3)
DO 55 JJ = 1,145
   CF45(JJ) = CF45(JJ)*1. - COS(PI/3.-ALPHA)*HT45(N45))
55 C45(N45) = C45(N45) + CF45(JJ)
C45(N45) = C45(N45),145
DO 56 JJ = 1,145
   DC45(N45) = DC45(N45) + (CF45(JJ) - C45(N45))*2
DC45(N45) = SQRT(DC45(N45)/(I45-1))
59 CLOSE(1)
60 CONTINUE
DO 80 I = 7,9
65 J = 0
OPEN(I,FILE = REDFIL(I),STATUS = 'OLD',IOSTAT = ION)
IF(ION.NE.0) THEN
  WRITE(*,*) REDFILE,I, 'NOT FOUND'
ELSE
  READ(I,*)HT60(I-6)
ENDIF
CLOSE(I)
J = I
OPEN(I,FILE = OUTFIL(I),STATUS = 'OLD',IOSTAT = IONM)
IF(IONM.NE.0) GO TO 79
69 I60 = I60 + 1
70 READ(1,1040)CF60(I60),ERRA,ERRB,DT
   J = J + 1
IF(ERRA.GT.0.04 OR. ERRB.GT.0.04 OR. ERRA.LT.0.0001 OR. ERRB.LT.0.0001 OR. DT.GT.0.0001) THEN
   IF(J.LT.7) THEN
     GOTO 70
   ENDIF
ENDIF
IF(J.LT.7) GOTO 69
IF(I60.LT.2) THEN
   I60 = I60 + 1
GOTO 79
ENDIF
I60 = I60 + 1
HT60(I60) = HT60(I-6)
DO 75 JJ = 1,160
   CF60(JJ) = CF60(JJ)*1. - COS(PI/3.-ALPHA)*HT60(I60))
75 C60(I60) = C60(I60) + CF60(JJ)
C60(I60) = C60(I60),REAL(I60)
DO 76 JJ = 1,160

Appendix C. PROGRAM PMCF.FOR

116
76 DC60(N60) = DC60(N60) + (CF60(JJ) - C60(N60))**2
DC60(N60) = SQRT(DC60(N60),(160-1))
79 CLOSE()
80 CONTINUE

C COMPUTE MAGNITUDE AND DIRECTION

OLDA = ALPHA
NTOT = NO + N45 + N60
150 CONTINUE
CALL LSQ(N0,N45,N60,C0,C45,C60,CX,CZ,AEL,BEL,DEL,DENOM)
CXBAR = CX
CZBAR = CZ
ALPHA = ATAN(CZ/CX)
WRITE(**) ALPHA = ALPHA*R2D
IF(ABS(ALPHA-OLDA).GT..05;R2D) THEN
DO 147 JJ = 1,NO
C0(JJ) = C0(JJ)*((1.-HT0(JJ)*COS(-ALPHA))/(1.-HT0(JJ)*COS(OLDA))
147 FACTOR = (1.-HT0(JJ)*COS(-ALPHA))
C 147 WRITE(,-)
DO 148 JJ = 1,N45
C45(JJ) = C45(JJ)*((1.-HT45(JJ)*COS(PI*.25-ALPHA))/(1.-HT45(JJ)*COS(PI*.25-OLDA))
148 FACTOR = 1.-HT45(JJ)*COS(PI*.25-ALPHA)
C 148 WRITE(,-)
DO 149 JJ = 1,N60
C60(JJ) = C60(JJ)*((1.-HT60(JJ)*COS(PI*3.-ALPHA))/(1.-HT60(JJ)*COS(PI*3.-OLDA))
149 FACTOR = (1.-HT60(JJ)*COS(PI*3.-ALPHA))
C 149 WRITE(,-)
150 CONTINUE
OLDA = ALPHA
GO TO 150
ENDIF

DO 151 JJ = 1,NO
DC0(JJ) = SQRT(DC0(JJ)*DC0(JJ) + ((1.-1.)/(1.-TAN(-ALPHA)*TAN(BETA))
1)*C0(JJ)**2)
DO 150 JJ = 1,NO
SIGCZ = SIGCZ + (C0(JJ)*TAN(ALPHA)-CZBAR)**2
150 SIGCX = SIGCX + (C0(JJ) - CXBAR)**2
DO 151 JJ = 1,N45
DC45(JJ) = SQRT(DC45(JJ)*DC45(JJ) + ((1.-1.)/(1.-TAN(PI*.25-ALPHA))
1)*C45(JJ)**2)
DO 150 JJ = 1,NO
SIGCX = SIGCX + (C45(JJ)*COS(PI*.25-ALPHA) - CXBAR)**2
150 SIGCZ = SIGCZ + (C45(JJ)*SIN(PI*.25-ALPHA) - CZBAR)**2
DO 151 JJ = 1,N60
DC60(JJ) = SQRT(DC60(JJ)*DC60(JJ) + ((1.-1.)/(1.-TAN(PI*3.-ALPHA))
1)*C60(JJ)**2)
DO 150 JJ = 1,NO
SIGCZ = SIGCZ + (C60(JJ)*COS(PI*3.-ALPHA) - CZBAR)**2
150 SIGCX = SIGCX + (C60(JJ)*SIN(PI*3.-ALPHA) - CXBAR)**2
SIGCX = SIGCX/NTOT
SIGCZ = SIGCZ/NTOT

***** ITERATE TO GET UNCERTAINTIES *****

DO 190 K = 1,NO
C(K) = C(K) - DC0(K)
CALL LSQ(N0,N45,N60,C0,C45,C60,CX,CZ,AEL,BEL,DEL,DENOM)
C(K) = C(K) - DC0(K)
DC0(K) = (CXBAR - CX)**2
190 DC0(K) = (CXBAR - CX)**2
DO 200 K = 1,N45
C45(K) = C45(K) + DC45(K)
CALL LSQ(N0,N45,N60,C0,C45,C60,CX,CZ,AEL,BEL,DEL,DENOM)
C45(K) = C45(K) - DC45(K)
DC45(K) = (CXBAR - CX)**2
200 DC45(K) = (CXBAR - CX)**2
DO 210 K = 1,NO
C60(K) = C60(K) + DC60(K)
CALL LSQ(N0,N45,N60,C0,C45,C60,CX,CZ,AEL,BEL,DEL,DENOM)

Appendix C. PROGRAM PMCF.FOR 117
C60(K) = C60(K) - DC60(K)
DCX60(K) = (CXBAR - CX)**2
210 DCZ60(K) = (CZBAR - CZ)**2

***** COMPUTE FINAL UNCERTAINTY ON CX AND CZ *****

DO 210 K = 1,N0
   DELX0 = DELX0 + DCX0(K)
220 DELZ0 = DELZ0 + DCZ0(K)
DO 230 K = 1,N45
   DELX45 = DELX45 + DCX45(K)
230 DELZ45 = DELZ45 + DCZ45(K)
DO 240 K = 1,N60
   DELX60 = DELX60 + DCX60(K)
240 DELZ60 = DELZ60 + DCZ60(K)

DCXBAR = SQRT(SIGCX + DELX0 + DELX45 + DELX60)
DCZBAR = SQRT(SIGCZ + DELZ0 + DELZ45 + DELZ60)

***** COMPUTE MAGNITUDE AND DIRECTION AND PRINT OUTPUT *****

CFMAG = SQRT(CXBAR*CXBAR - CZBAR*CZBAR)
ANGLE = ATAN(CZBAR*CXBAR + CXBAR*CZBAR)
DADX = CZBAR*(CZBAR*CZBAR + CXBAR*CXBAR)
DCANG = DCANG*180./PI
DCMDCX = CXBAR*CXBAR/CFMAG
DCMDCZ = CZBAR*CZBAR/CFMAG
DCM = SQRT(DCMDCX*DCBAR*DCBAR + DCMDCZ*DCZBAR*DCZBAR)
DCM = DCM*CFMAG*100.
DCXBAR = DCXBAR*100.
DCZBAR = DCZBAR*100.

SIGCX = SQRT(SIGCX)
SIGCZ = SQRT(SIGCZ)

OPEN(2,FILE = 'LPT1', STATUS = 'OLD')
WRITE(*,1030)PT,NO,N45,N60,CFMAG,ANGLE,DCM,DCANG
WRITE(*,1035)CXBAR,CZBAR,DCXBAR,DCZBAR,SIGCX,SIGCZ

DO 250 JJ = 1,3
   DCX0(JJ) = SQRT(DCX0(JJ))*CXBAR*100.
   DCZ0(JJ) = SQRT(DCZ0(JJ))*CZBAR*100.
250 WRITE(*,1036)JJ,DCX0(JJ),JJ,DCZ0(JJ)

CDO 260 JJ = 1,3
   DCX45(JJ) = SQRT(DCX45(JJ))*CXBAR*100.
   DCZ45(JJ) = SQRT(DCZ45(JJ))*CZBAR*100.
260 WRITE(*,1037)JJ,DCX45(JJ),JJ,DCZ45(JJ)

CDO 270 JJ = 1,3
   DCX60(JJ) = SQRT(DCX60(JJ))*CXBAR*100.
   DCZ60(JJ) = SQRT(DCZ60(JJ))*CZBAR*100.
270 WRITE(*,1038)JJ,DCX60(JJ),JJ,DCZ60(JJ)

WRITE(2,1030)PT,NO,N45,N60,CFMAG,ANGLE,DCM,DCANG
WRITE(2,1035)CXBAR,CZBAR,DCXBAR,DCZBAR,SIGCX,SIGCZ

DO 280 JJ = 1,3
280 WRITE(2,1036)JJ,DCX0(JJ),JJ,DCZ0(JJ)

CDO 290 JJ = 1,3
290 WRITE(2,1037)JJ,DCX45(JJ),JJ,DCZ45(JJ)

CDO 300 JJ = 1,3
300 WRITE(2,1038)JJ,DCX60(JJ),JJ,DCZ60(JJ)

CWRITE(2,1039)

CDO 310 JJ = 1,3
310 WRITE(2,1046)I,CO(I),I,DC0(I)
CDO 320 JJ = 1,3
320 WRITE(2,1047)I,CO5(I),I,DC5(I)
CDO 330 JJ = 1,3
330 WRITE(2,1048)I,CO6(I),I,DC6(I)

CLOSE(2)
END

C

SUBROUTINE LSQ (N0,N45,N60,CF00,CF45,CF60,CFX,CFZ,A,B,D,DENOM)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION CF00(3),CF45(3),CF60(3)

Appendix C. PROGRAM PMCF.FOR 118
PI = 4.*ATAN(1.)
COS45 = COS(PI*.25)
COS60 = COS(PI*.3.)
SIN60 = SIN(PI*.3.)
A = N0 + N45*.5 + N60*.25
B = N45*.5 + N60*SQRT(3.)*.25
D = N45*.5 + N60*.75
E = 0.
F = 0.
DO 2000 I = 1,N0
2000 E = E + CF60(I)
DO 2010 I = 1,N45
2010 E = E + CF45(I)*COS45
2020 F = F + CF45(I)*COS45
DO 2020 I = 1,N60
2020 F = F + CF60(I)*SIN60
DENOM = A*D - B*B
CFX = (D*E - B*F)/DENOM
CFZ = (A*F - B*E)/DENOM
RETURN
END